# Conditional Risk, Overconditioning, and the Performance of Momentum Strategies

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Recent empirical studies evaluate the performance of investment strategies using contemporaneously measured loadings to proxy for conditional risk. We demonstrate that such procedures lead to potentially large biases in alpha when payoffs are nonlinear. We combine lagged portfolio and component realized betas with standard instruments to improve performance analysis, and find that conditioning information reduces momentum alphas by 20-40% relative to unconditional estimates. Overconditioned alphas are up to 2.5 times larger than appropriately conditioned measures.

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**Abstract**

Recent empirical studies evaluate the performance of investment strategies using contemporaneously measured loadings to proxy for conditional risk. We demonstrate that such procedures lead to potentially large biases in alpha when payoffs are nonlinear. We combine lagged portfolio and component realized betas with standard instruments to improve performance analysis, and find that conditioning information reduces momentum alphas by 20-40% relative to unconditional estimates. Overconditioned alphas are up to 2.5 times larger than appropriately conditioned measures.

## 1. Introduction

Under the conditional CAPM, risk is defined as the conditional exposure to market returns given the information available to investors at the time of their portfolio decision. If an empiricist fails to account for predictable variation in risk, a bias can occur in evaluating the performance of investment strategies (Jensen, 1968; Grant, 1977; Dybvig and Ross, 1985) and in testing the pricing model (Hansen and Richard, 1987; Jagannathan and Wang, 1996). Standard empirical methods attempt to mitigate this *underconditioning* bias by permitting loadings to depend on data observable to investors, such as the lagged dividend yield, term spread, or other variables. For example, Shanken (1990) specifies beta as a linear function of lagged instruments, followed by Ferson and Schadt (1996), Ferson and Harvey (1999), the textbook exposition of Cochrane (2001), and others.[1]

In recent literature, an alternative approach dispenses with lagged instruments, and instead directly estimates factor loadings in windows contemporaneous with the returns to be risk-adjusted. For example, Grundy and Martin (2001, "GM") proxy for the month $\tau$ market and size betas of momentum portfolios using loadings estimated from monthly returns in the window $\tau$ to $\tau + 5$. They favor these estimates over lagged loadings because "the relevant risk to an investor... is the strategy's factor exposure during the investment window." (p. 43) Similarly, Ang, Chen, and Xing (2006, "ACX") argue:

> The CAPM predicts an increasing relationship between realized average returns and realized factor loadings, or contemporaneous expected returns and market betas. More generally, a multifactor model implies that we should observe patterns between average returns and sensitivities to different sources of risk over the same time period used to compute the average returns and the factor sensitivities. (p. 1201)

Lewellen and Nagel (2006, "LN") make the stronger claim of a new asset pricing test that circumvents the critique put forward by Hansen and Richard, explaining, "Our methodology... does not require any conditioning information. As long as betas are relatively stable within a month or quarter, simple CAPM regressions estimated over a short window – using no conditioning variables – provide direct estimates of assets' conditional alphas and betas." (p. 291)

Our paper introduces the idea of *overconditioning*, which occurs when an empiricist uses a conditional risk proxy, such as the contemporaneous realized beta, that is not entirely in the investor

[1] Numerous other studies specify CAPM or other loadings as a function of lagged instruments, and use a variety of moment conditions for estimation. See, for example, Avramov and Chordia (2006), Bollerslev, Engle, and Wooldridge (1988), Campbell (1987), Cochrane (1996), Duffee (2005), Ferson and Harvey (1991, 1993), Ferson, Kandel, and Stambaugh (1987), Harvey (1989), Lettau and Ludvigson (2001*b*), Petkova and Zhang (2005), Santos and Veronesi (2006), and Wang (2003).

information set. Note that any empirically calculated realized beta cannot be fully anticipated by investors.[2] The component orthogonal to their information ("noise") can be substantial and problematic in the short windows recommended by LN, even under the optimistic assumption that realized beta is an unbiased estimate of conditional risk. We show that if the noise in the empirical risk estimate covaries with unexpected market returns, the associated alpha is biased.

The key theoretical insight of our paper is that the magnitude of the overconditioning bias is directly tied to nonlinearity in the relation between asset and factor returns.[3] Intuitively, if an asset has payoffs that are concave (convex) in market returns, the noise in realized beta over a fixed window will be negatively (positively) correlated with the number and size of positive market surprises during the window. To be clear, many pricing models imply a linear correspondence between expected asset and factor returns, but the realized return relation may generally be nonlinear.[4] Thus, performance measures based on the conditional CAPM and other pricing models should be robust to payoff nonlinearities.

Evidence of payoff nonlinearities in stock returns is provided by Ang and Chen (2002) and ACX, who show that many securities covary differently with negative and positive market surprises. ACX sort days within a year by whether the excess market return is below or above its within-year average, and run market model regressions on the subsets of "down" and "up" days. They sort companies by the difference between the down and up betas and find considerable dispersion. For the highest quintile, this beta asymmetry is almost one. ACX ask whether stocks with large down betas earn abnormal profits.

Our interest in payoff nonlinearities has an almost entirely different motivation. Even in the absence of a risk premium associated with beta asymmetries (e.g., the conditional CAPM), payoff nonlinearities generate a bias in overconditioned alphas. Our goals in this paper are to (i) explain the overconditioning bias and demonstrate its empirical importance, (ii) develop an improved con-

[2] In idealized settings, continuous record asymptotics permit perfect observability of local quadratic variations and covariations (e.g., Foster and Nelson, 1991), but such environments are certainly more restrictive than necessary for a conditional CAPM to hold, and the limitations imposed by finite data in empirical work are important.

[3] A *payoff nonlinearity* occurs when, conditional on the *contemporaneous* factor return, the relation between the expected return on an asset and the realized factor return is nonlinear. For example, in a single-factor static setting one can linearly project an asset return onto the factor return. If the projection errors can be predicted by any function of the factor return, then payoffs are nonlinear.

[4] For example, under quadratic preferences the CAPM holds for arbitrary return specifications. Similarly, while early versions of the APT assume a strict factor structure that implies all asset payoffs are linear in factor returns, subsequent extensions are compatible with nonlinearities for an arbitrary number of individual assets provided that these average out in large portfolios. Specifically, Chamberlain and Rothschild (1983) generalize Ross (1976) by permitting an approximate factor structure with potentially correlated residuals. The residuals may then load on squared market returns, for example. Grinblatt and Titman (1985) provide an intuition for the Chamberlain and Rothschild result based on repackaging the initial assets into portfolios that obey the strict factor structure of Ross, implying that the average nonlinearity must converge to zero.

ditional performance methodology that avoids overconditioning while incorporating information from high-frequency realized betas, and (iii) use these methods to obtain more accurate estimates of conditional momentum performance.

We first evaluate the effects of payoff nonlinearities in a calibrated dynamic CAPM with beta asymmetry comparable to empirical values reported by ACX. Performance measures that use contemporaneous versus lagged risk proxies can systematically differ by as much as 40 basis points per month, suggesting an economically significant overconditioning bias.

To illustrate the importance of appropriate conditioning in an empirical setting, we reexamine the performance of momentum strategies, which buy recent winner stocks, sell recent losers, and earn large medium term profits (Jegadeesh and Titman, 1993). Recent theoretical research proposes that conditional risk could help to explain this apparent anomaly (Berk, Green, and Naik, 1999; GM; Johnson, 2002; Sagi and Seasholes, 2007). Empirically, GM and LN determine that momentum alphas increase or are unaffected by conditioning, but their conclusions are based on abnormal returns computed from contemporaneous risk loadings.[5] Moreover, despite large monthly turnover in momentum portfolios, GM and LN estimate betas at the portfolio level in windows ranging from several months to several years. More accurate methods of measuring conditional beta could thus lead to different conclusions regarding momentum performance.

Our empirical analysis shows that winner portfolios have more pronounced beta asymmetry than losers, implying that overconditioning should affect the long side of the strategy more than the short side. Confirming this prediction, momentum alphas are up to 75 basis points per month larger when based on contemporaneous versus lagged risk loadings, indicating a substantial overconditioning bias.

An independently important quantitative issue in our study is that compounding daily returns affects loser and winner portfolios differently. To obtain monthly alphas from regressions using daily returns, LN rescale estimated intercepts by the average number of trading days in a month. We develop an analytical approximation for the difference between monthly and rescaled daily (RD) average returns in terms of the monthly/daily variance ratio.[6] When daily returns are very persistent, as in the loser portfolio, compounding has strong effects and the RD average considerably understates the buy-and-hold average. The net impact in the winner minus loser portfolio is an RD bias as large as 30 basis points per month.

[5] GM also calculate alphas based on lagged risk loadings, but as described previously argue that contemporaneous loadings provide a better indicator of risk. They thus emphasize in their introduction that "hedging out the strategy's dynamic exposure to size and market factors... would have increased the mean monthly return to 1.34%" from 0.44%. These figures are based on hedging with realized loadings calculated from the investment period.

[6] Campbell, Lo, and MacKinlay (1996) discuss variance ratios and provide references.

The performance evaluation methodology we advocate combines standard instruments with lagged realized betas calculated at the portfolio and individual stock level, within a conditional return specification similar to Shanken (1990), Ferson and Schadt (1996), and Ferson and Harvey (1999). Unlike previous performance measures that use realized betas as direct proxies for conditional beta, treating the estimated betas as instruments corrects potential biases due to microstructure effects,[7] and combines information from different sources. Appropriately conditioned alphas are statistically significantly lower than unconditional alphas by approximately 20-40%. The overconditioned alphas recommended by LN are more than 2.5 times larger than appropriately conditioned estimates.

Korajczyk and Sadka (2004) argue that the high trading costs associated with momentum trading may substantially offset the strategy's estimated abnormal profits. Our findings complement their claims, since appropriate use of conditioning information significantly attenuates momentum alphas, and thus further reduces the risk- and cost-adjusted profitability of momentum trading. By contrast, overconditioned alpha estimates of almost 1.5% per month would make arguments based on trading costs seem less plausible.

Section 2 demonstrates overconditioning in a simple setting. Section 3 calibrates payoff nonlinearities in a dynamic CAPM to assess the economic importance of the overconditioning bias. Section 4 introduces momentum strategies, and Section 5 implements conditional CAPM performance measures. Section 6 extends the empirical analysis to the conditional Fama-French model. All proofs are in Appendix B.[8]

## 2. Overconditioning

For $t = 1, 2, \ldots$, let conditional expected excess returns on asset $i$ be

$$\mathbb{E}\left(R_{it} \middle| \mathcal{F}_{t-1}\right) = \alpha_{it}^{t-1} + \beta_{it}^{t-1}\mathbb{E}\left(R_{Mt} \middle| \mathcal{F}_{t-1}\right), \tag{2.1}$$

where $\{\mathcal{F}_t\}_{t=1}^{\infty}$ is a filtration, $R_{Mt}$ is the excess market return,

$$\beta_{it}^{t-1} \equiv \frac{Cov\left(R_{it}, R_{Mt} \middle| \mathcal{F}_{t-1}\right)}{Var\left(R_{Mt} \middle| \mathcal{F}_{t-1}\right)} \tag{2.2}$$

[7] Dimson (1979), Dimson and Marsh (1983) and Scholes and Williams (1977) discuss the impact of asynchronous trading on beta estimates.

[8] Studies that cite the conclusions of LN include Campbell and Vuolteenaho (2004), Daniel and Titman (2006), Fama and French (2006), and Petkova and Zhang (2005).

is a conditional beta, and $\alpha_{it}^{t-1}$ is a conditional intercept. If $\alpha_{it}^{t-1} = 0$ and $\mathcal{F}_{t-1}$ represents investor information, the conditional CAPM holds.

The *underconditioning* problem is well understood. Following Grant (1977) and Jagannathan and Wang (1996),[9] we take expectations of (2.1) to obtain

$$\bar{R}_i = \bar{\alpha}_i + Cov\left(\beta_{it}^{t-1}, R_{Mt}\right) + \bar{\beta}_i \bar{R}_M,$$

where $\bar{\alpha}_i \equiv \mathbb{E}\left(\alpha_{it}^{t-1}\right)$ is the mean conditional alpha, $\bar{\beta}_i \equiv \mathbb{E}\left(\beta_{it}^{t-1}\right)$ is the mean conditional beta, $\bar{R}_i \equiv \mathbb{E}\left(R_{it}\right)$, and $\bar{R}_M \equiv \mathbb{E}\left(R_{Mt}\right)$. For comparison, consider the unconditional market model $\bar{R}_i = \alpha_i^{UC} + \beta_i^{UC} \bar{R}_M$, where $\alpha_i^{UC}$ is the intercept, $\beta_i^{UC} \equiv Cov(R_{it}, R_{Mt})/\sigma_M^2$ is the beta, and $\sigma_M^2 \equiv Var\left(R_{Mt}\right)$. Failing to appropriately condition leads to the *unconditional alpha bias*

$$\alpha_i^{UC} - \bar{\alpha}_i = Cov\left(\beta_{it}^{t-1}, R_{Mt}\right) - \left(\beta_i^{UC} - \bar{\beta}_i\right) \bar{R}_M. \tag{2.3}$$

The first term captures the effects of market timing on returns holding the average conditional beta constant, and the second term reflects that unconditional beta is generally a biased measure of average risk.[10]

Our paper develops the complementary idea of *overconditioning*. Suppose an empiricist calculates a beta estimate equal to the investor's conditional beta plus error: $\hat{\beta}_{it} \equiv \beta_{it}^{t-1} + \nu_{\beta t}$, and decompose the error by projecting it onto investor information:

$$\nu_{\beta t} = \mathbb{E}(\nu_{\beta t} | \mathcal{F}_{t-1}) + \varepsilon_{\beta t}.$$

The first component of the decomposition represents underconditioning, and the second is due to overconditioning. To see this, note that the second term equals zero with probability one if the empiricist uses only information available to investors. By contrast, if $\hat{\beta}_{it}$ is a contemporaneous realized beta, then $\varepsilon_{\beta t}$ has strictly positive variance. We then show:

**Proposition 1.** *Let $\hat{\beta}_{it}$ be unbiased under the investor information set: $\mathbb{E}(\hat{\beta}_{it} | \mathcal{F}_{t-1}) = \beta_{it}^{t-1}$. The*

[9] See also Jensen (1968) who considers how market timing affects performance measures.

[10] The *beta bias* is

$$\beta_i^{UC} - \bar{\beta}_i = -\frac{\bar{R}_M}{\sigma_M^2} Cov\left(\beta_{it}^{t-1}, R_{Mt}\right) + \frac{1}{\sigma_M^2} Cov\left(\beta_{it}^{t-1}, R_{Mt}^2\right), \tag{2.4}$$

allowing the alpha bias to be rewritten

$$\alpha_i^{UC} - \bar{\alpha}_i = \left(1 + \frac{\bar{R}_M^2}{\sigma_M^2}\right) Cov\left(\beta_{it}^{t-1}, R_{Mt}\right) - \frac{\bar{R}_M}{\sigma_M^2} Cov\left(\beta_{it}^{t-1}, R_{Mt}^2\right). \tag{2.5}$$

These decompositions can be easily applied given an empirical proxy for $\beta_{it}^{t-1}$.

*overconditioned alpha is then* $\bar{\alpha}_i^{OC} \equiv \bar{R}_i - \mathbb{E}\left(\hat{\beta}_{it} R_{Mt}\right)$*, and the overconditioned alpha bias is* $\bar{\alpha}_i^{OC} - \bar{\alpha}_i = -Cov\left(\hat{\beta}_{it} - \beta_{it}^{t-1}, R_{Mt}\right)$.

Thus, even in the best case scenario that the contemporaneous realized beta is unbiased, an alpha calculated from the realized beta may be biased. Specifically, if the unpredictable part of realized beta covaries with the market return, the overconditioned alpha is a misleading measure of performance.

Recent literature provides abundant evidence that many assets covary differently with down and up markets (Ang and Chen, 2002; ACX; Hong, Tu, and Zhou, 2006). A simple example illustrates how beta asymmetry and overconditioning interact to produce a bias. Assume a static CAPM: $\bar{R}_i = \beta_i \bar{R}_M$ where $\beta_i \equiv Cov(R_i, R_M)/\sigma_M^2$. Let $S \in \{G, B\}$ be a variable that is not available to investors at time zero, but is observable ex post. For example, $G$ might be the event that excess market returns are greater than $\bar{R}_M$, and $B$ its complement. For $s \in \{G, B\}$, define the overconditioned betas

$$\beta_i^s \equiv \frac{Cov(R_i, R_M | S = s)}{Var(R_M | S = s)}. \tag{2.6}$$

If payoffs are linear then $\beta_i^G = \beta_i^B$, but as discussed previously the CAPM does not require linear payoffs.

We now consider the pitfalls associated with incorrectly using the contemporaneous information $S$ to evaluate performance. Denoting the overconditioned abnormal returns by $\alpha_i^s \equiv \mathbb{E}(R_i | S = s) - \beta_i^s \mathbb{E}(R_M | S = s)$, we show

**Proposition 2.** *The overconditioned abnormal returns are zero,* $\alpha_i^G = \alpha_i^B = 0$ *if and only if* $\beta_i^G = \beta_i^B$. *Furthermore,* $\mathbb{E}\left(\alpha_i^S\right) = \left[\beta_i - \mathbb{E}\left(\beta_i^S\right)\right] \bar{R}_M - Cov\left[\beta_i^S, \mathbb{E}(R_M | S)\right]$.

Thus, if contemporaneous information is used to test the CAPM and betas are asymmetric, then alphas are biased.

We illustrate this proposition in a simple four-state example. Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ describe the state space, where each event is equally likely. Excess returns $R_i(\Omega)$ and $R_M(\Omega)$ are chosen to satisfy $\bar{R}_M = 0.01$, $\bar{R}_i = 0.01$, and $\beta_i = 1$. The variable $S$ equals $B$ in states $\omega_1$ and $\omega_2$ where market returns are lower, and is $G$ in $\omega_3$ and $\omega_4$. We assume $\beta_i^B = 1.5$ and $\beta_i^G = 0.5$, consistent with a stock whose comovement is larger in down than up markets, and calculate the payoffs $R_M(\Omega)$ and $R_i(\Omega)$ as shown in Appendix A.1.

Figure 1 plots the returns in each of the four states and the conditional and unconditional regression lines. The CAPM holds with a slope of one, depicted by the solid line going through the origin. The conditional regressions correspond to the two dashed lines with slopes $\beta_i^B$ and $\beta_i^G$ and intercepts $\alpha_i^B = 0.017$ and $\alpha_i^G = 0.027$. As required by Proposition 2, the concave payoff structure in this example implies that $Cov\left[\beta_i^S, \mathbb{E}\left(R_M|\, S\right)\right] < 0$ and the expected overconditioned alpha is positive. If we had alternatively assumed $\beta_i^B < \beta_i^G$ then payoffs would be convex and the mean overconditioned alpha would be negative. Thus, when an empiricist overconditions by using information not available to investors, the size and direction of the resulting alpha bias depends on the degree of concavity or convexity in payoffs.

## 3. Conditioning Biases in a Calibrated Dynamic CAPM

We calibrate a dynamic CAPM with time-varying risk premia and conditional market loadings. Using simulated data, we analyze different performance measures. Substantial biases arise from failing to account for the conditional nature of the pricing relationship (underconditioning), or from using conditioning variables such as the ex post realized beta that are not in the investor information set (overconditioning).

### 3.1. Time-Varying Risk Premia, Asymmetric Betas, and Stock Returns

We model a portfolio excess return $R_{it}$ and a market excess return $R_{Mt}$ for $t = 0, 1, .., T$, where $t$ indexes observations at a short horizon such as one day. The relevant horizon of investors may be longer. To accommodate this, define $\tau(t) \equiv \lfloor t/n \rfloor + 1$, where $\lfloor x \rfloor$ denotes the integer part of $x$,[11] as an index into $n$-day months.

Immediately prior to each window $\tau = 1, 2, .., \tau(T)$, investors receive a signal $Z_{\tau-1} \in \{H, L\}$ about conditional returns in month $\tau$. Assuming stationarity, denote for $z \in \{H, L\}$ the conditional means

$$\begin{aligned} \bar{R}_M^z &\equiv \mathbb{E}\left(R_{Mt}|Z_{\tau(t)-1} = z\right) \\ \bar{R}_i^z &\equiv \mathbb{E}\left(R_{it}|Z_{\tau(t)-1} = z\right) \end{aligned}$$

and the conditional beta

$$\beta_i^z \equiv \frac{Cov\left(R_{it}, R_{Mt}|Z_{\tau(t)-1} = z\right)}{Var\left(R_{Mt}|Z_{\tau(t)-1} = z\right)}.$$

[11] To be precise, $\lfloor x \rfloor \equiv \max\{\, z \in \mathbb{Z}|\, z \leq x\}$ where $\mathbb{Z}$ is the set of integers.

The state variable $Z_\tau$ follows a symmetric regime-switching process:

$$\mathbb{P}\left(Z_\tau = z | Z_{\tau-1} = z\right) = \rho$$

for $z \in \{H, L\}$, where $0.5 \leq \rho \leq 1$ is the persistence parameter.

To model nonlinearity of $R_{it}$ in the realization of $R_{Mt}$, an additional variable $S_t \in \{G, B\}$ simultaneously affects the conditional distribution of returns:

$$\mathbb{P}\left(R_{it}, R_{Mt} | \, Z_{\tau(t)-1} = z, S_t = s\right) \sim \mathcal{N}\left[\left(\bar{R}_i^{zs}, \bar{R}_M^{zs}\right), \sum\nolimits^{zs}\right], \tag{3.1}$$

where $\bar{R}_i^{zs}$ and $\bar{R}_M^{zs}$ are the state-dependent means of $i$ and $M$, and

$$\sum\nolimits^{zs} \equiv \begin{bmatrix} \sigma_i^2 & \beta_i^{zs}\sigma_M^2 \\ \beta_i^{zs}\sigma_M^2 & \sigma_M^2 \end{bmatrix} \tag{3.2}$$

is the state-dependent covariance matrix. The variable $S_t$ is iid at a daily frequency and the states $G$ and $B$ occur with equal probability.

Distinguishing the roles played by $Z_\tau$ and $S_t$ is critical. The variable $Z_\tau$ helps investors to predict both market returns and the risk loading of portfolio $i$. The empirical literature establishes that market returns are predictable primarily at low frequencies (e.g., Lettau and Ludvigson, 2001a; Cochrane, 2001). We thus expect in empirical applications that candidate instruments for $Z_\tau$ should have a persistent component.

By contrast, $S_t$ is iid and not anticipated by investors. This accords with the difficulty of forecasting the daily "up market" and "down market" states used by ACX to calculate up and down betas. The up and down betas do not directly relate to predictability in market returns and loadings, but measure the tendency of an asset to respond differently to high versus low realized market returns. The variable $S_t$ thus operates at a high daily frequency, is not persistent,[12] and produces asymmetric betas.

We give a simple structure to the state-conditioned mean market returns:

$$\bar{R}_M^{zs} = \bar{R}_M + \Delta_M^Z \left(\mathbf{1}_{\left\{Z_{\tau(t)-1}=H\right\}} - \mathbf{1}_{\left\{Z_{\tau(t)-1}=L\right\}}\right) + \Delta_M^S \left(\mathbf{1}_{\{S_t=G\}} - \mathbf{1}_{\{S_t=B\}}\right), \tag{3.3}$$

where $\bar{R}_M$ is the unconditional mean, $\Delta_M^Z$ is the increment for the states $H$ and $L$, $\Delta_M^S$ is the

[12] The model can accommodate persistence in the latent state $S_t$, in which case investor learning is potentially important. We focus on the iid case for expositional clarity.

increment for the states $G$ and $B$, and for any event $A$ the indicator function $\mathbf{1}_A$ equals one if the event is true and zero otherwise. Similarly,

$$\beta_i^{zs} = \bar{\beta}_i + \Delta_\beta^Z \left(\mathbf{1}_{\{Z_{\tau(t)-1}=H\}} - \mathbf{1}_{\{Z_{\tau(t)-1}=L\}}\right) - \Delta_\beta^S \left(\mathbf{1}_{\{S_t=G\}} - \mathbf{1}_{\{S_t=B\}}\right) \tag{3.4}$$

where $\bar{\beta}_i$ is the average conditional beta and $(\Delta_\beta^Z, \Delta_\beta^S)$ are the beta increments corresponding to the state variables $Z$ and $S$ respectively. When $\Delta_\beta^S$ and $\Delta_M^S$ have the same sign, the stock covaries more strongly with down markets than up markets.

For simplicity, set

$$\bar{\beta}_i^z \equiv E(\beta_i^{ZS} | Z_{\tau(t)-1} = z) = \beta_i^z. \tag{3.5}$$

In general, the distinction between $\bar{\beta}_i^z$ (expected beta) and $\beta_i^z$ (conditional beta) is important, but enforcing (3.5) for the remainder of the Monte Carlo analysis eliminates in a reasonable way a parameter that would otherwise need to be specified, and focuses attention on the direct alpha bias caused by covariance between the conditional beta and the conditional market risk premium.

### 3.2. The Conditional CAPM

Let $\mathcal{F}_t \equiv \left\{\left(Z_{\tau(t')}, S_{t'}, R_{Mt'}, R_{it'}\right); t' \leq t\right\}$ denote investor information at date $t$. For all $1 \leq t \leq T$, the CAPM holds conditional on $t-1$ information:

$$\mathbb{E}\left(R_{it} | \mathcal{F}_{t-1}\right) = \beta_{it}^{t-1} \mathbb{E}\left(R_{Mt} | \mathcal{F}_{t-1}\right) \tag{3.6}$$

where $\beta_{it}^{t-1}$ is the conditional beta as defined in equation (2.2).

The variable $Z_{\tau(t)-1} \in \mathcal{F}_{t-1}$ is the only investor information useful for predicting the joint distribution of $R_{it}$ and $R_{Mt}$. We correspondingly show:

**Proposition 3.** *If the CAPM holds conditional on the investor information $\mathcal{F}_{t-1}$, then*

$$\bar{R}_i^z = \beta_i^z \bar{R}_M^z, \tag{3.7}$$

*where*

$$\bar{R}_M^z = \bar{R}_M + \Delta_M^Z \left(\mathbf{1}_{\{Z_{\tau(t)-1}=H\}} - \mathbf{1}_{\{Z_{\tau(t)-1}=L\}}\right), \tag{3.8}$$

*and*

$$\beta_i^z = \bar{\beta}_i + \Delta_\beta^Z \left(\mathbf{1}_{\{Z_{\tau(t)-1}=H\}} - \mathbf{1}_{\{Z_{\tau(t)-1}=L\}}\right). \tag{3.9}$$

*Furthermore, the return generating process for portfolio $i$ can be written as*

$$R_i^{zs} = \alpha_i^{zs} + \beta_i^{zs}(\bar{R}_M^{zs} + \varepsilon_M) + \varepsilon_i \tag{3.10}$$

*where*

$$\alpha_i^{zs} = \Delta_\beta^S \left[\Delta_M^S + \bar{R}_M^z \left(\mathbf{1}_{\{S_t=G\}} - \mathbf{1}_{\{S_t=B\}}\right)\right], \tag{3.11}$$

*and $\varepsilon_M$ and $\varepsilon_i$ are mean-zero, independent normal random variables with variances $\sigma_M^2$ and $\sigma_i^2 - (\beta_i^{zs}\sigma_M)^2$, respectively.*

Equations (3.10) and (3.11) show that using the contemporaneous information $S_t$ can lead to non-zero alphas even though the conditional CAPM holds. The overconditioned alpha $\alpha_i^{zs}$ is zero when the asset $i$ payoff is linear in $R_{Mt}$, i.e., $\Delta_\beta^S = 0$, and more generally is proportional to the beta asymmetry $\Delta_\beta^S$.

### 3.3. Case I: Nonlinear Payoffs without Conditioning Information

To focus attention on payoff nonlinearities, we consider in this subsection that $\Delta_M^Z = 0$ and $\Delta_\beta^Z = 0$, implying that the investor information $Z_{\tau(t)-1}$ is irrelevant. For $s \in \{G, B\}$ and $z \in \{H, L\}$, it then simplifies notation to denote $\beta_i^s \equiv \beta_i^{zs}$ and $\bar{R}_M^s \equiv \bar{R}_M^{zs}$. The slope and the risk premium of the conditional CAPM are state-independent and the unconditional CAPM holds.

Several parameters can be directly calibrated or assumed equal to reasonable values. We specify $\bar{\beta}_i = \beta_i = 1$, implying $\bar{R}_i^z = \bar{R}_M^z = \bar{R}_i = \bar{R}_M$. The unconditional daily mean of excess market returns is $\bar{R}_M = 0.0003$, or about 7.5% annually. We set $\sigma_M = 0.01$ per day and $\sigma_i = 0.02$ per day, which for reasonable values of $\Delta_M^S$ gives unconditional standard deviations slightly above 16% and 32% per year respectively.

#### 3.3.1. Beta Asymmetry Specification

We calibrate the remaining parameters $\Delta_\beta^S$ and $\Delta_M^S$ by approximately matching the empirical range of beta asymmetries. Bawa and Lindenberg (1977) introduce the down-market beta

$$\beta_i^- = \frac{Cov\left(R_{it}, R_{Mt} | \, R_{Mt} < \bar{R}_M\right)}{Var\left(R_{Mt} | \, R_{Mt} < \bar{R}_M\right)}, \tag{3.12}$$

and ACX calculate this statistic annually for NYSE stocks from 1963-2001 using down market subsamples of daily returns as described previously. ACX similarly define the up-market beta $\beta_i^+$ and estimate it using up market subsamples.

The down and up betas can be calculated analytically for our model, and relate to the probabilities of being in state $G$ or $B$ conditional on a down or up market. Let $P_G^- \equiv \mathbb{P}\left(S_t = G \middle| R_{Mt} < \bar{R}_M\right)$ and similarly define $P_G^+$. We then show:

**Proposition 4.** *The conditional probabilities of state $G$ satisfy $P_G^- = 1 - \Phi(k)$ and $P_G^+ = \Phi(k)$, where $\Phi(k)$ is the standard normal cumulative distribution function evaluated at $k = \Delta_M^S / \sigma_M$. The down and up betas are*

$$\beta_i^- = \beta_i + \Delta_\beta^S \frac{2\Phi(k) - 1}{1 + k^2 - \left(2\phi(k) + k\left(2\Phi(k) - 1\right)\right)^2} \tag{3.13}$$

*and*

$$\beta_i^+ = \beta_i - \Delta_\beta^S \frac{2\Phi(k) - 1}{1 + k^2 - \left(2\phi(k) + k\left(2\Phi(k) - 1\right)\right)^2}, \tag{3.14}$$

*where $\phi(k)$ is the standard normal probability density function.*

The down beta deviates from average beta by the product of $\Delta_\beta^S$ and an "asymmetry multiplier" that characterizes the informativeness of market returns for the latent state $S_t$. The multiplier depends only on the signal-to-noise ratio $k$: When $k = 0$, the multiplier and market-conditioned beta asymmetry are zero regardless of the state-conditioned beta asymmetry $\Delta_\beta^S$. The multiplier rises monotonically from 0 to 1.073 as $k$ increases to 1.131, and declines to 1 for $k > 1.131$. When $k$ is very large, the multiplier is approximately one and $\beta_i^- - \beta_i^+ \approx 2\Delta_\beta^S$. The maximal beta asymmetry corresponds to $k \approx 1.131$, where $\beta_i^- - \beta_i^+ \approx 2.146\Delta_\beta^S$.

Our calibration fixes $\Delta_\beta^S = 0.5$, which is sufficiently large to match the upper tail of empirical absolute beta asymmetries $\left|\beta_i^- - \beta_i^+\right|$. ACX sort stocks by beta asymmetry, and for the quintile with the largest difference (absolute as well as signed), the average difference is $\beta_i^- - \beta_i^+ = 1.45 - 0.37 = 1.08$. (See their Table 1, Panel F.) Our choice of $2.146\Delta_\beta^S = 1.073$ thus gives a maximal beta asymmetry similar to the top quintile of all NYSE stocks. By Proposition 4, the full range of empirical beta asymmetries can be captured by varying the single parameter $\Delta_M^S$.

We consider eleven values of $\Delta_M^S$ ranging from $-0.005$ to 0.02 per day, reported in Table 1. The second through fifth columns of the table give the resulting values of $\beta_i^-$, $\beta_i^+$, $P_G^-$, and $P_G^+$ in daily data, calculated analytically using Proposition 4. For small $\Delta_M^S$, the probabilities $P_G^-$ and $P_G^+$ are close to 0.5 and beta asymmetry is near zero. When $\Delta_M^S$ is positive, the down beta exceeds the up beta and payoffs are concave, while negative values of $\Delta_M^S$ reverse the up and down betas and give convex payoffs. As anticipated, a variety of payoff structures are produced by varying $\Delta_M^S$.

### 3.3.2. Simulated Returns and Alternative Conditioning: An Example

Among the specifications in Table 1, the case $\Delta_M^S = 0.005$ is a useful example with a large but reasonable beta asymmetry of $1.42 - 0.58 = 0.84$. This value falls between the averages of the first and second quintiles of beta asymmetries reported by ACX for all NYSE stocks. When $\Delta_M^S = 0.005$, the state-conditioned means are $\bar{R}_M^G = \bar{R}_M + \Delta_M^S = 0.0053$ and $\bar{R}_M^B = -0.0047$ per day. This variation may seem large, but to investors $\Delta_M^S$ is a small component of the noise in market returns. The market unconditional variance is $\sigma_M^2 + \left(\Delta_M^S\right)^2 = 0.01^2 + 0.005^2 \approx 0.0112^2$ per day, and the standard deviation is correspondingly about 17.7% per year. The binomial $S_t$ thus contributes 20% of the total variance, or about 9.5% of the unconditional standard deviation of market returns.

Figure 2 visually demonstrates the reasonable impacts of $S_t$ when $\Delta_M^S = 0.005$. Panel A plots 12,000 randomly chosen draws from the joint distribution of $(R_{it}, R_{Mt})$. The days when $S_t = B$ are marked in gray, and draws where $S_t = G$ are black. The CAPM holds, as demonstrated by the unconditional regression (solid line) going through the vertical axis at approximately zero with a slope of one. The marginal distributions of $R_{it}$ and $R_{Mt}$ are unimodal because $k = 0.005/0.01 = 1/2$ is not too large. As a result, the gray and black points overlap considerably, which can be confirmed by the values $P_G^- = 0.31$ and $P_G^+ = 0.69$ reported in Table 1. Despite the subtle nonlinearity in returns, the fitted state-conditioned regressions (dashed lines) clearly show the beta asymmetry with approximate slopes $\beta_i^G = 0.5$ and $\beta_i^B = 1.5$. Consistent with Proposition 3, the conditional intercepts are positive reflecting that the contemporaneous state $S_t$ is not appropriate conditioning information.

Panel B shows how market returns act as a proxy for the latent state $S_t$. The same points are displayed as in Panel A, but here gray dots correspond to down markets $R_{Mt} < \bar{R}_M$ and black dots represent up markets $R_{Mt} > \bar{R}_M$. The market-conditioned regression lines have slopes close to the population values $\beta_i^- = 1.42$ and $\beta_i^+ = 0.58$ reported in Table 1. Thus, conditioning on the contemporaneous market return biases alpha in the same manner as conditioning on the latent state itself.

Similar biases occur in monthly data. Every month $\tau$ has a random number of days $S_\tau$ in which $S_t = G$. Conditioning on $S_\tau$ below or above its mean, as displayed in Panel C, nonlinearity is present but weaker than in daily data due to averaging of the $S_t$ draws within months. A similar averaging effect occurs when conditioning on the observable market return $R_{M\tau}$. For each value of $\Delta_M^S$, we simulate a long sample of monthly returns and sort the months by whether the market return is below or above its mean. We estimate market model regressions on each subsample to obtain monthly down and up betas, reported in Table 1. These results confirm that monthly market-conditioned beta asymmetry is considerably smaller than daily asymmetry.

Consistent with the monthly beta asymmetry calculations, Panel D plots the same returns as Panel C but conditions on whether the market return is below or above its mean. Again, $R_{M\tau}$ acts as a partial proxy for the latent variable $S_\tau$. The slopes of the fitted conditional regression lines are close to their population values of $\beta_i^- = 1.09$ and $\beta_i^+ = 0.91$ reported in Table 1. Despite this apparently modest monthly beta asymmetry, both of the conditional intercepts are sizeable at about 0.4% per month.

It seems unlikely that any empiricist would interpret the intercepts from daily or monthly regressions conditioned on $R_{Mt} > \bar{R}_M$ as conditional CAPM alphas, but conceptually identical and perhaps less obvious biases can arise. Any variable correlated with the latent state $S_t$ can produce a contemporaneous conditioning bias as in the panels of Figure 2. Specifically, the realized beta calculated within a month is correlated with both $S_\tau$ and $R_{M\tau}$, and will thus lead to similar biases, as we now show.

### 3.3.3. Contemporaneous Portfolio (CP) Risk Adjustment

We consider an empiricist who calculates conditional CAPM alphas by partitioning the data into $\Theta$ windows of $N$ months (equivalently $nN$ days), where $\theta(t) = \lfloor t/(nN) \rfloor + 1$ indexes the windows and $N = 1, 3, 6$. We define two CP risk adjustment methods and compare their accuracy in simulated data with a benchmark unconditional alpha.

**The Rescaled Daily (RD) Conditional Alpha:** LN recommend running OLS regressions within each window $\theta$. We correspondingly specify:

$$R_{it} = \alpha_{i\theta}^{CPRD}/n + \beta_{i\theta}^{CP} R_{Mt} + \varepsilon_{it}, \qquad t \in \theta, \tag{3.15}$$

where LN interpret the rescaled intercept $\alpha_{i\theta}^{CPRD}$ as a conditional CAPM alpha and $\beta_{i\theta}^{CP}$ is the contemporaneous portfolio beta. The time-series average alpha is denoted $\bar{\alpha}_{iN}^{CPRD}$. The CPRD approach uses daily data within each subperiod to simultaneously estimate factor loadings and intercepts, and implicitly assumes that the relevant investor horizon for measuring abnormal returns is one day.

**The Buy-and-Hold (BH) Conditional Alpha:** In contrast to the RD method, most asset pricing tests assume that the pricing model holds over some longer horizon such as a month, quarter, or year. We incorporate the contemporaneous portfolio beta into a buy-and-hold performance measure by specifying:

$$\alpha_{i\theta}^{CPBH} \equiv \frac{1}{N}\left(R_{i\theta} - \beta_{i\theta}^{CP} R_{M\theta}\right), \tag{3.16}$$

where $\beta_{i\theta}^{CP}$ is estimated from (3.15), and $R_{i\theta}$ and $R_{M\theta}$ are excess buy-and-hold returns over the period $\theta$. The mean alpha over all windows is $\bar{\alpha}_{iN}^{CPBH}$.

**Unconditional (UC) Risk Adjustment** As a benchmark, for each value of $\Delta_M^S$ we estimate the standard unconditional time-series regressions

$$\begin{aligned} R_{it} &= \alpha_i^{UCRD}/n + \beta_i^{UCRD} R_{Mt} + \varepsilon_{it}, & t = 1,..,T, \quad (3.17)\\ R_{i\theta} &= N\alpha_{iN}^{UCBH} + \beta_{iN}^{UCBH} R_{M\theta} + \eta_{i\theta}, & \theta = 1,..,\theta(T), \quad (3.18) \end{aligned}$$

using daily and $N$-month data. The daily intercept is multiplied by $n$ and the $N$-month intercept is divided by $N$, similar to the conditional RD and BH alphas respectively.

### 3.3.4. Comparison of the Performance Measures, Case I

The final eight columns of Table 1 show average simulated alphas from CP and UC risk adjustment. For each value of $\Delta_M^S$, we simulate a single long sample of $10^8$ months of 21 daily returns $(R_{it}, R_{Mt})$. All statistics are essentially free of simulation error, and we do not report standard errors.

Both of the UC alphas (RD and BH) are indistinguishable from zero. The expected value of the RD alpha is exactly zero because our model imposes that the conditional CAPM holds at a daily horizon, and conditioning information is irrelevant when $\Delta_M^Z = \Delta_\beta^Z = 0$. Longstaff (1989) shows that when the CAPM holds for a given observation interval, it need not be satisfied at other horizons. This effect is small in our model since the UCBH alpha at a monthly horizon is also essentially zero, indicating that the monthly CAPM holds approximately. The RD and BH versions of the CP alphas are also practically identical in Table 1, further confirming insignificant differences between rescaled daily and monthly returns.

The results clearly show the impact of overconditioning. For both BH and RD, the moderate calibration $\Delta_M^S = 0.005$ results in an upward bias in alpha of 0.42 for monthly windows, 0.14 for quarterly windows, and 0.07 for semiannual windows, where we henceforth report all alphas in percent. Overconditioning has a large impact in small windows, and as $N$ grows the conditional regressions converge to the unconditional case. Larger nonlinearities $\Delta_M^S$ produce larger CP alpha biases.

### 3.4. Case II: Nonlinear Payoffs with Conditioning Information

We now consider the more general setting where $Z$ has predictive power for market returns and covariances, i.e. $\Delta_M^Z \neq 0$ and $\Delta_\beta^Z \neq 0$. We evaluate five sets of base parameters ($\Delta_M^Z$, $\Delta_\beta^Z$, $\Delta_M^S$, $\Delta_\beta^S$), detailed in Table 2. For each parameterization, we vary the persistence $\rho \in \{0.50, 0.75, 0.9\}$ of $Z_t$ to demonstrate the impact of short- versus long-horizon information.

Figure 3 shows $12,000$ randomly chosen draws from $(R_{it}, R_{Mt})$ in the third parameterization $(\Delta_M^Z, \Delta_\beta^Z, \Delta_M^S, \Delta_\beta^S)$= $(0.001, 0.2, 0.005, 0.5)$ with $\rho = 0.9$. Panels A and B display the full set of draws, while C and D isolate the days when $Z_{\tau(t)-1} = H$, and E and F isolate $Z_{\tau(t)-1} = L$. The left-hand-side panels (A, C, and E) condition on the latent state $S_t$, and the right-hand-side panels (B, D, and F) condition on $R_{Mt}$ above or is below its population mean. The figure shows that the conditioning variable $Z_{\tau(t)-1}$ tilts the properly conditioned beta upwards or downwards, while the state $S_t$ creates nonlinear payoffs. The regression lines conditioned on $Z_{\tau(t)-1}$ go approximately through zero, and the intercepts overconditioned on $S_t$ are positive.

For each combination of base parameters and $\rho$, we simulate as before a single long time series of $10^8$ months of 21 daily returns. We evaluate biases in the performance measures UC, CP, and two others now described.

#### 3.4.1. Lagged Portfolio (LP) Risk Adjustment

An ad-hoc method to avoid overconditioning is to risk adjust using the lagged CP loading $\beta_{i\theta}^{LP} \equiv \beta_{i,\theta-1}^{CP}$. The RD and BH alphas are

$$\alpha_{i\theta}^{LPRD} \equiv \frac{1}{N} \sum_{t \in \theta} \left[ R_{it} - \beta_{i\theta}^{LP} R_{Mt} \right], \tag{3.19}$$

$$\alpha_{i\theta}^{LPBH} \equiv \frac{1}{N} \left[ R_{i\theta} - \beta_{i\theta}^{LP} R_{M\theta} \right]. \tag{3.20}$$

Although GM base most of their conclusions about abnormal performance on a CP alpha, they calculate an LPBH alpha and refer to it as a "feasible hedged" return.

#### 3.4.2. $Z$-conditioned Risk Adjustment and Other Methods

As a benchmark, we consider that the econometrician directly observes the investor conditioning information $Z_{\tau(t)-1}$. If such data is available, it is clearly optimal.

Other performance evaluation methods are possible but not addressed in our Monte Carlo experiments. For example, we could linearly forecast CP beta using the LP beta and other lagged information. Additionally, the model above does not address the impacts of portfolio turnover or changing portfolio weights. We defer investigating these issues until our empirical analysis of momentum performance, which helps to maintain focus on overconditioning in this section. Finally, given the known Markov-switching structure of return dynamics in our model, an empiricist could employ optimal nonlinear filtering or smoothing to infer the latent conditional beta. Such procedures can potentially improve on linear forecasts, but depend critically on an appropriate specification for return nonlinearities and dynamics. In this paper we emphasize simple, easily implemented methods that incorporate useful information without overconditioning, and leave consideration of more structural approaches to future research.

### 3.4.3. Comparison of the Performance Measures, Case II

Table 2 shows average BH alphas from the UC, CP, LP, and $Z$-conditioned performance evaluation methods. The first base parameter set is the leading example from the previous subsection with moderate nonlinearity $(\Delta_M^S, \Delta_\beta^S) = (0.005, 0.5)$ and no investor predictability $(\Delta_M^Z, \Delta_\beta^Z) = (0, 0)$. The persistence $\rho$ is irrelevant, the UC, LP, and $Z$-conditioned measures are unbiased, and the CP alpha has positive bias declining in $N$, as discussed previously.

The second base parameterization captures the opposite case where investor conditioning is important $(\Delta_M^Z, \Delta_\beta^Z) = (0.001, 0.2)$ and payoffs are linear $(\Delta_M^S, \Delta_\beta^S) = (0, 0)$. By design, underconditioning is as problematic as overconditioning in the previous example, and the UC alpha is approximately 0.41. When investor information is not persistent ($\rho = 0.5$) LP conditioning is equivalent to UC with noise for any window size $N$, and the biases are very similar. As $\rho$ increases, LP conditioning becomes more effective, and larger window sizes $N$ dilute useful information. Because payoffs are linear, the CP measure is unbiased for all $\rho$ when $N = 1$, and for larger windows CP suffers from underconditioning.

The third base case combines the investor information from the second example $(\Delta_M^Z, \Delta_\beta^Z) = (0.001, 0.2)$ with the payoff nonlinearity from the first example $(\Delta_M^S, \Delta_\beta^S) = (0.005, 0.5)$. The dangers from overconditioning and underconditioning are now equal. For all persistence levels $\rho$ and $N = 1$, the biases of 0.43 are the same for UC (underconditioned but not overconditioned) and CP (overconditioned but not underconditioned). When $\rho = 0.5$, all performance measures except $Z$-conditioning trade off overconditioning vs. underconditioning, and have approximately equal alphas. As $\rho$ increases, larger windows become helpful for the CP method as the harmful overconditioned information is diluted and replaced with useful lagged information. By contrast, LP measures benefit from smaller windows with larger $\rho$, as these provide the most recent information.[13]

Relative to example three, the fourth and fifth base cases respectively increase ($\Delta_M^Z = 0.002$) or dampen ($\Delta_M^Z = 0.0005$) the importance of investor information while holding all else constant. Thus, underconditioning is more damaging than overconditioning in case four, while the opposite is true in case five, as reflected in the results.

Among all potential methods, only the true $Z$-conditioned alphas are reliably unbiased, which is a straightforward implication of the Hansen and Richard (1987) critique. In practical applications, empiricists must proceed without definitively knowing the true investor conditioning information. One clear message from our experiment is that CP risk proxies are not in the investor information set, and can cause substantial biases when used to evaluate abnormal performance. The only justifications for using CP risk loadings are: (i) the empiricist calculates alphas with respect to a pricing model predicated on a strict factor structure (i.e., the Ross APT), or (ii) the empiricist knows that the specific assets or strategies being evaluated have payoffs that are linear in the proposed factors. In most cases, neither condition is satisfied. Many asset pricing models do not require a strict factor structure, and there is abundant evidence of nonlinear payoffs in stock returns. Thus, contemporaneously measured factor loadings are not a panacea for the Hansen and Richard critique, as suggested by LN. The ad hoc lagged portfolio approach provides one simple way to avoid overconditioning, but the empiricist should still search broadly for appropriate empirical instruments, as we emphasize when assessing momentum performance in the remainder of the paper.

## 4. Momentum Investing: Raw Return and Risk Characteristics

Jegadeesh and Titman (1993) initiate the investigation of momentum strategies.[14] In this section, we define the momentum portfolios used in our study, and describe their return characteristics. Winner portfolios have stronger beta asymmetries, which Section 3 suggests will cause a bias in CP alphas. Loser portfolios have larger variance ratios (e.g., monthly/daily) which impacts RD performance measures.

[13] If we used lagged rolling windows to risk-adjust one month returns, i.e. $[\tau-6, \tau-1]$ beta to risk adjust month $\tau$ returns, this effect would be weakened. For simplicity, the experiment here uses lagged non-overlapping windows $[\tau-6, \tau-1]$ to risk-adjust returns over the same horizon length $[\tau, \tau+5]$.

[14] Jegadeesh and Titman (2001) confirm that momentum persists following their original study. Moskowitz and Grinblatt (1999) argue that momentum is an industry phenomenon, and Hou (2007) attributes this to slow information diffusion within industries, while GM dispute the importance of industry momentum. Lee and Swaminathan (2000) document that momentum is more prevalent in high turnover stocks. Hong, Lim, and Stein (2000) find that small firms with low analyst coverage exhibit higher momentum. Rouwenhorst (1998, 1999) and Griffin, Ji, and Martin (2003) document momentum in international stock markets. Chan, Jegadeesh, and Lakonishok (1996) and Chordia and Shivakumar (2006) show that return momentum is related to earnings momentum. Carhart (1997) and Daniel, Grinblatt, Titman, and Wermers (1997) use momentum as a control in assessing abnormal returns.

### 4.1. Momentum Portfolio Strategies

We consider three momentum strategies, denoted 6-d-h, with common 6 month formation periods but different delays $d$ and holding periods $h$, as is now explained. At the beginning of calendar month $\tau$, we sort stocks into deciles based on their return over the formation period $\tau - d - 6$ to $\tau - d - 1$. To be included in the sort, stocks must have (i) valid monthly returns on the CRSP database over the entire formation period, (ii) at least 12 additional valid monthly returns in the thirty months prior to formation, (iii) at least 15 non-missing daily returns in each month of the formation period. Immediately following the sort, the winner portfolio (W) makes a fixed \$1 investment with equal weights in the top decile stocks, and sells stocks that were added to the portfolio at the beginning of month $\tau - h$. The loser portfolio (L) is defined by similarly timed investments and liquidations in the bottom decile stocks. Momentum (WL) profits are the difference between W and L returns. The portfolios we consider are 6-0-6, 6-1-1, and 6-1-6, which aids comparison with LN (6-0-6) and GM (6-1-1).

To ensure sufficient data to calculate lagged market and Fama-French risk loadings, we begin analyzing momentum profits in January 1930 and end in December 2005. The portfolios are seasoned by implementing the strategies with holding period $h$ for $h - 1$ months prior to the sample start date. Our results are robust to reasonable variations of these portfolio formation rules.[15]

### 4.2. Raw Profits

Table 3 summarizes return means and standard deviations for the momentum strategies and CRSP value-weighted market index. The average returns at different horizons are rescaled to have similar magnitudes: the daily mean is multiplied by the average number of trading days in one month, denoted $n$;[16] monthly, quarterly and semi-annual means are divided by $N = 1, 3, 6$ respectively. The reported standard deviations of daily returns are similarly rescaled by the square root of $n$.

The momentum raw returns are uniformly large and positive. For example, the 6-0-6 average monthly profit is $1.61 - 1.07 = 0.54$, where we henceforth report all means and standard deviations in percentages. The quarterly and semi-annual averages for 6-0-6 are similar (0.52 and 0.64), while the daily rescaled mean is more than 50% larger (0.84). The 6-1-1 strategy has smaller profits and proportionately larger horizon effects, while the opposite is true for 6-1-6.

The scaled daily standard deviation of the market index is moderately smaller than the monthly

[15] We specifically confirm robustness to (i) including only NYSE and AMEX stocks, (ii) imposing a minimum price screen of \$1, (iii) restricting the sample period to January 1964-December 2005, and (iv) combinations of the above.

[16] The approximate number of days in a month is 24.5 for months prior to 1952, and 21 thereafter, due to the end of Saturday trading. The overall average is approximately 22 days in a month.

value (4.77 vs. 5.45). In winner portfolios, the horizon difference is somewhat larger (e.g., 5.65 vs. 7.52 for 6-0-6), and in loser portfolios the horizon difference is considerably larger (6.21 vs. 11.20).

### 4.3. The RD Bias in Raw Returns

We develop an analytical formula that approximates the difference between average RD and BH returns over an arbitrary horizon. Roll (1983) shows how compounded average daily returns deviate from mean buy-and-hold returns, which is a related but different comparison.[17] Our formula is useful for understanding the difference between daily mean returns and monthly, quarterly, or semi-annual profits, and also explains the difference between RD and BH methods of calculating alphas.

Let $r_{it} = \ln\left(R_{it}^{g}\right) \sim \mathcal{N}\left(\mu_i, \sigma_i^2\right)$ denote a series of daily log returns. We further assume that log returns aggregated over $nN$ days have a normal distribution with variance $\sigma_{iN}^2$, where $N$ is the relevant horizon in months. These assumptions are exactly satisfied if $r_{it}$ is a stationary ARMA$(p, q)$ process with Gaussian innovations, and approximately hold in more general cases. The buy-and-hold mean is $\bar{R}_{iN}^{gBH} \equiv \mathbb{E}(R_{i,t+1}^{g} \cdots R_{i,t+nN}^{g})$ and the rescaled daily mean is $\bar{R}_{iN}^{gRD} \equiv 1 + nN\mathbb{E}(R_{it}^{g} - 1)$. The RD statistic has no intrinsic interest to a typical investor, but is often interpreted as a performance measure, and we wish to understand its bias relative to the more relevant BH statistic. We show:

**Proposition 5.** *The approximate ratio of RD to BH mean returns is*

$$\nu_{iN} \equiv \frac{\bar{R}_{iN}^{gRD}}{\bar{R}_{iN}^{gBH}} \approx e^{nN\sigma_i^2(1-VR_{iN})/2}, \tag{4.1}$$

*where* $VR_{iN} \equiv \sigma_{iN}^2 / \left(nN\sigma_i^2\right)$ *is the variance ratio. The net return ratio is*

$$\nu_{iN}^{net} \equiv \frac{\bar{R}_{iN}^{gRD} - 1}{\bar{R}_{iN}^{gBH} - 1} \approx \nu_{iN} + \frac{\nu_{iN} - 1}{\bar{R}_{iN}^{gBH} - 1}. \tag{4.2}$$

The RD bias is thus determined by the daily mean $\mu_i$, the daily variance $\sigma_i^2$, and return autocorrelations as summarized by the variance ratio $VR_{iN}$. When the variance ratio is one, (e.g., if returns are iid), then $\nu_{iN} = \nu_{iN}^{net} = 1$. For portfolios, Lo and MacKinlay (1988) show that asynchronous trading typically leads to positive autocorrelations and variance ratios that exceed one. When $VR_{iN} > 1$,

[17] Other studies that consider horizon effects include Blume and Stambaugh (1983), who show how portfolio rebalancing impacts average daily returns in the presence of bid-ask bounce, and Canina, Michaely, Thaler, and Womack (1998), who also focus on high-frequency portfolio rebalancing. Bessembinder and Kalcheva (2007) show how bid-ask spreads produce biases in estimated liquidity premia, and recommend an easily implemented weighting scheme to correct this bias.

then $\nu_{iN}^{net} < \nu_{iN} < 1$ and $\bar{R}_{iN}^{gRD}$ has a downward bias relative to the BH average. All else constant, the net bias is more severe when $\sigma_i^2$ is high and when $\mu_i$ is low.

Table 4 presents monthly average BH and RD net returns for several standard portfolios, their exact RD biases, the approximate biases $\nu_{iN}^{net}$, and statistics relevant to the calculations. The approximate and actual biases are close for every portfolio, validating Proposition 5. In almost all cases the RD average is below the BH average, and the bias is more substantial for equal weighting than value weighting. The downward bias in RD is moderate for large cap portfolios (NYSE, S&P 500, Large, Large Growth, and Large Value portfolios), and can be substantial for small-cap portfolios. For example, the RD bias for small growth is approximately 30% (0.49/0.68 = 0.72). In general, the portfolios that weight illiquid stocks more heavily have more positive autocorrelations, larger variance ratios, and more pronounced RD biases.

Table 5 decomposes the RD bias for momentum portfolios and shows it is particularly severe for L, which has lower liquidity than W (e.g., Korajczyk and Sadka, 2004). For the 6-1-6 strategy, the loser RD mean is about one half of the BH mean (0.43/0.78 = 0.55) due to extremely high autocorrelations and a correspondingly large variance ratio. The winner RD bias is much smaller (1.60/1.74 = 0.92), and consequently the net WL profits are significantly overstated by RD relative to BH. The RD measures are difficult to interpret since they mix useful return information with microstructure effects such as high autocorrelations from asynchronous trading. By contrast, the BH statistics directly reflect returns at the relevant horizon.

### 4.4. Beta Measurement, Dynamic Risk, and Beta Asymmetries

Table 6, Panel A, shows unconditional betas for momentum strategies. For daily, monthly, quarterly, and semi-annual horizons, we report loadings from standard market model regressions. In daily data, we additionally use Dimson (1979) regressions with the lag structure suggested by LN:

$$R_{it} = a_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \frac{\beta_{i2}}{3} \sum_{k=2}^{4} R_{M,t-k} + \varepsilon_{it}, \tag{4.3}$$

where $R_{it}$ and $R_{M,t}$ are respectively excess returns on portfolio $i \in \{W, L, WL\}$ and the value-weighted CRSP index.[18] The Dimson "sum" beta is $\beta_{i0} + \beta_{i1} + \beta_{i2}$.

Dimson adjustment has a stronger impact on losers than winners, consistent with the lower liquidity of losers. Similarly, the difference between monthly betas and daily sum betas is larger for

[18] Our results are robust to alternative lead/lag specifications. In particular, we considered: (i) no Dimson leads or lags, and (ii) one lead of market returns in addition to the lags in (4.3).

losers than winners. Thus, WL loadings are larger for daily returns and without Dimson adjustments, and we henceforth always Dimson adjust when using daily returns. The unconditional WL loadings are then negative for all horizons.

Panel B reports average CP betas. Following LN, we form non-overlapping windows of $N \in \{1, 3, 6\}$ calendar months, and within each window estimate the regression (4.3) from daily returns. The CP beta is the sum beta associated with this regression. For W, the average CP betas are close to the daily UC loadings (e.g., for 6-0-6: 1.14, 1.18, and 1.21 vs. 1.16), while for L the average CP betas are smaller than the UC loadings (1.16, 1.22, and 1.24 vs. 1.38).

Figure 4 plots the time series of monthly 6-0-6 CP betas, showing both low-frequency movements and high-frequency variations. If the predictable low-frequency dynamics are correlated with expected market returns, then unconditional alphas suffer from underconditioning. Conversely, if the high-frequency changes are unpredictable and correlated with realized market returns, then overconditioning causes alphas to be biased.

Panels C and D of Table 6 show daily asymmetries without and with Dimson adjustments, and Panel E reports monthly values. The Dimson adjusted daily WL beta differences are uniformly positive (0.49 for 6-0-6, 0.60 for 6-1-1, and 0.41 for 6-1-6),[19] as are the monthly values (0.91 for 6-0-6, 1.18 for 6-1-1, and 0.74 for 6-1-6).[20] These beta asymmetries lead to overconditioning in momentum CP alphas, as we now show.

## 5. The Conditional CAPM and Momentum Performance

This section uses the conditional CAPM to assess momentum performance. Proper conditioning reduces alpha by a statistically significant 20-40% relative to unconditional levels. Overconditioned alphas can be more than 2.5 times as large as appropriately conditioned performance measures.

### 5.1. Unconditional Risk Adjustment

Table 7 reports UC alphas in column (i). Intercepts calculated from daily returns use the Dimson lag structure in (4.3) and are multiplied by $n$. Monthly, quarterly, and semi-annual alphas are from the market model regressions (3.18).

[19] Hong, Tu, and Zhou (2006) report similar asymmetries using a different sample and estimation technique. Ang and Chen (2002) and ACX do not report up or down betas for momentum portfolios.

[20] Table 6 shows larger asymmetries in monthly vs. daily returns, while the model in Section 3 implies that asymmetry should decline in the return horizon. There are of course measurement issues such as microstructure effects impacting the empirical results in Table 6, making a direct comparison imprecise. Nonetheless, it would be interesting to consider how the model in Section 3 could be modified to capture these horizon effects in beta asymmetries, and we leave this as a goal for future research.

Consistent with the negative market exposure of the WL portfolios reported in Table 6, UC risk-adjustment increases momentum performance relative to raw profits. For example, at a one month horizon, the 6-0-6 strategy has alphas of 0.57 for W, $-0.24$ for L, and a net of 0.81 for WL.

### 5.1.1. The RD Bias in Alphas

The daily return UC alpha is an RD performance measure, while the monthly and longer horizon alphas are BH measures. We can apply Proposition 4 to abnormal returns to show how the RD bias in raw returns translates into a bias in alphas. The alpha difference $\alpha_i^{UCRD} - \alpha_{iN}^{UCBH}$ for $N = 1, 3, 6$ is approximately equal to

$$(\nu_{iN}^{net} - 1)(\bar{R}_{iN}^{gBH} - 1) - \beta_i^{UC}(\nu_{MN}^{net} - 1)(\bar{R}_M^{gBH} - 1) - (\beta_i^{UC} - \beta_{iN}^{UC})\bar{R}_M^{gBH} \tag{5.1}$$

where $\beta_i^{UC}$ is the unconditional beta calculated from daily data, and $\beta_{iN}^{UC}$ are unconditional betas calculated from buy-and-hold returns over $N$ month windows. The first term approximates the RD bias in raw returns, and the second term is due to RD bias in market returns. The third term accounts for horizon differences in beta.

We illustrate this decomposition for the 6-0-6 strategy at a monthly horizon using information from Tables 3 and 6. Market returns have a small RD bias ($0.89/0.93 = 0.96$), and the average risk-free rate is 0.31. Losers have a BH return of 1.07, a substantial RD bias ($0.65/1.07 = 0.61$), and betas of $\beta_L^{UC} = 1.38$ and $\beta_{L1}^{UC} = 1.61$. Obtaining the same statistics for winners allows us to decompose the WL alpha bias into the RD bias in raw returns (0.30) less the effect of RD bias in market returns ($< -0.01$) less the impact of differences in monthly and daily betas (0.13), which approximates the exact bias $\alpha_{WL}^{UCRD} - \alpha_{WL1}^{UCBH} = 0.97 - 0.81 = 0.16$.

## 5.2. Contemporaneous Portfolio (CP) Risk Adjustment

Table 7 reports CP alphas using the RD (column ii) and BH (column iii) methodologies. Both approaches partition the data into windows of $N = 1, 3, 6$ months. Within each window $\theta$ and for each portfolio $i$, we run regression (4.3), obtaining the intercept $a_{i\theta}$ and the sum beta $\beta_{i\theta}^{CP}$. The RD alphas are the rescaled intercepts $\alpha_{i\theta}^{CPRD} = na_{i\theta}$, and their time-series mean is denoted $\bar{\alpha}_{iN}^{CPRD}$. The BH alphas apply the same risk adjustment to buy-and-hold returns, i.e., $\alpha_{i\theta}^{CPBH} = R_{i\theta} - \beta_{i\theta}^{CP} R_{M\theta}$, and have mean $\bar{\alpha}_{iN}^{CPBH}$.

The CPRD methodology follows LN exactly, and indicates larger WL alphas than UC alphas of the same horizon $N$. The effect is dramatic for $N = 1$ month (1.43 vs. 0.81 for 6-0-6), sizeable for quarterly intervals (1.16 vs. 0.89) and marginal for semi-annual windows (0.90 vs. 0.87). Almost all of the effect of CPRD conditioning on the WL portfolio is driven by the winner side. For example, the 6-0-6 winner alphas are 0.57 unconditionally, increasing to 1.18 for monthly CPRD. By contrast, in the L portfolio the UC and CPRD alphas are nearly identical ($-0.24$ vs. $-0.25$).

Comparing CPRD to UC alphas while holding $N$ constant mixes two effects. The CPRD alphas are based on daily returns, while UC measures are based on BH returns over $nN$ days. To isolate the difference between CP and UC risk-adjustment, we must compare all CPRD alphas to UC alphas at a daily horizon. The differences are then smaller than discussed in the previous paragraph, and for $N = 6$ all CPRD alphas for WL are at least marginally lower than the daily UC alpha.

Consistent with our results for raw returns, the RD bias in CP alphas (i.e., $\bar{\alpha}_{iN}^{CPRD} - \bar{\alpha}_{iN}^{CPBH}$) is negative for both W and L, and is more pronounced for L. As a consequence, the WL portfolio RD biases in CP alphas are positive. For example, the net RD bias is $1.43 - 1.15 = 0.28$ for the 6-0-6 strategy with $N = 1$, and does not vary substantially for different strategies and window sizes.

The RD bias in CP alphas has a similar decomposition to the RD bias in UC alphas discussed in Section 5.1.1. One important difference is that CPRD and CPBH use the same beta, so the third term in equation (5.1) becomes zero. Additionally, the second term becomes slightly more complicated for CP measures, due to covariance between CP beta and the potentially time-varying level of RD bias in market returns. The second term is generally small, however, and as a consequence the RD bias in CP alphas is closely approximated by the RD bias in raw returns. For example, in the 6-0-6 WL portfolio, the RD bias in CP alphas is 0.28 and the raw return bias is 0.30.

Holding the horizon $N$ constant, the CPBH and UC alphas use identical BH returns, and differ only by the betas they use for risk adjustment. For the 6-0-6 WL portfolio, the CPBH alpha is larger than UC for $N = 1$ (1.15 vs. 0.81), about the same for $N = 3$ (0.92 vs. 0.89), and smaller for $N = 6$ (0.67 vs. 0.87). As in Section 3, smaller horizons $N$ give more recent information, leading to less underconditioning but more overconditioning, making the results difficult to interpret. The next subsection uses lagged information to eliminate the overconditioning bias and provide a more accurate assessment of momentum performance under the conditional CAPM.

### 5.3. Lagged Portfolio (LP) Risk Adjustment

Define the lagged portfolio betas $\beta_{i\theta}^{LP} \equiv \beta_{i,\theta-1}^{CP}$. For $N = 1, 3, 6$, the RD alphas are $\alpha_{i\theta}^{LPRD} \equiv \frac{1}{N}\sum_{t\in\theta}\left[R_{it} - \beta_{i\theta}^{LP}R_{Mt}\right]$ and the BH alphas are $\alpha_{i\theta}^{LPBH} \equiv \frac{1}{N}\left[R_{i\theta} - \beta_{i\theta}^{LP}R_{M\theta}\right]$, presented in Table 7 columns (iv) and (v). The LP method eliminates overconditioning, and the alpha difference $\bar{\alpha}_{iN}^{CPBH} - \bar{\alpha}_{iN}^{LPBH}$ approximates the overconditioning bias.

The winner LP alphas are lower than the corresponding CP alphas, consistent with our overconditioning arguments (Sections 2-3) and the large winner beta asymmetry (Table 6). Overconditioning is dramatic for $N = 1$ (e.g., for 6-0-6, CPBH is 1.25 and LPBH is 0.55) and almost negligible for $N = 6$ (0.66 vs. 0.65). This pattern is consistent with the Monte Carlo results in Table 2. Long windows contain diluted and increasingly irrelevant conditioning information, and hence both CP and LP alphas approach UC for large $N$. By contrast, shorter windows result in large overconditioning for CP, but provide more accurate information when LP is used. These arguments suggest that the inverse relation between CP alpha and window length (1.25, 0.87, 0.66 for $N = 1, 3, 6$) is due to overconditioning, while the lower LP alphas with reversed horizon effects (0.55, 0.60, 0.65) show the effect of more appropriate conditioning.

The L portfolios are much less impacted by overconditioning, consistent with their smaller beta asymmetries in Table 6. For all strategies and horizons, the difference between CP and LP alphas is no larger than 0.10 for losers. Combining the large overconditioning bias in W with the near insensitivity of L, the WL portfolio has a positive bias that is very large for low $N$ (e.g., $\bar{\alpha}^{CPBH}_{WL,1} - \bar{\alpha}^{LPBH}_{WL,1} = 1.15 - 0.53 = 0.62$).

We draw particular attention to the LPBH alphas, which suggest that the conditional CAPM reduces momentum performance relative to the unconditional alpha for all horizons and risk measures, calling into question the conclusions of LN. For example, with $N = 1$ the CPRD methodology advocated by LN produces alphas that are considerably larger than UC (1.43 vs. 0.81 for 6-0-6, 1.43 vs. 0.57 for 6-1-1, and 1.69 vs. 1.14 for 6-1-6). By contrast, the LPBH alphas, which eliminate both the overconditioning and RD biases, are much smaller (0.53, 0.36, 0.87).

Despite the improvement in LP relative to CP, an important shortcoming in both approaches is that they do not account for changing portfolio composition. For example, the LP winner beta reflects W risk in previous periods, but momentum portfolios typically have high turnover and the betas of entering and exiting stocks need not be similar. In particular, GM show that betas of newly added winner and loser stocks vary with the market return in the formation period, due to selection. Nonetheless, neither GM nor LN account for portfolio turnover in calculating risk adjusted returns,[21] and these weaknesses are present in the CP and LP alphas. Since portfolio composition is an important determinant of momentum risk, we now develop a performance measure that accounts for the dynamics of portfolio holdings.

[21] GM proxy for month $\tau$ betas using portfolio loadings estimated in windows from months $\tau$ to $\tau + 5$ or from $\tau - 61$ to $\tau - 2$. (See their Table 1 and Sections 4.1-4.2.) Similarly, LN assume constant loadings over periods as long as twelve months.

### 5.4. Lagged Component (LC) Risk Adjustment

Fama and MacBeth (1973) test the CAPM using monthly rebalanced equal-weighted portfolios, and correspondingly calculate portfolio betas as equal-weighted averages of individual stock betas from prior windows. For buy-and-hold momentum strategies, a natural extension is to use a portfolio-weighted average of individual component loadings estimated from prior windows. This procedure accounts for turnover, and also adjusts for the fluctuating portfolio weights of individual stocks in buy-and-hold portfolios.

To implement the lagged component methodology, at the end of each calendar month $\tau - 1$ we first estimate betas of the individual stocks (components) that will belong to a portfolio in month $\tau$. The component loading regressions in our study are primarily based on two combinations of return frequencies and window lengths: (i) daily returns from the beginning of $\tau - 6$ to the end of $\tau - 1$ with the Dimson lag structure in (4.3); or (ii) monthly returns from $\tau - 36$ to $\tau - 1$. The corresponding portfolio loadings, denoted $\beta_{i\tau}^{LC6}$ and $\beta_{i\tau}^{LC36}$ respectively, are obtained by summing over components the product of (1) the component beta and (2) the beginning of month $\tau$ component portfolio weight. For lags $l = 6, 36$, the alphas are $\alpha_{i\tau}^{LCl} = R_{i\tau} - \beta_{i\tau}^{LCl} R_{M\tau}$ with time-series averages $\bar{\alpha}_i^{LCl}$, reported in Table 7 Panel A columns (vi) and (vii).

For all strategies, the LC net WL alphas are smaller than the one month LPBH alphas to which they are most comparable. The difference between LC and LPBH performance measures is most noticeable for 6-1-1, presumably because of its high turnover.

Panel B shows the average LC betas. The LC6 averages (e.g., 1.23 and 1.22 for 6-0-6 W and L) are larger than the one month CP beta averages (1.14 and 1.16, Table 6) by similar amounts for W and L. Hence, the reduction in WL alpha for LC6 relative to LPBH (0.43 vs. 0.53) is primarily due to a larger covariation between beta and the market return. By contrast, the 6-0-6 LC36 winner beta is smaller than the loser beta (1.25 vs. 1.35), which almost entirely explains the larger WL alpha for LC36 relative to LC6 (0.51 vs. 0.43). Microstructure issues may affect the average level of any realized beta, hence we now turn to a more general conditioning method that corrects biases and combines information in arbitrary risk predictors.

### 5.5. Forecast Component (FC) Risk Adjustment

The conditional performance measures CP, LP, and LC use fixed window betas as direct proxies for the unobserved conditional CAPM beta. An alternative approach specifies beta as a function of instruments, typically predictors of risk premia such as the dividend yield and T-bill rate. We combine these techniques by using LP and LC betas as additional instruments in conditional return regressions, within the context of two related approaches.

### 5.5.1. The One-step Method (FC1)

Following Shanken (1990), Ferson and Schadt (1996), and Ferson and Harvey (1999), we specify the conditional return regression:

$$R_{i\tau} = \alpha_i^{FC1} + \beta_i \left[ \begin{array}{cc} 1 & Z_{\tau-1} \end{array} \right] R_{M\tau} + e_{i\tau}, \tag{5.2}$$

where the conditional beta $\beta_{i\tau}^{FC1} \equiv \beta_i[\ 1 \quad Z_{\tau-1}\ ]$ uses as instruments any lagged conditioning information and $\tau$ indexes months.[22] Typical instruments are predictors of risk premia, and we consider the dividend yield (DY), term spread (TS), one-month T-bill rate (TB), and default spread (DS).[23] We additionally use as instruments LP and LC betas. If individual stock betas are greatly influenced by their portfolio membership (in vs. out), then the LP risk measures may be useful. On the other hand, if individual stock loadings are highly persistent and portfolio loadings change primarily due to fluctuating portfolio weights (including turnover), then the LC betas are likely to be more accurate. Consistent with the high turnover of momentum portfolios, we focus primarily on the lagged component betas LC6 and LC36, and discuss robustness to other choices.

The conditional return specification (5.2) includes as special cases the CP, LP, and LC methodologies discussed previously, by placing a weight of one on the corresponding CP, LP, or LC beta estimate, and zeros on all other instruments. The general specification has several advantages over direct proxying. First, permitting conditional beta to be a linear combination of prior window beta estimates and other potential predictors allows data-driven forecast averaging as opposed to an ad hoc choice of empirical proxy. Second, the general specification allows instruments to be scaled up or down, greatly mitigating concerns about using daily realized betas that may be attenuated towards zero due to asynchronous trading. Finally, the timing convention in the conditional regression clarifies that lagged instruments should be used, and hence directly inserting a CP beta in the

[22] The specification is identical to equation (4) in Ferson and Schadt. Shanken permits the intercept to also be linear in the instruments, and notes that the coefficients are zero under the null. Ferson and Harvey use the coefficient restrictions in a time-varying intercept specification to reject the conditional Fama-French model. We specify a constant intercept to maintain comparability with CP, LP, and LC alphas, and in Section 5.6 test for predictability in the residuals. Related empirical approaches specify time-varying betas in a GMM framework (e.g., Campbell, 1987; Ferson and Harvey, 1993; Harvey, 1989).

[23] DY is computed following Fama and French (1988). TS is from Robert Shiller's website http://www.econ.yale.edu/~shiller/data.htm, measured at the end of the previous year. DS is the difference between BAA and AAA corporate bond yields, obtained from the Federal Reserve, http://research.stlouisfed.org/fred2. TB is the 30-day T-bill yield from CRSP.

instrument set $Z_{\tau-1}$ is not advisable.

### 5.5.2. The Two-Step Method (FC2)

If one wishes to introduce CP betas into the conditional return regression, then appropriate instrumentation is required. Consider the first-stage predictive regression:

$$\beta_{i\tau}^{CP} = \gamma_{i0} + \gamma_{i1} Z_{\tau-1} + \varepsilon_{i\tau}. \tag{5.3}$$

The second stage return regression specifies conditional beta as a linear function of the fitted first-stage CP beta:

$$R_{i\tau} = \alpha_i^{FC2} + \left(\phi_{i0} + \phi_{i1} \widehat{\beta}_{i\tau}^{CP}\right) R_{M\tau} + u_{i\tau}. \tag{5.4}$$

The conditional beta is then $\beta_{i\tau}^{FC2} \equiv \phi_{i0} + \phi_{i1} \widehat{\beta}_{i\tau}^{CP}$.[24] If the fitted CP beta is an unbiased estimate of the conditional beta then $\phi_{i0} + \phi_{i1} = 1$, and if it is an efficient predictor then additionally $\phi_{i0} = 0$. Specifying $\phi_{i0}$ and $\phi_{i1}$ to be unconstrained recognizes that the fitted CP beta and conditional beta should be correlated, but due to microstructure biases in daily data and other potential horizon effects it may be optimal to rescale and translate the fitted realized beta.

The two-step regression is a special case of the one-step regression. Whereas FC1 allows conditional beta to be an unrestricted function of the instruments, FC2 requires the instruments to first be projected on the CP beta. As a consequence, under the FC2 null the one-step coefficients are the product of the FC2 first stage parameters and the second stage coefficient $\phi_{i1}$.

### 5.5.3. Forecast Component Empirical Results

Table 8 shows 6-0-6 FC regressions for different instruments. The columns report the FC2 beta and return regressions, and the FC1 alpha and $R^2$. In all cases, the FC1 and FC2 alphas are very close, and our discussion focuses on the two-step results.

With no instruments (specification 1), the first stage intercepts are by definition the mean CP betas (1.14 and 1.16 for W and L, Table 6), and the second stage fitted CP beta coefficients ($\phi_{i1} = 1.02$ and 1.39 for W and L) are ratios of the UC to average CP betas.[25] The FC1 and FC2

[24] Similar to our first stage regression, Ghysels and Jacquier (2006) regress CP betas on LP betas and other predictor variables. They do not consider LC betas, which may be justifiable given that their focus is on forecasting betas of industry portfolios, where turnover is generally low. Because of the emphasis of their paper, they also do not consider the second stage regression, which is critical for performance analysis, and also mitigates microstructure biases in realized betas calculated from daily data.

[25] The coefficients $\phi_0$ and $\phi_1$ are not separately identified in (1) because the fitted CP beta is constant.

return regression intercepts are the unconditional alphas previously reported in Table 7, and the net WL alpha is 0.81.

The standard instruments (2) are useful to predict CP betas, particularly for losers (for W and L, $R^2 = 2.85$ and 9.73 percent). The second stage fitted CP betas are highly significant, providing an efficient predictor of conditional beta for winners ($\phi_{i0}, \phi_{i1} = -0.12, 1.20$), and a downward biased but still useful predictor for losers $(0.51, 0.79)$. For W and L, the second stage $R^2$ increase by approximately two percentage points relative to (1), the alphas attenuate towards zero, and the net WL alpha decreases from 0.81 unconditionally to 0.70 conditionally.

Our paper emphasizes that lagged component betas are useful instruments for conditional return regressions. Relative to the standard predictors (2), the isolated LC6 beta (3) substantially improves the first stage CP beta regression ($R^2 = 26.4, 23.0$ for W and L), the second stage $R^2$ increase by 2.6 and 0.2 points for W and L with both alphas attenuating toward zero, and the WL alpha drops to 0.65.

Relative to LC6, the isolated LC36 beta (4) has less first stage predictive power ($R^2 = 19.2, 17.2$), and gives a marginally smaller WL alpha of 0.63. Combining LC6 and LC36 (5), both are significant with LC6 more heavily weighted in a ratio of approximately 2:1. Figure 5 shows the resulting conditional betas $\beta_{i\tau}^{FC2}$, which appear smoother than the CP betas in Figure 4. The WL alpha from this regression is 0.62.

Specification (6) combines the LC betas with instruments RU6 and RU36, the market runups over 6 and 36 month prior windows. To motivate these instruments, recall that GM show the formation period runup predicts W and L betas, due to selection. Additionally, when payoffs are nonlinear the beta estimated in any window can covary with contemporaneous market returns. Including market runup helps to control for this effect. The results show that RU6 positively predicts winner beta and RU36 negatively predicts loser beta with a small overall impact on $R^2$ and a WL alpha of 0.59.

Specification (7) combines LC betas with the standard instruments. For winners the LC beta coefficients are virtually unchanged, TB is driven out, and among the standard instruments only DY is significant. For losers, the total weight on LC betas remains similar but shifts towards LC36 (approximate 1:1 ratio), TS and DY are driven out, and the significance of TB and DS declines substantially. Combining all instruments in (8), the LC betas are stable and significant, the runup variables are driven out, and among the standard instruments only DY for winners and TB for losers remain significant. Relative to (1), the WL alpha falls by approximately 30% to 0.59.

As a robustness check, in untabulated results we also considered adding additional LC and LP betas to the regressions in Table 8. For example, the LP6 beta, calculated from six months of daily lagged portfolio returns, is significant independently, but is completely driven out when combined with LC6. In general, LC betas dominate LP betas, and the alphas reported in Table 8 with LC6 and LC36 betas are robust to adding component or portfolio betas measured over other horizons.

Table 9 abbreviates the forecast component analysis for 6-1-1 and 6-1-6 strategies. The results are qualitatively similar to those reported for 6-0-6, and the net alpha reductions from conditioning information are approximately 20% for 6-1-6 (0.93 vs. 1.14) and 40% for 6-1-1 (0.36 vs. 0.57).

We test the statistical significance of the reductions in alpha due to conditioning information using the GMM procedure outlined in Appendix A.2. For each conditional regression in Tables 8 and 9, an asterisk next to the WL alpha denotes significance at the 5% level. When the standard instruments alone are used, the decrease in alpha is significant only for the 6-1-1 strategy. In all specifications with at least one LC beta as an instrument, the conditional alpha is significantly lower than the unconditional alpha for all strategies.

These results contradict previous studies arguing that conditioning information increases momentum profits. GM and LN reach their conclusions using CP methods biased by overconditioning. LN argue that their study differs from previous implementations of the conditional CAPM primarily because they use a time-series method that imposes restrictions on risk premia not always enforced in cross-sectional regressions. Our study illuminates other problems in their methodology, and is not subject to their critique because we also use a time-series approach. We find that a standard conditional return regression with LC betas as instruments reduces momentum performance by approximately 20-40% relative to unconditional alphas.

### 5.6. Predicting Momentum Performance

Under the null hypothesis that the conditional CAPM holds, the time series of alphas should be unpredictable. Ferson and Harvey (1999) correspondingly specify the conditional three-factor intercept to be linear in lagged state variables, and test the collective significance of the coefficients. We similarly test whether the alphas (intercept plus residuals) from the regressions in Tables 8 and 9 are predictable.

Table 10 regresses 6-0-6 unconditional alphas (Panel A), conditional alphas (B) and their differences (C) on predictor variables. Panel B uses the FC2 version of regression (8) in Table 8, and the results are not highly sensitive to the choice of FC methodology or instrument set, provided at least one LC instrument is included.

The predictor variables for alpha are the standard instruments (DY, DS, TS, TB), the market runup RU36 and its square RU36$^2$, and five lags of alpha.[26] All $t$-statistics are based on Newey-West standard errors with five lags. In Panel A, the mean unconditional WL alpha is highly significant (regression 1), and the first two lagged alphas have significant (10% and 5%) negative coefficients (2). Among the standard predictors, only DS and TB are significant (3). The runup variables RU36 and RU36$^2$ have strong effects (4), consistent with Cooper, Gutierrez, and Hameed (2004). Combining predictors (5-7), the runup variables drive out the standard predictors and cause the magnitude and significance of lagged alphas to increase. The $R^2$ increase dramatically when runup and lagged alphas appear together.

Panel B considers a similar analysis for conditional alphas. Relative to A, the conditional mean is lower (1-2), and the $R^2$ for specifications (3-7) are slightly lower with coefficients generally closer to zero and marginally less significant. In predicting the difference between UC and conditional alphas, Panel C shows that only TS is significant. Specifications (1-2) confirm that the mean alpha reduction due to the conditional CAPM is significant.

### 5.7. Decomposing the Alpha Biases

The difference between conditional and unconditional alphas can be decomposed into a direct alpha bias and an indirect effect caused by beta bias using equation (2.3). Alternatively, equation (2.5) states the alpha bias as a sum of weighted covariances of conditional beta with $R_{Mt}$ and $R^2_{Mt}$.

Table 11 provides the data necessary for these decompositions using FC2 conditioning and the combinations of instruments considered previously. Panel B reports the average conditional betas $\bar{\beta}_i^{FC2}$ and their covariances with $R^2_M$. Panel C decomposes the UC bias $\alpha_i^{UC} - \bar{\alpha}_i^{FC2}$. For specifications (3)-(8) which include LC betas as instruments, the direct bias is always positive (2 to 8 basis points), and the beta bias effect (9 to 19 basis points) explains the majority of the decrease in conditional alpha relative to the unconditional benchmark. Using the information in Panels B and C, we can determine that most of the beta bias is contributed by the second term in equation (2.4), $Cov(\beta_{i\tau}^{FC2}, R^2_{M\tau})/\sigma^2_M$.

Similar calculations explain the differences between overconditioned and appropriately conditioned alphas. We show

[26]For comparison, Ferson and Harvey (1999) use the standard instruments to test alpha predictability in size and book-to-market portfolios. Chordia and Shivakumar (2002) argue that the standard instruments explain a large part of momentum profits, and Cooper, Gutierrez, and Hameed (2004) show that market runup helps to explain momentum performance.

**Proposition 6.** *The overconditioned alpha bias is*

$$\bar{\alpha}_{i1}^{CPBH} - \bar{\alpha}_{i}^{FCf} = -Cov\left(\beta_{i\tau}^{CP} - \beta_{i\tau}^{FCf}, R_{M\tau}\right) - \left(\bar{\beta}_{i1}^{CP} - \bar{\beta}_{i}^{FCf}\right)\bar{R}_{M},$$

*where* $f \in \{1, 2\}$ *and* $\bar{\beta}_{i}^{FCf} \equiv \mathbb{E}\left(\beta_{i\tau}^{FCf}\right)$.

This generalizes Proposition 1, which assumed that the overconditioned beta was unbiased with respect to the available information. In practice, mean CP betas are generally biased with respect to available information, and the second term of Proposition 6 accounts for this. The first term is identical to Proposition 1.

Panel D of Table 11 shows that the covariance between market returns and the difference of CP and FC betas is large and negative for winners (e.g., $-0.65$ in regression 5), and almost zero for losers, consistent with the beta asymmetries reported in Section 4. Additionally, the CP beta is biased downwards relative to the forecasted beta for both W and L. The magnitude of the beta bias is larger for losers, consistent with their lower liquidity. Applying Proposition 6, the overconditioned alpha bias for regression (5) is $-(-0.62) - 0.17 = 0.45$, which except for rounding error is identical to the directly calculated alpha difference of 0.46. The results are similar for other specifications. Thus, the difference in mean alphas is overwhelmingly explained by the fact that loser realized betas are insensitive to market conditions, while winner betas covary substantially more with down markets than up markets.

## 6. The Conditional Three-Factor Model and Momentum Performance

This section extends our analysis of momentum performance to conditional versions of the Fama-French (1993, "FF") three-factor model based on market, value, and size portfolios.[27]

### 6.1. Realized Loading Risk Adjustment

Table 12 shows the FF alphas obtained from the methods UC (column i), CP (ii-iii), LP (iv-v), and LC (vi-vii). Appendix A.3 provides calculation details. Consistent with prior research (e.g., Fama and French, 1996), UC risk adjustment produces larger momentum alphas under the FF model than the CAPM. For 6-0-6 at a one month horizon, the FF winner alpha is lower than the CAPM

[27] We obtain partial time-series for the three factors and historical book equity values from Ken French's website. Pre-formed daily factors prior to 1963 are not available, hence we create the pre-1963 daily factors following the procedure outlined by Fama and French (1993).

alpha reported in Table 7 (0.44 vs. 0.55), the loser alpha is lower by a greater margin ($-0.65$ vs. $-0.30$), and the net WL alpha increases by 0.29 to 1.10.

Overconditioning is again a significant problem for CP performance measures. For 6-0-6, the alpha difference between CP and LP exceeds 1.0 for one month windows, is about 0.6 for $N = 3$, and ranges from 0.1 to 0.2 for $N = 6$, where we again report all alpha differences in percentages and rescale by dividing by $N$. The RD bias is approximately 0.1-0.2 across all strategies and window sizes.

To calculate LC loadings and performance measures we again use either six months of lagged daily data (LC6) or 36 months of lagged monthly data (LC36) for each component in the W and L portfolios, using Dimson sum betas for LC6 as previously. The LC6 alphas (column vi) are moderately smaller than the LPBH alphas (0.73 vs. 0.82 for 6-0-6) and considerably smaller than UC (1.10). The LC36 method (vii) further reduces performance (0.45 for 6-0-6), but we interpret these alphas cautiously. The LC6 loadings capture recent movements in risk, but are calculated with daily data and, even with Dimson adjustments, are likely to be biased downwards more for the relatively illiquid loser side. The LC36 loadings help to address the asynchronous trading problem by using monthly data, but over the measurement period winner HML and SMB loadings are likely to decrease while loser loadings increase, and long window regressions will not capture these changes. Using the LC realized betas as instruments in the conditional return framework should help to correct these problems and provide an improved performance measure.

### 6.2. The Forecast Component Methodology with Three Factors

We focus on the one-step forecast component approach (FC1). The two-step approach uses as instruments in the return regression fitted values from first stage regressions of CP loadings on predictor variables. The FC2 results are similar to FC1, and are omitted for brevity.

The conditional three-factor regression is:

$$R_{i\tau} = \alpha_i^{FC1} + \sum_j \beta_{ij} F_{j\tau} \left( \begin{bmatrix} 1 & Z_{j,\tau-1} \end{bmatrix} \right) + \varepsilon_{i\tau},$$

where $j \in \{MKT, HML, SMB\}$ are the Fama-French factors. Table 13, Panel A presents results for 6-0-6. The unconditional regression (1) shows that loadings are uniformly larger for L than W (1.21 vs. 1.00 for MKT, 0.51 vs $-0.06$ for HML, and 1.52 vs. 0.88 for SMB). The standard instruments (2) appear especially useful for predicting HML loadings. The regression $R^2$ improve from 85.8 to 88.5 for W, and from 82.3 to 83.4 for L. The alphas for W and L attenuate toward zero, and the WL alpha falls to 0.93 from the unconditional 1.10.

Instrumenting with the LC loadings (3), both LC6 and LC36 are always highly significant for W with roughly equal weightings for all factors. For L, the weightings are higher on LC36 than LC6, and the latter are insignificant for HML and SMB. Relative to (1) and (2), the $R^2$ improve considerably, increasing to 93.3 and 86.6 for W and L. The alphas further attenuate toward zero for W and L, and the WL alpha is 0.85.

Combining the standard instruments and LC betas (4), the significance of the standard instruments generally moderates for SMB and HML, and is mixed for MKT. The LC betas appear to have more stable coefficients. Relative to (3), the $R^2$ improves marginally for W and is approximately constant for losers. The alphas for W and L attenuate slightly towards zero, and the WL alpha is 0.83.

Regression (4) is a reasonable estimate of the conditional 3-factor performance of the 6-0-6 momentum strategy. It avoids the considerable CP and RD biases inherent in previous methodologies and combines standard instruments with LC6 and LC36 betas. The ad hoc methods explored in Table 12 deviate from the FC1 alphas as expected: CP and RD methods overstate performance, while using LC6 and LC36 as direct risk proxies understates momentum performance.

We could of course incorporate other predictor variables into the analysis. Following our approach in Section 4, in untabulated results we add 6-month and 36-month factor runups to the regressions in Table 13. These additional instruments are somewhat helpful in predicting conditional loadings, but the WL alpha remains in all cases in the range of 0.83 to 0.87. Further, the differences between unconditional alphas and FC1 conditioned alphas are always significant when the LC betas are included as instruments. Panels B and C show similar results for the 6-1-1 and 6-0-6 strategies. We conclude that proper use of conditioning information reduces three-factor momentum performance by a statistically significant 20% to 25%, while overconditioned estimates can overstate performance by more than 2.5 times.

## 7. Conclusion

We study the bias caused by overconditioning, which occurs when an empiricist uses a risk proxy not in the information set of investors. If asset payoffs are nonlinear in factor returns, then overconditioned performance measures are generally biased. In a calibrated dynamic CAPM that matches reasonable levels of beta asymmetry, the overconditioning bias can be as large as 40 basis points per month.

In an empirical application, we show that inferences about momentum performance can be greatly influenced by overconditioning. Simply lagging the contemporaneously calculated realized beta by one month reduces the winner minus loser alpha by up to one percent per month. The purpose of conditioning is to capture predictable covariation between beta and risk premia. Since market returns are forecastable primarily at low frequencies, we should be suspicious of any performance measure that changes dramatically when the risk proxy is lagged by a period as short as one month.

We show that portfolio-weighted lagged risk loadings of individual stocks are valuable instruments in conditional performance analysis. Our lagged component betas account for both portfolio turnover and changing portfolio weights, and are robustly informative whether used in isolation or in combination with standard instruments and lagged portfolio loadings. Momentum alphas from conditional regressions using lagged component betas as instruments decrease by a statistically significant 20-40% relative to unconditional alphas. Overconditioned alphas are as much as 2.5 times larger than the appropriately conditioned measures.

Bias in overconditioned alphas occurs when the market return covaries with the difference between realized beta and forecasted conditional beta. This covariation provides an alternative measure of beta asymmetry to statistics used in the prior literature. For momentum portfolios, the surprise in loser realized beta is almost uncorrelated with market returns, while winner realized beta innovations and the market have strong negative covariation. In this paper, our focus has been to obtain appropriate conditional CAPM and three-factor model alphas in the presence of payoff nonlinearities. A natural next step in performance analysis is to consider settings where investors demand additional risk premia for assets like the winner portfolio that have more positive beta surprises when market returns are low.

# Appendix

## A. Details

### A.1. Section 2 Simple Example

We define four equidistant points centered around $\bar{R}_M$ such that the resulting variance of $R_M$ is equal to $\sigma_M^2$. The distance between any two neighboring points of $R_M$ is denoted by $x$. We want to solve for $\alpha_i^G$ and $\alpha_i^B$ and the returns of asset $i$ in the four states. These six unknowns require six conditions, two of which are provided by the definitions of the conditional betas (2.6). We thus require four additional equations.

The first two equations are the conditional alphas:

$$\begin{aligned} [C1]\ \alpha_i^G &= \mathbb{E}\left(R_i|S=G\right) - \beta_i^G \mathbb{E}\left(R_M|S=G\right) = R_i\left(\omega_3\right) + \frac{\beta_i^G x}{2} - \beta_i^G\left(\bar{R}_M + x\right) \\ [C2]\ \alpha_i^B &= \mathbb{E}\left(R_i|S=B\right) - \beta_i^B \mathbb{E}\left(R_M|S=B\right) = R_i\left(\omega_1\right) + \frac{\beta_i^B x}{2} - \beta_i^B\left(\bar{R}_M - x\right) \end{aligned}$$

Further, the unconditional CAPM must hold

$$[C3]\ \bar{R}_i = \beta_i \bar{R}_M.$$

We use the fact that the unconditional beta can be written as

$$\beta_i = \frac{Cov\left(R_i, R_M\right)}{Var\left(R_M\right)} = \frac{Cov\left(R_i, R_M\right)}{1.25x^2},$$

and

$$Cov\left(R_i, R_M\right) = \frac{1}{4}\sum_{j=1}^{4}\left(R_i\left(\omega_j\right) - \bar{R}_i\right)\left(\frac{-5+2j}{2}x\right),$$

which leads to

$$\beta_i = \frac{1}{5x}\left[2(R_i\left(\omega_3\right) - R_i\left(\omega_1\right)) + \frac{3}{2}\beta_i^G x - \frac{1}{2}\beta_i^B x\right]. \tag{A.1}$$

The CAPM can then be rewritten

$$\begin{aligned} & R_i\left(\omega_1\right)\left(\frac{1}{2} + \frac{2}{5x}\bar{R}_M\right) + R_i\left(\omega_3\right)\left(\frac{1}{2} - \frac{2}{5x}\bar{R}_M\right) \\ = \; & \beta_i^G x\left(\frac{3}{10x}\bar{R}_M - \frac{1}{4}\right) - \beta_i^B x\left(\frac{1}{10x}\bar{R}_M + \frac{1}{4}\right). \end{aligned}$$

In addition, we impose one of the following two restrictions:

$$\begin{aligned} [C4a]\ \alpha_i^G &= \alpha_i^B \\ [C4b]\ \beta_i &= \frac{\beta_i^B + \beta_i^G}{2} \end{aligned}$$

Under $C4a$ the conditional regression lines intersect at the at the $y$-axis. To impose $C4b$, we use the formula for unconditional beta (A.1), and the conditional regression lines intersect at $\bar{R}_M$. The example in Figure 1 imposes $C4b$, and implies that returns satisfy $R_M\left(\Omega\right) \equiv [-0.057, -0.012, 0.032, 0.077]$

and $R_i(\Omega) = [-0.068, -0.001, 0.044, 0.066]$.

### A.2. A GMM Test of the Difference in Alphas

We compare the alphas of a long-short position in portfolios $i = 1, 2$ under two different performance specifications $j = 1, 2$. Let

$$R_i = \begin{bmatrix} 1_T & \mathbf{X}_{ij} \end{bmatrix} \begin{bmatrix} \alpha_{ij} \\ \beta_{ij} \end{bmatrix} + \varepsilon_{ij},$$

where $1_T$, $R_i$, and $\varepsilon_{ij}$ are column vectors of length $T$, $\alpha_{ij}$ are scalars, $\mathbf{X}_{ij}$ are $T$ by $(k_{ij}-1)$ matrices, and $\beta_{ij}$ are column vectors of size $k_{ij}-1$.

Define the moment conditions

$$g \equiv \mathbb{E} \begin{bmatrix} R_1 - \alpha_{11} - \mathbf{X}_{11}\beta_{11} \\ (R_1 - \alpha_{11} - \mathbf{X}_{11}\beta_{11})' \mathbf{X}_{11} \\ R_2 - \alpha_{21} - \mathbf{X}_{21}\beta_{21} \\ (R_2 - \alpha_{21} - \mathbf{X}_{21}\beta_{21})' \mathbf{X}_{21} \\ R_1 - \alpha_{12} - \mathbf{X}_{12}\beta_{12} \\ (R_1 - \alpha_{12} - \mathbf{X}_{12}\beta_{12})' \mathbf{X}_{12} \\ R_2 - \alpha_{22} - \mathbf{X}_{22}\beta_{22} \\ (R_2 - \alpha_{22} - \mathbf{X}_{22}\beta_{22})' \mathbf{X}_{22} \end{bmatrix},$$

the coefficient vector

$$b \equiv \begin{bmatrix} \alpha_{11} & \beta'_{11} & \alpha_{21} & \beta'_{21} & \alpha_{12} & \beta'_{12} & \alpha_{22} & \beta'_{22} \end{bmatrix}',$$

and the matrix

$$d = \frac{\partial g}{\partial b} = \begin{bmatrix} \mathbf{D}_{11} & \mathbf{0}_{k_{11},k_{21}} & \mathbf{0}_{k_{11},k_{12}} & \mathbf{0}_{k_{11},k_{22}} \\ \mathbf{0}_{k_{21},k_{11}} & \mathbf{D}_{21} & \mathbf{0}_{k_{21},k_{12}} & \mathbf{0}_{k_{21},k_{22}} \\ \mathbf{0}_{k_{12},k_{11}} & \mathbf{0}_{k_{12},k_{21}} & \mathbf{D}_{12} & \mathbf{0}_{k_{12},k_{22}} \\ \mathbf{0}_{k_{22},k_{11}} & \mathbf{0}_{k_{22},k_{21}} & \mathbf{0}_{k_{22},k_{12}} & \mathbf{D}_{22} \end{bmatrix},$$

where $\mathbf{0}_{n1,n2}$ denotes a matrix of zeros of dimensions $n1$ by $n2$, and

$$\mathbf{D}_{ij} = -\begin{bmatrix} 1 & \mathbb{E}(\mathbf{X}_{ij}) \\ \mathbb{E}\left(\mathbf{X}'_{ij}\right) & \mathbb{E}\left(\mathbf{X}'_{ij}\mathbf{X}_{ij}\right) \end{bmatrix}$$

are symmetric squared matrices of size $k_{ij}$.

Using standard GMM results,

$$V \equiv Var\left(\hat{b}\right) = \frac{1}{T} d^{-1} S d^{-1},$$

where

$$S = \sum_{k=-\infty}^{\infty} \mathbb{E}\left(u_t u'_{t-k}\right)$$

and

$$u_t = \begin{bmatrix} \varepsilon_{11t} & \varepsilon_{11t}\mathbf{X}_{11t} & \varepsilon_{21t} & \varepsilon_{21t}\mathbf{X}_{21t} & \varepsilon_{12t} & \varepsilon_{12t}\mathbf{X}_{12t} & \varepsilon_{22t} & \varepsilon_{22t}\mathbf{X}_{22t} \end{bmatrix}'.$$

We estimate $\hat{V} = (1/T)\, d^{-1}\hat{S}d^{-1}$ following Newey and West (1987):

$$\hat{S} = \hat{\Omega}_0 + \sum_{k=1}^{m} \omega\,(k,m)\left(\hat{\Omega}_k + \hat{\Omega}_k'\right),$$

where

$$\hat{\Omega}_k = \frac{1}{T}\sum_{t=k+1}^{T} u_t u_{t-k}'$$

and $\omega\,(k,m)$ are the Bartlett kernel weights

$$\omega\,(k,m) = 1 - \frac{k}{m+1}.$$

Let $\alpha_j \equiv \alpha_{1j} - \alpha_{2j}$. The test statistic $\hat{\alpha}_2 - \hat{\alpha}_1$ is asymptotically normally distributed with a mean of $\alpha_2 - \alpha_1$ and a variance of $c'Vc$, where

$$c \equiv \left[\begin{array}{cccccccc} 1 & 0_{k_{11}-1}' & -1 & 0_{k_{21}-1}' & -1 & 0_{k_{12}-1}' & 1 & 0_{k_{22}-1}' \end{array}\right]'.$$

Applying this methodology to test the difference between conditional and unconditional momentum alphas, we set $R_1 = R_W$, $R_2 = R_L$, $\mathbf{X}_{11} = \mathbf{X}_{21} = R_M$, $\mathbf{X}_{12} = \left(R_M 1_{k_{12}}'\right) * \left[\begin{array}{cc} 1_T & Z_W \end{array}\right]$, and $\mathbf{X}_{22} = \left(R_M 1_{k_{22}}'\right) * \left[\begin{array}{cc} 1_T & Z_L \end{array}\right]$, where $*$ denotes element-by-element multiplication. To test the null hypothesis that the conditional alpha is greater than or equal to the unconditional alpha, we use a one-tailed test. We implement the Newey-West procedure with $m = 5$. Our results are unaffected by other choices of $m \leq 12$.

### A.3. Fama-French Loading Calculations and Performance Measures

To obtain conditional FF performance measures, in each non-overlapping window $\theta$ of length $N \in \{1,3,6\}$ months, we run a Fama-French daily regression using the same structure of Dimson adjustments for each factor $j \in \{MKT, HML, SMB\}$ as previously:

$$R_{it} = \alpha_{i\theta}^{CPRD}/n + \sum_j \left(\beta_{ij1\theta}F_{jt} + \beta_{ij2\theta}F_{j,t-1} + \frac{\beta_{ij3\theta}}{3}\sum_{k=2}^{4} F_{j,t-k}\right) + \varepsilon_{it}. \tag{A.2}$$

Denoting $\beta_{ij\theta}^{CP} \equiv \beta_{ij1\theta} + \beta_{ij2\theta} + \beta_{ij3\theta}$ as the sum beta from this regression, the CPBH alpha is

$$\alpha_{i\theta}^{CPBH} \equiv \frac{1}{N}\left(R_{i\theta} - \sum_j \beta_{ij\theta}^{CP} F_{j\theta}\right). \tag{A.3}$$

The LPRD performance measure is

$$\alpha_{i\theta}^{LPRD} \equiv \frac{1}{N}\sum_{t\in\theta}\left[R_{it} - \sum_j \beta_{ij\theta}^{LP} F_{jt}\right],$$

where $\beta_{ij\theta}^{LP} \equiv \beta_{ij,\theta-1}^{CP}$. The LPBH alpha is

$$\alpha_{i\theta}^{LPBH} \equiv \frac{1}{N}\left(R_{i\theta} - \sum_j \beta_{ij\theta}^{LP} F_{j\theta}\right).$$

As robustness checks, we verify in untabulated results that eliminating the Dimson lags in (A.2) and adding a Dimson lead (to account for asynchronous trading delays in the relatively illiquid long side of HML and SMB) do not substantially alter the results reported in Tables 12 and 13.

# B. Proofs

## B.1. Proof of Proposition 1

Note that $\beta_{it}^{t-1} = \mathbb{E}\left(\hat{\beta}_{it}\middle|\mathcal{F}_{t-1}\right)$. We rewrite the conditional expected return

$$\begin{aligned}
\mathbb{E}\left(R_{it}|\mathcal{F}_{t-1}\right) &= \alpha_{it}^{t-1} + \mathbb{E}\left(\hat{\beta}_{it}\middle|\mathcal{F}_{t-1}\right)\mathbb{E}\left(R_{Mt}|\mathcal{F}_{t-1}\right) \\
&= \alpha_{it}^{t-1} - Cov\left(\hat{\beta}_{it}, R_{Mt}\middle|\mathcal{F}_{t-1}\right) + \mathbb{E}\left(\hat{\beta}_{it}R_{Mt}\middle|\mathcal{F}_{t-1}\right).
\end{aligned}$$

The overconditioned alpha bias is

$$\begin{aligned}
\bar{\alpha}_i^{OC} - \bar{\alpha}_i &= -\mathbb{E}\left[Cov\left(\hat{\beta}_{it}, R_{Mt}\middle|\mathcal{F}_{t-1}\right)\right] \\
&= -\mathbb{E}\left[Cov\left(\varepsilon_{\beta t}, R_{Mt}|\mathcal{F}_{t-1}\right)\right] \\
&= -\mathbb{E}\left(\varepsilon_{\beta t}R_{Mt}\right) = -Cov\left(\varepsilon_{\beta t}, R_{Mt}\right).
\end{aligned}$$

## B.2. Proof of Proposition 2

Recall the definition $\alpha^s \equiv \mathbb{E}\left(R_i|s\right) - \beta^s\mathbb{E}\left(R_M|s\right)$. Taking expectations and using the CAPM yields:

$$\begin{aligned}
\mathbb{E}\left(\alpha_i^S\right) &\equiv \bar{R}_i - \mathbb{E}\left(\beta_i^S\right)\bar{R}_M - Cov\left[\beta_i^S, \mathbb{E}\left(R_M|S\right)\right] \\
&= \left[\beta_i - \mathbb{E}\left(\beta_i^S\right)\right]\bar{R}_M - Cov\left[\beta_i^S, \mathbb{E}\left(R_M|S\right)\right]
\end{aligned}$$

The remainder of the proof requires two technical assumptions. First, expected returns on $R_i$ must be continuous in the realized return on $R_M$, to avoid the special case where the conditional regression lines are parallel, i.e., $\beta_i^G = \beta_i^B$ with different conditional alphas. We further rule out the situation where $S$ is uncorrelated with $R_M$ and thus contains no relevant information. Ruling out these special cases, it directly follows that $\alpha_i^G = \alpha_i^B = 0$ if and only if $\beta_i^G = \beta_i^B$.

## B.3. Proof of Proposition 3

The market variance conditional on $Z$ is

$$Var(R_M|Z=z) = \sigma_M^2 + \left(\Delta_M^S\right)^2.$$

Express the portfolio return as a function of the market return:

$$R_i^{zs} = \alpha_i^{zs} + \beta_i^{zs}\left(\bar{R}_M^{zs} + \varepsilon_M\right) + \varepsilon_i. \tag{B.1}$$

We compute the covariance conditional on $Z$:

$$Cov\left(R_i, R_M | Z = z\right) = \beta_i^z \left[\sigma_M^2 + \left(\Delta_M^S\right)^2\right] + \frac{\Delta_M^S}{2}\left(\alpha_i^{zG} - \alpha_i^{zB} - 2\bar{R}_M^z \Delta_\beta^S\right),$$

which implies

$$\beta_i^z = \frac{Cov(R_i, R_M | Z = z)}{Var(R_M | Z = z)} = \beta_i^z + \frac{\frac{\Delta_M^S}{2}\left(\alpha_i^{zG} - \alpha_i^{zB} - 2\bar{R}_M^z \Delta_\beta^S\right)}{\sigma_M^2 + \left(\Delta_M^S\right)^2}.$$

For $\Delta_M^S \neq 0$, the state-contingent alphas must satisfy

$$\alpha_i^{zG} - \alpha_i^{zB} = 2\bar{R}_M^z \Delta_\beta^S. \tag{B.2}$$

Imposing the CAPM further restricts alphas since

$$\mathbb{E}\left(R_i | Z = z\right) = \beta_i^z \bar{R}_M^z + \frac{1}{2}\left(\alpha_i^{zG} + \alpha_i^{zB} - 2\Delta_M^S \Delta_\beta^S\right),$$

implying that

$$\mathbb{E}\left(\alpha_i | Z = z\right) = \frac{\alpha_i^{zG} + \alpha_i^{zB}}{2} = \Delta_M^S \Delta_\beta^S. \tag{B.3}$$

This completes the proof as we know the difference between overconditioned alphas (B.2) and the average alpha (B.3).

### B.4. Proof of Proposition 4

To derive analytical expressions for $\beta^-$and $\beta^+$ from the Markov Model in Section 3, we repeatedly use the following results from the normal distribution:

$$\begin{aligned}
\mathbb{E}\left(X | X < \alpha\right) &= \mu - \sigma \frac{\phi(\alpha)}{\Phi(\alpha)}, \\
Var\left(X | X < \alpha\right) &= \sigma^2 \left[1 - \alpha \frac{\phi(\alpha)}{\Phi(\alpha)} - \left(\frac{\phi(\alpha)}{\Phi(\alpha)}\right)^2\right].
\end{aligned}$$

We further recall the well known properties $\phi(-\alpha) = \phi(\alpha)$ and $\Phi(-\alpha) = 1 - \Phi(\alpha)$, and also note that for any random variables $X_1, X_2, Y$, and $S$

$$\begin{aligned}
\mathbb{E}\left(X_1 | Y\right) &= \mathbb{E}\left[\mathbb{E}\left(X_1 | Y, S\right) | Y\right] \\
Var\left(X_1 | Y\right) &= \mathbb{E}\left[Var\left(X_1 | Y, S\right) | Y\right] + Var\left[\mathbb{E}\left(X_1 | Y, S\right) | Y\right] \\
Cov\left(X_1, X_2 | Y\right) &= \mathbb{E}\left[Cov\left(X_1, X_2 | Y, S\right) | Y\right] + Cov\left[\mathbb{E}\left(X_1 | Y, S\right), \mathbb{E}\left(X_2 | Y, S\right) | Y\right].
\end{aligned}$$

To simplify notation, we suppress the argument $k$ when using the c.d.f. and p.d.f., i.e., $\Phi = \Phi(k)$ and $\phi = \phi(k)$. The Proposition assumes no relevant $Z$-information, and we hence omit all $Z$ subscripts. Let $R^-$ denote the event that $R^M < \bar{R}_M$.

A simple application of Bayes' Theorem shows that

$$\begin{aligned}
\mathbb{P}\left(S = B | R^-\right) &= \Phi \\
\mathbb{P}\left(S = G | R^-\right) &= 1 - \Phi.
\end{aligned}$$

We compute the expected values

$$\mathbb{E}\left(R^M\middle|\,R^-\right) = \bar{R}_M - k\sigma\left(2\Phi - 1\right) - 2\sigma\phi.$$

Using (B.1) and (B.2) from the proof of Proposition 3, we obtain

$$\mathbb{E}\left(R^i\middle|\,R^-\right) = \beta\mathbb{E}\left(R^M\middle|\,R^-\right) + \alpha^G - \Delta_\beta\left(\bar{R}_M + k\sigma\right).$$

The components of the variance are

$$\begin{aligned}
\mathbb{E}\left[Var\left(R^M\middle|\,R^-,\ S\right)\middle|\,R^-\right] &= \sigma^2\left[1 - \phi^2\left(\frac{1}{\Phi - \Phi^2}\right)\right] \\
Var\left[\mathbb{E}\left(R^M\middle|\,R^-,\ S\right)\middle|\,R^-\right] &= \sigma^2\left[\phi^2\left(\frac{1}{\Phi - \Phi^2} - 4\right) + 4k^2\left(\Phi - \Phi^2\right) - 4k\phi\left(2\Phi - 1\right)\right]
\end{aligned}$$

Combining these yields the conditional market variance

$$Var\left(R^M\middle|\,R^-\right) = \sigma^2\left[1 + k^2 - \left(2\phi + k\left(2\Phi - 1\right)\right)^2\right].$$

To compute the covariance, we again use (B.1) and (B.2) to obtain

$$Cov\left(R^i, R^M\middle|\,R^-\right) = \beta Var\left(R^M\middle|\,R^-\right) + \Delta_\beta\sigma^2\left(2\Phi - 1\right).$$

This yields the down beta

$$\beta^- = \beta + \Delta_\beta\frac{2\Phi\left(k\right) - 1}{1 + k^2 - \left(2\phi\left(k\right) + k\left(2\Phi\left(k\right) - 1\right)\right)^2}.$$

The derivation for $\beta^+$ follows the same steps.

### B.5. Proof of Proposition 5

We first observe that $\bar{R}^{gRD}_{iN} = 1 + nN\left(e^{\mu_i + \sigma_i^2/2} - 1\right) \approx e^{nN\left(\mu_i + \sigma_i^2/2\right)}$ where the approximation is most accurate for expected daily returns near one. The RD measure depends only on the first and second moments of daily returns. By contrast, buy-and-hold returns over a monthly horizon depend on how daily returns aggregate. In particular, by assumption of joint normality of daily log returns, the monthly log returns are also normally distributed, i.e., $r_{i1} + \cdots + r_{i,nN} \sim \mathcal{N}\left(nN\mu_i, \sigma^2_{iN}\right)$. As a consequence, the BH statistic is $\bar{R}^{gBH}_{iN} = e^{nN\mu_i + \sigma^2_{iN}/2}$. The ratio of the two statistics is

$$\begin{aligned}
\nu_{iN} &\equiv \frac{\bar{R}^{gRD}_{iN}}{\bar{R}^{gBH}_{iN}} \approx e^{\left(nN\sigma_i^2 - \sigma^2_{iN}\right)/2} \\
&= e^{nN\sigma_i^2\left(1 - VR_{iN}\right)/2}.
\end{aligned}$$

The ratio in the net returns is

$$\nu^{net}_{iN} \equiv \frac{\bar{R}^{gRD}_{iN} - 1}{\bar{R}^{gBH}_{iN} - 1} \approx \nu_{iN} + \frac{\nu_{iN} - 1}{\bar{R}^{gBH}_{iN} - 1}.$$

### B.6. Proof of Proposition 6

Applying the definitions of the alphas from equations (3.16) and (5.4) leads immediately to the decomposition.

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## TABLE 1. – Markov Model Unconditional CAPM: State Dependent Betas and Overconditioned Alphas

| $\Delta_M^S$ | Daily | | | | Monthly | | BH (%/m) | | | | RD (%/m) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (%/d) | $\beta^-$ | $\beta^+$ | $P_G^-$ | $P_G^+$ | $\beta^-$ | $\beta^+$ | $\alpha_1^{UC}$ | $\bar{\alpha}_1^{CP}$ | $\bar{\alpha}_3^{CP}$ | $\bar{\alpha}_6^{CP}$ | $\alpha_1^{UC}$ | $\bar{\alpha}_1^{CP}$ | $\bar{\alpha}_3^{CP}$ | $\bar{\alpha}_6^{CP}$ |
| 0.00 | 1.00 | 1.00 | 0.50 | 0.50 | 1.00 | 1.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 |
| 0.10 | 1.11 | 0.89 | 0.46 | 0.54 | 1.03 | 0.98 | -0.00 | 0.10 | 0.03 | 0.02 | -0.00 | 0.10 | 0.03 | 0.02 |
| 0.20 | 1.21 | 0.79 | 0.42 | 0.58 | 1.05 | 0.96 | -0.00 | 0.19 | 0.06 | 0.03 | -0.00 | 0.19 | 0.06 | 0.03 |
| 0.30 | 1.30 | 0.70 | 0.38 | 0.62 | 1.06 | 0.94 | 0.00 | 0.28 | 0.10 | 0.05 | 0.00 | 0.28 | 0.09 | 0.05 |
| 0.40 | 1.37 | 0.63 | 0.35 | 0.65 | 1.08 | 0.93 | -0.00 | 0.35 | 0.12 | 0.06 | -0.00 | 0.35 | 0.11 | 0.06 |
| 0.50 | 1.42 | 0.58 | 0.31 | 0.69 | 1.09 | 0.91 | 0.00 | 0.42 | 0.14 | 0.07 | 0.00 | 0.42 | 0.14 | 0.07 |
| 0.75 | 1.51 | 0.49 | 0.23 | 0.77 | 1.09 | 0.91 | 0.00 | 0.51 | 0.17 | 0.09 | 0.00 | 0.51 | 0.16 | 0.08 |
| 1.00 | 1.53 | 0.47 | 0.16 | 0.84 | 1.09 | 0.92 | -0.00 | 0.54 | 0.17 | 0.09 | -0.00 | 0.54 | 0.17 | 0.08 |
| 2.00 | 1.50 | 0.50 | 0.04 | 0.96 | 1.05 | 0.96 | 0.00 | 0.45 | 0.14 | 0.07 | 0.00 | 0.45 | 0.14 | 0.07 |
| -0.10 | 0.89 | 1.11 | 0.54 | 0.46 | 0.98 | 1.02 | -0.00 | -0.10 | -0.04 | -0.02 | -0.00 | -0.10 | -0.04 | -0.02 |
| -0.50 | 0.58 | 1.42 | 0.69 | 0.31 | 0.91 | 1.08 | 0.01 | -0.41 | -0.13 | -0.07 | 0.01 | -0.41 | -0.13 | -0.06 |

*Notes:* This table shows the effects of changes in $\Delta_M^S$ in the Markov model where $\Delta_M^Z = 0$, $\Delta_\beta^Z = 0$, and $\Delta_\beta^S = 0.5$. The values of $\beta^-$, $\beta^+$, $P_G^-$ and $P_G^+$ in daily data are calculated analytically using Proposition 4, where $\beta^-$ and $\beta^+$ are down and up betas, and $P_G^-$ and $P_G^+$ are respectively the probabilities of $S_t = G$ conditional on market returns being below or above the annual mean. The remaining statistics are obtained by simulating $10^8$ months of 21 daily returns. The monthly down and up betas $\beta^-$ and $\beta^+$ are market model loadings from subsamples of months where the market return is below or above its full sample mean. The buy-and-hold (BH) and rescaled daily (RD) alphas are reported in percent per month. Risk adjustment is done either unconditionally (UC) or using the contemporaneous portfolio method (CP) using non-overlapping windows of length $N = 1$, 3, or 6 months. Alphas and $\Delta_M^S$ are in percent.

## TABLE 2. – Markov Model Conditional CAPM: The Overconditioning Vs. Underconditioning Tradeoff

| Parameters | | | | | Alphas (%/m) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta_M^Z$ (%/d) | $\Delta_\beta^Z$ | $\Delta_M^S$ (%/d) | $\Delta_\beta^S$ | | $\rho = 0.5$ $N=1$ | $\rho = 0.5$ $N=3$ | $\rho = 0.5$ $N=6$ | $\rho = 0.75$ $N=1$ | $\rho = 0.75$ $N=3$ | $\rho = 0.75$ $N=6$ | $\rho = 0.9$ $N=1$ | $\rho = 0.9$ $N=3$ | $\rho = 0.9$ $N=6$ |
| 0.00 | 0.00 | 0.50 | 0.50 | UC | -0.00 | | | -0.00 | | | -0.00 | | |
| | | | | CP | 0.42 | 0.14 | 0.07 | 0.41 | 0.14 | 0.07 | 0.41 | 0.14 | 0.07 |
| | | | | LP | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 | -0.00 |
| | | | | Z | -0.00 | | | -0.00 | | | -0.00 | | |
| 0.10 | 0.20 | 0.00 | 0.00 | UC | 0.41 | | | 0.41 | | | 0.42 | | |
| | | | | CP | 0.00 | 0.29 | 0.38 | 0.00 | 0.18 | 0.29 | 0.01 | 0.08 | 0.16 |
| | | | | LP | 0.43 | 0.44 | 0.46 | 0.22 | 0.37 | 0.44 | 0.09 | 0.22 | 0.33 |
| | | | | Z | 0.00 | | | 0.00 | | | 0.00 | | |
| 0.10 | 0.20 | 0.50 | 0.50 | UC | 0.41 | | | 0.41 | | | 0.41 | | |
| | | | | CP | 0.42 | 0.43 | 0.45 | 0.42 | 0.31 | 0.35 | 0.42 | 0.22 | 0.23 |
| | | | | LP | 0.43 | 0.44 | 0.46 | 0.21 | 0.37 | 0.44 | 0.09 | 0.21 | 0.33 |
| | | | | Z | 0.00 | | | 0.00 | | | 0.00 | | |
| 0.20 | 0.20 | 0.50 | 0.50 | UC | 0.83 | | | 0.84 | | | 0.83 | | |
| | | | | CP | 0.42 | 0.72 | 0.84 | 0.43 | 0.50 | 0.66 | 0.42 | 0.31 | 0.42 |
| | | | | LP | 0.86 | 0.88 | 0.92 | 0.44 | 0.75 | 0.90 | 0.18 | 0.44 | 0.69 |
| | | | | Z | 0.00 | | | 0.00 | | | 0.00 | | |
| 0.05 | 0.20 | 0.50 | 0.50 | UC | 0.21 | | | 0.21 | | | 0.21 | | |
| | | | | CP | 0.42 | 0.28 | 0.26 | 0.42 | 0.23 | 0.22 | 0.42 | 0.18 | 0.15 |
| | | | | LP | 0.21 | 0.22 | 0.22 | 0.11 | 0.19 | 0.22 | 0.05 | 0.11 | 0.17 |
| | | | | Z | 0.00 | | | 0.00 | | | 0.00 | | |

*Notes:* We simulate$10^8$ months of 21 daily returns using the Markov model with parameters $\Delta_M^Z$, $\Delta_\beta^Z$, $\Delta_M^S$, $\Delta_\beta^S$, and $\rho$ taking the values in the table headings. We calculate buy-and-hold alphas rescaled to monthly equivalents for data windows of $N = 1, 3$, or 6 months, and the performance measures UC (unconditional), CP (contemporaneous portfolio), LP (lagged portfolio), or Z (investor conditioned).

## TABLE 3. – MOMENTUM PROFITS

| | Momentum Raw Profits | | | | | | | | | Market | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 6-0-6 | | | 6-1-1 | | | 6-1-6 | | | | |
| | WL | W | L | WL | W | L | WL | W | L | Excess | Raw |
| | A. Mean Returns | | | | | | | | | | |
| daily | 0.84 | 1.49 | 0.65 | 0.64 | 1.56 | 0.92 | 1.18 | 1.60 | 0.43 | 0.59 | 0.89 |
| 1 month | 0.54 | 1.61 | 1.07 | 0.24 | 1.68 | 1.44 | 0.91 | 1.73 | 0.83 | 0.62 | 0.93 |
| 3 month | 0.52 | 1.70 | 1.18 | 0.18 | 1.77 | 1.59 | 0.91 | 1.84 | 0.93 | 0.64 | 0.95 |
| 6 month | 0.64 | 1.73 | 1.10 | 0.26 | 1.80 | 1.53 | 1.04 | 1.88 | 0.84 | 0.62 | 0.94 |
| | B. Standard Deviations | | | | | | | | | | |
| daily | 3.96 | 5.65 | 6.21 | 5.02 | 5.81 | 6.56 | 3.83 | 5.69 | 6.15 | 4.77 | 4.77 |
| monthly | 7.13 | 7.52 | 11.20 | 8.35 | 7.58 | 12.00 | 6.75 | 7.67 | 10.97 | 5.46 | 5.45 |

*Notes:* This table reports means and standard deviations, in percent per month, of returns for momentum portfolios and the market over the sample period from January 1930 to December 2005. We consider three momentum strategies, denoted 6-d-h, with common 6 month formation periods but different delays $d$ and holding periods $h$. At the beginning of calendar month $\tau$, stocks are sorted into deciles based on their return over the formation period $\tau-d-6$ to $\tau-d-1$. To be included in the sort, stocks must have (i) valid monthly returns on the CRSP database over the entire formation period, (ii) at least 12 additional valid monthly returns in the thirty months prior to formation, (iii) at least 15 non-missing daily returns in each month of the formation period. Immediately following the sort, the winner portfolio (W) makes a fixed \$1 investment with equal weights in the top decile stocks, and sells stocks that were added to the portfolio at the beginning of month $\tau - h$. The loser portfolio (L) is defined by similarly timed investments and liquidations in the bottom decile stocks. Momentum (WL) profits are the difference between W and L returns. Market return is the CRSP value-weighted index. Daily returns are computed as average daily returns scaled by average number of days $n$ in one month. To obtain 1-, 3-, and 6-month measures, returns from the monthly CRSP file are compounded in overlapping windows of $N = 1, 3, 6$ months and divided by $N$. Panel B reports monthly standard deviations obtained from daily and monthly returns. The daily standard deviation is multiplied by $\sqrt{n}$.

Table 4. – The RD Bias in Market Index and Style Portfolios

| | Market Indices | | | | Size & B/M | | | | B/M | | Size | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | NYS | NDQ | NXN | SP | SV | SG | LV | LG | G | V | S | L |
| | | | | | A. Value-Weighted Portfolios | | | | | | | |
| BH | 0.96 | 1.05 | 0.95 | 0.97 | 1.60 | 0.68 | 1.07 | 0.87 | 0.82 | 1.37 | 1.21 | 0.88 |
| RD | 0.93 | 0.99 | 0.92 | 0.95 | 1.48 | 0.49 | 1.06 | 0.88 | 0.81 | 1.31 | 1.08 | 0.89 |
| RD/BH | 0.97 | 0.94 | 0.96 | 0.98 | 0.93 | 0.72 | 0.99 | 1.00 | 0.99 | 0.96 | 0.89 | 1.01 |
| $\sigma_i\sqrt{n}$ | 4.77 | 5.57 | 4.81 | 5.27 | 3.28 | 5.05 | 4.51 | 4.83 | 4.95 | 4.23 | 3.41 | 4.39 |
| $\sigma_{i1}$ | 5.32 | 6.53 | 5.42 | 5.55 | 5.68 | 8.03 | 4.69 | 4.69 | 5.12 | 5.22 | 6.15 | 4.18 |
| $VR_{i1}$ | 1.24 | 1.38 | 1.27 | 1.11 | 3.00 | 2.54 | 1.08 | 0.94 | 1.07 | 1.53 | 3.25 | 0.91 |
| $\nu_{i1}^{net}$ | 0.97 | 0.94 | 0.97 | 0.98 | 0.93 | 0.71 | 0.99 | 1.01 | 0.99 | 0.97 | 0.89 | 1.01 |
| $\rho_1$ | 9.90 | 9.70 | 10.03 | 5.26 | 31.51 | 28.38 | 8.59 | 8.36 | 11.38 | 16.13 | 36.00 | 5.97 |
| $\rho_2$ | -3.97 | -2.00 | -3.85 | -3.75 | 12.46 | 7.21 | -2.06 | -2.55 | -2.10 | 1.00 | 12.39 | -3.21 |
| $\rho_5$ | 1.38 | 0.13 | 1.36 | 0.75 | 11.08 | 8.21 | 0.90 | -0.94 | -0.59 | 3.28 | 12.95 | -0.56 |
| $\rho_{10}$ | 1.57 | 0.85 | 1.58 | 1.65 | 5.31 | 3.16 | 0.42 | -0.75 | -0.21 | 1.91 | 6.19 | -0.48 |
| $\rho_{15}$ | -0.34 | 0.78 | -0.27 | -0.37 | 5.86 | 5.48 | -0.11 | -0.10 | 0.28 | 1.64 | 7.54 | -0.26 |
| $\rho_{20}$ | 1.22 | 0.44 | 1.26 | 1.11 | 4.74 | 5.26 | 0.15 | -0.65 | -0.17 | 0.14 | 6.13 | -0.67 |
| | | | | | B. Equal-Weighted Portfolios | | | | | | | |
| BH | 1.84 | 2.23 | 2.12 | 1.32 | 2.92 | 1.77 | 1.19 | 0.95 | 1.20 | 2.90 | 2.50 | 0.94 |
| RD | 1.69 | 2.08 | 1.95 | 1.25 | 2.76 | 1.54 | 1.17 | 0.94 | 1.05 | 2.73 | 2.33 | 0.93 |
| RD/BH | 0.92 | 0.93 | 0.92 | 0.94 | 0.94 | 0.87 | 0.98 | 0.98 | 0.88 | 0.94 | 0.93 | 0.99 |
| $\sigma_i\sqrt{n}$ | 4.89 | 3.47 | 4.83 | 5.72 | 2.88 | 4.50 | 4.29 | 4.93 | 4.83 | 3.03 | 3.08 | 4.41 |
| $\sigma_{i1}$ | 6.87 | 6.22 | 7.14 | 6.73 | 5.81 | 7.97 | 4.62 | 5.15 | 7.24 | 5.81 | 6.23 | 4.48 |
| $VR_{i1}$ | 1.97 | 3.22 | 2.18 | 1.38 | 4.08 | 3.13 | 1.16 | 1.09 | 2.25 | 3.69 | 4.07 | 1.03 |
| $\nu_{i1}^{net}$ | 0.94 | 0.94 | 0.93 | 0.95 | 0.96 | 0.88 | 0.99 | 0.99 | 0.88 | 0.96 | 0.94 | 1.00 |
| $\rho_1$ | 23.38 | 31.02 | 24.75 | 10.31 | 39.02 | 34.39 | 11.40 | 13.20 | 26.40 | 35.67 | 42.55 | 10.40 |
| $\rho_2$ | -1.11 | 12.47 | 0.13 | -2.70 | 20.31 | 12.25 | -0.84 | -3.28 | 4.70 | 17.55 | 19.22 | -3.36 |
| $\rho_5$ | 5.03 | 12.15 | 6.24 | 1.78 | 16.82 | 12.93 | -0.92 | -0.88 | 6.14 | 14.78 | 18.36 | -0.37 |
| $\rho_{10}$ | 2.77 | 6.64 | 3.59 | 1.82 | 9.51 | 6.17 | 0.89 | -0.88 | 1.88 | 8.26 | 9.60 | -0.53 |
| $\rho_{15}$ | 1.00 | 6.63 | 1.59 | -0.02 | 9.42 | 7.00 | 0.26 | -0.09 | 3.86 | 8.43 | 10.06 | -0.05 |
| $\rho_{20}$ | 1.84 | 4.59 | 2.47 | 1.13 | 6.73 | 6.13 | 0.23 | -1.37 | 3.01 | 5.60 | 7.73 | -1.35 |

*Notes:* This table reports the return characteristics and RD bias of the value-weighted (Panel A) and equal-weighted (Panel B) market indexes and style portfolios. Returns, standard deviations, and autocorrelations are in percent. Market indices are NYSE (NYS), NASDAQ (NDQ), combined NYSE, AMEX, and NASDAQ (NXN), and SP500 (SP). The style portfolios are obtained from the Kenneth French data library and include small value (SV), small growth (SG), large value (LV), large growth (LG), growth (G), value (V), small (S), and large (L). BH denotes the monthly average buy-and-hold return computed by compounding daily returns, and RD denotes the average daily return multiplied by $n$, the average number of trading days in one month. The ratio RD/BH is the actual RD bias. $\sigma_i\sqrt{n}$ and $\sigma_{i1}$ are the standard deviations of log daily and log monthly returns. Variance ratios $VR_{i1}$ are calculated as $\sigma_{i1}^2/(\sigma_i^2 n)$. The approximate RD bias is given by $\nu_{i1}^{net}$ calculated from Proposition 5, and $\rho_k$ are autocorrelations of daily log returns at lag $k$. The sample period is January 1926 to December 2006 for market indices, and July 1963 to December 2006 for style portfolios.

Table 5. – THE RD BIAS IN MOMENTUM PORTFOLIOS

| | 6-0-6 | | 6-1-1 | | 6-1-6 | |
|---|---|---|---|---|---|---|
| | W | L | W | L | W | L |
| BH | 1.62 | 1.02 | 1.69 | 1.34 | 1.74 | 0.78 |
| RD | 1.49 | 0.65 | 1.56 | 0.92 | 1.60 | 0.43 |
| RD/BH | 0.92 | 0.64 | 0.92 | 0.69 | 0.92 | 0.55 |
| | | | | | | |
| $\sigma_i\sqrt{n}$ | 5.68 | 6.18 | 5.84 | 6.51 | 5.71 | 6.12 |
| $\sigma_{i1}$ | 7.53 | 10.32 | 7.62 | 10.88 | 7.61 | 10.19 |
| $VR_{i1}$ | 1.76 | 2.79 | 1.70 | 2.79 | 1.78 | 2.77 |
| $\nu_{i1}^{net}$ | 0.92 | 0.66 | 0.93 | 0.71 | 0.93 | 0.57 |
| | | | | | | |
| $\rho_1$ | 18.73 | 28.21 | 17.33 | 26.46 | 19.15 | 27.50 |
| $\rho_2$ | -1.20 | 8.53 | -0.72 | 9.32 | -1.53 | 8.29 |
| $\rho_5$ | 2.42 | 10.01 | 1.71 | 10.33 | 2.60 | 10.41 |
| $\rho_{10}$ | 2.04 | 4.44 | 2.00 | 4.66 | 1.93 | 4.55 |
| $\rho_{15}$ | 2.20 | 4.04 | 1.58 | 3.96 | 2.20 | 4.06 |
| $\rho_{20}$ | 1.76 | 1.12 | 2.01 | -0.28 | 1.33 | 1.64 |

*Notes:* This table reports the return characteristics and RD bias of momentum portfolios. Returns, standard deviations, and autocorrelations are in percent. BH denotes the monthly average buy-and-hold return computed by compounding daily returns, and RD denotes the average daily return multiplied by $n$, the average number of trading days in one month. The ratio RD/BH is the actual RD bias. $\sigma_i\sqrt{n}$ and $\sigma_{i1}$ are the standard deviations of log daily and log monthly returns. Variance ratios $VR_{i1}$ are calculated as $\sigma_{i1}^2/(\sigma_i^2 n)$. The approximate RD bias is given by $\nu_{i1}^{net}$ calculated from Proposition 5, and $\rho_k$ are autocorrelations of daily log returns at lag $k$. The sample period is January 1930 to December 2005.

## TABLE 6. – Momentum Betas

| | 6-0-6 | | | 6-1-1 | | | 6-1-6 | | |
|---|---|---|---|---|---|---|---|---|---|
| | WL | W | L | WL | W | L | WL | W | L |
| | | | | A. Unconditional Betas | | | | | |
| daily | 0.01 | 0.99 | 0.99 | 0.00 | 0.99 | 0.98 | 0.02 | 1.01 | 0.98 |
| daily sum | -0.22 | 1.16 | 1.38 | -0.26 | 1.15 | 1.41 | -0.19 | 1.17 | 1.36 |
| 1 month | -0.43 | 1.17 | 1.61 | -0.52 | 1.16 | 1.67 | -0.37 | 1.20 | 1.57 |
| 3 month | -0.76 | 1.28 | 2.04 | -1.10 | 1.21 | 2.31 | -0.61 | 1.35 | 1.95 |
| 6 month | -0.51 | 1.37 | 1.88 | -0.91 | 1.27 | 2.19 | -0.31 | 1.46 | 1.77 |
| | | | | B. Average CP Betas | | | | | |
| 1 month | -0.01 | 1.14 | 1.16 | -0.05 | 1.14 | 1.19 | 0.00 | 1.15 | 1.14 |
| 3 month | -0.04 | 1.18 | 1.22 | -0.06 | 1.18 | 1.24 | -0.02 | 1.19 | 1.21 |
| 6 month | -0.03 | 1.21 | 1.24 | -0.06 | 1.21 | 1.27 | -0.01 | 1.22 | 1.23 |
| | | | | C. Beta Asymmetry, Daily | | | | | |
| $\beta^-$ | 0.27 | 1.32 | 1.05 | 0.29 | 1.32 | 1.03 | 0.25 | 1.31 | 1.06 |
| $\beta^+$ | 0.06 | 0.96 | 0.90 | 0.06 | 0.97 | 0.91 | 0.09 | 0.97 | 0.89 |
| $\beta^- - \beta^+$ | 0.21 | 0.36 | 0.15 | 0.23 | 0.35 | 0.12 | 0.16 | 0.34 | 0.17 |
| | | | | D. Beta Asymmetry, Daily Sum | | | | | |
| $\beta^-$ | 0.20 | 1.51 | 1.31 | 0.23 | 1.52 | 1.29 | 0.19 | 1.50 | 1.31 |
| $\beta^+$ | -0.29 | 0.89 | 1.19 | -0.38 | 0.88 | 1.26 | -0.22 | 0.93 | 1.15 |
| $\beta^- - \beta^+$ | 0.49 | 0.61 | 0.12 | 0.60 | 0.64 | 0.04 | 0.41 | 0.57 | 0.16 |
| | | | | E. Beta Asymmetry, Monthly | | | | | |
| $\beta^-$ | -0.15 | 1.25 | 1.41 | -0.15 | 1.27 | 1.41 | -0.16 | 1.25 | 1.41 |
| $\beta^+$ | -1.07 | 0.94 | 2.01 | -1.32 | 0.87 | 2.19 | -0.90 | 1.02 | 1.92 |
| $\beta^- - \beta^+$ | 0.91 | 0.31 | -0.60 | 1.18 | 0.40 | -0.78 | 0.74 | 0.23 | -0.51 |

*Notes:* This table reports unconditional betas, contemporaneous portfolio betas, and beta asymmetry measures for momentum portfolios. In Panel A, daily unconditional betas are the slope coefficients from market model regressions on daily data. Daily sum betas are computed as the sum of the slope coefficients from regressing daily portfolio excess returns on excess market, its lag, and the average of lags 2 through 4 of excess market returns. $N$-month unconditional betas are from market model regressions using $N$-month returns. In Panel B, market model regressions are run on daily returns in nonoverlapping $N$-month windows, and the loadings in each window are averaged. In Panel C, $\beta^-$ and $\beta^+$ are market model loadings using subsamples of days within a year where excess market returns are respectively below and above the annual mean. The statistics $\beta^-$ and $\beta^+$ are calculated in every calendar year using daily data and then averaged. In Panel D, the regressions use the excess market return, its lag, and the average of lags 2 through 4 of excess market returns as regressors. The down and up betas $\beta^-$ and $\beta^+$ are the sum of the slope coefficients from these regressions, calculated in each year, and averaged across years. The monthly down and up betas in Panel E are calculated using monthly returns, sorting into subsamples based on whether the excess market return is below or above its full sample mean. The sample period is from January 1930 until December 2005. After 1963, we use the daily risk-free rate from Ken French's website. Prior to 1963, we use the monthly risk-free rate from his website to impute a daily equivalent.

## TABLE 7. – Momentum Portfolio CAPM Alphas and LC Betas

| | | | | Conditional Models | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Contemporaneous Portfolio (CP) | | | | | | Lagged Portfolio (LP) | | | | | | Lagged Component (LC) | | | | | |
| | (i) UC | | | (ii) RD | | | (iii) BH | | | (iv) RD | | | (v) BH | | | (vi) 6-month betas | | | (vii) 36-month betas | | |
| | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L |
| A. Alphas | | | | | | | | | | | | | | | | | | | | | |
| 6-0-6 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 0.97 | 0.50 | -0.47 | | | | | | | | | | | | | | | | | | |
| 1 month | 0.81 | 0.57 | -0.24 | 1.43 | 1.18 | -0.25 | 1.15 | 1.25 | 0.10 | 0.77 | 0.42 | -0.34 | 0.53 | 0.55 | 0.02 | 0.43 | 0.55 | 0.12 | 0.51 | 0.49 | -0.02 |
| 3 month | 0.89 | 0.56 | -0.33 | 1.16 | 0.77 | -0.39 | 0.92 | 0.87 | -0.05 | 0.80 | 0.47 | -0.33 | 0.56 | 0.60 | 0.04 | | | | | | |
| 6 month | 0.87 | 0.55 | -0.32 | 0.90 | 0.56 | -0.34 | 0.67 | 0.66 | -0.01 | 0.88 | 0.52 | -0.35 | 0.64 | 0.65 | 0.01 | | | | | | |
| 6-1-1 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 0.80 | 0.58 | -0.22 | | | | | | | | | | | | | | | | | | |
| 1 month | 0.57 | 0.66 | 0.09 | 1.43 | 1.29 | -0.13 | 1.08 | 1.37 | 0.29 | 0.65 | 0.49 | -0.16 | 0.36 | 0.62 | 0.26 | 0.15 | 0.65 | 0.51 | 0.26 | 0.59 | 0.33 |
| 3 month | 0.72 | 0.67 | -0.05 | 1.01 | 0.87 | -0.14 | 0.73 | 0.97 | 0.24 | 0.61 | 0.53 | -0.08 | 0.32 | 0.66 | 0.33 | | | | | | |
| 6 month | 0.67 | 0.66 | -0.01 | 0.60 | 0.58 | -0.02 | 0.32 | 0.68 | 0.35 | 0.73 | 0.63 | -0.10 | 0.44 | 0.75 | 0.31 | | | | | | |
| 6-1-6 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 1.30 | 0.61 | -0.69 | | | | | | | | | | | | | | | | | | |
| 1 month | 1.14 | 0.68 | -0.46 | 1.69 | 1.26 | -0.43 | 1.45 | 1.35 | -0.10 | 1.08 | 0.54 | -0.54 | 0.87 | 0.67 | -0.19 | 0.81 | 0.68 | -0.12 | 0.87 | 0.61 | -0.26 |
| 3 month | 1.20 | 0.66 | -0.54 | 1.51 | 0.90 | -0.61 | 1.29 | 1.00 | -0.29 | 1.13 | 0.58 | -0.55 | 0.91 | 0.71 | -0.20 | | | | | | |
| 6 month | 1.17 | 0.64 | -0.53 | 1.29 | 0.71 | -0.58 | 1.08 | 0.81 | -0.27 | 1.21 | 0.63 | -0.57 | 0.99 | 0.77 | -0.22 | | | | | | |
| B. LC Betas | | | | | | | | | | | | | | | | | | | | | |
| 6-0-6 Strategy | | | | | | | | | | | | | | | | 0.01 | 1.23 | 1.22 | -0.11 | 1.25 | 1.35 |
| 6-1-1 Strategy | | | | | | | | | | | | | | | | 0.05 | 1.24 | 1.18 | -0.09 | 1.25 | 1.34 |
| 6-1-6 Strategy | | | | | | | | | | | | | | | | 0.00 | 1.23 | 1.23 | -0.10 | 1.26 | 1.35 |

*Notes:* This table reports the unconditional and conditional CAPM alphas, in percent per month, and LP betas of winners (W), losers (L), and winners minus losers (WL) of the three momentum strategies. Unconditional (UC) daily alphas are intercepts from regression $R_{it} = \alpha_i^{UC}/n + \beta_{i0}R_{M,t} + \beta_{i1}R_{M,t-1} + (\beta_{i2}/3)\Sigma_{k=2}^4 R_{M,t-k} + \varepsilon_{it}$, where $R_{it}$, $i \in \{W, L\}$, and $R_{M,t}$ are daily excess portfolio and market returns, and $n$ is the average number of trading days in one month. $N$-month unconditional alphas are taken as intercepts from the regression $R_{i\theta} = N\alpha_i^{UC} + \beta_i R_{M,\theta} + \eta_{i\theta}$, where $R_{i\theta}$ and $R_{M,\theta}$ are $N$-month excess portfolio and market returns. To perform contemporaneous portfolio (CP) and lagged portfolio (LP) risk adjustment, data is partitioned into non-overlapping windows of length $N \in \{1, 3, 6\}$ months indexed by $\theta$, and the following regressions are run: $R_{i\theta} = \alpha_{i\theta}^{CPRD}/n + \beta_{i0\theta}R_{M,t} + \beta_{i1\theta}R_{M,t-1} + (\beta_{i2\theta}/3)\Sigma_{k=2}^4 R_{M,t-k} + \varepsilon_{it}$. CP rescaled daily (RD) alphas are average intercepts from these regressions. CP buy-and-hold (BH) alphas are averages over $\alpha_{i\theta}^{CPBH} = (1/N)(R_{i\theta} - \beta_{i\theta}^{CP}R_{M,\theta})$, where $\beta_{i\theta}^{CP} = \beta_{i0\theta} + \beta_{i1\theta} + \beta_{i2\theta}$. LPRD and LPBH alphas are calculated by averaging over $\alpha_{i\theta}^{LPRD} = (1/N)\Sigma_{t\in\theta}[R_{it} - \beta_{i\theta}^{LP}R_{M,t}]$ and $\alpha_{i\theta}^{LPBH} = (1/N)[R_{i\theta} - \beta_{i\theta}^{LP}R_{M,\theta}]$, where $\beta_{i\theta}^{LP} = \beta_{i,\theta-1}^{CP}$. To estimate the month $\tau$ lagged component (LC) risk exposure, at the end of each calendar month $\tau - 1$, we estimate betas of the individual stocks (components) that will belong to a portfolio in month $\tau$. The component loading estimations use either (i) daily returns from the beginning of $\tau - 6$ to the end of $\tau - 1$ with the Dimson lag structure; or (ii) monthly returns from $\tau - 36$ to $\tau - 1$. To calculate portfolio loadings, we sum over components the product of (1) the component beta and (2) the beginning of month $\tau$ component portfolio weight. For lags $l = 6, 36$, the reported LC alphas are averages over $\alpha_{i\tau}^{LCl} = R_{i\tau} - \beta_{i\tau}^{LCl}R_{M\tau}$. The sample period is from January 1930 until December 2005.

TABLE 8. – Momentum Portfolio Forecast Component Conditioning

| | | FC2 (Two-Step) | | | | | | | | | | | | | | FC1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Stage 1 Beta Regression | | | | | | | | | | Stage 2 Return Regression | | | | | |
| | | $\gamma_0$ | $\beta_\tau^{LC6}$ | $\beta_\tau^{LC36}$ | RU6 | RU36 | DY | TS | TB | DS | $R^2$ | $\alpha^{FC2}$ | $\phi_0$ | $\phi_1$ | $R^2$ | $\alpha^{FC1}$ | $R^2$ |
| (1) | W | 1.14 | | | | | | | | | | 0.57 | | 1.02 | 72.1 | 0.57 | 72.1 |
| | | [60] | | | | | | | | | | [4.3] | | [48] | | [4.3] | |
| | L | 1.16 | | | | | | | | | | -0.24 | | 1.39 | 60.9 | -0.24 | 60.9 |
| | | [50] | | | | | | | | | | [-1.0] | | [38] | | [-1.0] | |
| | WL | | | | | | | | | | | 0.81 | | | | 0.81 | |
| (2) | W | 1.41 | | | | | -4.58 | 1.13 | -23.7 | -1.89 | 2.85 | 0.53 | -0.12 | 1.20 | 73.8 | 0.56 | 74.1 |
| | | [21] | | | | | [-3.4] | [0.7] | [-2.7] | [-0.6] | | [4.1] | [-0.7] | [7.9] | | [4.3] | |
| | L | 1.39 | | | | | -3.14 | -7.93 | -86.9 | 18.2 | 9.73 | -0.17 | 0.51 | 0.79 | 62.7 | -0.14 | 63.3 |
| | | [18] | | | | | [-2.0] | [-4.2] | [-8.4] | [5.3] | | [-0.8] | [3.1] | [6.7] | | [-0.6] | |
| | WL | | | | | | | | | | | 0.70 | | | | 0.69 | |
| (3) | W | 0.20 | 0.76 | | | | | | | | 26.4 | 0.52 | 0.13 | 1.00 | 76.4 | 0.52 | 76.4 |
| | | [3.7] | [18] | | | | | | | | | [4.3] | [1.5] | [13] | | [4.3] | |
| | L | 0.30 | 0.70 | | | | | | | | 23.0 | -0.12 | 0.52 | 0.88 | 62.9 | -0.12 | 62.9 |
| | | [5.5] | [17] | | | | | | | | | [-0.5] | [3.2] | [7.1] | | [-0.5] | |
| | WL | | | | | | | | | | | 0.65* | | | | 0.65* | |
| (4) | W | 0.24 | | 0.73 | | | | | | | 19.2 | 0.49 | -0.05 | 1.14 | 76.0 | 0.49 | 76.0 |
| | | [3.7] | | [15] | | | | | | | | [4.0] | [-0.5] | [12] | | [4.0] | |
| | L | 0.03 | | 0.83 | | | | | | | 17.2 | -0.14 | 0.23 | 1.13 | 63.0 | -0.14 | 63.0 |
| | | [0.3] | | [14] | | | | | | | | [-0.6] | [1.2] | [7.2] | | [-0.6] | |
| | WL | | | | | | | | | | | 0.63* | | | | 0.63* | |
| (5) | W | 0.05 | 0.59 | 0.29 | | | | | | | 28.0 | 0.51 | 0.15 | 0.98 | 76.7 | 0.50 | 76.7 |
| | | [0.9] | [11] | [4.6] | | | | | | | | [4.2] | [1.9] | [14] | | [4.1] | |
| | L | 0.09 | 0.55 | 0.30 | | | | | | | 24.0 | -0.11 | 0.44 | 0.94 | 63.2 | -0.11 | 63.3 |
| | | [1.1] | [9.1] | [3.6] | | | | | | | | [-0.5] | [2.7] | [7.6] | | [-0.5] | |
| | WL | | | | | | | | | | | 0.62* | | | | 0.62* | |
| (6) | W | 0.10 | 0.62 | 0.22 | 0.32 | -0.02 | | | | | 28.3 | 0.50 | 0.21 | 0.95 | 77.1 | 0.48 | 77.2 |
| | | [1.5] | [11] | [3.1] | [2.4] | [-0.5] | | | | | | [4.1] | [2.9] | [14] | | [4.0] | |
| | L | 0.21 | 0.52 | 0.28 | -0.04 | -0.13 | | | | | 24.3 | -0.09 | 0.32 | 1.01 | 63.7 | -0.10 | 64.2 |
| | | [2.1] | [8.4] | [3.3] | [-0.3] | [-2.3] | | | | | | [-0.4] | [2.0] | [8.3] | | [-0.4] | |
| | WL | | | | | | | | | | | 0.59* | | | | 0.58* | |
| (7) | W | 0.18 | 0.57 | 0.29 | | | -2.82 | 1.25 | -2.34 | 0.62 | 28.4 | 0.50 | 0.26 | 0.91 | 77.0 | 0.52 | 77.1 |
| | | [2.0] | [10] | [4.7] | | | [-2.4] | [0.9] | [-0.3] | [0.2] | | [4.2] | [3.8] | [14] | | [4.3] | |
| | L | 0.26 | 0.42 | 0.37 | | | -1.02 | -2.17 | -44.8 | 7.20 | 25.9 | -0.10 | 0.27 | 1.01 | 64.2 | -0.05 | 65.2 |
| | | [2.3] | [6.5] | [4.3] | | | [-0.7] | [-1.2] | [-4.5] | [2.2] | | [-0.4] | [1.8] | [9.2] | | [-0.2] | |
| | WL | | | | | | | | | | | 0.60* | | | | 0.57* | |
| (8) | W | 0.21 | 0.59 | 0.25 | 0.22 | -0.04 | -2.38 | 1.16 | -1.85 | -0.20 | 28.5 | 0.50 | 0.29 | 0.89 | 77.1 | 0.51 | 77.4 |
| | | [2.2] | [10.0] | [3.5] | [1.6] | [-0.9] | [-2.0] | [0.8] | [-0.2] | [-0.1] | | [4.1] | [4.3] | [14] | | [4.2] | |
| | L | 0.38 | 0.40 | 0.35 | -0.12 | -0.08 | -1.40 | -1.99 | -44.5 | 5.67 | 26.0 | -0.09 | 0.28 | 0.99 | 64.2 | -0.06 | 65.2 |
| | | [2.8] | [6.2] | [3.9] | [-0.7] | [-1.3] | [-1.0] | [-1.1] | [-4.4] | [1.6] | | [-0.4] | [1.9] | [9.1] | | [-0.3] | |
| | WL | | | | | | | | | | | 0.59* | | | | 0.57* | |

*Notes:* This table reports the results for the forecast component (FC) conditioning method under the 6-0-6 momentum strategy. The first set of columns gives estimates, $t$-statistics, and adjusted $R^2$ values from the first stage beta prediction regression, $\beta_{i\tau}^{CP} = \gamma_{i0} + \gamma_{i1} Z_{\tau-1} + \varepsilon_{i\tau}$, where $i \in \{W, L\}$, $\tau$ indexes months, and instruments $Z_{\tau-1}$ include 6- and 36-month LC betas ($\beta_\tau^{LC6}$ and $\beta_\tau^{LC36}$), 6- and 36-month market runup (RU6 and RU36), dividend yield (DY), term spread (TS), 30-day T-bill rate (TB), and default spread (DS). The second set of columns presents the results from the second stage return regression $R_{i\tau} = \alpha_i^{FC2} + (\phi_{i0} + \phi_{i1}\widehat{\beta}_{i\tau}^{CP})R_{M\tau} + u_{i\tau}$. The third set of columns reports alphas and adjusted $R^2$ values from a single-step regression, $R_{i\tau} = \alpha_i^{FC1} + \beta_i[1 \quad Z_{\tau-1}]R_{M\tau} + \varepsilon_{i\tau}$. The performance measures $\alpha_i^{FC2}$ and $\alpha_i^{FC1}$ are in percent. Conditional winner minus loser FC alphas that are significantly smaller than UC alphas at the 5% level are marked with an asterisk. The sample period is from January 1930 to December 2005.

TABLE 9. – MOMENTUM PORTFOLIO FORECAST COMPONENT CONDITIONING

| | | FC2 (Two-Step) | | | | | | | | | | | | | | FC1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Stage 1 Beta Regression | | | | | | | | | | Stage 2 Return Regression | | | | | |
| | | $\gamma_0$ | $\beta_\tau^{LC6}$ | $\beta_\tau^{LC36}$ | RU6 | RU36 | DY | TS | TB | DS | $R^2$ | $\alpha^{FC2}$ | $\phi_0$ | $\phi_1$ | $R^2$ | $\alpha^{FC1}$ | $R^2$ |
| | | A. 6-1-1 Strategy | | | | | | | | | | | | | | | |
| (1) | W | 1.14 | | | | | | | | | | 0.66 | | 1.01 | 68.9 | 0.66 | 68.9 |
| | | [57] | | | | | | | | | | [4.6] | | [45] | | [4.6] | |
| | L | 1.19 | | | | | | | | | | 0.09 | | 1.40 | 57.8 | 0.09 | 57.8 |
| | | [47] | | | | | | | | | | [0.3] | | [35] | | [0.3] | |
| | WL | | | | | | | | | | | 0.57 | | | | 0.57 | |
| (2) | W | 1.43 | | | | | -4.67 | 0.74 | -27.9 | -1.70 | 2.73 | 0.62 | -0.11 | 1.18 | 70.6 | 0.64 | 70.7 |
| | | [20] | | | | | [-3.3] | [0.4] | [-3.0] | [-0.5] | | [4.5] | [-0.6] | [7.2] | | [4.6] | |
| | L | 1.37 | | | | | -2.32 | -6.97 | -83.3 | 18.5 | 7.90 | 0.15 | 0.52 | 0.81 | 59.6 | 0.20 | 60.3 |
| | | [16] | | | | | [-1.4] | [-3.3] | [-7.2] | [4.8] | | [0.6] | [2.8] | [6.4] | | [0.8] | |
| | WL | | | | | | | | | | | 0.46* | | | | 0.44* | |
| (3) | W | 0.05 | 0.59 | 0.30 | | | | | | | 31.2 | 0.60 | 0.23 | 0.92 | 74.0 | 0.60 | 74.0 |
| | | [0.8] | [11] | [5.0] | | | | | | | | [4.7] | [3.2] | [13] | | [4.6] | |
| | L | 0.09 | 0.58 | 0.31 | | | | | | | 24.5 | 0.23 | 0.44 | 0.94 | 60.7 | 0.23 | 60.8 |
| | | [1.2] | [8.6] | [3.7] | | | | | | | | [0.9] | [2.8] | [8.2] | | [0.9] | |
| | WL | | | | | | | | | | | 0.37* | | | | 0.37* | |
| (4) | W | 0.21 | 0.58 | 0.27 | 0.18 | -0.03 | -2.63 | 1.02 | 0.11 | -1.02 | 31.7 | 0.59 | 0.38 | 0.81 | 74.3 | 0.61 | 75.0 |
| | | [2.2] | [11] | [3.8] | [1.2] | [-0.6] | [-2.2] | [0.7] | [0.0] | [-0.4] | | [4.6] | [6.3] | [14] | | [4.8] | |
| | L | 0.31 | 0.42 | 0.40 | -0.07 | -0.10 | -0.74 | -0.60 | -44.9 | 5.85 | 26.7 | 0.23 | 0.37 | 0.91 | 61.5 | 0.26 | 62.4 |
| | | [2.2] | [5.9] | [4.3] | [-0.3] | [-1.6] | [-0.5] | [-0.3] | [-4.1] | [1.6] | | [0.9] | [2.5] | [9.4] | | [1.1] | |
| | WL | | | | | | | | | | | 0.36* | | | | 0.34* | |
| | | B. 6-1-6 Strategy | | | | | | | | | | | | | | | |
| (1) | W | 1.15 | | | | | | | | | | 0.68 | | 1.05 | 73.0 | 0.68 | 73.0 |
| | | [62] | | | | | | | | | | [5.1] | | [50] | | [5.1] | |
| | L | 1.14 | | | | | | | | | | -0.46 | | 1.37 | 60.9 | -0.46 | 60.9 |
| | | [50] | | | | | | | | | | [-2.0] | | [38] | | [-2.0] | |
| | WL | | | | | | | | | | | 1.14 | | | | 1.14 | |
| (2) | W | 1.41 | | | | | -4.14 | 0.79 | -24.2 | -2.07 | 2.65 | 0.64 | -0.21 | 1.30 | 74.7 | 0.68 | 75.3 |
| | | [22] | | | | | [-3.2] | [0.5] | [-2.8] | [-0.7] | | [4.9] | [-1.2] | [7.9] | | [5.3] | |
| | L | 1.39 | | | | | -3.34 | -7.80 | -86.8 | 18.0 | 9.80 | -0.40 | 0.52 | 0.77 | 62.6 | -0.37 | 63.1 |
| | | [18] | | | | | [-2.2] | [-4.2] | [-8.5] | [5.3] | | [-1.8] | [3.1] | [6.5] | | [-1.6] | |
| | WL | | | | | | | | | | | 1.03 | | | | 1.05 | |
| (3) | W | 0.07 | 0.60 | 0.28 | | | | | | | 29.3 | 0.62 | 0.18 | 0.97 | 77.7 | 0.62 | 77.7 |
| | | [1.1] | [11] | [4.6] | | | | | | | | [5.2] | [2.3] | [14] | | [5.1] | |
| | L | 0.11 | 0.56 | 0.26 | | | | | | | 23.9 | -0.34 | 0.40 | 0.97 | 63.4 | -0.34 | 63.6 |
| | | [1.4] | [9.5] | [3.3] | | | | | | | | [-1.5] | [2.6] | [7.8] | | [-1.5] | |
| | WL | | | | | | | | | | | 0.96* | | | | 0.96* | |
| (4) | W | 0.21 | 0.59 | 0.25 | 0.22 | -0.05 | -2.00 | 0.84 | -4.64 | -0.28 | 29.7 | 0.61 | 0.28 | 0.90 | 78.0 | 0.64 | 78.5 |
| | | [2.3] | [10] | [3.7] | [1.7] | [-1.1] | [-1.7] | [0.6] | [-0.6] | [-0.1] | | [5.1] | [4.1] | [14] | | [5.3] | |
| | L | 0.37 | 0.42 | 0.31 | -0.06 | -0.07 | -1.44 | -2.03 | -42.4 | 5.71 | 25.6 | -0.32 | 0.22 | 1.04 | 64.4 | -0.30 | 65.4 |
| | | [2.9] | [6.6] | [3.8] | [-0.4] | [-1.2] | [-1.0] | [-1.2] | [-4.2] | [1.7] | | [-1.4] | [1.5] | [9.4] | | [-1.4] | |
| | WL | | | | | | | | | | | 0.93* | | | | 0.93* | |

*Notes:* This table reports the results for the forecast component (FC) conditioning method under the 6-1-1 and 6-1-6 momentum strategies. The first set of columns gives estimates, $t$-statistics, and adjusted $R^2$ values from the first stage beta prediction regression, $\beta_{i\tau}^{CP} = \gamma_{i0} + \gamma_{i1} Z_{\tau-1} + \varepsilon_{i\tau}$, where $i \in \{W, L\}$, $\tau$ indexes months, and instruments $Z_{\tau-1}$ include 6- and 36-month LC betas ($\beta_\tau^{LC6}$ and $\beta_\tau^{LC36}$), 6- and 36-month market runup (RU6 and RU36), dividend yield (DY), term spread (TS), 30-day T-bill rate (TB), and default spread (DS). The second set of columns presents the results from the second stage return regression $R_{i\tau} = \alpha_i^{FC2} + (\phi_{i0} + \phi_{i1}\widehat{\beta}_{i\tau}^{CP}) R_{M\tau} + u_{i\tau}$. The third set of columns reports alphas and adjusted $R^2$ values from a single-step regression, $R_{i\tau} = \alpha_i^{FC1} + \beta_i [1\ Z_{\tau-1}] R_{M\tau} + \varepsilon_{i\tau}$. The performance measures $\alpha_i^{FC2}$ and $\alpha_i^{FC1}$ are in percent. Conditional winner minus loser FC alphas that are significantly smaller than UC alphas at the 5% level are marked with an asterisk. The sample period is from January 1930 to December 2005.

## TABLE 10. – Predicting Momentum Alphas

| | Intercept | Market Runup RU36 | Market Runup $RU36^2$ | Standard Instruments DY | Standard Instruments TS | Standard Instruments TB | Standard Instruments DS | Lagged Alpha $\alpha_{\tau-1}$ | Lagged Alpha $\alpha_{\tau-2}$ | Lagged Alpha $\alpha_{\tau-3}$ | Lagged Alpha $\alpha_{\tau-4}$ | Lagged Alpha $\alpha_{\tau-5}$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | A. Unconditional Alphas | | | | | | | |
| (1) | 0.81 | | | | | | | | | | | | |
| | [4.2] | | | | | | | | | | | | |
| (2) | 1.06 | | | | | | | -0.13 | -0.09 | -0.03 | 0.03 | -0.05 | 2.17 |
| | [3.9] | | | | | | | [-1.8] | [-2.1] | [-0.8] | [1.1] | [-1.3] | |
| (3) | 2.03 | | | -3.27 | 11.4 | 185.6 | -152.3 | | | | | | 3.02 |
| | [2.5] | | | [-0.2] | [0.8] | [2.5] | [-2.9] | | | | | | |
| (4) | 0.29 | 4.58 | -4.07 | | | | | | | | | | 3.08 |
| | [0.9] | [3.5] | [-3.5] | | | | | | | | | | |
| (5) | 1.63 | 2.41 | -2.48 | -7.17 | 6.39 | 103.1 | -93.3 | | | | | | 3.39 |
| | [1.9] | [1.9] | [-2.4] | [-0.5] | [0.4] | [1.3] | [-1.7] | | | | | | |
| (6) | 0.30 | 7.05 | -5.54 | | | | | -0.19 | -0.16 | -0.10 | -0.04 | -0.10 | 8.21 |
| | [0.8] | [4.0] | [-3.8] | | | | | [-3.0] | [-4.2] | [-2.9] | [-1.0] | [-2.3] | |
| (7) | 1.79 | 4.84 | -3.90 | -13.7 | -0.56 | 135.5 | -86.5 | -0.19 | -0.16 | -0.10 | -0.04 | -0.11 | 8.75 |
| | [1.9] | [2.9] | [-3.0] | [-0.9] | [0.0] | [1.3] | [-1.4] | [-3.1] | [-4.2] | [-3.0] | [-1.1] | [-2.4] | |
| | | | | | | B. FC2 Alphas | | | | | | | |
| (1) | 0.59 | | | | | | | | | | | | |
| | [3.5] | | | | | | | | | | | | |
| (2) | 0.74 | | | | | | | -0.15 | -0.05 | -0.03 | 0.06 | -0.04 | 2.53 |
| | [3.4] | | | | | | | [-2.8] | [-1.2] | [-0.7] | [1.3] | [-1.4] | |
| (3) | 1.46 | | | 0.22 | -1.02 | 155.8 | -118.9 | | | | | | 2.35 |
| | [2.5] | | | [0.0] | [-0.1] | [2.4] | [-3.1] | | | | | | |
| (4) | 0.16 | 3.58 | -3.09 | | | | | | | | | | 2.35 |
| | [0.6] | [3.9] | [-3.7] | | | | | | | | | | |
| (5) | 1.09 | 1.89 | -1.81 | -2.53 | -5.12 | 92.4 | -72.6 | | | | | | 2.54 |
| | [1.6] | [1.6] | [-1.9] | [-0.2] | [-0.4] | [1.3] | [-1.5] | | | | | | |
| (6) | 0.19 | 5.44 | -4.47 | | | | | -0.19 | -0.11 | -0.09 | 0.00 | -0.09 | 7.12 |
| | [0.6] | [4.1] | [-3.9] | | | | | [-3.7] | [-2.7] | [-2.0] | [0.1] | [-2.7] | |
| (7) | 1.40 | 3.95 | -3.32 | -12.0 | -19.7 | 92.6 | -56.5 | -0.20 | -0.12 | -0.10 | 0.00 | -0.10 | 7.70 |
| | [1.7] | [2.6] | [-2.7] | [-0.9] | [-1.4] | [1.1] | [-1.0] | [-4.0] | [-2.9] | [-2.2] | [-0.1] | [-2.8] | |
| | | | | | | C. Alpha Differences, $\alpha_{UC,t} - \alpha_{FC2,t}$ | | | | | | | |
| (1) | 0.22 | | | | | | | | | | | | |
| | [2.5] | | | | | | | | | | | | |
| (2) | 0.26 | | | | | | | 0.02 | -0.06 | -0.16 | 0.00 | 0.03 | 2.88 |
| | [2.8] | | | | | | | [0.2] | [-0.9] | [-2.4] | [0.0] | [0.8] | |
| (3) | 0.57 | | | -3.49 | 12.4 | 29.7 | -33.4 | | | | | | 0.82 |
| | [1.0] | | | [-0.5] | [2.1] | [0.8] | [-1.1] | | | | | | |
| (4) | 0.13 | 1.00 | -0.97 | | | | | | | | | | 0.70 |
| | [0.7] | [1.2] | [-1.4] | | | | | | | | | | |
| (5) | 0.54 | 0.52 | -0.67 | -4.64 | 11.5 | 10.7 | -20.7 | | | | | | 0.86 |
| | [1.0] | [0.8] | [-1.1] | [-0.6] | [1.9] | [0.2] | [-0.7] | | | | | | |
| (6) | 0.13 | 1.07 | -0.91 | | | | | 0.02 | -0.06 | -0.16 | 0.00 | 0.03 | 3.63 |
| | [0.7] | [1.2] | [-1.3] | | | | | [0.2] | [-0.9] | [-2.4] | [-0.1] | [0.7] | |
| (7) | 0.45 | 0.59 | -0.60 | -3.15 | 14.4 | 18.3 | -21.9 | 0.02 | -0.07 | -0.17 | -0.01 | 0.02 | 3.86 |
| | [0.9] | [0.8] | [-0.9] | [-0.4] | [2.1] | [0.4] | [-0.8] | [0.2] | [-1.0] | [-2.5] | [-0.2] | [0.5] | |

*Notes:* This table reports the estimates, $t$-statistics, and adjusted $R^2$ values from regressing momentum alphas, in percent per month, on predictor variables, which include 36-month market runup (RU36), squared 36-month market runup ($RU36^2$), dividend yield (DY), term spread (TS), 30-day T-bill (TB), default spread (DS), and five lags of alphas ($\alpha_{\tau-1}$ through $\alpha_{\tau-5}$). Panel A regressions use unconditional alphas $\alpha_{i\tau}^{UC} \equiv \alpha_i^{UC} + \eta_{i\tau}$ on the left-hand side. Panel B uses conditional alphas $\alpha_{i\tau}^{FC2}$ from the second step of the forecast component (FC) regression that employs 6- and 36-month LC betas, 6- and 36-month market runup, DY, TS, TB, and DS as instruments (regression 8 from Table 8). Panel C reports the results of regressing the difference $\alpha_{i\tau}^{UC} - \alpha_{i\tau}^{FC2}$ on the predictor variables. All $t$-statistics are calculated using Newey and West (1987) standard errors with five lags. The sample period is from January 1930 to December 2005.

TABLE 11. – DECOMPOSING THE ALPHA BIASES

| | | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| **A. Conditioning Variables** | | | | | | | | | |
| $\beta_i^{LC6}$ | | | | x | | x | x | x | x |
| $\beta_i^{LC36}$ | | | | | x | x | x | x | x |
| RU6, RU36 | | | | | | | x | | x |
| DY, TS, TB, DS | | | x | | | | | x | x |
| | | | | | | | | | |
| **B. Conditional Beta Moments** | | | | | | | | | |
| | W | 1.17 | 1.25 | 1.27 | 1.26 | 1.28 | 1.29 | 1.30 | 1.30 |
| $\bar{\beta}_{i\tau}^{FC2}$ | L | 1.61 | 1.43 | 1.54 | 1.54 | 1.53 | 1.48 | 1.44 | 1.43 |
| | WL | -0.43 | -0.17 | -0.27 | -0.28 | -0.25 | -0.19 | -0.14 | -0.13 |
| | | | | | | | | | |
| | W | 0.00 | -2.50 | -2.99 | -2.51 | -3.14 | -3.59 | -3.75 | -3.87 |
| $Cov(\beta_{i\tau}^{FC2}, R_M^2)$ | L | 0.00 | 5.26 | 1.93 | 1.96 | 2.19 | 3.56 | 5.04 | 5.28 |
| | WL | 0.00 | -7.76 | -4.92 | -4.48 | -5.33 | -7.14 | -8.79 | -9.15 |
| | | | | | | | | | |
| **C. Decomposition of the Underconditioning Bias** | | | | | | | | | |
| Direct UC Bias | W | 0.00 | -0.01 | -0.01 | 0.03 | 0.00 | 0.00 | -0.01 | -0.01 |
| $Cov(\beta_{i\tau}^{FC2}, R_{M\tau})$ | L | 0.00 | 0.05 | -0.07 | -0.05 | -0.08 | -0.07 | -0.03 | -0.03 |
| | WL | 0.00 | -0.06 | 0.06 | 0.08 | 0.07 | 0.07 | 0.02 | 0.02 |
| | | | | | | | | | |
| $-$ Beta Bias $\cdot \bar{R}_M$ | W | 0.00 | -0.05 | -0.06 | -0.05 | -0.07 | -0.08 | -0.08 | -0.08 |
| $(\beta_i^{UC} - \bar{\beta}_i^{FC2}) \cdot \bar{R}_M$ | L | 0.00 | 0.11 | 0.04 | 0.04 | 0.05 | 0.08 | 0.11 | 0.11 |
| | WL | 0.00 | -0.16 | -0.10 | -0.09 | -0.11 | -0.15 | -0.18 | -0.19 |
| | | | | | | | | | |
| $=$ UC Bias | W | 0.00 | 0.04 | 0.05 | 0.08 | 0.06 | 0.07 | 0.07 | 0.07 |
| $\alpha_i^{UC} - \bar{\alpha}_i^{FC2}$ | L | 0.00 | -0.06 | -0.11 | -0.10 | -0.12 | -0.15 | -0.14 | -0.14 |
| | WL | 0.00 | 0.10 | 0.16 | 0.18 | 0.19 | 0.22 | 0.21 | 0.22 |
| | | | | | | | | | |
| **D. Decomposition of the Overconditioning Bias** | | | | | | | | | |
| Direct OC Bias | W | 0.65 | 0.64 | 0.64 | 0.68 | 0.65 | 0.65 | 0.65 | 0.65 |
| $-Cov(\beta_{i\tau}^{CP} - \beta_{i\tau}^{FC2}, R_{M\tau})$ | L | 0.11 | 0.16 | 0.04 | 0.06 | 0.04 | 0.04 | 0.08 | 0.08 |
| | WL | 0.54 | 0.48 | 0.60 | 0.62 | 0.61 | 0.61 | 0.56 | 0.56 |
| | | | | | | | | | |
| $-$ Beta Bias $\cdot \bar{R}_M$ | W | -0.02 | -0.07 | -0.08 | -0.07 | -0.08 | -0.09 | -0.10 | -0.10 |
| $(\bar{\beta}_i^{CP} - \bar{\beta}_i^{FC2}) \cdot \bar{R}_M$ | L | -0.28 | -0.17 | -0.24 | -0.24 | -0.23 | -0.20 | -0.17 | -0.17 |
| | WL | 0.26 | 0.10 | 0.16 | 0.17 | 0.15 | 0.11 | 0.08 | 0.07 |
| | | | | | | | | | |
| $=$ OC Bias | W | 0.67 | 0.71 | 0.72 | 0.75 | 0.73 | 0.74 | 0.74 | 0.74 |
| $\alpha_{i\tau}^{CPBH} - \bar{\alpha}_i^{FC2}$ | L | 0.39 | 0.33 | 0.28 | 0.30 | 0.27 | 0.25 | 0.25 | 0.25 |
| | WL | 0.28 | 0.38 | 0.44 | 0.46 | 0.46 | 0.50 | 0.49 | 0.50 |

*Notes:* This table decomposes the underconditioning and overconditioning biases. Panel A lists the conditioning variables used in the forecast component (FC2) regressions: 6- and 36-month lagged component betas, 6- and 36-month market runups, and the standard instruments (DY, TS, TB, and DS). Panel B reports average FC2 conditional betas, and their covariances with squared market return. Panel C decomposes the alpha bias $\alpha_i^{UC} - \bar{\alpha}_i^{FC2}$ into the direct alpha bias $Cov\left(\beta_{i\tau}^{FC2}, R_{M\tau}\right)$ and the indirect alpha bias, $-(\beta_i^{UC} - \bar{\beta}_i^{FC2})\bar{R}_M$ caused by the beta bias, as in equation (2.3). Panel D decomposes the overconditioning bias $\alpha_i^{CPBH} - \bar{\alpha}_i^{FC2}$ into the direct OC bias $-Cov\left(\beta_{i\tau}^{CP} - \beta_{i\tau}^{FC2}, R_{M\tau}\right)$ and the beta bias, $-(\bar{\beta}_i^{CP} - \bar{\beta}_i^{FC2})\bar{R}_M$, as in Proposition 6. CP betas are calculated in windows of one month. The average excess market return $\bar{R}_M$ is 0.62% per month. The sample period is from January 1930 to December 2005.

## TABLE 12. – MOMENTUM PORTFOLIO 3-FACTOR MODEL ALPHAS

| | | | | Conditional Models | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Contemporaneous Portfolio (CP) | | | | | | Lagged Portfolio (LP) | | | | | | Lagged Component (LC) | | | | | |
| | (i) UC | | | (ii) RD | | | (iii) BH | | | (iv) RD | | | (v) BH | | | (vi) 6-month betas | | | (vii) 36-month betas | | |
| | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L | WL | W | L |
| 6-0-6 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 1.11 | 0.37 | -0.75 | | | | | | | | | | | | | | | | | | |
| 1 month | 1.10 | 0.44 | -0.65 | 2.07 | 0.90 | -1.17 | 1.88 | 0.91 | -0.98 | 1.06 | 0.16 | -0.89 | 0.82 | 0.29 | -0.52 | 0.71 | 0.32 | -0.39 | 0.45 | 0.10 | -0.35 |
| 3 month | 1.15 | 0.49 | -0.65 | 1.42 | 0.46 | -0.95 | 1.25 | 0.47 | -0.78 | 0.84 | 0.17 | -0.66 | 0.60 | 0.31 | -0.30 | | | | | | |
| 6 month | 1.05 | 0.42 | -0.63 | 0.88 | 0.25 | -0.63 | 0.74 | 0.26 | -0.48 | 1.08 | 0.20 | -0.88 | 0.84 | 0.33 | -0.51 | | | | | | |
| 6-1-1 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 0.93 | 0.44 | -0.49 | | | | | | | | | | | | | | | | | | |
| 1 month | 0.85 | 0.51 | -0.34 | 2.13 | 1.03 | -1.10 | 1.89 | 1.09 | -0.80 | 1.15 | 0.37 | -0.78 | 0.86 | 0.50 | -0.36 | 0.60 | 0.56 | -0.04 | 0.23 | 0.27 | 0.04 |
| 3 month | 1.03 | 0.60 | -0.44 | 1.26 | 0.56 | -0.69 | 1.06 | 0.58 | -0.48 | 0.79 | 0.28 | -0.51 | 0.50 | 0.41 | -0.09 | | | | | | |
| 6 month | 0.97 | 0.57 | -0.41 | 0.52 | 0.25 | -0.26 | 0.33 | 0.26 | -0.08 | 0.88 | 0.26 | -0.61 | 0.59 | 0.39 | -0.20 | | | | | | |
| 6-1-6 Strategy | | | | | | | | | | | | | | | | | | | | | |
| daily | 1.43 | 0.47 | -0.96 | | | | | | | | | | | | | | | | | | |
| 1 month | 1.40 | 0.54 | -0.87 | 2.22 | 0.92 | -1.30 | 2.13 | 0.94 | -1.19 | 1.31 | 0.25 | -1.06 | 1.09 | 0.38 | -0.71 | 1.07 | 0.44 | -0.63 | 0.77 | 0.20 | -0.56 |
| 3 month | 1.41 | 0.58 | -0.83 | 1.75 | 0.58 | -1.18 | 1.61 | 0.59 | -1.02 | 1.14 | 0.27 | -0.87 | 0.93 | 0.41 | -0.52 | | | | | | |
| 6 month | 1.26 | 0.48 | -0.79 | 1.28 | 0.40 | -0.88 | 1.16 | 0.41 | -0.75 | 1.43 | 0.32 | -1.11 | 1.21 | 0.45 | -0.76 | | | | | | |

*Notes:* This table reports the unconditional and conditional Fama-French alphas, in percent per month, of winners (W), losers (L), and winners minus losers (WL) of the three momentum strategies. Unconditional (UC) daily alphas are rescaled intercepts from the regression $R_{it} = \alpha_i^{UC}/n + \Sigma_j(\beta_{ij0}F_{jt} + \beta_{ij1}F_{j,t-1} + (\beta_{ij2}/3)\Sigma_{k=2}^{4}F_{j,t-k}) + \varepsilon_{it}$, where $R_{it}$, $i \in \{W, L\}$, and $F_{jt}$, $j \in \{MKT, HML, SMB\}$, are daily excess portfolio and factor returns, and $n$ is the average number of trading days in one month. $N$-month unconditional alphas are average intercepts from the regression $R_{i\theta} = N\alpha_i^{UC} + \Sigma_j(\beta_{ij}F_{j\theta}) + \eta_{i\theta}$, where $R_{i\theta}$ and $F_{j\theta}$ are $N$-month excess portfolio and factor returns. To perform contemporaneous portfolio (CP) and lagged portfolio (LP) risk adjustment, data is partitioned into non-overlapping windows of length $N \in \{1, 3, 6\}$ months indexed by $\theta$, and we run the regressions: $R_{it} = \alpha_{i\theta}^{CPRD}/n + \Sigma_j[\beta_{ij0\theta}F_{jt} + \beta_{ij1\theta}F_{j,t-1} + (\beta_{ij2\theta}/3)\Sigma_{k=2}^{4}F_{j,t-k}] + \varepsilon_{it}$. CP rescaled daily (RD) alphas are the average intercepts from these regressions. CP buy-and-hold (BH) alphas are averages over $\alpha_{i\theta}^{CPBH} = (1/N)\Sigma_j(R_{i\theta} - \beta_{ij\theta}^{CP}F_{j\theta})$, where $\beta_{ij\theta}^{CP} = \beta_{ij0\theta} + \beta_{ij1\theta} + \beta_{ij2\theta}$. LPRD and LPBH alphas are calculated by averaging over $\alpha_{i\theta}^{LPRD} = (1/N)\Sigma_{t\in\theta}\Sigma_j(R_{it} - \beta_{ij\theta}^{LP}F_{jt})$ and $\alpha_{i\theta}^{LPBH} = (1/N)\Sigma_j(R_{i\theta} - \beta_{ij\theta}^{LP}F_{j\theta})$, where $\beta_{ij\theta}^{LP} = \beta_{ij,\theta-1}^{CP}$. To estimate the month $\tau$ lagged component (LC) risk exposure, at the end of each calendar month $\tau - 1$, we estimate betas of the individual stocks (components) that will belong to a portfolio in month $\tau$. The component loading estimations use either (i) daily returns from the beginning of $\tau - 6$ to the end of $\tau - 1$ with the Dimson lag structure; or (ii) monthly returns from $\tau - 36$ to $\tau - 1$. To calculate portfolio loadings, we sum over components the product of (1) the component beta and (2) the beginning of month $\tau$ component portfolio weight. For lags $l = 6, 36$, the reported LC alphas are averages over $\alpha_{i\tau}^{LCl} = R_{i\tau} - \Sigma_j(\beta_{ij\tau}^{LCl}F_{j\tau})$. The sample period is from January 1930 until December 2005.

Table 13A. – Forecast Component Single-Stage Regression Results, 3-Factor Model, 6-0-6 Strategy

| | | | $MKT_\tau\times$ | | | | | | | $HML_\tau\times$ | | | | | | | $SMB_\tau\times$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha^{FC1}$ | 1 | $\beta^{LC6}_{MKT\tau}$ | $\beta^{LC36}_{MKT\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{HML\tau}$ | $\beta^{LC36}_{HML\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{SMB\tau}$ | $\beta^{LC36}_{SMB\tau}$ | DY | TS | TB | DS | $R^2$ |
| (1) | W | 0.44 | 1.00 | | | | | | | -0.06 | | | | | | | 0.88 | | | | | | | 85.8 |
| | | [4.7] | [53] | | | | | | | [-2.1] | | | | | | | [30] | | | | | | | |
| | L | -0.65 | 1.21 | | | | | | | 0.51 | | | | | | | 1.52 | | | | | | | 82.3 |
| | | [-4.1] | [39] | | | | | | | [12] | | | | | | | [31] | | | | | | | |
| | WL | 1.10 | | | | | | | | | | | | | | | | | | | | | | |
| (2) | W | 0.41 | 1.24 | | | -5.65 | 1.96 | -10.9 | 2.64 | 0.19 | | | 2.36 | 5.53 | -36.1 | -7.96 | 0.92 | | | -4.78 | 5.18 | 74.4 | -7.20 | 88.5 |
| | | [4.7] | [23] | | | [-5.0] | [1.3] | [-1.2] | [1.1] | [2.3] | | | [1.6] | [2.2] | [-2.5] | [-2.3] | [9.7] | | | [-2.9] | [1.9] | [4.6] | [-2.5] | |
| | L | -0.55 | 1.37 | | | -4.35 | -4.00 | -27.6 | 1.74 | 0.28 | | | 10.9 | -2.48 | -75.4 | -6.77 | 1.20 | | | 4.54 | -0.75 | 11.7 | 3.92 | 83.4 |
| | | [-3.5] | [14] | | | [-2.1] | [-1.4] | [-1.7] | [0.4] | [1.9] | | | [4.2] | [-0.6] | [-2.9] | [-1.1] | [7.1] | | | [1.5] | [-0.2] | [0.4] | [0.7] | |
| | WL | 0.97 | | | | | | | | | | | | | | | | | | | | | | |
| (3) | W | 0.36 | 0.04 | 0.46 | 0.51 | | | | | -0.17 | 0.41 | 0.35 | | | | | -0.03 | 0.44 | 0.43 | | | | | 93.3 |
| | | [5.5] | [0.7] | [6.9] | [7.1] | | | | | [-6.5] | [6.1] | [7.3] | | | | | [-0.6] | [6.7] | [6.2] | | | | | |
| | L | -0.49 | -0.18 | 0.33 | 0.80 | | | | | 0.19 | 0.10 | 0.82 | | | | | 1.27 | -0.05 | 0.29 | | | | | 86.6 |
| | | [-3.6] | [-1.3] | [3.8] | [6.6] | | | | | [3.5] | [1.1] | [8.8] | | | | | [9.8] | [-0.6] | [2.2] | | | | | |
| | WL | 0.85* | | | | | | | | | | | | | | | | | | | | | | |
| (4) | W | 0.33 | 0.35 | 0.35 | 0.46 | -3.22 | -1.24 | -12.8 | 4.43 | 0.00 | 0.37 | 0.42 | 0.25 | 3.05 | -4.87 | -8.77 | 0.00 | 0.31 | 0.51 | -1.06 | 6.81 | 31.1 | -3.78 | 94.0 |
| | | [5.3] | [3.9] | [4.9] | [6.3] | [-3.8] | [-1.1] | [-1.9] | [2.6] | [0.0] | [5.3] | [8.5] | [0.2] | [1.6] | [-0.4] | [-3.3] | [0.0] | [4.4] | [7.4] | [-0.8] | [3.4] | [2.6] | [-1.7] | |
| | L | -0.50 | -0.21 | 0.26 | 0.94 | -1.47 | 4.64 | 8.56 | -3.42 | 0.33 | 0.04 | 0.83 | 0.68 | -2.70 | -56.3 | 1.37 | 0.95 | -0.02 | 0.41 | 2.21 | -5.76 | -12.1 | 5.81 | 86.8 |
| | | [-3.6] | [-1.2] | [2.6] | [7.0] | [-0.8] | [1.8] | [0.6] | [-0.9] | [2.5] | [0.4] | [8.1] | [0.3] | [-0.7] | [-2.4] | [0.2] | [4.3] | [-0.2] | [2.9] | [0.8] | [-1.3] | [-0.5] | [1.2] | |
| | WL | 0.83* | | | | | | | | | | | | | | | | | | | | | | |

*Notes:* This table presents the results for the single-stage forecast component conditioning method under the 6-0-6 momentum strategy using the Fama-French three-factor model. Reported are the estimates, $t$-statistics, and adjusted $R^2$ from the regression $R_{i\tau} = \alpha_i^{FC1} + \Sigma_j(\beta_{ij}F_{j\tau}[1 \quad Z_{j,\tau-1}]) + \eta_{i\tau}$, where $R_{i\tau}$, $i \in \{W, L\}$, and $F_{j\tau}$, $j \in \{MKT, HML, SMB\}$, are monthly excess portfolio and factor returns, and instruments $Z_{\tau-1}$ include 6- and 36-month LC betas for the three factors, and the standard instruments (DY, TS, TB, DS). Alphas are in percent. Conditional winner minus loser FC alphas that are significantly smaller than UC alphas at the 5% level are marked with an asterisk. The sample period is from January 1930 to December 2005.

Table 13B. – Forecast Component Single-Stage Regression Results, 3-Factor Model, 6-1-1 Strategy

| | | | $MKT_\tau\times$ | | | | | | | $HML_\tau\times$ | | | | | | | $SMB_\tau\times$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha^{FC1}$ | 1 | $\beta^{LC6}_{MKT\tau}$ | $\beta^{LC36}_{MKT\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{HML\tau}$ | $\beta^{LC36}_{HML\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{SMB\tau}$ | $\beta^{LC36}_{SMB\tau}$ | DY | TS | TB | DS | $R^2$ |
| (1) | W | 0.51 | 0.97 | | | | | | | -0.03 | | | | | | | 0.90 | | | | | | | 83.1 |
| | | [4.9] | [47] | | | | | | | [-1.0] | | | | | | | [28] | | | | | | | |
| | L | -0.34 | 1.27 | | | | | | | 0.54 | | | | | | | 1.53 | | | | | | | 76.9 |
| | | [-1.7] | [33] | | | | | | | [10] | | | | | | | [25] | | | | | | | |
| | WL | 0.85 | | | | | | | | | | | | | | | | | | | | | | |
| (2) | W | 0.48 | 1.21 | | | -3.32 | 1.67 | -11.3 | -3.88 | 0.20 | | | 1.25 | 5.58 | -41.2 | -3.91 | 0.82 | | | -0.37 | 4.33 | 67.5 | -8.12 | 85.4 |
| | | [4.8] | [19] | | | [-2.6] | [0.9] | [-1.1] | [-1.4] | [2.3] | | | [0.8] | [2.0] | [-2.5] | [-1.0] | [7.6] | | | [-0.2] | [1.4] | [3.7] | [-2.4] | |
| | L | -0.24 | 1.45 | | | -7.59 | -3.41 | -17.7 | 7.92 | 0.09 | | | 15.9 | -0.89 | -52.5 | -11.8 | 1.20 | | | 4.03 | 1.92 | 14.4 | 4.03 | 78.2 |
| | | [-1.3] | [12] | | | [-3.1] | [-1.0] | [-0.9] | [1.5] | [0.5] | | | [5.0] | [-0.2] | [-1.6] | [-1.5] | [5.8] | | | [1.1] | [0.3] | [0.4] | [0.6] | |
| | WL | 0.72 | | | | | | | | | | | | | | | | | | | | | | |
| (3) | W | 0.52 | 0.17 | 0.36 | 0.47 | | | | | -0.14 | 0.32 | 0.39 | | | | | 0.09 | 0.27 | 0.50 | | | | | 92.1 |
| | | [7.1] | [3.1] | [6.1] | [6.8] | | | | | [-5.6] | [6.0] | [8.1] | | | | | [1.9] | [4.6] | [8.0] | | | | | |
| | L | -0.16 | -0.02 | 0.26 | 0.75 | | | | | 0.21 | -0.04 | 1.03 | | | | | 1.15 | 0.24 | 0.18 | | | | | 83.3 |
| | | [-1.0] | [-0.1] | [2.5] | [5.9] | | | | | [3.5] | [-0.4] | [11] | | | | | [9.0] | [2.2] | [1.5] | | | | | |
| | WL | 0.68 | | | | | | | | | | | | | | | | | | | | | | |
| (4) | W | 0.48 | 0.50 | 0.28 | 0.41 | -3.46 | -1.50 | -17.5 | 3.33 | 0.03 | 0.24 | 0.45 | 0.05 | 2.83 | -9.33 | -7.28 | -0.04 | 0.22 | 0.60 | -2.76 | 6.65 | 29.0 | 3.12 | 92.7 |
| | | [6.8] | [6.2] | [4.2] | [5.9] | [-3.8] | [-1.2] | [-2.4] | [1.7] | [0.4] | [4.3] | [9.2] | [0.0] | [1.4] | [-0.8] | [-2.5] | [-0.4] | [3.6] | [9.2] | [-2.0] | [2.9] | [2.2] | [1.3] | |
| | L | -0.19 | 0.01 | 0.17 | 0.84 | -2.82 | 5.75 | 17.1 | -0.93 | 0.20 | -0.06 | 1.02 | 4.17 | 0.71 | -40.9 | -3.02 | 0.70 | 0.34 | 0.28 | 6.12 | -8.17 | -23.2 | 4.90 | 83.6 |
| | | [-1.2] | [0.1] | [1.6] | [6.2] | [-1.3] | [1.9] | [1.0] | [-0.2] | [1.3] | [-0.5] | [11] | [1.4] | [0.1] | [-1.5] | [-0.4] | [2.9] | [3.0] | [2.3] | [1.9] | [-1.5] | [-0.7] | [0.9] | |
| | WL | 0.68 | | | | | | | | | | | | | | | | | | | | | | |

*Notes:* This table presents the results for the single-stage forecast component conditioning method under the 6-1-1 momentum strategy using the Fama-French three-factor model. Reported are the estimates, $t$-statistics, and adjusted $R^2$ from the regression $R_{i\tau} = \alpha_i^{FC1} + \Sigma_j(\beta_{ij}F_{j\tau}[1 \quad Z_{j,\tau-1}]) + \eta_{i\tau}$, where $R_{i\tau}$, $i \in \{W, L\}$, and $F_{j\tau}$, $j \in \{MKT, HML, SMB\}$, are monthly excess portfolio and factor returns, and instruments $Z_{\tau-1}$ include 6- and 36-month LC betas for the three factors, and the standard instruments (DY, TS, TB, DS). Alphas are in percent. Conditional winner minus loser FC alphas that are significantly smaller than UC alphas at the 5% level are marked with an asterisk. The sample period is from January 1930 to December 2005.

## Table 13C. – Forecast Component Single-Stage Regression Results, 3-Factor Model, 6-1-6 Strategy

| | | | $MKT_\tau \times$ | | | | | | | $HML_\tau \times$ | | | | | | | $SMB_\tau \times$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\alpha^{FC1}$ | 1 | $\beta^{LC6}_{MKT\tau}$ | $\beta^{LC36}_{MKT\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{HML\tau}$ | $\beta^{LC36}_{HML\tau}$ | DY | TS | TB | DS | 1 | $\beta^{LC6}_{SMB\tau}$ | $\beta^{LC36}_{SMB\tau}$ | DY | TS | TB | DS | $R^2$ |
| (1) | W | 0.54 | 1.02 | | | | | | | -0.04 | | | | | | | 0.92 | | | | | | | 87.3 |
| | | [5.9] | [56] | | | | | | | [-1.5] | | | | | | | [32] | | | | | | | |
| | L | -0.87 | 1.18 | | | | | | | 0.50 | | | | | | | 1.49 | | | | | | | 82.4 |
| | | [-5.6] | [39] | | | | | | | [11] | | | | | | | [31] | | | | | | | |
| | WL | 1.40 | | | | | | | | | | | | | | | | | | | | | | |
| (2) | W | 0.52 | 1.27 | | | -6.56 | 1.33 | -10.7 | 4.76 | 0.22 | | | 2.00 | 5.07 | -38.3 | -8.03 | 0.87 | | | -3.81 | 6.16 | 75.0 | -5.02 | 89.4 |
| | | [6.1] | [24] | | | [-5.9] | [0.9] | [-1.2] | [2.1] | [2.8] | | | [1.4] | [2.1] | [-2.7] | [-2.3] | [9.4] | | | [-2.4] | [2.3] | [4.7] | [-1.8] | |
| | L | -0.78 | 1.31 | | | -2.90 | -3.41 | -28.8 | 0.02 | 0.29 | | | 10.5 | -1.58 | -74.3 | -7.92 | 1.23 | | | 3.86 | -1.17 | 9.61 | 3.75 | 83.3 |
| | | [-5.1] | [14] | | | [-1.5] | [-1.2] | [-1.8] | [0.0] | [2.1] | | | [4.1] | [-0.4] | [-2.9] | [-1.3] | [7.4] | | | [1.3] | [-0.2] | [0.3] | [0.7] | |
| | WL | 1.30 | | | | | | | | | | | | | | | | | | | | | | |
| (3) | W | 0.45 | 0.04 | 0.42 | 0.57 | | | | | -0.16 | 0.49 | 0.39 | | | | | 0.01 | 0.42 | 0.42 | | | | | 94.0 |
| | | [7.0] | [0.6] | [6.2] | [7.8] | | | | | [-6.7] | [7.1] | [7.6] | | | | | [0.3] | [6.2] | [6.1] | | | | | |
| | L | -0.71 | -0.26 | 0.45 | 0.76 | | | | | 0.20 | 0.19 | 0.57 | | | | | 1.28 | -0.03 | 0.25 | | | | | 86.5 |
| | | [-5.3] | [-2.0] | [5.2] | [6.5] | | | | | [3.8] | [2.2] | [6.6] | | | | | [10] | [-0.3] | [2.0] | | | | | |
| | WL | 1.16* | | | | | | | | | | | | | | | | | | | | | | |
| (4) | W | 0.44 | 0.29 | 0.32 | 0.54 | -3.19 | -1.55 | -11.5 | 5.20 | -0.04 | 0.44 | 0.42 | 0.31 | 3.30 | -3.26 | -6.78 | -0.02 | 0.33 | 0.47 | 0.64 | 6.99 | 30.2 | -5.57 | 94.5 |
| | | [7.0] | [3.3] | [4.5] | [7.2] | [-3.8] | [-1.4] | [-1.8] | [3.1] | [-0.6] | [6.0] | [7.9] | [0.3] | [1.8] | [-0.3] | [-2.6] | [-0.2] | [4.7] | [6.8] | [0.5] | [3.5] | [2.6] | [-2.6] | |
| | L | -0.72 | -0.28 | 0.35 | 0.91 | -1.47 | 4.62 | 9.15 | -2.45 | 0.38 | 0.13 | 0.63 | 0.55 | -1.80 | -55.6 | -1.60 | 0.94 | -0.01 | 0.38 | 1.96 | -5.15 | -10.7 | 7.01 | 86.7 |
| | | [-5.2] | [-1.7] | [3.6] | [7.0] | [-0.8] | [1.8] | [0.6] | [-0.7] | [2.9] | [1.5] | [6.2] | [0.2] | [-0.5] | [-2.4] | [-0.3] | [4.4] | [-0.1] | [2.8] | [0.7] | [-1.2] | [-0.4] | [1.4] | |
| | WL | 1.16* | | | | | | | | | | | | | | | | | | | | | | |

*Notes:* This table presents the results for the single-stage forecast component conditioning method under the 6-1-6 momentum strategy using the Fama-French three-factor model. Reported are the estimates, $t$-statistics, and adjusted $R^2$ from the regression $R_{i\tau} = \alpha_i^{FC1} + \Sigma_j(\beta_{ij}F_{j\tau}[1 \quad Z_{j,\tau-1}]) + \eta_{i\tau}$, where $R_{i\tau}$, $i \in \{W, L\}$, and $F_{j\tau}$, $j \in \{MKT, HML, SMB\}$, are monthly excess portfolio and factor returns, and instruments $Z_{\tau-1}$ include 6- and 36-month LC betas for the three factors, and the standard instruments (DY, TS, TB, DS). Alphas are in percent. Conditional winner minus loser FC alphas that are significantly smaller than UC alphas at the 5% level are marked with an asterisk. The sample period is from January 1930 to December 2005.

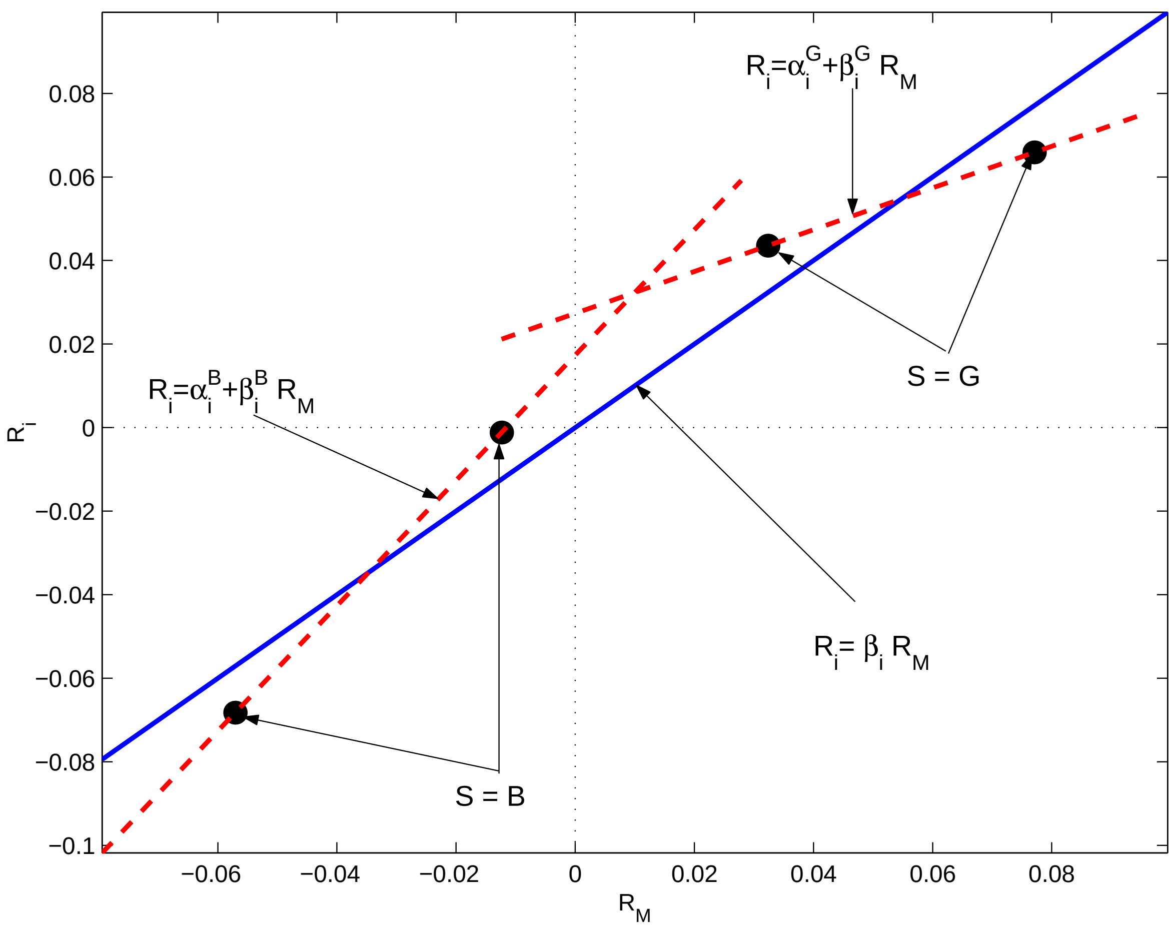


**Figure 1. − Overconditioning in a 4-state Example.** This figure plots portfolio returns against the market return to illustrate overconditioning in a 4-state example. The solid line passing through the origin shows the investor-conditioned pricing relation, while the dashed lines represent the nonlinearity in payoffs, or the overconditioned pricing relations. Returns are $R_M(\Omega) \equiv [-0.057, -0.012, 0.032, 0.077]$ and $R_i(\Omega) = [-0.068, -0.001, 0.044, 0.066]$, and conditional betas are $\beta_i^B = 1.5$ and $\beta_i^G = 0.5$.

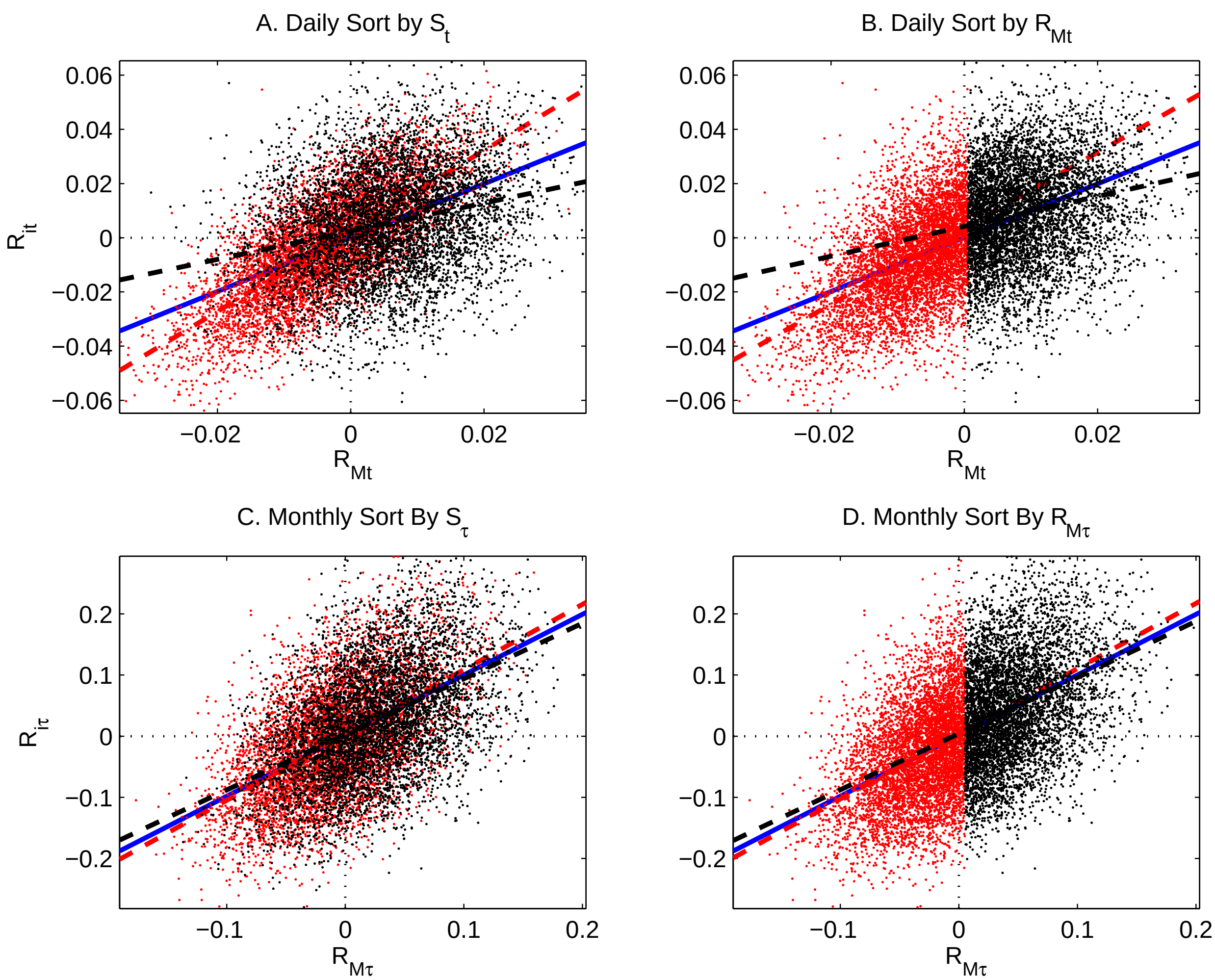


**Figure 2.** – **Overconditioning in Daily and Monthly Returns.** Each Panel of this figure plots $12,000$ randomly chosen draws of returns under the unconditional Markov Model with $\Delta_M^S = 0.005$. Panels A and B plot daily returns, and Panels C and D show monthly returns. In each Panel, the returns are split into two groups by the state $S_t$ (Panel A), the daily market return (Panel B), the number of $S_t = G$ states within a month (Panel C), or the monthly market return (Panel D). The solid lines represent the unconditional pricing relationships, while the dashed lines show the estimated pricing relationships within each subset of data.

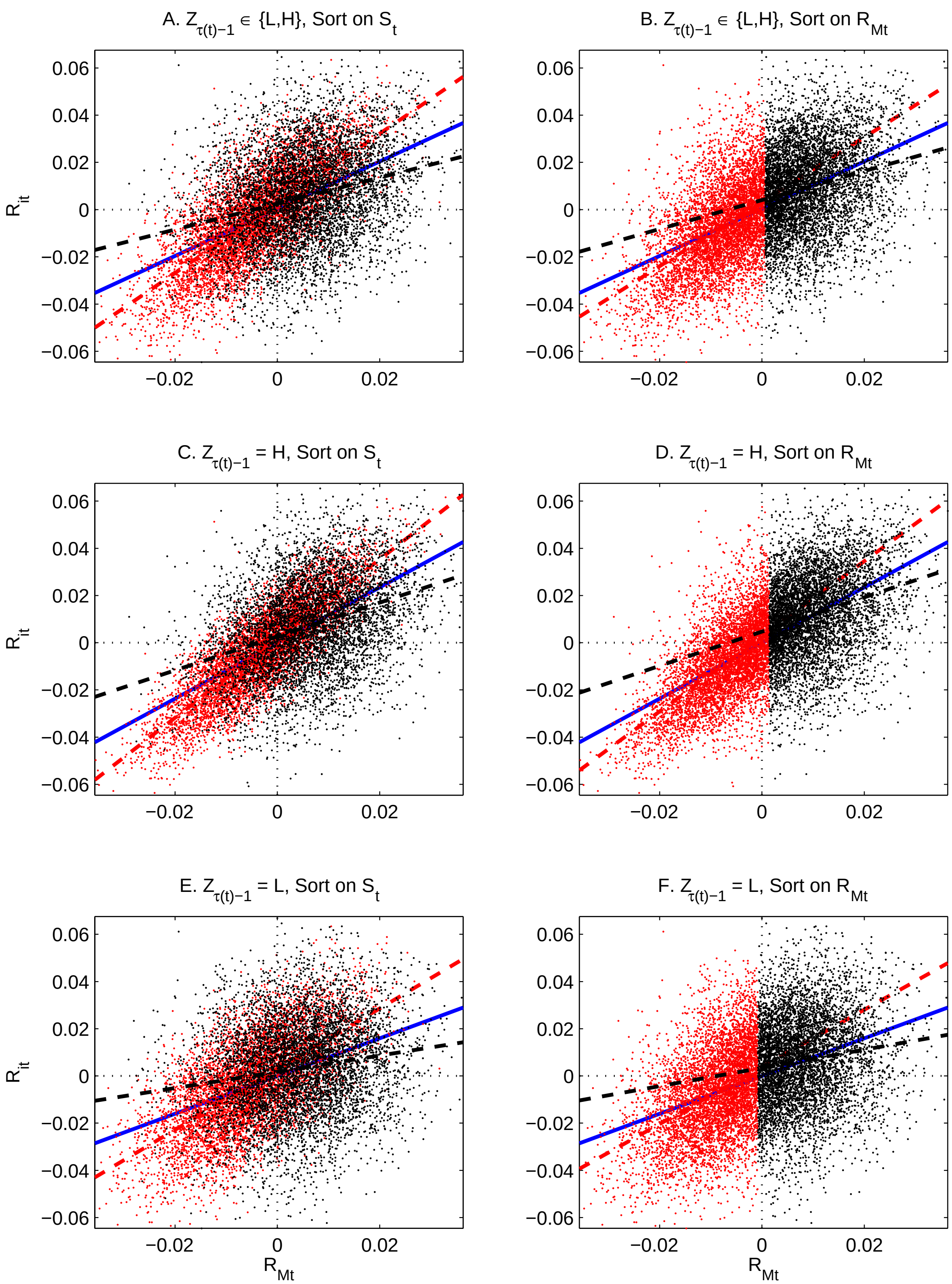


**Figure 3.** – **Overconditioning in the Conditional CAPM, Daily Returns.** This figure shows $12,000$ randomly chosen draws from $(R_{it}, R_{Mt})$ for the parametrization $(\Delta_M^Z, \Delta_\beta^Z, \Delta_M^S, \Delta_\beta^S) = (0.001, 0.2, 0.005, 0.5)$ with $\rho = 0.9$. Panels A and B show the full set of draws, while C and D isolate the days when $Z_{\tau(t)-1} = H$, and E and F isolate $Z_{\tau(t)-1} = L$. The left-hand-side panels (A, C, and E) condition on the latent state $S_t$, and the right-hand-side panels (B, D, and F) condition on $R_{Mt}$ above or is below its population mean.

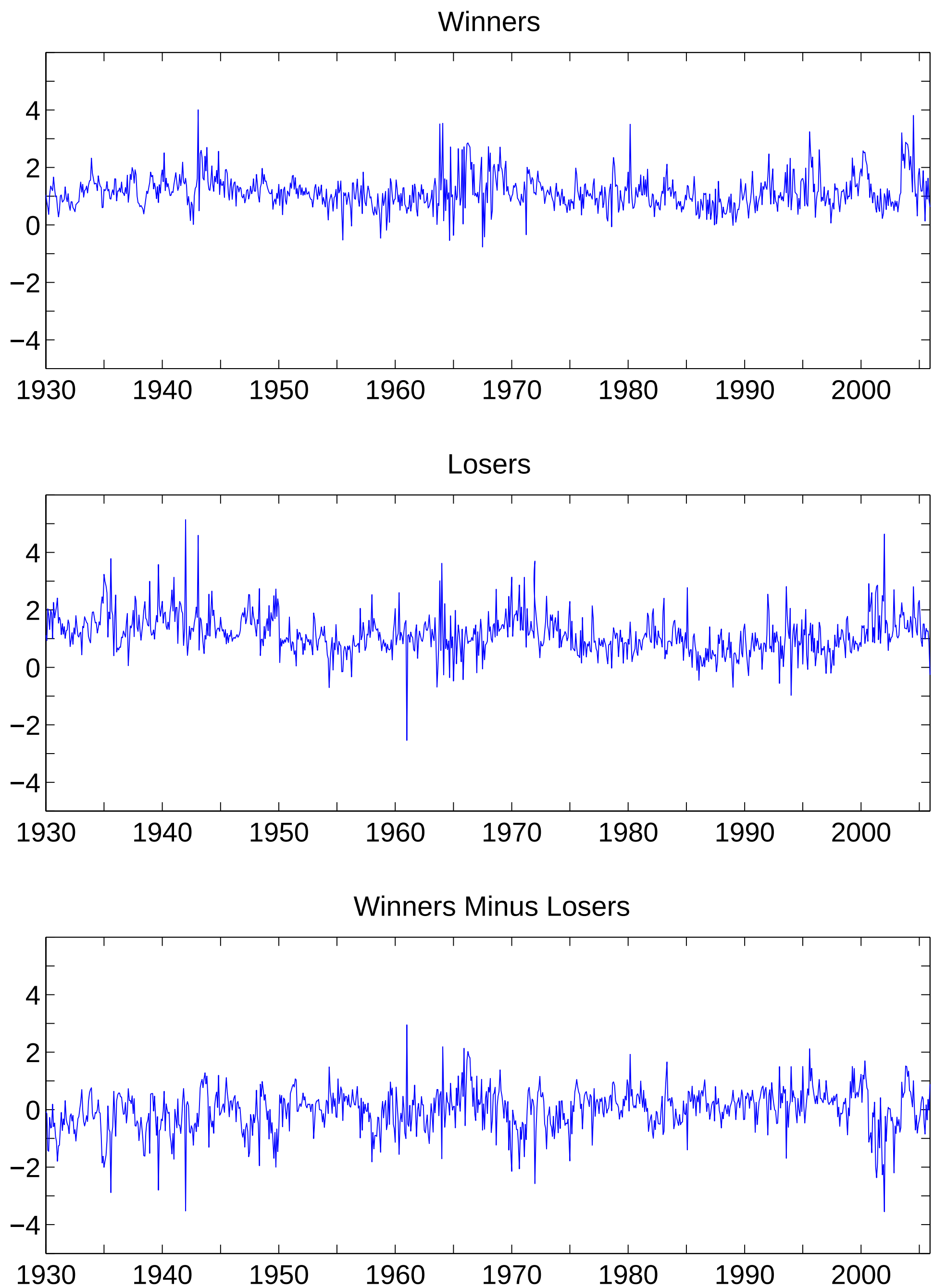


**Figure 4. – Contemporaneous Portfolio 1-Month Betas.** This figure shows CP betas of winners, losers, and winners minus losers for the 6-0-6 momentum strategy. The betas are estimated in each calendar month as the sum of the three slope coefficients from regressing excess returns on excess market return, its lag, and the average of lags 2 through 4 of market excess return. The sample period is from January 1930 to December 2005.

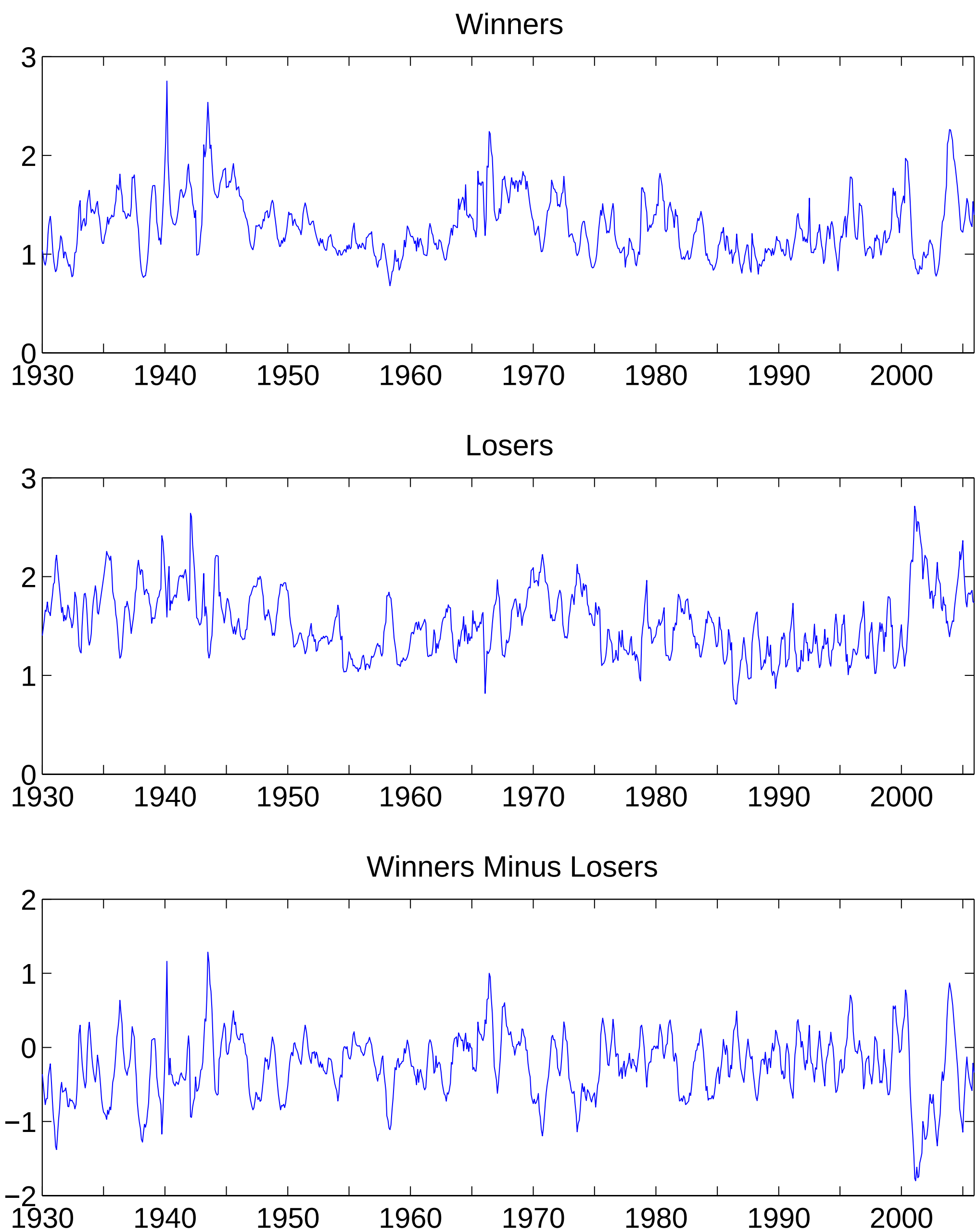


**Figure 5. – Forecast Component Betas.** This figure shows FC betas of winners, losers, and winners minus losers for the 6-0-6 momentum strategy. Instruments are 6 and 36 month LC betas. The sample period is from January 1930 to December 2005.