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HOUSING CYCLES

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Abstract

Within metropolitan areas, housing prices display high levels of volatility and mean reversion. For example, a five-year increase in prices of $10,000 above the metropolitan area trend is associated with a $3,200 decrease relative to the trend over the next five years. This paper argues that various macroeconomic and financial attempts to understand price dynamics, such using price-to-income ratios or assessing prices as the net present value of rents, are generally flawed both conceptually and empirically. Instead, we put forward a dynamic version of an Alonso-Rosen spatial equilibrium model where house prices reflect the willingness to pay for the wages of different metropolitan areas. A calibration of this model suggests that it can do well at explaining mean reversion and the relationship between prices and incomes, but less well at explaining the high variance of housing price changes, particularly within high cost areas. This failure might either be the result of underestimating the true year-to-year variation in relevant local economic conditions or the result of unobserved factors driving the willingness to pay for housing in different areas.

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I. Introduction

There are at least four noteworthy stylized facts about house price changes over the past three decades. First, the disparity of prices across metropolitan areas has increased dramatically, with the most expensive cities in 1970 experiencing much higher growth rates since then (Glaeser, Gyourko, Saks, 2005; Gyourko, Mayer, Sinai, 2006). Second, house price volatility is high and is one of the striking facts about the housing cycle. The standard deviation in one-year real house price changes in our sample of metropolitan areas is $11,134, with that for three-year changes being $26,346. Third, the correlation of price changes across metropolitan areas is modest. Less than eight percent of the variation in price levels can be explained by common year effects, with barely more than one-quarter of the variation in price changes accounted for by year effects. Fourth, housing prices display local cycles with short-term persistence and long-term mean reversion (Case and Shiller, 1989, Hendershott and Abraham, 1991). Using Office of Federal Housing Enterprise (OFHEO) repeat sales indices for over 100 metropolitan areas since 1980, we find that a one dollar increase in local prices during one year is associated with a 71 cent increase in the next year. A one dollar increase in local market prices over five years is associated with a 32 cent decrease over the next five years. Section II documents and discusses these stylized facts in greater detail.

There are two prevailing paradigms for understanding the patterns of housing prices. The urban economics paradigm follows Alonso (1962) and Rosen (1979), and views prices as the result of a spatial equilibrium in which land values are driven by the demand for one place versus another. This paradigm dominates the literature on static prices, but is used far less in the analysis of price changes. The alternative paradigm, which is far more common in discussions of housing price dynamics, is rooted in macroeconomics and finance. This literature thinks of housing as a durable good with its demand driven by macroeconomic variables such as national income and interest rates.

In Section III of this paper, we argue that the ability of the financial approach to explain the dynamic patterns in housing prices is modest. For example, changes in interest rates and aggregate income have little ability to explain the variation in housing prices in the data. In many regions of the country, the correlation between these variables and price changes is almost zero. Moreover, macroeconomic-based reasoning—such as the idea that there is a natural ratio of price-to-income—collides both with the microeconomics of housing and with the data.

Section IV presents a dynamic model based on Rosen (1979) and Roback (1982). Our framework differs from the canonical model in that it incorporates supply, and by doing so, it is able to form predictions about price levels, not just differences in prices across space. Changes in prices are driven by changes in productivity and demand for local amenities. While it is no surprise that this model can generate the key stylized facts of the housing market, the main contribution of the model is that it can be calibrated to test whether reasonable estimates of the model's key parameters generate the variation seen in the data.

In Section V, we estimate the model's core parameters including the variance and shocks to local productivity, the extent that supply costs rise with new production and the heterogeneity of preferences for a particular locale. In Section VI, we investigate whether these parameters can explain the levels and trends of housing prices, the variance of housing price changes and the mean reversion of prices. We also look at whether the spectacular housing cycles in the east and west coasts of America can be understood with changes in productivity or in the demand for coastal amenities.

The model works well at explaining some key moments of the data, but it fails to deliver the volatility of high frequency price changes. For example, the model predicts the observed level of mean reversion almost perfectly. In the model, one quarter of the mean reversion is the result of new supply and three-quarters results from mean reversion of observed local income levels. The model does a good, but somewhat weaker, job of explaining the observed connection between local income levels and housing prices. The model can also explain the long term trends in local prices.

The largest failure of the model is in delivering the observed variance of local housing price changes, particularly in the high appreciation markets. Actual price volatility is multiples of our estimate of what it should be based on the observed level of local income changes. One explanation for this is that we are underestimating the true variation of income changes for marginal buyers. A second explanation is that omitted factors—such as local tax rates or amenities—are varying in ways not yet reflected in our analysis. A third explanation is that less rational factors are driving the movement of housing prices. All will be explored in the next version of the paper.

II. Stylized Facts about Housing Markets and Prices

In this section, we review four major facts about housing markets in recent decades: the increasing divergence of prices across markets, the high levels of price volatility in some markets, the low correlations of price changes across metropolitan areas, and the presence of cycles marked by short-run persistence of price increases and longer-term mean reversion. The growing divergence of prices is shown in Figure 1 which plots the distribution of median metropolitan area house values for 1950, 1970, 1990, and 2000.[1] Dispersion across metropolitan areas increased slightly between 1950 and 1970, but the 1970 distribution still looks much like that for 1950, only proportionately right-shifted. This is not the case for the past three decades, as many different markets clearly have been appreciating at very different rates over the last three decades.

This rising dispersion is partially the result of higher appreciation rates in high cost areas. Different metropolitan areas had different trends and these trends were positively correlated with initial price levels. Figure 2 depicts the 60 percent correlation between median housing price in 1980 and growth in prices between 1980 and 2005. A $10,000 higher median price in 1980 is associated with just over an 8 percent greater amount of price appreciation over the ensuing quarter century. Given the $41,146 standard deviation in real median prices in 1980 for our sample, the area that is one standard deviation more expensive than the mean in 1980 appreciated at about 1 percent per annum higher rate. Real price appreciation over this period averaged about 1.5 percent per year, so this is an economically significant difference.

The second fact is the relatively high amount of local price volatility. For our sample of markets, the standard deviations of 1- and 3-year house price changes are $11,134 and $26,346, respectively. Just as price volatility in the stock market remains an abiding puzzle in the finance literature, the same holds for the housing market, and is just as important to understand as are the patterns of mean reversion and momentum. This volatility is concentrated in those markets that have had sharp increases in prices over the

[1] This figure is taken from Glaeser, Gyourko, and Saks (2006).

last 25 years. Figure 3 visually highlights the 78 percent correlation across metropolitan areas between their average annual house price change (in dollars) and the average absolute difference from their mean one-year price change (i.e. the average value of $\left|H_{t+1} - H_t - (H_{2004} - H_{1980})/24\right|$). The higher a market's typical annual price change, the greater the typical deviation from the average.

The third important fact is that these price changes are not highly correlated across metropolitan areas. For example, regressing house prices for our panel of metropolitan areas on a vector of year dummies finds that only 8 percent of the variation in prices over the past quarter century can be explained by factors that are common across markets within a given year. The analogous regression using one-year price changes yields a R^2=0.28. Local market fixed effects alone can account for over three-quarters of all variation in house price levels across metropolitan areas over time. These results are more indicative of a series of distinct local economies linked by migration rather than a single national housing market, and suggest the need for a dynamic urban model to better understand the nature of house prices across markets over time.

Next, we turn to the cyclical behavior of housing cycles, by which we mean short-run persistence and long-run mean reversion. Because there are complex issues regarding the estimation of time series variables, we begin with a simple presentation of longer-term mean reversion. Using U.S. Census data across all presently defined 374 metropolitan areas, the correlation between the growth in the logarithm of median housing values in the 1990s and the growth in the logarithm of median housing values in the 1980s is -71 percent. A one log point increase in price growth in the 1980s is associated with a 0.5 percent decrease in the 1990s. Figure 4 documents the somewhat smaller correlation across the set of metropolitan areas for which we have OFHEO repeat sales indices that attempt to control for changes in the quality of the housing stock. In this sample, the correlation coefficient is -48 percent and a one log point increase in price growth in the 1980s is associated -0.29 log point decrease over the next ten years.[2]

Table 1 provides more systematic evidence on mean reversion and momentum across cities using only the OFHEO data. We work with absolute price changes rather than changes in the logarithm of prices in order to be compatible with the model in Sections IV and V, but our empirical results are not sensitive to such changes in functional form. Since the OFHEO index only gives us price increases relative to a base year, we convert this into an implied price by using the median housing value in the metropolitan area in 1980 as a base price in the metropolitan area. Unless otherwise noted, our sample is a set of 115 metropolitan areas for which we have continuously defined price data from 1980-2005.

The first results are estimates from a regression of the current change in prices on the lag change in prices as shown in equation (1)

$$(1)\quad Price_{t+j} - Price_t = \alpha_{MSA} + \gamma_{Year} + \beta\left(Price_t - Price_{t-j}\right),$$

for j equal to one, three and five years. Because fixed effects estimates such as these which remove market-specific averages can be biased (with spurious mean reversion produced especially when the number of time periods is relatively low), we report

[2] The difference between Census and OFHEO results reflect the actual data series, not the sample of cities. When the Census sample is restricted to the same cities that are in our OFHEO sample, the correlation remains -70 percent and the coefficient on lagged growth remains about -0.5. The difference presumably reflects the fact that greater quality upgrading in the 1980s is associated with reduced quality upgrading in the 1990s.

Arellano-Bond estimates which use lagged values of the dependent variable (price changes) as instruments.[3]

Over a one-year period, the coefficient is 0.71, implying that a one dollar increase in prices during one period is associated with a 71 cent increase over the next year. Similar results obtain if this regression is run using the changes in the logarithm of price instead of changes in the price level, so transforming the variable does not matter. This momentum echoes the findings of Case and Shiller (1991), and is evidence of a particular form of predictability in the housing market. This type of high frequency result is sensitive to any smoothing procedures used in creating the price series, and we cannot be confident that this positive correlation is not the result of smoothing or other measurement-related issue artificially inducing a positive correlation across one-year periods.

In the second column of Table 1, we find a three-year coefficient of 0.27 for the Arellano-Bond estimator. Again, this represents significant persistence of housing price increases. In the third column, we finally see the mean reversion that was so evident in the decadal plot above. The Arellano-Bond estimate is -0.32, which implies that one-third of every dollar gained over a five year period is lost over the next five years.[4]

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[3] See their 1991 paper for the details. More specifically, we use the "xtabond" Stata command with year and area fixed effects.

[4] The OLS estimate with year and metro fixed effects is -0.39 (standard error of 0.04), so there is evidence of a bias towards finding mean reversion in such specifications. We also addressed concerns about spurious mean reversion by estimating specifications without metropolitan area fixed effects. If we estimate the following equation, $Price_{t+5} - Price_t = \alpha + \gamma_{Year} + \beta(Price_t - Price_{t-5})$, the mean reversion coefficient drops to -0.11 and becomes only marginally significant. However, as soon as we include percent of adults with college degrees as a control, the coefficient becomes -0.18 with a t-statistic of three. If we estimate the same change regression using the logarithm of prices instead of the levels, the coefficient is -0.20 (-0.22 with the college graduate control) and has a t-statistic of four. Thus, mean reversion is stronger with metropolitan area fixed effects, but it also appears without them.

The second row reports results for the analogous specifications using rent data on apartments collected by an industry consultant and data provider, REIS Inc. Their data cover only a limited number of metropolitan areas (46 in our sample), but they show similar patterns of short-run momentum and longer-run mean reversion. The mean reversion over longer horizons suggests this is not exclusively a feature of the asset market, but reflects something about the nature of demand for cities.

In sum, housing prices, like many asset values, display short-run persistence and long-run mean reversion (Cutler, Poterba and Summers (1991)). Moreover, mean reversion over the five year time horizon is remarkably similar in magnitudes to that observed in the financial economics literature (Fama and French (1988)). Unfortunately, the short time horizons for which we have constant quality data at less than decadal frequencies makes it difficult to know whether this mean reversion is a permanent feature of urban life or whether it represents the impact of shocks that are specific to the post-1980 time period.

While housing prices reflect the price of living in a city, understanding the overall market also requires insight into quantities or the number of people living in the city. At decadal frequencies, the most salient fact is the enormous persistence of population growth rates (Glaeser and Shapiro, 2003). Figure 5 illustrates the very strong 73 percent correlation in the change of the logarithm of population in the 1990s and that same variable in the 1980s. A one log point increase in log population growth in the 1980s is associated with a 0.58 log point increase in log population growth in the 1990s. A similar relationship holds for changes in the number of homes: a one log point increase in the number of homes during the 1980s is associated with a 0.5 log point increase in the number of homes in the 1990s. Clearly, a model in which both prices and quantities are driven by shocks to demand will have trouble accounting for why price increases in the 1980s are correlated with price decreases in the 1990s, while population increases in the 1980s are correlated with population increases in the 1990s.

Unfortunately, there is no good population data at higher frequencies so we turn instead to two other measures of "quantity": employment and housing units. Employment figures come from the Bureau of Labor Statistics. For employment, we estimate: $E_{t+j} - E_t = \alpha_{MSA} + \gamma_{Year} + \beta(E_t - E_{t-j})$ where E_t refers to the logarithm of employment at time t. These results are reported in the third row of Table 1, with the Arellano-Bond results for one, three and five-year changes being 0.33, 0.28, and -0.08, respectively. The negative coefficient on the five-year changes not only is small, it is statistically insignificant. The final set of results reported in Table 1 use housing permits estimated in the following regression:

$Log(Permits_t^{t+j}) = \alpha_{MSA} + \gamma_{Year} + \beta Log(Permits_{t-j}^{t})$, where $Permits_{t-j}^{t}$ refers to the number of permits issued between time t-j and time t. The one-three and five year Arellano-Bond coefficient estimates are 0.79, 0.58, and 0.10, respectively. Note that there is no evidence here that employment or permits mean revert over any time horizon. Prices mean revert over horizons as short as five years, but quantities do not.

III. Alternative Approaches to Understanding House Price Dynamics

The macroeconomics approach to housing typically focuses on demand-side forces— including income, interest rates, tax rates and demographics— that are thought to drive the nationwide demand for housing.[5] The hallmark of the urban economics approach to housing is a focus on housing prices as reflecting the cost of living in one place versus another. Neither approach has historically paid much attention to housing supply beyond assuming a simple construction technology which varies little over time and space

Income and Housing Prices

In popular discussion of housing prices, the ratio between income and housing value is seen as a useful measure capturing something like the fundamental demand for housing. This conclusion would be justified if, for example, housing supply was fixed and utility was Cobb-Douglas. In that case, spending on housing would always be a fixed share of total income, and if supply was fixed, changes in income would raise prices proportionately (and, by assumption, not impact quantities). This logic can be questioned in many ways (e.g., housing supply is not fixed), but from the perspective of the urban economics approach to housing, the biggest drawback of this reasoning is that it treats housing as an ordinary consumer durable, not as the price for living in a particular location.

The urban economics approach derives housing prices by first assuming that consumers are indifferent across space, either at every point in time or over the course of their lifetimes. Typically, housing price are assumed to move until $Utility(W_i - R_i, A_i)$ is constant across space, where W refers to wages, R refers to rents or housing costs and A refers to amenities in each city indexed by i. We will ignore amenities here, but return to

[5] Perhaps the most famous paper relating demographics to house price is Mankiw and Weil (1990). There are many others relating interest rates, incomes, and construction costs to house prices. See Poterba (1984) and Hendershott (19xx) for early examples of this literature.

them in the next section. We will also assume throughout this discussion that housing quantities can be assumed to be everywhere equal to one.

If there are two locations, with wages W_1 and W_2, and if a house of cost H requires annual costs of (r+m)H (where r reflects the real interest rate and m reflects maintenance costs), then $H_2 - H_1 = \frac{W_2 - W_1}{r + m}$ if the world was completely static with no price or wage changes. Stated differently, differences in housing prices should be proportional to the change in wages. If r+m equaled 0.1, and housing prices in one location were $100,000 and wages equaled $50,000, then if wages were $100,000 in a second location, housing prices in that second locale should equal $600,000. In the first location, the price-to-income ratio is two and in the second the price-to-income ratio is six. According to the urban economics approach to housing, even wildly different price-to-income ratios are not an indication of bubbles or disequilibrium or anything other than the working of a spatial market for housing, where income differences are offset by housing price differences.[6]

In principle, these two views of the world can be tested with cross-metropolitan area variation. The constant price-to-income ratio view of the world suggests that if housing prices are regressed on household income, the coefficient should be equal the national average price-to-income ratio, which was 2.39 across our sample of 115 metropolitan areas in 2000. The urban economics approach suggests a coefficient of

[6] Median family incomes are clustered pretty tightly along the interquartile range of our metropolitan areas, but the differences widen considerably in the tails. According to the 2000 census, median family income for the 75th percentile metropolitan area was only $11,657 (or 22 percent) higher than that for the 25th percentile area. However, the gap was much larger, at nearly #32,000 (or 64 percent) higher, between the 90th and 10th percentile areas in terms of family income.

1/(r+m), which given most estimates of real rates, local property taxes and maintenance costs should be at least 5.

A simple regression of real house prices on median family income in the year 2000 yields: *House Price$_i$ = -150,873 + 5.47*Income$_i$ + ε_i*,

$$\underset{(0.59)}{}$$

where house price is the median housing value from the 2000 census and income is the median family income from the same source. A one dollar increase in family income is associated with \$5.47 higher house prices according to this linear model and the R^2=0.47. The coefficient is on the low end of the estimates predicted by the urban approach, but it is at least double the approach suggested by a constant price-to-income ratio.[7] The price-to-income ratio is not constant across cities, but instead prices seem to rise in way that suggests that one extra dollar of housing price is associated with about 18 cents of extra housing costs.

To make a comparison between the two approaches using aggregate time series, we pin down price in at least one location, by assuming that housing in that location is elastically supplied at a cost C_t and that wages are $\underline{W}_t$. If wages are treated as being exogenous in the other locations (W_{it}), then housing prices in each other location will equal $H_{it} = C_t + \frac{W_{it} - \underline{W}_t}{r+m}$ and average housing costs will be $C_t + \frac{\hat{W}_t - \underline{W}_t}{r+m}$ where $\hat{W}_t$ is the average income in the country. This calculation only suggests a constant price-to-income ratio if construction costs and wages in the reservation locale rise one-for-one with income. If construction costs go up by less than income, as they have over the last 30 years, then this will push the national price-to-income ratio downward. If income in

[7] The r-squared from a regression based on the constant price-to-income ratio assumption where the slope equals 2.39 (the mean ratio) finds a much lower R^2 of 0.31.

the reservation locale goes up by less than the average income in the country (which also appears to be the case), then this will push the price-to-income ratio upward.

We now turn to the historical relationship between national income and housing prices in our sample. When we regress local house prices in market i in year t ($HP_{i,t}$) on real GDP per capita as in $HP_{i,t} = \alpha + \beta * RealGDP_t + \varepsilon_{i,t}$, the R^2 is only 3%, with the estimated coefficient of β implying that an addition billion dollars in real GDP is associated with \$3.28 higher house prices.[8] If this regression is run with both variables logged, the elasticity of house price with respect to GDP is only 0.07 (standard error = 0.03 when clustering by year). Thus, the marginal effect of added national economic activity on local house prices is small on average. The impact is even more negligible if we stop the sample in the year 2000. The R^2 from that regression is 0.001, with the estimated β being 0.81 (and only marginally significant with a t-statistic of 1.81). Excluding California markets from the sample also reduces the estimated effect. A billion dollars of real GDP is associated with only \$2.11 higher house prices for the 49 other states over the full sample period.

Adding each metropolitan area's house price in 1980 ($HP_{i,80}$) as well as the interaction of this early year house price with national real GDP as in

$$HP_{i,t} = \alpha + \beta * HP_{i,80} + \gamma * RealGDP_t + \delta(HP_{i,80} * RealGDP_t) + \varepsilon_{i,t}$$

does find that the more expensive house price markets are more sensitive to the national business cycle, but the economic relevance is small. The estimate of β is 0.82 (standard error of 0.05), that for γ is -7.20 (0.73), and δ is 0.000091 (0.000011). The marginal effect of real GDP is computed as $-7.20 + 0.000091 * HP_{i,80}$. Using the mean annual

[8] There are 3,132 observations in this regression and we use the OFHEO-based real price series discussed above.

change in real GDP since 1975 of $450 billion and plugging in values from the distribution of 1980 house prices finds that the median market's price is only 0.5% higher (as a percentage of 1980 value) if national GDP rises by $450 billion. The market in the 90th percentile of the 1980 price distribution experiences a 1.7% higher price. This is more than triple the marginal impact of the median value market, but the economic importance remains small.

Interest Rates and House Prices

A second macroeconomic variable that is commonly thought to be a major driver of national housing pirces in interest rates. In this case, there is no disagreement that falling interest rates should cause an increase in the willingness to pay. The formula $C_t + \frac{\hat{W}_t - \underline{W}_t}{r+m}$ also makes this clear. However, the popular press and recent academic research have implied interest rates have played crucial roles in influencing values, especially during the recent boom, but there are two reasons to question the view that interest rates are particularly influential empirically: low estimated coefficients and low explanatory power when housing prices are regressed on interest rates.[9]

Regressing house prices for our panel of OFHEO markets on the real interest rate for a 10 year Treasury bond as in $HP_{i,t} = \alpha + \beta*RealRate10yr_t + \varepsilon_{i,t}$ does yield a statistically significant negative estimate for β, but the economic significance is

[9] Reports in the popular press are far too numerous to catalogue. One only needs to google the phrase 'interest rates + house prices' to find numerous reports attributing the recent boom to low rates and forecasting a general price collapse if rates keep rising. On the academic front, McCarthy and Peach (2004), Himmelberg, Mayer, and Sinai (2005), and Smith and Smith (2006) each use relatively low interest rates to largely or partially account for why current prices are not a bubble.

marginal.[10] A 100 basis point increase in the real rate being is associated with only a $5,856 lower price. Moreover, variation in long rates explains only 2% of the variation in house prices across markets over time.[11]

There are two explanations why the house price-interest rate relationship is much weaker in reality than is generally conceived publicly. First, housing economics tells us that there cannot be capitalization of higher interest rates (or anything else for that matter) in markets with perfectly elastic supplies of units. Assuming an infinite elasticity is admittedly extreme, but there are many markets in the United States such as Atlanta, Charlotte, Phoenix (most of the sun belt, in fact; see the discussion below on supply) in which new supply is readily brought to market. In places like these, prices pretty much are pinned down by the physical cost of construction and variability in interest rates can affect house prices only to the extent they impact that cost of construction.[12]

However, even in coastal housing markets such as the Bay Area and New York, where supply is inelastic, the correlation between interest rates and housing prices is modest over longer periods of time. In principle, a 100 basis point increase in the mortgage rate from 6% to 7% raises the monthly cost of a $480,000 30-year mortgage (80 percent of San Franscisco's median price of $600,000) by $297 and is associated with $106,920 in added debt service payments over the life of the loan. Despite this large number, we do not see a significant relationship between long rates and house prices in

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[10] The real rate is calculated as the nominal rate less the inflation forecast reported in the Livingston survey. This is identical the method used in Himmelberg, Mayer, and Sinai (2005), and we thank Todd Sinai for providing these data.

[11] This regression also contains 3,132 observations and the standard set of metropolitan areas. In this case, we use all available price and interest rate data back to 1975. [The panel is unbalanced prior to 1980, but this does not affect the results in any material way.] In addition, the results are not materially affected if we use the log of real house prices as the dependent variable. In that case, a 100 basis point higher ten year real rate is associated with 3.81 percent lower prices, with the R^2 still only 2 percent.

[12] There certainly is a capital cost to new construction, but it is relatively small. Data on physical construction costs show that labor alone represents about two-thirds of the total (Gyourko and Saiz (2006)).

the coastal markets with the most inelastic supplies of housing. Figure 6 plots the OFHEO log real price index for San Francisco against the ten year real rate. In this market and other like it, the correlation weakens considerably when one looks past the last few years of the recent boom.[13]

While an elastic supply schedule cannot account for the paucity of capitalization here, the option to refinance can. If the owner facing the hypothetical 100 basis point increase in rates thought it could refinance at the lower 6% rate within five years, the added debt service payments would be no more than \$17,820 which is just under 3% of house price.[14] Thus, for changes in interest rates to have a really large impact on house prices in supply constrained markets, people would have to expect they would not be able to refinance for a very long time.

While we will not analyze mean reversion in interest rates[15], there is no doubt that long real rates are relatively volatile as documented in Figure 6. Over the past 30 years, the average absolute annual change in the ten year real rate has been 76 basis points, and the standard deviation about that change is 75 basis points.[16] Federal Reserve Board data based on responses to questions in the *Survey of Consumer Finances* (*SCF*) and other special surveys confirm that refinancing has become quite widespread, although this was not always the case. For example, data from the 1977 *SCF* showed that only 8% of owners had refinanced the loan on their current home. Later surveys from 1994 and 1997

[13] When the log of the real OFHEO index is used as the dependent variable, the coefficient is -4.79(standard error=3.21 for a t-statistic of -1.49) and the R^2 is 0.08. Similar results obtain for other markets thought to have inelastic supply schedules.
[14] The \$17,820 figure represents the (undiscounted) additional debt service payment stream if one refinanced at 6% on the last day of year five.
[15] There is growing evidence for mean reversion in interest rates (e.g., see Wu and Zhang (1996)), but this is a very complex issue because it involves the full term structure and is clearly beyond the scope of this paper.
[16] The average annual change in 10 year rates over this time period is only four basis points, so the positive and negative moves are fairly symmetric.

reported 45% and 49%, respectively, had refinanced at least once. Federal Reserve Board staff link this upsurge to changes in underwriting technology that has lowered the costs of refinancing, as well as the secular decline in long rates over the past few years. The dollar-weighted average remaining maturity just before refinancing was 23 years and 10 months according to the latest Federal Reserve Board analysis of *SCF* data, suggesting that owners typically are able to refinance within five to ten years of taking out their mortgage.[17]

The Net Present Value Approach to Housing Prices

The purely financial approach to housing prices sees an analogy between stock prices and housing prices: if stock prices reflect the net present value of future dividends, then housing prices should equal the net present value of future rents. More precisely, a representative consumer has the choice of renting a unit at cost R(t) or purchasing the same unit at cost H(t). If the person buys the home, he pays maintenance costs of M(t). The nominal interest rate is i(t), where $i(t) = r(t) + \pi(t)$, with r(t) representing the real rate and $\pi(t)$ reflecting expected inflation. As interest is deductible, the owner only pays $1-\tau$ times i(t), where τ represents the effective tax rate. We ignore local property taxes and many other complications of the user cost formula because our purpose is to illustrate general features of the model rather than to calibrate it exactly.

Given these assumptions, $(1-\tau)i(t)H(t) + M(t) - E\big(H(t+1) - H(t)\big) = R(t)$ results as a core indifference relationship. If $E\big(H(t+1)\big) = (1 + a(t) + \pi(t))H(t)$, where a(t) is expected real appreciation, then

[17] See the reports by Brady, Canner, and Maki (July 2000) and Canner, Dynan, and Passmore (December 2002) in the *Federal Reserve Bulletin* for added detail on these numbers and others.

$$(2)\quad H(t)=\frac{R(t)-M(t)}{(1-\tau)r(t)-\tau\pi(t)-a(t)}$$

holds. If the parameters R(t), M(t), (r) and τ are known, and if the value of a(t) is based on the long-run appreciation rate of housing, then (2) yields a value of housing predicted by rents.[18]

There are three related problems associated with using this type of calculation: (1) it is logically inconsistent for the net present value relationship to hold for both landlords and tenants; (2) this logical inconsistency is explained by unobserved maintenance costs and unobserved demand for the benefits of living in owner-occupied housing that make this type of calculation impractical to use with any precision; and (3) the stock of owned and rented units are very different, which further confounds attempts to use this calculation.

To more clearly illustrate the first point, consider a potential landlord who must be indifferent between buying and not buying a house. If

$R(t+j)-M(t+j)=(1+a)^{j}(R(t)-M(t))$ for all j and

$\lim_{j\to\infty}E((1+(1-\tau)r)^{-j}H(t+j))=0$, then (2) can be rewritten as

$H(t)=\dfrac{(1+(1-\tau)r)(R(t)-M(t))}{(1-\tau)r-a}$. For a landlord to be willing to purchase a house and then rent it out, the purchase price must equal the discounted value of future rents, where the discount rate has no adjustment for the tax benefit of being an owner-occupier so that

$$(3)\,H(t)=\sum_{j}\frac{R(t+j)-M_{L}(t+j)}{(1+r)^{j}}=\frac{(1+r)(R(t)-M_{L}(t))}{r-a},$$

[18] It is noteworthy that Poterba (1984), who introduceding this approach, neither considers the own/rent margin nor equates the utility flow from owning with the observed rental price of a house. He employs the user cost formula to figure out the cost to owners, which then shifts the demand for the quantity of housing.

where $M_L(t)$ reflects the landlord's maintenance costs.

If maintenance costs were the same for owner-occupiers and landlords, the former would be willing to pay $\frac{(1+(1-\tau)r)(r-a)}{(1+r)((1-\tau)r-a)}$ times what landlords would pay for the same house. If interest rates are five percent, appreciation is two percent, and the tax rate is twenty percent, then this ratio equals 1.49, so a renter who wants to become an owner-occupier would be willing to outbid the landlord by almost 50 percent. If housing prices are at the right level to make a potential owner-occupier indifferent between buying and renting, then they are too low for a landlord to be willing to buy and rent out the unit.[19]

We can reconcile these inconsistent indifference relationships by assuming that maintenance costs differ severely between landlords and renters or that there are unobserved benefits to owning that are not observed. However, once we think that unobserved benefits and costs are important enough to justify a 49 percent difference in the willingness to pay, we conclude that the net present value relationship is far too imprecise to be used to evaluate the appropriateness of house prices.

In addition, while some authors (Smith and Smith, 2006) have worked extremely hard to find comparable renter and owner-occupied housing, the two types of housing are

[19] Another way to think about this problem is to assume that both owner-occupiers and landlords live for exactly one period and can invest and receive returns equal to r. All net income of the landlord is taxed at a rate τ; the homeowner's return on equity is not taxed. An owner who has Y dollars at time t, occupies his own home and then sells it at the start of period t+1 for a deterministic price H(t+1) will have $(1+r(1-\tau))(Y-H(t))+H(t+1)-M(t)$ at time t+1. If that same individual rents, his net wealth will equal $(1+r(1-\tau))Y-R(t)$, so renting dominates owning $H(t)>\frac{H(t+1)+R(t)-M(t)}{1+r(1-\tau)}$. A landlord will earn $(1-\tau)rH(t)$ if he invests in the asset market and $(1-\tau)((H(t+1)-H(t)-M_L(t)+R(t))$ if he invests in a house which he then rents out. As such, it only makes sense to buy a house and rent it out if $\frac{H(t+1)+R(t)-M_L(t)}{1+r}>H(t)$.

generally are quite different in both physical structure and location within a city. Eighty-five percent of single-family detached homes are owned and eighty-five percent of multi-family units are rented (Glaeser and Shapiro (2004)). Presumably, the extremely strong correlation between ownership type and structure type results from the fact that agency problems mean that maintenance costs are minimized by having one owner for each single-family structure. The view that rents and housing prices are best seen as the costs of very different housing stocks is further supported by the fact that owner-occupied housing is generally much further from the urban core and by the modest correlation (53 percent in our sample) between rental costs and housing prices within metropolitan areas.

Of course, there will be some sort of indifference relationship between these two types of housing, albeit one that is not easy to quantify in the way suggested by equation (2), both because of omitted costs and benefits and because there are even more limits to arbitrage in this market than there are in financial markets (Shleifer and Vishny, 1997). For example, risk aversion ensures that people will turn down opportunities to delay purchase or sale even if housing prices can be reliably predicted to move in one direction.

Suppose there is an individual who knows that he will eventually own a home in a given area, and to make things simple, assume that he can either buy at time t or wait until time t+1. Initially ignoring risk aversion, we further assume this person is maximizing $E(V(Wealth_{t+1}))$, where $Wealth_{t+1}$ refers to wealth net of housing costs. If the person buys at time t, he receives $V((1+(1-\tau)r)(Y-H(t))-M(t))$. If he rents at time t and then buys, he receives $E(V((1+(1-\tau)r)Y-R(t)-H(t+1)))$. If $H(t+1)=H(t)+\overline{H(t+1)-H(t)}+\varepsilon(t)$, where $\varepsilon(t)$ is mean zero and $\overline{H(t+1)-H(t)}$ is the predictable component of the change in housing price, then using a second order Taylor series approximation, waiting makes sense if and only if

$$(4)\ \left(1-\tau\right)rH(t)-\overline{H(t+1)-H(t)}-R(t)+M(t)>\frac{\sigma Var(\varepsilon)}{\left(2-z\sigma\right)\left((1+(1-\tau)r)(Y-H(t))-M(t)\right)}$$

where $\sigma=-\frac{\left((1+(1-\tau)r)(Y-H(t))-M(t)\right)V''\left((1+(1-\tau)r)(Y-H(t))-M(t)\right)}{V'\left((1+(1-\tau)r)(Y-H(t))-M(t)\right)}$ and

$$z=\frac{\left(1-\tau\right)rH(t)-\overline{H(t+1)-H(t)}-R(t)+M(t)}{\left((1+(1-\tau)r)(Y-H(t))-M(t)\right)}.$$

The standard deviation of annual housing price changes for the 115 markets in our data for which we have consistently defined prices since 1980 is \$11,134. If the coefficient of relative risk aversion is 2 and if we assume that non-home wealth of the person who buys at time t is \$50,000, then the expected gains from waiting would need to be at least \$2,700. Essentially, the plausible gulf between user cost and rent has increased by more than \$200 per month because of risk aversion. Higher levels of risk aversion cause the willingness to buy now to increase proportionately even with expected short term losses. As we estimate that only 10 percent of the observations in our sample face expected losses over the next year of more than -\$3,091, the potential gains from waiting one year to buy in a declining market are almost always overwhelmed by the added risk taken on in waiting to buy.

Risk aversion point will also limit arbitrage among those people who know that they are about to sell, since waiting forces them to take on more risk from fluctuating housing prices. Among this group, wealth will be higher and this will increase the ability to arbitrage rising prices by refraining from selling houses. However, the non-pecuniary costs of waiting are likely to be much larger, and considering how many people never sell their homes, it may be that at this end there also is less ability to arbitrage.

IV. A Dynamic Model of Housing Prices

Because the financial approach to homeownership is not well suited to comprehending the heterogeneity we see across local housing markets, we turn to a dynamic version of the urban model first formally introduced by Rosen (1979) and Roback (1982). In doing so, we abstract from interest rate changes, assuming that r(t)=r, a fixed real rate. We will also assume that inflation is zero. This is not because we believe these variables literally have no influence on local house prices, but to simplify the model so that we can determine how much explanatory power the urban economics framework provides for understanding house prices and their cyclical behavior.

From the perspective of urban economics, the fundamentals are not rents, but the productivity and amenities of an area. These fundamentals then move wages and the willingness to pay to live in an area, so that housing demand interacts with housing supply to form price. This Rosen-Roback approach assumes that consumers are indifferent across space, either at every point in time or over the course of their lifetimes. As discussed earlier, the instantaneous spatial indifference condition requires that $Utility(W, A, R) = \underline{U}$, where W refers to wages, A refers to amenities, R refers to the cost of housing and $\underline{U}$ is reservation utility.

We now consider a dynamic model in which people buy homes, rather than rent, so we will assume that there are two locales: a city and a reservation locale. The utility flow for person i living in the city during period t is $W(i,t) + A(i,t) - R(t)$, and people discount utility at a constant interest rate. We also assume that there is a reservation locale that delivers utility of $\underline{U}(i,t)$ and where housing always costs "C", which represents a constant physical cost of construction. We further assume that $W(i,t) + A(i,t) - \underline{U}(i,t) = D(t) + \theta(i)$, so that if people are buying during each period, the indifference condition becomes $H(t) = D(t) + \theta(i^*) + \frac{rC}{1+r} + \frac{E(H(t+1))}{1+r}$.

We assume that $D(t) + \theta(i^*) = \overline{D} + qt + x(t) - \alpha N(t)$, where $\overline{D} + qt$ are deterministic demand factors, x(t) includes shocks to either labor productivity or tastes, and α captures the fact that wages may fall with population and that the marginal resident's taste for living in this city will decline with the number of residents. When we later turn specifically to wages, we assume that $W(i,t) + A(i,t) - \underline{U}(i,t) = D(t) + \theta(i)$. This equation gives us that a permanent increase in population is associated with a $-\frac{1+r}{r}\alpha$ decrease in housing prices.

Housing does not depreciate in this model, so that $N(t) = N(t-1) + I(t)$ where I(t) is the housing built in each period and N(t) is the stock of housing at time t. To capture investment costs, we assume that there is free entry of builders, all of whom must pay $C + c_0 t + c_1 I(t)$ for each new housing unit that is constructed. These investment decisions for time t are made based on time t-1 information, so firms will build housing to the point where $C + c_0 t + c_1 I(t)$ equals expected housing price. These costs are meant to capture both the physical costs of new construction and the legal and administrative costs associated with land use regulation and land assembly. One can extend the model either by assuming that the overall population increases construction costs or that construction costs increase with both the level of investment and the rate of change of investment (as in Rosen and Topel, 1988). The latter assumption creates a slower adjustment of housing construction to shocks.

We assume that $x(t) = \delta x(t-1) + \varepsilon(t)$, where $0 < \delta < 1$ and the $\varepsilon(t)$ are i.i.d. with mean 0. We also use the notation: $\hat{H}(t) = C + c_0 t + c_1\left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right)$,

$\hat{I}(t) = \frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)} = \hat{I}$ and $\hat{N}(t) = \frac{\overline{D}}{\alpha} + \frac{(1+r)(\alpha c_0 - rc_1 q) + r^2 c_0 c_1}{\alpha^2 (1+r)^2} + \left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right)t$,

with these quantities representing constant growth steady state housing prices, investment and population (respectively) of this city if x(t)=0. In this model, secular trends in the demand for an area are reflected in the steady state level of prices, but not in a steady state increase in growth of prices because population grows exactly enough to offset rising demand. Price rises only with increases in the costs of construction. With these basics, we turn to the solution to the model:

Proposition 1a: At time t, housing prices equal

$H(t) = \hat{H}(t) + \frac{1+r}{1+r-\delta\phi}x(t) - \frac{\alpha(1+r)}{1+r-\phi}\left(N(t) - \hat{N}(t)\right)$, and investment equals

$I(t+1) = \hat{I} + \frac{\delta(1+r)(\phi-\delta)}{(1+r-\delta)(1-\delta)c_1 - \delta(1+r)\alpha}x(t) - (1-\phi)\left(N(t) - \hat{N}(t)\right)$, where

$$0 < \phi = \frac{(2+r)c_1 + (1+r)\alpha - \sqrt{r^2 c_1^2 + 2(2+r)(1+r)\alpha c_1 + (1+r)^2\alpha^2}}{2c_1} < 1.$$

A shock to the desirability of a locale immediately increases prices by $\frac{1+r}{1+r-\delta\phi}$ times the flow increase in the amount of utility from the area. If we let the ratio $\frac{\alpha}{c_1}$ go to zero reflecting increasingly inelastic supply of housing, then this coefficient goes to $\frac{1+r}{1-\delta+r}$, which is the coefficient on shocks if the housing supply in the area is fixed.

Proposition 1a also gives us predictions about the long-run changes in housing prices:

$$H(t+j)-H(t)=\frac{(1-\phi)c_0+(1+r)q}{1+r-\phi}j+\frac{1+r}{1+r-\delta\phi}(x(t+j)-x(t))-\frac{\alpha(1+r)}{1+r-\phi}\left(N(t+j)-N(t)\right)$$

.

Holding actual population changes constant, the model predicts that prices should be increasing with rising supply costs, rising demand, and increases in the underlying shock, and decreasing in the amount of new construction. [All proofs are in the appendix.]

Proposition 1b: At time t, the expected values of time t+j housing price, construction and population for all j>0 equal:

$$\hat{H}(t+j)-c_1\left(\frac{\delta(1+r)\left(\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)\right)x(t)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}+\phi^{j-1}(1-\phi)\left(N(t)-\hat{N}(t)\right)\right),$$

$$\hat{I}(t+j)-\left(\frac{\delta(1+r)\left(\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)\right)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}x(t)+\phi^{j-1}(1-\phi)\left(N(t)-\hat{N}(t)\right)\right) \text{ and}$$

$$\left(1-\phi^j\right)\hat{N}(t)+\phi^j N(t)+\frac{\delta(\delta^j-\phi^j)(1+r)x(t)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}\ .$$

If x(t)=0, then population will converge monotonically to its steady state level. Prices and investment will converge on their steady state levels from below if initial population is higher than steady state and converge from above if initial population is below steady state. The rate of convergence is determined by ϕ which is a function of r and the ratio $\frac{\alpha}{c_1}$. Higher levels of $\frac{\alpha}{c_1}$ will cause ϕ to monotonically decrease and convergence to speed up.[20]

The impact of a shock x(t) is explored in the next proposition:

Proposition 2: If $N(t) = \hat{N}(t)$, $x(t) > 0$, and $\phi \neq \delta$, then investment and housing prices will initially be higher than steady state levels, but time t expected values of time t+j construction and housing prices will lie below steady state levels when $j > 1 + \ln\left(\frac{1-\phi}{1-\delta}\right) \Big/ \ln\left(\frac{\delta}{\phi}\right)$. The situation is symmetric if x(t)<0.

Proposition 2 highlights that this model not only delivers mean reversion, but overshooting. Construction responds strongly to a positive shock, but eventually this means that prices and construction are lower than they would have been without the shock because the shock has disappeared because of the assumed mean reversion, yet the extra units are still there. If the shocks were permanent, then a positive shock would always lead to an increased level of production and housing prices at all future points.

The main value of this model is as a calibration tool, and Proposition 3 tells us about the time period needed for a shock to work its way through the system and the amount of new housing that will be produced over that time period.

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[20] Since ϕ is the smaller root of $x^2 - ((1+r)\frac{\alpha}{c_1} + 2 + r)x + (1+r) = 0$, the assertion follows from the fact that for $b, c > 0$ and $b^2 > 4c$, the smaller root $x^* = \frac{1}{2}\left(b - \sqrt{b^2 - 4c}\right)$ of $x^2 - bx + c = 0$ has $\frac{\partial x^*}{\partial b} = \frac{1}{2}\left(1 - \frac{b}{\sqrt{b^2 - 4c}}\right) < 0$.

Proposition 3: (a) If at time *t*, $N(t)=\hat{N}(t)$ and $x(t)>0$, and there are no further shocks to x after time t, then at time $t+j^*$ the expected excess of housing price above its trend will fall to $\lambda \in [0,1]$ times its time *t* level, where j^* satisfies $\frac{1-\lambda}{c_1\delta(1+r-\delta\phi)}=\frac{\phi^{j^*-1}(1-\phi)-\delta^{j^*-1}(1-\delta)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}$. (b) The increase in the number of units produced at time t+j will equal $\frac{\delta(\phi^j-\delta^j)}{c_1\left(1+\delta^j(1-\delta)-\delta\phi^{j-1}(1-\phi)-\delta/\phi\right)}$ times x(t).

The first part of Proposition 3 tells us how quickly we should expect shocks to housing prices to disappear as a function of the parameters of the system. The second part tells us how much new housing will be built to achieve any given amount of price response to an initial shock.

The model also delivers formulae (contained in Appendix 1) for the variance of price changes, and the mean reversion of those changes. We use those formulae is Section VI, but first we turn to the estimation of the model's key parameters.

V. Estimating the Parameters

While the model has a number of parameters, most of these are eliminated when we control for area-specific means or trends. In all of our calibrations, we will consider variance and coefficients only relative to metropolitan area fixed effects and year fixed effects. With those controls, the moments of high frequency fluctuation require five parameters: the real interest rate (r), the mean reversion of economic shocks (δ), the variation of those shocks ($Variance(\varepsilon)$), the degree to which demand declines with city population (α), and the degree to which construction responds to higher costs (c_1).

Of these parameters, the interest rate is the least significant in determining magnitudes. We will generally work with a value of .05 for r, but results are not particularly sensitive to alternative values of this parameter. We will estimate a common value of δ across cities, but will allow c_1 to vary. We will also allow α to take on a range of values reflecting our inability to estimate this parameter accurately.

The Response of Wages to Population and the Mean Reversion of Wages: δ and σ_ε^2

We assume that $W(i,t) = \omega_0 + \omega_1 t + x(t) - \gamma N(t)$ so that it is wages, not amenities, that have high frequency variation. Thus $W(i,t+j) - W(i,t) = \omega_1 j + x(t+j) - x(t) - \gamma\alpha(N(t+j) - N(t))$. To use income data to assess the variance of ε and its mean reversion, we must adjust wage changes for local trends and for changes in N(t) which reflects shifts in labor supply. We adjust for local trends (and national shocks) by looking at wage changes first controlling for year and area fixed effects. The correction for annual changes in labor supply is slightly more difficult both because we do not observe annual shifts in labor supply and because of uncertainty about the value of γ. While these make things difficult, the magnitude of annual changes in labor supply is sufficiently small that our estimates of δ and the variance of ε do not depend much of these assumptions.

The parameter γ reflects the impact that an extra member of the labor force has on the wage rate. Since the customary literature uses labor demand elasticities, which are $\frac{Labor\ \ Force}{Wage}\frac{\partial Wage}{\partial Labor\ \ Force}$, we will have to isolate the latter term in this expression which is of most interest to us by multiplying by the ratio of the wage to the labor force in each market. We use the Bureau of Economic Analysis measure of personal income per capita as our measure of wages. For our sample of metropolitan areas, the mean was \$26,965 (in \$2000) in 1990 which is in the middle of our sample period. Mean employment in 1990 across these metropolitan areas was 539,215, so our ratio of wage to the labor force is about 0.05 (~26,965/539,215).

As for the labor demand elasticity, there is not a strong concensus. Much of the recent work in this area has focused on the impact of immigrants on wages, where immigrants create a potential shock to local labor force that can be used to identify labor supply. Most of the estimates are statistically indistinguishable from zero (e.g., Card, 1991). Borjas (2003) finds a higher estimate of -.3, although this is at the national level. We will take -0.3 to 0.0 as our range of estimates, which means that the range of values for $\gamma\alpha$ that we consider is between 0 and -0.02 (i.e., -0.3*0.05~0.02). In the simulations discussed below, we use a value of 0 and of -0.02.

Next, we must multiply this value times a measure of the available annual labor force. Unfortunately, high frequency data on the labor force is not available at the metropolitan area level. Using data sets like the *Current Population Survey* is impossible because of the small sample sizes for individual areas, so we turn to housing permit data instead. The change in the labor force is proxied by the ratio of the permit share in each year of a decade times a predicted change in the labor force in the metropolitan area over that decade.[21] To translate our housing-based number into a measure of labor force change, we multiply by 1.26—the ratio of labor force to housing units in the 1990 census—to provide an annual estimate of the change in the total labor force in the area in

[21] For the purposes of this calculation, we assume the years since 2000 are part of the 1990s.

each year. The mean of this labor force change variable, denoted $dhouseadj_{i,t}$, is 4,529 with a standard deviation of 8,028.

Adjusted wage change estimates ($dwageadj_{i,t}$) then are computed for different time horizons. For the 1-year case, this variable is defined as follows:

$dwageadj_{i,t} = dHHPersonal\ Income_{i,t} - \gamma * dhouseadj_{i,t-1}$,

where unadjusted wages in each area i and year t are measured by per capita BEA personal income that is scaled up by 2.5 to reflect a household-level value ($dHHPersonalIncome_{i,t}$), the dhouseadj variable is our estimate of the change in the labor force. Note, also, that we use a lagged value of this variable when computing the wage change.[22]

Table 2 reports our estimates of the variance in the wages shock and the mean reversion of that shock based on one and three year estimates of the variance of x(t+j)-x(t). The variance of a one year change in x(t) equals $(1-\delta)^2 Var(x(t)) + Var(\varepsilon)$. The variance of a three year change in x(t) equals $(1-\delta^3)^2 Var(x(t)) + \frac{1-\delta^6}{1-\delta^2} Var(\varepsilon)$. For x(t) to have its steady state variance level, it must be that $Var(x(t)) = \frac{Var(\varepsilon)}{1-\delta^2}$, and we assume that this condition holds so that the variance of one year changes in x(t) equals $2Var(\varepsilon)/(1+\delta)$ and the variance of three year changes in x(t) equals $2(1-\delta^3)Var(\varepsilon)/(1-\delta^2)$.

The impact of different values of labor demand appears to be relatively small, presumably because the implied labor supply doesn't change that much from year to year.

[22] For longer horizons, we use the same formula, but include additional lags of the dhouseadj variable to capture the changes to the local labor force that occur over multiple years.

In addition, the reported variances are associated with standard deviations of $1,350, which does not seem unreasonable for wages.

The Response of Construction to Housing Prices: c_1

The parameter c_1 is the extent to which construction costs rise with the number of new units that are being built, but it can also be understood as the inverse of new construction with respect to expected future prices. As such, it can be estimated using the relationship between prices and new construction. The usual approach in this literature is to use a logarithmic formulation where the logarithm of the number of new units is regressed on the logarithm of price. The logarithmic functional form is attractive because it yields much more precise estimates than the linear estimates that we use in the next section. Using this methodology, Topel and Rosen (1988) estimate a housing supply elasticity of approximately 2.

We can use a similar approach and estimate an ordinary least squares regression with city and year fixed effects that yields the following results,

$$Log(Permits_{i,t}) = \underset{(0.13)}{0.68} Log(HP_{i,t}) + \gamma MSA_i + \delta Year_t + \varepsilon_{i,t},$$

where there are 2,899 observations and the R^2=0.87. This specification has, of course, a potentially severe endogeneity problem because we are regressing a quantity variable on price. A second problem is that we are using actual price rather than the expectation of next year's price.

We can instrument for price using three labor demand shift variables that we describe more fully in the appendix. Briefly, we constructed three instruments for local income that represent shocks to specific business sectors. The first is an interaction of the real price of oil per barrel with local employment share in the energy sector in each MSA as of 1980 (the beginning of the sample period).[23] The second instrument is created by interacting a technology firm earnings index with the local share of employment in technology industries as of 1980. The earnings index uses annual operating earnings as reported in the CRSP/Compustat merged data base. The firms used were those classified as technology companies each year in the S&P500 Index, with firm weights based on each company's share of aggregate index market capitalization (in the previous year).[24] The third instrument is very similar except it is for the pharmaceutical industry. That is, an earnings index is interacted with firm operating earnings drawn from the CRSP/Compustat files.

With these instruments we estimate:

$$Log(Permits_{i,t}) = \underset{(0.39)}{2.12} Log(HP_{i,t}) + \gamma MSA_i + \delta Year_t + \varepsilon_{i,t},$$

using the same sample. The coefficient has gone up, as basic economic intuition suggested it would, since the reverse causality problem introduces a downward bias. While we are not confident of this estimate, we do believe that 0.6-2.1 provide a reasonable bound on the magnitude of this elasticity. Moreover, the coefficient of 2 is quite close to that estimated by Rosen and Topel (1988).

To transform this elasticity into a linear relationship between price and the number of new units, we must multiply this by the ratio of the number of new units to price. The median price in our sample is about \$105,000. The median number of new units is 4,783. The tenth percentile of the number of new units (as reflected in annual

[23] See the Appendix for further detail, including the specific employment subsectors used in creating this and the other two instruments.

[24] Standard & Poor's does not classify firms on a consistent basis prior to 1990, but Jeremy Siegel has done so for his research on long-run stock returns. We rely on Siegel's judgment and are extremely grateful for his provision of these data which go back to 1980. See the appendix for further detail.

permits) is 1,487 and the 90^{th} percentile is 18,685. Using these numbers gets one a range a range for c_1 of between 3 and 16 dollars. Matching up different values from the price distribution and using data from the tails of the permitting distribution can generate a range of from \$2 and \$30, where the \$2 figure reflects the relatively easy supply conditions of a high supply market like Houston and the \$30 value reflects constrained building conditions in many coastal coastal markets.

Using a linear specification, if one includes the California metropolitan areas, it is difficult to find any statistically or economically meaningful relationship between permits and prices. Hence, our estimate of c_1 from a linear model is based on a 96 metropolitan area sample that excludes the MSAs in California. To try to capture the range of impacts across areas that readily versus only sparingly allow new building, both the level of real house prices and real house price interacted with the share of permits issued between 1975-1979 as a fraction of the preexisting housing stock in 1970 are included as regressors. More specifically, we estimate the following equation

$$Permits_{i,t} = \alpha HP_{i,t} + \beta(HP_{i,t} * PermitShare7579_i) + \gamma MSA_i + \delta Year_t + \varepsilon_{i,t},$$

where $Permits_{i,t}$ represents total housing permits issues in metropolitan area i in year t, HP reflects real house prices using the OFHEO index as described above, $PermitShare7579_i$ captures the permit share prior to our estimation period for each area, and MSA and Year are vectors of metropolitan area and year dummies.

Because of the potentially severe endogenity involved in regressing a quantity on a price, we again instrumented using the three labor demand shift variables described above.[25] The results as follows: α=-0.025(0.081); β=1.190(0.651). The level effect is small and insignificantly different from zero, while the interaction term is fairly large, although only marginally significant. The marginal effect of a change in house price is given by the following: -0.025 + 1.190*PermitShare7579$_i$. For the interquartile range of our data, the impacts range from .10 to .25. From the 10th percentile market (in terms of prior period permit share) to the 90th percentile market, the impact ranges from .05 to .36. Thus, in terms of the c_1 parameter, values range from about 2 to over 20. These numbers correspond to these given by the elasticity estimates and so we will use this range.

The Response of Housing Prices to New Construction: α and ϕ

We have the least guidance in our choice about α, which we based on empirical estimates of the elasticity of demand for housing. Generally, such demand elasticities have a range between 0 and 2. However, we still need to make two transformations to derive α from demand elasticities. The first is to transform an elasticity in a levels estimate; second, we need to adjust for the fact that α gives the impact of more housing on the flow of utility, not on the stock price of housing. To transform the elasticities into levels coefficients, we multiply by the ratio of price to population. Which ratio value to use is not clear because it varies a good bit across housing booms and busts. Experimentation showed that an extreme range for the derivative of housing prices respect to population is from 0 to 3.

Since α reflects the response of the flow of utility to population size in steady state, the impact of a permanent increase in population will equal $\frac{(1+r)\alpha}{r}$ times that

[25] The procedure is more complicated because of the interaction term. There are six instruments in this case, as we create the interactions of our shifters with the preexisting permit share. The instruments are not quite as powerful in this case, with their t-statistics in the first stage not exceeding 8.

increase. Therefore, the value of α is found by multiplying the estimated housing demand effects by $\frac{r}{1+r}$. This yields a range of estimates of α going from 0 to 0.15. A value of 0.05 implies that for every 1,000 extra homes sold, the marginal purchaser likes living in the area $50 less per year.

For our calibration, the independent values of α and c_1 only matter in their influence on ϕ, which is in fact only a function of the ratio of these two parameters (increasing in c_1 and decreasing in α). If α varies from 0 to 0.2 and c_1 varies from 2 to 20, the range of ϕ is between 0.744 and 1. If $\alpha = .05$ and $c_1 = 10$, then our estimate of ϕ is 0.95. At $\alpha = .05$ and $c_1 = 2$, ϕ equals 0.87. The lower bound on ϕ that we think is reasonable is 0.82 which occurs when $\alpha = .1$ and $c_1 = 2$. Our best guess of ϕ is 0.92 occurs with $c_1 = 10$ and $\alpha = .1$.

VI. The Calibrated Model

We now see whether the calibrated model actually resembles the observed moments of the data. We first consider the relationship between prices and income shocks. We then turn to the level of mean reversion. Finally, we consider the variance of one- and five-year housing price changes.

The Response of Prices to Income Shocks: $\frac{1+r}{1+r-\delta\phi}$

The relationship between an income shock (x(t)) and housing prices is predicted by the model to be $\frac{1+r}{1+r-\delta\phi}$. If r=0.05, $\delta = .88$ and ϕ equals .75, then this parameter equals 2.7, which is the extreme lower bound predicted by our model. If ϕ equals 0.82, then the predicted relationship rises to 3.2. If ϕ equals 0.92, then the predicted relationship rises to 4.3, and if ϕ equals 0.95, then the predicted relationship is 5.

We estimate this parameter using the three instruments for income shocks that we used to look at housing supply elasticities, but first we show the basic OLS estimate if housing prices is regressed on BEA income. We first transform the BEA's per capita value into a household-level number by multiplying by 2.5. Then, we estimate the equation

$$H_{i,t} = \alpha_i + \beta_j{*}Trend + \gamma_t + \delta Income_{i,t} + \varepsilon_{i,t} \ ,$$

where $H_{i,t}$ represents housing prices in market i and year t formed as described above with the OFHEO real price index and the 1980 census median housing value, α_i is a vector of metropolitan area dummy variables, β_j is a vector of census division dummy variables[26] and Trend represents the number of years since 1980 (the beginning of our sample period), γ_t is a vector of year dummies, $Income_{i,t}$ is the adjusted-BEA income measure that varies across markets and over time, and ε is the standard error term. The OLS estimate of δ is \$3.05 (standard error of 0.32 when clustering by metropolitan area). More generally, the R^2 in this specification is 0.94, and as year and metropolitan area fixed effects together account for 89 percent of the variation in house prices, controlling for personal income explains about 45 percent of the variation in housing prices within metropolitan areas.

Of course, the OLS estimate is suspect because of the mutual endogeneity of prices and wages. When we instrument, the R^2 in the first stage is 0.94, and all three instruments have significant explanatory power, with none having a t-statistic below 11.

[26] There are nine dichotomous dummy variables for each census division. Because we are working in levels of prices and incomes in this specification, our model indicates that the data must be detrended.

With these instruments, the coefficient on income falls to $2.54 (standard error of 0.43) in our second stage.[27]

These estimates certainly are at the low end of our predicted values, and they are compatible only with an extremely low value of ϕ. Still, the fit isn't too bad and we may be underestimating the true empirical relationship between income and housing prices because of mismeasurement of income.

The Mean Reversion of Price Changes

We focus on the mean reversion of five year price changes, or covariance of five year change in price with the five year lag of that variable divided by the variance of the five year lag. As always in this calibration section, we consider only effects that have been normalized to eliminate metropolitan area and year fixed effects. The formula for mean reversion is contained in an appendix, but its value essentially depends only on the estimate of ϕ. When ϕ equals 0.8, the model predicts a mean reversion coefficient of -0.41. When ϕ equals 0.9, the model predicts a mean reversion coefficient of -0.31 and when ϕ equals 1, the model predicts a mean reversion coefficient of -0.24.

Our observed level of mean reversion is given in Section II where a five year increase in prices of one dollar is estimated to depress changes in prices over the next five years by 32 cents. Somewhat remarkably, this is almost exactly the level of mean reversion predicted by the model when ϕ equals 0.9. As such, the simple dynamic Alonso-Rosen model does a remarkably good job of predicting the mean reversion that we see in the data.

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[27] Full results are available upon request.

This mean reversion predicted by the model comes ultimately from two sources: the mean reversion of income and the supply of new housing. The model allows us to estimate the extent of mean reversion that is the result of new housing because we can calculate the level of mean reversion that would occur as the value of c_1 goes to infinity. In this case, the value of ϕ approaches 1 and the predicted level of mean reversion approaches -0.24, which is 75 percent of the mean reversion that we see in the data. This experiment suggests that one quarter of the mean reversion that we see in prices is the result of new construction and three quarters reflects the mean reversion of shocks to local income.

The Variance of One and Five Year Price Changes

Finally, we come to the most important moments in the data—the variation of one- and five-year price changes. We have estimated these variances correcting for year and metropolitan area fixed effects for three different samples: our entire metropolitan area sample, those areas with average growth in prices that is less than $5,000 per year and those areas with average growth in prices that is less than $1,550 per year (the median in the sample).

Table 3 reports actual variances, which clearly are much larger for those metropolitan areas with high appreciation rates. The variances predicted by our model are again a function of ϕ and are given in Table 4. Comparing the two tables makes it clear that we have little chance of explaining the entire sample. Only in the extreme case where $\phi = 1$ can we even predict the one year housing price changes. The five year housing price changes are over three times the predicted value even in that case.

In the restricted samples, we can readily predict the one-year price changes even with the lower values of ϕ. The five-year changes are compatible with the very high values of ϕ for the sample with average annual growth is below $5,000. The five year growth is compatible with more modest values of ϕ for the sample where average growth is below $1,550.

VII. Conclusion

We conclude from these results that the model does a decent job of matching the moments of the data for at least one half of the United States. Both the level of mean reversion and the variation of prices are in line with the predictions of the level. In another quarter of the country, the predictions are somewhat more strained but at least still compatible with reasonable parameter values.

However, in the areas of the country which have seen the fastest housing price growth over the past 25 years, the model completely fails to deliver the extremely high levels of variation in price changes that we see in the data. This remains the most pressing puzzle for future research. It is possible that we are seriously underestimating the income volatility of the marginal buyer by using BEA income values, which aggregates across all owners and renters. That is a pressing research task that will be addressed in the next version of the paper. Presently, we ignore amenities on the grounds that they are unlikely to be changing at relatively high frequencies. However, it may also pay to reexamine that assumption of good data can be found. Failing those two changes, unchanging results will point us in the direction of an area that may not be explicable using basic economic tools.

References (to be completed).

Table 1: Time Series Patterns in House Prices, Rents, Employment, and Permits
Arellano-Bond Estimates of Coefficients on Lagged Dependent Variable
1, 3, & 5 year horizons

Dependent Variable	*1-year changes*	*3-year changes*	*5-year changes*
House Price Change	0.71 (0.01) N=2819	0.27 (0.04) N=690	-0.32 (0.07) N=345
Rent Change	0.27 (0.03) N=1007	0.27 (0.08) N=274	-0.64 (0.17) N=91
Change in the Logarithm of Employment	0.33 (0.02) N=2643	0.28 (0.05) N=575	-0.08 (0.07) N=230
Logarithm of New Permits	0.79 (0.02) N=2875	0.58 (0.06) N=690	0.10 (0.06) N=460

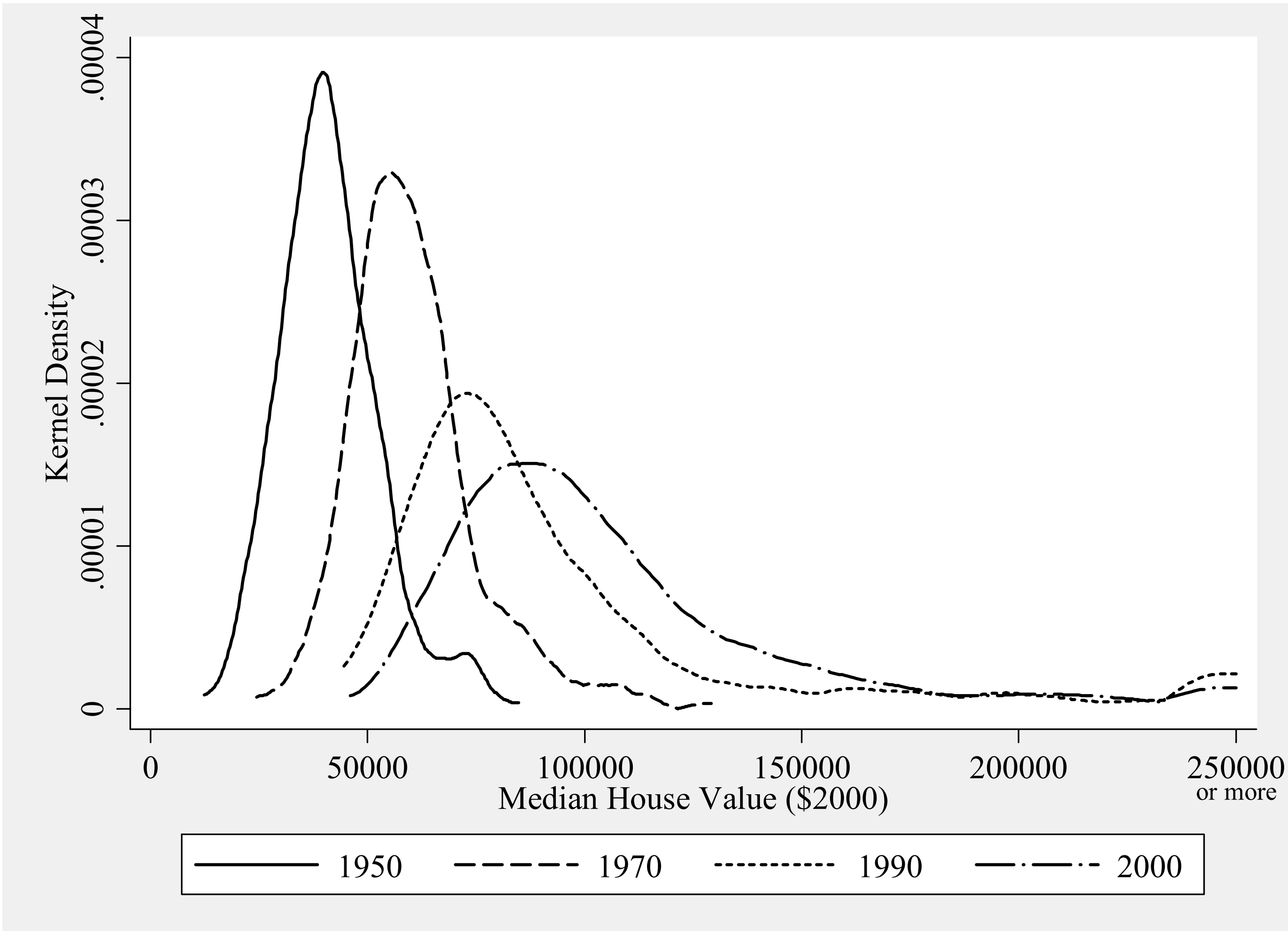
Notes:
1. Sample for house price, employment, and permit specifications is 115 metropolitan area sample described in text.
2. Sample for rent specification is 46 metropolitan areas tracked by REIS.

Table 2: Wage Shocks and Mean Reversion				
Labor Demand Parameter:	Variance of one-year wage changes	Variance of three-year wage changes	Implied Value of δ	Implied Variance of ε
$\gamma = 0$	$1,745,041	$4,778,596	.91	$1,805,000
$\gamma = -0.02$	$1,763,584	$4,648,336	.88	$1,847,000

Table 3: Price Volatility, One- and Five-Year Horizons			
Time Length	Entire Sample	Sample with Mean Growth <5000	Sample with Mean Growth <1550
Actual Variances			
One Year Housing Price Change	79,210,000	21,200,000	8,019,000
Five Year Home Price Change	1,047,363,769	278,000,000	90,600,000

Table 4: Price Volatility, One- and Five-Year Horizons: Model Variances			
Time Length	$\phi = .8$	$\phi = .9$	$\phi = 1$
One Year Housing Price Change	20,000,000	35,000,000	75,000,000
Five Year Home Price Change	63,000,000	125,000,000	295,000,000

Figure 1: The Growing Dispersion in House Prices Across Markets (from Glaeser, Gyourko, & Saks, 2006)



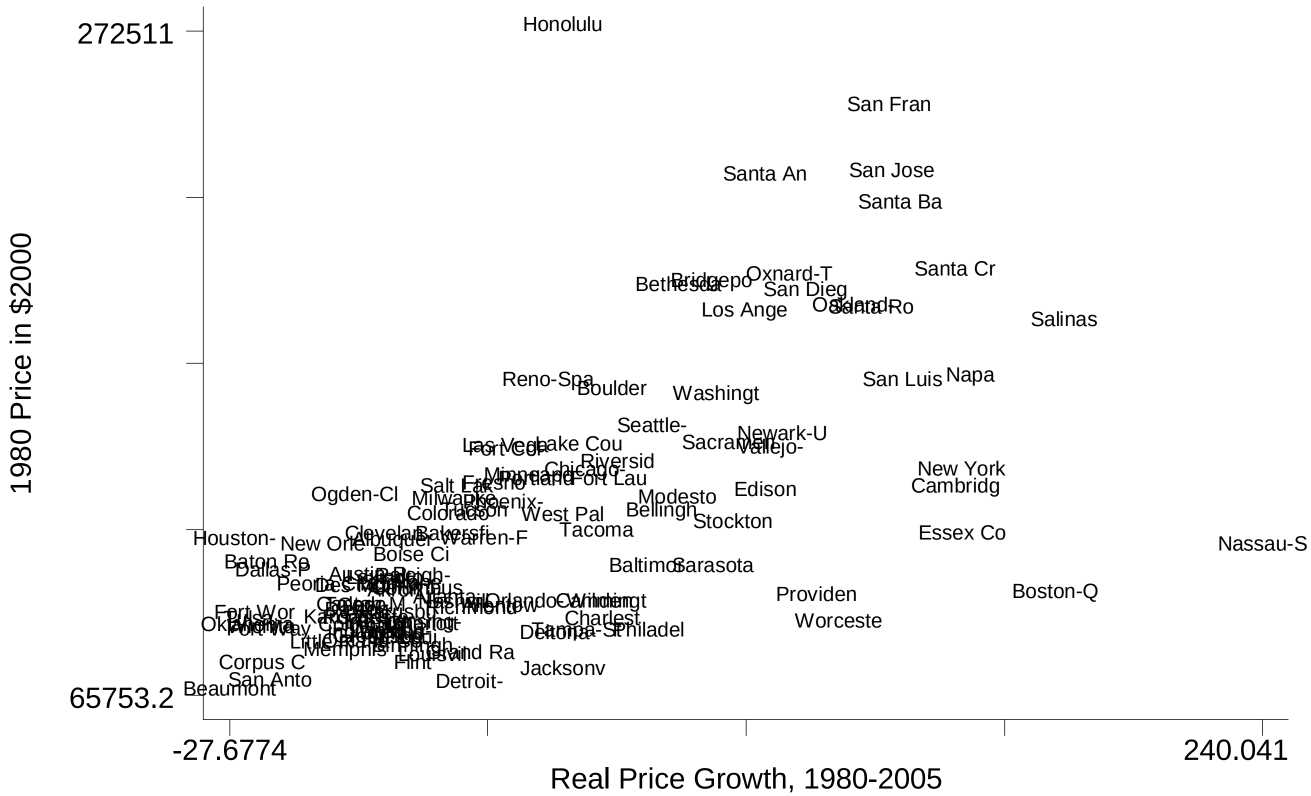


Figure 2: High House Price Levels and Appreciation Rates, 1980-2005

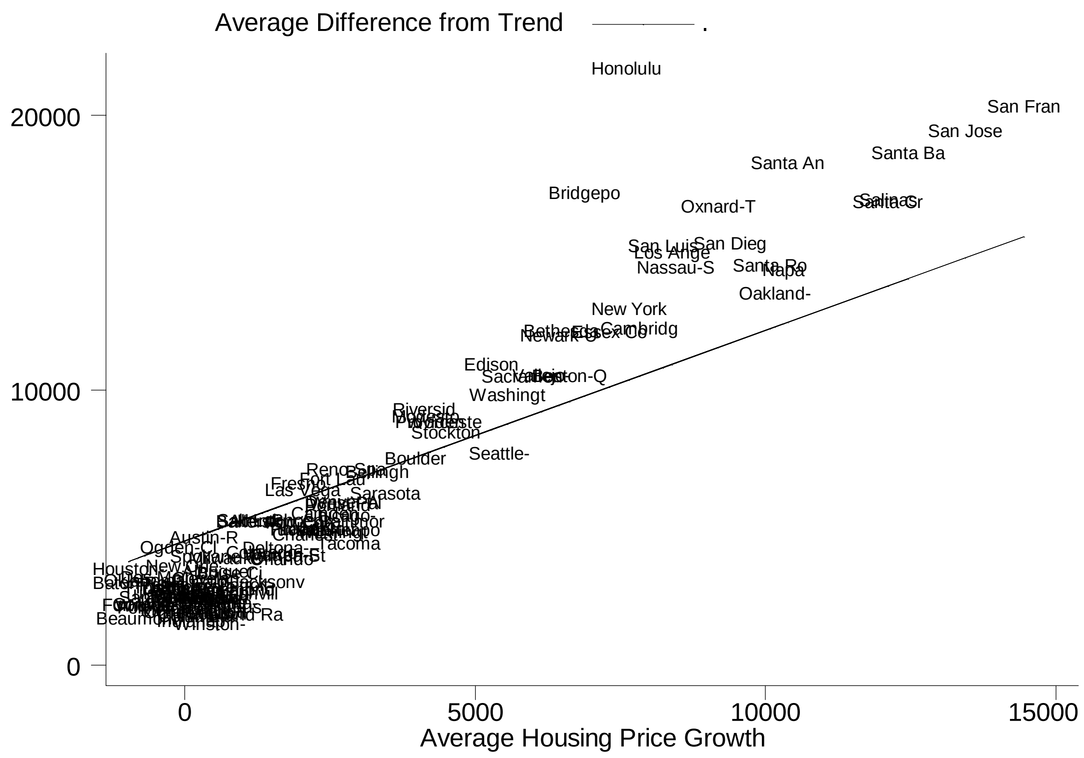


Figure 3: Price Change Volatility is Greater in Higher Price Growth Markets

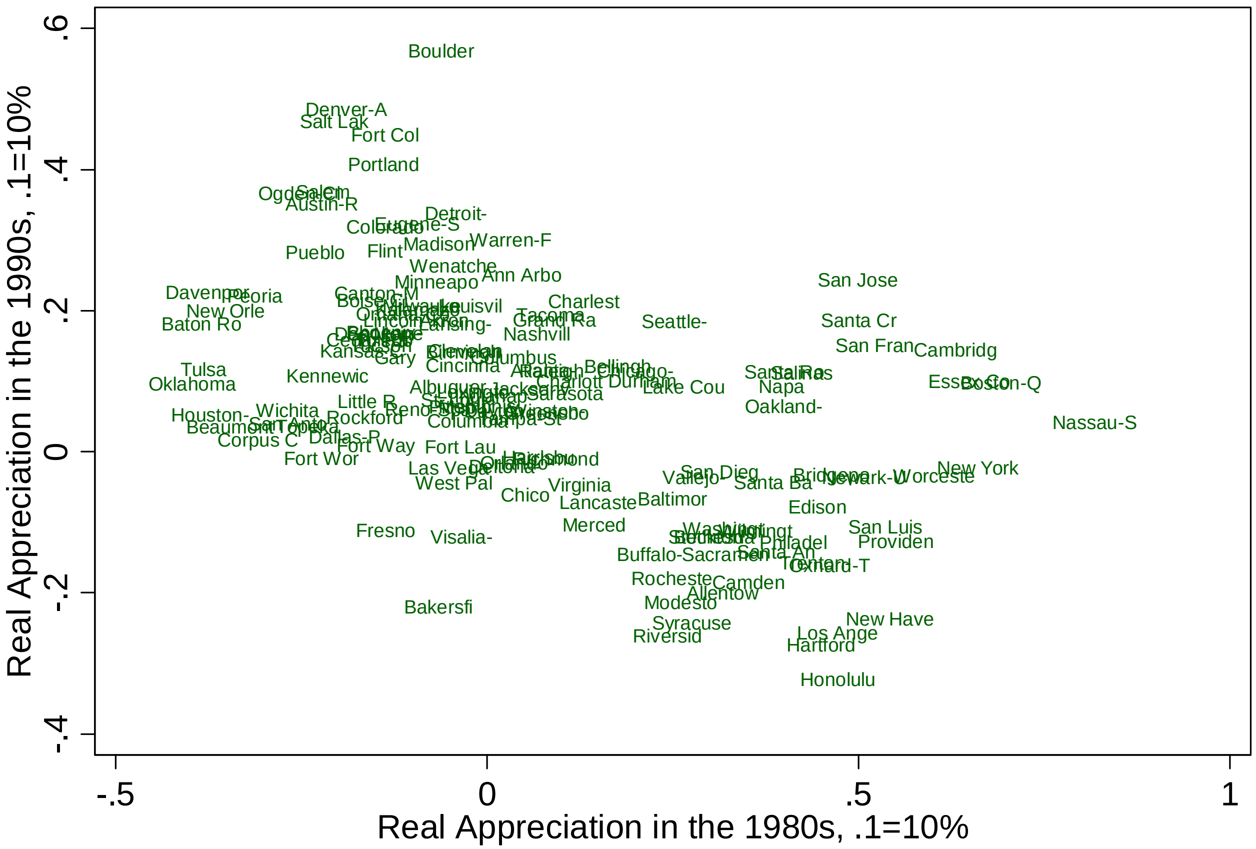


Figure 4: Real House Price Appreciation in the 1980s and 1990s

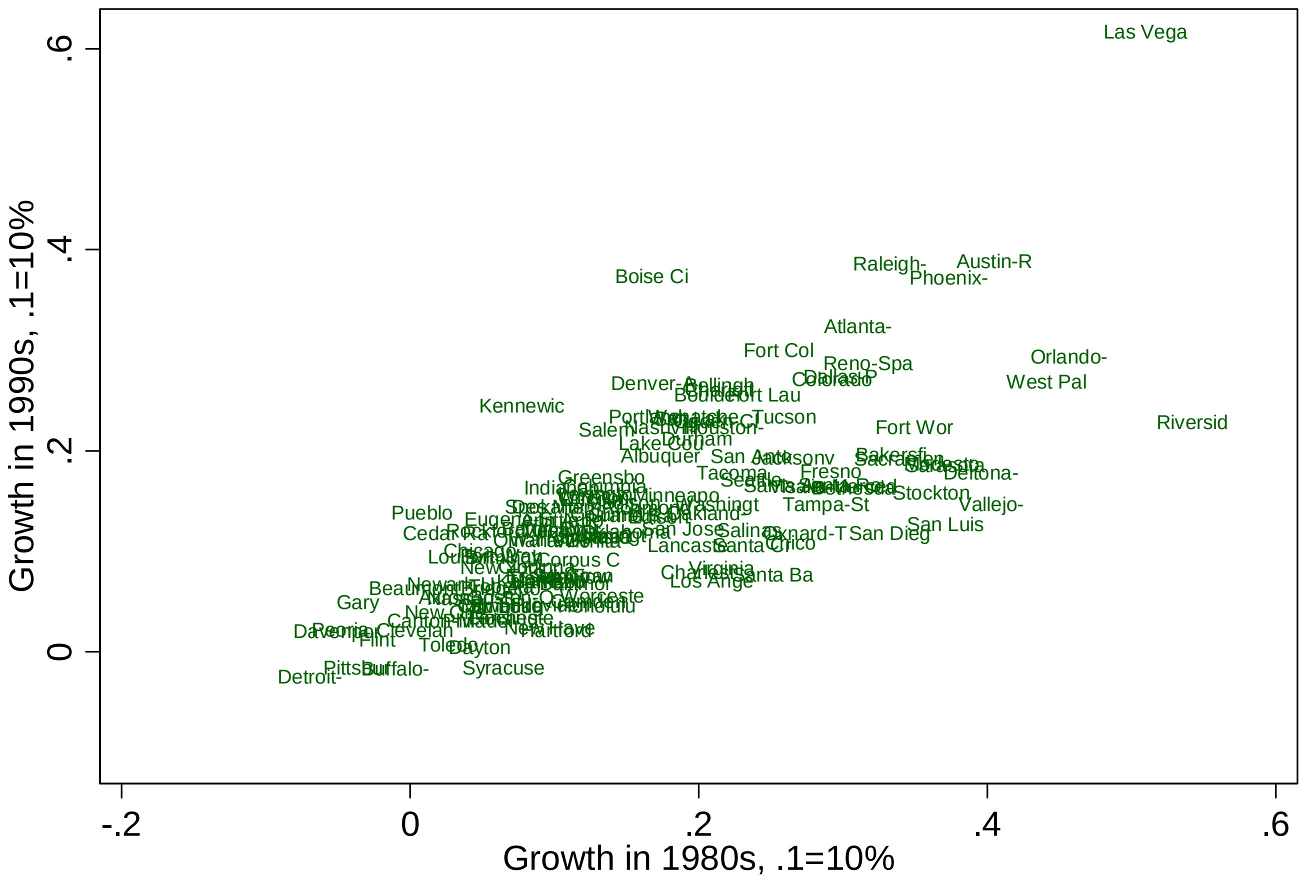


Figure 5: Population Growth in the 1980s and 1990s

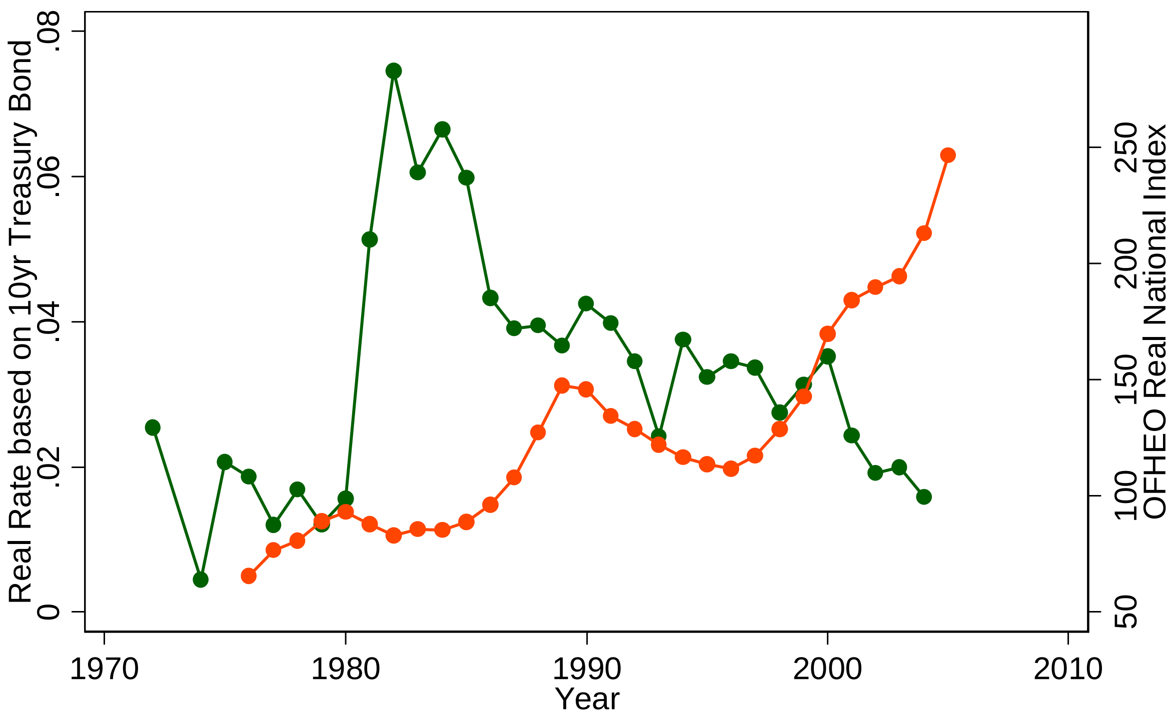


Figure 6: 10 Year Real Rate Vs. San Francisco Real OFHEO Index

Appendix: Instrument Creation

Three instruments for BEA personal income were used in instrumental variable regressions. The first instrument is an interaction of real oil prices with the 1980 share of employment in the energy sector for each metropolitan area. The real price of oil was defined as the marketed first sales price of domestic crude oil. The price was in chained (in 2000 dollars) and calculated by using gross domestic product implicit price deflators. The data was obtained from the US Energy Information Administration . (http://www.eia.doe.gov/emeu/aer/txt/ptb0518.html). The SIC codes used in the creation of the local employment share are listed in the table below.

The second instrument is an interaction of a technology company earnings index with the 1980 share of employment in technology business sectors for each MSA. The SIC codes used in the creation of the Tech share are listed in the table below. The tech index was created as follows. First, the companies in the index were based on data provided by Jeremy Siegel, who has classified firms in the S&P500 Index by business sector for each year dating back to 1980. Annual operating earnings for each company was obtained from the CRSP/Compustat merged database. Firm earnings were weighted by each company's share of the market capitalization of all firms in the index during the previous year. By weighting with a ratio of individual company market capitalization to aggregate industry capitalization, we lower the probability that weights will be affected by factors such as interest rates and the like. The index itself was summed over all relevant companies as follows:

$$\text{techindex}_t = \sum_I marketcapshare_{t-1} \bullet earnings_t .$$

Because of various idiosyncracies in how firms can report even audited earnings, we dropped outliers. In a given year, if earnings per share was more than two standard deviations above or below the sample average, the company was not included in the index. The SIC codes used in the creation of the employment shares are listed in the table below.

The third instrument is an interaction of a pharmaceutical company earnings index with the 1980 share of employment in the drug sector for each MSA. Earnings index creation is analogous to that described for the technology sector, and we rely on again on firm data provided by Jeremy Siegel. The index itself was summed over relevant companies as follows.

$$\text{pharmaindex}_t = \sum_I marketcapshare_{t-1} \bullet earnings_t \,.$$

The SIC codes used in the creation of local employment shares are listed in the table below.

Employment Sectors Used for Employment Shares

Variable name	1972 SIC Code	Industry Description
Energy	12 13	Mining: Oil and Natural Gas
	29	Petroleum Refining and Related Industries
Tech	7372	Computer Programming and Software Services
	7374	Data processing
	7379	Computer Services
	7391	Research and Development Labs
	8071	Medical Laboratories
Pharma	2831	Biological Product Manufactures
	2833	Medicinal and Botanical Product Manufacturing
	2834	Pharmaceutical Manufacturers

Appendix: Proofs of Propositions

Proof of Proposition 1: We use the change of variables $I(t) = m(t) + \hat{I}(t)$,

$N(t) = n(t) + \hat{N}(t)$, and $H(t) = z(t) + \hat{H}(t)$. The core pricing equation

$H(t) = \overline{D} + qt + x(t) - \alpha N(t) + \frac{rC}{1+r} + \frac{E(H(t+1))}{1+r}$ is then rewritten as

$$z(t) + C + c_0 t + c_1\left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right) = \overline{D} + qt + x(t)$$
$$-\alpha\left(n(t) + \frac{\overline{D}}{\alpha} + \frac{c_0 - rc_1 q/\alpha + r^2 c_0 c_1/\alpha(1+r)}{\alpha(1+r)} + \left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right)t\right) + \frac{rC}{1+r} + \text{, which}$$
$$\frac{E(z(t+1)) + C + c_0(t+1) + c_1\left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right)}{1+r}$$

simplifies to $z(t) = x(t) - \alpha n(t) + \frac{E(z(t+1))}{1+r}$. The optimality condition for production,

$C + c_0(t+1) + c_1 I(t+1) = E(H(t+1))$,

becomes

$$C + c_0(t+1) + c_1\left(m(t+1) + \frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right) = E(z(t+1)) + C + c_0(t+1) + c_1\left(\frac{q}{\alpha} - \frac{rc_0}{\alpha(1+r)}\right)$$

or $c_1 m(t+1) = E(z(t+1))$. We have $n(t) = n(t-1) + m(t)$. Together these equations

imply that $(\alpha + c_1)m(t) = \delta x(t-1) - \alpha n(t-1) + \frac{c_1 E_{t-1}(m(t+1))}{1+r}$, and substituting $t+1$ for t

and taking expectations at $t-1$ gives

$\alpha m(t) = \delta^2 x(t-1) - \alpha n(t-1) - (\alpha + c_1)E_{t-1}(m(t+1)) + \frac{c_1 E_{t-1}(m(t+2))}{1+r}$, which when used

with the first equation to eliminate $\alpha n(t-1)$ yields

$\delta(1-\delta)(1+r)x(t-1) = (1+r)c_1 m(t) - ((1+r)\alpha + (2+r)c_1)E_{t-1}(m(t+1)) + c_1 E_{t-1}(m(t+2))$. Substituting $t+j$ for t and taking expectations at $t-1$ then gives

$$\begin{aligned}(1-\delta)(1+r)\delta^{j+1}x(t-1) &= (1+r)c_1 E_{t-1}(m(t+j)) - \\ &((1+r)\alpha + (2+r)c_1)E_{t-1}(m(t+j+1)) + c_1 E_{t-1}(m(t+j+2))\end{aligned}$$

.This then implies a solution of the form $E_{t-1}(m(t+j)) = \phi^j K(t-1) + \delta^j Q(t-1)$. Substitution shows that the second-order equation is satisfied if

$Q(t-1) = \dfrac{(1-\delta)\delta(1+r)}{(1+r)c_1 - ((1+r)\alpha + (2+r)c_1)\delta + c_1\delta^2} x(t-1)$ and ϕ solves

$0 = (1+r)c_1 - ((1+r)\alpha + (2+r)c_1)\phi + c_1\phi^2$. In this case,

$$\phi = \frac{(2+r)c_1 + (1+r)\alpha - \sqrt{r^2 c_1^2 + 2(2+r)(1+r)\alpha c_1 + (1+r)^2\alpha^2}}{2c_1} = \frac{(2+r)c_1 + (1+r)\alpha - \omega}{2c_1}$$

; note that $\phi < 1$ because $\omega = \sqrt{r^2 c_1^2 + 2(2+r)(1+r)\alpha c_1 + (1+r)^2\alpha^2} > rc_1 + (1+r)\alpha$,

and $\phi > 0$ because

$\left((2+r)c_1 + (1+r)\alpha\right)^2 = (2+r)^2 c_1{}^2 + 2(2+r)(1+r)\alpha c_1 + (1+r)^2\alpha^2 > \omega^2$.

Thus $E_{t-1}(m(t+j)) = \phi^j K(t-1) + \dfrac{\delta^j \delta(1-\delta)(1+r)x(t-1)}{\left((1+r)c_1 - ((1+r)\alpha + (2+r)c_1)\delta + c_1\delta^2\right)}$.To find K,

we repeatedly substitute $E_{t-1}(m(t+j)) = \phi^j K(t-1) + \delta^j Q(t-1)$ into the first period

equality

$(\alpha + c_1)E_{t-1}(m(t+1)) = \delta^2 x(t-1) - \alpha n(t-1) - \alpha m(t) + \dfrac{c_1 E_{t-1}(m(t+2))}{1+r}$, getting

$$\begin{aligned}&\left(\alpha(1+r) + (\alpha + c_1)(1+r)\phi - c_1\phi^2\right)K(t-1) = \\ &(1+r)\delta^2 x(t-1) - \left(\alpha(1+r) + (\alpha + c_1)(1+r)\delta - c_1\delta^2\right)Q(t-1) - \alpha(1+r)n(t-1)\end{aligned}.$$

Using the earlier expression for $Q(t-1)$ and the relation

$c_1\phi^2 = ((1+r)\alpha + (2+r)c_1)\phi - (1+r)c_1$, we conclude that

$K(t-1)=\dfrac{(1+r)^2\alpha\delta x(t-1)}{\left(c_1(1+r-\delta)(1-\delta)-(1+r)\alpha\delta\right)\left(c_1\phi-(1+r)(\alpha+c_1)\right)}+\dfrac{\alpha(1+r)n(t-1)}{c_1\phi-(1+r)(\alpha+c_1)}$. To

solve for current price, we substitute the solutions for Q and K into

$z(t)=x(t)-\alpha n(t)+\dfrac{E(z(t+1))}{1+r}=x(t)-\alpha n(t)+\dfrac{c_1(K(t)+Q(t))}{1+r}$. The coefficient on $x(t)$

is

$$1+\frac{(1-\delta)\delta c_1}{(1+r)c_1-((1+r)\alpha+(2+r)c_1)\delta+c_1\delta^2}+\frac{(1+r)\alpha\delta c_1}{\left(c_1(1+r-\delta)(1-\delta)-(1+r)\alpha\delta\right)\left(c_1\phi-(1+r)(\alpha+c_1)\right)}$$
$$=\frac{(1+r)\left((1+r)\delta\alpha^2-(1-3\delta+r(1-2\delta)+\delta\phi)\alpha c_1-(1-\delta)(1+r-\phi)c_1^2\right)}{\left((1+r)\alpha\delta-(1-\delta)(1+r-\delta)c_1\right)\left((\alpha+c_1)(1+r)-c_1\phi\right)}\frac{1+r-\delta\phi}{1+r-\delta\phi}$$
$$=\frac{(1+r)\left((1+r)\alpha\delta-(1-\delta)(1+r-\delta)c_1\right)\left((\alpha+c_1)(1+r)-c_1\phi\right)}{\left((1+r)\alpha\delta-(1-\delta)(1+r-\delta)c_1\right)\left((\alpha+c_1)(1+r)-c_1\phi\right)\left(1+r-\delta\phi\right)}=\frac{1+r}{1+r-\delta\phi}$$

and the coefficient on $n(t)$ is

$\dfrac{\alpha c_1}{c_1\phi-(1+r)(\alpha+c_1)}-\alpha=\alpha\left(\dfrac{((1+r)\alpha+(2+r)c_1)\phi-c_1\phi^2}{c_1\phi^2-(1+r)(\alpha+c_1)\phi}\right)=-\dfrac{\alpha(1+r)}{1+r-\phi}$, where we have

used the equation $\phi^2=((1+r)\dfrac{\alpha}{c_1}+2+r)\phi-1-r$. Thus

$z(t)=\dfrac{1+r}{1+r-\delta\phi}x(t)-\dfrac{\alpha(1+r)}{1+r-\phi}n(t)$, giving

$H(t)=\hat{H}(t)+\dfrac{1+r}{1+r-\delta\phi}x(t)-\dfrac{\alpha(1+r)}{1+r-\phi}\left(N(t)-\hat{N}(t)\right)$, as desired.

Expected construction is then

$$E_t(m(t+j)) = \phi^{j-1}K(t) + \delta^{j-1}Q(t)$$
$$= \left(\begin{array}{l} \dfrac{\phi^{j-1}(1+r)^2\alpha\delta}{\left(c_1(1+r-\delta)(1-\delta)-(1+r)\alpha\delta\right)\left(c_1\phi-(1+r)(\alpha+c_1)\right)} \\ +\dfrac{\delta^{j-1}(1-\delta)\delta(1+r)}{(1+r)c_1-((1+r)\alpha+(2+r)c_1)\delta+c_1\delta^2} \end{array} \right) x(t) + \frac{\phi^{j-1}\alpha(1+r)}{c_1\phi-(1+r)(\alpha+c_1)} n(t)$$
$$= \left(\begin{array}{l} \dfrac{\phi^{j-1}(1+r)^2\alpha\delta}{\left(c_1(1+r-\delta)(1-\delta)-(1+r)\alpha\delta\right)\left(c_1\phi-(1+r)(\alpha+c_1)\right)}\dfrac{1-\phi}{1-\phi} \\ +\dfrac{\delta^{j-1}(1-\delta)\delta(1+r)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} \end{array} \right) x(t) + \frac{\phi^{j-1}\alpha(1+r)}{c_1\phi-(1+r)(\alpha+c_1)}\frac{1-\phi}{1-\phi} n(t)$$
$$= \frac{\delta(1+r)\left(\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)\right)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} x(t) - \phi^{j-1}(1-\phi)n(t)$$

, and expected population

is

$$E_t(n(t+j)) = n(t) + \sum_{i=1}^{j} E_t(m(t+i)) = n(t) + \sum_{i=1}^{j}\left(\frac{\delta(1+r)\left(\delta^{i-1}(1-\delta)-\phi^{i-1}(1-\phi)\right)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} x(t) - \phi^{i-1}(1-\phi)n(t) \right)$$
$$= \frac{\delta(1+r)(\phi^j-\delta^j)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} x(t) + \phi^j n(t)$$

. Expected housing price is

$$E_t(z(t+j)) = c_1 E_t(m(t+j)) = c_1\left(\frac{\delta(1+r)\left(\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)\right)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} x(t) - \phi^{j-1}(1-\phi)n(t) \right).$$

Adding back the trends, we arrive that the equations of Proposition 1(b). Finally, since $I(t+1)$ is known at time *t*, we can use the expectation equation from 1(b) to conclude that $I(t+1) = \hat{I} + \dfrac{\delta(1+r)(\phi-\delta)}{(1+r-\delta)(1-\delta)c_1-\delta(1+r)\alpha} x(t) - (1-\phi)\left(N(t)-\hat{N}(t)\right).$

Proof of Proposition 2:

The condition $\delta > \phi$ is equivalent to $\delta(1+r)\alpha > (1+r-\delta)(1-\delta)c_1$. Indeed, $\delta > \phi$ reduces to $(1+r)\alpha + (2+r)c_1 - 2\delta c_1 < \sqrt{r^2 c_1^2 + 2(1+r)(2+r)\alpha c_1 + (1+r)^2 \alpha^2}$, and since each side is positive we may square each side to obtain $\delta(1+r)\alpha > (1+r-\delta)(1-\delta)c_1$. Under a positive shock *x*(*t*) at time *t*, housing prices are certainly higher than steady state at time t. After time t, the difference between expected housing prices at t+j and steady state housing prices at time t+j equals c_1 times the difference between expected investment at time t+j and steady state investment at time t+j so it is sufficient to examine the dynamics of construction. At time t+1, the difference between construction and steady state construction equals $\frac{\delta(1+r)(\delta-\phi)x(t)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1} > 0$, so expected prices are also higher than the steady state.

After period one, the difference between expected construction and steady state construction equals $\frac{\delta(1+r)x(t)\left(\phi^{j-1}(1-\phi)-\delta^{j-1}(1-\delta)\right)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}$. If $\delta > \phi$, then $\delta(1+r)\alpha > (1+r-\delta)(1-\delta)c_1$, so this is positive if and only if $\frac{1-\phi}{1-\delta} > \left(\frac{\delta}{\phi}\right)^{j-1}$ which will be true if and only if $\ln\left(\frac{1-\phi}{1-\delta}\right) \Big/ \ln\left(\frac{\delta}{\phi}\right) > j-1$. If $\delta < \phi$, then $\delta(1+r)\alpha < (1+r-\delta)(1-\delta)c_1$, so this is positive if and only if $\delta^{j-1}(1-\delta) > \phi^{j-1}(1-\phi)$ or $\ln\left(\frac{1-\delta}{1-\phi}\right) \Big/ \ln\left(\frac{\phi}{\delta}\right) > j-1$. Thus for high enough levels of j, both investment and prices will lie below steady state levels. The symmetry when x(t)<0 is obvious.

Proof of Proposition 3: Given a y unit increase in housing price above trend at time t, we deduce that $x(t)=\frac{1+r-\delta\phi}{1+r}y$. Then at time $t+j^*$,

$$(1-\lambda)y=E_t(z(t+j^*))$$
$$=\frac{c_1\delta(1+r)\left(\phi^{j^*-1}(1-\phi)-\delta^{j^*-1}(1-\delta)\right)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}x(t)=\frac{c_1\delta\left(\phi^{j^*-1}(1-\phi)-\delta^{j^*-1}(1-\delta)\right)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}(1+r-\delta\phi)y,$$

and the desired formula drops out. Over j periods, the price change following an initial shock will equal x(t) times:

$$\frac{(2+r)c_1+(1+r)\alpha+\omega}{(2-2\delta+r)c_1+(1+r)\alpha+\omega}+\frac{c_1\delta(1+r)\left(\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)\right)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}=\frac{c_1(1+r)\left(1+\delta^j(1-\delta)-\delta\phi^{j-1}(1-\phi)-\delta/\phi\right)}{\left((1-\delta+r)(1-\delta)c_1-\delta(1+r)\alpha\right)}$$

, and over the same time period the number of extra units built will equal $\frac{\delta(\delta^j-\phi^j)(1+r)x(t)}{\delta(1+r)\alpha-(1+r-\delta)(1-\delta)c_1}$, so the ratio of number of new units to decline in price will equal

$$\frac{\delta(\phi^j-\delta^j)}{c_1\left(1+\delta^j(1-\delta)-\delta\phi^{j-1}(1-\phi)-\delta/\phi\right)}.$$