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Foreign Investment and Domestic Productivity*

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(Preliminary draft. Please do not cite without permission)

Abstract

This paper provides both empirical and theoretical evidence of the presence of positive horizontal spillover associated with foreign direct investment (FDI). Based on a newly collected firm level data of Bangladesh garment sector, this paper shows that not only are firms with foreign equity more productive, but also that the productivity improvement of these foreign firms raises the productivity of domestic firms in the same industry. This horizontal spillover effect of FDI is further explained in a theoretical model with heterogenous firms. In this model, the productivity of domestic firms depends their learning ability and the productivity of the FDI firms in the industry. In equilibrium, the productivity of FDI firms affects the productivity of domestic firms through improving the entire productivity distribution of domestic firms, and through weeding out inefficient domestic firms as market competition is toughened. Using the firm survey data, a conditional Weibull distribution of the productivity of domestic firms is estimated and the calibrated results are shown to support the model.

Keywords: Foreign direct investment, heterogenous firms, horizontal productivity spillover, Weibull distribution.

JEL Classification: E24, F12, J51

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*The author thanks Derek Chen, Svetlana Demidova, Rob Feenstra, Ana Margarida Fernandes, Caroline Freund, Beata Javorcik, Kala Krishna, Sandeep Mahajan, Marcelo Olarreaga, Zaidi Sattar, Jim Tybout and seminar participants at Penn State University for comments. The findings, interpretations, and conclusions expressed in this paper are entirely those of the author, and do not necessarily represent the view of the World Bank, its Executive Directors, or the countries they represent.

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1 Introduction

Conventional wisdom dictates that firms with foreign equity, commonly referred to as the foreign direct investment (FDI) firms, tend to be more productive. This could be due to the firm specific tangible assets such as exclusive technology and product designs, or the intangible know-how embodied in foreign equity such as marketing, networking and input sourcing. Such assets may be more readily available in big multinational corporations (MNCs). As such, being part of MNCs allow the local subsidiaries with foreign equity to gain access to these assets, which in turn enable them to produce more output given the same level of inputs, resulting in a higher level of total factor productivity (TFP) than the solely domestic owned firms. Such a hypothesis has some empirical support based on samples of Venezuela and Lithuania manufacturing firms studied in Aitken and Harrison (1999) and Javorcik (2004). It is plausible to expect that domestic firms may benefit from the productivity of these foreign subsidiaries through some *horizontal spillover effects*. For example, domestic firms may observe and adopt the best practice of FDI firms and become more productive over time; some well-trained workers may leave the FDI firms to join or set up domestic firms with the techniques they have learned from the FDI firms; or FDI firms may significantly increase the capacity of the domestic industry which attract international buyers to set up buying houses in the host countries and cut down advertisement, search and networking costs of the domestic firms. Unfortunately, while all these horizontal spillover channels of FDI are intuitive in theory, they are not widely supported by empirical evidence.

Since Aitken and Harrison (1999), most papers in the literature have found that, instead of positive productivity spillover, the *presence* of FDI firms in fact hurt the domestic firms as they intensify the competition and force the latter to produce at a suboptimal scale. Such a negative horizontal spillover effect due to the presence of FDI firms in the industry, which is also called the "market stealing effect," is further believed to be related to the learning ability of domestic firms. Indeed, only upon focusing on a sample of U.K. firms which are believed to have high absorptive capacity, Haskel, Pereira and Slaughter (2002) find a small but statistically significant evidence for the positive spillover hypothesis. All of these studies have been focusing on the effect of the presence of FDI in the industry on the TFP of domestic firms within the same industry.

The objective of this paper is to study both empirically and theoretically the potential horizontal productivity spillover of FDI firms to domestic firms, beyond the physical presence of FDI firms. Instead of relating the *presence* of FDI firms to the productivity of domestic firms in the same industry, this paper links the *productivity* of FDI firms to the productivity of domestic firms. The intuition is simple. Positive spillover effects of FDI firms, if exist, must be due to their superior productivity. Thus, instead of using the conventional measure of the presence of FDI firms, which is more of a market competition measure, average productivity of FDI in an industry will better measure productivity spillover effects of FDI.

The empirical evidence of this paper is based on a newly collected representative random sample of firms in the garment sector of Bangladesh, from 1999 to 2003.[1] The quality of the data set is good as firm specific prices of materials and main products are included.

[1] The survey was collected by the World Bank with the support from the Bangladeshi government, CIDA and DFID.

This allows more accurate measures of firms' outputs and inputs, as oppose to what have been done in the literature which is to use a common industry level price index which hides the heterogeneity of firms within an industry. More importantly, the use of industry price indexes to deflate revenue and costs of firms may cause systematic bias in the estimation of firm productivity that is correlated with the productivity of FDI firms. This paper is able to avoid such a problem by using the firm specific price indexes for the estimation of firm productivity.

To establish positive horizontal spillover effects, this paper first shows that, in a between-estimation, controlling for location, industry and year effects, FDI firms are 20 percent more productive than the domestic firms. In addition, the productivity of domestic firms is shown to increase significantly with the average productivity of FDI firms within the same industry-year (weighted by employment share and foreign equity share). The positive horizontal spillover effect is nontrivial – for every 10 percent increase in the productivity level of FDI in the industry, productivity of domestic firms increases by 3.3 percent. This is the most direct evidence of productivity spillover effects of FDI firms.

The remaining part of the paper focuses on providing a theoretical explanation regarding how productivity of FDI firms may affect productivity of domestic firms. The model builds on the earlier endogenous growth model (Romer, 1990) to explain productivity spillover, as well as the recent monopolistic competition model with heterogenous firms (Melitz, 2003) to explain market selections due to the endogenous entry and exit of firms. In the model, all domestic firms are endowed with a random but different learning ability, which affects their realized productivity given the non-rival productivity progress made by FDI firms. Domestic firms with a higher learning ability will be able to absorb more and thus more productive. As such, the equilibrium average productivity of domestic firms depends on the productivity of FDI firms through two channels. The first is the spillover channel – productivity growth in FDI firms improve the whole distribution of the productivity of domestic firms such that all domestic firms are now more productive. The second is the selection channel – the improved productivity distribution of domestic firms will attract entry, and thus toughen the market place which weed out inefficient firms and leads to a higher cutoff productivity in equilibrium. Both channels – the better productivity distribution and the higher cutoff productivity – together cause the industry to be more productive and efficient overall, which explains the positive relationship between the average productivity of FDI firms and domestic firms in the sample.

To calibrate the theoretical model, we specify a Weibull productivity distribution function. Given that ex-post we only observe surviving firms and not the whole distribution of firms, we estimate the conditional Weibull distribution using the sample productivity distribution generated nonparametrically by Kernel density estimation. The results support the theoretical model where productivity of FDI firms is shown to act like a scaling parameter which leads to stochastic dominating movements of the productivity distribution of domestic firms, both conditional and unconditionally. Based on the estimated cutoff productivity, we infer that the fixed production cost is about \$30,000, entry fixed cost is about \$800,000, and an average profit margin of 8.4%.

The rest of the paper is organized as follows: Section 2 presents an overview of Bangladesh garment sector, with data description and the estimation of firm productivity. Section 3 presents some reduced form relationship between productivity and ownership, and the effects of FDI productivity on domestic firm productivity. Section 4 presents the theoretical model, follows by the empirical strategy and results in Section 5. Section 6 concludes the paper.

2 Data description

The firm level survey was conducted from the period of November 2004 to April 2005, which covers a stratified random sample of 350 firms, which is about 10% of the total population of the garment firms currently operating in Bangladesh. the sample was stratified to reflect the population distribution of firms by size, by sub-sector and by location. After cleaning up the data to exclude outliers and firms with incomplete information, there are a total of 231 firms in the unbalanced final panel data set of 1026 observations, spanning the years 1999 to 2003. In this unbalanced panel data set, the composition of sub-sector is 24 percent in knitwear, 8 percent in sweaters and 68 percent in woven garments, roughly reflects the population of firms in the garment sector. Among the sampled firms, 13% have positive foreign equity, while the remaining 87% are purely domestic owned. Moreover, 15% of the sampled firms are in the Dhaka and Chittagong export processing zones (EPZs), 63% in Dhaka and 15% in Chittagong. We will rely on the differences in sub-sector and year, controlling for location, to identify horizontal spillover effects.

Table 1 presents the sample means of the key variables by foreign versus domestic firms. It is clear that FDI firms are in general larger in sales, in exports, they purchase more material inputs, including imported materials, they hire more employees, including production workers. FDI firms also have larger capital stock and investment. All these suggest that FDI firms are larger in scale and presumably more profitable and productive. To formally study the productivity superiority of FDI firms, and the possible productivity spillover to domestic firms, we will need to first estimate firm level productivity. The estimated firm productivity is then related to the ownership of the firms, and subsequently the relationship between productivity of domestic and FDI firms in the same sub-sector will be statistically examined.

2.1 Estimating firm productivity

To formally study the overall productivity of firms, we need to estimate firm production function, taking into account total factor usage per unit of output. In the firm survey we asked firms to provide the annual increase in the main product price and the main material input price. The firm level price information allows us to construct firm level price indexes of output and material, which we use to deflate sales and material costs to obtain values of real output and material input. Firm specific output price is particularly important for the construction of firm TFP. As in heterogeneous firm models, price of firm is negatively correlated with firm's productivity. To the extent that productivity of FDI firms are higher than that of domestic firms which drags down the industry aggregate price index, using the industry price index to deflate individual firm sales will unavoidable make domestic firms appear more productive, and introduce the spurious positive correlation between the productivity of domestic firms and that of FDI firms. By using the firm specific price index, we are able to avoid this. Moreover, domestic materials are more expensive than imported materials, we express all materials in terms of imported materials by proportionately inflating the usage of domestic materials to reflect the price differential.[2]

Firm productivity is estimated by fitting the following production function, where in logs, output of firm i in year t, Y_{it}, is linearly related to labor, L_{it}, materials, M_{it}, and capital stock, K_{it}. Any part of Y_{it} that is not explained by the three factors of production is attributed to productivity, ϕ_{it}, which varies by firms and years. In other words, if we regress $\ln Y_{it}$ on $\ln L_{it}$, $\ln M_{it}$ and $\ln K_{it}$ using ordinary least squares (OLS) estimation, the regression errors are the firms productivity, $\ln \phi_{it}$.

$$\begin{aligned} Y_{it} &= \phi_{it} L_{it}^{\alpha_L} M_{it}^{\alpha_M} K_{it}^{\alpha_K}, \\ \ln Y_{it} &= \ln \phi_{it} + \alpha_L \ln L_{it} + \alpha_M \ln M_{it} + \alpha_K \ln K_{it}. \end{aligned}$$

However, firm's input choices are likely to be endogenous. How many workers to hire, how many unit of fabrics to purchase, and how many new machines to set up each year depend on the realized productivity of the firms, which is known only to the firms, but not the researchers. Such input endogeneity will bias OLS estimates of the coefficients of labor and materials, α_L and α_M, upward. In addition, if larger and older firms tend to stay in business despite low productivity, while younger and smaller firms tend to quit easier,

[2] This is necessary as only the more productive firms may find it profitable to use the more expensive domestic textile materials in order to obtain tariff preference in the EU market. Without the adjustment, the total costs of material used by these firms may be larger even though the quantity used is not. This will make these firms appear less productive than they really are.

such entry/exit decision of the firm will bias OLS estimates of the coefficient of capital, α_K, downward.

To address input endogeneity bias and selectivity bias, we follow a 3-step nonlinear estimation methodology developed by Olley and Pakes (1996). Olley and Pakes (1996) assume that in each year t, the unobserved productivity, $\ln \phi_{it}$, is the only state variable which follows a common exogenous Markov process, which, jointly with fixed input K_{it}, determines the entry/exit decision and investment demand, $\ln I_{it}$, of the firms. Considering only the Markov perfect Nash equilibriums, which firms expectation matches the realization of future productivity, we can then use a polynomial function of $\ln I_{it}$ and $\ln K_{it}$, to infer the unobserved productivity, $\ln \phi_{it}$.[3] The polynomial function is assumed to be common across all firms in all years.

However, part of the firm productivity is likely to be specific to firms and years. For example, FDI firms are likely to be more productive as they have access to the MNCs know-how. This may lead to firm specific investment that is not captured in the common polynomial function. Similarly, when there is a productivity progress that is common to all firms in a specific year due to, say, positive FDI productivity spillover which cause all firms to be more productive, we may see a surge in investment beyond the specified polynomial function. Thus, in this context, it is necessary for us to allow for firm and year specific productivity and investment movement. Another benefit of including firm and year fixed effects in the regression is to controlling for firm specific fraudulent accounting practise which

[3] This is possible because, given $\ln K_{it}$, $\ln I_{it}$ is an increasing function of $\ln A_{it}$, which makes the function invertible.

overstate costs and understate sales, may cause firm specific bias estimation. Similarly, by including year fixed effects, we are able to control for demand and macro shocks that are common across all firms within a year. We therefore modify the three stage nonlinear estimation of the above production function due to Olley and Pakes to include firm and year fixed effects. Furthermore, given that older firms are more likely to stay in business despite temporary down turn in business, we also control for firm age in the estimation.

In step one, we control for input endogeneity by regressing $\ln Y_{it}$ on $\ln L_{it}$, $\ln M_{it}$, a full set of firm and year fixed effects and a 3rd order polynomial function of real investment, $\ln I_{it}$ and capital, $\ln K_{it}$. The full set of firm and year fixed effects, together with the polynomial function is used to control for the unobserved productivity, $\ln \phi_{it}$. The estimated coefficients on labor and materials, $\hat{\alpha}_L$ and $\hat{\alpha}_M$, are consistent.[4]

In step two, we estimate the entry/exit decision of the firms using a Probit regression on a 3rd order polynomial function of investment, capital and age, controlling for year, region and industry fixed effects. This regression yields the propensity for a firm to stay in business. Finally, in step three, we regress $\ln Y_{it} - \hat{\alpha}_L \ln L_{it} - \hat{\alpha}_M \ln M_{it}$, on age, capital, and a 3rd order polynomial function of propensity of survival and $E(\ln Y_{it}) - \hat{\alpha}_L \ln L_{it} - \hat{\alpha}_M \ln M_{it}$,

[4] Firms' real investment, I_{it}, is obtained by deflating nominal investment by the GDP deflator for domestic fixed capital formation in Bangladesh in the respective years. Capital is constructed by summing real investment over the years using perpetual inventory method with an annual depreciation rate, of 10 percent:

$$\begin{aligned} K_{it} &= K_{it-1}(1-\delta) + I_{it}, \\ K_{i0} &= \frac{1}{2}\left(F_{i1} + \frac{I_{it}}{\delta}\right), \end{aligned}$$

with initial capital stock being constructed using average between firm's first year fixed asset, F, and the infinite sum series of investment prior to the first year, assuming that the growth rate of investment of 0 and depreciation rate of 10 percent.

and a full set of firm and year fixed effects. This last-stage nonlinear regression gives us a consistently estimated coefficient on capital, $\hat{\alpha}_K$.

With these estimates, we constructed firm productivity according to the following equations:

$$\begin{aligned} \ln \hat{\phi}_{it} &= \ln Y_{it} - \hat{\alpha}_L \ln L_{it} - \hat{\alpha}_M \ln M_{it} - \hat{\alpha}_K \ln K_{it}, \\ \hat{\phi}_{it} &= \exp\left(\ln Y_{it} - \hat{\alpha}_L \ln L_{it} - \hat{\alpha}_M \ln M_{it} - \hat{\alpha}_K \ln K_{it}\right). \end{aligned}$$

This procedure gives us the estimated firm productivity, which by construction will have components that are firm and year specific. We then relate $\hat{\phi}_{it}$ to the productivity of foreign firms to test the spillover hypothesis. We also use the estimated firm productivity to generate the sample distribution of firm productivity using Kernel density estimation.

Table 2 presents the productivity estimating procedure. Column (1) shows the OLS estimation with no correction on endogeneity, selectivity, firm or year fixed effects. These estimates are likely to be biased. Column (2) shows the within estimates with firm and year fixed effects. While these estimates are robust to factors such as location which is specific to a firm and macroeconomic climate which is specific to a year, year to year variation of productivity within firm will still bias our estimates. Column (3) reports the first step Olley-Pakes procedure, where a 3rd order polynomial function of investment, capital and age is included, in addition to firm and year fixed effects, to control for within firm year to year changes in the unobserved firm productivity. This procedure corrects for input endogeneity, which reduces the upward bias relative to the OLS estimates. The consistent estimates, $\hat{\alpha}_L$ and $\hat{\alpha}_M$, are 0.255 and 0.715, respectively. Without correcting for selectivity, the estimated coefficient of capital will be too low.

Column (4) presents result from step three, which we control for selectivity bias by including a 3rd order polynomial function of the estimated survival probability and the net fitted output. The consistent estimate for $\hat{\alpha}_K$ is 0.021. All these coefficients are statistically significant, and are in line with the estimates in the literature.

Figure 1 presents the nonparametrically generated kernel density distribution of productivity for all firms. Given the long tail to the right, it does resemble the Weibull density function with $\beta > 1$. We will formally test for this in the later section.

3 Reduced Form Results

3.1 Are FDI firms more productive?

Using the estimated firm TFP, at sample mean, without considering other factors, productivity of firms with foreign equity is about 23 percent higher than purely domestically owned firms. What could have explained such productivity advantage of FDI firms? Column (1) of Table 3 regress the estimated $\ln \hat{\phi}_{it}$ of firms on a FDI indicator variable, controlling for industry, year and location fixed effects. This is to isolate the effect of foreign ownership from the influences of sub-sector, investment climate of the locations, and the macro economic shock in each year. Given that ownership seldom change within firms in our sample, between-firms variation in foreign ownership is used to identify the effect of FDI dummy on productivity. The result shows that a FDI firm is still about 20% more productive than a domestic firm in the same industry, location and year. This shows that the effect of foreign equity on firm productivity is independent on the location of the firms, the sub-industry of the firms and the macro economic fluctuations. Columns (2) and (3) further include age and export destinations of the firms in both the between and the OLS regressions. It is clear that FDI firms do have a higher level of productivity, even after we take into account export destinations and thus potential demand shocks that the firms may face, as well as the experience of the firms as proxied by age. Moreover, the OLS results show that firms export to US tend to be more productive.

Columns (4) to (6) repeat the exercise by using the actual foreign equity share in the regressions instead of a FDI dummy variable. The results are strikingly similar. This could be because most of the FDI firms in Bangladesh garment sector have 100 percent foreign equity, only 7 FDI firms are jointly venture firms with foreign equity no less than 25 percent.

There may be a concern that the FDI dummy or equity share variable is endogenous due to the cherry picking behavior of the parent firms – MNCs actively select the more productive domestic firms to buy up or to set up joint ventures. This will lead to the reverse causality between TFP and the foreign ownership variables. However, according to Bangladesh Investment Board, in the case of the garment sector, this is not an issue. Most FDI firms are green field investments, which makes the issue of cherry picking not relevant.

Thus overall there is convincing and statistical significant evidence suggesting that FDI firms are more productive than otherwise identical domestic firms operating in Bangladesh. This result is robust after taking into account the effects of locations, sub-sectors, macro fluctuation, export destinations and experience. Figure 2 shows the productivity distribution of domestic firms and FDI firms. It is clear that FDI firms have higher productivity than domestic firms.

3.2 Productivity Spillover: Can Domestic Firms Benefit from FDI Firms?

Many countries provide special incentives such as tax holidays or subsidies, and import duty exemptions to attract FDI, with the assumptions that the presence of FDI will benefit the domestic economy through the some unmeasured "spillover effects." To date, there is evidence of "vertical" spillover effects through the contact of domestic upstream suppliers with the downstream FDI firms (Javorcik, 2004), evidence of "horizontal" spillover effects however have been quite elusive.

To study whether such effects exists in Bangladesh's garment sector, we first relate the estimated TFP of the domestic firms to the presence of FDI firms in the sub-sector. Presence of FDI firms in industry j, FP_{jt}, is captured by the share of employment of FDI firms collectively in the industry in a given year, adjusted by the percentage of foreign ownership of FDI firms, FS_{it}, for all firm i in industry j. This measure of the influence of FDI firms has been used in the literature (Aitken and Harrison, 1999).

$$FP_{jt} = \frac{\sum_{i \in j} L_{it} FS_{it}}{\sum_{i \in j} L_{it}}.$$

However, share of FDI firms in the industry may capture at least three opposing forces when it is related to the productivity of domestic firms. First, is the market stealing effect – the expansion of FDI firms in the industry causes the intra-marginal domestic firms to cut back in their production and, in the presence of increasing returns to scale, makes the local firms appear less productive. Second, is the selection effect – the expansion of FDI firms in the industry pushes out inefficient marginal domestic firms, increases the average productivity of the industry. Third, is the spillover effect – the expansion of FDI firms is driven by the productivity gains of FDI firms, which may benefit domestic firms and increase their productivity. In addition, when spillovers do happen, it is possible that domestic firms learn so much from the FDI firms, they may take over some market shares of the latter. Thus, the overall net effect of the presence of FDI on the productivity on domestic firms is unclear, and most importantly, it may not capture the positive spillover effects due to the productivity gains of FDI firms.

For this reason, we further relate the estimated productivity of domestic firms to the average productivity of FDI firms in the same industry and year. In order to capture the economic influence of the productivity of FDI firms, we weight the TFP of FDI firms with the share of foreign equity and the share of employment in the industry. Weighting by capital or output would not change the results.

$$\ln \phi_{jt}^{FDI} = \frac{\sum_{i \in j} L_{it} FS_{it} \ln \phi_{it}}{\sum_{i \in j} L_{it}} = \left(\sum_{i \in j} \frac{L_{it} FS_{it}}{\sum_{i \in j} L_{it} FS_{it}} \ln \phi_{it} \right) FP_{jt}.$$

Thus, by construction, the average productivity of FDI firms in an industry depends on the productivity of individual FDI firms, and the share of the FDI firms in the industry. An increase in the average productivitry of FDI firms in an industry may therefore be driven by increases in the productivity of the individual FDI firms or the increase in the presence of FDI in the industry. By controlling for the presence of FDI firms, we will then be able to isolate the effect of the productivity gains in FDI firms on the productivity of domestic firms. Moreover, given that both the presence of FDI in the industry and the productivity of FDI firms in the industry do not vary within each firm observation, and are specific to each industry-year, we have aggregate variables in micro unit, which will artificially deflate the standard errors of the firm level panel regression (Moulton, 1990). We correct for such problem nonparametrically by clustering the standard errors of the regressions by industry-year.

Table 4 presents the regression results. Column (1) shows that controlling for firm and year fixed effects, productivity of domestic firms increases with the presence of FDI firms in the sub-industry. However, while the effect is positive, it is not statistically significant. This demonstrates that the opposing forces wash out the effect of FDI presence on the productivity of domestic firms and is quite in line with the finding of the previous literature. This result is also robust to the inclusion of other control variables such as age and export destinations in Column (2). The more interesting result is presented in Columns (3) where we find positive and significant effects when we relate the productivity of domestic firms to that of the average FDI firms. Based on the average foreign presence of 26 percent, for every 10 percent increase in the average productivity of FDI firms, the productivity level of domestic firms in the same industry improves by 0.4 percent. This result is robust to controlling for export shares and age of the firms as shown in Column (4).

As mentioned above, the increase in the average productivity of FDI firms in an industry may be due to true productivity gains of FDI firms, or it can be driven by the increase in the presence of FDI in the industry. To isolate the effect of the former, which captures spillover effects, we further include the presence of FDI in Column (5). In this specification, controlling for the market share of FDI in the industry, the spillover effects of the productivity of FDI firms is shown to remains robustly positive. Moreover, the magnitude of spillover is significantly larger once we control for the market share of FDI firms – for every 10 percent increase in the average productivity of FDI firms, productivity of domestic firms increase by 3.3 percent.[5]

There may be a concern that the positive relationship between the average productivity of FDI firms and domestic firms found is purely driven by the selection effect – productivity improvement of FDI firms intensifies market competition and thus weeds out the marginal inefficient domestic firms such that, in equilibrium, the average productivity of those surviving firms are higher. This is possible even if the share of FDI in the industry remains constant as the surviving firms take over all the market shares of the exiting inefficient firms. In fact when we regress the survival dummy variable on the average productivity of FDI firms in a fixed effect logit regression, we find a very strong negative correlation suggesting a higher exit rate for those industries where the productivity of FDI firms are higher.[6] However, if this is the only channel by which the productivity of FDI firms may be positively correlated with the productivity of domestic firms, then we would not expect productivity of

[5] In Column (5), the coefficient of FDI presence is strongly negative. This does not imply that the presence of FDI decreases the productivity of domestic firms in the same industry, given that the average productivity of FDI in the industry also depends on the presence of FDI. The overall effect of FDI presence on the productivity of local firms, conditional on the average productivity of FDI remains constant, is -3.122+1.246*2.732=0.280 (standard error is 0.098), where the sample average FDI productivity of 2.732 is used to evaluate the effect.

[6] Result is available upon request.

those more productive firms, those intra-marginal firms that are not affected by the selection mechanism, to be affected by the productivity of FDI firms. Column (6) relates productivity of FDI firms to the productivity of a subset of domestic firms, whose productivity is above 25 percentile in 1999. The positive relationship between the productivity of FDI firms and domestic firms survives, which further strengthens the positive spillover hypothesis.

Column (7) relates the average productivity of domestic firms to the productivity of FDI firms in the full sample of FDI firms to test whether there is any evidence of reverse spillover effects from domestic firms to FDI firms. The average productivity of domestic firms is constructed in a similar way as the average productivity of FDI firms in the same industry-year. Result shows that there is a positive but insignificant effect, which do not support the reverse spillover hypothesis.

Finally, the positive relationship between the average productivity of FDI firms and the productivity of domestic firms within the same industry year may simply reflect the overall productivity of the industry which may be affected by other industry-year specific variables such as trade costs and R&D expenses. In other words, the horizontal spillover effects could be spurious and are driven by other industry-year specific variables that are correlated with the productivity of both foreign and domestic firms. Column (8) addresses this issue. To control for industry-year specific trade costs, industry export to EU and US are included. As shown in Demidova, Kee and Krishna (2006), due to differences in the rules of origin requirement, the US is a much tougher market than the EU for Bangladeshi garment exporters. As such the total industry export to US is larger when firms, both domestic and foreign, within the industry are more productive. This may explain the positive relationship between the average productivity of FDI firms and the productivity of domestic firms within the same industry year. The reverse should hold for the EU. The log of industry export to US and EU are included in Column (8) to control for this potential bias. Similar story holds for the overall R&D expenditure of the industry. The larger is the R&D expenditure, the higher is the productivity of all firms in the industry, including the FDI firms, which may explain the positive relationship between the productivity of FDI firms and the productivity of domestic firms. The log of the total industry R&D expense is also included in (8), together with own R&D which may affect firms' productivity. Finally, if horizontal spillover effects exist, we may expect the productivity of domestic firms to relate to the lagged value of average productivity of FDI firms, in addition to the contemporaneous relationship, as learning of domestic firms may take time to bear fruits. The lagged value of the average productivity of FDI is also included in Column (8).

Result presented in (8) shows that it is important to control for other industry-year specific variables such as export to US and EU and the overall R&D of the industry. The coefficients of the total industry export to US and R&D expenditure are positive and statistically significant. The coefficients of the total industry export to EU and own R&D expenditure have the right sign but not statistically significant. The positive horizontal spillover effects of FDI firms nonetheless survive. Not only is the contemporaneous relationship positive and significant, the lag of the average productivity of FDI firms also increases the productivity of domestic firms which indicates that learning of domestic firms may take more than one year to realize.

Overall, results presented in Table 4 show that there are sufficient statistically evidence suggesting that domestic firms may benefit from the productivity growth of FDI firms in their industry. Thus, not only are FDI firms more productive than domestic firms, productivity growth of FDI firms may spillover to the domestic economy to benefit the domestic firms.

The remaining part of the paper aims to provide a theoretical explanation as to why and how productivity progress of FDI firms may benefit productivity of domestic firms. The theoretical results do not hinge on a particular distribution function, but for the purpose of testing the model in a structural approach, we specify a Weibull distribution in the following empirical section.

4 Theoretical model

In this model we assume that FDI firms, through in-house R&D activities in the headquarters, are more productive. Knowledge generated by the FDI firms are nonrival, can be learned by the domestic firms. The learning ability of domestic firms, which are ex-ante identical, depends on some random probability draw from a common distribution, such that ex-post the higher are the firms' the learning ability, the more they can learn from the FDI firms, and the more productive they are. This model thus builds on the earlier endogenous growth model (Romer, 1990) to explain productivity spillover, as well as the recent heterogenous firm model (Melitz, 2003) to explain productivity differences among firms. By combining this two models, we show that productivity of domestic firms depends on the productivity of FDI firms through two channels. The first is the spillover channel – productivity growth in FDI firms improve the whole distribution of the productivity of domestic firms such that all domestic firms are now more productive. The second is the selection channel – the improved productivity distribution of domestic firms will attract entry, and leads to a more competitive industry, which has a lower aggregate price index. This will toughen the market place and lead to a higher cutoff productivity of the remaining firms by weeding out inefficient firms. The higher cutoff productivity, combined with the improve productivity distribution, results in an industry that is overall more productive and efficient.

4.1 Productivity distributions

We start by assuming that there are a fixed mass, M^f, of FDI firms operating in the industry. This could be due to government restrictions which limit the number of FDI firms in the economy. The reason FDI firms are located in this host country presumably is to take advantage of the lower labor costs and possible trade preferences, so as to use the economy as an export platform to service the world market. Productivity of these FDI firms depend on the R&D investment of their headquarters each period, and are taken as exogenous. Together, they define the nonrival stock of knowledge available in the industry, which is the average productivity of the FDI firms,

$$\theta = \left(E\left[\left(\phi^f \right)^{\sigma-1} \right] \right)^{\frac{1}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ is the elasticity of substitution to be defined below. We assume that due to low technical capability, domestic firms do not invest in R&D activities and therefore do not participate in the generation of the stock of knowledge.

Productivity of domestic firms are determined by both the stock of knowledge available in the industry, θ, and a random learning ability draw of the firms, e_i, according to the continuous and differentiable distribution function, $F(.)$ and the density function, $f(.)$,

$$\begin{aligned} \phi_i &= \theta e_i, \text{ where } \theta > 0 \text{ and} \\ e_i &\sim F(e). \end{aligned} \tag{2}$$

We further assume that the hazard function of e is non-decreasing,

$$h(e) = \frac{f(e)}{1 - F(e)}, \text{ with } h'(e) \geq 0. \tag{3}$$

This assumption implies that conditional of e being observed, the probability of observing a learning ability higher than e is higher as e increases. With the non-decreasing hazard function of e, we are ready to show that as θ increases due to productivity progress of FDI firms, the unconditional and conditional productivity distributions of domestic firms stochastically improve. By stochastic improvement we meant that the probability distribution after θ increases first order stochastic dominates the productivity distribution before θ increases.

Proposition 1 *Given $\phi = \theta e$, where $\theta > 0$, $e \sim F(.)$, F is continuous and differentiable such that $F'(e) = f(e)$, $h(e) = \frac{f(e)}{1-F(e)}$ is non-decreasing, and $f(e|e > e^*) = \frac{f(e)}{1-F(e^*)}$,*

1. *the cumulative density function (CDF) of ϕ is*

$$G(\phi; \theta) = F\left(\frac{\phi}{\theta}\right), \text{ with } \frac{\partial G(\phi; \theta)}{\partial \theta} < 0; \tag{4}$$

2. *the probability density function (PDF) of ϕ is*

$$g(\phi; \theta) = \frac{1}{\theta} f\left(\frac{\phi}{\theta}\right); \tag{5}$$

3. for $\phi > \phi^*$, the conditional PDF of ϕ is

$$\mu(\phi;\theta) = \frac{1}{\theta} f(e|e > e^*), \ \text{where } \phi^* = \theta e^*;$$

4. for $\phi > \phi^*$, the conditional CDF of ϕ is

$$\Phi(\phi;\theta) = \frac{F\left(\frac{\phi}{\theta}\right) - F\left(\frac{\phi^*}{\theta}\right)}{1 - F\left(\frac{\phi^*}{\theta}\right)}, \ \text{with } \frac{\partial \Phi(\phi;\theta)}{\partial \theta} < 0 \tag{6}$$

Proof. Given $\phi = \theta e$,

1. $G(\phi;\theta) = \Pr(\phi_i < \phi) = \Pr(\theta e_i < \phi) = \Pr\left(e_i < \frac{\phi}{\theta}\right) = F\left(\frac{\phi}{\theta}\right)$. F is continuous and differentiable implies G is continuous and differentiable, and $\frac{\partial G(\phi;\theta)}{\partial \theta} = \frac{\partial F\left(\frac{\phi}{\theta}\right)}{\partial \theta} = -\frac{\phi}{\theta^2} f\left(\frac{\phi}{\theta}\right) < 0$.

2. $g(\phi;\theta) = \frac{\partial G(\phi;\theta)}{\partial \phi} = \frac{\partial F\left(\frac{\phi}{\theta}\right)}{\partial \phi} = \frac{1}{\theta} f\left(\frac{\phi}{\theta}\right)$.

3. $\mu(\phi;\theta)$ is the conditional PDF function of ϕ, with $\phi > \phi^*$, and $\phi^* = \theta e^*$, for some e^*. Thus, by definition,

$$\begin{aligned} \mu(\phi;\theta) &\equiv \begin{cases} \frac{g(\phi;\theta)}{\Pr(\phi_i > \phi^*)}, \ \forall \phi > \phi^* \\ 0, \ \forall \phi \leq \phi^* \end{cases}, \\ &= \begin{cases} \frac{g(\phi;\theta)}{1 - G(\phi^*;\theta)}, \ \forall \phi > \phi^* \\ 0, \ \forall \phi \leq \phi^* \end{cases} = \begin{cases} \frac{\frac{1}{\theta} f(e)}{1 - F(e^*)}, \ \forall e > e^* \\ 0, \ \forall e \leq e^* \end{cases} = \begin{cases} \frac{1}{\theta} f(e|e > e^*), \ \forall e > e^* \\ 0, \ \forall e \leq e^* \end{cases}. \end{aligned}$$

4. $\Phi(\phi;\theta)$ is the conditional CDF function of ϕ, with $\phi > \phi^*$, and $\phi^* = \theta e^*$, for some e^*.

Thus, by definition,

$$\begin{aligned}
\Phi(\phi;\theta) &\equiv \int_0^{\phi} \mu(t;\theta)\,dt \\
&= \int_{\phi^*}^{\phi} \frac{g(t;\theta)}{1-G(\phi^*;\theta)}dt \\
&= \frac{G(\phi;\theta)-G(\phi^*;\theta)}{1-G(\phi^*;\theta)} \\
&= \frac{F\left(\frac{\phi}{\theta}\right)-F\left(\frac{\phi^*}{\theta}\right)}{1-F\left(\frac{\phi^*}{\theta}\right)}, \text{ from 1.} \\
\frac{\partial\Phi(\phi;\theta)}{\partial\theta} &= \frac{\left[-\frac{\phi}{\theta^2}f\left(\frac{\phi}{\theta}\right)+\frac{\phi^*}{\theta^2}f\left(\frac{\phi^*}{\theta}\right)\right]\left[1-F\left(\frac{\phi^*}{\theta}\right)\right]-\frac{\phi^*}{\theta^2}f\left(\frac{\phi^*}{\theta}\right)\left[F\left(\frac{\phi}{\theta}\right)-F\left(\frac{\phi^*}{\theta}\right)\right]}{\left[1-F\left(\frac{\phi^*}{\theta}\right)\right]^2} \\
&= \frac{-\frac{\phi}{\theta^2}f\left(\frac{\phi}{\theta}\right)\left[1-F\left(\frac{\phi^*}{\theta}\right)\right]+\frac{\phi^*}{\theta^2}f\left(\frac{\phi^*}{\theta}\right)\left[1-F\left(\frac{\phi}{\theta}\right)\right]}{\left[1-F\left(\frac{\phi^*}{\theta}\right)\right]^2} \\
&= \frac{\left[-\phi h(e)+\phi^* h(e^*)\right]\left[1-F\left(\frac{\phi}{\theta}\right)\right]}{\theta^2\left[1-F\left(\frac{\phi^*}{\theta}\right)\right]} < 0,
\end{aligned}$$

given that $\phi > \phi^*$, $e > e^*$ and $h'(e) \geq 0 \Rightarrow -\phi h(e) + \phi^* h(e^*) < 0$.

In words, as θ increases, the conditional CDF of ϕ, for $\phi > \phi^*$, decreases.

■

Thus, given (2) and the distribution function of learning ability, when there is a productivity improvement in FDI firms which increases the stock of available knowledge, the probability distribution of productivity of the domestic firms is stochastically improved such that the probability of a higher productivity draw is higher. In addition, conditional on an observed minimum productivity, the probability of observing a higher productivity is also higher with the increase in the available knowledge stock. All these results come in handy in the later sections when we analyze the comparative statics of the equilibrium. We first formally define first order stochastic dominance and likelihood ratio dominance (which is also known as uniform conditional stochastic dominance, see Müller and Stoyan, 2002).

Definition 1 *Let $\phi_1 \sim G_1$ and $\phi_2 \sim G_2$ be two random variables. ϕ_1 first order stochastic dominates ϕ_2, denoted $\phi_1 \succ_{FSD} \phi_2$ if and only if:*

1. *$G_1(\phi) \leq G_2(\phi)$ for all ϕ in the common support with strict inequality for some ϕ; Or*
2. *$E[u(\phi_1)] \geq E[u(\phi_2)]$ for all function u with $u' \geq 0$.*

From Definition 1 it is clear that if $G_1(\phi) < G_2(\phi)$, it is necessary that $E[\phi_1] > E[\phi_2]$. In other words, if $\phi_1 \succ_{FSD} \phi_2$ it is true that the expected value of ϕ_1 is larger than the expected value of ϕ_2.

Definition 2 *Let $\phi_1 \sim G_1$ and $\phi_2 \sim G_2$ be two random variables, G_1 and G_2 are both continuous and differentiable. ϕ_1 likelihood ratio dominates ϕ_2, denoted $\phi_1 \succ_{LRD} \phi_2$ if and only if:*

1. $\frac{\partial \frac{g_1}{g_2}}{\partial \phi} > 0$
2. *$[\phi_1|\phi_1 \in A] \succ_{FSD} [\phi_2|\phi_2 \in A]$, for $Prob(\phi_1 \in A) > 0$ and $Prob(\phi_2 \in A) > 0$.*
3. *$E[u(\phi_1)|\phi_1 \in A] \geq E[u(\phi_2)|\phi_1 \in A]$ for all function u with $u' \geq 0$.*

The first part of Definition 2 states that G_1 likelihood ratio dominates G_2, if and only if the likelihood ratio of G_1 relative to G_2 is non-decreasing. This is equivalent to stating that the conditional distribution of G_1 first order stochastic dominates the conditional distribution of G_2, which immediately implies that the conditional expectation of G_1 is necessary greater than G_2 from Definition 1. Thus it is clear that, if we define set A as $\{\phi|\phi > \phi^*\}$, the last part of Definition 2 implies that if G_1 likelihood ratio dominates G_2 then it is necessary that $E[\phi_1|\phi > \phi^*] \geq E[\phi_2|\phi > \phi^*]$, for any cutoff ϕ^*.

Lemma 1 *Given all the assumptions in Proposition 1, if* $\theta_0 < \theta_1$, *then*

1. $G(\phi;\theta_0) > G(\phi;\theta_1)$;
2. $\phi(\theta_1) \succ_{FSD} \phi(\theta_0)$;
3. $E[\phi(\theta_0)] < E[\phi(\theta_1)]$;
4. $\Phi(\phi;\theta_0) > \Phi(\phi;\theta_1)$;
5. $\phi(\theta_1) \succ_{LRD} \phi(\theta_0)$;
6. $E[\phi(\theta_0)\,|\phi > \phi^*] < E[\phi(\theta_1)\,|\phi > \phi^*]$.

Proof. Given $0 < \theta_0 < \theta_1$ and $G(\phi;\theta)$ and $\mu(\phi;\theta)$ are continuous and differentiable from Proposition 1.

1. According to (4), $\frac{\partial G(\phi;\theta)}{\partial \theta} < 0$. Thus, $\theta_0 < \theta_1$ implies $G(\phi;\theta_0) > G(\phi;\theta_1)$.

2. By Definition 1, $G(\phi;\theta_0) > G(\phi;\theta_1)$ implies $\phi(\theta_1) \succ_{FSD} \phi(\theta_0)$.

3. Given $\phi(\theta_1) \succ_{FSD} \phi(\theta_0)$, Definition 1 implies $E[\phi(\theta_0)] < E[\phi(\theta_1)]$.

4. According to (6), $\frac{\partial \Phi(\phi;\theta)}{\partial \theta} < 0$. Thus, $\theta_0 < \theta_1$ implies $\Phi(\phi;\theta_0) > \Phi(\phi;\theta_1)$.

5. From Definition 1, $\Phi(\phi;\theta_0) > \Phi(\phi;\theta_1)$ iff $\phi(\theta_1) \succ_{FSD} \phi(\theta_0)$ iff $\phi(\theta_1) \succ_{LRD} \phi(\theta_0)$, by Definition 2.

6. Given $\phi(\theta_1) \succ_{LRD} \phi(\theta_0)$, let $A = \{\phi|\phi > \phi^*\}$, Definition 2 implies $E[\phi(\theta_0)\,|\phi > \phi^*] < E[\phi(\theta_1)\,|\phi > \phi^*]$.

$\blacksquare$

We use Weibull distribution as an example to illustrate our results. Assuming F follows a Weibull distribution with the shape parameter, β, where

$$\begin{aligned} F(e) &= 1 - \exp\left(-e^{\beta}\right), \\ f(e) &= \beta e^{\beta-1} \exp\left(-e^{\beta}\right), \text{ and} \\ h(e) &= \frac{\beta e^{\beta-1} \exp\left(-e^{\beta}\right)}{\exp\left(-e^{\beta}\right)} = \beta e^{\beta-1}, \text{ with} \\ h'(e) &= \beta(\beta-1) e^{\beta-2} \geq 0 \text{ if } \beta \geq 1. \end{aligned}$$

Then $\phi = e\theta$ implies,

$$\begin{aligned} G(\phi;\theta) &= 1 - \exp\left(-\left(\frac{\phi}{\theta}\right)^{\beta}\right), \\ g(\phi;\theta) &= \frac{\beta}{\theta}\left(\frac{\phi}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{\phi}{\theta}\right)^{\beta}\right), \\ \mu(\phi;\theta) &= \frac{\frac{\beta}{\theta}\left(\frac{\phi}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{\phi}{\theta}\right)^{\beta}\right)}{\exp\left(-\left(\frac{\phi^*}{\theta}\right)^{\beta}\right)} = \frac{\beta}{\theta}\left(\frac{\phi}{\theta}\right)^{\beta-1} \exp\left(\left(\frac{\phi^*}{\theta}\right)^{\beta} - \left(\frac{\phi}{\theta}\right)^{\beta}\right), \text{ and} \\ \Phi(\phi;\theta) &= \frac{-\exp\left(-\left(\frac{\phi}{\theta}\right)^{\beta}\right) + \exp\left(-\left(\frac{\phi^*}{\theta}\right)^{\beta}\right)}{\exp\left(-\left(\frac{\phi^*}{\theta}\right)^{\beta}\right)} = 1 - \exp\left(\left(\frac{\phi^*}{\theta}\right)^{\beta} - \left(\frac{\phi}{\theta}\right)^{\beta}\right). \end{aligned}$$

Figure 3 shows that as θ increases from 2 to 3, the CDF of ϕ shifts to the right which implies improvement in productivity distribution such that $\phi(3)$ first order stochastic dominates $\phi(2)$. The unconditional expected values of $\phi(2)$ and $\phi(3)$ is 2 and 3, respectively, if $\beta = 1$. Conditional on $\phi^* = 1$, the expected values of $\phi(2)$ and $\phi(3)$ is 3 and 4, respectively.

In summary, given that firms productivity is positively and linearly depends on the stock of knowledge generated by FDI firms and the random learning ability draw, productivity progress in FDI firms will spillover to domestic firms by improving the distribution of firm productivity such that not only is the probability of getting a high productivity draw is now higher, both the conditional and unconditional average productivity of domestic firms are also higher with the productivity growth in FDI firms.

4.2 Demand

The following set up is similar to Melitz (2003) so we will keep it brief. We assume that a representative consumer has the following Dixit-Stiglitz CES utility function define over a continuum variety of a heterogenous good, indexed by $\omega \in \Omega$, where $q(\omega)$ is the quantity of variety ω consumed:

$$\begin{aligned} U &= Q = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}}, \text{ with } \rho \in (0,1), \text{ and} \\ \sigma &= \frac{1}{1-\rho} > 1, \end{aligned}$$

is the elasticity of substitution between the varieties. The cost of consuming all available variety of the heterogenous goods defined the aggregate price index of the industry, with price of each variety ω is $p(\omega)$,

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

By Sheppard's lemma, the partial derivative of P with respect to $p(\omega)$ gives us the derived demand for variety ω per unit of Q. So the individual demand and expenditure for variety

ω are

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma}, \tag{7}$$

$$r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma}, \tag{8}$$

where $R = PQ = \int\limits_{\omega\in\Omega} r(\omega)\, d\omega$ denotes the aggregate expenditure on the heterogenous good.

4.3 Production

There are a continuum of domestic and FDI firms, each chooses to produce a different variety ω. To produce ω, all firms use only labor input with a common fixed cost, f and a constant marginal cost. Domestic firms are however different according to their learning ability, e. Given the learning ability and the stock of available knowledge depends on the productivity of FDI firms, the productivity of domestic firms is $\phi(\theta) = e\theta$. The higher is their learning ability, the higher is their productivity, and the lower is the marginal cost, $\frac{1}{\phi(\theta)}$, where wages are set to one. Given the residual demand for each variety, ω, all domestic firms have a constant markup rule such that price of each variety is set to be inversely related to the productivity and the elasticity of substitution,

$$p(\phi) = \frac{1}{\rho\phi(\theta)}, \tag{9}$$

It can then be shown that firm's profit is

$$\pi(\phi(\theta)) = \frac{r(\phi(\theta))}{\sigma} - f$$

where $r(\phi(\theta))$ is the revenue of firm with productivity $\phi(\theta)$. Both $r(\phi(\theta))$ and $\pi(\phi(\theta))$ depend on the aggregate price and revenue,

$$\begin{aligned} r(\phi(\theta)) &= R(\rho\phi(\theta)P)^{\sigma-1}, \\ \pi(\phi(\theta)) &= \frac{R(\rho\phi(\theta)P)^{\sigma-1}}{\sigma} - f. \end{aligned}$$

FDI firms have similar production and pricing rules, except the productivity of FDI firms are taken as exogenous. As shown in Melitz (2003), it is very convenient to know the following properties:

$$\frac{q(\phi_1(\theta))}{q(\phi_2(\theta))} = \left(\frac{\phi_1(\theta)}{\phi_2(\theta)}\right)^{\sigma}, \text{ and } \frac{r(\phi_1(\theta))}{r(\phi_2(\theta))} = \left(\frac{\phi_1(\theta)}{\phi_2(\theta)}\right)^{\sigma-1}. \tag{10}$$

Thus a more productive firm would have a lower price, a larger quantity of output, and higher the revenue and profit.

4.4 Industry

An equilibrium of the industry of the heterogenous good is defined by a mass M^d of domestic firms, and an equilibrium distribution of productivity, $\mu(\phi;\theta)$. The total mass of firms including FDI firms is now

$$M = M^f + M^d.$$

In such an equilibrium, the aggregate price index is given by

$$P(\theta) = \left[\int_0^{\infty} p(\phi(\theta))^{1-\sigma} M\mu(\phi;\theta)\,d\phi\right]^{\frac{1}{1-\sigma}} = M^{\frac{-1}{\sigma-1}}\left[\rho\left(\frac{M^d}{M}\int_0^{\infty}\phi(\theta)^{\sigma-1}\mu(\phi;\theta)\,d\phi + \frac{M^f}{M}\theta^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\right]^{-1}$$

using (9) where $p(\phi(\theta)) = (\rho\phi(\theta))^{-1}$. The industry the average productivity level is the weighted average between the productivity of the domestic and FDI firms,

$$\begin{aligned}
\tilde{\phi}^{d}(\theta) &= \left[\int_{0}^{\infty} \phi(\theta)^{\sigma-1} \mu(\phi;\theta)\, d\phi\right]^{\frac{1}{\sigma-1}}, && (11)\\
\tilde{\phi}(\theta) &= \left[\frac{M^{d}}{M}\int_{0}^{\infty} \phi(\theta)^{\sigma-1} \mu(\phi;\theta)\, d\phi + \frac{M^{f}}{M}\theta^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{M^{d}}{M}\tilde{\phi}^{d}(\theta)^{\sigma-1} + \frac{M^{f}}{M}\theta^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. && (12)
\end{aligned}$$

Then the aggregate price index can further be reduced to

$$P(\theta) = M^{\frac{-1}{\sigma-1}} p\left(\tilde{\phi}(\theta)\right) = \left[M^{\frac{1}{\sigma-1}}\rho\tilde{\phi}(\theta)\right]^{-1}. \quad (13)$$

Thus an increase in the average productivity, $\tilde{\phi}(\theta)$, will lead to a decrease in the aggregate price index, $P(\theta)$. It can further be shown that $\tilde{\phi}(\theta)$ is crucial in summarizing all the other aggregate variables:

$$\begin{aligned}
Q(\theta) &= M^{\frac{1}{\rho}} q\left(\tilde{\phi}(\theta)\right), \\
R(\theta) &= PQ = Mr\left(\tilde{\phi}(\theta)\right), \\
\Pi(\theta) &= M\pi\left(\tilde{\phi}(\theta)\right),
\end{aligned}$$

where $r\left(\tilde{\phi}(\theta)\right)$ and $\pi\left(\tilde{\phi}(\theta)\right)$ are the average revenue and profit, $\bar{r} = R/M$ and $\bar{\pi} = \Pi/M$. Thus, if we can establish the effects of θ on $\tilde{\phi}(\theta)$, we can study the effects of θ on all the aggregate variables. We can further define the average revenue and profit of domestic firms, $\bar{r}^{d} = r\left(\tilde{\phi}^{d}(\theta)\right)$ and $\bar{\pi}^{d} = \pi\left(\tilde{\phi}^{d}(\theta)\right)$.

4.5 Equilibrium

At any point in time there is a mass M^d of firm operating in the industry. A fraction of these firms, δM^d, with $\delta < 1$, are randomly subject to some adverse shock and die immediately. At the same time, there are infinitely many prospective entrants waiting to join the industry. These entry and exit of firms generate the endogenously determined productivity distribution of the industry.

Prior to entry, all domestic firms are identical. To enter, these firms must spend a sunk entry fixed cost, f_e, which allows them to draw learning ability from the common distribution function $F(e)$. Those firms that have a low learning ability draw such that $e < e^*$, where e^* is some cutoff learning ability, may find it not worth to stay in the market to produce and face a production fixed cost of f. Thus, upon entering, only $1 - F(e^*)$ of entrants will choose to stay and produce and make non-negative profits. Prior to entry, all the prospective domestic entrants must therefore compare the expected value of an infinite stream of profit upon a successful entry which may be subjected to the adverse shock of probability δ, to f_e to determine whether they should enter or not. At equilibrium, the expected value of profits of domestic firms must equal to f_e which defines the free entry condition:[7]

$$(1 - F(e^*)) \frac{\bar{\pi}^d}{\delta} = f_e.$$

Given θ, we can also define the cutoff productivity, $\phi^* = e^*\theta$, and re-express the free entry

[7] Note that from the point of view of domestic firms, the expected profit depends on the average productivity of the domestic firms, not the average productivity of the industry. The latter is a weighted average between the average productivity of domestic and foreign firms.

condition as

$$\bar{\pi}^d = \frac{\delta f_e}{1 - G(\phi^*; \theta)}. \tag{14}$$

Moreover, since free entry condition must hold in equilibrium, (14) implies that the cutoff productivity must depend on the stock of knowledge, $\phi^*(\theta)$.

The equilibrium distribution of learning ability, $f(e|e \geq e^*)$, is therefore defined by the cutoff learning ability, e^*, conditional on successful entry, i.e. $e \geq e^*$:

$$f(e|e \geq e^*) = \begin{cases} \frac{f(e)}{1-F(e^*)}, \text{ if } e \geq e^* \\ 0, \text{ otherwise.} \end{cases}.$$

Or equivalently, the equilibrium distribution of productivity, $\mu(\phi; \theta)$ would also depend on the cutoff productivity, ϕ^*, conditional on successful entry:

$$\mu(\phi; \theta) = \begin{cases} \frac{g(\phi;\theta)}{1-G(\phi^*;\theta)}, \text{ if } \phi \geq \phi^* \\ 0, \text{ otherwise.} \end{cases}$$

$\mu(\phi; \theta)$ further helps us define the average domestic and industry productivity as a function of the cutoff productivity according to (11) and (12) :

$$\tilde{\phi}^d(\theta) = \left[\frac{1}{1 - G(\phi^*; \theta)} \int_{\phi^*}^{\infty} \phi(\theta)^{\sigma-1} g(\phi; \theta) \, d\phi\right]^{\frac{1}{\sigma-1}} \tag{15}$$

$$\tilde{\phi}(\theta) = \left[\frac{M^d}{M}\tilde{\phi}^d(\theta)^{\sigma-1} + \frac{M^f}{M}\theta^{\sigma-1}\right]^{\frac{1}{\sigma-1}} = \left[\frac{M^d}{M}E\left(\phi(\theta)^{\sigma-1} | \phi \geq \phi^*\right) + \frac{M^f}{M}\theta^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{16}$$

In addition to the free entry condition, which links the average profit of domestic firms to the cutoff productivity and stock of knowledge, in the equilibrium, entry will only stop when the marginal domestic firm is indifferent between producing or not. This is the zero cutoff profit condition, which helps us pin down the cutoff learning ability and productivity:

$$\pi^d(\phi^*; \theta) = 0 \Longleftrightarrow r^d(\phi^*; \theta) = \sigma f. \tag{17}$$

We use the property listed in (10) to re-express the zero cutoff profit condition in terms of average domestic profits:

$$\begin{aligned}
\bar{\pi}^d &= \frac{r\left(\tilde{\phi}^d(\theta)\right)}{\sigma} - f \\
&= \frac{r(\phi^*;\theta)}{\sigma}\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} - f \qquad (18)\\
\bar{\pi}^d &= f\left[\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} - 1\right]. \qquad (19)
\end{aligned}$$

Thus the free entry condition (14) and zero cutoff profit condition (19) jointly determine the equilibrium pair $\phi^*(\theta)$ and $\bar{\pi}^d(\theta)$.

$$\phi^*(\theta) = \left\{\phi \mid \frac{\delta f_e}{1 - G(\phi^*;\theta)} = f\left[\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} - 1\right]\right\}. \qquad (20)$$

We will show the existence and uniqueness of equilibrium by showing that the left-hand side of (20) is monotonically decreasing with respect to ϕ, and it will equal to the constant $\frac{\delta f_e}{f}$ which defines the cutoff productivity, $\phi^*(\theta)$.

Proposition 2 *Let* $\tilde{\phi}^d(\theta) = \left[\frac{1}{1-G(\phi^*;\theta)}\int_{\phi^*}^{\infty}\phi(\theta)^{\sigma-1} g(\phi;\theta)\,d\phi\right]^{\frac{1}{\sigma-1}}$, $k(\phi^*;\theta) = \left[\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} - 1\right]$ *and* $z(\phi^*;\theta) = (1 - G(\phi^*;\theta))\,k(\phi^*;\theta)$.

1. $k(\phi^*;\theta) > 0$;
2. $\frac{\partial\tilde{\phi}^d(\phi^*;\theta)}{\partial\phi^*} > 0$;
3. $\frac{\partial k(\phi^*;\theta)}{\partial\phi^*} > 0$, *if* $\sigma < 1 + \phi^*\frac{g(\phi^*;\theta)}{1-G(\phi^*;\theta)}\frac{k}{k+1}$;
4. $\frac{\partial z(\phi^*;\theta)}{\partial\phi^*} < 0$.

Proof. The following is quite straightforward:

1. $\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} = \frac{1}{1-G(\phi^*;\theta)}\int_{\phi^*}^{\infty}\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} g(\phi;\theta)\,d\phi$, where $\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} > 1$, $\forall\phi \in [\phi^*,\infty)$. Thus $\left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} > \frac{1}{1-G(\phi^*;\theta)}\int_{\phi^*}^{\infty} g(\phi;\theta)\,d\phi = \frac{1-G(\phi^*;\theta)}{1-G(\phi^*;\theta)} = 1$. This implies that $k(\phi^*;\theta) > 0$, i.e. the ratio of average to marginal productivity is greater than 1.

2. $\frac{\partial\tilde{\phi}^d(\phi^*;\theta)}{\partial\phi^*} = \frac{1}{\sigma-1}\tilde{\phi}^d(\phi^*;\theta)^{2-\sigma}\frac{g(\phi^*;\theta)}{1-G(\phi^*;\theta)}\left[\tilde{\phi}^d(\theta)^{\sigma-1} - (\phi^*)^{\sigma-1}\right] = \frac{1}{\sigma-1}\tilde{\phi}^d\frac{g}{1-G}\frac{k}{k+1} > 0$, i.e. an increase in the marginal productivity always increases the average productivity.

3. $\frac{\partial k(\phi^*;\theta)}{\partial\phi^*} = (\sigma-1)(k+1)^{\frac{\sigma-2}{\sigma-1}}\left[\frac{\tilde{\phi}'\phi^*-\tilde{\phi}}{(\phi^*)^2}\right] = \frac{(\sigma-1)(k+1)}{\phi^*}\left[\frac{1}{\sigma-1}\phi^*\frac{g}{1-G}\frac{k}{k+1} - 1\right] > 0$ if $\sigma < 1 + \phi^*\frac{g(\phi^*;\theta)}{1-G(\phi^*;\theta)}\frac{k}{k+1}$, i.e. an increase in the marginal productivity leads to an increase in the ratio of average to marginal productivity if the elasticity of substitution between goods is bounded above (relatively small).

4. $\frac{\partial z(\phi^*;\theta)}{\partial\phi^*} = -gk + (1-G)\frac{(\sigma-1)(k+1)}{\phi^*}\left[\frac{1}{\sigma-1}\phi^*\frac{g}{1-G}\frac{k}{k+1} - 1\right] = -(1-G)\frac{(\sigma-1)(k+1)}{\phi^*} < 0.$

■

The last point of the above proposition shows that $z(\phi^*;\theta)$ is monotonically decreasing with respect to ϕ^*. Given that $\lim\limits_{\phi^*\to 0} z(\phi^*;\theta) = \lim\limits_{\phi^*\to 0} k(\phi^*;\theta) = \infty$, and $\lim\limits_{\phi^*\to\infty} z(\phi^*;\theta) = 0$, and given that $z(\phi^*;\theta)$ is continuous for $\phi^* > 0$, there must exist a unique $\phi^* \in (0,\infty)$ such that (20) is satisfied. Figure 4 illustrates the equilibrium cutoff productivity. Thus the solution to both the free entry condition and zero cutoff profit condition give us the equilibrium ϕ^* and $\bar{\pi}^d$ in terms of θ, f, f_e, δ, and σ. With ϕ^* and $\bar{\pi}^d$ we can solve for $\tilde{\phi}^d$ and $\bar{r}^d$.

To fully solve the model for M^d and therefore M, we would need to first solve for $\bar{r}^f = r(\theta)$ using (10) :

$$\bar{r}^f = \bar{r}^d\left(\frac{\theta}{\tilde{\phi}^d}\right)^{\sigma-1} = \sigma\left(\bar{\pi}^d + f\right)\left(\frac{\theta}{\tilde{\phi}^d}\right)^{\sigma-1} = \sigma f\left(\frac{\theta}{\phi^*}\right)^{\sigma-1}.$$

Thus the average revenue of FDI firms depends on the ratio of its average productivity to the productivity of the marginal domestic firms. Given θ, an increase in the cutoff productivity will decrease the average productivity of FDI firms. Similarly, we can solve of the average revenue of domestic firms as the following:

$$\bar{r}^d = \sigma\left(\bar{\pi}^d + f\right) = \sigma f \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{\sigma-1},$$

which shows that the average revenue of domestic firms depend on the ratio of the average domestic productivity to the cutoff productivity.

Total revenue of the industry must equal to the total labor earning (with wages set to one) which helps us solve for M^d and M, given fixed M^f :

$$\begin{aligned}
R &= M^d \bar{r}^d + M^f \bar{r}^f \\
L &= \sigma f \left[M^d \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{\sigma-1} + M^f \left(\frac{\theta}{\phi^*}\right)^{\sigma-1}\right] \Rightarrow \\
M^d &= \frac{L}{\sigma f \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{\sigma-1}} - M^f \left(\frac{\theta}{\tilde{\phi}^d}\right)^{\sigma-1} \text{ and } M = \frac{L}{\sigma f \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{\sigma-1}} + M^f \left[1 - \left(\frac{\theta}{\tilde{\phi}^d}\right)^{\sigma-1}\right]
\end{aligned}$$

Once we pin down M^d and M, we can then solve for the average industry productivity $\tilde{\phi}$, according to (16) and aggregate price index, P, according to (13). The rest of the aggregate variables are thus straightforward to solve.

For our earlier the example of G equals a Weibull distribution with $\beta = 1$ and $\theta = 2$, we set $f_e = 10$, $f = 5$, $\delta = 0.1$ and $\sigma = 2$. It can be solved that ϕ^* is 2.653. Given ϕ^*, $\bar{\pi}$ and $\tilde{\phi}^d$ can be solved from (14) and (**??**), as 3.769 and 4.653 respectively. Assuming $L = 1$, $M^f = 1$, M^d and M are 0.702 and 1.702. The average industry productivity and aggregate price index are therefore 2.174 and 0.860, respectively.

4.6 Productivity progress of FDI firms

Now consider an equilibrium if the stock of knowledge generated by FDI firms is higher exogenously. Given learning ability of domestic firms, from Proposition 1 we know that a higher θ causes the unconditional productivity distribution and the conditional productivity distribution of domestic firms to stochastically improve, as $\partial G/\partial\theta < 0$, and $\partial\Phi/\partial\theta < 0$. This leads to a higher average productivity, $\tilde{\phi}^d$, which is the conditional expected value of ϕ given $\phi > \phi^*$, for any given level of ϕ^*. How would this affect the equilibrium? First, given ϕ^*, an increase in θ shifts $z(\phi^*;\theta) = (1 - G(\phi^*;\theta))\, k(\phi^*;\theta)$ up, which captures the spillover effect of the productivity of FDI firms as all firms are now more productive.

Moreover, with $z(\phi^*;\theta)$ shifting up, in equilibrium, ϕ^* is now higher, since the average profit of domestic firms increases which attract entry, intensifies market competition and leads a higher equilibrium cutoff productivity. In other words, $\phi^{*\prime}(\theta) > 0$. The increase in ϕ^* further increases the average productivity of domestic firms due to the exiting of inefficient firms and thus captures the selection effect of the increase in θ. The extent to which ϕ^* and $\tilde{\phi}^d$ increase due to the increase in θ depends on the elasticity of substitution between variety, σ, and the distribution function, G. Given G, if goods are relatively homogeneous, i.e. if σ is very large, then an increase in θ will have a larger effect on ϕ^* and $\tilde{\phi}^d$. In other words, the selection effect due to an increase in θ is stronger when goods are more homogeneous. This would likely lead to an equilibrium with less domestic variety, M^d, and a smaller overall variety, M. On the other hand, if σ is small, especially when σ is very close to one, then an increase in θ will have a very small effect on ϕ^*, $\tilde{\phi}^d$, $\tilde{\phi}^d/\phi^*$ and thus, M^d and

M. At the limit when σ is one, M^d and M are both constant and are independent on θ. In all cases, regardless of the movement of domestic variety, average industry productivity, $\tilde{\phi}$, unambiguously increases and the aggregate price index, $P(\theta)$ unambiguously decreases. This captures the selection effect due to productivity improvement of FDI firms.

Overall, the spillover effect and the selection effect reinforce each other such that the equilibrium $\phi^*, \tilde{\phi}^d$ and $\tilde{\phi}$ are higher, which indicate that the industry on average has become more productive and efficient, through both overall improvement of productivity distribution and exiting of inefficient firms facing a higher cutoff productivity.

Proposition 3 *Let* $\tilde{\phi}^d(\phi^*;\theta) = \left(\frac{1}{1-G(\phi^*;\theta)}\int_{\phi^*}^{\infty}\phi^{\sigma-1}g(\phi;\theta)\,d\phi\right)^{\frac{1}{\sigma-1}}$, $k(\phi^*;\theta) = \left(\frac{\tilde{\phi}^d(\theta)}{\phi^*(\theta)}\right)^{\sigma-1} - 1$ *and* $z(\phi^*;\theta) = (1-G(\phi^*;\theta))\,k(\phi^*;\theta)$.

1. $\int_{\phi^*}^{\infty}\left(\frac{\phi}{\phi^*}\right)^{\sigma-1}\left(\frac{\partial g}{\partial\theta} + \frac{g^*}{1-G^*}\frac{\partial G^*}{\partial\theta}\right)d\phi > 0$, *for given* ϕ^*;
2. $\frac{\partial\phi^*}{\partial\theta} > 0$;
3. $\frac{\partial\tilde{\phi}^d}{\partial\theta} > 0$;
4. $\frac{\partial\left(\tilde{\phi}^d/\phi^*\right)}{\partial\theta} > 0$ *if* $\frac{\tilde{\phi}^d}{\sigma-1}\frac{g^*}{1-G^*}\frac{k}{k+1} > 1$;
5. $\frac{\partial M^d}{\partial\theta} < 0$ *if* $\frac{\partial\left(\tilde{\phi}^d/\phi^*\right)}{\partial\theta} > 0$ *and* $\frac{\partial\tilde{\phi}^d}{\partial\theta}\frac{\theta}{\tilde{\phi}^d} < 1$;

Proof. Note that $\tilde{\phi}^{\sigma-1} = E\left(\phi^{\sigma-1}|\phi > \phi^*;\theta\right)$.

1. $\frac{\partial\tilde{\phi}^d}{\partial\theta}|_{\phi^*} = \frac{1}{\sigma-1}\left(\tilde{\phi}^d\right)^{2-\sigma}\left(\frac{1}{1-G^*}\left(\tilde{\phi}^d\right)^{\sigma-1}\frac{\partial G^*}{\partial\theta} + \frac{1}{1-G^*}\int_{\phi^*}^{\infty}\phi^{\sigma-1}\frac{\partial g}{\partial\theta}d\phi\right)$

   $= \frac{\tilde{\phi}^d}{(\sigma-1)(k+1)(1-G^*)}\int_{\phi^*}^{\infty}\left(\frac{\phi}{\phi^*}\right)^{\sigma-1}\left(\frac{\partial g}{\partial\theta} + \frac{g}{1-G^*}\frac{\partial G^*}{\partial\theta}\right)d\phi > 0$, by Lemma 1.

   Thus, $\int_{\phi^*}^{\infty}\left(\frac{\phi}{\phi^*}\right)^{\sigma-1}\left(\frac{\partial g}{\partial\theta} + \frac{g}{1-G^*}\frac{\partial G^*}{\partial\theta}\right)d\phi > 0$.

2. To find $\frac{\partial \phi^*}{\partial \theta}$, we apply implicit function theorem by totally differentiate (20) with respect to θ :

$$\begin{aligned}
\frac{\delta f_e}{f} &= \left(1 - G\left(\phi^*; \theta\right)\right) k\left(\phi^*; \theta\right) \\
\frac{\delta f_e}{f} &= \int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} g\left(\phi; \theta\right) d\phi - 1 + G\left(\phi^*; \theta\right) \Rightarrow \\
\frac{\partial \phi^*}{\partial \theta} &= \frac{\phi^*}{\left(\sigma-1\right)\left(1-G^*\right)\left(k+1\right)} \left[\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \frac{\partial g}{\partial \theta} d\phi + \frac{\partial G^*}{\partial \theta}\right] \\
&> \frac{\phi^*}{\left(\sigma-1\right)\left(1-G^*\right)\left(k+1\right)} \left[-\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \frac{g}{1-G^*} d\phi + 1\right] \frac{\partial G^*}{\partial \theta}, \text{ given } \frac{\partial \tilde{\phi}^d}{\partial \theta}|_{\phi^*} > 0 \\
&= -\frac{\phi^* k}{\left(\sigma-1\right)\left(1-G^*\right)\left(k+1\right)} \frac{\partial G^*}{\partial \theta} > 0, \text{ given } \frac{\partial G^*}{\partial \theta} < 0.
\end{aligned}$$

3. $\frac{\partial \tilde{\phi}^d}{\partial \theta} = \frac{1}{\sigma-1} \left(\tilde{\phi}^d\right)^{2-\sigma} \left(\frac{g^*}{1-G^*} \left(\tilde{\phi}^d\right)^{\sigma-1} \frac{\partial \phi^*}{\partial \theta} - \frac{g^*}{1-G^*} \left(\phi^*\right)^{\sigma-1} \frac{\partial \phi^*}{\partial \theta} + \frac{1}{1-G^*} \left(\tilde{\phi}^d\right)^{\sigma-1} \frac{\partial G^*}{\partial \theta} + \frac{1}{1-G^*} \int_{\phi^*}^{\infty} \phi^{\sigma-1} \frac{\partial g}{\partial \theta} d\phi\right.$

$= \frac{\tilde{\phi}^d}{(\sigma-1)(1-G^*)(k+1)} \left[g^* k \frac{\partial \phi^*}{\partial \theta} + \int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \left(\frac{\partial g}{\partial \theta} + \frac{g}{1-G^*} \frac{\partial G^*}{\partial \theta}\right) d\phi\right] > 0$, by 1.

$\frac{\partial \tilde{\phi}^d}{\partial \theta} \frac{\theta}{\tilde{\phi}^d} = \frac{\theta}{(\sigma-1)(1-G^*)(k+1)} \left[g^* k \frac{\partial \phi^*}{\partial \theta} + \int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \left(\frac{\partial g}{\partial \theta} + \frac{g}{1-G^*} \frac{\partial G^*}{\partial \theta}\right) d\phi\right] \lesseqgtr 1$

4. $\frac{\partial\left(\tilde{\phi}^d/\phi^*\right)}{\partial \theta} > 0$ if $\frac{\partial \tilde{\phi}^d}{\partial \theta} > \frac{\partial \phi^*}{\partial \theta}$.

$\frac{\partial \tilde{\phi}^d/\partial \theta}{\partial \phi^*/\partial \theta} = \frac{\tilde{\phi}^d}{\sigma-1} \frac{g^*}{1-G^*} \frac{k}{k+1} + \frac{\tilde{\phi}^d}{\phi^*} \frac{\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \left(\frac{\partial g}{\partial \theta} + \frac{g}{1-G^*} \frac{\partial G^*}{\partial \theta}\right) d\phi}{\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \frac{\partial g}{\partial \theta} d\phi + \frac{\partial G^*}{\partial \theta}} = \frac{\tilde{\phi}^d}{\sigma-1} \frac{g^*}{1-G^*} \frac{k}{k+1} + \frac{\tilde{\phi}^d}{\phi^*} \frac{\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \frac{\partial g}{\partial \theta} d\phi + \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{\sigma-1} \frac{\partial G^*}{\partial \theta}}{\int_{\phi^*}^{\infty} \left(\frac{\phi}{\phi^*}\right)^{\sigma-1} \frac{\partial g}{\partial \theta} d\phi + \frac{\partial G^*}{\partial \theta}} >$ 1 if $\frac{\tilde{\phi}^d}{\sigma-1} \frac{g^*}{1-G^*} \frac{k}{k+1} > 1$.

5. $\frac{\partial M^d}{\partial \theta} = -\left(\sigma-1\right) \left[\frac{L}{\sigma f} \left(\frac{\tilde{\phi}^d}{\phi^*}\right)^{-\sigma} \frac{\partial\left(\tilde{\phi}^d/\phi^*\right)}{\partial \theta} + M^f \left(\frac{\theta}{\tilde{\phi}^d}\right)^{\sigma-2} \left(\frac{1-\left(\theta/\tilde{\phi}^d\right) \partial \tilde{\phi}^d/\partial \theta}{\tilde{\phi}^d}\right)\right] < 0$ if $\frac{\partial\left(\tilde{\phi}^d/\phi^*\right)}{\partial \theta} >$ 0 and $\frac{\partial \tilde{\phi}^d}{\partial \theta} \frac{\theta}{\tilde{\phi}^d} < 1$.

■

Thus, increases in θ causes the curve $z\left(\phi^*; \theta\right)$ to move up. Given $\delta f_e/f$, the equilibrium cutoff productivity, ϕ^* increases as a result. The elasticity of ϕ^* with respect to θ depends positively on σ. As σ approaches infinity, goods are more homogeneous, the effect of θ on ϕ^* increases. On the other hand, if goods are very different as σ approaches one, the effect of θ on ϕ^* decreases.

The theoretical results above further suggest the following. The aggregate price index is lower due to a higher θ shows that if $P(\theta)$ is used to deflate domestic firms revenue, it will over estimate firms productivity and introduce a positive spurious correlation between θ and the estimated productivity of domestic firms. Moreover, given that θ has an ambiguous effect on M^d and M shows that the share of FDI in the industry is probably not a good measurement for θ, especially in the context to capture the horizontal spillover effects, $\frac{\partial \tilde{\phi}^d}{\partial \theta}$.

5 Empirical strategy

To test the model, we formally fit the structural model by estimating the productivity distribution function in the especially case of Weibull distribution. This structural approach will directly link the average productivity of FDI firms to the sample productivity distribution of domestic firms, by estimating the shape parameter, β, and the cutoff productivity, ϕ^*.

Specifically, $G(\phi;\theta)$ is a Weibull distribution, such that

$$G(\phi;\theta) = 1 - \exp\left(-\left(\frac{\phi}{\theta}\right)^{\beta}\right), \text{ with } \beta > 1.$$

Empirically, we don't observe the whole distribution of ϕ, since only those firms that have $\phi > \phi^*$ operate in the market. In other words, we only observe the conditional distribution,

$$\Phi(\phi;\theta) = 1 - \exp\left[\left(\frac{\phi^*}{\theta}\right)^{\beta} - \left(\frac{\phi}{\theta}\right)^{\beta}\right], \ \phi > \phi^* \text{ and } \beta > 1. \tag{21}$$

With the estimated Kernel density distribution, we construct sample conditional distribution of domestic firm productivity, and estimate (21) using domestic firm productivity, ϕ, and the average productivity of FDI firms, θ. If the data fits the model well, we will be able to estimate ϕ^* and β from the nonlinear regression. For our theoretical model to explain the data, we expect β to be greater than one, and ϕ^* to be smaller than all the surviving firms' productivity. We will be able to test these hypotheses with the estimated distribution function.

5.1 Estimating Weibull Distribution and Model Calibration

To estimate (21) , we construct the sample distribution of productivity using the Kernel density estimation. Table 5 presents the results of the nonlinear estimation. When we pool all domestic firms in all industries together, the estimated β is 1.313 and is statistically significant. The estimated cutoff productivity is 1.857 and is also statistically significant. The null hypotheses that $\beta > 1$ and $\phi^* < \min \phi$ cannot be rejected.

We calibrate the model using the estimated elasticity of substitution obtained from Broda and Weinstein (2004), which is about 2, together with the estimated β and ϕ^*. First, with β, ϕ^* and σ we can infer from the model that $\tilde{\phi}^d$ is about 17.5. This is slightly higher than the simple average productivity of the sample, which is 14.1. With $\tilde{\phi}^d$, we can further infer that $k = \left(\tilde{\phi}^d/\phi^*\right)^{\sigma-1} - 1 = 8.42$, and $1 - G(\phi^*; \theta) = 0.95$. We can then obtain $\delta f_e/f$ equals to 8 from (4). Thus the estimation shows that firms in the garment sector of Bangladesh face a relatively high entry fixed cost. To get a good sense of the entry fixed cost, we use (17) which shows that revenue of the marginal firm equals to σf. In our sample, the lowest revenue is about \$60,000, given that σ is 2, f is therefore no greater than \$30,000.[8] This implies that the discounted entry fixed cost, δf_e is about \$240,000, and the expected average profit is \$252,600, from (19). Relative to the average revenue of the domestic firms in the industry, which is about \$3 million, the calibrated profit seems reasonable as it implies a profit margin of 8.4%, and is quite close to the sample average profit of \$285,000.

The size of entry fixed cost depends on the failure rate, δ. According to the membership information of Bangladesh Garment Manufacturers and Exporters Association, currently there are about 3,451 garment firms registered as operating in Bangladesh. However, according to the customs data, there are only 2,387 firms that are actively producing and exporting. Given that Bangladeshi garments are mostly for exports, we infer that the failure rate of is about 30% which implies that the entry fixed cost is about \$800,000, or 44% of the reported fixed assets of the survey firms.

Overall it appears that the data fits the model reasonably well, with the average productivity of FDI firms acting as the scale parameter, θ, such that when θ increases, the whole distribution of productivity of domestic firms moves to the right, which triggers a selection effect due to the exiting of inefficient firms and increases the average industry productivity furthermore.

[8] This is after we remove some outliers from the sample.

6 Conclusion

This paper provides both empirical and theoretical evidence of horizontal spillovers of FDI. Based on a newly collected firm level data set of the Bangladeshi garment sector, this paper shows that not only are FDI firms more productive, but also that the productivity progress of FDI firms raises the productivity of domestic firms. This horizontal spillover of FDI is further explained in a theoretical model with heterogenous firms. Productivity of domestic firms depends on a random learning ability draw and the productivity of FDI firms. Using the firm survey data, a conditional Weibull distribution of the productivity of domestic firms is estimated and is shown to support the model through calibration.

There are three possible reasons why previous studies found weak evidence of horizontal spillover effects. First is the limitation of data. Due to the lacking of firm specific price indexes on output and materials, previous studies have been using industry prices indexes. As shown in the model, aggregate price index of output decreases as productivity of FDI firms increases, using industry price index to deflate sales of domestic firms will cause the estimated productivity of domestic firms to be artificially higher, which results in a spurious positive correlation between the presence of FDI firms and the productivity of domestic firms. Similarly, using industry price index of material to deflate material costs may cause the local firms to appear less productive if the presence of FDI firms reduces the industry index. This leads to a spurious negative correlation between FDI presence of the productivity of domestic firms. The overall impact of the use of industry prices therefore depends on which bias is bigger. This could explain why, when relating foreign presence to domestic productivity, some papers found a positive effect and other found a negative or no effect. The second reason is the limitation of the foreign presence variable in capturing productivity spillover. As shown in the model, when the average productivity of FDI firms increases, share of FDI firms may increase, decrease or remain constant. It depends on elasticity of substitution and the underlying distribution function of domestic productivity. In addition, increase in the presence of FDI may capture market stealing effect, selection effect, in addition to spillover effects. All these effects affect domestic productivity in opposite directions. This could explain why the previous literature found mixed results, as conceptually we cannot isolate spillover effects of productivity by relating foreign presence with the productivity of domestic firms. Finally, unlike previous literature which focuses on all industries within the manufacturing sector, this paper looks more in depth into one industry – the garment industry in Bangladesh – which many have considered as a success story of development. Positive spillover effects of FDI in Bangladesh's garment industry may or may not be able to generalize to other countries or industries.

In a broader context, this paper provides a theoretical model to explain potential productivity differences among countries. Countries may have different aggregate productivity that is not only driven by the selection mechanism as in Melitz (2003), but because of the inherent differences in the distribution of productivity or learning ability. Such inherent differences in productivity distribution, could be purely exogenous as in Eaton and Kortum (2002), or could be endogenously determined as in the case of this paper. By allowing trade costs or FDI policies to affect the productivity distribution, this paper presents a fresh way to link trade or FDI policies to affect aggregate productivity and long term growth.

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Table 1: Sample Averages

	domestic firms	FDI firms
sales	2648.9	10500.0
export sales	2589.3	10400.0
materials	1880.4	7568.2
imported materials	1045.7	5500.6
employee (number)	593.7	1565.6
wage bill	313.9	1076.0
investment	195.7	243.1
capital	1047.0	3963.2
age (year)	6.8	6.5

Notes: All values are in US$000, except employee and age.

Table 2: Dependent variable: Log of output

	(1) OLS	(2) Within	(3) Olley-Pakes	(4) Olley-Pakes
Materials	0.688***	0.716***	0.715***	0.715***
	(0.037)	(0.065)	(0.065)	(0.065)
Labor	0.283***	0.245***	0.255***	0.255***
	(0.036)	(0.087)	(0.089)	(0.089)
Capital	0.029***	0.017	0.018	0.021*
	(0.008)	(0.022)	(0.249)	(0.011)
Age			-0.184	0.030*
			(0.315)	(0.019)
Investment			0.140	
			(0.113)	
Endogeneity correction[1]	No	No	Yes	Yes
Selectivity correction[2]	No	No	No	Yes
Firm fixed effects	No	Yes	Yes	Yes
Year fixed effects	No	Yes	Yes	Yes
Observations	1027	1027	1027	795

Notes: Heteroskadasticity corrected white robust standard errors in parentheses.

[1]A 3rd order polynomial function of age, capital and investment are included.

[2]A 3rd order polynomial function of propensity to stay in business and the fitted output net of labor and capital are included.

Table 3: Dependent variable: Log of TFP

	(1) Between	(2) Between	(3) OLS	(4) Between	(5) Between	(6) OLS
FDI dummy variable	0.201*	0.183*	0.240***			
	(0.109)	(0.097)	(0.086)			
Foreign equity share				0.214*	0.196*	0.254***
				(0.113)	(0.101)	(0.088)
Age		0.001	0.000		0.001	0.000
		(0.003)	(0.001)		(0.003)	(0.001)
Export share of US		0.258	0.244***		0.260	0.249***
		(0.164)	(0.063)		(0.164)	(0.064)
Export share of EU		0.151	0.130**		0.151	0.132**
		(0.158)	(0.054)		(0.158)	(0.054)
Region fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1027	1013	1013	1027	1013	1013

Notes: Asymptotic standard errors in parentheses in (1), (2), (4) and (5).
Heteroskadasticity corrected white robust standard errors in parentheses in (3) and (6).
Total number of firms in the unbalanced panel is 232 in (1) and (4), and 227 for the rest .
Dependent variable is constructed based on (4) of Table 2.

Table 4: Dependent variable: Log of TFP

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
	Within	Within	Within	Within	Within	Within	Within	Wit
FDI share in industry	0.312	0.335			-3.122***	-3.075***		
	(0.222)	(0.222)			(0.622)	(0.548)		
Productivity of FDI			0.135**	0.143**	1.246***	1.058***		0.161*
firms in industry			(0.063)	(0.063)	(0.228)	(0.217)		(0.0
Age		-0.004***		-0.005***	-0.009***	-0.010***	-0.181***	0.0
		(0.002)		(0.001)	(0.002)	(0.002)	(0.012)	(0.0
Export share of US		0.008		0.008	0.012	0.010	1.274**	0.1
		(0.128)		(0.127)	(0.127)	(0.159)	(0.528)	(0.0
Export share of EU		-0.081		-0.081	-0.071	-0.094	1.163**	0.1
		(0.145)		(0.145)	(0.145)	(0.211)	(0.437)	(0.0
Productivity of local							0.394	
firms in industry							(0.228)	
Lag productivity of								0.214*
FDI firms in industry								(0.0
Log of industry total								0.122*
export to US								(0.0
Log of industry total								-0.0
export to EU								(0.0
Log of own R&D								0.0
								(0.0
Log of industry total								0.02
R&D								(0.0
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Y
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Y
Observations	878	878	878	878	878	605	135	5

Notes: Both FDI presence and productivity are specific to industry and year.
To correct for correlation of errors within industry-year, we cluster the standard errors in parentheses for each industry-year.
(1) - (5) consist of an unbalanced panel of 196 wholly domestic owned firms. (6) consists of the more productive domestic firms. (7) only includes FDI firms. (1)-(7) are for 1999-2003.
(8) consists of an unbalanced panel of 162 wholly domestic owned firms, 2000-2003.

Table 5: Conditional Weibull Distribution

β	1.313***
	(0.122)
ϕ^*	1.857***
	(0.542)
Observation	878
$F(2,876)$	1621.32

Figure 1: Cumulative Distribution of Productivity

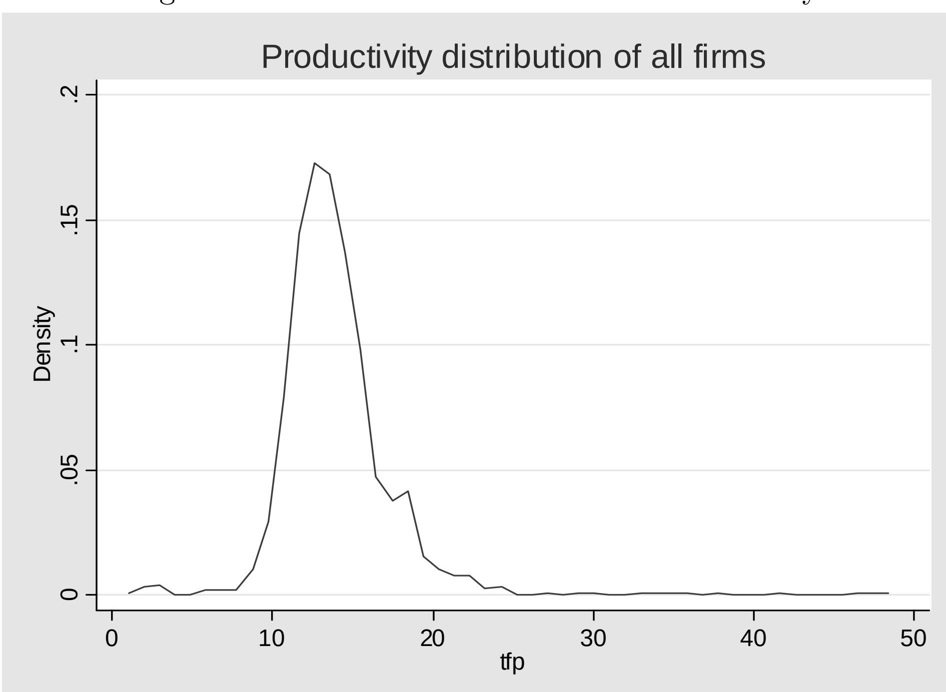

Figure 2: Cumulative Distribution of Productivity

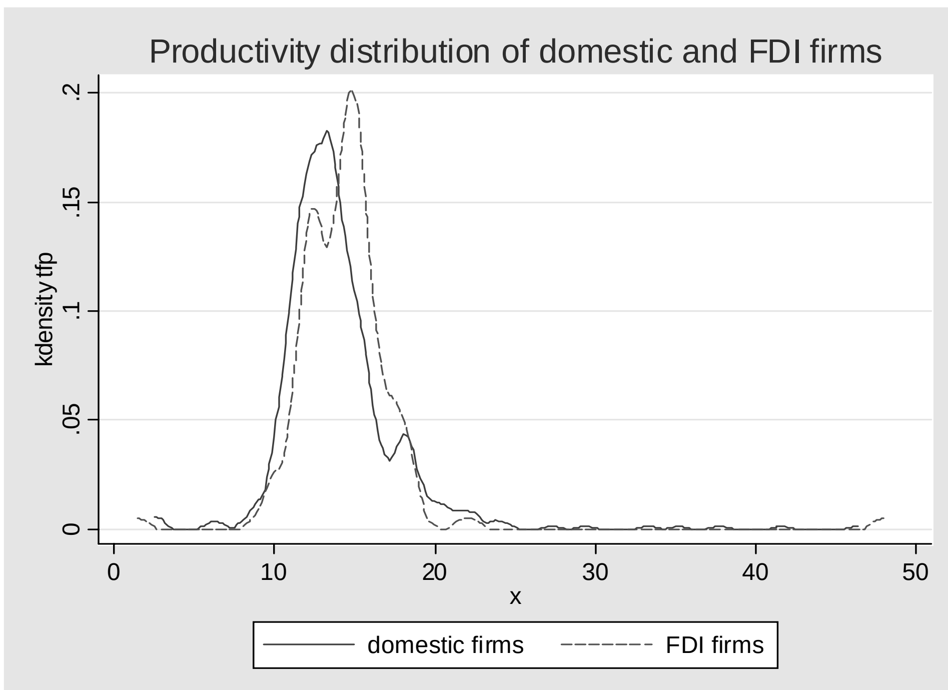

Figure 3: Cumulative Distribution of Productivity

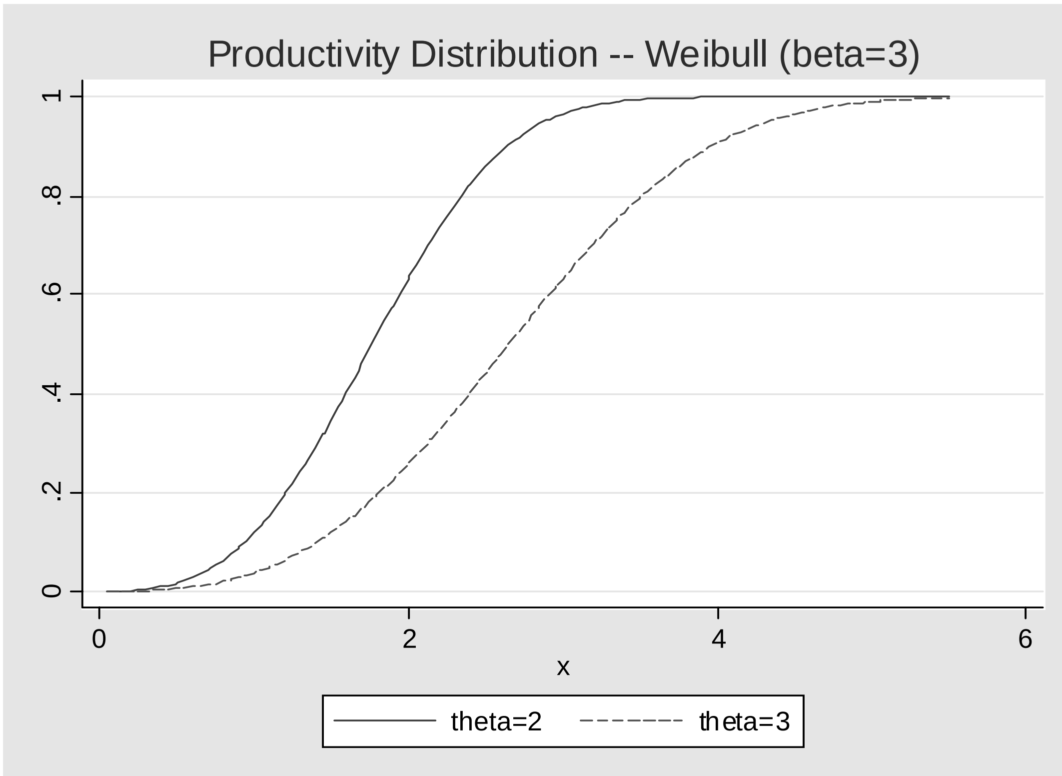

Figure 4: Cutoff Productivity in Equilibrium