Fiscal Policy and Macroeconomic Uncertainty in Developing Countries: The Tale of the Tormented Insurer

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Abstract

Governments in developing countries often behave like a "tormented insurer," trying to use non-state-contingent debt instruments to avoid cuts in payments to private agents despite large fluctuations in public revenues. In the data, average public debt-GDP ratios decline as the variability of revenues increases, primary balances and current expenditures follow cyclical patterns sharply at odds with the countercyclical patterns of industrial countries, and the cyclical variability of public expenditures exceeds that of private expenditures by a wide margin. This paper proposes a model of a small open economy with incomplete markets that can rationalize this behavior. In the model, a fiscal authority makes optimal expenditure and debt plans given shocks to output and revenues, and private agents make optimal consumption and asset accumulation plans. Quantitative analysis of the model calibrated to Mexico yields a negative relationship between average public debt and revenue variability similar to the one observed in the data. The model also mimics Mexico's GDP correlations of government purchases and the primary balance. The ratio of public-to-private expenditures fluctuates widely and the implied welfare costs dwarf conventional estimates of negligible benefits of risk sharing and consumption smoothing.

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1 Introduction

The empirical regularities of fiscal policy in developing countries differ from those observed in the industrial world in four key respects:

(1) The ratios of public revenue to GDP are significantly smaller on average and substantially more volatile, as Figures 1a and 1b show.

(2) Average public debt-GDP ratios decline as the variability of public revenue ratios increases (see Figure 2), so the higher variability of revenues in developing countries is associated with lower average debt ratios.

(3) The cyclical variability of government expenditures exceeds that of private expenditures by large margins (see Figure 3).

(4) Fiscal policy is countercyclical in industrial countries but acyclical or slightly procyclical in developing countries. In particular, GDP and the primary fiscal balance (government expenditures) are positively (negatively) correlated over the business cycle in industrial countries, while in developing countries the GDP correlation of the primary balance (government expenditures) is close to zero or slightly negative (positive). Talvi and Végh (2005) and Gavin and Perotti (1997) first documented these facts, and studies by Catao and Sutton (2002), Kaminsky, Reinhart and Végh (2004) and Alessina and Tabellini (2005) provide detailed cross-country evidence.

This paper argues that the striking differences in the stylized facts of fiscal policy in developing economies may result from the combination of frictions in financial markets and imperfections in their own structure of government revenues and outlays. In particular, this paper models fiscal authorities as playing the role of a "tormented insurer," who tries to maintain a relatively smooth stream of payments to the private sector (i.e., provide a form of social insurance) in the face of substantial, non-insurable fiscal revenue risk and having access only to a non-state-contingent debt instrument.

Mendoza and Oviedo (2004) show how a simple, partial-equilibrium model of a tormented insurer that tries to keep fiscal outlays constant at an exogenous level, given a Markov process of fiscal revenues, delivers results that combine the notion of a "natural debt limit" from the precautionary savings literature with Barro's (1979) prediction re-

garding the Random Walk behavior of public debt in his tax smoothing framework. The government keeps its outlays constant as long as the history of revenue realizations and the dynamics of debt result in a debt level above the annuity value of the "worst," or "catastrophic," level of fiscal revenues (i.e., the government's natural debt limit). When, and if, this limit is reached, government outlays adjust to a "fiscal crisis" (or "lowest tolerable") level.[1] In the long run, the public debt-GDP ratio has dynamics that are determined by initial conditions and lacks a well-defined limiting distribution. Depending on the initial debt ratio, public debt ends up hitting either the natural debt limit or being retired entirely (with zero debt imposed as a lower bound at which the government rebates its revenue to the private sector).

This basic setup cannot say much about procyclical fiscal policy, since both revenues and outlays are exogenous, or about the implications of the tormented insurer's behavior for the private sector's plans, equilibrium allocations, and the overall welfare of the economy. Yet, it does illustrate the potential for the tormented insurance framework to account for the observed negative relationship between average debt ratios and the variability of government revenues: Governments with more volatile revenues have tighter natural debt limits and, under incomplete asset markets, require more precautionary savings, and hence tolerate less debt. This result suggests also that traditional approaches to evaluate fiscal solvency in developing countries ignoring aggregate uncertainty can be very misleading.

This paper reformulates the tormented insurer's problem in a dynamic, stochastic general equilibrium framework. The paper focuses in particular on the competitive equilibrium of an incomplete-markets economy in which the government chooses optimal plans for public debt and government expenditures facing two exogenous sources of revenue volatility. The first source are the cyclical variations in the economy's output. The second source are fluctuations in an "implied tax" that captures policy shocks affecting public revenues as well as shocks affecting key exogenous determinants of fiscal revenues in developing countries.

[1] This level of outlays can be set to zero without loss of generality. In this case, the government's natural debt limit allows the largest debt and satisfies the same definition as in Aiyagari (1994). A positive (and more realistic) crisis level of government outlays results in a tighter natural debt limit, and the limit is tighter the smaller the cut in outlays, so that countries that are perceived as capable of stronger fiscal adjustment can support higher debt ratios.

Among the latter, fluctuations in real world commodity prices are particularly important because commodity exports are a major source of government revenues in many developing nations. The government issues one-period, non-state contingent bonds with a return perfectly arbitraged with the return on foreign bonds of the same type. Domestic agents make optimal consumption and savings plans facing the volatility of their after-tax income and using as vehicles of saving domestic public debt and international bonds. In contrast with the basic setup in Mendoza and Oviedo (2004), the model features well-defined limiting distributions of public and foreign debt, and yields predictions about the cyclical behavior of the primary balance and government purchases and the welfare implications of the government's actions.

The model features two forms of asset market incompleteness. The first one is the standard "external" asset market incompleteness typical of small open economy models: the economy as a whole experiences idiosyncratic income fluctuations and has only access to a world market of non-state-contingent bonds. The second one is "domestic" asset market incompleteness. If the government could issue state contingent debt, or enact state-contingent, non-distorting taxes, it could attain a domestic social planner's optimum in which the incomes of the government and the private sector are pooled (so that the relevant constraint is the resource constraint of the economy as a whole) and the marginal utilities of public and private spending are equalized across time and states of nature. In this case, cyclical fluctuations in fiscal revenues and after-tax private income would not alter the distribution of wealth between the government and the private sector.[2] In the tormented insurer's world, however, the implied tax process splits the economy's income across the private and public sectors, and the government can only issue non-state-contingent debt. Hence, the fiscal authority cannot replicate the domestic social optimum.

The equilibrium of the model is represented in recursive form as a Markov perfect equilibrium (MPE) so as to provide a mechanism for computing the state-contingent dynamics of wealth of the private and public sectors. The government (private sector) formulates its

[2]Because the first form of market incompleteness is not eliminated even if the government can issue domestic debt contingent on fiscal revenues, this social optimum does not correspond to the Arrow-Debreu complete markets equilibrium.

optimal spending and financing plans taking as given a conjecture of the private sector's (government's) optimal plans, but behaving competitively so that both agents move simultaneously and take all relevant prices as given. The MPE is attained when the optimal plans chosen by the government (private sector) are consistent with the conjecture of the government's (private sector's) optimal plans under which the private sector (government) formulates its plans. In this MPE, fluctuations in fiscal revenue and after-tax private income change the distribution of wealth across the private and public sectors.

The quantitative predictions of the model are examined by conducting a set of numerical experiments calibrated to Mexican data. The results show that the model, when calibrated to capture the low average and high volatility of Mexico's public revenues, makes progress in explaining the other three stylized facts that distinguish fiscal policy in developing countries from that of industrial nations. In particular, the model replicates the inverse relationship between average debt ratios and fiscal revenue volatility found in the data, and generates GDP correlations for government purchases and the primary balance very similar to the ones estimated for Mexico.

A comparison of the domestic social optimum with the MPE shows that domestic asset market incompleteness has important implications for equilibrium allocations and welfare. The volatility of expenditures is significantly higher in the MPE than in the social optimum, even though both result in similar long-run averages of private and public expenditures, and this translates into welfare costs of market incompleteness that are several orders of magnitude larger than standard results in the literature. The costs range from 1.6 to 4.9 percent in terms of the long-run average of a compensating variation in a time-and-state-invariant level of private consumption that equates national expected lifetime utility across the social optimum and the MPE. In the literature, the welfare costs of eliminating asset markets for purposes of consumption smoothing and/or risk sharing with standard preferences are generally below 1/10th of a percent (see, for example, Lucas (1987) or Mendoza (1991)) .

This paper forms part of a growing literature examining fiscal policy in environments with incomplete asset markets, including, among others, the studies by Aiyagari, Marcet, Sargent, and Seppälä (2002), Aguiar, Amador and Gopinath (2005), Celasun, Durdun and

Ostry (2005), Schmitt-Grohé and Uribe (2002), and Yakadina and Kumhof (2005). This paper differs in its focus on the split between domestic vis-a-vis international asset market incompleteness, and on the analysis of optimal debt and expenditure policies for a given stochastic process of revenues (instead of solving a Ramsey optimal taxation problem). This approach seems more in line with the developing countries' heavy reliance on commodity exports as a large source of fiscal revenue, and with their limited ability to fine-tune conventional direct and indirect tax rates with the aim to improve risk sharing.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 conducts the quantitative analysis. Section 4 concludes.

2 The Model

Consider a small open economy with stochastic endowment income and limited opportunities for risk sharing and consumption smoothing because financial markets are incomplete. This economy is inhabited by two infinitely-lived agents: a representative household and a government. The government receives stochastic public revenues that are affected by two sources of uncertainty: fluctuations in the economy's endowment income and fluctuations in an implied tax rate that represents fluctuations in tax policy and in other key exogenous determinants of fiscal revenues. The government's total non-interest outlays include current expenditures (i.e., purchases of goods and services), which will be chosen optimally, and transfer payments to the private sector, which are kept at a deterministic, constant level for simplicity. The government can sell one-period, non-state contingent bonds to the private sector to finance primary fiscal deficits. On the side of the private sector, households collect stochastic after-tax income, which is affected by the same sources of uncertainty as fiscal revenues. Households make optimal intertemporal plans and they have access to the domestic market of public bonds and to a world market of one-period, non-state contingent bonds. These two bonds are perfect substitutes, and the gross real rate of return on both equals R. The combination of market incompleteness and income uncertainty induces both agents to undertake precautionary saving in order to self-insure against endowment and tax

shocks.

The variables that describe the state of the economy at any point in time are the stock of public debt, b^g_{t-1}, the net foreign asset position, b^I_{t-1} and a pair of realizations of the shocks, $\epsilon_t = (\epsilon^y_t, \epsilon^\tau_t)$, where ϵ^y_t is the shock to national income and ϵ^τ_t is the implied tax shock. Thus, the state of the economy at date t is given by the triple $s_t = (b^g_{t-1}, b^I_{t-1}, \epsilon_t)$.

The economy's endowment income, $y_t \exp(\epsilon^y_t)$, is the product of a deterministic trend component, y_t, and a cyclical component, $\exp(\epsilon^y_t)$. The trend component grows at the constant, exogenous, gross rate γ and all equilibrium allocations follow this common trend. Following Carroll (2004), the analysis focuses, without loss of generality, on a detrended representation of the model in which: (a) all allocations are expressed as ratios of y_t and (b) the subjective discount factor and the gross return on assets are adjusted so that the solutions of the detrended model can be mapped into the equivalent solutions of the growing economy.[3] These adjustments imply an effective discount factor given by $\beta\gamma^{1-\sigma}$, where β is the subjective discount factor and σ is the coefficient of relative risk aversion, and an effective gross return on assets $\mathcal{R} \equiv R/\gamma$.

The implied tax will be calibrated so that, taking GDP as the overall tax base, the stochastic process of total public revenues in the model matches one taken from actual data. The implied tax *rate* is $\tau^y \exp(\epsilon^\tau_t)$ and it includes an average tax rate τ as a share of GDP and an implied tax shock $\exp(\epsilon^\tau_t)$. Note that, to the extent that this shock reflects the effect on public revenues of fluctuations in world commodity prices, it is analogous to a terms-of-trade shock.

The shocks to GDP and implied taxes are represented by a joint Markov process defined by an $NS \times 2$ matrix $\mathcal{E}$ containing NS realizations of the pair $\epsilon = (\epsilon^y, \epsilon^\tau)$ and an $NS \times NS$ transition probability matrix P with typical element $P_{ij} = \text{Prob}(\epsilon_t = \epsilon_j | \epsilon_{t-1} = \epsilon_i)$, where $i, j = 1, ..., NS$ index the NS realizations of the pair ϵ, and $\sum_{j=1}^{NS} P_{ij} = 1$, $\forall i = 1, ..., NS$.

[3] Carroll also shows how to generalize this setup to include i.i.d. shocks to the trend component, which would add to the model permanent shocks as those studied by Aguiar and Gopinath (2004).

2.1 The Government's Problem

The government chooses sequences of (detrended) expenditures and debt issues $\{g_t, b_t^g\}_{t=0}^{\infty}$ so as to maximize a standard CRRA expected utility function:

$$E_0 \left[\sum_{t=0}^{\infty} \left(\beta \gamma^{1-\sigma}\right)^t \frac{g_t^{1-\sigma}}{1-\sigma} \right]; \qquad \sigma \neq 1; \quad \sigma > 0 \tag{1a}$$

subject to the following government budget constraint:

$$g_t + z + \mathcal{R} b_{t-1}^g \leq b_t^g + \exp(\epsilon_t^y) \tau^y \exp(\epsilon_t^\tau) \tag{1b}$$

where g_t represents government expenditures at time t, z are time-and-state-invariant transfer payments to the private sector, $\mathcal{R} \equiv R/\gamma$ is the growth-adjusted interest rate, and b_t^g is the stock of public debt chosen at time t.

Given that the marginal utility of public expenditures goes to infinity as g_t approaches zero from above, the government never chooses a plan that leaves it exposed to the risk of facing less than strictly positive expenditures in all dates and states of nature. As a result, the government imposes on itself a "natural debt limit," which is given by the annuity value of the lowest Markov realization of fiscal revenues net of transfers $\frac{\min[\exp(\epsilon^y)\tau \exp(\epsilon^\tau)] - z}{\mathcal{R}-1}$. This upper bound on debt is exactly analogous to the concept of the natural debt limit introduced by Aiyagari (1994) in the heterogenous agents-precautionary savings literature. Following Aiyagari, the debt constraint faced by the government can be expressed more generally as an upper bound ϕ^g that satisfies:

$$b_t^g \leq \phi^g \leq \frac{\min\left[\exp(\epsilon^y)\tau \exp(\epsilon^\tau)\right] - z}{\mathcal{R} - 1} \tag{1c}$$

Hence, the government's debt constraint can be set at the natural debt limit, or at an arbitrarily tighter limit, which Aiyagari (1994) labeled an ad-hoc debt limit. This ad- hoc debt limit can be justified as a form of natural debt limit implied by a constraint requiring government purchases not to fall below an exogenous minimum level at any date and state of nature.

Mendoza and Oviedo (2004) show that the government's natural debt limit also implies a credible commitment to remain "able" to repay (i.e., to have enough resources to repay) the debt at all times. This is because the natural debt limit represents the stock of debt the government can honor even if it draws the worst realization of fiscal revenue "almost surely." Mendoza and Oviedo obtain this debt limit as an exogenous requirement that an effectual insurer, defined as the one that keeps government outlays smooth for the longest possible time, would want to meet. In this case, allowing situations where debt could exceed this limit would imply that there would be sequences of fiscal revenues $\{(\exp \epsilon_t^y)\tau \exp(\epsilon_t^\tau)\}_{t=0}^T$ with non-zero probability of occurrence under which the government cannot repay its debt even by setting $g_T = 0$ at some date T. In the model of this paper, the CRRA form of the government's payoff function enforces the natural debt limit as an endogenous feature of the model.

The optimality conditions of the government's problem are the budget constraint (1b), the debt constraint (1c), and the following Euler equation:

$$g_t^{-\sigma} = \beta\gamma^{1-\sigma}\mathcal{R}\mathrm{E}\left[g_{t+1}^{-\sigma}\right] + \mu_t^g \tag{1d}$$

where μ_t^g is the non-negative Lagrange multiplier on the debt constraint. The above Euler equation has the standard interpretation of equating the marginal cost and benefit of sacrificing a unit of public expenditures at date t, with the caveat that if the debt constraint binds the multiplier in the right-hand-side is positive. Note, however, that when ϕ^g is set equal to the natural debt limit, the CRRA utility function implies that $\mu_t^g = 0$ at equilibrium for all t.

2.2 The Household's Problem

The household 's problem is analogous to the government's problem. The choice variables are stochastic sequences of consumption and bond holdings $\{c_t, b_t^g, b_t^I\}_{t=0}^\infty$ that maximize expected lifetime utility:

$$E_0\left[\sum_{t=0}^{\infty}\left(\beta\gamma^{1-\sigma}\right)^t \frac{c_t^{1-\sigma}}{1-\sigma}\right] \tag{2a}$$

subject to the following budget constraint:

$$c_t + x + (b_t^g + b_t^I) \leq \exp(\epsilon_t^y)\left[1 - \tau \exp(\epsilon_t^\tau)\right] + \mathcal{R}(b_{t-1}^g + b_{t-1}^I) + z \tag{2b}$$

This budget constraint restricts the sum of consumption, an exogenous, invariant level of private absorption x (which is used in the calibration to represent investment expenditures), and purchases of public and foreign bonds not to exceed the household's resources. The latter are given by the sum of after-tax private income, financial income, and government transfers.

Since the household's payoff function has the same CRRA form as the government's, the household's problem features a similar natural debt limit on its total bond position $(b_t^g + b_t^I)$ given by the annuity value of the lowest Markov realization of after tax income, adjusted to add transfers and subtract x. Alternatively, for a given Markov plan of domestic public debt given by the policy function $b_t^g = \hat{b}^g(b^g, b^I, \epsilon)$, the household faces a natural limit on external debt that results in the following constraint on foreign bond holdings:

$$b_t^I \geq \phi^I \geq -\frac{\min\left[\exp(\epsilon^y)\left[1 - \tau \exp(\epsilon^\tau)\right] + \mathcal{R}b^g - b^{g\prime}(b^g, b^I, \epsilon)\right] + z - x}{\mathcal{R} - 1} \tag{2c}$$

The right-most expression in this constraint is the natural debt limit on foreign debt. Hence, condition (2c) allows for the external debt constraint to be set at its natural debt limit or at a tighter ad-hoc debt limit. As before, the ad-hoc debt limit can be thought of as implied by an exogenous constraint requiring consumption never to fall below a pre-determined minimum level.

The optimality conditions of the private sector are the budget constraint (2b), the external debt constraint (2c), and the following Euler equation:

$$c_t^{-\sigma} = \beta\gamma^{1-\sigma}\mathcal{R}\mathrm{E}\left[c_{t+1}^{-\sigma}\right] + \mu_t^c \tag{2d}$$

where μ_t^c is the non-negative Lagrange multiplier on the external debt constraint that, as in the case of the government, is equal to zero for all t if ϕ^I is set at the natural debt limit. This Euler equation is also standard, with the caveat of the multiplier on the borrowing

constraint, and it states that the private sector equates the marginal benefit and cost of an additional unit of consumption expenditures.

Given the perfect substitutability between b_t^g and b_t^I, the household is indifferent between the two assets and their returns are perfectly arbitraged. Still, the composition of the household's portfolio is well defined at equilibrium because the government has a well-defined supply of debt that it desires to issue each period. In general, the government is indifferent between placing this debt at home or abroad, since the interest rate is the same, but the model assumes that all domestic debt is held by domestic residents. Except for the dynamics of net foreign assets, this assumption is innocuous because the government could be viewed as placing any fraction θ of its debt abroad, and the household would then formulate optimal plans for total assets $b_t = b_t^I + (1-\theta)b_t^g$. This reformulation of the problem would yield identical expenditure allocations for both agents and the same levels of welfare. Foreign and domestic debt markets could be segmented by introducing additional frictions into the financial setup of the model, but the aim of the analysis is to highlight the implications of the incompleteness of asset markets that emerge in the tormented insurer's framework even when domestic public debt and foreign bonds are perfect substitutes.

The payoff functions of the government and the private sector do not depend directly on each other's actions. An alternative formulation could introduce public expenditures (or a fraction of them) in the private utility function to explore a richer set of strategic interactions. If the household's utility function is separable in public and private expenditures, and the assumptions of non-distorting taxation and simultaneous moves are maintained, the optimal plans would be the same as in the current specification. Another interesting extension would be to explore different discount rates across the private and public sector in order to capture the fact that governments generally have shorter life horizons than private agents.

2.3 Competitive Equilibrium

Definition (CE) The competitive equilibrium of the economy is characterized by stochastic sequences representing the allocations of private and public expenditures, government debt,

and private net foreign asset holdings, $\{c_t, g_t, b_t^g, b_t^I\}_{t=0}^{\infty}$, such that:

i) $\{g_t, b_t^g\}_{t=0}^{\infty}$ solve the government's problem.

ii) $\{c_t, b_t^g, b_t^I\}_{t=0}^{\infty}$ solve the household's problem.

iii) The following economy-wide resource constraint holds:

$$c_t + x + g_t \leq \exp(\epsilon_t^y) + \mathcal{R} b_{t-1}^I - b_t^I \tag{3}$$

2.4 Markov Perfect Equilibrium

Solving for the above competitive equilibrium is difficult because the incompleteness of asset markets prevents the private and public sectors from pooling risk, and as a result the equilibrium cannot be represented as the solution to a social planner's problem. The non-insurable shocks faced by the private and public sectors lead to fluctuations in the "domestic distribution of wealth" (i.e., the wealth distribution of the government vis-a-vis the private sector). These fluctuations, and the implied absence of risk pooling across the two sectors, are reflected in fluctuations in the ratio of marginal utilities of expenditures, or, given the CRRA structure of preferences, in the ratio c_t/g_t.

Under these conditions, solving for the competitive equilibrium requires adopting a solution strategy that can capture the state-contingent wealth dynamics accurately. The strategy adopted here is to solve for the equilibrium as a Markov perfect equilibrium where the government and the private sector behave competitively by taking all relevant prices as given and by making simultaneous moves. Each agent formulates optimal plans taking as given a conjecture of the other agent's optimal plans, and the agents do not internalize the effect of their actions on each other's choices. The equilibrium is a Nash equilibrium of a two-player dynamic game with uncertainty. The solution strategy involves representing the game in recursive form and solving it by backward induction.

The two agent's optimization problems are expressed in recursive form as follows. At the beginning of each period, agents observe the state of the economy $s = (b^g, b^I, \epsilon)$. The

state space of the Markov shocks includes the NS possible realizations of the pair $\epsilon = (\epsilon^y, \epsilon^\tau)$ defined earlier. The state space of domestic and external debt is defined by discrete grids with NBG and NBI nodes respectively. Hence, the state space of public debt is $b^g \in B^g = \{b_1^g < b_2^g < ... < b_{NBG}^g = \phi^g\}$ and the one for net foreign assets is $b^I \in B^I = \left\{b_1^I = \phi^I < b_2^I < ... < b_{NBI}^g\right\}$. Each agent takes as given a conjectured decision rule for the other agent's optimal plans. The government conjectures that the household's decision rule is $b^{I\prime} = \tilde{b}^I(b^g, b^I, \epsilon)$ and the household conjectures that the government's decision rule is $b^{g\prime} = \tilde{b}^g(b^g, b^I, \epsilon)$. Given these, each agent finds an optimal decision rule that solves the Bellman equation representing their individual optimization problems.

The Bellman equation for the government's optimization problem, given the conjecture $\tilde{b}^I(b^g, b^I, \epsilon)$, is:

$$V(b^g, b^I, \epsilon) = \max_{b^{g\prime} \in B^g, g} \left\{ \frac{g^{1-\sigma}}{(1-\sigma)} + \beta\, \gamma^{1-\sigma} \mathrm{E}\left[V(b^{g\prime}, \tilde{b}^I(b^g, b^I, \epsilon), \epsilon') \right] \right\} \tag{4}$$

$$\text{s.t.:} \quad g + z + \mathcal{R} b^g \leq b^{g\prime} + \exp(\epsilon^y)\tau^y \exp(\epsilon^\tau)$$

$$b^{g\prime} \leq \phi^g$$

The solution to this dynamic programming problem yields a decision rule for the government's debt $\hat{b}^g(b^g, b^I, \epsilon)$.

The Bellman equation for the private sector's optimization problem, given the conjecture $\tilde{b}^g(b^g, b^I, \epsilon)$, is:

$$W(b^g, b^I, \epsilon) = \max_{b^{I\prime} \in B^I, c} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta\, \gamma^{1-\sigma} \mathrm{E}\left[W(\tilde{b}^g(b^g, b^I, \epsilon), b^{I\prime}, \epsilon') \right] \right\} \tag{5}$$

$$\text{s.t.:} \quad c + x + \tilde{b}^g(b^g, b^I, \epsilon) + b^{I\prime} \leq \exp(\epsilon^y)\left[1 - \tau \exp(\epsilon^\tau)\right] + \mathcal{R}(b^g + b^I) + z$$

$$b^{I\prime} \geq \phi^I$$

The solution to this dynamic programming problem yields a decision rule for the private sector of the form $\hat{b}^I(b^g, b^I, \epsilon)$.

Definition (MPE) A Markov perfect equilibrium for the small open economy is a pair of value functions V and W, a pair of decision rules $\hat{b}^g$ and $\hat{b}^I$, and a pair of conjectures $\tilde{b}^g$

and $\tilde{b}^I$ such that:

i) Given the conjecture $\tilde{b}^I$, V solves the Bellman equation (4), and $\hat{b}^g$ is the associated optimal policy rule.

ii) Given the conjecture $\tilde{b}^g$, W solves the Bellman equation (5), and $\hat{b}^I$ is the associated optimal policy rule.

iii) The conjectured and optimal decision rules satisfy $\hat{b}^g(\cdot) = \tilde{b}^g(\cdot)$ and $\hat{b}^I(\cdot) = \tilde{b}^I(\cdot)$

Conditions i) to iii) imply that, at equilibrium, each agent's conjecture of the other agent's optimal decisions matches the decisions that the other agent actually finds optimal to choose. Note that the computation of this MPE is simplified by the fact that, while the government's actions affect the household's dynamic programming problem, the household's choices do not affect the government's optimal plans. Hence, the MPE can be computed by solving first the government's Bellman equation, and then imposing the resulting public debt decision rule on the household's problem. This feature of the model follows from the lack of feedback from actions of the private sector on the government's payoff and constraints, which in turn results from the simplifying assumptions making fiscal revenues independent of the actions of the private sector.[4] The rationale for these assumptions is to show that even in this case, in which the strategic interaction between the two agents is simplified substantially, the incompleteness of domestic financial markets has important consequences. Extending the analysis to the case in which there is two-way feedback between the government's and the private sector's plans is straightforward, albeit computationally intensive.

It is straightforward to show that a Markov perfect equilibrium, if it exists, is a competitive equilibrium for the small open economy. Consider first the Bellman equations (4) and (5). Using the standard Benveniste-Sheinkman equation, it follows that the first order conditions of the Bellman equations imply that the Euler equations of the competitive equilibrium, eqs. (1d) and (2d), hold. Finally, the budget constraints of the Bellman equations yield the economy-wide resource constraint (3) of the competitive equilibrium.

[4]For example, if the government taxes consumption, the consumption decisions of the private sector would affect the government budget constraint, and hence the two Bellman equations would need to be solved simultaneously.

2.5 Domestic Social Optimum

As explained in the Introduction, the main distortion preventing the tormented insurer from implementing fiscal policies that support perfect risk pooling across the private and public sectors is the incompleteness of domestic asset markets. The quantitative analysis of the next section explores the effects of this distortion on allocations and welfare by comparing the outcome of the MPE with that of a social optimum in which domestic asset markets are assumed to be rich enough to support perfect domestic risk pooling. The domestic economy as a whole still faces incomplete markets vis-a-vis the rest of the world because GDP fluctuations represent non-insurable, idiosyncratic income shocks for the small open economy.

The social planner's problem that yields the domestic social optimum is similar to the standard Negishi-Mantel social planner's representation of the complete markets equilibrium as a weighted sum of individual utilities subject to the resource constraint. The one key difference is that, because the international asset markets remain incomplete, the outcome of the social planner's problem defined here does not correspond to the complete-markets equilibrium of the small open economy. The domestic social optimum is defined as the sequences of allocations $\{c_t, g_t\}_{t=0}^{\infty}$ that maximize the weighted sum of the government and private payoffs,

$$E_0 \left\{ \eta \sum_{t=0}^{\infty} \left(\beta \gamma^{1-\sigma}\right)^t \frac{c_t^{1-\sigma}}{1-\sigma} + (1-\eta) \sum_{t=0}^{\infty} \left(\beta \gamma^{1-\sigma}\right)^t \frac{g_t^{1-\sigma}}{1-\sigma} \right\}, \tag{6a}$$

where $\eta > 0$ and $(1-\eta)$ are the weights assigned to the private and public sector payoffs respectively, subject to the small open economy's resource constraint:

$$c_t + g_t + x + b_t^I \leq \exp(\epsilon^y) + \mathcal{R} b_{t-1}^I \tag{6b}$$

and the following borrowing constraint on net foreign assets:

$$b^{I\prime} \geq \phi^{SO} \geq -\frac{\min\left[\exp(\epsilon^y)\right] - x}{\mathcal{R} - 1} \tag{6c}$$

where the quotient in the right-hand-side of this borrowing constraint is the social planner's natural debt limit.

In this equilibrium with perfect domestic risk pooling, the marginal utilities of public and private expenditures are equalized across states and over time. Given CRRA utility functions, this translates into an equilibrium condition that determines a time- and state-invariant expenditure ratio:

$$\frac{g_t}{c_t} = \left(\frac{1-\eta}{\eta}\right)^{1/\sigma} \tag{6d}$$

Note that c_t and g_t still fluctuate because the external market incompleteness still exists and it does not allow the small open economy to fully insure away its macroeconomic risk.

The recursive form of the social planner's problem is given by the following dynamic programming problem:

$$V^{so}(b^I, \epsilon^y) = \max_{b^{I\prime} \in B^I, c, g} \left[\eta \left(\frac{c^{1-\sigma}}{1-\sigma} \right) + (1-\eta) \left(\frac{g^{1-\sigma}}{1-\sigma} \right) + \beta \gamma^{1-\sigma} \mathrm{E} \left[V^{so}(b^{I\prime}, \epsilon^{y\prime}) \right] \right] \tag{6e}$$

subject to the resource constraint (6b) and the external debt constraint (6c).

If the government had access to state contingent tax or debt instruments, it could implement the above domestic social optimum as a competitive equilibrium. State contingent taxes could work as follows: Assume that the public and private debt limits are set at their natural debt limits, public debt is set to zero at all times, and the government introduces a set of state-contingent income taxes $\tau(\epsilon_t^y, \epsilon_t^\tau) = (g_t^* + z) / (\exp(\epsilon_t^y)\tau^y \exp(\epsilon_t^\tau))$, where starred variables represent optimal allocations of the social planner's problem. This policy would support the social optimum because: (a) $\{g_t^*\}_{t=0}^\infty$ would satisfy the government budget constraint and the government's Euler equation; (b) $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$ would satisfy the private sector's budget constraint and its Euler equation; and (c) the resource constraint holds. That the Euler equations are satisfied is obvious from the fact that the same Euler equations hold for the social planner. The government budget constraint obviously holds given the definition of the tax rule. The budget constraint of the private sector holds for $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^\infty$ because the resource constraint holds for the social optimum, the government's budget constraint holds, and there is no public debt.

Similarly, the social optimum could be obtained as a competitive equilibrium with a policy setting income taxes to zero and issuing state-contingent public debt (i.e., domestic one-period Arrow securities) to effectively implement state contingent lump-sum taxes $LST_t = g_t^* + z$. This policy would tax away from households, in a lump-sum fashion, exactly the resources needed to pay for the time-invariant transfers and the social optimum sequence $\{g_t^*\}_{t=0}^{\infty}$. The private sector would then find it optimal to choose the allocations $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^{\infty}$ because $\{c_t^*\}_{t=0}^{\infty}$ satisfies its Euler equation and the lump-sum tax leaves just enough disposable income for the private sector to satisfy its budget constraint by choosing $\left\{c_t^*, b_t^{I*}\right\}_{t=0}^{\infty}$.

3 Quantitative Findings

The numerical solutions to the MPE are obtained by iterating to convergence on the Bellman equations (4) and (5) on the discrete state space containing NBG possible public debt positions, NBI possible net foreign asset positions, and NS pairs of income and tax shocks. The algorithm is executed using NBG=200, NBI=200 and NS=9, so the discrete state space is of dimensions $200 \times 200 \times 9$.[5] As long as there is no feedback from the private sector choices to the government's Bellman equation, the algorithm can be simplified by solving first the government's problem as a stand-alone value-function-iteration problem, and then using the resulting optimal decision rule for public debt as the conjectured decision rule for the same variable in the Bellman equation of the private sector.

3.1 Baseline Calibration

The quantitative analysis uses a baseline calibration to Mexican data at an annual frequency. The calibration has two components. First, a Markov representation of the stochastic process of output and implied taxes observed in the data. Second,a set of parameter values set so that long-run averages of variables in the model match their counterparts in

[5] We explore the robustness of the results to enlarging the state space or conforming it with finer grids by solving the model with a state space of dimensions $400 \times 400 \times 9$. The results are largely robust, with the ergodic means, standard deviations, and autocorrelations differing by at most 5 percent.

Mexican data, or taken as standard from the Real Business Cycle (RBC) literature. Table 1 summarizes the calibration parameters.

The joint Markov process driving the shocks to endowment income and implied taxes is constructed using annual data for Mexico's real GDP and total fiscal revenues for the period 1980-2004. The implied tax-rate series, TX, is constructed by computing the ratio of total public revenue to GDP. The data for GDP and TX are then expressed in per capita terms, logged, and detrended using the Hodrick-Prescott filter to obtain their cyclical components, $\widehat{GDP}$ and $\widehat{TX}$. Panel (a) of Table 2 reports the unconditional moments of these two time series. The joint process driving the endowment and implied tax shocks is obtained by estimating a VAR(1), $\widehat{X}_t = \Phi \widehat{X}_{t-1} + \zeta_t$, where $\widehat{X}_t = (\widehat{GDP}_t, \widehat{TX}_t)'$, Φ is the 2×2 matrix of autocorrelation coefficients, and ζ_t is an i.i.d. random vector with covariance matrix Σ_ζ. The estimation results of this VAR are as follows:[6]

$$\hat{\Phi} = \begin{pmatrix} 0.2789 & 0.4389 \\ (1.4495) & (1.2614) \\ & \\ -0.1489 & 0.6004 \\ (-1.5643) & (3.4867) \end{pmatrix}; \qquad \hat{\Sigma}_\zeta = \begin{pmatrix} 0.000727 & -0.000268 \\ -0.000268 & 0.002378 \end{pmatrix}$$

Since the MPE is solved using a discrete state space, the VAR representation of the shocks needs to be converted into a discrete Markov process. This is done using Tauchen's (1991) quadrature method, setting the Markov chain to carry nine *pairs* of shocks to GDP and taxes (i.e., nine states in total). Given that the off-diagonal elements of $\hat{\Phi}$ are not statistically different from zero, we use the diagonal version of $\hat{\Phi}$ and $\hat{\Sigma}_\zeta$ as inputs for Tauchen's algorithm. The resulting set of Markov realizations and their associated long-run probabilities are reported in the Appendix. Panel (b) of Table 2 shows the unconditional moments of GDP and taxes produced by the Markov chain. These do not match exactly the moments from the data in Panel (a) because of the approximation error of the Markov

[6] t-statistics are shown in parenthesis.

chain with 9 states, but the standard deviation, correlation and autocorrelation coefficients are close to their empirical counterparts.[7]

The growth rate is set to $\gamma = 1.0088$, which is the average growth rate of Mexico's real GDP per capita between 1980 and 2000 computed using data from the World Bank's *World Development Indicators*. In the same source and sample, the average investment-GDP ratio yields $x = 0.226$. The mean income tax rate is set to $\tau = 0.256$, which is the 1980-2002 average ratio of total public revenue to GDP using data at current prices from Mexico's INEGI Bank of Economic Information (available at *http://dgcnesyp.inegi.gob.mx/cgi-win/bdieintsi.exe*). The time-invariant government transfers are set at $z = 0.111$, which is the difference between the average ratio of total non-interest government outlays to GDP (from the same *INEGI* source) and the average government expenditures-GDP ratio, $g = 0.0978$, from *World Development Indicators*.

The real interest rate is set to $R = 1.0986$, which is the average EMBI+ real return on Mexican sovereign debt for the 1994-2002 period computed using the data from Neumeyer and Perri (2005). This rate includes both the risk-free rate as well as the default risk premium. The model does not consider default explicitly, but using an interest rate that is more representative of the actual rate at which the Mexican government borrows is more reasonable than simply applying the real rate on U.S. T-bills.

Regarding preference parameters, the value of the coefficient of relative risk aversion is $\sigma = 2$, which is the usual value in RBC models. Unlike in the RBC literature, however, the subjective discount factor β cannot be set simply to match the inverse of the growth-adjusted gross interest rate because precautionary savings makes asset holdings diverge to infinity in this case, as agents try to attain a non-stochastic consumption stream in the face of non-diversifiable income shocks. In models without long-run growth (see Aiyagari (1994), Huggett (1993) and Ljungqvuist and Sargent (2004, ch. 17)), the condition $\beta R < 1$ yields a well-defined unique invariant distribution of asset holdings. In models with growth, Carroll

[7]The approximation improves as NS, but at a high cost in computing time. The drawback of working with a limited number of states is that, by construction, natural debt limits are produced with the lowest realizations of income supported by the Markov chain, and the extent to which these realizations reflect adverse outcomes that are truly relevant (i.e., that have nontrivial probability) depends on the number of states in the chain.

(2004) shows that the condition required to prevent asset holdings from diverging to infinity is: $\beta R \times \max[(1/R)^{\sigma}, (1/\gamma)^{\sigma}] < 1$.

Precautionary savings also implies that the averages of the model's stochastic stationary state vary with the choice of parameters, particularly those that appear in Carroll's condition, the Markov process of shocks, and the choice of ϕ^g. Figure 4 shows the response of the long-run average of public debt in the MPE to increments in β that make the expression $\beta R \times \max[(1/R)^{\sigma}, (1/\gamma)^{\sigma}]$ in Carroll's condition converge to 1 from below. The graph is constructed for three values of σ (2, 3 and 5), using the described Markov process of shocks, the values of R and γ set above, and setting ϕ^g equal to the government's natural debt limit obtained using (1c). The figure is plotted with the horizontal axis inverted to show that it yields the same concave relationship as the Aiyagari-Hugget class of models (with the elasticity of average public "assets" going to infinity as β increases).[8]

Since each long-run average of public debt has an associated long-run average of public expenditures through the government budget constraint, it follows that the higher the average of public debt in Figure 4, the lower the corresponding average of government expenditures (as more resources are allocated to debt service). The baseline calibration uses this result to set the value of β. In particular, β is set such that, given the calibrated values of R, γ and σ and with ϕ^g set at the government's natural debt limit, Carroll's condition for the model to have a well-defined stochastic steady state holds *and* the long-run average of government purchases equals Mexico's average of 9.78 percent of GDP. The resulting value is $\beta = 0.925$ (for which Carroll's condition yields $\beta R \gamma^{-\sigma} = 0.9984 < 1$).

The grids of public debt and foreign assets can now be specified using the information of the baseline calibration. The upper bound of the public debt grid is its natural debt limit. Given the values of R, γ, z and the lowest realization of government revenue supported by the states of income and tax shocks in the Markov process, equation (1c) yields $b^g_{200} = \phi^g = 1.318$ (or about 132 percent of GDP). The lower bound of public debt is set to zero ($b^g_1 = 0$) which implies that the government cannot become a net creditor (i.e., hold negative debt

[8]Formally, public debt should go to -∞ as Carroll's condition approaches 1 from below. Since the upper bound of public debt is set to zero, however, the entire mass of the ergodic distribution of debt concentrates at this upper bound as β raises to levels for which Carroll's condition is "close enough" to 1.

positions). This constraint does not play an important role because it binds only with 0.51 percent probability in the long run.

Given the optimal decision rule for public debt, the values of R, γ, z and x, and the lowest realization of private disposable income (defined as $\exp(\epsilon^y)\left[1-\tau\exp(\epsilon^\tau)\right]+\mathcal{R}b^g-b^{g\prime}(b^g,b^I,\epsilon)$) supported by the states in the Markov process, equation (2c) yields a natural debt limit on net foreign assets of -7.105 (more than 7 times larger than GDP). However, the MPE obtained with such a large *maximum* external debt ratio yields an *average* foreign debt ratio that is also too large (at 200 percent of GDP), and consequently the corresponding long-run average consumption ratio is too low relative to the average in Mexican data (53 percent in the MPE v. 66.9 percent in the data). Hence, instead of using the natural debt limit for the lower bound of external assets, we set an ad-hoc debt limit such that the baseline MPE yields an average private consumption ratio consistent with the data. The ad-hoc debt limit that satisfies this criterion is $b_1^I=\phi^I=-0.5$. The upper bound for b^I is chosen so that its long-run probability is approximately zero (without any binding constraint) and has no effect on the moments of the ergodic distribution. The resulting upper bound is $b_{200}^I=0.1$.

3.2 Cyclical Co-movements in the Baseline Scenario

Table 3 shows the statistical moments that characterize cyclical co-movements in the ergodic distribution of the MPE. All moments in the table correspond to the model's detrended variables, which are ratios relative to GDP. The mean value of GNP is lower than that for GDP (which by construction is equal to 1) because the economy is paying interest to the rest of the world on a stock of net foreign assets of about 36 percent of GDP (in line with the estimates for Mexico obtained by Lane and Milesi-Ferretti (1999)). This debt position and the value of R, imply that the economy runs a trade surplus of 3.2 percent of GDP on average in the long run. The average public debt ratio is equal to 53 percent of GDP, and hence the mean *total* asset position of the household (i.e., $\mathrm{E}[b_t^g+b_t^I]$) is equal to 17 percent of GDP.

Figures 5.a and 5.b show the limiting distributions of public debt and international

assets. These figures show that the support of the equilibrium allocations of public debt and net external assets differ sharply from the corresponding debt limits. The long-run mean of public debt (external assets), is 79 (14) percentage points lower (higher) than its natural debt limit. The substantial difference between the mean of public debt and the government's natural debt limit shows that it is optimal for a government acting as a tormented insurer *not* to use a large portion of its borrowing capacity on average. There are non-zero-probability histories of adverse revenue outcomes that can lead the government to hold large amounts of debt in the stochastic steady state, of up to 130 percent of GDP, but the endogenous probability of observing these outcomes is very low (see Figure 5a). Outcomes with higher debt ratios are not consistent with optimal behavior in the economy's long-run competitive equilibrium.

The variability and co-movement indicators in Table 3 illustrate the challenges that the private and public sectors face in trying to smooth consumption in the competitive equilibrium of this economy with incomplete markets. Private consumption is more volatile than after-tax income and government expenditures are more volatile than fiscal revenues. On the other hand, the correlations of private and public expenditures with the relevant income measures (after-tax income and fiscal revenues respectively) are quite low, so precautionary savings does help agents lower the correlation between their incomes and their expenditures in the long run. Still, because asset markets are incomplete, they cannot fully hedge idiosyncratic income risk, and they end up enduring higher levels of variability for expenditures than for incomes, even though the correlations between expenditures and incomes are low.

The effects of imperfect risk pooling are also observed in the relative variability of private and public expenditures. If the government had access to state-contingent fiscal instruments, the ratio of private to public expenditures would be constant across time and states of nature. In contrast, in the model's competitive equilibrium the coefficient of variation of government expenditures is nearly 6 times larger than that for private consumption, and the coefficient of variation of the ratio g_t/c_t is 33 percent.

The low, positive correlation between GDP and government expenditures (at 0.02) matches the estimate of this correlation observed in the data (see Appendix Table 5 in Kaminsky et al. (2004)). As noted in the Introduction, this pattern of non-negative cor-

relations between government purchases and GDP is one of the defining features of the puzzling phenomenon of procyclical fiscal policies in developing economies. For the United States, for example, Kaminsky et al. report an estimate of the same correlation of -0.37, and the average of their estimates for all G7 countries is about -0.2.

The model is also able to replicate a second key feature of procyclical fiscal policies in developing countries: the primary fiscal balance is nearly uncorrelated with output, instead of displaying the marked positive correlations observed in industrial countries. Alesina and Tabellini (2005) estimate regressions of the (differenced) primary balance-GDP ratio on the cyclical components of GDP and terms of trade, and the lagged primary balance-GDP ratio, and find that the "beta" coefficients on GDP are significantly higher for industrial economies than for developing countries. The average of their beta coefficients for industrial OECD countries is 0.26, while their average for Latin American and the Caribbean is -0.13 (see Table 1 of Alessina and Tabellini's paper). Their beta coefficient for Mexico is -0.094. Using a stochastic simulation of the model's decision rules to generate data for 20,000 periods, and estimating the analog of their regression, the model produces a beta coefficient of 0.097 (with a negligible standard error of 0.00326). It is difficult to assess the accuracy of this match between the betas produced by the model and the data because Alessina and Tabellini do not report country-specific standard errors. They do note that their beta coefficients have generally large standard errors, and that many of the developing country betas are not statistically different from zero. Hence, this suggests that it is possible that again the model matches not just the general feature that developing countries tend to have betas much closer to zero than industrial countries, but that the model could match closely Mexico's true beta.

The autocorrelations of public debt and net foreign assets are very close to 1, and in addition, public and private expenditures are highly serially correlated. These results are consistent with the high autocorrelation of assets typical of models of incomplete markets and precautionary saving with non-state-contingent assets, and they are also in line with the findings of Aiyagari et al. (2002), who found that the solution to the Ramsey optimal taxation problem with incomplete markets and exogenous government expenditures yields near-random-walk behavior in optimal taxes and debt. Here, tax revenue is exogenous and

hence optimal public expenditures, instead of optimal taxes, and debt display near-unit-root behavior.[9] As Aiyagari et al. also note, the high autocorrelation of public debt is a feature that is in line with the predictions of Barro's (1979) classic work on tax smoothing.

It is interesting to compare the outcome of the model's competitive equilibrium with the predictions of the simpler partial equilibrium model of Mendoza and Oviedo (2004). In the partial equilibrium setup, the government keeps outlays constant at an exogenous ad-hoc level, except when a sufficiently long sequence of adverse revenue realizations puts the economy in a state of "fiscal crisis," at which the natural debt limit binds and the fiscal authority therefore cuts its outlays to an ad-hoc crisis level. In that setup, given a Markov process of revenue realizations, simulated time paths of public debt always diverge eventually to either zero (i.e., the no-government-assets constraint) or to the natural debt limit, depending on the initial debt ratio–truly matching Barro's (1979) result stating that the long-run behavior of debt is fully determined by initial conditions.

Unfortunately, the partial equilibrium setup of Mendoza and Oviedo does not support a well-defined long-run distribution of debt, and as a result it is of limited use for assessing the long-run dynamics of public debt. In contrast, the competitive equilibrium of the model of this paper has a unique, invariant limited distribution. When perturbed by an initial shock, all endogenous (detrended) variables eventually revert to their means as the effect of the shock vanishes. This unique distribution yields precise predictions about the long-run moments and the time-series dynamics of public debt, and the rest of the model's endogenous variables, in the long run and the short run for any given set of initial conditions. It is also true, however, that the near-random-walk patterns of public debt, net foreign assets, and expenditures imply that the mean-reverting dynamics of these variables can take a very long time after an exogenous shock to GDP or implied taxes hits the economy.

Figures 6.a and 6.b plot the Markov forecast functions of public debt and net foreign

[9] Aiyagari et al. also need limits on government assets (i.e., negative debt) to recover Barro's predictions, because otherwise the optimal taxes are set to zero an all expenditures are financed with a large enough "war chest" of precautionary savings. In contrast, the model of this paper needs only to satisfy Carroll's stationarity condition to have a well-defined ergodic distribution of assets. Limits on government assets are required only if one is interested in particular long-run equilibria that match particular features of the data (as was the case in the baseline calibration that uses the natural debt limit as a lower bound for debt and 0 as an upper bound to match the observed average GDP share of government expenditures).

assets starting from the mean value of the latter and an initial public debt ratio of 0.634, roughly 10 percentage points above the mean in the ergodic distribution. The plots show nine forecast functions (i.e., non-linear impulse responses conditional on the initial asset ratios and a date-0 pair of shocks) corresponding to each of the nine pairs of shocks in the states of the Markov chain of GDP and implied tax shocks. Thus, each plot has nine forecast functions, one for each of the nine possible initial states: $s = (b_0^g = 0.634,\ b_0^I = \mathrm{E}[b^I], \epsilon_j)$ for $j = 1, .., 9$. Although the nine forecast functions start at $b^g = 0.634$, which is visually imperceptible in the graph given that the time horizon goes through $t = 2000$,the nine realizations of ϵ give rise to nine different values of b^g in the second period of the figure. One way to interpret these forecast functions is to view them as plotting the expected responses of public debt and foreign assets given a date-0 unanticipated shock (e.g. the rescue of failing banks) that puts the stock of public debt 10 percentage points above its long-run average.

Figures 6a and 6b illustrate the effects of the high persistence of assets discussed above, in the form of the very low speed of convergence of the public debt and net foreign asset ratios to their corresponding long-run averages. This is near-Random-Walk behavior, but is clearly *not* a true Random Walk, as in Barro (1979), because the competitive equilibrium of the tormented insurer model features a unique invariant distribution of assets, even though the effects of purely transitory shocks on asset holdings can take very long to fade away.

Figure 7 shows a sample of 10 simulated time series of the public debt-GDP ratio for the same starting state s_0 described above. For each simulation, a 50-period time series of realizations of ϵ_t is drawn from the Markov matrix of realizations $\mathcal{E}$ detailed in the Appendix. As the length of the simulations grows sufficiently large, the expected value of b_t^g computed with each of the 10 time series converges to the mean public debt ratio in the limiting distribution. As each time series fluctuates over time, however, the effect of the high persistence of asset holdings is reflected in the wide range of debt ratios that are consistent with the competitive equilibrium starting from the same initial conditions.

Figures 8-10 show impulse-response functions to tax and GDP shocks constructed by estimating a standard, unrestricted VAR(1) model using data generated from the model in a stochastic time-series simulation that runs for 20,100 periods, discarding the first 100

periods. This simulation uses the optimal asset accumulation rules of the MPE and the Markov chain of the exogenous shocks. The impulse response functions show responses to Cholesky one-standard-deviation, positive innovations to the tax and GDP processes starting from initial conditions equal, by construction, to the averages of public debt and foreign assets.

Figure 8 shows the impulse responses of the g_t/c_t ratio for tax and output shocks. The plots are truncated at 20 periods, even though if plotted for a sufficiently long sample, the two of them return to the zero line because of the mean-reverting nature of the model's stochastic steady state. Both a positive output and a positive tax shock raise the g_t/c_t ratio above the long-run mean; but the impact effect of a tax shock is around 10 times larger than that of the output shock, and the difference gets larger before beginning its slow-moving reversion to the zero line. These results reflect the fact that the positive output shock increases the incomes of both the private and public sector, but the tax shock moves the two agent's incomes in opposite directions. Hence, the tax shock implies larger foregone opportunities for efficient risk sharing because of the incompleteness of asset markets. Government revenues rise while private disposable income falls, and this opposing moves result in persistent differences in expenditure patterns that imply relatively higher (lower) government (private) expenditures for a long period of time. In contrast, the domestic social optimum would imply a constant g_t/c_t ratio, which corresponds to the zero line in Figure 8.

Figure 9 provides further details on the impulse response functions of the rest of the model's endogenous variables to a positive tax shock. Public debt and net foreign assets exhibit highly persistent declines. Private consumption declines on impact and then recovers very gradually, while public expenditures rise on impact and continue to increase until they level off about 6 periods after the tax shock hits. The current account falls as private agents borrow to moderate the consumption effect of the adverse shock to disposable income. Conversely, the primary fiscal balance increases on impact and then falls gradually, as the government reacts to its positive revenue shock by saving to transfer part of its extra income for future expenditures. The persistence of the effects shown in the impulse response functions of assets and expenditures illustrate once more the wealth effects induced by the incompleteness of asset markets. The tax shock plays no role in the domestic social optimum,

since the social planner pools the income of the public and private sectors, thus effectively providing full insurance against pure implied tax shocks, so again the zero line is the relevant point for comparing the responses of the competitive equilibrium with those of the social optimum.

Figure 10 shows the impulse-response functions that follow a positive, one-standard-deviation innovation to the output process in the competitive equilibrium and the domestic social optimum. The output shock is non-insurable risk for both economies, because both have access to the same set of incomplete international asset markets. Hence, the output shock does not result in large differences in the responses of macroeconomic variables. Still, the plots show that the wealth effects of the asset market incompleteness are not exactly identical in the two economies. In particular, private consumption (public consumption) is always lower (higher) in the competitive equilibrium than in the social optimum. Note also that public expenditures increase on impact, illustrating again the procyclical pattern of government purchases predicted by the model.

3.3 Welfare Costs of the Tormented Insurer's Problem

This section compares the competitive equilibrium allocations and welfare resulting from the MPE with those produced by the domestic social optimum. Welfare comparisons are conducted following the approach introduced by Lucas (1987), which is based on computing compensating variations in time- and state-invariant (i.e., stationary) consumption levels that represent particular levels of expected lifetime utility under alternative environments (in this case, between the MPE and the domestic social optimum). The aim is to convert the ordinal units of the payoff functions into cardinal measures that can be used for quantitative welfare comparisons.

To construct the welfare measure for the domestic social optimum, define $\check{c}^{SO}(b^g, b^I, \epsilon)$ as a stationary consumption function that represents the *collective* welfare of the public and

private sectors. This welfare measure satisfies:[10]

$$\frac{\check{c}^{SO}(b^g, b^I, \epsilon)^{1-\sigma}}{(1-\sigma)(1-\beta)} = V^{so}(b^g, b^I, \epsilon)$$

A comparable measure of *collective* welfare of the two sectors in the MPE is constructed by defining the stationary consumption function $\bar{c}^{MPE}(b^g, b^I, \epsilon)$ that satisfies the following condition:

$$\frac{\bar{c}^{MPE}(b^g, b^I, \epsilon)^{1-\sigma}}{(1-\sigma)(1-\beta)} = (1-\eta)V(b^g, b^I, \epsilon) + \eta W(b^g, b^I, \epsilon)$$

where V and W are the value functions defined in (4) and (5). There are also stationary consumption functions that measure the *individual* welfare of each sector, and these are given by the function $\bar{g}(b^g, b^I, \epsilon)$ that yields the same payoff as $V(b^g, b^I, \epsilon)$ and the function $\bar{c}(b^g, b^I, \epsilon)$ that yields the same payoff as $W(b^g, b^I, \epsilon)$.

The welfare costs of domestic asset market incompleteness are measured by the percent differences between $\check{c}^{SO}(b^g, b^I, \epsilon)$ and $\bar{c}^{MPE}(b^g, b^I, \epsilon)$. The minimum and maximum differences across the 360,000 triples (b^g, b^I, ϵ) in the state space are 0.0007 and 41.45 percent respectively. These are difficult to interpret, however, because they do not take into account the long-run probability of the particular coordinate in the state space to which they correspond. Hence, it makes more sense to evaluate welfare effects by comparing expected welfare costs computed using the limiting distributions of the MPE or the domestic social optimum. Using the limiting distribution of the MPE, the expected welfare cost is:

$$\sum_{(b^g, b^I, \epsilon) \in B^g \times B^I \times s} \Pi_{\infty}^{MPE}(b^g, b^I, \epsilon) \left(\frac{\check{c}^{SO}(b^g, b^I, \epsilon)}{\bar{c}^{MPE}(b^g, b^I, \epsilon)} - 1 \right)$$

where $\Pi_{\infty}^{MPE}(b^g, b^I, \epsilon)$ is the endogenous ergodic distribution of public debt, foreign assets and the Markov shocks in the MPE. The second alternative measure of expected welfare costs uses the limiting distribution of the social optimum, $\Pi_{\infty}^{SO}(b^g, b^I, \epsilon)$, instead of $\Pi_{\infty}^{MPE}(b^g, b^I, \epsilon)$. A problem with this second measure is that these probabilities are not defined for the

[10] These welfare-equivalent stationary consumption levels are computed for each pair (b^I, ϵ^y) in the social optimum, and for each triple (b^g, b^I, ϵ) in the MPE. To express the stationary consumption levels of the social optimum in terms of the state triples of the MPE, note that neither b^g nor ϵ^τ affect the social planner's problem so that $\check{c}^{SO}(b^g, b^I, \epsilon)$ could equivalently be defined as $\check{c}^{SO}(b^I, \epsilon^y)$.

grid B^g, since public debt does not enter into the social planner's problem. To make this probability measure cover the state space for all triples (b^g, b^I, ϵ), the probability of each pair (b^I, ϵ) is distributed evenly across each of the NBG elements of the B^g grid.

The average welfare costs yield striking results. Using the ergodic distribution of the domestic social optimum, the benefits of eliminating the tormented insurer's problem are equivalent to an increase of 4.9 percent in the trend level of consumption per capita. If we use the ergodic distribution of the MPE instead, the increment in trend consumption per capita reaches 1.6 percent. These welfare gains from improving risk sharing dwarf the negligible measures of the benefits of consumption smoothing and risk sharing obtained in standard RBC models, and are comparable to those obtained in the quantitative analysis of the efficiency gains of replacing capital income taxes with consumption taxes (see, for example, Lucas (1996), Mendoza (1991), Mendoza and Tesar (1998)).

The main reason for the marked difference with previous measures of the welfare gains of risk sharing is that the tormented insurer's problem deviates sharply from the representative agent environment of the standard RBC models. In the competitive equilibrium examined here, the income process of the government is significantly different from that faced by the private sector, and the two agents can only used non-state-contingent debt to smooth consumption and self-insure. As a result, the ratio of g_t/c_t (a proxy for shifts in the distribution of wealth between the private and public sectors) fluctuates widely over the business cycle, and the fluctuations in government expenditures are particularly costly. The latter is due in part to the fact that the average of g_t is nearly 7 times smaller than the average of c_t, and the curvature of the CRRA payoff function with a common σ parameter is more pronounced and yields larger utility changes for the fluctuations in g_t.

Table 4 compares the long-run moments of macroeconomic aggregates in the MPE and the social optimum to offer more insight on the determinants of the large welfare costs of imperfect risk sharing in the model. The mean values of the variables are almost identical in the two economies, so the welfare costs do not arise because the social planner has access to resources not available to the agents in the competitive equilibrium. The welfare costs arise instead because when the government can access state-contingent debt or taxes, the ability to implement perfect risk pooling results in public and private expenditure allocations that

fluctuate much less than when the economy lacks state-contingent fiscal instruments. As Table 4 shows, the standard deviation of public expenditures (private consumption) is 13.5 (2.3) times higher in the competitive equilibrium than in domestic social optimum.

3.4 The Revenue Process and Public Debt Dynamics

The properties of the stochastic process of public revenue are a key determinant of the equilibrium dynamics of public debt that the model produces. The significantly larger effects of the tax shock compared to those of the GDP shock found in the impulse response analysis already illustrate this fact. This subsection explores further the dependence of the quantitative predictions of the MPE on the stochastic properties of the implied tax process. The analysis focuses on the effects of changes in the mean, variance, and autocorrelation of the implied tax rate.

Altering the mean implied tax rate changes not only the ability of the government to provide public goods, but also the nature of the insurance problem and the distortions separating the competitive equilibrium from the domestic social optimum. Figure 11 shows three limiting distributions of public debt in the MPE, one for each of three values of the mean tax rate: $\tau = 0.20$, $\tau = 0.256$ (as in the baseline case), and $\tau = 0.30$. The figure shows that the mean and variance of public debt grow with the tax rate because the ability of the government to rely on financial markets to smooth their expenditures improves with the government's capacity to collect revenue. The figure also illustrates the sensitivity of the government's natural debt limit to the tax rate: six percentage points difference in the mean value of fiscal revenue (between 0.256 and 0.20) reduce the natural debt limit of b^g by 55 percentage points.

The effects of changes in the mean tax rate on the business cycle moments of other macroeconomic aggregates are illustrated in Figure 12. Each plot in Figure 12 displays the mean tax rate on the horizontal axis, and the mean and standard deviation of a different variable of the MPE on the vertical axes (the left axis shows the means and the left axis shows the standard deviations). The mean and the standard deviation of public debt, public expenditures, the ratio g/c , and the primary fiscal balance increase as the mean tax rate

rises. In contrast, the mean of private expenditures falls but they also become more variable.

The changes in the moments of the g/c ratio can be used to illustrate how changes in the mean tax rate affect the behavior of the competitive equilibrium relative to the domestic social optimum. At one end, if the tax rate were zero, the government could not provide any public expenditures or issue any debt (since its natural debt limit would be negative, as it would still need to make time-invariant transfer payments). In this case, the ratio of public to private expenditures goes to zero, as all the wealth is concentrated in the private sector, and the welfare loss of the competitive equilibrium relative to the social optimum is infinitely large. At the other end, if the tax rate were 1, the private sector still retains some income in the form of government transfers, but its net-of-tax endowment income is zero, and hence its ability to borrow is significantly reduced and its wealth is significantly smaller than the public sector's. The collective welfare cost relative to the domestic social optimum would be very large because now the private sector ends up with consumption allocations very distant from those obtained in that optimum. These observations suggest that the welfare costs of domestic market incompleteness are a non-monotonic function of the mean tax rate, with larger costs at the extremes, when the implied tax is very high or very low, than for values in between.

Figures 13 and 14 conduct sensitivity experiments for the effects of changes in the variability and persistence of the implied tax process (keeping the mean tax rate equal to its baseline value of 0.256). Figure 13 (14) shows how the means (standard deviations) of the endogenous variables change as the autocorrelation and the variance of the implied tax process increase. The x-axis of each plot in these graphs displays the autocorrelation and variance of the innovations to the implied tax as a ratio relative to the value of these parameters in the baseline scenario. Hence, when $x = 1$, the results shown in the figures correspond to the means (Figure 13) and standard deviations (Figure 14) of the variables obtained in the baseline.[11]

Figure 13 shows that when the tax shocks are more persistent, or when the variance of their innovations is higher, the mean of public debt falls. Increasing the persistence or the variance of innovations rises the volatility of the tax shocks, so the model predicts more

[11] Note that, by construction, the two lines in each pane intersect when $x = 1$.

limited access of the government to debt markets. As the mean of fiscal revenue remains constant in this exercise, the long-run average of public expenditures increases when less resources are spent on debt service. Figure 13 also shows that the means of consumption and the primary balance fall, and that of the g/c ratio increase, as the variability or persistence of the tax process rise. In contrast, Figure 14 shows that increases in the variability or persistence of the implied tax process affect the standard deviations of all variables in the same direction, making all of them increase as the autocorrelation or variance of the tax shocks rises.

Finally, we examine the model's ability to account for the inverse relationship between average public debt ratios and coefficients of variability of public revenues found in the data To this end, we added to Figure 3 "artificial" observations obtained by solving the model for 45 different values of the coefficient of variation of public revenues taken to approximate those observed in the international data. In each case, the model yield a endogenous long-run average for the public debt-GDP ratio.[12] These artifical observations are identified by asterisks in Figure 3, and the continuous curve in the same plot represents the corresponding logarithmic regression line. As the plot shows, the model is consistent with the data in predicting that (a) the long-run average debt ratio is a negative function of the variability of fiscal revenues, and (b) this relationship is non-linear, with the ability to sustain average debt ratios declining at a faster rate as the coefficient of variation of public revenues increases. A comparison of the model's regression line with that produced with actual data shows, however, that the model's mean debt-revenue variability curve is steeper than that observed in the data.

4 Concluding Remarks

Fiscal policy in developing countries differs sharply from that of industrial countries in four key respects: (1) public revenue-GDP ratios are much smaller and significantly more volatile, (2) countries with more variable revenue ratios support lower average debt ratios,

[12] Each of the 45 coefficients of variation of public revenues was generated by forming a grid containing five values of the element (2,2) of the autocorrelation matrix $\hat{\Phi}$ and nine values of the element (2,2) of the covariance matrix $\hat{\Sigma}_\epsilon$ passed to the Tauchen's algorithm described in the Baseline Calibration of Section 3.

(3) the cyclical variability of government expenditures exceeds that of private expenditures by large margins, and (4) fiscal policy follows acyclical or procyclical patterns, with GDP correlations of the primary balance (government expenditures) close to zero or slightly negative (positive).

This paper proposes a model of fiscal policy in small open economies with incomplete asset markets that can rationalize these facts. The model characterizes optimal debt and expenditure policies in an environment in which the government tends to act as a "tormented insurer," seeking to keep its expenditures relatively smooth despite low and volatile revenues and debt markets limited to non-state-contingent debt. The competitive equilibrium exhibits shifts in the distribution of wealth across the private and public sector, which are solved for in recursive form as a Markov perfect equilibrium.

Domestic asset market incompleteness has important implications for welfare and for optimal public debt and government expenditure choices. The welfare costs arising from this form of market incompleteness are evaluated by comparing the MPE with the domestic social optimum attained by a planner who can pool the fiscal risk, and thus equate the marginal utilities of public and private expenditures across states and over time. The average welfare costs of imperfect domestic risk sharing are large, with an average gain of 4.9 percent in a utility-equivalent compensating variation in stationary consumption calculated with the limiting distribution of the MPE, or 1.6 percent if the stationary distribution of the social optimum is used instead. Costs of these magnitude dwarf the negligible costs of imperfect risk sharing and cyclical variability of consumption obtained with conventional RBC models.

The model is able to explain why countries suffering from higher fiscal risk support lower average public debt-GDP ratios. In particular, the model is consistent with international data in producing a negative, non-linear relationship between the variability of fiscal revenues and the average public debt ratios. The model also matches the GDP-correlations of government expenditures and the primary fiscal balance found in data for Mexico, and is consistent with the data in producing higher cyclical variability in public expenditures than in private expenditures.

The implications of a modification of the MPE in which the government shifts from the

implied tax on GDP to a non-state-contingent consumption tax are studied in an ongoing extension of this paper. This modification adds complexity to the MPE by introducing two-way feedback between the plans of the private and public sector via an endogenous tax base determined by the private sector's consumption plans. The presumption is that the consumption tax may be more desirable from a social welfare perspective because private consumption smoothing can act as an endogenous stabilizer that yields more stable fiscal revenues than the implied tax (i.e., than commodity export revenues). In turn, reduced revenue variability implies more access to debt for the government and improved ability to self-insure and smooth expenditures.

Appendix: Markov Chain for GDP and Implied Tax Rate

The Markov chain representing the dynamics of the exogenous state variables, aggregate income (or GDP) and implied tax rates, is characterized by the (9×9) state-transition matrix P, and the (9×2) matrix $\mathcal{E}$ containing all possible realizations of the income and tax shocks, $\epsilon = (\epsilon^y, \epsilon^\tau)$. These two matrices along with the vector $\pi_\infty^{\mathcal{E}}$ that represents the unconditional stationary probability distribution of the ϵ pairs are shown next:

$$P = \begin{pmatrix} 0.1853 & 0.3210 & 0.0348 & 0.1496 & 0.2592 & 0.0281 & 0.0075 & 0.0131 & 0.0014 \\ 0.0988 & 0.3950 & 0.0988 & 0.0652 & 0.2609 & 0.0652 & 0.0027 & 0.0108 & 0.0027 \\ 0.0412 & 0.3807 & 0.2197 & 0.0223 & 0.2057 & 0.1187 & 0.0008 & 0.0069 & 0.0040 \\ 0.0464 & 0.0804 & 0.0087 & 0.2268 & 0.3929 & 0.0425 & 0.0693 & 0.1201 & 0.0130 \\ 0.0278 & 0.1111 & 0.0278 & 0.1111 & 0.4444 & 0.1111 & 0.0278 & 0.1111 & 0.0278 \\ 0.0130 & 0.1201 & 0.0693 & 0.0425 & 0.3929 & 0.2268 & 0.0087 & 0.0804 & 0.0464 \\ 0.0040 & 0.0069 & 0.0008 & 0.1187 & 0.2057 & 0.0223 & 0.2197 & 0.3807 & 0.0412 \\ 0.0027 & 0.0108 & 0.0027 & 0.0652 & 0.2609 & 0.0652 & 0.0988 & 0.3950 & 0.0988 \\ 0.0014 & 0.0131 & 0.0075 & 0.0281 & 0.2592 & 0.1496 & 0.0348 & 0.3210 & 0.1853 \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} -0.0467 & -0.0655 \\ 0.0000 & -0.0827 \\ 0.0467 & -0.0999 \\ -0.0467 & 0.0172 \\ 0.0000 & 0.0000 \\ 0.0467 & -0.0172 \\ -0.0467 & 0.0999 \\ 0.0000 & 0.0827 \\ 0.0467 & 0.0655 \end{pmatrix}; \qquad \pi_\infty^{\mathcal{E}} = \begin{pmatrix} 0.0397 \\ 0.1477 \\ 0.0434 \\ 0.0968 \\ 0.3448 \\ 0.0968 \\ 0.0434 \\ 0.1477 \\ 0.0397 \end{pmatrix}$$

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Table 1: Calibration of the Model to the Mexican Economy

Notation	Parameter / Variable	Value
β	Discount factor	0.925
γ	Gross growth rate	1.00888
ϕ^g	Natural debt limit on public debt	1.3183
ϕ	Ad-hoc debt limit on international assets	-0.5000
σ	Coefficient of relative risk aversion	2.0000
τ	Mean income-tax rate	0.2558
R	Gross world interest rate	1.0986
x	Private investment expenditures	0.2262
z	Government transfers	0.1114
-	Minimum value of government debt	0.0000
-	Maximum value of international assets	0.1000

Table 2: Unconditional Moments of GDP and the Implied Tax Rate. Mexican Annual Data, 1980-2004 and Unconditional Moments of the Markov Chain

Statistic	Mexican data (a)		Markov chain (b)	
	GDP	Implied tax rate	GDP	Implied tax rate
Standard deviation	0.02948	0.06027	0.02781	0.05689
Minimum	-0.07073	-0.12294	-0.04670	-0.09991
Maximum	0.05018	0.01080	0.04670	0.09991
Cross correlation	-0.24172	-0.24172	-0.19786	-0.19786
Autocorrelation	0.351	0.535	0.278	0.576

Table 3: Moments of Macroeconomic Aggregates in the Ergodic Distribution of the Markov Perfect Equilibrium (Baseline Calibration)

Variable (x)	E[x]	$\sigma(x)$	cv(x)	$\rho(x)$	$\rho(x, y_i)$, where $y_i =$		
					GDP	After-tax inc.	Fiscal revenue
GDP	1.00	2.80	2.80	0.28	1.00	0.86	0.28
GNP	0.97	3.03	3.13	0.42	0.95	0.82	0.26
Atfer-tax income	0.74	2.78	3.73	0.38	0.86	1.00	-0.25
Consumption	0.64	3.20	4.97	0.97	0.20	0.24	-0.07
International assets	-0.36	10.73	-29.72	0.98	0.08	0.08	0.00
Total assets	0.17	32.20	188.27	1.00	0.02	0.05	-0.05
Gov. expenditures	0.10	2.83	29.07	1.00	0.02	-0.04	0.12
Tax rate	0.26	1.47	5.73	0.58	-0.20	-0.68	0.88
Fisc revenue	0.26	1.50	5.86	0.53	0.28	-0.25	1.00
Primary fiscal bce.	0.05	3.05	64.47	0.90	0.12	-0.09	0.38
Public debt	0.53	30.51	57.37	1.00	0.00	0.02	-0.05
Gov. expend. / cons.	0.15	5.07	33.01	1.00	-0.01	-0.07	0.11
Trade balance	0.03	2.55	79.43	0.34	0.82	0.68	0.28
Current account	0.00	2.27	-	0.24	0.96	0.80	0.31

Note: For each variable x, E[x] is the mean, $\sigma(x)$ is the percentage standard deviation, cv(x) is the percentage coefficient of variation, $\rho(x)$ the autocorrelation, and $\rho(x, y_i)$ is the cross-correlation with the variable y_i indicated in the respective column headings.

Table 4: Comparison of Limiting Moments of Macroeconomic Aggregates: Markov Perfect Equilibrium versus Social Optimum

Variable (x)	Markov Perfect Equilibrium					Social Optimum				
	E[x]	$\sigma(x)$	cv(x)	$\rho(x)$	$\rho(x,y)$	E[x]	$\sigma(x)$	cv(x)	$\rho(x)$	$\rho(x,y)$
GDP	1.00	2.80	2.80	0.28	1.00	1.00	2.80	2.80	0.28	1.00
GNP	0.97	3.03	3.13	0.42	0.95	0.97	3.01	3.11	0.41	0.96
Consumption	0.64	3.20	4.97	0.97	0.20	0.64	1.39	2.16	0.89	0.46
Gov. expenditures	0.10	2.83	29.07	1.00	0.02	0.10	0.21	2.16	0.89	0.46
Gov. expenditures/Consumption	0.15	5.07	33.01	1.00	-0.01	15.00	0.00	0.00	-	-
Trade balance	0.03	2.55	79.43	0.34	0.82	0.03	2.51	75.35	0.33	0.82
Current account	0.00	2.27	-	0.24	0.96	0.00	2.25	-	0.23	0.95
International assets	-0.36	10.73	-29.72	0.98	0.08	-0.37	10.01	-26.76	0.97	0.08

Note: For each variable x, E[x] is the mean, $\sigma(x)$ is the percentage standard deviation, cv(x) is the percentage coefficient of variation, $\rho(x)$ the autocorrelation, and $\rho(x,y)$ is the cross-correlation with GDP.

Figure 1: Average Public Revenue-GDP Ratios in Emerging and Industrial Countries: 1990-2002

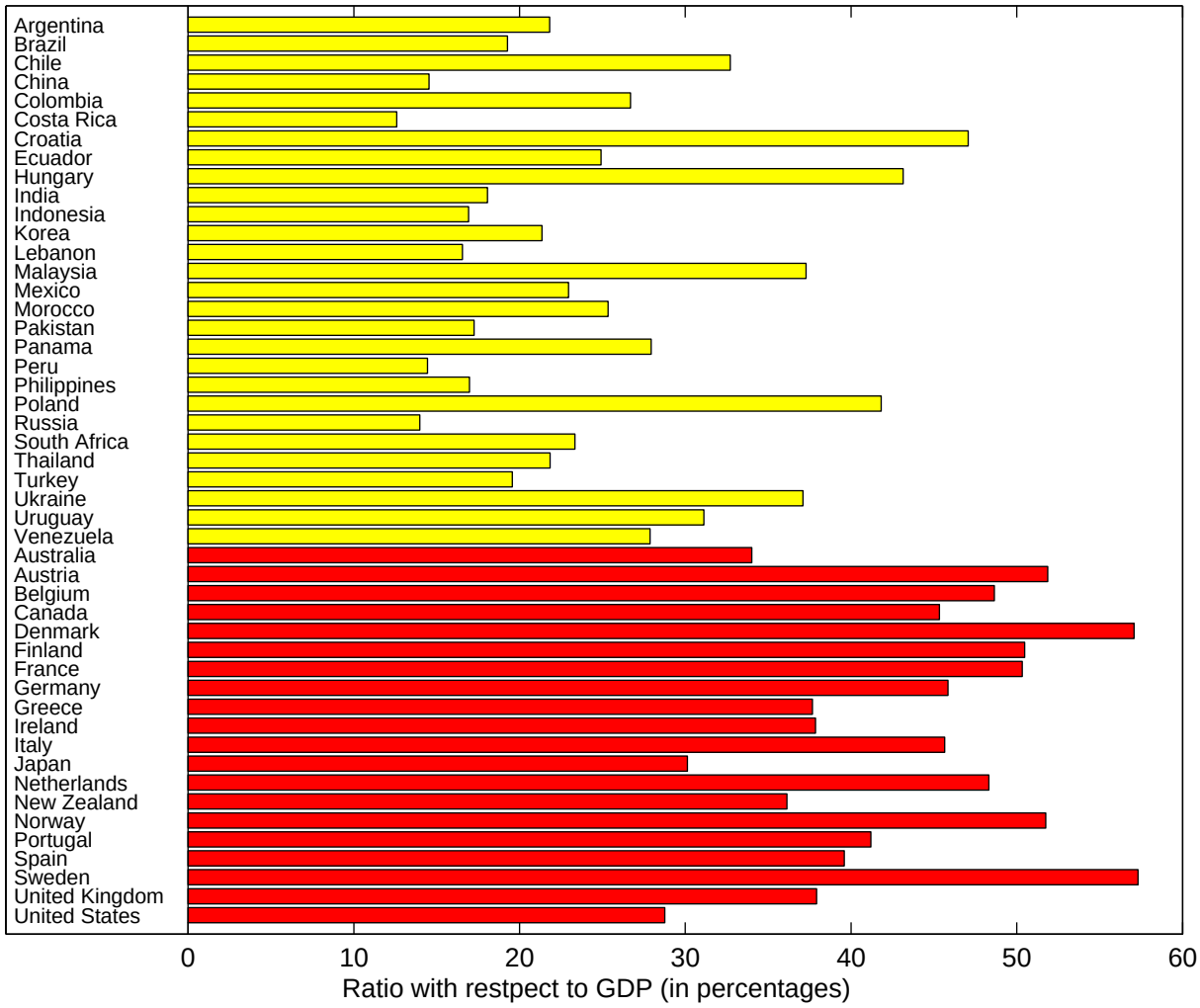


Figure 2: Coefficients of Variation of Public Revenue-GDP Ratios in Emerging and Industrial Countries: 1990-2002

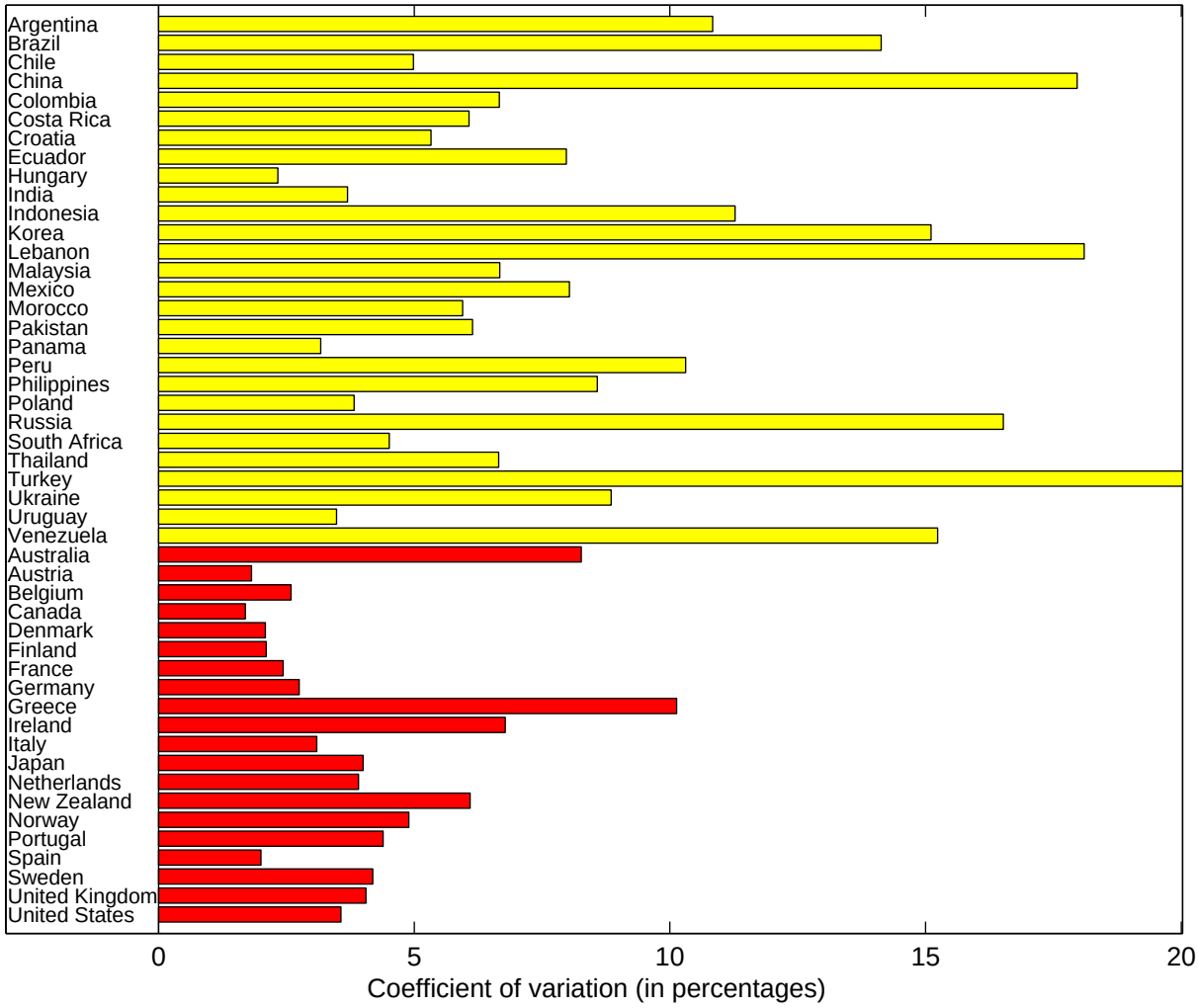


Figure 2. Standard Deviations of Difference in Cyclical Components of Public & Private Expenditures in Percent of United States (1980-2003)

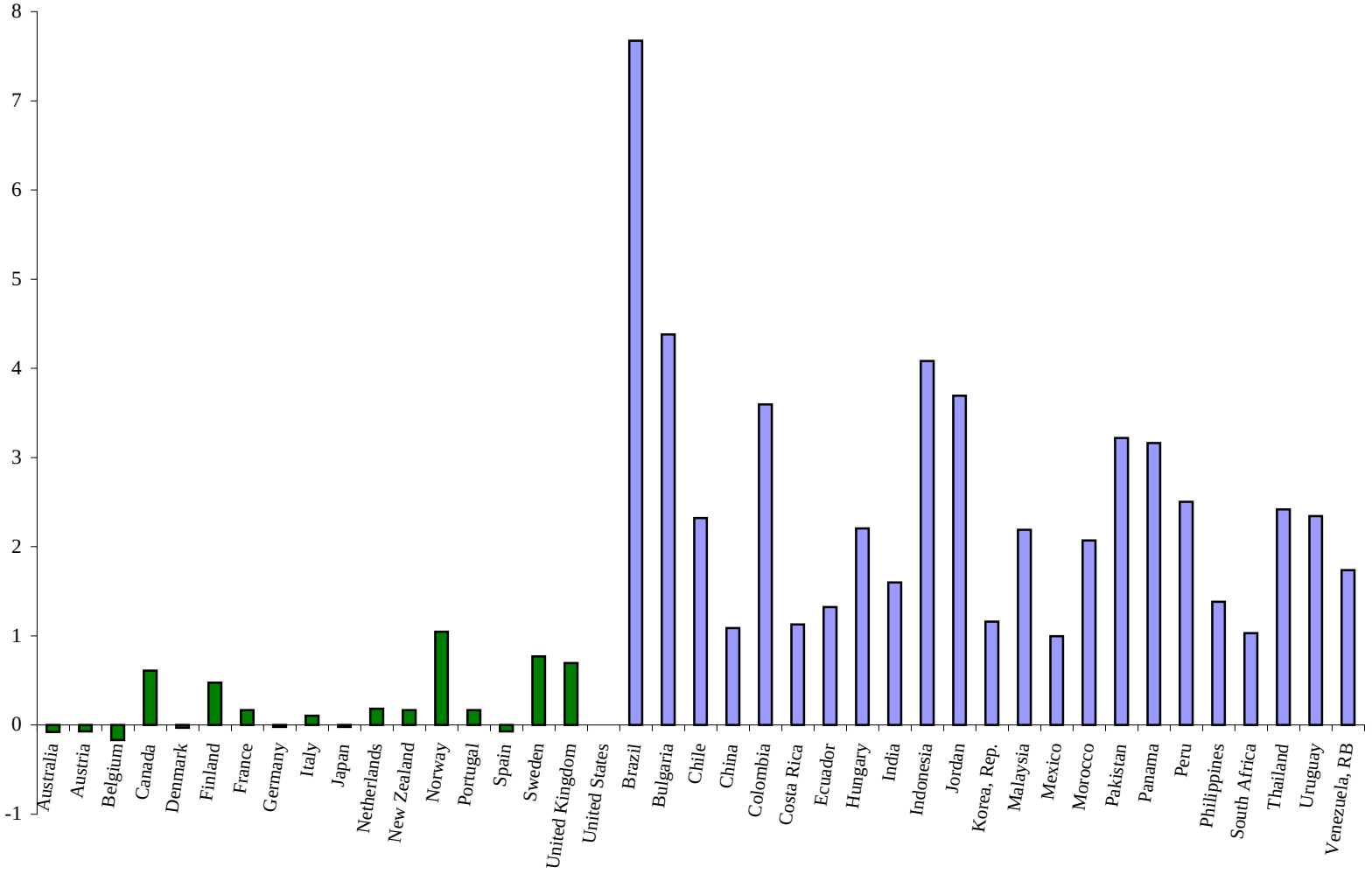


Figure 3: Volatility of Fiscal Revenues and Average Public Debt Ratios

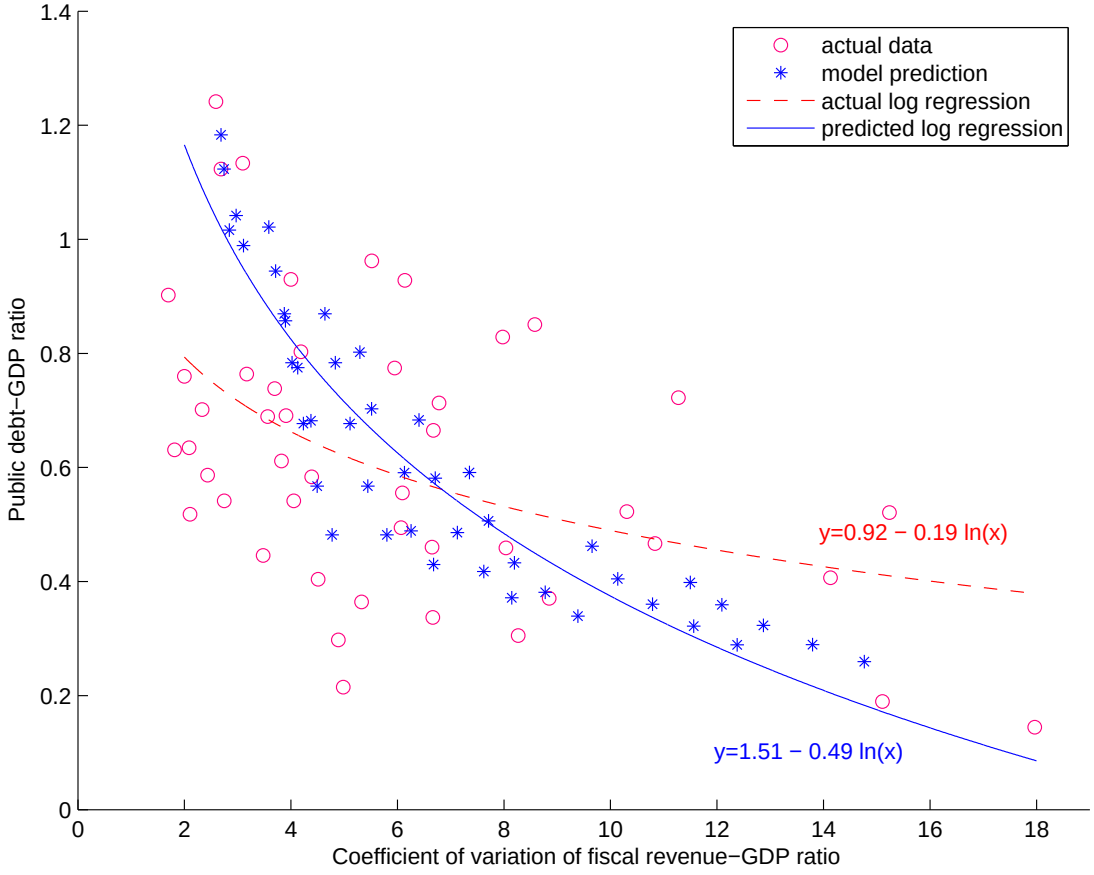
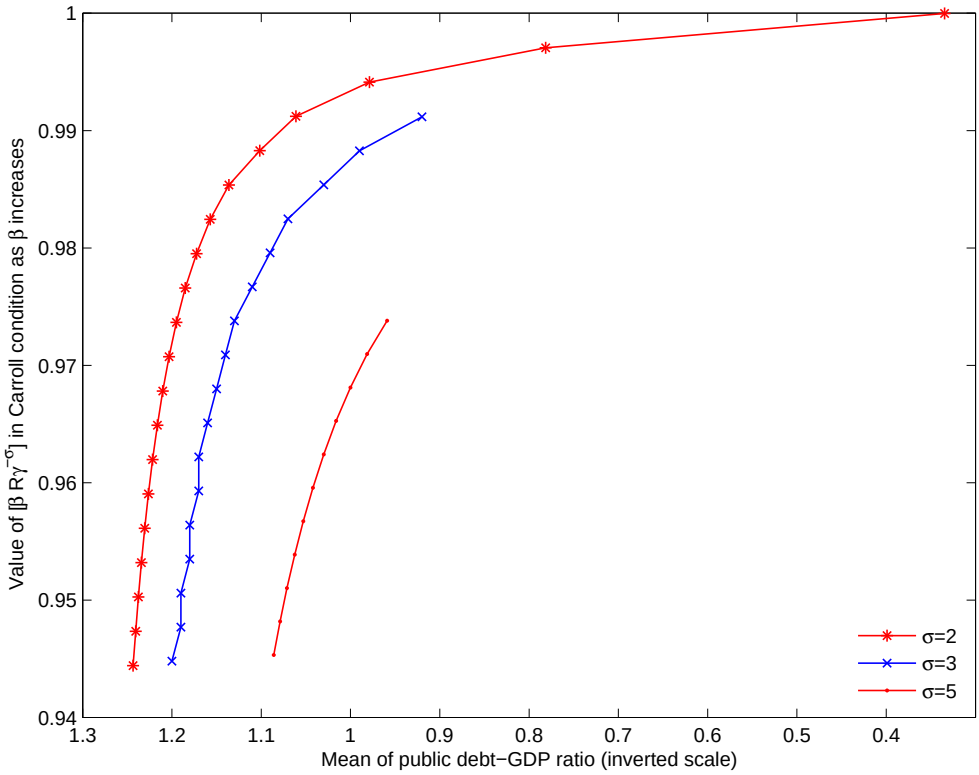
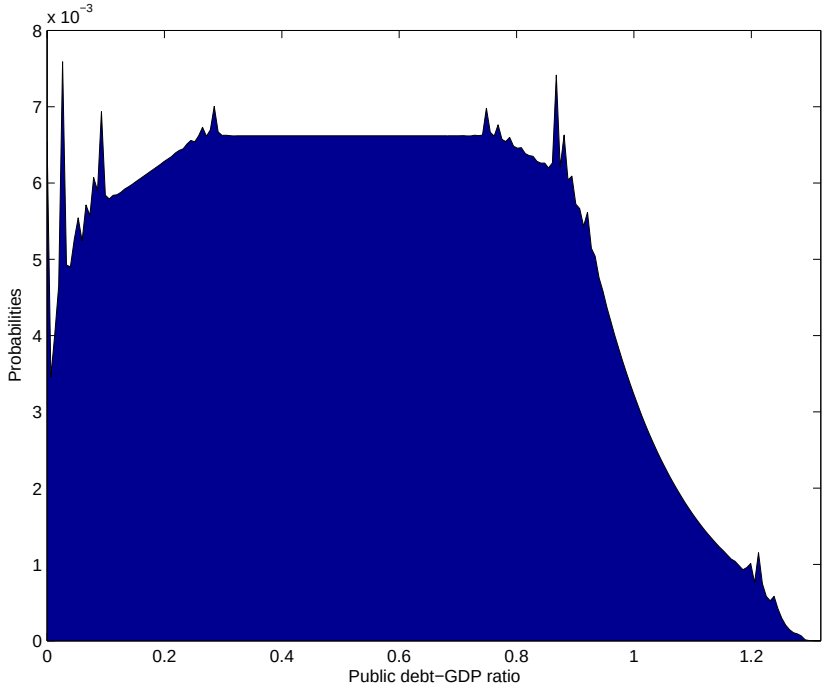


Figure 4: Carroll's Condition, Discount Factor and Mean Value of Public Debt-GDP Ratio

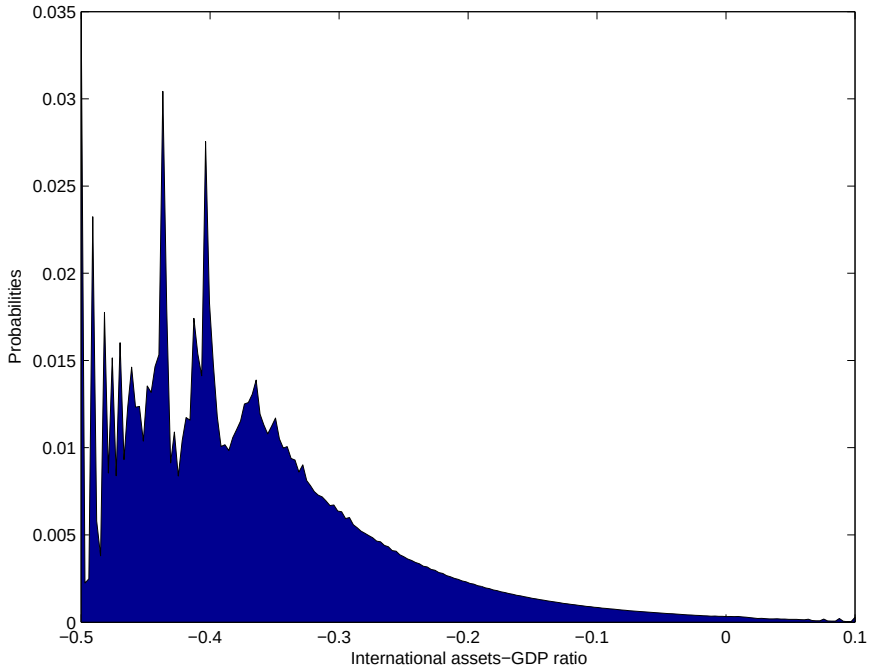


Note: Carroll's (2004) condition required to prevent public debt diverging to $-\infty$ is $\beta R \times \max[R^{-\sigma}, \gamma^{-\sigma}] < 1$. In the baseline calibration, $\gamma = 1.00888$, $\sigma = 2$, $R = 1.0986$, and $\beta = 0.925$, so the condition is satisfied as $\beta R \gamma^{-\sigma} = 0.9984$. The graph is constructed for three values of σ, keeping unchanged the values of γ and R, and considering values of $\beta \in [0.875, 0.92647]$. The natural debt limit of public debt in the baseline calibration is 1.3183.

Figure 5: Marginal Distribution of Public Debt and International Assets in the Limiting Distribution of the MPE under Baseline Calibration

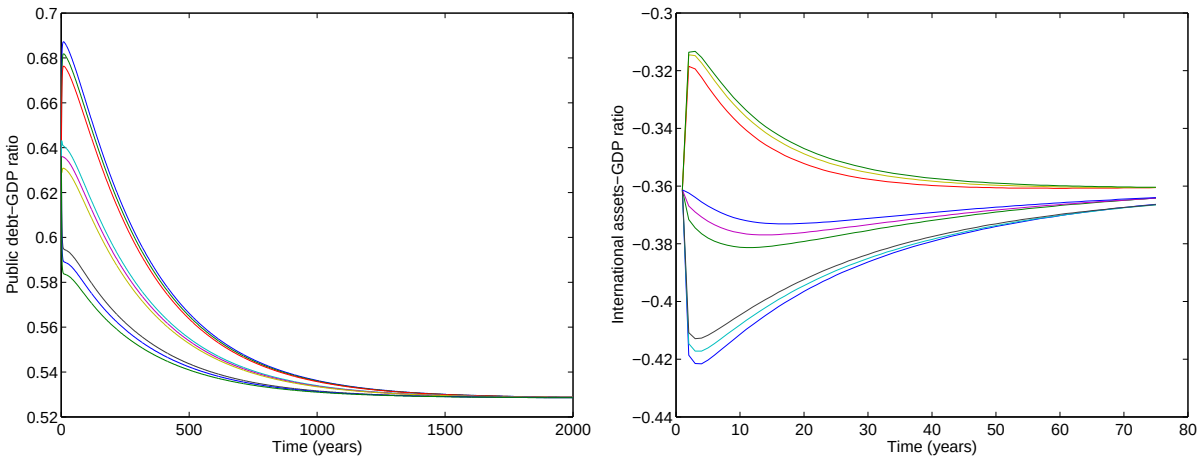


(a) Public Debt



(b) International Assets

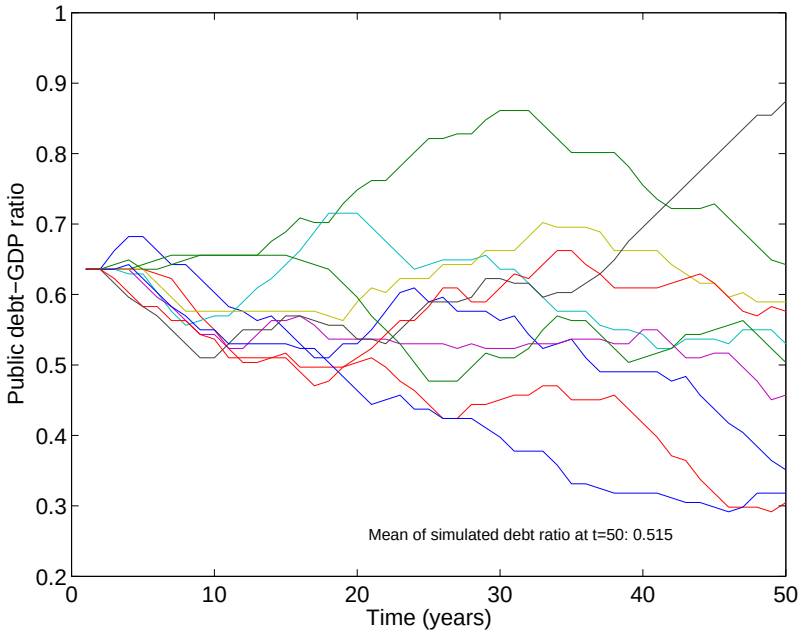
Figure 6: Forecasting Functions of Assets



(a) Public Debt (b) International Assets

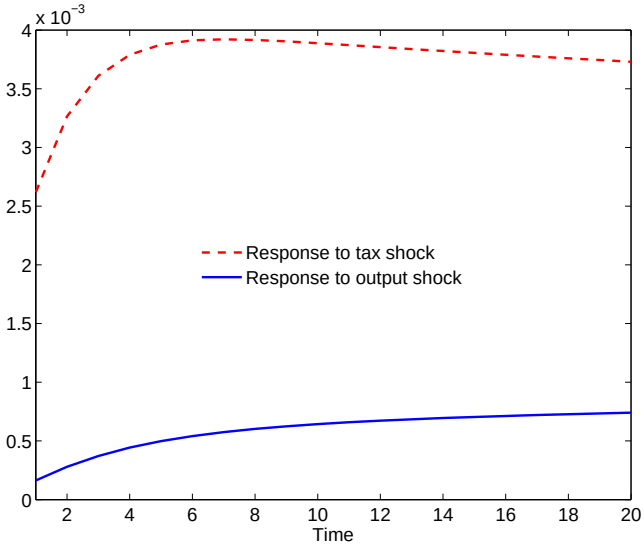
Note: Starting value of public debt is equal to 0.634 (10 percentage points above its mean value in the limiting distribution); starting value of international assets is equal to its mean value in the limiting distribution.

Figure 7: Simulations of Public Debt Using Simulations of the Markov Chain and Optimal Policy Functions



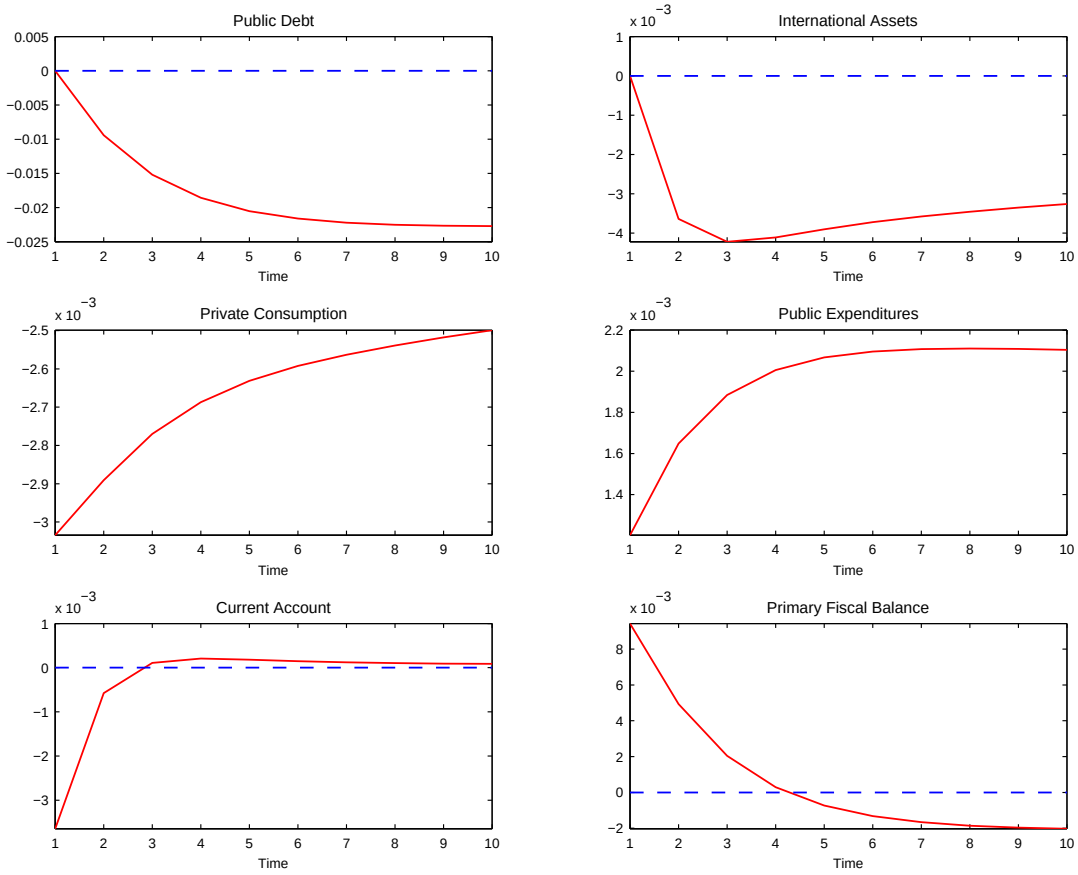
Note: Starting value of public debt is equal to 0.634 (10 percentage points above its mean value in the limiting distribution).

Figure 8: Impulse Response Functions of the Government Expenditures-Consumption Ratio



Note: Estimation errors are orthogonalized using the Choleski decomposition of the covariance matrix of the residuals so the innovations are equal to one standard deviation of the orthogonalized covariance matrix of errors.

Figure 9: Impulse Response Functions to Tax Shocks in the Competitive Equilibrium



Note: Estimation errors are orthogonalized using the Choleski decomposition of the covariance matrix of the residuals so the innovations are equal to one standard deviation of the orthogonalized covariance matrix of errors.

Figure 10 Impulse Response Functi**ons to an Output Shock in the**
Competitive Equilibrium and the Domestic Social Optimum
(responses to Cholesky One S.D. Innovations)

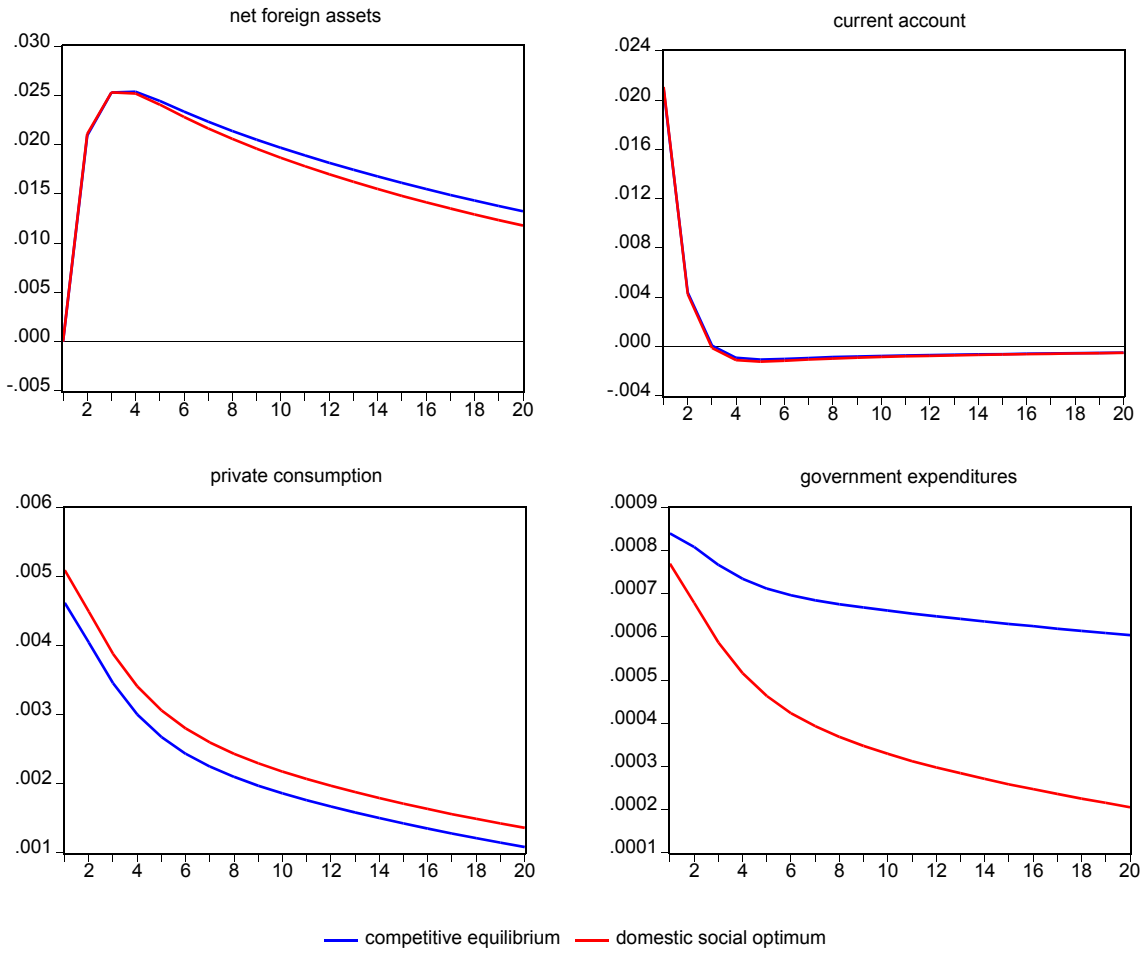


Figure 11: Mean of Implied Income-Tax Rate and the Limiting Marginal Distribution of Public Debt

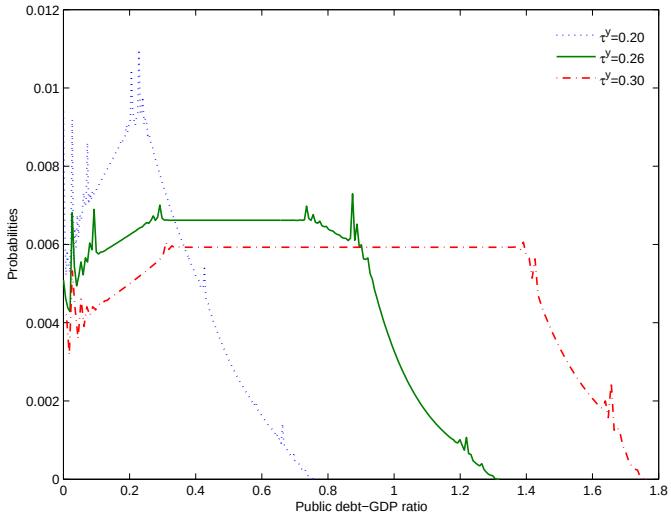
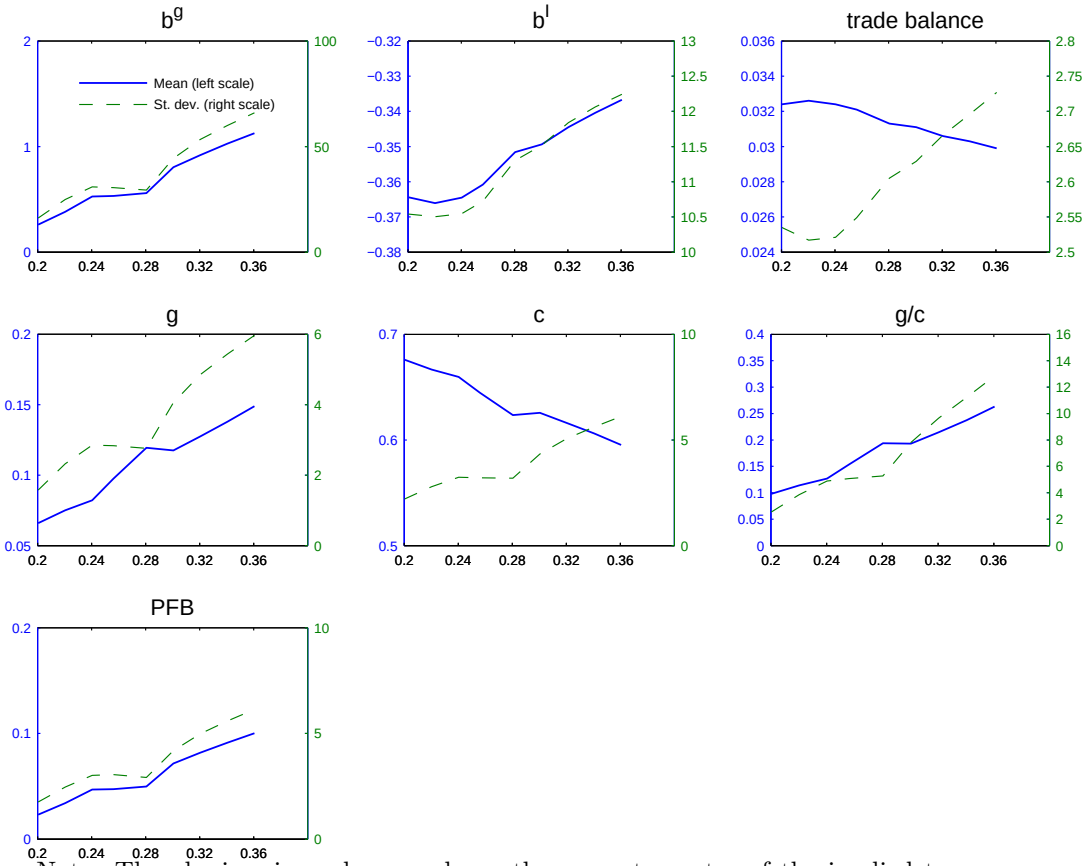
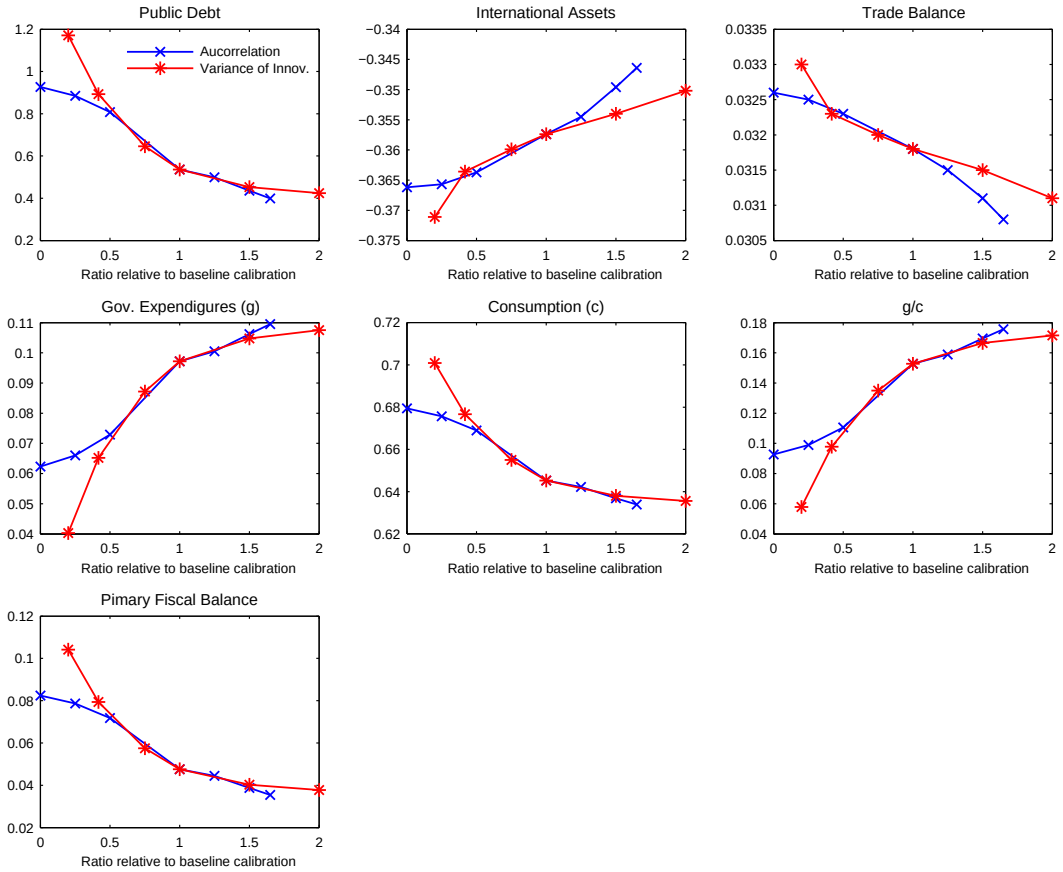


Figure 12: Mean of Implied Income-Tax Rate and the Mean and Standard Deviation of Macroeconomic Aggregates in the MPE



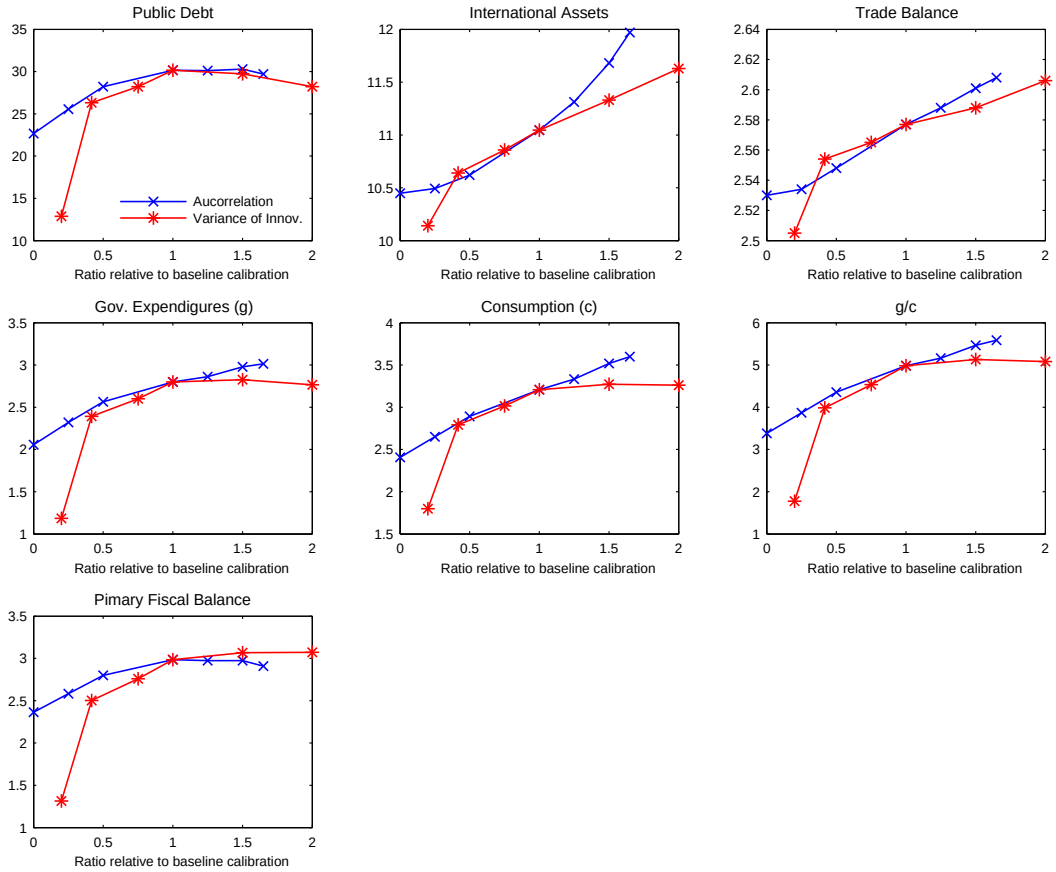
Note: The abscissa in each pane shows the mean tax rates of the implied tax process and the two ordinates show, the mean value of each variable on the left-hand-side scale and the standard deviation of each variable on the right-hand-side scale.

Figure 13: Autocorrelation and Variance of the Innovations to the Tax Process: Effect on the *Means* of Macroeconomic Ratios in the Markov Perfect Equilibrium



Note: The x-axes show the values of the autocorrelation and variance of the innovations to the tax process measured with respect the values of these parameters of the tax process in the baseline calibration. When $x = 1$, the variance of the innovations to the tax process takes its steady state value equal to 0.002378 and the autocorrelation is equal to 0.60.

Figure 14: Autocorrelation and Variance of the Innovations to the Tax Process: Effect on the *Standard Deviations* of Macroeconomic Ratios in the Markov Perfect Equilibrium



Note: The x-axes show the values of the autocorrelation and variance of the innovations to the tax process measured with respect the values of these parameters of the tax process in the baseline calibration. When $x = 1$, the variance of the innovations to the tax process takes its steady state value equal to 0.002378 and the autocorrelation is equal to 0.60.