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Stock-Flow Matching Model: A Quantitative Analysis*

Robert Shimer

University of Chicago and NBER

shimer@uchicago.edu

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Abstract

This paper explores the ability of the stock-flow matching model to generate realistic business cycle frequency fluctuations in unemployment, job vacancies, and labor flows. The model's behavior is very similar to that in Shimer (2006) and fits the data significantly better than a comparable search and matching model (Shimer, 2005).

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1 Introduction

This paper explores the ability of the stock-flow matching model (Taylor, 1995; Coles and Muthoo, 1998; Coles and Smith, 1998) to generate realistic business cycle frequency fluctuations in unemployment, job vacancies, and labor flows. The model's premise is that the labor market is characterized by heterogeneous workers and jobs. Most workers would be unproductive if forced to take a random job and the suitability of a worker for a job is independent across worker-job pairs. When a worker becomes unemployed, she examines the stock of vacant jobs. If she is suitable for one of them, she takes it; otherwise, she joins the stock of unemployed workers and waits until a firm creates a suitable job. Thus in equilibrium the inflow of new workers matches with the stock of available jobs and symmetrically the stock of unemployed workers matches with the inflow of new jobs.

I show that the stock-flow matching model generates realistic unemployment-vacancy (Beveridge curve) dynamics in response to productivity shocks. Moreover, the transition rate from unemployment to employment, the job-finding rate, is strongly correlated with the vacancy-unemployment (v-u) ratio, although the theoretical correlation, about 0.8, is smaller than the empirical correlation of 0.95. Finally, the model explains between a quarter and a third of the fluctuations in the job finding rate and the v-u ratio in response to quantitatively plausible productivity shocks, considerably more than a similarly-calibrated search and matching model (Shimer, 2005).

There are several reasons I am interested in the behavior of this model. First, in Shimer (2006), I developed a "mismatch" model of distinct labor markets—geographic areas or occupations—with perfect competition within labor markets but limited mobility of workers and jobs across labor markets. In equilibrium, unemployed workers exist in labor markets with insufficient jobs and job vacancies in markets with excess jobs. Aggregate shocks then lead to fluctuations in the same labor market variables. One can think of the mismatch model as one with ex ante heterogeneity, a perfect correlation in the suitability of workers for jobs. The stock-flow matching model is the one with ex post heterogeneity, no correlation in the suitability of workers for jobs. This paper then represents a robustness check on the conclusions of that earlier work. The important conclusions carry through.

Second, the state space in the stock-flow matching model is small relative to the mismatch model, potentially facilitating some interesting extensions. In the mismatch model, the state is the distribution of workers and jobs across labor markets. Under a strong assumption, exogenous mobility, it is possible to characterize the equilibrium when the economy is subject to aggregate shocks. In the stock-flow matching model, the state of the economy is simply the unemployment and vacancy rates. In future versions of this paper, I plan to examine how labor market conditions affect the willingness of workers to accept jobs that are a less-than-perfect match.

Finally, reality probably contains elements of each model. Across broad labor market, the mismatch assumption of segmented labor markets is possibly realistic. However, within narrower labor markets, the idiosyncratic matching problem captured by the stock-flow matching model may be more relevant. The fact that the models' behavior is so similar is therefore reassuring and suggests that the results in both papers reflect a more general approximate aggregation theorem.

The outline of this paper is as follows: Section 2 describes the basic model. Section 3 analyzes the determination of unemployment and vacancies as a function of the number of active jobs in the economy. Section 4 discusses the determination of the number of jobs both in a centralized and a decentralized economy. Section 5 explains how I calibrate the model and Section 6 discusses the results.

2 Model

I study a continuous time, infinite horizon model. At any point in time t, there is a measure $M = 1$ of workers and a measure $N(t)$ of jobs. While the measure of workers is exogenous, the measure of jobs will be determined endogenously by firms' job creation decision.

Workers are risk-neutral and infinitely-lived. They can be either unemployed, obtaining leisure z, or employed in a job producing $p(t) > z$ units of output. Productivity $p(t)$ follows an first order Markov process and is the sole driving force in this economy. More precisely, there is an aggregate shock at rate λ, at which point the new value of productivity p' is drawn from some distribution that depends on current productivity. Let $\mathbb{E}_p X_{p'}$ denote the expected value of some variable X following the next productivity shock, conditional on the current state p.

Most worker-job matches are totally unproductive, while a few have the potential to produce output. Which matches are productive is independent across worker-job pairs and so for any worker looking at any random set of jobs, the number of suitable jobs is a Poisson random variable. In particular, within a measure ν of jobs, a worker has at least one productive job with probability $1 - e^{-\alpha\nu}$. The parameter α measures the extent of search frictions in the model economy.

Firms are also risk-neutral and infinitely-lived. They can create an unlimited number of jobs by paying a sunk-cost $k > 0$, while existing jobs end according to a Poisson process with arrival rate s. Immediately upon creating a job, a firm observes whether any unemployed worker can produce with it. If so, the job is filled and production begins. Otherwise, the job remains vacant until either it ends or until an employed worker loses her job but finds she can take this vacancy. Thus at any point in time unemployed workers and vacant jobs coexist, but no unemployed worker can produce with any vacant job.

I characterize the characterize the solution to the problem of a social planner who is interested in maximizing the expected present value of output in the economy and who can instruct firms when to create jobs. I later consider how this can be decentralized.

3 Beveridge Curve

I start by examining how the measure of active jobs $N(t)$ determines which workers are matched with which jobs, which workers are unemployed, and which jobs are vacant. Any variation in the number of jobs then induces variation in unemployment and vacancies, the Beveridge curve. I finish the section by comparing the empirical and model-generated Beveridge curves.

3.1 Computing Unemployment and Vacancies

Order the workers $i \in [0, 1]$ according to the amount of time since they last lost a job, so worker 1 just lost her job. Similarly order the jobs $j \in [0, N(t)]$ according to the amount of time since they entered, with job $N(t)$ the newest entrant. If a positive measure of jobs entered at the some instant, any ordering of those jobs is permitted.

Then match workers to jobs sequentially, giving worker 0 the opportunity to match first. Since there are $\mu_0(t) = N(t)$ jobs available, she has a match with probability $1 - e^{-\alpha N(t)}$. In this event, she takes the lowest productive match, the one that entered at the earliest date. Proceeding sequentially, when worker i has the opportunity to match, there are $\mu_i(t)$ available jobs and so she has a match with probability $1 - e^{-\alpha \mu_i(t)}$. This implies $\partial \mu_i(t)/\partial i = -1 + e^{-\alpha \mu_i(t)}$ and so solving the differential equation gives

$$\mu_i(t) = \frac{1}{\alpha} \log \left(e^{\alpha i} + e^{\alpha N(t)} - 1 \right) - i. \tag{1}$$

Worker i is unemployed with probability $e^{-\alpha\mu_i(t)} = \frac{e^{\alpha i}}{e^{\alpha i}+e^{\alpha N(t)}-1}$ and so the unemployment rate is

$$U(t) = \int_0^1 e^{-\alpha\mu_i(t)}di = \frac{1}{\alpha}\log\left(e^{\alpha}+e^{\alpha N(t)}-1\right) - N(t), \tag{2}$$

while the number of vacancies is just equal to the number of jobs left after worker 1 enters,

$$V(t) = \mu_1(t) = \frac{1}{\alpha}\log\left(e^{\alpha}+e^{\alpha N(t)}-1\right) - 1. \tag{3}$$

Note that the number of employed workers $1-U(t)$ is equal to the number of filled jobs $N(t)-V(t)$ and both depend only on the number of jobs $N(t)$.

Also note the symmetry of the characterization. The matching of workers to jobs would have been unchanged if we gave the lowest-named job the opportunity to match first and then matched jobs to workers in order. In particular, the probability job j is vacant is

$$v_j = \frac{e^{\alpha j}}{e^{\alpha}+e^{\alpha j}-1}. \tag{4}$$

This matching is stable in the following sense: First, if a new job enters, it immediately hires a worker if it has a match with one of the unemployed workers, with probability $1-e^{-\alpha U(t)}$. This is equal both to the probability that job $N(t)$ is filled, $\frac{e^{\alpha}-1}{e^{\alpha N(t)}+e^{\alpha}-1}$, and to marginal effect of entry on unemployment, $-\partial U(t)/\partial N(t)$, as one would expect.

Second, suppose an arbitrary job j exits. If the firm was vacant, no one loses or changes jobs, consistent with the initial matching. If it was filled, with probability $1-v_j$, the displaced worker immediately moves to the end of the queue. From the fact that she was matched with job j, we know that she cannot match with any vacant job $j' \in [0,j)$; however, the worker may be able to match with any of the $\int_j^{N(t)} v_{j'}dj'$ remaining vacancies. It follows that the probability a worker becomes unemployed when job j exits is the product of these two probabilities:

$$(1-v_j)e^{-\alpha\int_j^{N(t)} v_{j'}dj'} = \frac{e^{\alpha}-1}{e^{\alpha N(t)}+e^{\alpha}-1} \equiv \delta_{N(t)}, \tag{5}$$

independent of the job's identity. Curiously, the probability a job is filled by a worker with no other employment possibilities does not depend on the age of the job, only on the total number of jobs in existence. I will use this fact later.

The probability that a job exiting leads to a worker getting displaced, δ, is equal to the probability that a new entrant immediately hires a worker, $1-e^{-\alpha U(t)}$, so the simultaneous entry and exit of a job does not affect unemployment and vacancies. Moreover, the job's exit and the reordering of workers implies that the resulting matching pattern is consistent with the outcome of original matching algorithm; worker 1 is unemployed only if she does not have a match among the $V(t)$ vacancies.

I summarize these results as follows:

Proposition 1 *Suppose at some time t_0, $U(t_0)$ and $V(t_0)$ satisfy equations (2) and (3). Then, provided that jobs enter sequentially and unemployed workers and vacancies are matched whenever possible between time t_0 and t_1, $U(t_1)$ and $V(t_1)$ solve the same pair of equations, regardless of the evolution of $N(t)$.*

It is worth noting that a more sophisticated matching procedure may result in fewer unemployed workers and vacancies. In particular, there may be a worker i employed in job j but productive in a vacant job j' and an unemployed worker i' who is productive in job j. The stock-flow matching algorithm does not permit worker i' to take job j and worker i to take job j' since newly unemployed workers only look at the set of vacant jobs. If there are substantial unmodeled search or turnover costs, it may not be advantageous to try to take advantage of such matching opportunities.

3.2 Theory and Evidence

I can eliminate $N(t)$ between equations (2) and (3) to get

$$V(t) = \frac{1}{\alpha} \log\left(\frac{1 - e^{-\alpha}}{1 - e^{-\alpha U(t)}}\right), \tag{6}$$

the theoretical Beveridge curve.

I compare this with U.S. data on unemployment and job vacancies. The Bureau of Labor Statistics (BLS) uses the Current Population Survey (CPS) to measure the unemployment rate each month. The CPS measures employment and unemployment using a household questionnaire designed to determine whether an individual is working or, if she is not working, available for and actively seeking work. The ratio of unemployment to the sum of unemployment and employment is the unemployment rate.

Since December 2000, the BLS has measured job vacancies using the JOLTS. This is the most reliable time series for vacancies in the U.S.. According to the BLS, "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting

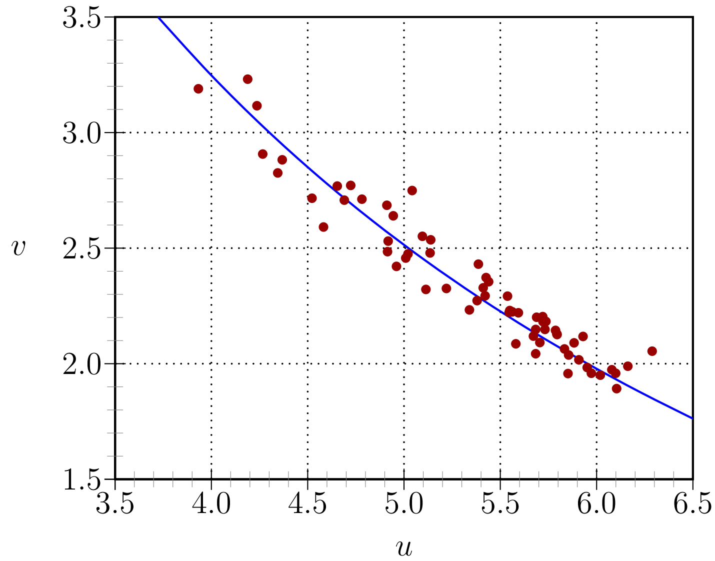

Figure 1: The brown dots show U.S. monthly data from December 2000 to April 2006. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the model generated Beveridge curve with $\alpha = 19.2$

means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods."[1] I measure the vacancy rate as the ratio of vacancies to vacancies plus employment. The brown dots in Figure 1 show the strong negative correlation between unemployment and vacancies over this time period, the empirical Beveridge curve.

In an average month from December 2000 to April 2006, the geometric mean of the unemployment and vacancy rates were 5.33% and 2.33%, respectively. Using equation (6), this implies $\alpha = 19.2$. The blue line in Figure 1 shows the modeled-generated Beveridge curve. The fit of the model to the data is excellent and virtually indistinguishable from Figure 1 in Shimer (2006). The fact that the level of the model-generated Beveridge curve fits the data reflects a judicious choice of α. But the fact that the slope and curvature of the model-generated Beveridge curve also fits the data comes from the structure of the model. That the results are so similar in the mismatch and stock-flow matching models suggests that the Beveridge curve may simply be an aggregation phenomenon.

[1]See BLS news release, July 30, 2002, available at `http://www.bls.gov/jlt/jlt_nr1.pdf`

4 Determination of the Number of Jobs

I now solve the problem of a social planner who chooses the number of jobs $N(t)$ in order to maximize the expected present value of output net of job creation costs. I show that the planner's solution is characterized by a function mapping current productivity p into a target number of jobs N_p^*. If the actual number of jobs is below the target, the planner adjusts up to the target instantaneously. If it is above the target, the planner permits the number of jobs to decline with exit, at rate s, until the target is reached. I then discuss the decentralization of the social optimum. Finally, I show how to solve for the targets numerically.

4.1 Planner's Problem

Let $W_p(N)$ denote the expected present value of net output when current productivity is p and the current number of jobs is N. I represent the planner's problem recursively as

$$rW_p(N) = \max_{g \geq 0} \; p(1 - u(N)) + zu(N) - kg \\ + W_p'(N)(g - sN) + \lambda \mathbb{E}_p\big(W_{p'}(N) - W_p(N)\big) \quad (7)$$

Here g is the gross increase in the number of jobs and

$$u(N) = \frac{1}{\alpha} \log \left(e^{\alpha} + e^{\alpha N} - 1\right) - N, \quad (8)$$

solves equation (8). The flow value of the planner, $rW_p(N)$, can be divided into three terms. First is current net output, p for each of the $1 - u(N)$ employed workers, z for each of the $u(N)$ unemployed workers, and $-k$ for each job created. Second is the future increases in in $W_p(N)$ coming from any net increase in the number of jobs, the difference between gross job creation and deprecation, $g - sN$. Third is the possibility of an aggregate shock, with arrival rate λ, at which point the planner anticipates a capital gain $\mathbb{E}_p\big(W_{p'}(N) - W_p(N)\big)$.

The first order condition for the gross amount of job creation conditional on the current state (p, N) is

$$g_p(N) \geq 0, \quad W_p'(N) \leq k, \quad \text{and} \quad g_p(N)(W_p'(N) - k) = 0. \quad (9)$$

That is, whenever the marginal value of a job is smaller than k, gross job creation is zero and conversely, if some jobs are being created, the marginal value of a job must equal its cost.

The envelope condition is

$$
\begin{aligned}
rW_p'(N) = (p - z)\big(1 - e^{-\alpha u(N)}\big) - sW_p'(N) \\
+ W_p''(N)(g_p(N) - sN) + \lambda \mathbb{E}_p\big(W_{p'}'(N) - W_p'(N)\big),
\end{aligned} \tag{10}
$$

where I use the fact that $u'(N) = -\frac{e^{\alpha}-1}{e^{\alpha}+e^{\alpha N}-1} = -\big(1 - e^{-\alpha u(N)}\big)$. Combining the first order and envelope conditions, we can define the targets N_p^* as follows. First, if $N > N_p^*$, no new jobs are created, $g_p(N) = 0$ so

$$
(r + s + \lambda)W_p'(N) = (p - z)\big(1 - e^{-\alpha u(N)}\big) - W_p''(N)sN + \lambda \mathbb{E}_p W_{p'}'(N). \tag{11}
$$

Second, if $N = N_p^*$, $g_p(N) = sN$ and $W_p'(N) = k$, so the envelope condition reduces to

$$
(r + s + \lambda)k = (p - z)\big(1 - e^{-\alpha u(N_p^*)}\big) + \lambda \mathbb{E}_p W_{p'}'(N_p^*). \tag{12}
$$

Finally, if $N < N_p^*$, entry immediately drives N up to N_p^*.

4.2 Decentralization

Before discussing how to solve for the targets N_p^*, I briefly mention how to decentralize the planner's solution. Whenever a job is filled by a worker with no opportunities among the vacancies, the worker is paid her value of leisure z. When a job is filled by a worker with at least one opportunity, the worker receives her marginal product $p(t)$. For example, if a single firm enters and hires a worker from the stock of unemployed, that worker will, at least initially, be paid the value of leisure; however, if firm later creates a suitable job vacancy that goes unfilled, the wage will later increase.

To see that this decentralizes the optimum, recall from equation (5) that the probability job j is filled by a worker with no other job opportunities is $\delta_N = 1 - e^{-\alpha u(N)}$, independent of the age of the job. Then let $J_p(N)$ denote the expected value of a job, vacant or filled, when the aggregate state is (p, N). If $N > N_p^*$, there is no job creation and so $\dot{N} = -sN$. Then

$$
rJ_p(N) = (p - z)\big(1 - e^{-\alpha u(N)}\big) - sJ_p(N) - J_p'(N)sN + \lambda \mathbb{E}_p\big(J_{p'}(N) - J_p(N)\big). \tag{13}
$$

The current value of a job is $p - z$ if it is filled by a worker with no opportunities among the vacancies and zero otherwise. The job ends at rate s, the number of other jobs decreases at rate sN, and the aggregate state may change at rate λ. Alternatively, if $N = N_p^*$, job creation balances job destruction and the expected value of a job must equal to the creation cost k. Then the Bellman equation is

$$rk = (p - z)\big(1 - e^{-\alpha u(N_p^*)}\big) - sk + \lambda \mathbb{E}_p\big(J_{p'}(N_p^*) - k\big). \tag{14}$$

Finally, if $N < N_p^*$, entry drives N up to N_p^* immediately. Note that in this case firms do not care about the order in which their jobs enter. Although this may affect the probability of hiring a worker, if two jobs could have hired the same worker but one job is left vacant, the other job is forced to pay the high wage p.

To prove that these wages decentralize the social optimum, simply note that if $J_p(N) \equiv W_p'(N)$, equation (11) is equivalent to equation (13) and equation (12) is equivalent to equation (14). A version of the Mortensen (1982) rule is optimal: to give firms the proper incentive to enter the market, they must receive the full marginal product of a filled job when the worker would otherwise be unemployed and nothing otherwise.

4.3 Solution Method

In what follows I assume that productivity is an increasing function of a latent variable y, p_y, while the latent variable in turn follows a homoskedastic first-order autoregressive process and lives in a countable set Y. This ensures that the thresholds N_p^* are increasing in p. For notational simplicity, define the value functions and thresholds directly in terms of the latent variables: $\mathcal{N}_y^* \equiv N_{p_y}^*$ and $\mathcal{J}_y^*(N) \equiv J_{p_y}^*(N)$.

More precisely, let

$$Y \equiv \{-n\Delta, -(n-1)\Delta, \ldots, 0, \ldots, (n-1)\Delta, n\Delta\},$$

where $\Delta > 0$ is the step size and $2n + 1 \geq 3$ is the number of grid points. When a shock hits, at rate λ, the new value y' either moves up or down by one grid point:

$$y' = \begin{cases} y + \Delta \\ y - \Delta \end{cases} \text{with probability} \begin{cases} \frac{1}{2}\left(1 - \frac{y}{n\Delta}\right) \\ \frac{1}{2}\left(1 + \frac{y}{n\Delta}\right) \end{cases}. \tag{15}$$

Note that although the step size is constant, the probability that $y' = y + \Delta$ is smaller when y is larger, falling from 1 at $y = -n\Delta$ to zero at $y = n\Delta$. One can prove that y follows a first-order autoregressive process with drift $-\gamma y$ where $\gamma = \lambda/n$ and instantaneous variance $\sigma^2 = \lambda\Delta^2$. In the limit as $n \to \infty$, the stochastic process for y converges to an Ornstein-Uhlenbeck process. See Shimer (2005) for details.

To solve the model, I start with the smallest value $y = -n\Delta$ with associated threshold $\mathcal{N}^*_{-n\Delta}$. Following an aggregate shock, productivity increases by one step with certainty and so the target number of job increases discretely. If the number of jobs was at the target $\mathcal{N}^*_{-n\Delta}$ before the shock, the marginal value of a job is k both before and after the shock, $\mathcal{J}_{-n\Delta}(\mathcal{N}^*_{-n\Delta}) = \mathcal{J}_{-(n-1)\Delta}(\mathcal{N}^*_{-n\Delta}) = k$. Then the envelope condition (14) reduces to

$$(r+s)k = (p_{-n\Delta} - z)\big(1 - e^{-\alpha u(\mathcal{N}^*_{-n\Delta})}\big). \tag{16}$$

Solve this explicitly for $\mathcal{N}^*_{-n\Delta}$.

Now compute the remaining thresholds by induction. For any $y > -n\Delta$, $y \in Y$, suppose we have computed $\mathcal{J}_{y'}(\mathcal{N}^*_{y-\Delta})$ for all $y' < y$, $y' \in Y$. Equation (13) implies that for $N \in [\mathcal{N}^*_{y-\Delta}, \mathcal{N}^*_y]$ and $y' < y$,

$$\begin{aligned}(r+s+\lambda)\mathcal{J}_{y'}(N) = (p_{y'} - z)\big(1 - e^{-\alpha u(N)}\big) - \mathcal{J}'_{y'}(N)sN \\ + \frac{\lambda}{2}\left(1 + \frac{y'}{n\Delta}\right)\mathcal{J}_{y'-\Delta}(N) + \frac{\lambda}{2}\left(1 - \frac{y'}{n\Delta}\right)\mathcal{J}_{y'+\Delta}(N)\end{aligned}$$

with $\mathcal{J}_y(N) = k$ by the free entry condition. Using the terminal conditions provided, solve this system of differential equations for $\mathcal{J}_{y'}(N)$, $N \in [\mathcal{N}^*_{y-\Delta}, \mathcal{N}^*_y]$ for all $y' < y$, $y' \in Y$. Finally, equation (14) gives

$$(r+s)k = (p_y - z)\big(1 - e^{-\alpha u(\mathcal{N}^*_y)}\big) + \frac{\lambda}{2}\left(1 + \frac{y}{n\Delta}\right)\big(\mathcal{J}_{y-\Delta}(\mathcal{N}^*_y) - k\big),$$

where I use the fact that $\mathcal{J}_{y+\Delta}(\mathcal{N}^*_y) = k$ to eliminate the term coming from a positive shock. We can solve this for $\mathcal{N}^*(y)$. This gives all the terms needed for the next induction step.

5 Calibration

I calibrate the model to match salient facts about the U.S. economy. The model is in continuous time and so I normalize a time period to represent a quarter. I set the quarterly discount rate at $r = 0.012$ and the separation rate at $s = 0.1$ (Shimer, 2005). I fix $\alpha = 19.2$

to match the location of the Beveridge curve (see Section 3.2). For the productivity process, I let

$$p_y = e^y + (1 - e^y)\left(z + \frac{(r+s)k}{1 - e^{-\alpha}}\right).$$

That is, $p_y - z - \frac{(r+s)k}{1-e^{-\alpha}}$ follows a geometric random walk. The lower bound on productivity ensures that, even in the worst possible state, $y = -n\Delta$, the unemployment rate stays between 0 and 1; see equation (16). At the mean value of $y = 0$, I normalize productivity to $p_0 = 1$ and set the value of leisure to $z = 0.4$. As in the search model, this is a critical parameter for the volatility of aggregate productivity (Hagedorn and Manovskii, 2005). I set $k = 3.56389$, which implies a 5.7 percent unemployment rate in the deterministic steady state with $p = 1$; this matches the mean unemployment rate during the post-war period.

I allow for 2001 productivity states ($n = 1000$) and verify that my results are insensitive to this choice. I calibrate the remaining parameters to match moments in the U.S. labor productivity process, $\lambda = 86.6$ and $\Delta = 0.00634$. These parameters, or more precisely $\gamma = \lambda/n = 0.0866$ and $\sigma = \Delta\sqrt{\lambda} = 0.059$, determine the autocorrelation and standard deviation of y.

To characterize the equilibrium, I start by computing the type-dependent thresholds N_p^*. I then choose an initial value for $y(0)$ and $N(0)$ and select the timing of the first shock t, an exponentially-distributed random variable with mean $1/\lambda$. I then compute the number of jobs at time t: if $N(0) \leq e^{st}\mathcal{N}_{y(0)}^*$, $N(t) = \mathcal{N}_{y(0)}^*$; otherwise, $N(t) = e^{-st}N(0)$ as the number of jobs decays with exits. I also compute the number of unemployed workers who finds jobs during the interval $[0, t]$, $m(t)$:

- if $N(0) < \mathcal{N}_{y(0)}^*$,

$$m(t) = u(N(0)) - u(\mathcal{N}_{y(0)}^*) + s\mathcal{N}_{y(0)}^*\big(1 - e^{-\alpha u(\mathcal{N}_{y(0)}^*)}\big)t.$$

  $u(N(0)) - u(\mathcal{N}_{y(0)}^*)$ unemployed workers find jobs immediately as the number of jobs jumps up to $\mathcal{N}_{y(0)}^*$. During the remaining t periods, jobs enter to balance exits, at rate $s\mathcal{N}_{y(0)}^*$, and hire an unemployed worker if one is suitable, with probability $1 - e^{-\alpha u(\mathcal{N}_{y(0)}^*)}$.

- if $\mathcal{N}_{y(0)}^* \leq N(0) \leq e^{st}\mathcal{N}_{y(0)}^*$,

$$m(t) = s\mathcal{N}_{y(0)}^*\big(1 - e^{-\alpha u(\mathcal{N}_{y(0)}^*)}\big)\left(t - \frac{\log(N(0)/\mathcal{N}_{y(0)}^*)}{s}\right).$$

For the first $\log\left(N(0)/\mathcal{N}^*_{y(0)}\right)/s$ periods, there is no entry and so no unemployed workers find jobs. Thereafter, unemployed workers find jobs when a job enters to balance an exit and finds a suitable unemployed worker.

- if $N(0) > e^{st}\mathcal{N}^*_{y(0)}$, $m(t) = 0$ since there is no entry before time t.

I next choose the value of $y(t)$ according to equation (15), find the timing of the next shock, compute the incremental number of unemployed workers who find a job before the shock hits, and repeat.

At the end of each month (1/3 of a period), I record unemployment, vacancies, cumulative matches, and productivity. I measure the job finding probability F for unemployed workers as the ratio of the number of matches during a month to the number of unemployed workers at the start of the month. I throw away the first 25,000 years of data to remove the effect of initial conditions. Every subsequent 53 years of model-generated data gives one sample. I take quarterly averages of monthly data and express all variables as log deviation from an HP filter with parameter 10^5, a relatively low frequency filter.[2] I create 100,000 samples in order to precisely estimate model moments and bootstrap model standard errors. I compare these results with the U.S. data reported in Shimer (2005), Table 1, and repeated here in Table 1 for convenience.

6 Results

Table 2 summarizes the model generated data. The last column shows the driving force, labor productivity. By construction, I exactly match the standard deviation and quarterly autocorrelation in U.S. data. The remaining numbers reflect results driven by the structure of the model. As expected, the model delivers a strong negative correlation between unemployment and vacancies; the correlation is imperfect only because of nonlinearities. Vacancies are somewhat more volatile than unemployment both in the model and in the data, with the model explaining about 35 percent of the observed fluctuations in both the v-u ratio and the job finding probability. Mortensen and Nagypal (2005) argue that productivity shocks in fact should not explain all of the observed fluctuations in the v-u ratio since the empirical

[2] In Shimer (2005), I record the quarterly data directly. Taking quarterly averages of monthly observations makes the model-generated data more similar to U.S. data. The main effect that this has is to increase the autocorrelation of variables by reducing high-frequency noise. I offset this here by using a lower autocorrelation for productivity.

Summary Statistics, quarterly U.S. data, 1951 to 2003

		U	V	V/U	F	p
Standard Deviation		0.190	0.202	0.382	0.118	0.020
Quarterly Autocorrelation		0.936	0.940	0.941	0.908	0.878
	U	1	−0.894	−0.971	−0.949	−0.408
	V	—	1	0.975	0.897	0.364
Correlation Matrix	V/U	—	—	1	0.948	0.396
	F	—	—	—	1	0.396
	p	—	—	—	—	1

Table 1: Seasonally adjusted unemployment U is constructed by the BLS from the CPS. Seasonally adjusted Help Wanted Index V is constructed by the Conference Board. Job finding probability $F_t \equiv 1 - (U_{t+1} - U^s_{t+1})/U_t$ is constructed from seasonally adjusted unemployment (U_t) and short-term unemployment (U^s_t) data, computed by the BLS from the CPS and corrected for CPS redesign in 1994. U, V, and F are quarterly averages of monthly data. p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviation from an HP trend with smoothing parameter 10^5.

correlation between productivity and the v-u ratio is only 0.4; by their metric, the stock-flow matching model explains nearly all of the productivity-induced fluctuations in the v-u ratio.

The model's weakest point is the low autocorrelation of the job finding probability, or equivalently the low correlation between this variable and unemployment and vacancies. The empirical autocorrelation is 0.91, while the theoretical correlation is just above 0.5. This low autocorrelation is intrinsic to the structure of the model: the job finding probability fluctuates with the inflow rate into unemployment, i.e. in response to *changes* in the number of jobs. In contrast, vacancies and unemployment depend on the stock of jobs. This leads to a correlation between the job finding probability and both the level and change in the v-u ratio. Coles and Petrongolo (2003) argue that this offers a way to test the stock-flow matching model; however, U.S. data the correlation in levels is remarkably strong. One possible way to reconcile model and data would be to make the marginal cost of job creation increasing in gross job creation g; this should dampen the sharp transitory fluctuations in the job finding probability.

Despite this, the model generates a "reduced-form matching function"—a relationship between the job finding probability and the v-u ratio—that is similar to the one in U.S. data. Empirically, a one percent increase in the v-u ratio is associated with a 0.28 percent increase in the job finding probability. The corresponding theoretical elasticity is about 0.22.

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		U	V	V/U	F	p
Standard Deviation		0.061 (0.008)	0.077 (0.010)	0.137 (0.018)	0.041 (0.004)	0.020 (0.003)
Quarterly Autocorrelation		0.878 (0.030)	0.878 (0.030)	0.878 (0.030)	0.525 (0.086)	0.878 (0.030)
Correlation Matrix	U	1	−0.999 (0.000)	−1.000 (0.000)	−0.716 (0.053)	−0.999 (0.001)
	V	—	1	1.000 (0.000)	0.717 (0.053)	0.995 (0.002)
	V/U	—	—	1	0.717 (0.053)	0.997 (0.002)
	F	—	—	—	1	0.714 (0.052)
	p	—	—	—	—	1

Table 2: Results from simulating the model. All variables are reported as log deviation from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 100,000 model simulations—are reported in parenthesis. See the text for details on the calibration.

Moreover, one can test the constant elasticity assumption both in the theory and the data by regressing the log job finding probability on the log v-u ratio and its square. The quadratic term is insignificant at conventional confidence levels in the data and significant only twice in 100,000 simulations of the model.

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