

# On the Aggregate and Distributional Implications of Productivity Differences across Countries<sup>†</sup>

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February 2006

Abstract

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We develop a quantitative theory of human capital with heterogeneous agents in order to assess the sources of cross-country income differences. The cross-sectional implications of the theory and U.S. data are used to restrict the parameters of human capital technology. We then assess the model's ability to explain the cross-country data. Our quantitative model generates a total-factor-productivity (TFP) elasticity of output per worker of 2.8. This implies that a factor of 3 difference in TFP is amplified through physical and human capital accumulation to generate a factor of 20 difference in output per worker – as observed in the data between rich and poor countries. The implied difference in TFP is in the range of estimates from micro studies. The theory suggests that using Mincer returns to measure human capital understates human capital differences across countries by a factor of 2. The cross-country differences in human capital implied by the theory are consistent with evidence from earnings of immigrants in the United States. We also find that TFP has substantial effects on cross-sectional inequality and intergenerational mobility and that public education policies can have important aggregate and distributional implications.

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Keywords: output per worker, TFP, human capital, heterogeneity, inequality, mobility.

JEL Classification: O1.

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# 1 Introduction

One of the most important challenges faced by economists is to explain the observed large differences in per capita income across countries. In this paper, we develop a quantitative theory of heterogeneous agents to assess the importance of human capital stocks and total factor productivity in accounting for income differences across countries. In order to circumvent the lack of conclusive micro evidence on the parameters of human capital technology – which are crucial for the quantitative implications of the theory – we use cross-sectional data for the United States to indirectly infer these parameters. Building a theory of heterogeneity enables us to systematically compare cross-sectional statistics on earnings and schooling in the model and the data. We then assess the model’s ability to explain broad aspects of the data across countries. In particular, we study economies that differ in their total factor productivity (TFP) and evaluate the quantitative impact of TFP on returns to schooling, quality of schooling, human capital accumulation, and output. Furthermore, our approach allows us to evaluate the distributional consequences of TFP differences across countries, which are yet to be explored by researchers interested in development issues.

We develop a heterogeneous-agent model of physical and human capital accumulation. Every period, physical and human capital are used to produce a single good with a constant-returns-to-scale technology. The economy is populated by overlapping generations of people who live for 5 periods and are altruistic toward their descendants. People are heterogeneous in skills and physical assets and face idiosyncratic (uninsurable) uncertainty about the earnings potential (learning ability) of their descendants. Human capital investment includes children’s time and parental expenditures. Parents cannot borrow to finance investment in

human capital. Parental expenditures on education affect the *quality* of education of their children. The quantitative importance of TFP on income differences across countries hinges on the importance of human capital quality. We use our quantitative theory to measure the importance of this component of human capital. We calibrate our benchmark economy to U.S. data. Our strategy is to restrict parameter values so that the equilibrium of the model matches a set of aggregate and cross-sectional targets for the U.S. economy. In particular, we discipline the importance of human capital quality by restricting the economy to match cross-sectional observations on earnings and schooling in the United States as well as the proportion of goods in the total cost of human capital investment. Since the calibrated benchmark economy produces economic statistics that are consistent with the cross-sectional evidence for the United States, we argue that the model is a good quantitative theory of *within-country* inequality.

Our quantitative model produces a TFP elasticity of output per worker of almost 2.8. This implies that in order for the model to produce a factor of 20 difference in output per worker – as observed in the data between rich and poor countries – a factor of 3 difference in TFP is needed. This implied difference in TFP is in the range of estimates from micro studies (see for instance Prescott, 1998). Hence, relatively small differences in productivity are amplified through physical and human capital accumulation to generate large differences in output per worker across countries. The theory suggests that using Mincer returns to measure human capital understates human capital differences across countries by a factor of 2. Not only the model supports large differences in human capital across countries, but also it implies differences in human capital quality that are consistent with (i) the evidence from earnings of immigrants in the United States, (ii) the cross-country evidence on estimates of

Mincer returns, and *(iii)* the evidence on the relationship between average years of schooling and per-capita income across countries.

Our approach of using a heterogeneous-agent model allows us to study the distributional consequences of TFP differences across countries. Economic theory suggests that TFP affects both the return and the cost of investment in human capital. Hence, whether TFP increases or decreases inequality and mobility across countries is a quantitative question. We find that countries with lower TFP exhibit substantially more cross-sectional inequality and intergenerational persistence of inequality. Furthermore, we show that public education policies also have important aggregate and distributional implications.

We emphasize that, in our analysis, all differences in output per worker are ultimately generated by differences in TFP. As a result, our paper is about the magnitude of TFP differences that could generate the observed income differences in the data and not about whether differences in income are due to either factor accumulation or TFP. In this sense, the policy prescriptions that can be derived from our model are close in spirit to the work of Parente and Prescott (1999, 2000), Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999). We find, however, that differences in TFP are strongly amplified through their effects on human capital accumulation, in particular, on the unmeasured quality of human capital.

Our paper is closely related to Manuelli and Seshadri (2005), who infer human capital differences across countries using life-cycle human capital theory. Our paper differs from Manuelli and Seshadri (2005) in two important respects. First, we restrict the parameters of human capital technology using cross-sectional heterogeneity in the United States at a point in time whereas Manuelli and Seshadri use life-cycle data with a representative-agent

framework. We view the two approaches as complementary in providing measures of human capital stocks across countries. Second, our paper addresses the distributional impact of TFP differences across countries.

The paper proceeds as follows. In the next section we describe a simple human capital accumulation problem to illustrate the main features of our theory. Section 3 describes in detail the economic environment. The benchmark economy is calibrated to U.S. data in section 4 and its properties are discussed in section 5. Section 6 evaluates the aggregate and distributional impact of TFP differences across countries, studies sensitivity of the results, and evaluates the consequences of cross-country differences in public education. The paper concludes in section 7.

## 2 Simple Illustration

In this section, we consider a simple model of human capital accumulation in order to illustrate that the quantitative implications of TFP differences across countries hinge on the specification of human capital technology. We also use this simple framework to motivate our approach of using cross-sectional heterogeneity within a country to restrict the parameters governing human capital accumulation. Consider an economy populated by an infinitely lived representative household with standard preferences over consumption. The household is endowed with one unit of productive time each period and a positive level of human capital at date 0. At each date, output is produced with a technology linear in human capital services, with total factor productivity  $A$ . Assume that human capital accumulation requires time  $s$  and expenditures in education  $e$  as inputs. In order to simplify the analysis,

we assume that human capital depreciates fully during the period.

A planner then solves the following problem:

$$\max_{\{c_t, e_t, s_t, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

s.t.

$$c_t + e_t = Ah_t(1 - s_t), \quad t = 0, 1, \dots$$

$$h_{t+1} = (s_t^\eta e_t^{1-\eta})^\xi, \quad t = 0, 1, \dots$$

$$c_t, e_t \geq 0, \quad s_t \in [0, 1], \quad t = 0, 1, \dots$$

with  $\xi, \eta \in (0, 1)$  and  $h_0 > 0$  given. The steady-state solution to this problem has a simple form:

$$h_s = \left\{ \left[ \frac{A(1-\eta)}{\eta} \right]^{(1-\eta)\xi} \frac{\beta\eta\xi}{1+\beta\eta\xi} \right\}^{\frac{1}{1-(1-\eta)\xi}}.$$

The steady-state level of human capital depends positively on the TFP parameter  $A$  as long as the elasticity of expenditures in human capital accumulation is positive, which requires a time share  $\eta$  less than 1. Intuitively, when human capital accumulation only requires time inputs ( $\eta = 1$  which implies  $e_t = 0$ ), the level of TFP equally affects the return and cost of human capital and, as a result, it does not affect human capital accumulation.

**Cross-country Implications** In order to make comparisons across countries, we need to make some assumptions regarding the parameters of the model. Assuming that the values of parameters  $\xi$ ,  $\eta$ , and  $\beta$  are the same across countries, relative output per worker between

any two countries  $i$  and  $j$  is

$$\frac{y_i}{y_j} = \underbrace{\left(\frac{A_i}{A_j}\right)}_{\text{direct}} \times \underbrace{\left(\frac{A_i}{A_j}\right)^{\frac{(1-\eta)\xi}{1-(1-\eta)\xi}}}_{\text{indirect}} = \left(\frac{A_i}{A_j}\right)^{\frac{1}{1-(1-\eta)\xi}}.$$

Therefore, TFP differences across countries have a direct impact on output per worker and an indirect impact through human capital accumulation. The TFP elasticities of human capital and output per worker are  $\frac{(1-\eta)\xi}{1-(1-\eta)\xi}$  and  $\frac{1}{1-(1-\eta)\xi}$ . These elasticities are determined by the elasticity of expenditures in the human capital production technology,  $(1-\eta)\xi$ . The quantitative importance of this elasticity should not be overlooked. In order for the model to produce a factor of 20 difference in output per worker between any two countries, a factor of 3.3 difference in TFP between these countries would be needed if  $(1-\eta)\xi = 0.6$  (i.e., TFP elasticity of output per worker equal to 2.5), while a factor of 1.35 difference in TFP would be needed if  $(1-\eta)\xi = 0.9$  (i.e., TFP elasticity of output per worker equal to 10).<sup>1</sup> Notice that when human capital investment consists only of time ( $\eta = 1$ ), TFP does not affect human capital accumulation and differences in TFP translate one-to-one into differences in output per worker. Hence, the quantitative importance of TFP on output per worker hinges on the expenditure elasticity of human capital.

**Cross-sectional Heterogeneity** We have shown that the expenditure elasticity of human capital,  $(1-\eta)\xi$ , determines the quantitative impact of TFP on human capital accumulation and output per worker. More generally, the quantitative implications of TFP differences also depend on the share of time and goods used as inputs to human capital. The quantitative

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<sup>1</sup>Recall that a factor of 20 difference in output per worker is roughly the difference in the data between the richest and poorest 10 percent of the countries (see Heston, Summers, and Aten, 2002).

analysis in Section 5 uses estimates from Kendrick (1976) to restrict the expenditure share. Unfortunately, there is no conclusive micro evidence on the parameters determining the elasticity parameters of human capital accumulation ( $\eta$  and  $\xi$ ).<sup>2</sup> In light of this difficulty, and following Erosa and Koreshkova (forthcoming), we build a framework with heterogeneous agents and use the cross-sectional implications of the theory in order to parameterize the human capital technology. To motivate this approach, notice that in a competitive decentralization of the above planner's problem, log-earnings of a person with human capital  $h$  is:

$$\log(Ah) = b_0 + \frac{1}{1 - (1 - \eta)\xi} \log(s),$$

where  $b_0 = \log(A) + \frac{(1-\eta)\xi}{1-(1-\eta)\xi} \log\left(\frac{A(1-\eta)}{\eta}\right)$  is a constant and  $\frac{1}{1-(1-\eta)\xi}$  represents the schooling elasticity of income. The theory thus implies that the parameters determining the amplification effect of cross-country TFP differences also determine the schooling elasticity of income. This is important because following the influential work of Mincer (1974), there is a large number of empirical studies estimating the effects of schooling on income. The above representative-agent framework does not impose restrictions on the relationship between schooling and income because schooling and income do not vary across individuals. The task is then to build a quantitative theory with heterogeneity in schooling and earnings that can be matched to the data. Then, the cross-sectional implications of the theory can be compared to U.S. data. In particular, we can compare the schooling elasticity of earnings in our quantitative theory with the findings in empirical studies of the U.S. economy.<sup>3</sup>

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<sup>2</sup>The survey in Browning, Hansen, and Heckman (1999) suggests a wide range of estimates from micro evidence for  $\eta\xi$ , between 0.5 and almost 1. Similarly, there is wide variation in estimates for the individual shares of time and goods.

<sup>3</sup>In this simple framework, the Mincer return –which is the change of log wages on years of schooling– can be calculated using the Chain Rule as the derivative of log wages on log schooling times the derivative



We end this section by noting that the intercept term  $b_0$  in the log-earnings and schooling relationship in this simple model depends on the TFP parameter  $A$ . Thus, ignoring this intercept term in measuring human capital stocks across countries (i.e., using only Mincer returns to estimate human capital across countries) can produce misleading results. Since differences in the intercept term can be broadly interpreted as capturing differences in quality of education across countries, measures of human capital using Mincer returns do not capture all differences in human capital across countries (see for instance the discussion of this issue in Klenow and Rodriguez-Clare, 1997). We use our quantitative framework to evaluate the importance of this omission in estimates of human capital stocks across countries using Mincer regressions. In the next section, we present a model with heterogeneous agents and human capital accumulation that builds on the basic insights from this section.

### 3 Economic Environment

We develop a quantitative general-equilibrium heterogeneous-agent model of physical and human capital accumulation in order to study the implications of TFP differences on inequality, mobility, and output per worker across countries. We consider an economy populated by overlapping generations of people who are altruistic toward their descendants. People are heterogeneous in skills and physical assets and face idiosyncratic (uninsurable) uncertainty about their labor earnings. Investment in human capital involves the investment of children's time and expenditures by parents that affect the quality of the human capital of their children. Parents cannot borrow to finance investment in human capital. Since we focus on

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of log schooling on schooling. As a result, the Mincer return is given by  $\frac{1}{(1-(1-\eta)\xi)} \frac{1}{s}$ .

steady states, we omit time subscripts in the description of the model. We denote with a prime variables corresponding to the period following the current period.

**Production Technologies** Output is produced with a constant returns to scale technology,

$$Y = AK^\alpha H^{1-\alpha}, \quad 0 < \alpha < 1, \quad (1)$$

where  $Y$  denotes output,  $K$  represents physical capital services,  $H$  stands for aggregate human capital services, and  $A$  is total factor productivity (TFP). Output can be consumed  $C$ , invested in physical capital  $X$ , and invested in human capital  $E$ . Feasibility requires  $C + X + E = Y$ . Physical capital is accumulated according to

$$K' = (1 - \delta)K + A_k X, \quad A_k \leq 1,$$

where  $A_k$  is a parameter determining the productivity of investment in physical capital (i.e., the effectiveness with which current period output can be transformed into capital available for production in the following period). The aggregate human capital  $H$  is given by the sum of human capital services across all individuals (labor is supplied inelastically). We discuss how human capital is accumulated when presenting the decision problem of the household.

**Market Structure** Firms take factor prices as given and maximize profits by choosing the demand for factor inputs:

$$\max_{K, H > 0} \{AK^\alpha H^{1-\alpha} - wH - (r + \delta)K\}. \quad (2)$$

**Demographic Structure** There is a large number of dynasties (mass one). The economy is populated by overlapping generations of people who live for 5 periods and are altruistic toward their descendants. The model period is set to 16 years, which is roughly the total number of years spent on education by a person with a college degree. People live three periods as adults and two periods as children. Panel A of Table 1 summarizes the demographic structure in the model and the mapping between age in the model and real age in the data. In order to match individual life expectancy in the model to that observed in the United States, we introduce an exogenous probability of survival from period 4 to period 5,  $\phi$ . A household is composed of a parent-child pair in the first two stages and a retired adult in the last stage. These three stages of the life cycle of households are described in Panel B of Table 1.

Table 1: Demographic Structure and Life-cycle Stages of Households

Panel A: Demographic Structure

Model Age	Real Age	Name
1	6-21	child
2	22-37	old child
3	38-53	young adult
4	54-69	old adult
5	70-85	retired adult

Panel B: Life-cycle Stages of Households

Stage	Adult	Adult's Age	Child	Child's Age
1	young	38-53	child	6-21
2	old	54-69	old child	22-37
3	retired	70-85	—	—

**Decision Problem of the Household** All decisions of the household are made by the parent. We assume that markets are imperfect in that households cannot perfectly insure against labor market risk and they cannot borrow. The state of a young parent is given by a triple  $(z, h, q)$ : earnings ability  $z$ , human capital  $h$ , and parental transfer  $q$  received from the previous household in the dynasty line. Households maximize discounted lifetime utility of all future generations in the dynasty. Young parents choose consumption  $c_y$ , assets  $a'_y$ , time spent in school by their children  $s$  (where  $1 - s$  is working time of the children), and resources spent on the quality of education of their children  $e$ . A parent who provides his child with  $s$  years of schooling and a quality of education  $e$  incurs expenditures of  $e + (w\bar{l} - p)s$ , where  $w\bar{l}$  is a cost per year of education (which is assumed to depend on the market wage rate) and  $p$  denotes public education expenditures (or subsidies) per year of education. We take a broad view of human capital and interpret the quality of education  $e$  as including non-education expenditures (such as child-rearing and health care) that enhance future earnings of children. Because we abstract from investment in human capital on the job, we capture the life-cycle growth in wages by assuming exogenous life-cycle productivity parameters  $(\psi_c, \psi_y, \psi_o)$  for children, young adults, and old adults. The productivity of old children is normalized to one.

Young parents face uncertainty regarding the ability of their children  $z'$ . Human capital of children is given by

$$h' = z' (s^\eta e^{1-\eta})^\xi,$$

where  $z'$  evolves according to a discrete Markov transition matrix  $Q(z, z')$  and is realized in the second stage of the household's life cycle.

In the second stage, the household consists of a child with earnings  $wh'$  and a parent with earnings  $\psi_owh$ . Old parents decide savings for retirement  $a'_o$ , consumption  $c_o$ , and an intergenerational transfer  $q'$  for the next household in the dynasty. Retired people consume their savings.

The decision problem of a young household can be written using the dynamic programming language as follows:

$$v(z, h, q) = \max_{c_y, e, s, h', a'_y, [c_o, c_r, a'_o, q'](z')} \left\{ U(c_y) + \beta \sum_{z'} Q(z, z') [U(c_o) + \beta Ev] \right\}, \quad (3)$$

subject to

$$c_y + a'_y + e + (w\bar{l} - p)s = (1 - \tau) [\psi_y wh + w\Psi_c(1 - s) + rq] + q,$$

$$c_o(z') + a'_o(z') + q'(z') = (1 - \tau) [\psi_owh + wh'(z') + ra'_y] + a'_y,$$

$$c_r(z') = (1 - \tau)ra'_o(z') + a'_o(z'),$$

$$e + (w\bar{l} - p)s \geq 0,$$

$$\Psi_c = \psi_c (s^\eta e^{1-\eta})^\xi,$$

$$h' = z' (s^\eta e^{1-\eta})^\xi,$$

$$a_y, a_o, q'(z') \geq 0, \quad s \in [0, 1],$$

where

$$Ev = [\phi (U(c_r) + v(z', h', q')) + (1 - \phi)v(z', h', q' + c_r)].$$

The parameter  $\phi$  is the probability of survival for a retired adult. Since old parents know

the ability of their children when making consumption, saving, and bequest decisions, these choices are expressed as contingent on their children's ability  $z'$  in the dynamic programming problem of young parents. We denote by  $g^i(z, h, q)$  for  $i = \{c_y, e, s, h', a'_y\}$ ,  $g^j(z, h, q; z')$  for  $j = \{c_o, c_r, a'_o, q'\}$  the decision rules implied by (3).

The decision rules of households and the transition matrix  $Q$  imply a mapping from the distribution of adult households in a given period to the distribution of adult households two periods later (since a new household is formed every two periods in a dynasty line):

$$\mu'(z', h', q') = T(\mu(z, h, q)), \quad \forall(z, h, q). \quad (4)$$

**Public Education** Public education expenditures are financed with a proportional tax  $\tau$  on household's income. Public and private expenditures are perfect substitutes in the production of human capital.

**Definition of Equilibrium** A stationary recursive competitive equilibrium is a list of functions:  $v(z, h, q)$ ,  $g^i(z, h, q)$  for  $i = \{c_y, e, s, h', a'_y\}$ ,  $g^j(z, h, q; z')$  for  $j = \{c_o, c_r, a'_o, q'\}$  for adult households, a distribution function  $\mu(z, h, q)$ , demand of factor inputs by firms  $K^d$ ,  $H^d$ , prices  $w$  and  $r$ , and government expenditures in education  $p$ , such that (i) Given prices and  $p$ ,  $v$  solves (3) and the implied policy functions from this problem  $g$  are optimal; (ii) Given prices,  $K^d$  and  $H^d$  solve the firm's problem in (2); (iii)  $\mu$  is time invariant satisfying (4); (iv) the government budget balances  $p \int g^s(z, h, q) d\mu(z, h, q) = \tau Y$  and (v) markets clear,

letting  $x = (z, h, q)$  :

$$\int \left( q + g^{a'_y}(x) + \sum_{z'} Q(z, z') g^{a'_o}(x; z') \right) d\mu(x) = K^d,$$

$$\int \left[ (\psi_y + \psi_o)h + \Psi_c(x)(1 - g^s(x)) + \sum_{z'} Q(z, z') g^{h'}(x; z') \right] d\mu(x) = H^d.$$

## 4 Calibration

As discussed in Section 2, the aggregate implications of TFP differences across countries in our model hinge on the parameters determining human capital accumulation. Our calibration strategy is to restrict these parameters using cross-sectional heterogeneity of schooling and earnings in the data for the United States.

### 4.1 Parameters and Targets

We calibrate our benchmark economy (B.E.) to data for the United States. We assume a period is 16 years. Because we are interested in comparisons across countries, the level of technology in our benchmark economy is effectively a normalization. Therefore, we set  $A = A_k = 1$ . The mapping between parameters and targets in the data is multidimensional, and we thus solve for parameter values jointly. We divide the discussion of calibration into parameters that relate to preferences, demographics, and production of goods and parameters that relate to human capital accumulation. A summary of parameter values and data targets is provided in Table 2.

Table 2: Parameters and Data Targets

Parameter		Value	Target	U.S.	B.E.
CRRA	$\sigma$	2	Empirical literature	—	—
Discount factor	$\beta^{1/16}$	0.94	Interest rate (%)	5	5
Survival probability	$\phi$	0.4	Life expectancy at birth (years)	76	76
Capital share	$\alpha$	0.33	Capital income share	0.33	0.33
Annual depreciation	$\delta$	0.07	Investment to output	0.2	0.2
H.C. time share	$\eta$	0.66	Share of labor in total ed. cost	0.9	0.9
H.C. RTS	$\xi$	0.79	Mincer returns to schooling (%)	10	10
Schooling cost	$\bar{l}$	0.89	Average years of schooling	12.9	12.9
Tax rate on income	$\tau$	0.039	Public Education (% of GDP)	3.9	3.9
Child's productivity	$\psi_c$	0.13	Percentage with college degree	24	24
Young adult's productivity	$\psi_y$	1.4	Relative earnings	1.4	1.4
Old adult's productivity	$\psi_o$	1.08	Relative earnings	1.57	1.57
Ability variance	$\sigma_z$	(0.51) <sup>2</sup>	VAR(log-earnings)	0.36	0.36
Ability correlation	$\rho_z$	0.17	CORR(log-earnings)	0.5	0.5

**Preferences, Demographics, and Production of Goods** We set the relative-risk-aversion parameter  $\sigma$  to 2. There is not a direct empirical counterpart for this parameter in the empirical literature since our model period is 16 years and there is an infinite inter-temporal substitution of consumption within a period. However, we consider a value of  $\sigma$  that is in the range of values considered in quantitative studies with heterogeneous agents. (See Keane and Wolpin, 2001 and Restuccia and Urrutia, 2004 for discussions of these estimates.) The discount factor  $\beta$  is set to target an annual interest rate of 5 percent which is roughly the return on capital in the U.S. economy. (See Poterba, 1997.)<sup>4</sup> In our model, retired adults live until age 85. The National Center for Health Statistics (2004) reports that in 1990 the average life expectancy at birth in the United States was 76 years. Therefore, we calibrate the probability of survival for retired adults ( $\phi$ ) to 0.4 so that the life expectancy at birth in

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<sup>4</sup>Average return on non-financial corporate capital net of taxes in 1990-96.



our model matches 76 years. The capital-share parameter is set to 0.33, consistent with the capital income share in the U.S. economy from the National Income and Products Accounts. The depreciation rate  $\delta$  is selected to match an investment to output ratio of 20 percent as documented in the Economic Report of the President (2004).<sup>5</sup>

**Human Capital Accumulation** Recall that the human capital technology is given by  $h' = z' (s^\eta e^{1-\eta})^\xi$ , where  $s$  denotes schooling time and  $e$  denotes educational expenditures. We need to specify two elasticity parameters:  $\eta$  and  $\xi$ . Ability follows an  $AR(1)$  process (in logs):

$$\log(z') = \rho_z \log(z) + \epsilon_z,$$

where  $\epsilon_z \sim N(0, \sigma_z)$ . In our computations, we approximate this stochastic process with a discrete first-order Markov chain that takes 7 possible values for ability  $z$ . We use the approximation procedure in Tauchen (1986) to compute transition probabilities. This procedure involves selecting two additional parameter values:  $\rho_z$  and  $\sigma_z$ . There are five additional parameters affecting human capital accumulation: Schooling cost  $\bar{l}$ , tax rate on income  $\tau$  (that determines public education subsidies in equilibrium  $p$ ), and life-cycle productivity parameters  $(\psi_c, \psi_y, \psi_o)$  affecting relative labor earnings of children, young adults, and old adults. Our calibration procedure restricts the values of these 9 parameters so that the equilibrium of the model matches the following 9 targets from U.S. data:

1. Intergenerational correlation of log-earnings of 0.5 from Mulligan (1997). (See also excellent surveys of the empirical literature on the intergenerational correlation of

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<sup>5</sup>We obtain a similar target if instead we take the average of the investment to output ratio in the PWT6.1 for the period 1990 to 1996, see Heston, et al., 2002.

- earnings by Stokey, 1998 and Solon, 1999.)
2. Variance of log permanent earnings of 0.36. (See Mulligan 1997 and 1999.)
  3. Average years of schooling of 12.9 from the U.S. Department of Education (2004) in 1990. (See also Barro and Lee, 1996).
  4. The distribution of people across education categories in 1990 as follows: 24 percent of people with a college degree or more from the Historical Tables of the U.S. Census Bureau (2004).
  5. Public education expenditures as a fraction of GDP of 3.9 percent from the Statistical Abstract of the United States (1999). In computing this statistic in the data, we treat as public expenditures all state and federal expenditures. We exclude public local expenditures in education because these expenditures are closely tied to property values and therefore to the income of parents. (See Restuccia and Urrutia, 2004 for a discussion.)
  6. The ratio of earnings for full-time, year-round workers of ages 35-54 to ages 25-34 of 1.40 in 2003 from the U.S. Census Bureau, Historical Income Tables.
  7. The ratio of earnings for full-time, year-round workers of ages 55-64 to ages 25-34 of 1.57 in 2003 from the U.S. Census Bureau, Historical Income Tables.
  8. Mincer returns to schooling of 10 percent. Heckman et al. (2005) report a Mincer return of between 10 to 13 percent during the period 1980 to 1990. Psacharopoulos (1994) estimates a Mincer return of 10 percent for the United States for the period

1990-95. Because Psacharopoulos also provides data on Mincer returns for a large set of countries, we follow Bils and Klenow (2000) in using Psacharopoulos' estimate for the U.S. economy. In our model, we measure returns to education by regressing log-wages on years of education:

$$\log(wh'_i) = b_0 + b_1 (16s_i) + u_i,$$

where  $b_1$  gives the Mincer returns to schooling in our economy.

9. The share of labor inputs in the total cost of investment in education. Kendrick (1976) estimates this share to be 90 percent for the U.S. economy.

## 4.2 Discussion

In our model, heterogeneity in earnings across people arises from uninsurable idiosyncratic earnings (ability) shocks. The cross-sectional inequality in parental resources is partially transmitted to the next generation through unequal investment in human and physical capital. The inequality in parental investment occurs for two reasons: borrowing constraints and heterogeneity in the schedules of expected marginal returns to education. Three parameters characterizing human capital accumulation – returns to scale  $\xi$ , time share  $\eta$ , and a resource cost parameter  $\bar{l}$  – affect the extent to which heterogeneity in parental resources transmits to the offspring generation.

**Returns to Scale** The relationship between earnings and schooling (in logs) across people in our model,  $\log(whz) = \log(w) + \log(z) + \xi[\eta \log(s) + (1-\eta) \log(e)]$ , depends to a large extent

on the returns-to-scale parameter  $\xi$  of human capital accumulation. Although the Mincer returns to schooling (coefficient of log wages on years of schooling) does not correspond exactly with returns to schooling in our model, the average value of the Mincer return increases with  $\xi$ , and therefore it represents a useful target for calibrating the returns-to-scale parameter in the benchmark economy. The parameter  $\xi$  has an asymmetric effect on the returns to schooling at different points of the distribution of education: A higher  $\xi$  increases the return at high levels of human capital investment but it decreases the return at low levels. However, the distribution of education in the benchmark economy is such that the average returns to education increases with  $\xi$ . More importantly, a higher returns-to-scale parameter increases the education expenditure on goods per unit of schooling time, and thus, increases the average percentage wage gain per unit of schooling time. The asymmetric impact of changes in  $\xi$  on human capital investment leads to an increase in the endogenous variation of earnings and the persistence of earnings inequality across generations.

**Time Share** Our model explicitly incorporates a schooling time decision because the best available cross-sectional data on human capital investment is reported in terms of years of schooling. Hence, our calibration of the human capital technology draws on schooling observations. Similar to the returns-to-scale parameter that controls how differences in expected learning ability affect overall investment decisions in human capital, the time elasticity parameter  $\eta$  controls the proportion of investment accomplished via the time input as opposed to the expenditure input. Hence, the time elasticity parameter  $\eta$  is restricted to match the expenditure share of time in education. Notice that a higher  $\eta$  increases the variance of education. The variance of earnings, however, may not increase because the expenditure

share in human capital  $(1 - \eta)$  falls. A higher  $\eta$  reduces the persistence of earnings inequality across generations due to the borrowing constraint since households are homogeneous in their endowment of time.

**Cost of Education and Public Subsidy** In addition to foregone earnings, schooling time has a resource cost:  $\bar{l}$  units of market human capital services per unit of schooling time. A portion of this cost is subsidized by the government at the rate  $p$  per unit of schooling time. An increase in the resource cost of education lowers the desired amount of schooling time for all agents. As a result, our calibration restricts  $p$  and  $\bar{l}$  so that in equilibrium the model reproduces both the fraction of expenditures provided by the government as well as the average level of education in the U.S. economy. Notice that public education has important distributional consequences in our theory since it tends to equalize investment in school across households. Because we use cross-sectional heterogeneity in earnings and schooling to restrict the human capital technology, it is important that we do not abstract from the distributional impact of public education in our benchmark economy.

## 5 Properties of the Benchmark Economy

In this section, we describe relevant statistics in the benchmark economy that were not used as targets in the calibration. We show that the model is consistent with several dimensions of heterogeneity in the data. We conclude that the model is a good quantitative theory of *within*-country heterogeneity.

**Distribution of Schooling** According to the U.S. Department of Education (2004) the proportion of people in 1990 between 25 and 34 years of age (all sexes and races) with primary schooling (1st to 8th grade) as their highest education attainment was 4 percent, with secondary schooling (9th to 12th grade) – 50 percent, and with college education (4 years of college or more) – 24 percent. Our model matches these statistics reasonably well as documented in Table 3.<sup>6</sup>

Table 3: Education and Earnings – Model and Data

	Schooling Dist.		Rel. Earnings		Mincer Ret.(%)	
	Model	Data	Model	Data	Model	Data
Primary	0.08	0.04	0.66	0.62	15.6	21.8
Secondary	0.28	0.50	1.00	1.00	9.3	11.5
College	0.24	0.24	1.78	1.70	10.3	9.6

Mincer returns data from Willis (1986).

**Schooling and Earnings** The model matches the joint distribution of earnings and schooling in the data well. From the U.S. Department of Education (2004), in 1998, relative to earnings of male high-school graduates, males with college education earned, on average, 70 percent more; those with some college earned 17 percent more; and those with primary schooling earned 48 percent less. The respective earnings ratios for the model are reported in Table 3. Recall that the model was calibrated to match the average Mincer returns to education. In Table 3, we also show that the model matches the distribution of Mincer returns in the data reasonably well.

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<sup>6</sup>We note, however, that time in school is a continuous variable in our model, making its comparison with the data non-trivial. In particular, the distribution of schooling in the data has clear spikes at levels of education where an educational degree is completed.

**Expenditures on Education** The share of GDP spent on education increases with the returns to scale parameter in the human capital production function as discussed in our calibration section. In light of that discussion, it is interesting to compare the proportion of GDP in the form of educational expenditures in our model with the data. In the Statistical Abstract of the United States (2005), educational expenditures amount to 7.2 percent of GDP in 1990, where 3.9 percentage points are government expenditures (i.e., including federal and state components but excluding local government expenditures). Haveman and Wolfe (1995) report that expenditures on children aged 0-18 are as large as 14.5 percent of GDP. This share includes not only public investment, but also private costs, such as food, housing, transportation and foregone parental earnings in child care. Parental costs are about 10 percentage points of this total. In our model, total education expenditures correspond to  $(e + w\bar{l}s)$  aggregated over all people. In the benchmark economy, total expenditures on education amount to 12 percent of GDP, a figure that lies in the range cited above.

## 6 Quantitative Results

We use our quantitative theory to assess the aggregate and distributional consequences of TFP differences across countries. Changes in TFP affect human capital accumulation since human capital investment requires goods in our calibrated model economy (see the discussion in Section 2). The question we address in this section is about the *quantitative* magnitude of this effect. We find that TFP has a large effect on human capital accumulation and output even though goods represent only a small proportion of the total cost of education in our benchmark economy (around 10 percent). Moreover, TFP has substantial effects on

economic mobility and inequality within a country.

## 6.1 Aggregate Implications

We assume that countries are identical in terms of preferences and technologies except for their level of TFP in the production of goods. Then, by construction, all cross-country differences in output per worker in our model are generated by differences in TFP. Since TFP has an indirect effect on output per worker through factor accumulation, we investigate the degree to which the impact of TFP on output per worker is amplified by factor accumulation and the relative contribution of physical and human capital accumulation. To illustrate the magnitude of this amplification effect we compare aggregate statistics from these economies. Relative to the benchmark economy, economies with relative TFP levels of 1/2 and 1/3 observe relative output per worker of 1/6 and 1/21 and relative human capital of 1/3 and 1/4. Low relative TFP leads to low average years of schooling and high Mincer returns to schooling.

**Amplification Effect** A simple way of measuring the amplification effect of TFP in our calibrated model economy is as follows. First, note that changes in TFP induce a linear relationship (with slope equal to 1) between log output and log physical capital. This result is a consequence of the fact that in Bewley-type economies (dynastic economies with uninsurable idiosyncratic risk), the equilibrium interest rate is close to the rate of time preference (see for instance Aiyagari, 1994 and Fuster, 2000). As a result, in equilibrium the marginal product of capital is close to the rate of time preference plus the depreciation rate of capital, i.e.,  $\frac{\partial y}{\partial k} = \alpha \frac{y}{k} \approx \rho + \delta$ . Using this relationship to solve for  $k$  as a function



of output we obtain  $k = c_k y$  for some constant  $c_k$ . Second, as indicated in Figure 1, the model implies a linear relationship between log human capital and log output as TFP varies across economies, but the slope of this relationship is less than one. Using this observation, we write human capital as a function of output as  $\log(h) = c_h + \gamma \log(y)$ , which implies that  $h = \exp(c_h)y^\gamma$ . Substituting the expressions derived for  $k$  and  $h$  (in terms of  $y$ ) in the production function of goods and solving for  $y$  we obtain

$$y = c_y A^{\frac{1}{(1-\alpha)(1-\gamma)}}, \quad (5)$$

for some constant  $c_y$ . Then, the TFP elasticity of output per worker in our model is  $\frac{1}{(1-\alpha)(1-\gamma)}$ . In the benchmark economy,  $\alpha = 0.33$  and  $\gamma = 0.46$  (as indicated by the slope coefficient in Figure 1). As a result, the TFP elasticity of output per worker is equal to 2.77. It follows that if TFP differs by a factor of 2 between two economies, the model implies that their output per worker would differ by a factor of  $2^{2.77} = 6.8$ . Another way of expressing this result is to compute the TFP differences required in the model to generate a given difference in output per worker between two countries. From equation (5), the ratio of output per worker between any arbitrary economies  $i$  and  $j$  is related to their relative TFP levels:

$$\frac{y_i}{y_j} = \left( \frac{A_i}{A_j} \right)^{\frac{1}{(1-\alpha)(1-\gamma)}}.$$

Using an elasticity of 2.77 from our previous calculations, it follows that an output ratio of 20 can be generated by a TFP ratio of 2.94. We thus conclude that our calibrated model implies a large amplification effect of TFP differences across countries. Moreover, we note

that the amplification effect provided by physical capital is  $\frac{1}{1-\alpha} = 1.49$  and the one provided by human capital is  $\frac{1}{1-\gamma} = 1.85$ . Human capital thus represents an important source of amplification.

**Human Capital and Mincer Returns** To the extent that schooling quality affects the intercept term in a Mincer regression (as discussed in Section 2), the use of estimated Mincer returns to measure human capital stocks across countries may underestimate differences in human capital across countries. Since Mincer returns are frequently used to measure human capital in growth accounting exercises, it is of interest to assess the importance of this bias using our calibrated model economy. To this end, we use Mincer returns to measure human capital across model economies that differ on their TFP levels. We consider country-specific Mincer returns and allow each year of schooling to have a different return, depending on whether the year of schooling corresponds to primary, secondary, or college education. We add across people using the population share in each schooling category to obtain an aggregate measure of human capital per worker. We report results in Table 4. Whereas the economy with relative TFP of 1/3 has a human capital equal to 0.25 (relative to the benchmark economy), the Mincer measure would imply a human capital of 0.5 (half the difference in our model). We conclude that Mincer returns underestimate human capital differences across countries by a large margin.

Table 4: Human Capital across Economies

Relative TFP	1	1/2	1/3
(1) Human Capital Ratio	1	0.45	0.25
(2) Mincer H.C. Ratio	1	0.69	0.51
Ratio of (2) to (1)	1	1.5	2.0

**Schooling Quality** Our quantitative theory implies that schooling quality is important for understanding differences in human capital and output per worker across countries. This result raises the question: Are the schooling-quality differences implied by our theory reasonable? While there are no reliable cross-country measures of schooling quality, the literature has used the empirical evidence on earnings of immigrants as an indirect approach to measuring human capital differences across countries. Therefore, it is of interest to compare our findings with those of Borjas (1987). Looking at immigrant wages in the United States, Borjas estimates that, on average, the wage that a worker with a given amount of education earns in the United States is 0.12 percent higher when the income per person in the immigrant's country of origin is 1 percent higher.

Table 5 shows that the average earnings of a person in the benchmark economy is between 3.5 and 4 times the average earnings of a similar worker in the economy with relative TFP level of  $1/2$  (depending on the schooling level of the person) and it is more than 7 times the earnings of an equally educated worker in a country with relative TFP level of  $1/3$ . The earnings ratio is largest for people with primary education. The bulk of cross-country earnings differences can be attributed to differences in relative prices. If a person from the economy with relative TFP of  $1/2$  were to migrate to the benchmark economy, the wage rate of this person would increase by a factor of 2.8. If the immigrant comes from an economy with relative TFP of  $1/3$ , his wage rate would increase by a factor of 5.2. Our model is thus consistent with the observed migration pressures from poor to rich countries. On average, immigrants in the benchmark economy would not earn the same as natives with the same school years because of the differences in the quality of schooling (as captured by the expenditure on education goods). Native workers with primary and college education in the

benchmark economy earn between 20 to 40 percent more than potential immigrants with same level of schooling and born in the economy with relative TFP of 1/2. The information in Table 5 can be used to obtain an estimate of the income elasticity of schooling quality (by schooling level) across countries as follows:

$$\eta_{quality,y} = \frac{\log(\frac{H_1}{H_j})}{\log(\frac{Y_1}{Y_j})},$$

where  $H_1$  and  $Y_1$  stand for human capital and per capita income in the benchmark economy (relative TFP of 1) and  $j$  represents a country with relative TFP of  $j$ . When considering potential immigrants with secondary education from economies with relative TFP of 1/2 and 1/3, we obtain schooling-quality elasticities between 0.10 and 0.11 which are close to the 0.12 estimate in Borjas (1987).

Table 5: Quality of Education

Education Level	Earnings Ratio*	Wage Ratio*	Quality Ratio*	Elasticity
Panel A: relative TFP= 1/2				
Primary	4.0	2.8	1.4	0.18
Secondary	3.5	2.8	1.2	0.10
Some college	3.5	2.8	1.2	0.10
Panel B: relative TFP= 1/3				
Primary	10.6	5.2	2.0	0.23
Secondary	7.4	5.2	1.4	0.11
Some college	7.1	5.2	1.4	0.11

\*Ratio of benchmark economy to economies with relative TFP of 1/2 and 1/3.

## 6.2 Sensitivity Analysis

Our baseline calibration targeted an expenditure share of time inputs of 90 percent as reported by Kendrick (1976). We acknowledge that, despite Kendrick’s careful analysis, it may be difficult to accurately measure inputs into human capital accumulation. Since the magnitude of our quantitative results hinge on the importance of time inputs, a sensitivity analysis along this dimension is warranted.<sup>7</sup> To this end, we consider three alternative calibrations to our benchmark economy. In all the calibrated economies, we maintain the calibration targets of our benchmark economy except for the share of time inputs in education costs, which we set at 85, 95, and 100 percent (instead of 90 percent in the baseline calibration). Our goal is to evaluate the effects of TFP under different assumptions about the importance of time inputs in human capital accumulation. We then use data to discriminate among the alternative specifications.

The three calibrated model economies do as well as the benchmark economy in matching the targets discussed in section 4 (see Table 2).<sup>8</sup> In other words, all the economies match the data targets well, including the distributional statistics. Therefore, the economies considered are equally good quantitative theories of the U.S. income distribution. However, there are a number of dimensions where these economies perform differently. We discuss these differences in detail.

To start, we compute the TFP elasticity of output per worker in each of the calibrated model economies and report the results in Table 6. When human capital requires only time

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<sup>7</sup>There is a related discussion in the taxation literature where the tax effect on human capital accumulation hinges on the importance of goods in the production of human capital. For instance, see Trostel (1993) and Davies and Whalley (1989).

<sup>8</sup>The parameter values needed to match the targets are available from the authors upon request.

inputs, the only source of amplification works through physical capital and it implies a TFP elasticity of 1.49. When the share of time inputs is 95 percent, the TFP elasticity of income is 1.84. This statistic increases to 2.8 in our baseline calibration and to 3.63 when time inputs represent 85 percent. Given these elasticity estimates, the amplification effect of TFP differences varies substantially across economies. A factor of 3 difference in TFP implies a factor of 5.1 difference in output per worker in the specification with no goods inputs, whereas it implies a factor of 54 difference in output per worker when the share of time inputs is 85 percent. To put it differently, the TFP ratio needed to generate an output ratio of 20 between two economies is 7.5 in the time-only economy, 2.9 in the baseline calibration, and 2.3 when time inputs are 85 percent.

Table 6: Time Share and Amplification

	Time-share Target (%)			
	100	95	90	85
TFP Elasticity	1.49	1.84	2.77	3.63
Output Ratio	5.1	7.6	20.8	54.0
TFP Ratio	7.5	5.1	2.9	2.3
Schooling-Quality Elasticity:				
Primary	0	0.08	0.18	0.30
Secondary	0	0.07	0.10	0.16

Given a TFP elasticity of output per worker, the output ratio is the one implied by a factor of 3 difference in TFP and the TFP ratio is what it is needed for a factor of 20 difference in output per worker. The schooling-quality elasticity is computed using data for the economies with relative TFP of 1 and 1/2.

We discriminate among the calibrated model economies as follows. For each of the specifications, we obtain observations for average years of schooling and output per worker by simulating economies that vary in their relative levels of TFP. In Figure 2, we plot

cross-country data on schooling and income, taken from Barro and Lee (1996) and Heston, Summers, and Aten (2002), together with data generated by simulating the calibrated model economies. We find that the specification with a time share of 100 percent implies that average years of schooling do not vary with income, an implication that is at odds with the cross-country data. Intuitively, when human capital only requires time inputs, a change in TFP affects equally the benefits and costs of human capital accumulation. When human capital requires goods, however, an increase in TFP increases the benefits proportionally more than the costs of human capital accumulation, leading to an increase in the average years of schooling. Figure 2 also reveals that our baseline calibration (time share of 90 percent) does a good job of reproducing the observed pattern between schooling and income across countries. The economy with a time share of 85 percent also does a good job of reproducing this pattern.

In Figure 3, we plot cross-country data on Mincer returns and schooling, as reported in Psacharopoulos (1994). Note that Mincer returns tend to be low in countries with high average years of schooling. Our simulations reveal that goods inputs in human capital are needed to match the negative association between average years of schooling and Mincer returns across countries. Our baseline calibration does a good job of reproducing the pattern in the data. However, the economies with a time share of 85 and 95 percent also do a good job of reproducing this pattern. The time-only model implies that average years of schooling and Mincer returns do not vary across economies, an implication that is inconsistent with the cross-country data.

Goods inputs in human capital accumulation are also necessary for generating schooling-quality differences across countries. Since in the time-only economy there are no cross-

country differences in schooling quality, potential immigrants would earn the same amount as natives regardless of their country of origin, an implication that is *inconsistent* with the empirical findings of Borjas (1987) and Hendricks (2002) on the relative earnings of immigrants. Recall that, using data from immigrants, Borjas estimated an income elasticity of schooling quality of 0.12. Table 6 reveals that the specification with a time share of 95 percent generates too little cross-country differences in schooling quality relative to Borjas' estimate. In this economy, the schooling-quality elasticity for people with secondary education is 0.07, which is substantially lower than the 0.12 value estimated by Borjas.<sup>9</sup> In our baseline calibration, with a time share of 90 percent, the schooling elasticity of income for people with secondary education is 0.10, a value close to the 0.12 estimate. When the time share is 85 percent, the schooling elasticity of income is 0.16. Overall, we conclude that the data seems to be consistent with a time share closer to 90 percent than to 85 or 95 percent.

### 6.3 Literature Discussion

We discuss our findings relative to important papers in the literature. In particular, we relate our results with those of Bils and Klenow (2000) [hereafter BK], Mankiw, Romer and Weil (1992) [hereafter MRW], Manuelli and Seshadri (2005) [hereafter MS], and Hendricks (2002).

BK argue that MRW may have overstated the importance of human capital in accounting for cross-country income differences by focusing on a one-sector model with no distinction between the production of goods and human capital. Since, according to Kendrick's (1976)

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<sup>9</sup>Recall that most immigrants in the United States have a higher level of education than primary education.



study, time inputs represent 90 percent of the total costs of human capital accumulation, BK consider a two-sector model in which the production of human capital only requires time inputs. Given that in fact education does require some goods (such as computers, books, buildings, paper, and pencils) the following question arises: Is it important to take goods inputs into account when evaluating the consequences of TFP differences across countries? Our findings could not be more striking. By calibrating our benchmark economy to the estimates in Kendrick (1976), with goods accounting for only 10 percent of the cost of human capital investment, we find that human capital still implies a large amplification effect of TFP differences across countries. In fact, the amplification effect in our model is larger than the one implied in MRW.

MRW consider a one-sector growth model with  $Y = C + I_K + I_H = AK^\alpha H^\beta L^{1-\alpha-\beta}$ , where  $\alpha = 0.30$  and  $\beta = 0.28$ . Then, the ratio of output per worker across countries differing in TFP can be expressed as:

$$\frac{y_h}{y_l} = \frac{A_h}{A_l} \left(\frac{A_h}{A_l}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{A_h}{A_l}\right)^{\frac{\beta}{1-\alpha-\beta}} = \left(\frac{A_h}{A_l}\right)^{\frac{1}{1-\alpha-\beta}},$$

where the subscripts  $h$  and  $l$  stand for high and low TFP. In MRW, differences in TFP are amplified by a factor of  $\frac{1}{1-\alpha-\beta} = \frac{1}{1-0.30-0.28} = 2.38$ . Thus, the amplification effect in our baseline calibration is 16 percent larger than the one implied by MRW. This finding may seem paradoxical: While MRW advocate that factor accumulation can account for most of the cross-country income differences, we find that TFP differences of a factor of 3 are needed for explaining the large variation of per capita income across countries. How can we reconcile these findings? The explanation is, as pointed by Klenow and Rodriguez-Clare (1997), that

MRW overstate the cross-country variation in human capital when doing their accounting exercise.<sup>10</sup>

MS use a calibrated model economy to evaluate the importance of human capital for understanding cross-country income differences. Their approach differs from ours in that they use a representative-agent life-cycle model. They assume that all wage growth over the life cycle is due to investment in human capital rather than capital deepening or technological progress. They calibrate the parameters of the human capital technology to match the age profile of wages in the data. This produces a TFP elasticity of output per worker of 6.6, which is substantially larger than the 2.77 elasticity in our baseline calibration.<sup>11</sup> The discrepancy between these elasticities is not minor: While MS find that factor of 20 differences in output per worker can be explained with a TFP difference of 60 percent, our results point to a TFP difference of 200 percent. Alternatively, an amplification effect of 2.77 in our baseline calibration implies that an annual rate of TFP growth of 0.65 percent accounts for the post-war output growth in the United States (about 1.8 percent a year), whereas the amplification effect found by MS requires a much lower annual rate of technological progress (0.27 percent).

The sensitivity analysis in section 6.2 reveals that the model economy calibrated to a time-share target in the range of 90 to 85 percent can account for the cross-country evidence on schooling and income, the cross-country evidence on Mincer returns to schooling, and is consistent with evidence on immigrants' earnings from Borjas (1987). It follows that an

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<sup>10</sup>Klenow and Rodriguez Clare (1997) argue that primary enrollment rates vary much less across countries than secondary enrollment rates. Thus, by using secondary enrollment as a measure of human capital investment, MRW overstate the variance of human capital across countries.

<sup>11</sup>Manuelli and Seshadri (2005) report a TFP elasticity of output per worker of 9 when both TFP and demographic factors are allowed to vary across countries. We estimate the elasticity to be 6.6 when demographic factors are kept constant to U.S. levels using the results in Table 4, page 24.

amplification effect of relative TFP differences in the range from 2.77 to 3.6 is plausible and that the cross-country variation in output per worker can be explained with relative TFP differences in the range from 2.3 to 3. In a closely related study that follows a different methodology from ours, Hendricks (2002) also concludes that TFP differences of a factor of 3 are needed to account for the cross-country data. Hendricks performs a growth accounting exercise without assuming a specific functional form for human capital accumulation by directly measuring cross-country differences in school quality using data on relative earnings (adjusted by schooling levels) of immigrants in the United States. To generate comparable statistics from our model, we simulate immigrants from four potential source countries differing with respect to their TFP. For each source country, we select immigrants with an average level of schooling consistent with the data reported in Hendricks. We assume that, conditional on the level of schooling, immigrants are randomly drawn from the distribution of ability types in the source country. We find that an immigrant from a country with an income in the range of 10 to 20 percent of U.S. income, has relative earnings of about 84 percent of a similarly schooled U.S. worker, which is close to the value of 83 percent reported by Hendricks (see Figure 4).

We conclude that our benchmark economy is roughly consistent with Hendrick's data. Importantly, the economy with a time share of 85 percent implies relative earnings of immigrants that are too low compared to the data (see fourth panel in Figure 4). One interpretation of this result is that this economy generates differences in the quality of human capital that are too large compared to the data. An alternative interpretation is that the assumption that immigrants are randomly drawn from the talent distribution (conditional on schooling levels) is not correct. Instead, if immigrants are positively selected, the implications of the

model economy with a time share of 85 percent could well be consistent with the data. Nevertheless, this economy would imply an amplification effect of 3.6, which is still substantially below the value of 6.6 found by MS. An amplification effect of 6.6 would obviously imply much larger quality differences in schooling across countries than the ones obtained in our baseline calibration. We thus side with Hendricks in concluding that accounting for the observed cross-country income differences on the basis of human and physical capital alone would require implausibly large degrees of self-selection in unobserved skills among immigrants. Moreover, our findings suggest that TFP is more important than is apparent in Hendrick's careful analysis since TFP differences can account for most of the cross-country variation in average years of schooling and in schooling quality.

## 6.4 Distributional Implications

We use our theory to evaluate how TFP affects cross-sectional inequality and intergenerational mobility within countries. We find that TFP has substantial distributional implications in our calibrated model economy. The quantitative consequences of TFP for distributional statistics hinge on the share of time inputs in the cost of human capital investment. In particular, when investment in human capital only requires time inputs, we find that TFP has no effect on cross-sectional inequality and intergenerational mobility.

We proceed by simulating model economies that differ from the benchmark economy in their levels of TFP. Low relative TFP is associated with high cross-sectional inequality (in terms of earnings, income, and consumption) and with low intergenerational mobility of earnings. (See Figure 5.) The effects of TFP on intergenerational mobility are particularly

striking: The intergenerational persistence of earnings increases from 0.5 in the benchmark economy to 0.64 in the economy with relative TFP of  $1/3$ . That these results hinge on the share of time inputs follows from the fact that TFP does not have distributional consequences in the economy with only time inputs. The explanation for this finding is simple. People across the income distribution are homogeneous in terms of their endowment of time but not in their wealth. When human capital accumulation requires goods, a decrease in TFP reduces more strongly the incentives to accumulate human capital among poor people than among rich people. This effect leads to higher cross-sectional inequality and lower intergenerational mobility.

Although there is little systematic data on inequality and mobility for a wide array of countries, we think that the implications of our model conform well with the conventional view that poor countries tend to be more unequal and less mobile than rich countries. (See for instance the survey by Solon, 2002.) In addition, our simulated economies only differ in a single dimension with respect to the benchmark economy – their levels of TFP. Dimensions in which countries can potentially differ include wage setting institutions, and public support for education, among others. These features would enrich the patterns of cross-sectional inequality and mobility across countries implied by our theory.<sup>12</sup> We address this issue in the next subsection, which studies economies that differ on the level of TFP and public support for education.

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<sup>12</sup>There is a large empirical literature on the relationship between cross-sectional inequality and income (or growth) across countries. There is an open debate in this literature about the exact relationship between inequality and development. See for instance Persson and Tabellini (1994), Alesina and Rodrik (1994), Barro (2000), Deininger and Squire (1998), and Forbes (2000).

## 6.5 Differences in Public Education

Countries differ on their overall public support for education. For instance, public expenditures on education range from 1 to 7 percent of GDP across countries according to data from the World Development Indicators. In our benchmark economy, we set the target for this statistic to 3.9 percent as in the U.S. economy. To the extent that subsidies to education affect incentives for acquiring education, it is of interest to study the aggregate and distributional implications of alternative institutional arrangements supporting education together with differences in TFP across economies.

**Public Education Expenditures** We proceed by simulating economies that differ with respect to public support for education and the level of TFP. In particular, we study economies with (relative) TFP of 1,  $1/2$ , and  $1/3$  and public expenditures on education over GDP of 1.95, 3.9, and 5.9 percent. Table 7 summarizes the results of these experiments. Changes in the amount of public education expenditures generate sizeable aggregate and distributional effects in the benchmark economy (relative TFP of 1). Increasing public support for education in the benchmark economy from 3.9 to 5.9 percent of GDP leads to a 9 percent increase in output per worker through an increase in human and physical capital accumulation. Interestingly, changes in public education have larger aggregate consequences in poorer economies. For instance, in the economy with relative TFP of  $1/3$ , the same increase in public education expenditures over GDP from 3.9 to 5.9 percent leads to a 23 percent increase in output per worker. Changes in public support for education also have important consequences for inequality and mobility. In the benchmark economy, a change in public education expenditures over GDP from 3.9 to 5.9 percent leads to a decrease in the Gini

coefficient of earnings from 0.32 to 0.30 and a decrease in the intergenerational correlation of log earnings from 0.50 to 0.39. Unlike the aggregate effects, the distributional implications of this policy change are *smaller* in poorer economies. In the economy with relative TFP of 1/3, the Gini coefficient of earnings does not change and the intergenerational correlation of earnings decreases slightly (from 0.64 to 0.62). Our experiments also reveal that changes in public support for education produces substantial variation in average years of schooling and Mincer returns across economies (see Table 7).

Table 7: Public Education across Relative TFP Economies

Pub. Ed. Exp. (% of GDP)	2.0	3.9	5.9
Panel A: Relative TFP 1			
Relative $Y$	0.85	1.00	1.09
Average Years of Schooling	9.6	12.9	15.1
Returns to Schooling (%)	11.4	10.0	11.1
Inequality (Gini earnings)	0.34	0.32	0.30
Persistence (int. corr. log earnings)	0.57	0.50	0.39
Panel B: Relative TFP 1/2			
Relative $Y$	1/8	1/6	1/5
Average Years of Schooling	4.8	7.1	9.8
Average Returns to Schooling (%)	20.6	14.5	11.3
Inequality (Gini earnings)	0.36	0.35	0.34
Persistence (int. corr. log earnings)	0.63	0.61	0.58
Panel C: Relative TFP 1/3			
Relative $Y$	1/27	1/21	1/17
Average Years of Schooling	2.7	4.3	6.1
Returns to Schooling (%)	34.3	22.7	16.4
Inequality (Gini earnings)	0.37	0.36	0.36
Persistence (int. corr. log earnings)	0.66	0.64	0.62

Relative  $Y$  is output per worker relative to the value of the benchmark economy.

**Public Expenditures by Level of Education** In our baseline calibration, the public subsidy to education is proportional to the time spent in school. The evidence suggests that

poor countries tend to subsidize post-secondary education to a larger extent than primary and secondary education. Table 8 shows data on the proportion of public education expenditures per student relative to GDP per capita across countries. In light of this observation, we investigate the aggregate and distributional consequences of public subsidies to education that are relatively more generous at higher levels of education. To this end, we simulate the economy with relative TFP of  $1/2$ , assuming that the subsidy on education is an increasing function of time in school:  $p(s) = p_0 + p_1 s$ . (Recall that  $p_1 = 0$  in our baseline calibration.) We select the slope parameter  $p_1$  so that the average student receives the same amount of subsidy as in the benchmark specification (flat subsidy).

Table 8: Public Education Expenditures per Student

Countries	Education Level		
	Primary	Secondary	Tertiary
U.S.	21	24	32
Spain	19	25	21
Korea	17	15	7
Japan	22	21	17
Brazil	11	10	59
Bolivia	11	10	45
Colombia	15	16	41
Mexico	12	14	45
South Africa	14	18	54
Zimbabwe	13	20	201

Data expressed as percentage of GDP per capita. Data is for the year 2000 except for Korea and Mexico that is 1999.

Source: World Development Indicators.

Relative to the baseline policy, in the economy with relative TFP of  $1/2$ , a scheme of an increasing subsidy rate with years of education leads to an increase in output per worker of around 6 percent, an increase in the average years of schooling from 7 to 8.1, an increase



in the proportion of people completing college from 5 to 22 percent, and a decrease in the average Mincer returns to schooling from 14 to 11 percent. Human capital investment and output per worker increase at the cost of a substantial increase in economic inequality and intergenerational persistence.

To summarize, differences in TFP in conjunction with disparate education policies can play an important role in accounting for the cross-country variation in output per worker, human capital investment (average years of schooling and Mincer returns), and distributional statistics.

## 6.6 Productivity of Investment Goods

Our quantitative analysis has focused on TFP as the main force driving output per worker differences across countries. One implication of this assumption in the context of a neoclassical growth model is that the physical capital to output ratio is roughly constant across economies. In the data, the capital to output ratio differs across countries and the literature has suggested productivity differences in the production of investment goods as one of the explanations (e.g. Restuccia and Urrutia, 2001). Hence, it is of interest to ask whether our results change if, in addition to TFP differences, we allow for productivity differences in the investment-goods sector. We simulate an economy that features a productivity of investment goods that is  $1/4$  of the benchmark economy, i.e.,  $A_k = 1/4$ . This productivity difference is roughly consistent with the difference in the relative price of capital between rich and poor countries (see for instance Jones, 1994 and Restuccia and Urrutia, 2001). In addition, we reduce the TFP level of this economy so that output per worker relative to

the benchmark economy is the same as in the economy with relative TFP of  $1/3$  (and no difference in the productivity of investment goods). A relative TFP of  $3/5$  in this economy produces an output per worker that is  $1/21$  of the benchmark economy. Table 9 reports statistics for this economy and the economy with relative TFP of  $1/3$  in the baseline specification. We observe that the aggregate and distributional statistics are quite similar except for the capital-to-output ratio, which in the economy with low productivity of investment goods is  $1/5$  of the value in the benchmark economy. We conclude that the aggregate and distributional implications of our model are robust to the source of productivity differences. Low investment in human capital in poor economies is driven by low wage rates and not by whether low wage rates are the result of low TFP, low physical capital to output ratio, or a combination of both.

Table 9: Sectoral TFP Differences in the Model

Relative TFP ( $A$ )	1	$1/3$	$3/5$
Rel. Sector TFP ( $A_k$ )	1	1	$1/4$
Rel. $Y$	1	$1/21$	$1/21$
Rel. $H$	1	$1/4$	$1/4$
$K/Y$	2.8	2.7	0.6
Rel. $K/Y$	1	1	$1/5$
Average Years of Schooling	12.9	4.3	4.0
Returns to Schooling (%)	10.0	22.6	24.3
Inequality (Gini earnings)	0.32	0.36	0.36
Persistence (int. corr. log earnings)	0.50	0.64	0.645

## 7 Conclusions

Rich and poor countries differ in their average output per worker by roughly a factor of 20. While mechanically large differences in TFP could explain these differences, they would

not be supported by the available micro evidence. In this paper, we develop a quantitative theory of human capital with heterogeneous agents in order to assess the sources of cross-country income differences. A model with heterogeneous agents allows us to restrict the parameters of human capital technology. In particular, we systematically compare the cross-sectional implications of the theory with U.S. data. We show that the model constitutes a good quantitative theory of earnings and schooling inequality in the United States. In this model, relatively small differences in TFP translate into large differences in output per worker. Our quantitative model produces a TFP elasticity of output per worker of 2.8. This implies that in order for the model to produce a factor of 20 difference in output per worker, a factor of 3 difference in TFP is needed. This implied difference in TFP is in the range of estimates from micro studies. The theory suggests that using Mincer returns to measure human capital understates human capital differences across countries by a factor of 2. Not only does the model support large differences in human capital across countries, but also it implies differences in human capital quality that are consistent with (i) the evidence from earnings of immigrants in the United States, (ii) the cross-country evidence on estimates of Mincer returns, and (iii) the evidence on the relationship between average years of schooling and per-capita income across countries. We also find that TFP has substantial effects on cross-sectional inequality and intergenerational mobility. We show that differences in TFP in conjunction with disparate education policies can play an important role in accounting for the cross-country variation in output per worker, human capital investment, and distributional statistics.

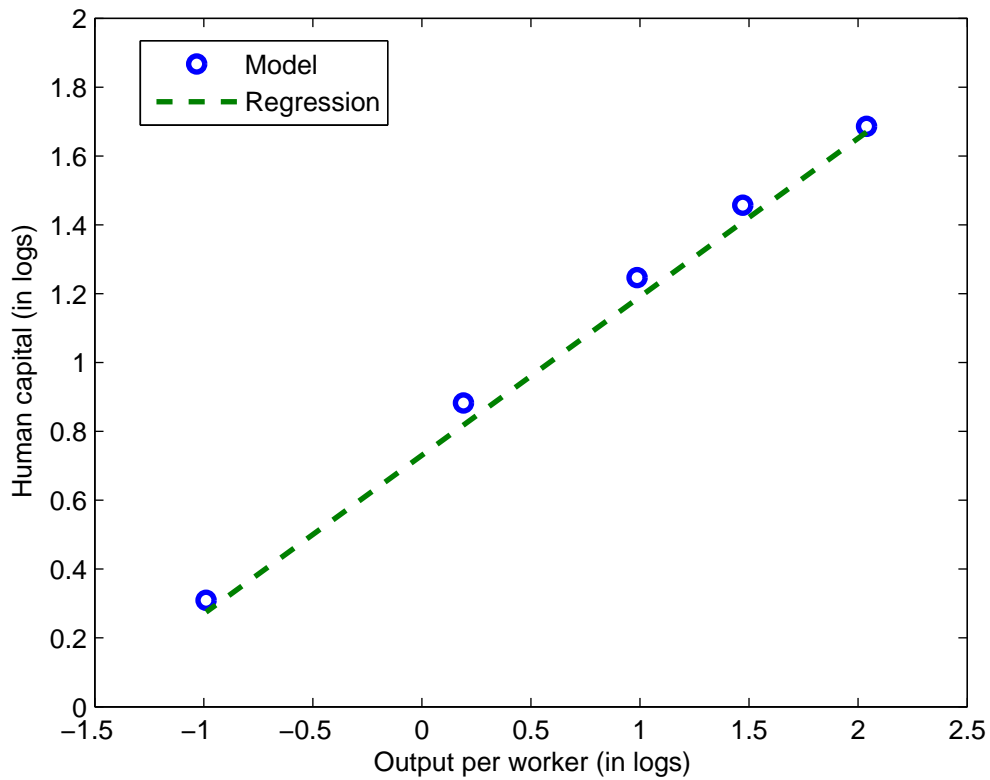
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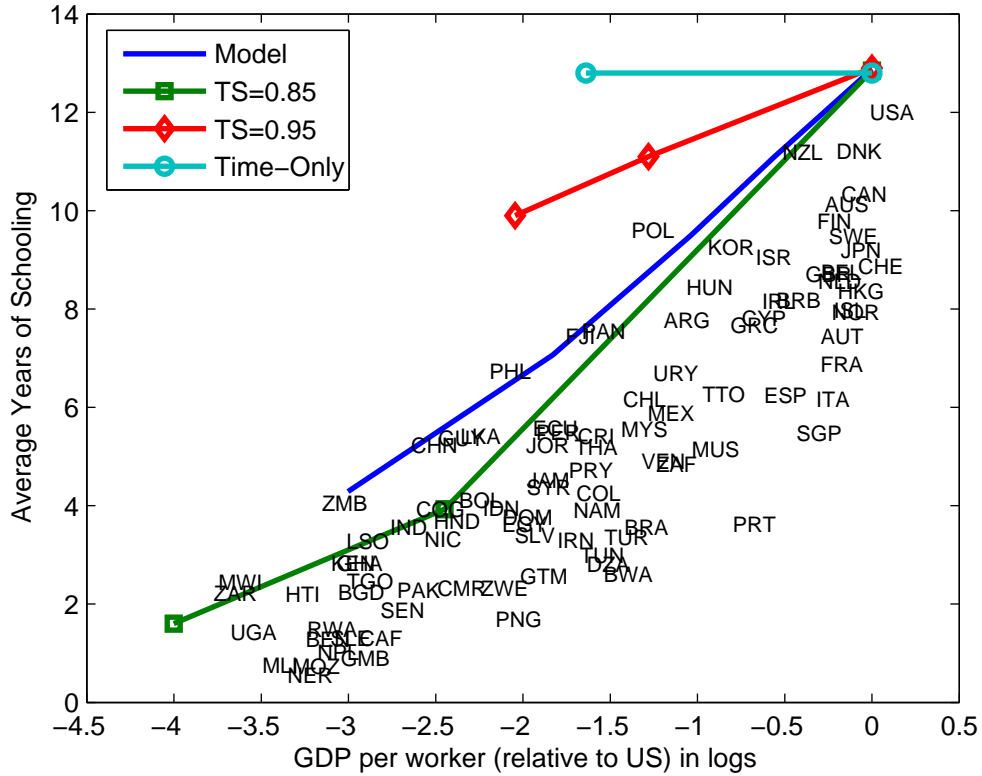
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Figure 1: Human Capital and Output



Model refers to economies with relative TFP of 1, 0.8, 2/3, 1/2, and 1/3 in our baseline calibration. Regression refers to an OLS regression of log human capital on log output with a constant term resulting in:  $\log(H) = 0.7307 + 0.4605 \log(Y)$ .

Figure 2: Schooling and Output – Data vs. Model

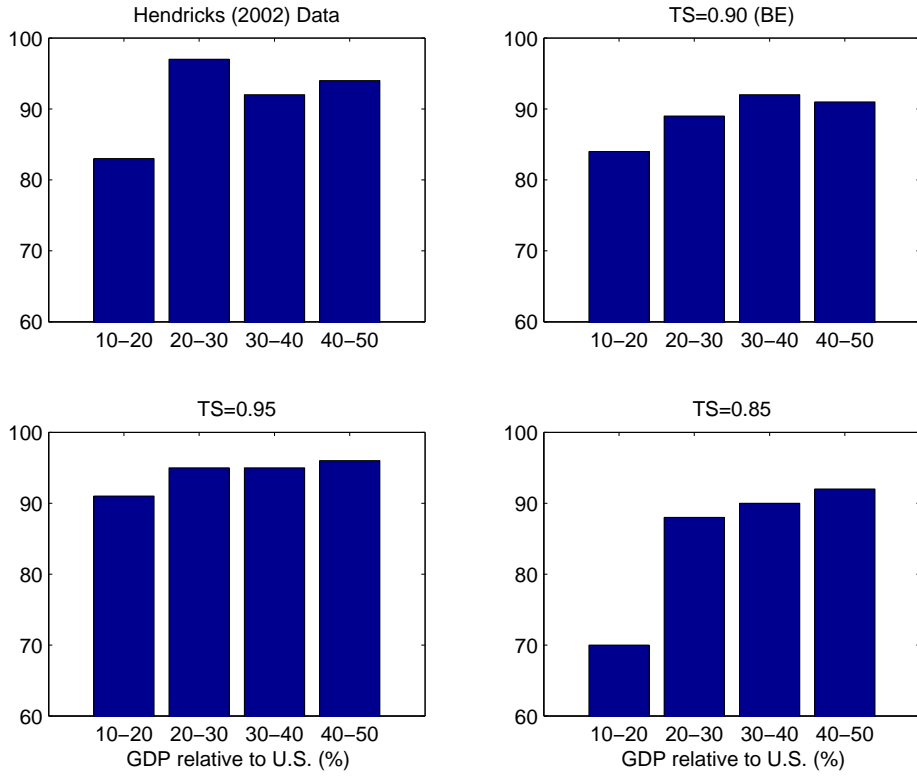


Data are from Barro and Lee (1996) and Heston, Summers, and Aten (2002). We take averages (five-year intervals) of GDP per worker in the data. This figure focuses on the averaged data for 1990. Model refers to economies with relative TFP of 1, 0.8, 2/3, 1/2, and 1/3 in our baseline calibration.  $TS = z$  refers to economies with relative TFP of 1, 1/2, and 1/3 for a re-calibration of the model to a time share target of  $z$  percent. Time-only model refers to economies with relative TFP of 1 and 1/3 when human capital accumulation features only time inputs, i.e.,  $\eta = 1$ .



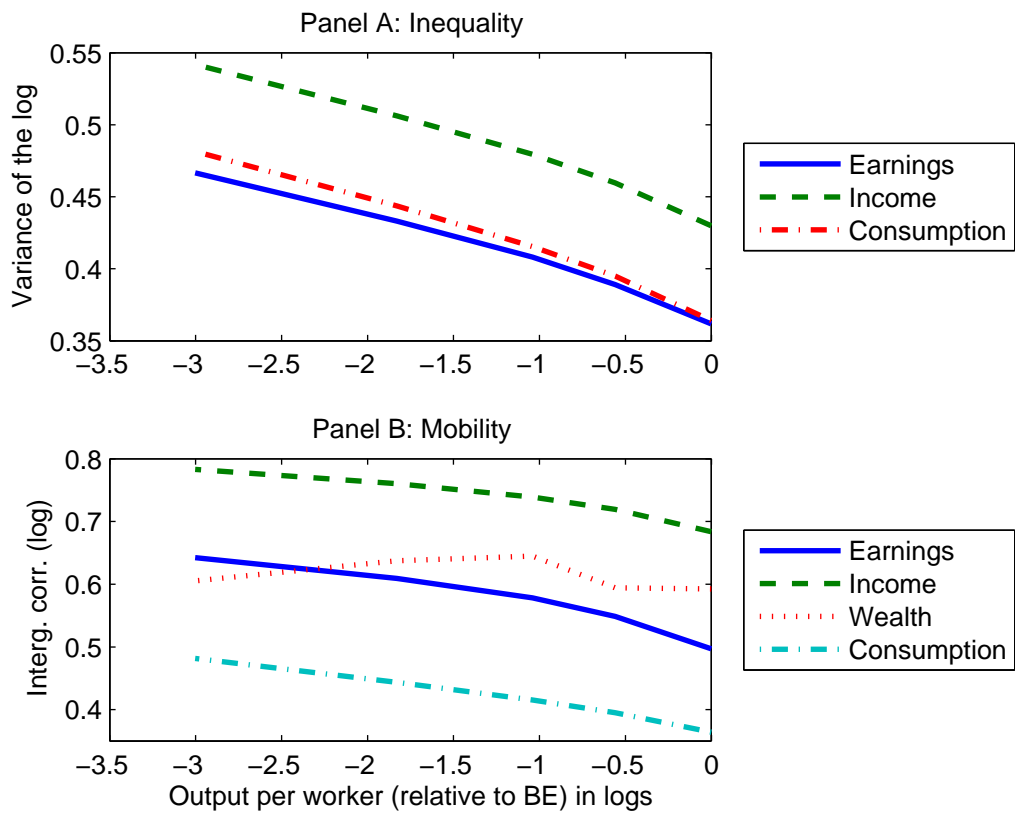


Figure 4: Relative Earnings of Immigrants



Data on relative earnings of immigrants across countries is from Hendricks (2002) and is adjusted by the level of schooling of the immigrant population. Comparable statistics are computed from the model for different values of the share of time in the total cost of education. According to Hendricks' data, the average years of schooling of the immigrant population to the United States from countries whose GDP per capita is between 10-20, 20-30, 30-40, and 40-50 percent of the United States is 12.8, 12.5, 12.8, and 11.7. We use this data to calculate relative earnings in the model for workers with the same average years of education.

Figure 5: Distributional Implications



Economies with relative TFP of 1, 0.8, 2/3, 1/2, and 1/3 in our baseline calibration.