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Intermediation, Capital Immobility, and Asset Prices

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Abstract

We introduce intermediation frictions into a Lucas (1978) asset pricing model in order to study the effects of low capital in the intermediary sector on asset prices. Our model shows that low intermediary capital can increase risk premia, Sharpe ratios, volatility and comovement among intermediated assets. Reductions in intermediary capital also lead to a flight-to-quality in which intermediaries' investors withdraw their funds and purchase bonds. The model thereby replicates observed asset market behavior during aggregate liquidity crises. In a dynamic context, we show that intermediaries will hedge against periods of low capital, and a liquidity factor driving asset returns arises from such hedging. We calibrate the model to quantify the asset market effects of low intermediary capital.

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1 Introduction

It is by now widely accepted that shocks to the capital of intermediaries (hedge funds, market makers, trading desks, etc.) can have a causal effect on asset prices. A significant data point in support of this view is the hedge fund crisis of the fall of 1998. Adverse shocks, beginning with the Russian default, led to mounting losses among hedge funds. Investors grew to question the safety of the hedge fund positions. Rather than injecting fresh capital into the affected institutions, investors pulled back into short-term and liquid assets. Funds were forced to liquidate positions, leading to substantial price effects in many of the markets where hedge funds specialized. Spreads and risk premia rose as prices fell; volatility increased; and market liquidity fell. Bottlenecks in the movement of capital emerged as sophisticated parts of the financial system were compromised while other sectors of the economy were relatively unaffected.[1,2]

The events of the fall of 1998 are puzzling when viewed from a neoclassical perspective, in which financial intermediaries are a frictionless conduit through which households gain access to a larger investment opportunity set. In the neoclassical model, if the capital of an intermediary falls in a way that shrinks the investment opportunity set of its end investors, these investors simply restore the capital of the intermediary, or bypass the affected intermediary and inject capital directly into the affected markets. In contrast to the experience of the fall of 1998, neoclassical models hypothesize that capital moves extremely quickly across markets and institutions so as to render intermediation capital irrelevant for asset prices.

This suggests that in order to understand how intermediation capital may affect asset prices we need a model in which intermediary capital and household capital are, at times, not perfectly fungible. Recent models of financial intermediation are promising in this regard. Holmstrom and Tirole (1997) and Allen and Gale (2005) present general equilibrium models which motivate an intermediation sector, and role for intermediary capital, from first principles. In Holmstrom and Tirole, the role for intermediary capital arises endogenously because of a moral hazard problem. In general equilibrium, shocks to intermediary capital are transmitted to the asset market. In Allen and Gale, intermediaries arise as efficient providers of liquidity services to households. The liquid holdings of the intermediaries play a key role in the general equilibrium determination of asset prices.

We build on this literature and develop a model to study the importance of intermediation capital for asset prices. Our model has an intermediation sector and a household sector. Households lack the knowledge to invest directly in a set of risky complex assets, and choose to delegate such investment to intermediaries that are managed by skilled "specialists." The main friction we introduce is a capital constraint: because

[1] See Caballero and Krishnamurthy (2006) for an analysis of the macroeconomic consequences of capital bottlenecks during flight to quality episodes.

[2] Other important asset markets, such as the equity or housing market, were relatively unaffected by the turmoil. The dichotomous behavior of asset markets suggests that the problem was hedge fund capital specifically, and not capital more generally. Investors did not bypass the distressed hedge funds in a way as to undo any asset price impact of the hedge fund actions. They also did not restore the hedge funds' capital.

of an agency problem, specialists must use their own wealth to coinvest with the households in forming an intermediary. The coinvestment places some of the specialist's wealth at stake in the intermediary, and thereby provides the specialist with appropriate investment incentives. The intermediary-household relationship in our paper bears a close resemblance to the model of Holmstrom and Tirole.

In the recent literature referenced above, asset pricing consequences are derived within stylized one or two period models. In contrast, our paper introduces intermediation into a canonical asset pricing framework. We place the household-intermediary relationship within a general equilibrium that resembles the endowment economies commonly studied in the equilibrium asset pricing literature. The risky assets are represented by a Lucas (1978) tree, which pays a dividend governed by a geometric Brownian motion process. Subject to the intermediation friction, markets are complete. In general equilibrium, we show that the aggregate capital of the intermediary sector is an important state variable in determining asset prices.

When there is a large enough amount of intermediary capital that capital constraints do not bind in equilibrium, households are indirectly able to hold their desired portfolio of risky assets, and the intermediation frictions play little role in determining asset prices. The economy resembles one without any agency problems. Risk is shared between specialists and households, and risk premia are small.

Starting from the level of intermediary capital where the supply of intermediation is large, if we imagine reducing intermediary capital, there comes a threshold for intermediary capital where capital constraints bind in equilibrium. As we reduce capital below this level, households withdraw funds from intermediaries, recognizing that not doing so would trigger an agency problem; i.e. specialists would not have a sufficient stake to provide investment incentives. Households reduce (indirect) participation in the risky asset. The specialists increase leverage to absorb more of this risky asset and shocks to dividends are borne predominantly by specialists.

In the latter case, dividend shocks have a larger effect on asset prices, leading to higher conditional volatility. This reinforces the effect of capital constraints, as dividend shocks lead to greater fluctuations in the capital levels of intermediaries, creating an amplification mechanism. Risk premia and Sharpe ratios rise.

One advantage of our multi-period model of asset trading over a static model of asset trade is that our approach moves research closer towards a quantifiable equilibrium model of market illiquidity. In this spirit, we calibrate our model and illustrate the size of the capital effects. We find that while risk premia when capital constraints do not bind are 3% in our baseline calibration, when constraints place intermediaries near bankruptcy the risk premium rises to close to 7%. Asset price volatility rises from 10% in the unconstrained region to 18%, when intermediaries are near bankruptcy. As intermediary capital falls, households withdraw funds from intermediaries and purchase riskless bonds. The increased demand for bonds drives down the riskless interest rate from 1.5% when intermediaries are unconstrained to 0.40% when intermediaries are near bankruptcy.

The patterns that our model generates when intermediary capital is low are consistent with episodes of market illiquidity such as the events of the fall of 1998. Indeed, our model suggest a link between market liquidity and intermediary capital. When intermediary capital is low, there are effectively less buyers of risky assets as households do not fully participate in the risky asset markets. We show that during the low capital periods, correlation across intermediated assets rises, consistent with the fall of 1998 events. We also show that decreases in asset prices leads household to withdraw funds from intermediaries and invest in riskless bonds, consistent with a flight to quality.

A number of recent papers have documented that aggregate liquidity risk is a priced factor (see Amihud, 2002, Acharya and Pedersen, 2003, Pastor and Stambaugh, 2003, and Sadka, 2003), suggesting that the marginal investor is averse to assets that pay little when aggregate liquidity is low. Interpreting our low intermediary capital states as low aggregate liquidity states, we find a natural reason why the marginal investor in our economy should be averse to low intermediary capital: Since the marginal investor in our model is the specialist who has low consumption and bears substantial risk when intermediary capital is low, the marginal investor will be averse to aggregate liquidity risk. We characterize the specialist's demand for hedging against declines in aggregate liquidity. We also show that the aversion to liquidity risk gives rise to a two-factor cross-sectional asset pricing model, with positive loadings on both the market as well as shocks to intermediary capital.

In our model, the specialists (intermediaries) are the marginal investor in the risky asset market. There is a growing empirical literature suggesting that intermediaries are the marginal investor in many specialized asset markets. These studies include, research on the mortgage-backed securities market (Gabaix, Krishnamurthy, and Vigneron, 2005), the corporate bond market (Collin-Dufresne, Goldstein, Martin, 2001), the credit default swap market (Berndt, et. al., 2004), the catastrophe insurance market (Froot and O'Connell, 1999, 2001), and the options market (Bates, 2003; Garleanu, Pedersen, and Poteshman, 2005). These studies reiterate the relevance of intermediation for asset prices. However they paint a richer picture of intermediation than is captured in our model, as they suggest that the capital of intermediaries specialized in particular markets is relevant for those particular markets. In our model, the entire intermediary sector specializes in the same set of assets. Our model takes a broad brush approach to the relationship between intermediation capital and asset prices.

There is also a growing theoretical literature drawing the connection between intermediary capital and asset prices.[3] Allen and Gale (1994) present a model in which the amount of "cash" of the marginal investor affects asset prices. This "cash-in-the-market" can be linked to the balance sheet position of intermediaries, and Allen and Gale (2005) draw such a connection more explicitly. Holmstrom and Tirole (1997) also present an explicitly micro-founded model in which there is a role for intermediary capital, and changes in this capital

[3]These papers belong to a larger literature on the connection between financial intermediation and asset pricing. Some of the papers in the literature include Allen and Gorton (1993), Brennan (1993), Dow and Gorton (1994). Grossman and Zhou (1996), Shleifer and Vishny (1997), Cuoco and Kaniel (2001), and, Dasgupta, Prat and Verardo (2005).

affect asset prices (the interest rate in Holmstrom and Tirole). As noted above, the micro-foundation of intermediation in our model draws from the Holmstrom and Tirole model. Relative to Holmstrom and Tirole, our model has aggregate uncertainty and is a dynamic model of asset trade.

Xiong (2001) develops a model to study capital effects in a dynamic setting. His model also delivers the amplification effect that arises in our paper (see also Kiyotaki and Moore, 1997, and Krishnamurthy, 2003). Unlike our paper, Xiong does not explicitly model an intermediation sector, treating such a sector with a short-hand log utility preference assumption.

Gromb and Vayanos (2002) and Liu and Longstaff (2004) study settings in which an arbitrageur with limited wealth and facing a capital constraint trades to exploit a high Sharpe-ratio investment. Liu and Longstaff show that the capital constraint can substantially affect the arbitrageur's optimal trading strategy. Gromb and Vayanos show that the capital constraints can have important asset pricing effects. Both of these papers point to the importance of a capital effect for asset pricing.

Our paper draws the connection between financial intermediation and aggregate liquidity risk in a dynamic model. Vayanos (2005) performs a similar exercise. He introduces an open-ending friction, rather than the capital constraint of our model, into a model of intermediation. His model generates interesting implications for the premium on liquidity and volatility risk.[4]

The remainder of this paper is devoted to developing a model to assess the importance of intermediary capital for market liquidity and asset pricing.

2 The Model: Capital Constraints and Asset Prices

We consider an infinite horizon, continuous time, economy with a single perishable consumption good, which we will use as the numeraire. There are two assets, a riskless bond in zero net supply, and a stock that pays a risky dividend. We normalize the total supply of stocks to be one unit.

The stock pays a dividend of D_t per unit time, where $\{D_t : 0 \leq t < \infty\}$ follows a Geometric Brownian Motion,

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t \quad \text{given} \quad D_0. \tag{1}$$

$g > 0$ and $\sigma > 0$ are constants. Throughout this paper $Z = \{Z_t : 0 \leq t < \infty\}$ is a standard Brownian motion on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with an augmented filtration $\{\mathcal{F}_t : 0 \leq t < \infty\}$ generated by the Brownian motion Z.

We denote the progressively measurable processes $\{P_t : 0 \leq t < \infty\}$ and $\{r_t : 0 \leq t < \infty\}$ as the stock price and interest rate processes, respectively.

[4] Brunnermeier and Pedersen (2005) show how the interaction between volatility risk and capital effects can create multiple equilibria in the asset market.

There are two classes of agents in the economy, households and specialists. We are interested in studying an intermediation relationship between households and specialists. To this end, we assume that the risky asset payoff comprises of a set of complex investment strategies that the specialist has a comparative advantage in managing. As in the literature on limited market participation (e.g., Mankiw and Zeldes, 1991, Allen and Gale, 1994, Basak and Cuoco, 1998, Vissing-Jorgensen, 2002, Brav, Constantinides, and Geczy, 2002, Guvenen, 2005), we assume that the household can directly invest only in the bond market. For example, in the limited participation literature this assumption is often motivated by appealing to "informational" transaction costs that households face when investing in the stock market.

We depart from the limited participation literature by allowing specialists to invest in the risky asset on behalf of the households. The specialist and household strike up a beneficial intermediation relationship whereby the household allocates some funds to the specialists who invest in the risky asset on the household's behalf. We may imagine that the intermediary is a hedge fund, mutual fund, or a bank.

Both households and specialists are infinitely lived. The specialists have concave preferences over date t consumption, c_t,

$$\int_0^\infty e^{-\rho t} u(c_t) dt \qquad \rho > 0;$$

we consider a CRRA instantaneous utility function with parameter γ for the specialists.

The households have log preferences over date t consumption c_t^h,

$$\int_0^\infty e^{-\rho^h t} ln\ c_t^h\ dt \qquad \rho^h > 0.$$

We next turn to describing the intermediation relationship.

2.1 Intermediation

We envision the following market structure for intermediation. At every t, each specialist is randomly matched with a household. The specialist and household then potentially enter into an intermediation relationship. These interactions occur instantaneously and result in a continuum of (identical) one-to-one relationships. After intermediation decisions are taken, specialists trade in a Walrasian stock and bond market, and the household trades in only the bond market. At $t + dt$ the match is broken, and the intermediation market repeats itself.[5]

Suppose that a specialist raises funds from households to manage an intermediary of size T. He makes investment decisions that result in a stochastic portfolio return of $\widetilde{dR_t}$. The specialist's portfolio is denoted as α^s shares of stock and α^b in bonds.

Assumption 1 (Moral Hazard)

The specialist makes an unobserved portfolio choice decision, (α^s, α^b), and an unobserved due-diligence deci-

[5] The instantaneous matching structure means that all contracts are short-term. In principle, a long term contract could improve allocations.

sion, $e \in \{0, 1\}$. For any given portfolio, if the specialist shirks and sets $e = 0$, the return on the portfolio falls by $x\ dt$, but the specialist gets a private benefit (in units of the consumption good) of $bT\ dt$. We assume that $x > b > 0$ so that choosing $e = 1$ maximizes total surplus.

We think of shirking as failing to execute trades in an efficient manner. If one specialist shirks and his portfolio return falls by xdt, the other investors in the risky asset collectively gain xdt. Since each specialist is infinitesimal, the other specialists' gain is infinitesimal. Shirking only leads to transfers and not a change in the aggregate endowment.

As is usual, we focus on financial contracts that implement high effort (along the equilibrium path, shirking does not occur). Specialists have to be given incentives to put in effort.

We restrict attention to the following type of financial arrangement. We assume that the specialist raises funds from the household in the form of both (instantaneous) debt and equity contracts. The instantaneous debt contract is riskless and pays interest rate r_t, while the return on the equity contract depends on the specialist's decisions.

As the return on the equity contract depends on specialist effort, we assume that incentives for effort are provided through the specialist owning a portion of the equity of the intermediary:

Assumption 2 (Ownership Stake)
Contracts specify a pair (β, C). $\beta \geq 0$ is the percentage stake of the equity that the specialist must own. $C \geq 0$ is a fee the specialist receives for managing the intermediary.

We denote B^h as the debt of the intermediary and E^I and E^h as the equity held by specialist and household, respectively. T is equal to $B^h + E^h + E^I$, and $\beta = \frac{E^I}{E^h + E^I}$.

We focus on these simple affine contracts in order to retain tractability when embedding our intermediation model into an equilibrium asset pricing framework. Of course, in general, the optimal contract may be a complex non-linear function of returns. A realistic example of a non-linear contract is an option contract that only compensates the specialist if the return is sufficiently high.

An intermediation contract specifies (β, C) that maximizes the utility of the household, given Assumptions 1 and 2. We solve for the optimal contract assuming that the household has all of the bargaining power and makes a take-it or leave-it offer to the specialist.[6]

Proposition 1 *The intermediation contract between a household and specialist involves:*

1. *Specialists are subject to a capital constraint. They can raise equity capital of, at most, $E^h = m \times E^I$ from the households, where $m = \frac{x}{b} - 1$ is a strictly positive constant.*

[6]In the case where the bargaining power is intermediate we find that $C > 0$. While we can also solve this case in our model, it is more cumbersome and does not affect the results substantively.

2. *Households may also lend to the intermediary with instantaneous debt contracts of B^h at interest rate r_t. B^h is not subject to any constraints.*

3. *Both specialists and households receive the return $\widetilde{dR}_t - \frac{B^h}{T} r_t dt$ on their equity contributions to the intermediary.*

4. $C = 0$.

Proof. See Appendix A. ■

The optimal contract involves an equity capital requirement. The requirement of a having an ownership stake in investment captures the explicit and implicit incentives across many modes of intermediation.

To illustrate the factors behind this capital requirement, let us fix the total fund size $T = 1$ and consider how β is determined. On the one hand if the specialist shirks, he gains bdt, a private benefit proportional to the total funds under management. On the other hand if the specialist shirks, the total return on the fund falls by a fixed amount xdt, but the specialist only bears β proportion of this cost. Loosely speaking this suggests an incentive constraint that compares $\beta\widetilde{dR}_t$ against $\beta\widetilde{dR}_t - \beta x dt + bdt$. In Appendix A, we show formally that the incentive constraint requires,

$$-\beta x + b \geq 0 \quad \Rightarrow \quad \beta \geq b/x.$$

The capital requirement is increasing in the cost of effort (b), and decreasing in the penalty to shirking (x).

Our modeling of intermediation and the derivation of the capital requirement closely follows Holmstrom and Tirole (1997). Unlike Holmstrom and Tirole we make assumptions to restrict the contract space. These assumptions are necessary in order to maintain tractability when placing the contracting problem within an asset pricing framework. But our resulting contracts do resemble those that Holmstrom and Tirole derive.

A household invests with a particular intermediary by purchasing both its bonds and its equity. As we have described above, after intermediation decisions are taken, households are no longer allowed to participate in the risky asset market (limited market participation) but may choose to enter a Walrasian bond market to purchase more bonds. Since either the bonds from a particular intermediary or bonds purchased in the Walrasian bond market are identical and riskless, the composition of this bond investment is indeterminate and irrelevant for equilibrium. Without loss of generality, we set B^h equal to zero and assume that the household only purchases bonds in the Walrasian bond market.

Given these investment opportunities, households face a choice over how much funds to invest in the equity contracts of the intermediary and how much to invest in the riskless bond. We make an assumption on household beliefs that pins down this portfolio choice decision in a simple fashion:

Assumption 3 (Household beliefs)

Households have static beliefs[7] *over the returns delivered by specialists who do not shirk:*

$$\bar{\pi} = E[\widetilde{dR_t}] - r_t \quad and, \quad \bar{\sigma}^2 = Var[\widetilde{dR_t}].$$

The mean and variance satisfy, (1) $\bar{\pi} = \bar{\sigma}^2$*, and (2)* $\bar{\pi} < x$.

We imagine that households have no ability to understand time-varying returns in the marketplace. This assumption is in keeping with our premise that households do not understand the complex investment strategies pursued by hedge funds and other intermediaries. Condition (1) implies that a log investor with these beliefs will choose to invest upto 100% of his wealth with the intermediary. Condition (2) implies that the investor would rather invest in the riskless bond than invest with a specialist who shirks.

The model we have outlined has the characteristic that when the specialists' wealth falls, their ability to intermediate funds of the households also falls. This reduces the aggregate demand for stocks, requiring an increase in the equilibrium return on the stock in order to clear the stock market. On the other hand, the disintermediation results in households increasing their demand for the bond, leading to a fall in the equilibrium interest rate.

Note that the model is asymmetric, as a sufficient rise in the wealth of specialists may lead to a situation where all of the funds of the households are being intermediated, so that further changes in intermediary capital does not affect intermediation.

Finally, as the wealth of specialists falls toward zero, intermediation falls. We define a "bankruptcy" condition at the point where specialist wealth falls to zero. The bankruptcy state is an exogenous boundary condition for our dynamic economy.[8]

Assumption 4 (Bankruptcy)

If specialist wealth falls to zero, an intermediary is declared bankrupt and a "bankruptcy court" steps in.

- *The intermediary's assets are liquidated to the debt-holders and debt is retired.*

[7] This assumption ensures that households wish to invest 100% of their wealth with the intermediary as long as the intermediary is not capital constrained. It is possible to substitute the static beliefs assumption with a more standard setup where the household observes time variation in the risk/return tradeoff of the stock market if we impose a no-borrowing constraint on the household. In our parameterizations we focuse on cases where $\gamma > 1$ so that typically the required return for the specialist is higher than that of the household. As a result, equilibrium returns are such that the household will wish to invest at least 100% of his wealth with the intermediary. The no-borrowing constraint ensures that the household does not go into debt in order to invest more money with the intermediary.

[8] In principal the model is well specified without this bankruptcy boundary. If specialists were not allowed to go bankrupt, they will trade in a way that their wealth always remains above zero (although wealth may approach zero). By defining a bankruptcy state in which the specialists are provided a positive level of consumption, we alter the specialists trading incentives so that zero traded wealth becomes possible. Without the bankruptcy boundary, the behavior of asset prices when specialist wealth approaches zero becomes extremely idiosyncratic, as interest rates fall towards negative infinity to clear the bond market, and the stock price may rise because of the falling discount rate. We avoid these idiosyncracies by explicitly specifying a boundary condition.

- *The court restructures the strategies underlying the risky asset so that the dividend is no longer risky, but yields a fixed and smaller dividend stream of δD_τ, where τ is the date of bankruptcy and $0 < \delta < 1$.*
- *Shares are given to debt-holders. The specialist is restricted from trading shares or starting an intermediary thereafter.*
- *Households who receive shares in bankruptcy are assessed a tax on their asset income at rate rate ϕ. The tax proceeds are given to specialists to fund consumption for the rest of time. We set $\delta = 1 - \phi$ which ensures that the consumption of the specialist does not jump in bankruptcy.*

2.2 Household heterogeneity

We study two versions of the model. Our baseline case is the model we have just outlined. As we will show, this model has the property that if the economy is currently in a state where each of the specialists' capital constraint does not bind, then the economy will never transition to a state where the capital constraints do come to bind. Such a model is useful for characterizing asset prices conditional on some initial condition, but has an important drawback. The model cannot speak to how the anticipation of tight constraints will affect behavior in a state where constraints do not bind. Thus dynamic linkages of asset prices are incomplete.

We also study a variation of the model just presented that does not suffer this problem. We assume that each household is made up of a pair of agents, a "stock investor" and a "debt investor" . At the beginning of date t, the households divide their wealth of w^h between the two members of the household in fractions λ (debt investor) and $1 - \lambda$ (stock investor). The debt investor uses his wealth to only puchase riskless debt. The stock investor behaves like the households we have described so far. He is matched with an intermediary and delegates a portion of his wealth subject to the intermediation constraints. We envision that at the end of each infinitesimal time interval the household aggregates the wealth from each member before making its consumption decision.

Our modeling of the household borrows from Lucas (1990)'s worker/shopper model. The modeling device of using a two-member household is a simple way to introduce heterogeneity among households without substantially complicating the analysis.

Notice that this model with $\lambda = 0$ is our baseline model. The model analysis for the case of $\lambda > 0$ is substantively the same as the case of $\lambda = 0$. We develop equations for the $\lambda = 0$ case in the text, and present the $\lambda > 0$ case in the Appendix C. In following numerical solutions we study the case of $\lambda = 0$ and a case with a small positive $\lambda = 0.02$.

2.3 Decisions and equilibrium

At every t specialists and households interact and strike up intermediation relationships. After the intermediation decision, agents trade in the asset and goods market to achieve their desired portfolio shares and consumption rates. Given our assumption on household beliefs, the investment decision of the household is trivially to invest any remaining wealth (after giving money to the intermediary and consuming) directly in bonds. The specialist chooses a portfolio of stock and bond and the consumption rate to maximize his lifetime utility.

The decision problem of a household is to choose his consumption rate and funds for delegation, given his wealth, $w_t^h > 0$. We denote X_t as the fraction of wealth that is invested with the intermediaries. Then,

$$T^h = w_t^h X_t = \min\left\{m w_t^I, w_t^h\right\},$$

i.e. the household delegates the maximum possible funds to the intermediary given household beliefs (see Assumption 3). Then the return on the household's wealth is,

$$\widehat{dR_t} = (1 - X_t) r_t dt + X_t \widetilde{dR}_t,$$

where $\widetilde{dR}_t$ is the cumulative return process delivered by intermediaries. The optimization problem for a household, given some w_0^h, is:

$$\max_{\{c_t^h\}} E\left[\int_0^\tau e^{-\rho^h t} ln\ c_t^h\ dt\right] \quad s.t. \quad dw_t^h = -c_t^h dt + w_t^h \widehat{dR_t}, \tag{2}$$

where τ is the first time the economy reaches the bankruptcy boundary.

The specialist chooses his consumption rate c_t, his portfolio (α_t^s and α_t^b), as well as a quantity of intermediation services (Q_t) to offer to households.

$$\max_{\{c_t, \alpha_t^s, \alpha_t^b, Q_t\}} E\left[\int_0^\tau e^{-\rho t} u(c_t)\ dt\right] \tag{3}$$

subject to the budget constraint,

$$dw_t^I = -c_t dt + w_t^I \widetilde{dR}_t$$

and the capital constraint,

$$Q_t \leq m w_t^I = m\left(\alpha_t^b + \alpha_t^s P_t\right).$$

w_t^I corresponds to the specialist's contribution T^I. The return delivered by intermediaries depends on the specialist's portfolio selection:

$$\widetilde{dR}_t = \frac{1}{w_t^I}\left[\alpha_t^b r_t dt + \alpha_t^s \left(D_t dt + dP_t\right)\right].$$

Definition 1 *An* equilibrium *is a set progressively measurable price processes P_t and r_t, and decisions $(Q_t, X_t, c_t, c_t^h, \alpha_t^s, \alpha_t^b)$ such that,*

1. *Given the price processes, decisions solve (2) and (3).*

2. The stock market clears:

$$\left(w_t^I + w_t^h X_t\right) \frac{\alpha_t^s}{w_t^I} = 1.$$

3. The bond market clears:

$$\left(w_t^I + w_t^h X_t\right) \frac{\alpha_t^b}{w_t^I} + w_t^h (1 - X_t) = 0. \tag{4}$$

4. The intermediation market clears:

$$Q_t = w_t^h X_t.$$

5. The goods market clears:

$$c_t + c_t^h = D_t.$$

The market clearing conditions for the stock market requires that the proportion of stocks in the specialists portfolio ($\frac{\alpha_t^s}{w_t^I}$), applied to the total funds under intermediation (specialist wealth plus intermediated funds), must add up to the total supply of stocks. The condition for the bond market is similar to that of stocks, but reflects that households may hold bonds directly, and that total bond holdings must sum to zero.

3 Solution

We derive the equilibrium by conjecturing a candidate pricing function and price process and then solving agents' decision problem given these prices. We then verify that given agent decisions, market clearing conditions recover the conjectured pricing function and price process.

3.1 State variables and candidate price functions

We look for a stationary Markov equilibrium where the state variables are (y_t, D_t), where $y_t \equiv \frac{w_t^h}{D_t}$ is the dividend scaled wealth of the household. Our economy with only specialists is a standard CRRA/GBM economy that has been fully analyzed in the literature. For this economy, the only state variable is D_t. Moreover, the economy scales up and down linearly with D_t. We thereby guess that D_t is a state variable, and that the economy scales with D_t.

Intermediation frictions imply that the distribution of wealth between households and specialists affects equilibrium. For example, whether capital constraints bind or not depends on the relative wealth of households and specialists. We conjecture that we need one state variable to capture the distribution of wealth in the economy. We have some freedom in choosing how to define the distribution state variable. It turns out that using the households' wealth is convenient for the analysis.

We conjecture that the equilibrium evolution of y_t may be written as an Ito process which solves the following Stochastic Differential Equation,

$$dy_t = \mu_y dt + \sigma_y dZ_t, \tag{5}$$

where the drift μ_y and the diffusion σ_y are well-behaved functions of y (to ensure the existence and the uniqueness of solution $\{y_t : 0 \leq t \leq \tau\}$).[9]

We also conjecture that we can write the equilibrium stock price as,

$$P_t = D_t F(y_t) \tag{6}$$

where $F : \mathbb{R} \rightarrow \mathbb{R}$ is twice continuously differentiable on its relevant domain. $F(y)$ is the price/dividend ratio of the stock.

We derive relations for the three unknown functions, $F(y)$, μ_y and σ_y.

3.2 Specialist and household decisions

We write the total return on the stock as,

$$dR_t = \frac{D_t dt + dP_t}{P_t}.$$

Applying Ito's Lemma to (6) yields the total return process for the stock:

$$dR_t = \left(g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 + \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma\right)dt + \left(\sigma + \frac{F'}{F}\sigma_y\right)dZ_t, \tag{7}$$

where (and in the analysis that follow) we omit the argument in $F(y)$, $F'(y)$ and $F''(y)$ for brevity.

Given this return process, optimality for the specialist gives us the standard consumption-based asset pricing relations:

$$-\rho dt - \gamma E_t\left[\frac{dc_t}{c_t}\right] + \frac{1}{2}\gamma(\gamma+1)Var_t\left[\frac{dc_t}{c_t}\right] + E_t\left[dR_t\right] = \gamma Cov_t\left[\frac{dc_t}{c_t}, dR_t\right] \tag{8}$$

where c_t is the consumption rate of the specialist. The Euler equation is valid for all $t \leq \tau$ (the time the economy hits the bankruptcy boundary) since the specialist is always marginal in trading assets in the economy. Assumption 4 ensures that the Euler equation is also valid at $t = \tau$. We use $E_t[\cdot]$ as the conditional expectation operator, and $Cov_t[\cdot,\cdot]$ ($Var_t[\cdot]$) as the conditional covariance (variance) operator.

For the short-term bond, since the bond price is always one,[10]

$$r_t dt = \rho dt + \gamma E_t\left[\frac{dc_t}{c_t}\right] - \frac{\gamma(\gamma+1)}{2}Var_t\left[\frac{dc_t}{c_t}\right]$$

Consider the household next. It is easily verified that for a log investor, the optimal consumption rate is,

$$c_t^h = \rho^h w_t^h,$$

[9] After bankruptcy, or $t > \tau$, the economy stays at the point $y = \frac{\delta}{\rho^h}$.

[10] The debt that household hold is riskless for $t < \tau$. But at $t = \tau$, intermediaries may default on their debt which makes household debt risky. In this case, the interest rate we describe will deviate from that of the riskless debt. We may imagine that the short-term interest rate we derive is on a repo loan between intermediaries, which is always repaid before any debts due to households. Such a repo loan will always be riskless.

regardless of the stochastic process for $\widehat{dR_t}$.[11]

3.3 Market clearing

Market clearing from the goods market is that,

$$c_t + c_t^h = D_t.$$

From our analysis of the household's problem, we can write c_t^h as a function of the state variables. Using market clearing, we infer the consumption rate of the specialist to be

$$c_t = D_t - \rho^h w_t^h = D_t(1 - \rho^h y_t).$$

In equilibrium, the stochastic processes c_t and R_t must jointly satisfy the Euler equation of the specialist. Applying Ito's Lemma to c_t we can rewrite the Euler equation for the specialist to find:[12]

Proposition 2 *The equilibrium Price/Dividend ratio $F(y)$ satisfies the ordinary differential equation (ODE),*

$$\begin{aligned} g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 \quad + \quad & \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma = \rho + \gamma g - \frac{\gamma\rho^h}{1-\rho^h y}(\mu_y + \sigma_y\sigma) \\ & + \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma + \frac{F'}{F}\sigma_y\right) - \frac{1}{2}\gamma(\gamma+1)\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)^2 \end{aligned} \tag{10}$$

Proposition 2 is an ODE that $F(y)$ must satisfy. μ_y and σ_y are unknown functions in this ODE and describe the dynamics of the households' wealth along the equilibrium path. We next turn to describing these dynamics.

We denote $\theta_s(y)$ and $\theta_b(y)$ as the stock and bond holdings (direct plus indirect) of the household, where $w_t^h = \theta_b(y_t) + \theta_s(y_t)P_t$.

3.4 Unconstrained intermediation

Consider a state (y_t, D_t) where every specialist has sufficient wealth that the capital constraint on intermediation does not bind. The unconstrained intermediation case arises when,

$$m w_t^I \geq w_t^h.$$

[11] The Euler equation for the log investor is:

$$-\rho^h dt - E_t\left[\frac{dc_t^h}{c_t^h}\right] + Var_t\left[\frac{dc_t^h}{c_t^h}\right] + E_t\left[\widehat{dR_t}\right] = Cov_t\left[\frac{dc_t^h}{c_t^h}, \widehat{dR_t}\right]$$

The solution $c_t^h = \rho^h w_t^h$ satisfies the Euler equation since $dc_t^h/c_t^h = dw_t^h/w_t^h = -\rho^h dt + \widehat{dR_t}$.

[12]

$$\begin{aligned} \frac{dc_t}{c_t} &= \frac{dD_t}{D_t} - \frac{\rho^h dy_t}{1-\rho^h y_t} - \frac{\rho^h}{1-\rho^h y}Cov_t\left[dy_t, \frac{dD_t}{D_t}\right] \\ &= \left(g - \frac{\rho^h}{1-\rho^h y_t}(\mu_y + \sigma_y\sigma)\right)dt + \left(\sigma - \frac{\rho^h}{1-\rho^h y_t}\sigma_y\right)dZ_t \end{aligned} \tag{9}$$

Since the capital constraint on intermediation does not bind, the household knows that specialists have a sufficient stake in the intermediary that the specialist will not shirk. Then, by Assumption 3, the household chooses to delegate 100% of its wealth to the intermediary. Through the intermediary, the specialist invests all of the wealth of the economy and chooses identical portfolio shares of stocks and bonds for each unit of this wealth.

As the bond is in zero net supply, the equilibrium portfolio share in bonds must be zero. All of the economy's wealth is invested in stocks. Then the households stock holdings are,

$$\theta_s(y) = \frac{w^h}{P} = \frac{y}{F(y)}.$$

Consider how a shock to dividends affects the wealth shares in this economy. Since both household and specialists own only stocks, a dividend shock leads to an equal percentage change in the wealth of both specialist and household. Shocks do not alter the distribution of wealth, which means that the wealth distribution state variable, y, is not affected by dividend shocks.

Formally,

Lemma 1 *When intermediation is not capital constrained, we have*

$$\sigma_y = 0 \quad \textit{and,} \quad \mu_y = \frac{1}{1-\theta_s F'}\left(\theta_s - \rho^h y\right).$$

Proof. For households, the change in y reflects any capital gains in the stock and any changes in the asset positions, i.e.,

$$dy = \theta_s dF + F d\theta_s.$$

We apply Ito's Lemma to $F(y)$ to expand dF. The second term reflects changes in the household's asset position from dividend inflows and household consumption, $DFd\theta_s = D\theta_s dt - \rho^h w^h dt$. Combining we arrive at the expressions in the Lemma. ■

In the unconstrained region, shocks have an equal effect on household and specialists. It is easy to see therefore that if the economy currently is in a state (y_t, D_t) where intermediation is not constrained, then y evolves deterministically and there is no path for which the economy will ever become constrained.[13] This perfect alignment of portfolios is broken in the model where $\lambda > 0$. In the latter case, the "debt" household always demands some debt. Since debt is in zero net supply, the specialist and household portfolios are no longer aligned. We provide the expressions for the wealth dynamics for the $\lambda > 0$ case in the Appendix and show that $\sigma_y > 0$.

When $\lambda = 0$, we arrive at a stark result:

Proposition 3 *When $y < y^c$, the equilibrium risk premium on the stock is constant:*

$$E_t[dR_t] - r_t dt = \gamma\sigma^2 dt.$$

[13]Strictly speaking this statement holds when $\mu_y < 0$, i.e., the household sector diminishes over time which requires a parameter restriction on ρ, ρ^h, γ, g, and σ^2, which ensures the uniqueness of our equilibrium when $\lambda = 0$ (see Apendix B for details).

Proof. The risk premium on the stock is given by $\gamma Cov_t\left[\frac{dc}{c}, dR\right]$. When $\sigma_y = 0$, the diffusion terms in both (7) and (9) are σ. ■

The risk premium in the unconstrained case corresponds exactly to what we would derive in an economy with the specialist intermediary as the representative agent, but without intermediation frictions. This result provides a counterpoint to the results we derive next for the case where intermediation is constrained.

3.5 Constrained intermediation

Intermediaries are capital constrained when $mw_t^I < w_t^h$. We can solve for the cutoff point between unconstrained and constrained intermediation as the point where $mw_t^I = w_t^h$. Since $w_t^I + w_t^h = P_t$, we can solve for this cutoff in terms of y as,

$$y^c = \frac{m}{m+1} F(y^c). \tag{11}$$

This equation implicitly defines the point when capital constraints arise. For $0 < y \leq y^c$, intermediation is not capital constrained. For $y > y^c$, intermediation is capital constrained. *(Find a place to define $w^{I,c}/D = F(y^c) - y^c$ since later on we will use it.)*

When specialist wealth falls (or household wealth rises) so that the economy falls into the constrained region, household do not invest 100% of their wealth with specialists. Households recognize that delegating more wealth to the intermediary will dilute the specialist's stake in the intermediary's return and violate the specialist's incentive compatibility constraint.

Households delegate the maximum possible to the specialist and invest the rest of their wealth in the riskless bond, $\theta_b = w^h - \theta_s P > 0$. The positive bond holding in the constrained region play a key role in the economy's dynamics because it breaks the perfect alignment between specialist and household portfolios (since bonds are in zero net supply).

We can solve explicitly for the household stock position of θ_s. Since all stocks are held through the intermediaries, $\theta_s(y) = m\alpha^s$, where α^s is the stock holdings of the specialists. But since, $m\alpha^s + \alpha^s = 1$ (i.e., all stocks have to be held through intermediaries), we find

$$\theta_s(y) = \frac{m}{1+m}.$$

Note that the stock holdings of both specialist and household are constant in this region. The stock holdings increase towards one as m rises. When m is large, the specialist is required to hold only a small stake in the intermediary, so that risk sharing is improved.

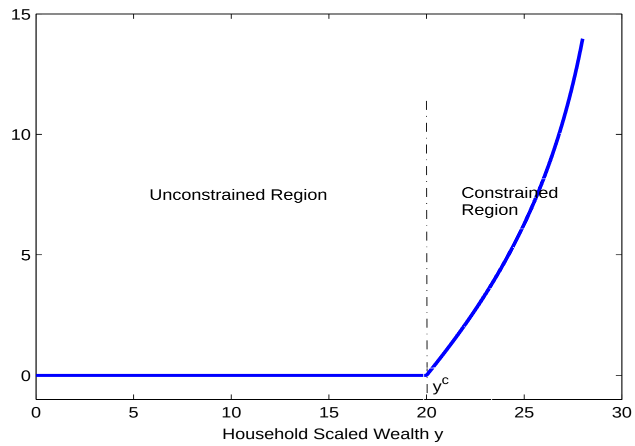
Given these stock holdings, we find that the bond holdings of the household are,

$$\hat{\theta}_b \equiv \frac{\theta_b}{D} = y - \theta_s F(y)$$

It is convenient to scale the bond-holdings by dividends for the analysis.

Figure 1 graphs the scaled bond-holdings as function of y. We discuss the parameter choices behind this figure in the next section. We note from the figure that the bond-holdings are zero in the unconstrained region, and increase sharply in the constrained region.

Figure 1: Scaled Bond-holdings



The scaled bond-holdings ($\hat{\theta}_b$) are graphed as a function of scaled household wealth (y). The parameters are $\rho = 0.015$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 1$, $\alpha = 0.7$, $\lambda = 0$.

Lemma 2 *When $y > y^c$ so that the intermediary sector is capital constrained,we have*

$$\sigma_y = -\frac{\hat{\theta}_b}{1-\theta_s F'}\sigma$$

and,

$$\mu_y = \frac{1}{1-\theta_s F'}\left(\theta_s + (r+\sigma^2-g)\hat{\theta}_b - \rho^h y + \frac{1}{2}\theta_s F''\sigma_y^2\right)$$

where,

$$r = \rho + \gamma\left(g - \frac{\rho^h(\mu_y + \sigma\sigma_y)}{1-\rho^h y}\right) - \frac{\gamma(\gamma+1)}{2}\left(\sigma - \frac{\rho^h\sigma_y}{1-\rho^h y}\right)^2.$$

Proof. The household's scaled wealth is

$$y = \theta_s F + \hat{\theta}_b.$$

Differentiating y we find,

$$\begin{aligned} dy &= \theta_s dF - \frac{\theta_b}{D^2}dD + \frac{\theta_b}{D^3}Var_t[dD] + Fd\theta_s + d\hat{\theta}_b \\ &= \theta_s dF - \hat{\theta}_b\frac{dD}{D} + \hat{\theta}_b\sigma^2 dt + (\theta_s + r\hat{\theta}_b - \rho^h y)dt. \end{aligned}$$

We have used the accounting identity, $Fd\theta_s + d\hat{\theta}_b = (\theta_s + r\hat{\theta}_b - \rho^h y)dt$, that governs changes in the households' assets position to go from the first to the second line. Substituting in for dF we arrive at the statements of the Lemma. ■

σ_y is less than zero in the constrained region,[14] and increasingly so as the households' scaled bond holding $\hat{\theta}_b > 0$ rises. In other words, a negative innovation to D, or a negative shock to the stock market, increases the households scaled wealth y and tightens the capital constraint.

In the constrained region, households withdraw funds from intermediaries recognizing that not doing so will lead intermediaries to shirk on the due-diligence decision. They withdraw funds until the point that the intermediaries capital constraint is met.

But in withdrawing funds from intermediaries, households also reduce their indirect participation in the stock market, while increasing bond holdings. Specialists absorb these changes by increasing their borrowing to buy the stock.

$$w^h = \frac{m}{m+1}P + \theta_b \text{ and } w^I = \frac{1}{m+1}P - \theta_b.$$

Intermediaries have a leveraged position in the risky asset in the constrained region. Since $\theta_b > 0$, a shock to dividends produces a muted reaction in the households' wealth, w^h, and a more amplified reaction in the specialists' wealth. As a result, $\sigma_y < 0$ and increasingly so as θ_b rises.

The increased volatility in the specialists' consumption leads to a larger effective risk aversion and induces a higher risk premium on the stock. In this sense, the risk aversion of the specialist endogenously increases as capital constraints are tightened:

Proposition 4 *In the constrained region the risk premium on the risky asset is:*

$$E_t[dR_t] - r_t dt = \gamma\sigma^2 \left(1 + \frac{\rho^h}{1-\rho^h y}\frac{1}{1-\theta_s F'}\hat{\theta}_b\right)\left(1 - \frac{F'}{F(1-\theta_s F')}\hat{\theta}_b\right)dt$$

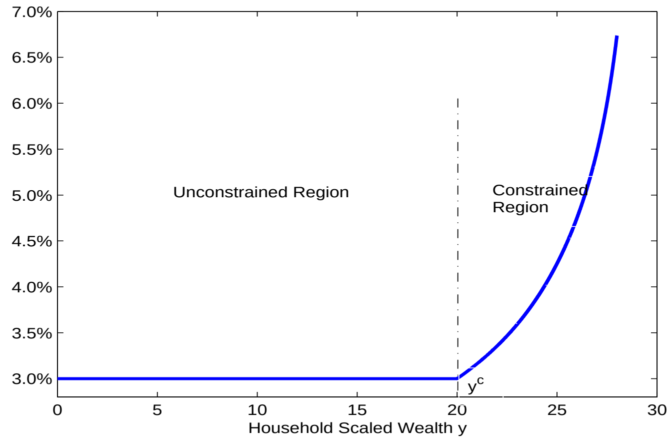
If $F' < 0$ then the risk premium is unambiguously higher in the constrained region than the unconstrained region.

The first term in parentheses in the expression for the risk premium captures the volatility of the pricing kernel and the second term in parentheses is the loading of the stock's return on the pricing kernel. When $F' < 0$, both effects contribute to a higher risk premium.

Figure 2 graphs the risk premium as a function of y. The pattern of the risk premium resembles the pattern pictured earlier for the scaled bond-holdings.

[14] We can show that $1-\theta_s F'$ is always positive. To see this, note that $G(y) \equiv \frac{1}{1-\theta_s F'}$ (defined in equation (19) of the Appendix) is positive when $y = y^c$. Suppose that G goes to positive infinity. The RHS is dominated by the fourth term which is negative, while on the LHS G' has a positive coefficient. Hence $G' < 0$, contradiction.

Figure 2: Risk Premium



The risk premium is graphed as a function of scaled household wealth (y). The parameters are $\rho = 0.015$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 1$, $\alpha = 0.7$, $\lambda = 0$.

3.6 Bankrupt intermediaries

At the point $w^I = P - w^h = 0$, intermediaries go bankrupt and the economy is defined as in Assumption 4. The bankruptcy threshold is implicitly defined by the equation,

$$y^b = F(y^b).$$

Given our assumption on bankruptcy, the households own the risky asset after this event but receive a lower dividend stream. Since the households have log preferences with discount rate of ρ^h, we have

$$F(y^b) = \frac{\delta}{\rho^h}.$$

This equation specifies one boundary condition for the ODE of Proposition 2.

4 Analysis of Equilibrium

The ODE of Proposition 2 does not admit an analytical solution. In this section, we solve the model numerically and analyze the equilibrium. We present numerical solutions of the model for both the $\lambda = 0$ case as well as the $\lambda > 0$ case.

The solution method is detailed in Appendix B. We solve the ODE subject to boundary conditions at $y = 0$ (fully unconstrained intermediation) and $y = y^b$ (bankrupt intermediaries).

4.1 Conditional asset pricing

We note at the outset that our analysis emphasizes the behavior of asset prices *conditional* on a given dividend and intermediation state (D_t, y_t). In particular, we study how the degree of capital constraints affects equilibrium. We do not explain how the economy arrives at a given state. Given any initial condition, the economy of our model eventually converges to one of two points: either the specialist has all of the wealth, or the household has all of the wealth. This aspect of the model is a well known property of two-agent models. Although there are assumptions that can be introduced (e.g., rebirth) to ensure a non-degenerate steady-state distribution, we have chosen not to complicate the model with such auxiliary assumptions in order to retain focus on the intermediation effects of the current model.

4.2 Parameters

Table 1: Baseline Parameters

Panel A: Intermediation		
m	Intermediation multiplier	0.5 - 3
λ	Debt ratio	0, 0.02
$F(y^b)/F(0)$	Bankruptcy boundary	70%
Panel B: Preferences and Cashflows		
g	Dividend growth	2%
σ	Dividend volatility	10%
ρ^h	Time discount rate of household	2.5%
ρ	Time discount rate of specialist	1.5 - 2%
γ	RRA of specialist	3

PARAMETERIZING m

The intermediation multiplier m is the focus of our model. m measures the capital requirement of an intermediary. Increasing m means that for every dollar of specialist wealth, the specialist can attract more funds from households. There are two main effects of m that arise in our model.

The primary effect of increasing m, for a given wealth distribution, is to relax intermediaries' capital constraint (a "constraint effect"). Thus, increasing m reduces intermediation frictions and increases capital mobility. In the limit, if we take m towards infinity, households participate fully in the risky asset market regardless of the specialist wealth.

For moderate values of m, a second effect arises. If we focus on the states of the world where intermediaries are constrained, a \$1 fall in specialist wealth leads to an \$$m$ outflow of household funds from intermediaries. Higher values of m thereby make intermediation more sensitive to capital shocks in the constrained region and lead to more dramatic bankruptcy effects (a "sensitivity effect"). We illustrate these effects of m in our numerical solutions.

Within our model m represents the capital requirement/performance share of the manager-equity holders of an intermediary. Typical hedge fund contracts call for managers to be paid $20-25\%$ of the return on assets under management. Note that $\frac{1}{1+m}$ is the analogous quantity in our model. Thus, our model suggests a value of m equal to three to parameterize a hedge fund contract paying 25%.

We consider two different parameterization of m in our model. One captures the hedge funds case noted above, in which we choose m equal to three. Moreover we are most interested in studying how this case may shed light on dramatic asset pricing effects, such as those evidenced in the fall of 1998 crises. We focus on the region of the state space where the economy is sufficiently capital constrained that intermediaries are leveraged ($\theta_b > 0$, see Figure 1).

The second parameterization aims to consider intermediaries more broadly to include banks, mutuals funds, hedge funds, and insurance companies. For the broad class of intermediaries, it is not possible to discern the value of m from contractual arrangements. Indeed, in many of these cases, implicit contracts underly intermediation. For example, mutual funds do not have capital requirements and contractually receive minimal performance-related compensation. However, it is well established that implicit contracts through the performance/inflows relation do provide performance related compensation to a fund's manager. Moreover, the reputational capital of mutual fund company is to some extent at stake in each mutual fund. In this case, the capital requirement of our model captures an implicit contract underlying fund management. For our second parameterization, the single m parameter captures the capital effects across many modes of intermediation.

Allen (2001) reports that in the 1990s, 50% of total wealth was intermediated while 50% was directly invested in financial assets. In our model, specialists play two roles. First, they directly invest their wealth in the risky asset. Second, every specialist intermediates some of the funds of the passive household, and all households are only exposed to the risky asset through intermediaries.

However, our model is isophormic to one in which some specialists only invest their own funds, and other specialists invest their own funds as well as intermediate the funds of households. Imagine that a fraction κ of the specialists ran intermediaries, and $1-\kappa$ only made direct investments. All specialist agents begin with the same wealth, and the intermediaries are subject to a capital requirement of M. Then, the wealth of these two types of specialists will evolve identically because both agents choose the same portfolios (this holds only for the case we consider where $C=0$). In aggregate the supply of intermediation is given by κM times the wealth of specialists. If we define $m=\kappa M$, this variation reduces back to the model we have solved.

Rather than base m on an observation about contracts, we choose m to reflect the balance of funds in and outside intermediation, based on Allen (2001). We interpret the specialists as the direct investors of

Allen, and the households as the intermediated investors. Thus, if specialists control half of the wealth of the economy and intermediate the wealth of the other half of the economy, m should be around one. As noted above this does not mean that intermediaries have a 50% capital requirement. It may mean, for example, that intermediaries have a 10% capital requirement, with $\kappa = 11\%$ $\left(\kappa = \frac{\frac{1-0.5}{0.5}}{\frac{1-0.1}{0.1}}\right)$.

REST OF INTERMEDIATION PARAMETERS

The parameter λ affects the transition probability between the unconstrained states and the constrained states. For $\lambda > 0$ negative shocks to dividends shift wealth from specialists to households.

For the case of $m = 1$, we set $\lambda = 0$. This case illustrates the pure m effect of the model. For the larger m case where we model hedge funds, we set $\lambda = 0.02$. This parameter choice allows for some transition of paths, but still allows that m effects dominate the determination of equilibrium.

We choose δ so that the price/dividend ratio at $y = y^b$ is 70% of the price/dividend ratio at $y = 0$. In some sense, the choice of δ also defines the range of the price/dividend ratio. In the data, the price/dividend ratio on the stock market varies substantially from maximium to minumum: the minimum is roughly 30% of the maximum. We are being relatively conservative with this choice of $F(y^b)$. For ease of interpretation, we transform the parameter δ to α, where α is defined as $\frac{F(y^b)}{F(0)}$.[15] We provide sensitivity analysis based on alternative values of $F(y^b)/F(0)$ (we denote this ratio as α).

PREFERENCES AND CASHFLOW PARAMETERS

The rest of the parameters govern cashflows and preferences, and are fairly standard. We set $g = 2\%$. This number is in the range of estimates for economic growth, consumption growth, and stock-market dividend growth. We choose $\sigma = 10\%$. The number matches the dividend volatility of the stock market. Volatility of consumption growth is an order of magnitude smaller. Since our model concerns the financial position of intermediaries, matching σ to financial volatility seems appropriate.[16]

We choose $\rho^h = 2.5\%$ and values of ρ between $1.5 - 2\%$. These values of the time preference parameter produces a riskless interest rate around $1 - 1.5\%$ in the unconstrained region. We use a different ρ than ρ^h because ρ^h primarily affects the economy by changing the bankruptcy boundary condition ($F(y^b) = \frac{\delta}{\rho^h}$), while ρ primarily affects the riskless interest rate.

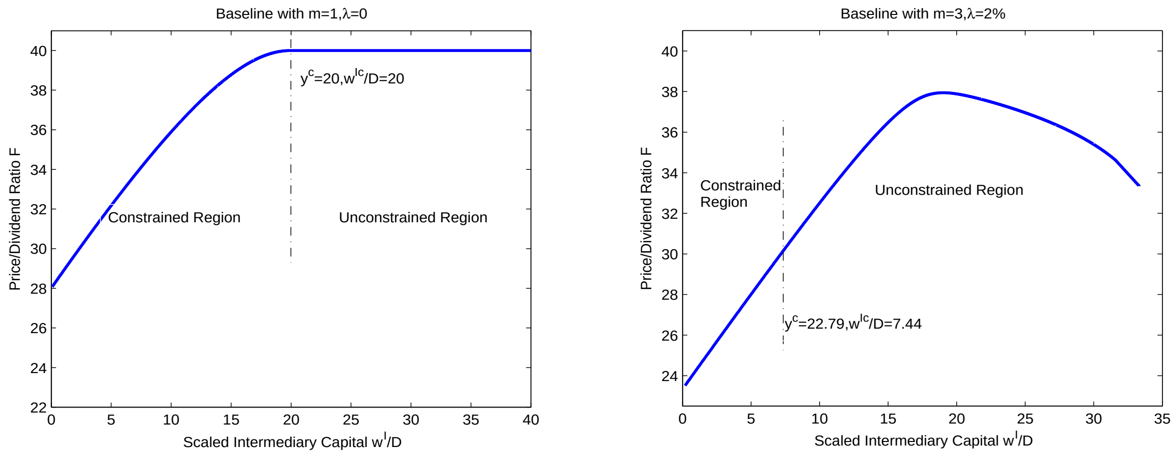
We set the risk-aversion parameter for the specialists to be $\gamma = 3$. Both the ρ and γ numbers are in the range that is typical in the literature.

[15] Since $F(0) = \frac{1}{\rho + g(\gamma-1) + \frac{\gamma(1-\gamma)}{2}\sigma^2}$ (see Appendix B) and $F(y^b) = \frac{\delta}{\rho^h}$, we have $\alpha = \delta\left(\frac{\rho+g(\gamma-1)}{\rho^h} + \frac{\gamma(1-\gamma)}{2\rho^h}\sigma^2\right)$. Under baseline parameters, $\delta = 70\%$ ($\rho = 1.5\%$) or $\delta = 58\%$ ($\rho = 2\%$).

[16] For example, we could imagine that there is a class of households who receive a fixed labor income each period that must be added to aggregate dividends when computing aggregate consumption. This will tend to reduce consumption volatility below 10%.

4.3 Price/dividend ratio

Figure 3: Price/Dividend Ratio



The equilibrium price/dividend ratio is graphed against the scaled-intermediary capital (w^I/D). Parameters for the left panel are $\rho = 0.015$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 1$, $\alpha = 0.7$, $\lambda = 0$. Parameters for the right panel are $\rho = 0.02$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 3$, $\alpha = 0.7$, $\lambda = 0.02$.

Figure 3 graphs the price/dividend ratio for each baseline parameters The x-axis on this graph is scaled-intermediary capital (w^I/D) rather than the state variable y that is used in our derivation (as well as previous graphs). w^I/D is a monotone transformation of y, so that the graphs present the same results. Using w^I/D allows us to more directly illustrate how a change in intermediary capital affects the equilibrium.

There are three effects that determine the shape of $F(\cdot)$ in Figure 3: the bankruptcy boundary condition, an interest rate effect, and a risk premium effect. When $w^I = 0$ (or $y = y^b$), $F(\cdot)$ is pinned down by the bankruptcy boundary condition. For highes values of w^I, $F(\cdot)$ is determined by the discounted value of the dividend stream, accounting for the possibility that at bankruptcy, the dividend stream ends and is replaced by $F(y^b)$. Thus close to $w^I = 0$, the bankruptcy boundary weighs heavily in the determination of $F(\cdot)$. However, for larger values of w^I, the chances of hitting bankruptcy are sufficiently small that the discount rates applied to the dividend stream play the larger role.

$F(\cdot)$ is graphed for two baseline parameterizations in Figure 3: The figure in the left-hand panel corresponds to the case of $m = 1$ and $\lambda = 0$; the figure on the right corresponds to $m = 3$ and $\lambda = 0.02$. As noted earlier, a larger m leads to a smaller constrained region (right panel)

In the unconstrained region of the left panel, $F(\cdot)$ is constant. When $\lambda = 0$ there is no chance of the economy moving from the unconstrained to the constrained region. As a result the lower values of $F(\cdot)$ in the constrained region never enter when calculating $F(\cdot)$ in the unconstrained region. In addition, as we have noted earlier, the risk premium is constant in the unconstrained region. Finally, as we show below, our parameter choices lead the interest rate to be constant in the unconstrained region (see Figure 4). Together, these factors lead $F(\cdot)$ to be constant in the unconstrained region.

As we have remarked (see Proposition 4), the risk premium rises moving from the unconstrained region to the constrained region. The fall in $F(\cdot)$ in the constrained region, to the right of $w^I/D = 20$, is in part due to this rising risk premium.

In the right-hand panel, $F(\cdot)$ has a more pronounced shape. First, note that $F(\cdot)$ is falling even to the right of $w^{I,c}$. This occurs because when $\lambda > 0$, the economy has some chance of entering the constrained region and the prices in the constrained region weigh into the computation of $F(\cdot)$ in the unconstrained region. The fall in $F(\cdot)$ in the unconstrained region occurs because the interest rate falls through the unconstrained region.

4.4 Risk and return: $m = 1$ baseline

Figure 4 graphs the risk premium and interest rate for the case of $m = 1$ as well as a variation with $m = 0.6$. The general message from the figure is clear: when the intermediary sector is capital constrained, the risk premium is high and the interest rate low; when the intermediary sector is unconstrained, the risk premium is low while the interest rate is high.

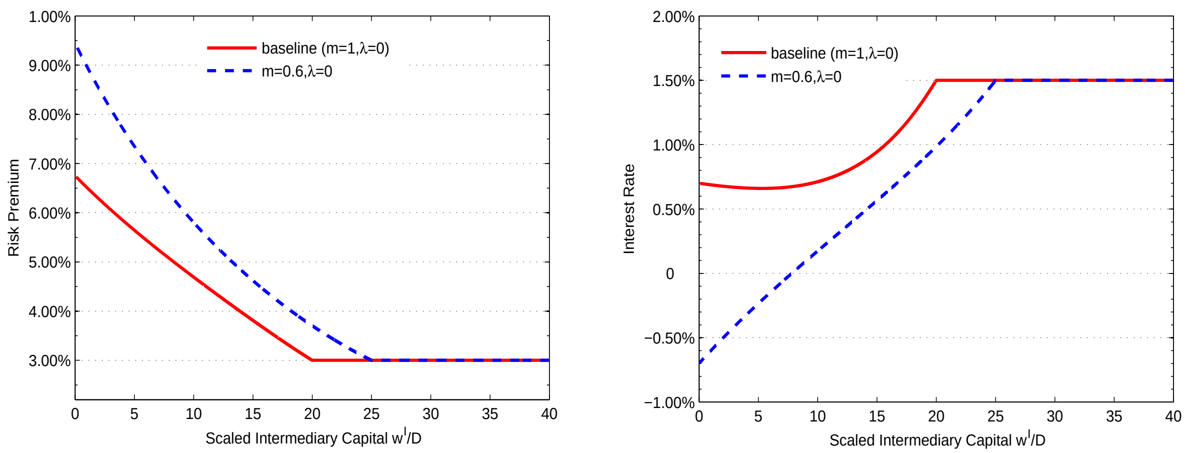
For the baseline case with $\lambda = 0$ and $m = 1$, the risk premium rises from 3% to about 6.8%, while the interest rate drops from 1.5% to 0.7%. If we focus on the point where constraints begin to bind ($w^I/D = 20$), then a 10% fall in intermediary capital raises the risk premium about 31 bps and lowers the interest rate about 27 bps.

The risk premium rises because the specialist is marginal in pricing the risky asset, and as the specialist becomes more constrained, his consumption volatility endogenously rises causing his effective risk aversion to rise. The fall in the interest rate is because, as intermediary capital falls, households withdraw funds from intermediaries and purchase the riskless bond. This extra demand for bonds lowers the interest rate.

The graphs present risk premia and interest rates for the cases of $m = 1$ and $m = 0.6$. As we have noted, a larger m relaxes the capital constraint, since with any given level of capital the specialist can intermediate more funds when m is larger. The larger the m, the smaller the size of the constrained region.

The left-panel of Figure 5 graphs the specialist's debt position as a function of intermediary capital, for both the $m = 1$ and $m = 0.6$ cases. As noted above, when $m = 1$, intermediary capital has to fall further in order to enter the constrained region (the solid line is to the left of the dashed line). Thus, for every level of intermediary capital, the debt position is smaller when $m = 1$. Notice that the solid and dashed

Figure 4: Risk Premium and Interest Rate



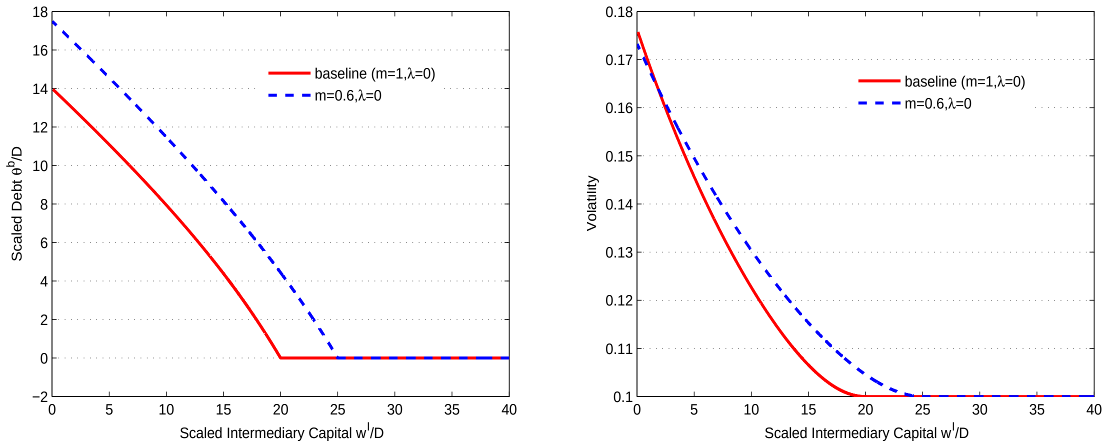
Risk premium (left panel) and interest rate (right panel) are graphed against scaled-specialist wealth (w^I/D). Two parameter sets are illustrated in the graphs. The solid lines correspond to $\rho = 0.015$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 1$, $\alpha = 0.7$, $\lambda = 0$. The dashed lines correspond to a change from $m = 1$ to $m = 0.6$.

curves pictured in the figure have a similar slope. Indeed the primary difference between the two curves is the point ($w^{I,c}/D$) delineating the constrained region. The larger debt position due to a smaller m tends to make specialist consumption more volatile and contributes to the higher risk premium in the small m case (see Figure 4). This is the constraints effect we allude to earler.

There is a second effect of m in the equilibrium. The right-panel of Figure 5 graphs the price volatility of the risky asset as a function of intermediary capital. Note that, upon entering the constrained region, volatility rises in both cases, but rises faster for the $m = 1$ case than the $m = 0.6$ case.

Price volatility rises in the constrained region for two reasons. First, as specialist consumption experiences more fluctuations in the constrained region, $|\sigma_y|$ rises and the pricing kernel becomes more volatile. Thus the discount rates applied to the risky asset's cash flows are more variable resulting in higher price volatility. Second, since the constrained region is smaller when m is larger, $F(\cdot)$ falls more dramatically over the constrained region. The larger slope for the $m = 1$ case means that $F(y)$ is more sensitive to y in the constrained region (i.e., a larger F'). Then shocks to the distribution of wealth have a magnified effect on price changes. This

Figure 5: Debt and Volatility



In the left panel, $\hat{\theta}_b$ is graphed as a function of scaled intermediary capital (w^I/D). In the right panel, the volatility of the risky asset is graphed as a function of scaled intermediary capital. The parameters are $\rho = 0.015$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $\alpha = 0.7$, $\lambda = 0$. $m = 1$ is pictured as the solid line, and $m = 0.6$ is pictured as the dashed line.

second effect (a sensitivity effect) also contributes to higher price volatility.

The latter effect is behind the difference between the $m = 1$ and $m = 0.6$ case. Since the constrained region is smaller when m is higher, the risky asset price falls faster for larger m. Thus, shocks have a larger effect on prices leading to higher price volatility in the constrained region.

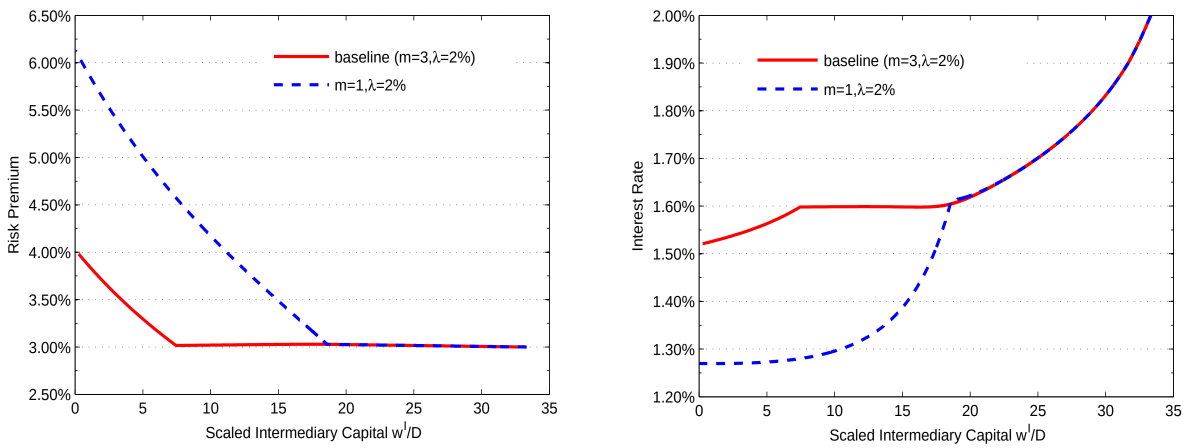
Taken together, these effects lead to a smaller absolute risk premium when m is smaller, but a risk premium that is more sensitive to changes in intermediary capital. In Figure 4, we note that slope of the risk premium, upon entering the constrained region, is higher when $m = 1$ than when $m = 0.6$.

4.5 Risk and return: $m = 3$ baseline

Figure 6 graphs the risk premium and interest rate for the parameterization with larger m ($m = 3$) and $\lambda > 0$. We also picture $m = 1$ for comparison. The patterns in the figure are qualitatively similar to those illustrated earlier. The larger m case leads to a much smaller constrained region. This leads to a smaller rise in the risk premium in the constrained region. The interest rate falls through the constrained region.

One difference relative to the earlier parameterization is that the interest rate also falls through the unconstrained region. Indeed, the falling interest rate is behind the falling price/dividend ratio in the unconstrained region of Figure 3. The interest rate falls because when $\lambda > 0$ the households always desire a positive holding of bonds, roughly proportional to their wealth of y. The specialist takes a short position in the bond to clear the bond market. As y rises, the short position rises, and the interest rate has to fall in order to induce the specialist to short the bond. This line of reasoning is similar to that which we offered earlier for the fall in the interest rate in the constrained region, where the specialist has an increasingly leveraged position.

Figure 6: Risk Premium and Interest Rate



Risk premium (left-panel) and interest rate (right panel) are graphed against scaled-specialist wealth (w^I/D). Two parameter sets are illustrated in the graphs. The solid lines correspond to $\rho = 0.02$, $\rho^h = 0.025$, $\gamma = 3$, $g = 2\%$, $\sigma = 10\%$, $m = 3$, $\alpha = 0.7$, $\lambda = 0.02$. The dashed lines correspond to a change from $m = 3$ to $m = 1$.

5 Liquidity Crisis

In this section we discuss how our model sheds light on a liquidity crisis. We focus on the constrained region of the economy in which leverage is high and shocks have more dramatic capital effects.

5.1 Capital shocks and flight to quality

In the constrained region, the specialist's wealth is,

$$w^I = \frac{1}{m+1}P - \theta_b$$

We can study how a shock that moves the price of the risky asset affects specialist wealth by computing the elasticity of wealth with respect to the price of the risky asset:[17]

$$\frac{dw^I}{w^I} = \frac{dP}{P}\frac{1}{1-(m+1)\frac{\hat{\theta}_b}{F}}. \tag{12}$$

A larger value of θ_b increases the effect of price shocks on specialist's capital. This is a pure leverage effect. Moving further into the constrained region, leverage rises, and the elasticity rises.

A decrease in intermediary capital, in the constrained region, also reduces the amount of funds that households delegate through intermediaries. A drop of \$1 in intermediary capital reduces delegation by $m\times$ \$1. In percentage terms, the elasticity of delegated funds with respect to the risky asset price is equal to the elasticity in (12)

Figure 7 graphs the fund size elasticity, focusing purely on the constrained region. The x-axis on this graph has been modified to be the degree of capital constraints, measured as $\frac{mw^I}{w^h}$. This serves to present a clearer comparison across the different m's.

In the unconstrained region, the elasticity is one (this is the right-most point on the graphs, where $\frac{mw^I}{w^h} = 1$; to the right of this point the elasticity remains one). A one percent fall in the price mechanically reduces the total assets under intermediation by one percent. In the constrained region, the elasticity rises as shocks to specialist wealth create a multiplier effect.

We can see from equation (12) that for a given $\hat{\theta}_b$, larger values of m increases the leverage effect and lead to larger capital shocks. Although a larger value of m means that the specialist has a smaller absolute stake in the intermediary (hence an exposure of $\frac{1}{m+1}$), to be in the constrained region, the specialist's wealth must be small. This means that a larger m leads to a greater percentage exposure of specialist wealth to stock prices, which results in a higher elasticity. This sensitivity effect of m is also visible in the figure. Note also that while increasing m increases the elasticity, it does so only slightly relative to the effect of being deeper into the capital constrained region (further left on the figure).

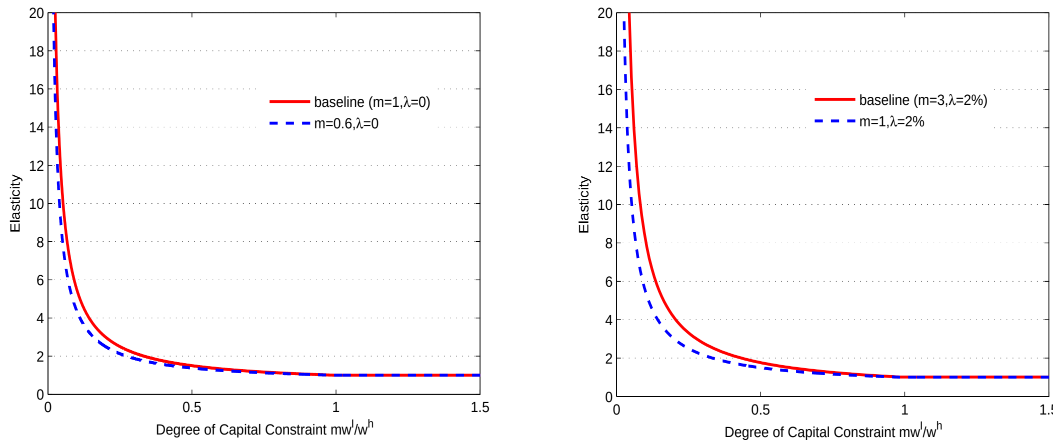
The patterns of disintermediation presented in Figure 7 are consistent with a "flight to quality." Household withdraw funds from intermediaries and increase their investment in bonds in response to negative price shocks. The increased bond demand leads to a fall in the equilibrium interest rate in the constrained region.

5.2 Market liquidity and asset comovement

In the capital constrained region, an individual specialist who may want to sell some risky asset faces buyers with reduced capital. Additionally, since households reduce their (indirect) participation in the risky asset market, the set of buyers of the risky asset effectively shrinks in the constrained region. In this sense, the

[17]This is a partial equilibrium exercise, since P is a function of endogenous variables, and a shock that moves P will also move θ_b. Computing the elasticity with respect to the underlying exogenous shocks yields a similar picture to that presented.

Figure 7: Fund Size Elasticity



The elasticity of intermediary size with respect to the price of the risky asset is graphed against the degree of capital constraints ($\frac{mw^I}{w^h}$). The elasticity increases sharply in the constrained region.

market for the risky asset "dries up.". On the other hand, if a specialist wished to sell some bonds, then the potential buyers include both specialists as well as households. Thus the bond is more liquid than the stock.

There are further connections we can draw between low intermediary capital and aggregate illiquidity periods.[18] As we have already seen, a negative shock in the constrained region leads to a rise in risk premia, volatility, and fall in interest rate. Our model also generates the increase in comovement of assets that many papers have documented as an empirical regularity during periods of low aggregate liquidity (see for example, Chordia, Roll, and Subrahmanyam, 2000). To show this within our model, we extend the model slightly.

We consider the pricing of a new risky asset, j, in the economy. Investment in this asset is also subject to the delegation friction. The intermediated asset-j is infinitesimal relative to the original economy in the sense that equilibrium is not affected by the introduction of this asset.[19]

We assume intermediated asset j has dividends,

$$\frac{dD^j}{D^j} = g^j dt + \sigma^j dZ_t + \hat{\sigma}^j d\hat{Z}_t^j \quad \text{where} \quad Cov_t(dZ_t, d\hat{Z}_t) = 0$$

where dZ_t is the common factor modeled earlier, and $\hat{Z}_t^{\,j}$ is a second Brownian Motion, orthogonal to Z_t,

[18] Eisfeldt (2004) presents a model in which aggregate liquidity varies across the business cycle. In Eisfeldt's model, during bad times, adverse selection is high leading to higher illiquidity of assets.

[19] If the asset was traded by both households and specialists then its introduction will have an effect on equilibrium, since the market is incomplete. However, introducing an intermediated asset will not alter equilibrium.

which captures asset-j's idiosyncratic variation. As before, we guess that $P_t^j = D_t^j F^j(y_t)$, and define

$$dR_t^j = \frac{D_t^j dt + dP_t^j}{P_t^j}.$$

The price of asset-j must satisfy the Euler equation for the specialist:

$$E_t[d(e^{-\rho^h t} c_t^{-\gamma} R_t^j)] = 0.$$

Repeating the steps as before we derive the following 2^{nd}-order ODE for asset j's price/dividend ratio $F^j(y)$:[20]

$$\begin{aligned}
\frac{E_t\left[dR_t^j\right]}{dt} - r_t &= g^j + \frac{F^{j\prime}}{F^j}\left(\mu_y + \sigma_y \sigma^j\right) + \frac{1}{2}\frac{F^{j\prime\prime}}{F^j}\sigma_y^2 + \frac{1}{F^j} - r_t \\
&= \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma^j + \frac{F^{j\prime}}{F^j}\sigma_y\right)
\end{aligned}$$

The above equation is just the risk premium on asset j. In the unconstrained region, $\sigma_y = 0$ (since $\hat{\theta}_b = 0$), so that the risk premium is $\gamma\sigma\sigma^j$, which is positive if $\sigma^j > 0$. But in the constrained region $\sigma_y = \frac{-\hat{\theta}_b\sigma}{1-\theta_s F'} < 0$, the risk premium rises, and unambiguously so for the case of $F^{j\prime} < 0$.

Since asset-j is subject to the delegation friction, the pool of buyers for the asset shrinks when capital constraints bind making asset-j more illiquid. Thus, when the capital constraint binds and the aggregate market becomes less liquid, all intermediated assets become less liquid and yield a higher expected return.

Note also that there is a spillover to asset-j from the risky asset in the constrained region. A negative innovation to D increases y and $\hat{\theta}_b$.

Since the diffusion term on dR^j is $\left(\sigma^j + \frac{F^{j\prime}}{F^j}\sigma_y\right) dZ_t + \hat{\sigma}^j d\hat{Z}_t^j$, we can calculate the correlation between asset j and the market return as[21]

$$Corr_t\left[dR^j, dR\right] = \left(1 + \frac{\hat{\sigma}^{j2}}{\left(\sigma^j + \frac{F^{j\prime}}{F^j}\sigma_y\right)^2}\right)^{-1/2} \times sign\left(\sigma^j + \frac{F^{j\prime}}{F^j}\sigma_y\right).$$

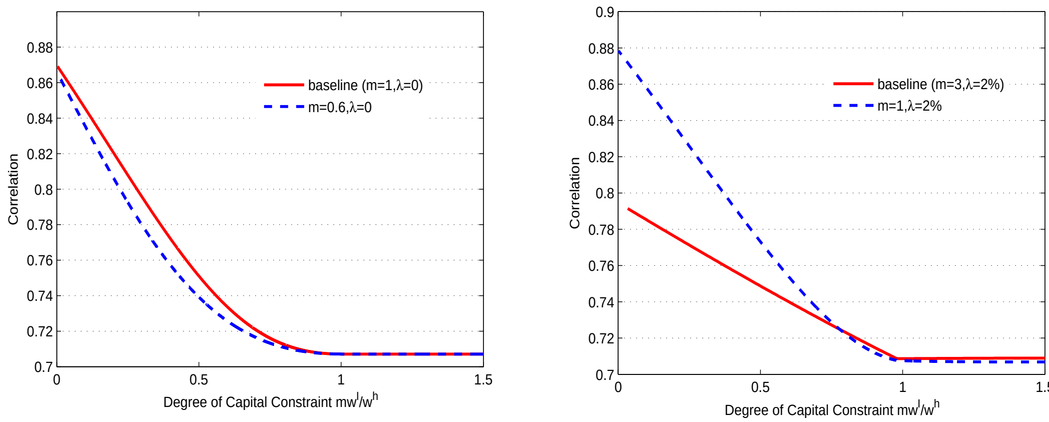
In the capital constrained region, the correlation between intermediated assets tends to rise. Consider the asset where $g^j = g$, $\sigma^j = \sigma$, and $\delta^j = \delta$; this is an asset which is a noisy version of the market asset. For this case, $F^j(y) = F(y)$. In the unconstrained region, the correlation is constant, while in the constrained region the correlation rises.

Figure 8 illustrates this effect for the case where $\hat{\sigma}^j = 0.1$ and our various parametererizations. The correlations are close to constant in the unconstrained regions (the points to the right of $\frac{mw^I}{w^h} = 1$; hence we truncate the plots at $\frac{mw^I}{w^h} = 1.5$). In the constrained region, the correlations rise monotonically. The figure

[20]The boundary conditions for the ODE are, $F^j(0) = \frac{1}{\rho + \gamma g - g^j - \frac{\gamma(1+\gamma)\sigma^2}{2} + \gamma\sigma\sigma^j}$, and $F^j(y^b) = \frac{\delta^j}{\rho^h + g - \sigma^2 - g^j + \sigma\sigma^j}$, where δ^j is the potentical loss in bankruptcy for asset-j.

[21]We compute the correlation between intermediated asset-j and the market asset. The empirical literature documents co-movement among intermediated assets. It is straightforward to introduce two infinitesimal intermediated assets and compute their pairwise correlation. The correlation will follow a similar pattern to the one we have graphed.

Figure 8: Correlation



The correlation between asset j and the market is graphed against the degree of capital constraints, for various parameterizations. When the economy enters the constrained region, the correlation rises as the capital constraint drives all prices.

also illustrates the effect of m. A larger m leads to an initially sharper rise in correlations. This effect is for the same reason that price volatility rises faster when m is larger – that is, the constrained region is smaller, leading to more dramatic effects for both the market asset as well as asset j (the sensitivity effect). These assets thereby become more correlated. In the right panel we see that increasing m lowers the correlations over the entire constrained range. This occurs because the constraints effect we have noted earlier dominates in this case.

In the fall of 1998 hedge fund crisis, the correlation of prices among intermediated asset rose. In many cases, the correlations rose so dramatically that the risk-management models of hedge funds failed. When intermediation capital falls, the investors in intermediated assets are simultaneously affected across all of their markets. In general equilibrium, the prices of these intermediated assets exhibit the comovement we observe in practice. Indeed, such comovement is symptomatic of an aggregate liquidity shortage.

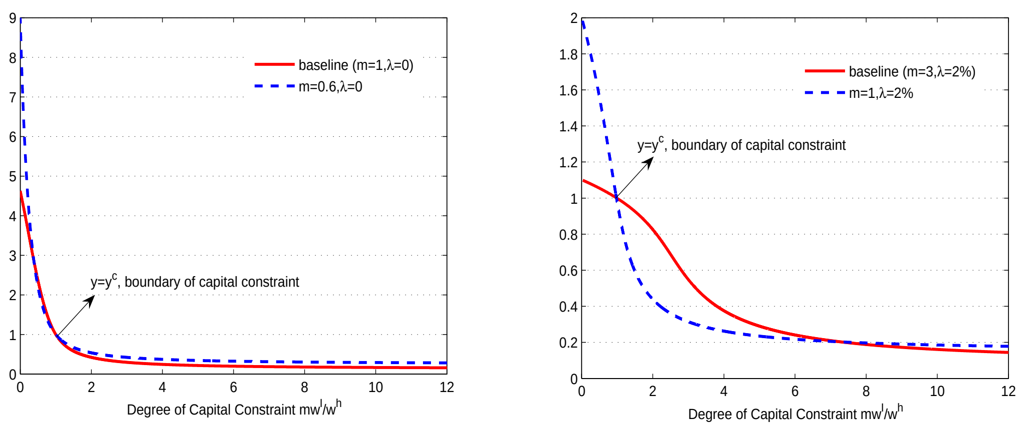
5.3 Hedging declines in liquidity

The effects we have presented so far show that asset prices are strongly affected, conditional on intermediaries being capital constrained. The largest effects arise when intermediaries are near bankruptcy, with almost no asset pricing effects when intermediaries are unconstrained.

In part, the latter result is because the measures we have presented are instantaneous asset prices measurements. In this subsection we consider how an agent, while currently in the unconstrained region, prices assets that make payments in the constrained region.

This exercise is interesting because a number of recent papers have provided evidence of a liquidity factor governing asset returns (see Amihud, 2002, Acharya and Pedersen, 2003, Pastor and Stambaugh, 2003, and Sadka, 2003), suggesting that the marginal investor is particularly averse to times of low aggregate liquidity conditions. In our model, low liquidity corresponds to times when intermediation capital is low and the economy is constrained. The marginal investor is the specialist, and times of low intermediary capital are particularly bad times for specialists. Thus there is a natural reason to expect that the marginal investor will be averse to assets that pay little in low aggregate liquidity states.

Figure 9: Liquidity Valuation



The quantity $\frac{u'(c(D_t,mw^I/w^h))}{u'(c(D_t,mw^{I,c}/w^{h,c}))}$ is graphed as a function of mw^I/w^h. This quantity is equal to one at the boundary of the constrained region, above one in the constrained region, and below one in the unconstrained region.

Figure 9 graphs the quantity $\frac{u'(c(D_t,mw^I/w^h))}{u'(c(D_t,mw^{I,c}/w^{h,c}))}$ as a function of mw^I/w^h. This quantity corresponds to the following measure: Fix a value of dividends at D_t and suppose that the specialist is currently at the boundary of the constrained region. The measure corresponds to the valuation of an asset that pays one unit of consumption in state $(D_t, mw^I/w^h)$ in terms of current consumption (at $w^{I,c}$), after accounting for the probability of that state. Since the specialist is averse to declines in intermediary capital, he pays a premium for states with low intermediary capital (constrained states). He pays a discount for states with high intermediary capital. The figure illustrates that the specialist values assets that hedge against declines in aggregate liquidity (intermediary capital).

One caveat in reading Figure 9 is that the constrained states cannot occur when currently in the unconstrained states for the $\lambda = 0$ case. Thus the left panel of the figure is only suggestive of the liquidity valuations. The right panel, where $\lambda > 0$, does not suffer this problem.

The figure also illustrates the effects of m. The liquidity valuation is primarily driven by the effect that lower m means a tighter capital constraint. The tightness of the capital constraint in turn leads the specialist to value assets that relax the constraint.

5.4 Liquidity factor

The preceding exercise suggests that the specialist is averse to reductions in aggregate liquidity and may rationalize a liquidity factor for asset prices. However, since our model is driven by a single source of uncertainty (the one-dimensional Brownian motion governing dividends), changes in both the risky asset price and intermediary capital are perfectly correlated. This makes it difficult to clarify the role of a liquidity factor for asset returns separate from the market factor.

As we have explained, the mechanics of our model does suggest such a factor. But the effect is not apparent, as it is multiplicative rather than additive, in our model. Shocks to dividends have an amplified effect on the economy because they affect both the cash-flows expected on the risky asset as well as the aggregate liquidity of intermediaries. In a standard model, shocks to dividends also affect the economy, but the amplification due to the change in liquidity will be absent.[22]

To clarify that our model does indeed have a separate liquidity effect, we develop an "additive" model. We consider the following thought experiment. We perturb our model by adding a second shock process that is orthogonal to dividends but which directly affects intermediary capital. We then trace the effects of this second factor on asset returns. The exercise gives us some understanding of the separate, additive, role of intermediary capital risk, without working out a full-blown two-factor model.

We suppose that nature randomly redistributes a small amount of wealth between intermediaries and households. The redistribution of w^h amounts to $\sigma_1 w^h dZ_{1,t}$, where $Z_{1,t}$ is orthogonal to Z_t and $\sigma_1/\sigma \to 0$. Thus this second shock process is small compared to the primary dividend process. Without loss of generality we assume $\sigma_1 > 0$.

For the households with log preferences, since this distribution only affects the actual return on their wealth (and σ_1 is small), their consumption policy remains $c^h = \rho^h w^h$. Also, as σ_1 is small, the equilibrium price/dividend ratio, $F(\cdot)$, remains unchanged.

Then, we can directly rewrite dy as,

$$dy = \mu_y dt + \sigma_y dZ_t - \sigma_1 y dZ_{1,t} \qquad \sigma_1 > 0.$$

[22] The amplification effect present in our model is also highlighted in the model of Xiong (2001). There is also a literature in macroeconomics that emphasizes the importance of collateral amplification effects in explaining business cycles (see Kiyotaki and Moore, 1997, or Krishnamurthy, 2003).

Recall that our derivations suggest that $\sigma_y < 0$ in the constrained region; i.e., a negative shock to dividends transfers capital away from intermediaries and towards the households. Similarly, a negative innovation in $dZ_{1,t}$ redistributes wealth away from specialists and towards households. Thus, $dZ_{1,t} < 0$ has the interpretation of an exogenous negative shock to intermediary capital, with no direct effect on the aggregate dividend process.

Also, note that the size of the intermediary shock is proportional to y. We make this assumption for two reasons. First, recall that σ_y is increasing in y. For ease of comparison, we choose the new shock to also increase in y. Second, the proportionality to y keeps the new shock in terms of a percentage so that it remains relatively "small" for all values of y.

Our expressions for dR and dc/c remain as before, with the redefined dy. Thus we find that the risk premium on the risky asset is now,

$$E_t\left[dR_t\right] - r_t dt = \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma - \frac{F'}{F}\sigma_y\right)dt - \gamma\frac{F'}{F}\frac{\rho^h}{1-\rho^h y}\left(\sigma_1 y\right)^2 dt. \tag{13}$$

The latter term on the right-hand side is new relative to our previous expressions. This term reflects the effect of shocks to intermediary capital on the aggregate market return. $dZ_{1,t}$ has no direct effect on dividends and hence on stock prices. It affects prices through a liquidity channel. Endogenously the risk aversion of the specialist is affected by y, and hence stock prices and consumption are affected by $dZ_{1,t}$.

Notice also that the new term resembles the quadratic component of the first risk premium term on the right hand side (i.e. if $\sigma_1 y$ was equal to σ_y, then the new term is exactly the quadratic component of the first term). This relation clarifies that the quadratic component is the "multiplier" that our model generates due to the liquidity channel.

We next consider pricing an intermediated asset, whose dividend process loads on $dZ_{1,t}$. We introduce an infinitesimal amount of asset-j with dividend process,

$$\frac{dD_t^j}{D_t^j} = g^j dt + \sigma^j dZ_t + \sigma_1^j dZ_{1,t}.$$

We consider the limiting case where $\frac{\sigma_1^j}{\sigma^j} \to 0$. We also assume that $g^j = g$ and $\sigma^j = \sigma$. As before, these assumptions imply,

$$P_t^j = D_t^j F(y_t).$$

Defining dR^j as the instantaneous return on asset-j, we find that the risk premium satisfies:

$$E_t\left[dR_t^j\right] - r_t dt = \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma - \frac{F'}{F}\sigma_y\right)dt + \gamma\frac{\rho^h}{1-\rho^h y}\left(\sigma_1^j - \frac{F'}{F}\sigma_1 y\right)\sigma_1 y dt. \tag{14}$$

Combining equation (13) with (14) we find:

$$E_t\left[dR^j\right] - r_t dt = \overbrace{E_t\left[dR_t\right] - r_t dt}^{\text{Market}} \;+\; \overbrace{\gamma\frac{\rho^h}{1-\rho^h y}\sigma_1 y\ \sigma_1^j dt}^{\text{Liquidity}}. \tag{15}$$

The first term on the right hand side of (15) is the return on asset-j for bearing market risk. Since $g^j = g$ and $\sigma^j = \sigma$, asset-j's return has a β of one with the market return. As result, one component of asset-j's risk premium is equal to the market risk premium. The second term on the right hand side of (15) is the risk premium for exposure to the new liquidity factor. The risk premium is positive and increasing in σ_1^j, the loading on the liquidity factor. The risk premium also reflects the dependence on y that we have highlighted earlier. As y rises, the leverage of the specialist endogenously rises, and as a result his effective risk aversion increases. Thus, the risk premium also rises.

The thought experiment we have conducted makes clear that our model rationalizes a liquidity factor but through a multiplier mechanism rather than an additive mechanism. In an additive model, the risk premium is proportional to $\sigma_1 y$, the shocks to specialist consumption induced by the orthogonal shock to intermediation capital. In our multiplicative model, the volatility of specialist consumption is,

$$\sigma - \frac{\rho^h}{1 - \rho^h y}\sigma_y.$$

In the constrained region, $\sigma_y < 0$. The entire increase in consumption volatility is due to the effect of shocks to intermediation capital, and for the reasons outlined above, can give rise to a liquidity factor for asset returns.

6 Conclusion

We have presented a model to study the effects of capital constraints in the intermediary sector on asset prices. Capital effects arise because (1) households lack the knowledge to participate in the risky asset; and, (2) intermediary capital determines the endogenous amount of exposure that households have to the risky asset. The model builds on an explicit microeconomic foundation for intermediation. The model is also cast within a dynamic economy in which one can articulate the dynamic effects of capital constraints on asset prices. We show that the model can help to explain the behavior of asset markets during aggregate liquidity events.

There are a number of interesting directions to take this research. First, the model we have presented has a degenerate steady-state distribution, which means that we cannot meaningfully simulate the model. We can alter the model, say by allowing rebirth of bankrupt intermediaries, in order to solve this problem. Then we can study the average effects of liquidity events on asset prices. Within the current model, we have tried simulating paths for 100 years given an initial condition, in order to get a sense of these average effects. Our investigations so far suggest that with only dividend shocks, the economy does not spend enough time in the constrained region to arrive at large asset price effects. However, we think it is plausible that there are other sources of shocks that affect intermediation – for example, investors become concerned that the moral hazard problem becomes more severe at times, or households are subject to liquidity shocks as in Allen and Gale (2005) – that may also affect dynamics. Studying a model with a richer shock structure also seems interesting for this research.

A second avenue of research is to expand the number of traded assets. Currently the only non-intermediated asset in the model is the riskless bond. However, in practice, even unsophisticated households have the knowledge to invest in many risky assets directly. It will be interesting to introduce a second asset in positive supply in which households can directly invest, and study the differential asset pricing effects across these different asset classes. This exercise seems particularly relevant in light of the evidence in the fall of 1998 that it was primarily the asset classes invested in by hedge funds that were affected during the crises. We intend to investigate these issues more fully in future work.

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A Appendix A: Proof of Proposition 1

Denote the value functions of household and specialist as $J^h\left(w^h;Y\right)$ and $J^I\left(w^I;Y\right)$ where w^h and w^I are household's and specialist's wealth respectively, and Y is the relevant state variable vector. We assume the value functions are smooth enough, and note that the marginal values of wealth $J_w^h\left(w^h;Y\right)$ and $J_w^I\left(w^I;Y\right)$ must be positive.

The IC constraint for the specialist is:

$$\begin{aligned}E_t \quad & \left[J^I(\ (w^I - E^I)(1+\widetilde{dR_t}) + E^I + C + \beta(T\widetilde{dR_t} - B^h r_t dt); Y)\right] \geq \\ & E_t\left[J^I(\ (w - E^I)(1+\widetilde{dR_t}) + E^I + C + \beta(T(\widetilde{dR_t} - xdt - B^h r_t dt) + Tbdt; Y)\right]\end{aligned}$$

where $E^I \leq w^I$. Carrying through a Taylor expansion (note that 2^{nd} order terms cancel on both sides), we find that

$$T(\beta x - b)J_w^I \geq 0$$

or,

$$\frac{E^I}{E^h + E^I} \geq \frac{b}{x} \Rightarrow E^I\left(\frac{x}{b} - 1\right) \geq E^h.$$

We define $m \equiv \frac{x}{b} - 1$. Since $E^I \leq w^I$ and $E^h \leq mw^I$ we arrive at the capital constraint of the Proposition. $C = 0$ follows from the full bargaining power of the household. Q.E.D.

B Appendix B: ODE solution for $\lambda = 0$ Case

In this appendix, we detail the solution of the ODE that characterizes the equilibrium. We analyze our ODE based on state variable y, i.e., the scaled households wealth. Since under our parameterization the equilibrium scaled specialists wealth $w^I/D = F(y) - y$ is a monotone transform of y, in the main text we plot $F(\cdot)$ against w^I/D to highlight the effect of intermediary capital.

B.1 ODE

Recall

$$\begin{aligned}g + \frac{F'}{F}\mu_y + \frac{1}{2}\frac{F''}{F}\sigma_y^2 \quad + \quad & \frac{1}{F} + \frac{F'}{F}\sigma_y\sigma = \rho + \gamma g - \frac{\gamma\rho^h}{1-\rho^h y}(\mu_y + \sigma_y\sigma) \\ & + \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma + \frac{F'}{F}\sigma_y\right) - \frac{1}{2}\gamma(\gamma+1)\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)^2\end{aligned} \tag{16}$$

and $\sigma_y = -\frac{\hat{\theta}_b}{1-\theta_s F'}\sigma$, $\mu_y = \frac{1}{1-\theta_s F'}\left(\theta_s + (r + \sigma^2 - g)\hat{\theta}_b - \rho^h y + \frac{1}{2}\theta_s F''\sigma_y^2\right)$.

B.1.1 Unconstrained Region: $y < y^c$

In this region $\theta_s = \frac{y}{F}$ and $\hat{\theta}_b = 0$; hence $\sigma_y = 0$ and $\mu_y = \frac{1}{1-\theta_s F'}\left(\theta_s - \rho^h y\right)$, which yields the following first-order ODE to solve on the region $y \in [0, y^c]$:

$$\left(\frac{F'}{F} + \frac{\gamma\rho^h}{1-\rho^h y}\right)\left(\frac{1}{F} - \rho^h\right) = \left(\frac{F'}{F} - \frac{1}{y}\right)\left(\frac{1}{F} - \left(\rho + g(\gamma-1) + \frac{\gamma\sigma^2}{2}(1-\gamma)\right)\right) \tag{17}$$

B.1.2 Constrained Region: $y^c \le y \le y^b$

In this region $\mu_y = \frac{1}{1-\theta_s F'}\left(\theta_s + (r+\sigma^2-g)\hat{\theta}_b - \rho^h y + \frac{1}{2}\theta_s F''\sigma_y^2\right)$ and $\sigma_y = -\frac{\hat{\theta}_b}{1-\theta_s F'}\sigma$, where $\theta_s = \frac{m}{1+m}$. Substituting for μ_y, we find,

$$\begin{aligned} & \left(\frac{F'}{F} + \frac{\gamma\rho^h}{1-\rho^h y}\right)\left(\frac{1}{1-\theta_s F'}\right)\left(\theta_s + \hat{\theta}_b(r-g) - \rho^h y\right) + \frac{1}{F} \\ & \quad + \frac{1}{2}F''\sigma_y^2\left(\frac{1}{1-\theta_s F'}\right)\left(\frac{1}{F} + \theta_s\frac{\gamma\rho^h}{1-\rho^h y}\right) \\ = \quad & \rho + g(\gamma-1) + \gamma\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)\left(\sigma + \frac{F'}{F}\sigma_y\right) \\ & \quad - \frac{1}{2}\gamma(\gamma+1)\left(\sigma - \frac{\rho^h}{1-\rho^h y}\sigma_y\right)^2 \end{aligned} \tag{18}$$

where,

$$\begin{aligned} r \quad = \quad & \rho + g\gamma - \frac{\rho^h\gamma}{1-\rho^h y}\frac{\theta_s + (r-g)\hat{\theta}_b - \rho^h y + \frac{\sigma^2}{2}\theta_s F''\frac{\hat{\theta}_b^2}{(1-\theta_s F')^2}}{1-\theta_s F'} \\ & - \frac{\gamma(\gamma+1)\sigma^2}{2}\left(1 + \frac{\rho^h\hat{\theta}_b}{1-\rho^h y}\frac{1}{1-\theta_s F'}\right)^2 \end{aligned}$$

We define a function, $G(y) \equiv \frac{1}{1-\theta_s F'}$; with this definition, we can write $G' = \theta_s G^2 F''$, and

$$\sigma_y = -\frac{\hat{\theta}_b}{1-\theta_s F'}\sigma = -\hat{\theta}_b\sigma G.$$

Then rewriting (18),

$$\begin{aligned} G' \quad \frac{(\hat{\theta}_B\sigma)^2}{2} \quad G \quad & \left(\frac{1}{\theta_s F} + \frac{\gamma\rho^h}{1-\rho^h y}\right) = \rho + g(\gamma-1) - \frac{1}{F} \\ & + \frac{1}{2}\gamma\sigma^2\left(1 + \frac{\rho^h}{1-\rho^h y}\hat{\theta}_B G\right)\left(\frac{2\left(y - G\hat{\theta}_b\right)}{\theta_s F} - (1+\gamma)\frac{1-\rho^h y + \rho^h G\hat{\theta}_b}{1-\rho^h y}\right) \\ & - \left(\frac{G-1}{\theta_s F} + \frac{\gamma\rho^h}{1-\rho^h y}G\right)\left(\theta_s + \hat{\theta}_b(r-g) - \rho^h y\right) \end{aligned}$$

and

$$r = \frac{\rho + g\gamma - \frac{\rho^h\gamma G}{1-\rho^h y}\left(\theta_s - g\hat{\theta}_b - \rho^h y + \frac{\sigma^2}{2}G'\hat{\theta}_b^2\right) - \frac{\gamma(\gamma+1)\sigma^2}{2}\left(1 + \frac{\rho^h\hat{\theta}_b G}{1-\rho^h y}\right)^2}{1 + \frac{\rho^h\gamma G\hat{\theta}_b}{1-\rho^h y}}.$$

We combine these two pieces, using the relation, $\hat{\theta}_b\left(\frac{G-1}{\theta_s F}+\frac{\gamma\rho^h G}{1-\rho^h y}\right)=-\frac{y-G\hat{\theta}_b}{\theta_s F}+\frac{1-\rho^h y+\gamma\rho^h G\hat{\theta}_b}{1-\rho^h y}$, and arrive at a final expression of the ODE:

$$\begin{aligned}
& G'\frac{(\hat{\theta}_b\sigma)^2}{2}\frac{G}{\theta_s F}\left(\frac{1+\rho^h y(\gamma-1)}{1-\rho^h y+\rho^h\gamma G\hat{\theta}_b}\right) \qquad (19)\\
= \; & \rho+g(\gamma-1)-\frac{1}{F}+\frac{\gamma(1-\gamma)\sigma^2}{2}\left(1+\frac{\rho^h G\hat{\theta}_b}{1-\rho^h y}\right)\frac{y-G\hat{\theta}_b}{\theta_s F}\left[\frac{1-\rho^h y-\rho^h G\hat{\theta}_b}{1-\rho^h y+\rho^h\gamma G\hat{\theta}_b}\right]\\
& -\left(\frac{\left(1-\rho^h y\right)(G-1)}{\theta_s F}+\gamma\rho^h G\right)\frac{\theta_s+\hat{\theta}_b(g(\gamma-1)+\rho)-\rho^h y}{1-\rho^h y+\rho^h\gamma G\hat{\theta}_b}.
\end{aligned}$$

Since,

$$F'=\frac{1}{\theta_s}\frac{G-1}{G},$$

we arrive at a system of first-order ODE's (in F and G) to solve on the region $y\in\left[y^c,y^b\right]$.

When $m=0$, the ODE is:

$$\begin{aligned}
F''\frac{(y\sigma)^2}{2F} \;=\; & \rho+g(\gamma-1)-\frac{1}{F}+\frac{\gamma(1-\gamma)\sigma^2}{2}\left(\frac{1}{1-\rho^h y}\right)\left[\frac{1-2\rho^h y}{1+\rho^h y(\gamma-1)}\right]\\
& -\left(\frac{F'}{F}\left(1-\rho^h y\right)+\gamma\rho^h\right)\frac{yg(\gamma-1)+\left(\rho-\rho^h\right)y}{1+\rho^h y(\gamma-1)}.
\end{aligned}$$

B.2 Boundary Conditions and Numerical Approach

B.2.1 Unconstrained Region: $0<y<y^c$

First, we solve F for $y\in(0,y^c]$. From (17), we can derive $F(0)=\frac{1}{\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)}$. Note that $F(0)$ is the Price/Dividend ratio when the economy only comprises of specialists (the households have zero wealth). Although we never reach this limit, when y is small the economy must be "close" to the limiting case, in order to rule out Ponzi games (see below). Then, continuity of F at the origin requires $F(0+)=F(0)$. Utilizing this continuity, the singularity at $y=0$ allows us to derive

$$F'(0)=\gamma\left(1-\rho^h F(0)\right);$$

and the sign of $F'(0)$ depends on the sign of $\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)-\rho^h$.

The behavior of the solution to our 1^{st}−order ODE (17) depends crucially on the above parameterization conditions. For better understanding the dynamic economic evolution behind the solution of (17), it is helpful to have the next Lemma.

Lemma 3 *Suppose there is no delegation friction in this economy and $\rho^h\neq\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)$. Then there does not exist $y^*\in\left(0,\frac{1}{\rho^h}\right)$ to have $F(y^*)=\frac{1}{\rho^h}$.*

Proof. Note that the change of (scaled) household wealth, μ_y, is proportional to $\theta_s-\rho^h y=y(\frac{1}{F}-\rho^h)$ (her dividend income less her consumption). Suppose that there exists y^* so that $F(y^*)=1/\rho^h$, which implies that $\mu_y = 0$ and the economy stays at (or approaches to) y^* with $F(y^*) = 1/\rho^h$. But we know that in the absorbing state the equilibrium price/dividend ratio implied by the active specialist is $F(0)$ (note that then specialist just consumes $1 - \rho^h y^*$ fraction of total endowment, or converges to this limit), which leads to a contradiction. ■

Now we are ready to discuss our three cases, where the idea is similar to the "saddle-path" argument. Assume away delegation frictions; so the relevant domain for ODE (17) is $y \in [0, 1/\rho^h]$. Note that the ending point $1/\rho^h$ represents the world where households are wealthy enough to consume everything.

1. $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1-\gamma) < \rho^h$. In this case there is a unique solution for F satisfying the continuity condition $F(0+) = F(0)$, $F'(y) < 0$, and $F(1/\rho^h) = 1/\rho^h$. Since $F(0) > 1/\rho^h$ it is not difficult to check that in this solution we have $dy/dt = \mu_y < 0$, which implies that the households' wealth diminishes over time. Intuitively, more patient specialists save more relative to households, hence the wealth is slowly transfered from households to the intermediary sector. Now for any $y > 0$, $F(y)$ is determined by the future price, which is $F(0)$.

   We find other solutions diverges ($F(0+) = +\infty$ or $-\infty$) on the singular point $y = 0$. $F(0+) = -\infty$ is easily ruled out by no-arbitrage, or the result that F must stay above $1/\rho^h$ (Lemma 3). On the other hand, since y_t converges to 0 deterministicly when $t \to \infty$, this implies that in those solutions with $F(0+) = +\infty$, specialists are playing Ponzi game with each other.

2. $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1-\gamma) > \rho^h$. Now since speciliasts consume aggresively, the passive households become wealthier over time, or $\frac{dy}{dt} = \mu_y > 0$.[23] It is because $F(0) < 1/\rho^h$ and there is no $F(y)$ to be above $1/\rho^h$. In this case there are continuum of solutions for F satisfying the continuity condition $F(0+) = F(0)$, and $F'(0) > 0$. However, under this parameter assumption, $y = 1/\rho^h$ plays the same role as $y = 0$ in the first case. As a result, among these solutions only one of them will land at $F(1/\rho^h) = 1/\rho^h$, and other solutions diverges at this ending point. Similar arguments as in Case 1 imply that this is the only equilibrium solution in this case (without capital constraints).

3. $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1-\gamma) = \rho^h$. In this case constant function $F(y) = \frac{1}{\rho + g(\gamma-1) + \frac{\gamma\sigma^2}{2}(1-\gamma)} = \frac{1}{\rho^h}$ solves (17), and $\mu_y = 0$. Both agents consume at the same velocity, so any wealth point y works as an absorbing state.

For illustrative purpose we set our baseline case parameters with $\lambda = 0$ to satisfy $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1-\gamma) = \rho^h$, hence $F(y) = 1/\rho^h$ for all y's. When $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1-\gamma) < \rho^h$, in the unconstrained region $y_t \to 0$ deterministically with time, and each $F(y)$ for $y \in (0, y^c]$ is pinned down by the future stock price $F(0)$ through (17). As we show below, this condition ensures the uniqueness of the equilibrium with captial constraint.

[23] Even though $F' > 0$, it is easy to check that in solution to (17) $1 - \frac{y}{F}F'$ stays bounded below from 0.

B.2.2 Constrained Region: $y^c \leq y \leq y^b$

From the unconstrained region, we obtain a boundary at $y^c = \frac{m}{m+1} F(y^c)$. We require continuity of F, $F(y^c+) = F(y^c)$. At the other boundary, $y^b = F\left(y^b\right) = \delta/\rho^h$. Hence, our problem is a 2^{nd}-order ODE with two boundary conditions, and the shooting method is the natural solution method to employ. Specifically, starting from the point $\left(\delta/\rho^h, \delta/\rho^h\right)$ on the $y - F$ plot we attempt different values for $\phi = \widetilde{F}'\left(y^b\right)$ and integrate equation (18) down to y^c; when $\widetilde{F}(y^c)$ hits $F(y^c)$ (in fact, until the solution pastes smoothly to the unconstrained region solution), we pin down the desired solution. (Or, if using w^I/D as state variable, the two ending points are $\left(0, \delta/\rho^h\right)$ (bankrutcy) and $(F(y^c) - y^c, F(y^c))$.) We use $Matlab\_7$'s built-in solver $ode15s$ with tolerance level 10^{-12} to numerically solve our problem; other solvers, such as $ode45$, $ode15i$ etc., deliver almost identical results once we find the correct ϕ.

When $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma) < \rho^h$, so the specialists dominate the economy in the end. Specifically, starting from any $y \in \left(y^c, y^b\right)$, the economy either hits the bankruptcy state y^b, or another absorbing state y^c which leads the economy deterministically to $y = 0$.

What happens when $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma) > \rho^h$? Because in this case y tends to grow with time, (y^c, F^c) is no longer pinned down by $F(0)$. When there is no capital constraint, it is determinend by $y = 1/\rho^h > y^c$ (see Section B.2.1). However, in the presence of capital constraint, (y^c, F^c) is indeterminate. Intuitively, now on the $y - F$ plot, the bankruptcy state $\left(\delta/\rho^h, \delta/\rho^h\right)$ and the capital constraint threshold point (y^c, F^c) become the remote future for any y between y^c and y^b, and the endogenous boundary (y^c, F^c) gives rise to a multiplicity of solutions. More specifically, there could be continuum of equilibrium price/dividend ratio F's for $y \in \left[y^c, y^b\right]$: for any (y^c, F^c) with $y^c = \frac{m}{1+m} F^c$ and $F^c < 1/\rho^h$, combining with the bankruptcy state $\left(\delta/\rho^h, \delta/\rho^h\right)$ we could determine an equilibrium evolution path for F. In any of these solutions, if we start from $y < y^c$, then the households' wealth shrinks and the economy slowly slides into the capital constrained region $\left[y^c, y^b\right]$. Once the capital constrained region is reached, the economy stays inside until it hits the bankruptcy state.

However, as shown in the Appendix C, once the debt investors are introduced (i.e., $\lambda > 0$) we still obtain a unique equilibrium even in the case of $\rho + g(\gamma - 1) + \frac{\gamma\sigma^2}{2}(1 - \gamma) > \rho^h$. Note that now our economy could move between both constrained and unconstrained regions smoothly, and as a result the non-dying 2^{nd} order term in the ODE ensures a unique solution.

C Appendix C: $\lambda > 0$ Case

C.1 Modified ODE

The only difference between this model and the one in our main text is in the household's assets positions. We arrive at the same 2^{nd}-order ODE after simply substituting the new scaled bond position $\widehat{\theta}_b^{sb}$ and the new stock position θ_s^{sb} into those expressions in Lemma 2, where the superscript "sb" stands for "stock - bond investors".

1. If $mw^I \geq (1-\lambda) w^h$ then all of the stock investor's wealth $(1-\lambda) w^h$ is intermediated, while the debt investor purchases $\lambda w^h = \lambda y D$ of the riskless bond. The specialist's positions are α_b and α_s. The relations, $w^I = P - w^h = D(F-y)$ along with the intermediation constraints and market clearing in the short-term bond imply that,

$$\frac{F-y+(1-\lambda)y}{F-y}\alpha_b + \lambda y D = 0,$$

   where the first term is the total bond position of the intermediary (including holdings of stock investors), and the second term $\lambda y D$ is the bond position directly held by the debt investor. So $\alpha_b = -\frac{\lambda y(F-y)}{F-\lambda y}D$, and the stock investor's indirect bond holding is $\frac{(1-\lambda)y}{F-y}\alpha_b = -\frac{\lambda(1-\lambda)y^2}{F-\lambda y}D$. Therefore the household's total scaled bond holding is

$$\widehat{\theta}_b^{sb} = -\frac{\lambda(1-\lambda)y^2}{F-\lambda y} + \lambda y = \lambda y \frac{F-y}{F-\lambda y}.$$

   Applying a similar argument to α_s, we have $\theta_s^{sb} = \frac{(1-\lambda)y}{F-y}\alpha_s = \frac{(1-\lambda)y}{F-\lambda y}$.

   The intermediation sector is capital constrained when $mw^I = (1-\lambda) w^h$, or $m(F-y) = (1-\lambda) y$. We can solve for this cutoff as

$$y^c = \frac{m}{1-\lambda+m}F(y^c).$$

   and the economy is in the unconstrained region if $0 < y \leq y^c$.

2. If $mw^I < (1-\lambda) w^h$ (or $y > y^c$) then the intermediation sector is capital constrained. The stock investor will invest mw^I into the intermediary, and put the rest of his wealth $(1-\lambda) w^h - mw^I$ into the riskless bond; the debt investor still invests λw^h into the short-term bond. Since all stocks are held through intermediaries, and the ratio of intermediated funds to the specialist's capital is always m, Proposition 1 tells us that $\theta_s^{sb} = \frac{m}{1+m}$. Hence $\widehat{\theta}_b^{sb} = y - \frac{m}{1+m}F(y)$.

3. Intermediaries go bankrupt when $w^I = P - w^h = 0$. The bankruptcy threshold is implicitly defined by the equation $y^b = F(y^b) = \delta/\rho^h$.

C.2 Solution Method

As discussed, our ODE (19) and the boundary conditions $F(0) = \frac{1}{\rho+g(\gamma-1)+\frac{\gamma\sigma^2}{2}(1-\gamma)}$ and $F\left(y^b\right) = \alpha F(0)$ remain unchanged. Similar to the previous method, we could attempt different $\phi = F'\left(y^b\right)$ so that the solution to (19) lands at $F(0)$. It turns out that, due to the high nonlinearity of (19) and the singularity at $y=0$, the $ode15s$ in this backward-shooting scheme diverges easily for y close to 2 under our baseline paramters. Note that this observation implies the uniqueness of our solution.

To oversome this issue, we adopt a "forward-shooting and line-connecting" method. More specifically, fix $\epsilon > 0$ and call $\widetilde{F}$ as the attempted solution. For each trial $\phi \equiv \widetilde{F}'(\epsilon)$ we the set $\widetilde{F}(\epsilon) = F(0) + \phi\epsilon$. Since $\left(\epsilon, \widetilde{F}(\epsilon)\right)$ is off from the singularity, via attemping different ϕ's we could apply the standard shooting method to obtain the desired solution F that lands at $\left(y^b, F\left(y^b\right)\right)$. For $y < \epsilon$, we simply approximate the solution by a line connecting $(0, F(0))$ and $(\epsilon, F(\epsilon))$, and by construction this line meets the solution F tangentially for $\epsilon \leq y \leq y^b$. In other words, we solve F on $\left[\epsilon, y^b\right]$ with a smooth pasting condition for $F'(\epsilon) = \frac{F(\epsilon)-F(0)}{\epsilon}$ and a value matching condition for $F\left(y^b\right) = \delta/\rho^h$.

For the positive λ baseline parameters, we use $\epsilon = 3$ which gives error bouded by 3×10^{-5} for $y < \epsilon$, and different ϵ's deliver almost identical solutions for $y \geq 5$. Because we are mainly interested in the solution behavior near y^c and onwards, our main calibration results are free of the approximation errors caused by the choice of ϵ. Finally we find that, in fact, these errors are at the same magnitude as those generated by the capital constraint around y^c (3.5×10^{-5}).