Informational Assumptions on Income Processes and Consumption Dynamics In the Buffer Stock Model of Savings *

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This version: June 26, 2006

Abstract

Idiosyncratic household income is typically assumed to consist of several components. While the total income is observed and is often modelled as an integrated moving average process, individual components are not observed directly. In the literature, econometricians typically assume that household income is the sum of a random walk permanent component and a transitory component, with uncorrelated permanent and transitory shocks. This characterization is not innocuous since households may have better information on individual income components than econometricians do. I show that, for the same reduced form model of income, different models for the income components lead to sizeably different estimates of the marginal propensity to consume (MPC) out of shocks to current and lagged income, and the volatility of consumption changes relative to income changes in data *generated* by an infinite horizon buffer stock model. I further suggest that the MPC out of shocks to current and lagged income estimated from *empirical* micro data should help identify parameters of individual components of the income process, including the correlation between transitory and permanent shocks. I use a structural life cycle model of consumption to estimate the parameters of the income process by the method of simulated moments (MSM). I find statistically significant negative contemporaneous correlation between permanent and transitory shocks and reasonable, precise estimates for the time discount factor and the relative risk aversion parameter.

KEYWORDS: Unobserved Components Models, Income Processes, Buffer Stock Model of Savings, Method of Simulated Moments.

JEL CLASSIFICATIONS: C15, C61, D91, E21.

*I owe special thanks to Bent Sørensen for extensive comments, advice and encouragement. I am also grateful to Chinhui Juhn, Maria Luengo-Prado, Chris Murray, and Nat Wilcox. For helpful comments, I thank seminar participants at University of Houston, University of Alberta, University of Connecticut, New Economic School and EERC.

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1 Introduction

Households face a variety of income shocks. Promotions, layoffs, long term and temporary unemployment, health shocks, and lump-sum bonuses are a few in the list of events that make disposable household income volatile. In a world of imperfect insurance markets, idiosyncratic labor market risk is important for household decisions over consumption and savings, portfolio choice, and even the choice of career. Economists refer to persistent, or long lasting shocks as permanent, and temporary, or short-lived shocks as transitory. Correct identification of permanent versus transitory shocks is important for the prediction of economic behavior. The permanent income hypothesis (PIH), for example, predicts that households adjust consumption fully to the newly arrived permanent shocks, and change consumption only by the annuity value of the transitory shocks, a very small adjustment in economic terms. Some empirical studies of income processes find that household income may be modelled as an integrated time series process with a strong mean-reverting, low-order moving average component.[1] Consistent with this finding, econometricians typically assume that household income may be represented by the sum of a permanent random walk component and a short-lived transitory component, with no correlation between transitory and permanent income shocks.[2] Obviously, households may have better information about income components, and therefore about the stochastic processes that govern the dynamics of each component.[3] For different reduced form models of aggregate income, Quah (1990) shows that there exists a decomposition of income into permanent and transitory components that helps solve the PIH "excess smoothness" puzzle.[4] Thus, Quah (1990) implicitly shows that the "correct" decomposition of income is the one that helps reconcile the joint dynamics of consumption and income with the PIH predictions. This decomposition of income into its components, which can be reasonably assumed to be known to households, may or may not coincide with the decomposition done by econometricians.

In this paper I explore an idea similar to that in Quah (1990, 1992) in the context of the buffer

[1] See, e.g., Abowd and Card (1989), Meghir and Pistaferri (2004).

[2] Notable examples are Carroll and Samwick (1997) and Meghir and Pistaferri (2004). They split income changes into permanent and transitory parts, and, under the assumption of orthogonality between permanent and transitory shocks, identify and estimate household or group-specific volatility of permanent and transitory shocks.

[3] In general, any decomposition of non-stationary income processes done by the econometricians is not unique. E.g., Quah (1992) shows how to decompose an integrated time series process into permanent and transitory components of different relative sizes.

[4] If income is non-stationary and income growth exhibits positive serial correlation—as supported by aggregate data—the PIH predicts that consumption should change by an amount greater than the value of the current income shock. Consequently, consumption growth should be more volatile than income growth. Consumption growth in aggregate data, though, is much less volatile than income growth. Therefore consumption growth is said to be "excessively smooth" relative to income growth. See, e.g., Deaton (1992).

stock model of savings. I simulate an infinite horizon buffer stock model for different unobserved components (UC) decompositions of the same reduced form income process, and analyze the simulated economy at the aggregate and household levels. Specifically, I estimate the volatility of consumption growth relative to the volatility of income growth and the MPC out of shocks to current and lagged income at different levels of aggregation of simulated data. Throughout the paper I refer to the ratio of the volatility of consumption growth to the volatility of income growth as excess smoothness, and refer to the MPC out of shocks to lagged income as excess sensitivity.[5] I find that a bigger size of the permanent component within the same reduced form income process implies a statistically significantly larger MPC out of current income shocks at the aggregate and micro levels, larger excess sensitivity and smaller excess smoothness at the micro level.

Having established these results, I suggest that the MPC and the excess sensitivity estimated from empirical micro data should help identify parameters of the income process, including the correlation between permanent and transitory shocks. Importantly, this correlation cannot be identified from the univariate dynamics of integrated moving average processes[6] and must show how well the orthogonal decomposition of income done by econometricians describes the joint dynamics of household consumption and income. In other words, the estimate of the correlation between structural shocks should reveal the extent to what (income) information sets of econometricians may differ from the ones held by households.

I estimate parameters of the income process by the method of simulated moments (MSM). Using a buffer stock model, I simulate the MPC, the excess sensitivity, the persistence of income, and consumption profile over the life cycle and match them to the same moments constructed from the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX) data. To my knowledge, this is the first paper that attempts to back out parameters of the income process using estimation of a structural life cycle model of consumption. I find significantly negative contemporaneous correlation between transitory and permanent income shocks of about –0.4, and precise estimates of the time discount factor and the relative risk

[5] In aggregate data, current consumption growth is sensitive to lagged information of consumers, typically measured by lagged income growth. Hall's (1978) version of the PIH predicts that consumption changes are not sensitive at all to lagged information. Thus, consumption growth is said to exhibit "excess sensitivity" to lagged income growth. Following the literature, I measure "excess sensitivity" with the response of consumption growth to lagged income growth.

[6] Zero covariance between permanent and transitory shocks is also typically assumed in the literature modelling income processes at the aggregate level (e.g., Clark (1987)). Reduced form dynamics for aggregate income processes is richer than reduced form dynamics of household income processes, and requires more complicated models for the transitory component. Morley et al. (2003) show that if quarterly US GDP follows ARIMA(2,1,2); permanent component is a random walk; and transitory component is an AR(2), the covariance between permanent and transitory shocks can be identified from the univariate dynamics of GDP.

aversion parameter. The hypothesis of zero covariance between structural income shocks can be easily rejected.

As in the literature on household income processes, the estimated volatility of transitory shocks is found to be larger than the volatility of permanent shocks. Taken together, the estimates of permanent and transitory volatility, the time discount factor, and the relative risk aversion parameter imply the existence of a strong precautionary motive in household choices of consumption and savings over the life cycle.

Arguably, a better model of the household income process, consistent with empirical data on consumption and income, helps identify the behavioral parameters of a structural life cycle model better. For the buffer stock economies, I show that consumption dynamics at household and macro levels critically depends on the structure of the income process. Thus, correct identification of the components of the income process enhances specification of the consumption function of a life cycle dynamic optimization problem. This, in turn, can prove to be important for understanding wealth accumulation, portfolio choice, and other households' life cycle choices jointly determined with consumption.

The paper is organized in the following way. I elaborate further on the main idea of this paper in Section 2. In Section 3, I discuss decomposition of MaCurdy's (1982) ARIMA(0,1,1) income process into permanent and transitory components of different relative sizes. I then simulate an infinite horizon buffer stock model and present results on the sensitivity of (simulated) consumption to informational assumptions. In Section 4, I lay out the procedure of estimating the income and behavioral parameters by the method of simulated moments (MSM). In Section 5, I estimate the volatility of permanent shocks from a univariate dynamics of income. In Section 6, I discuss the main results, possible biases and potential extensions. Section 7 concludes.

2 Information Sets of Econometricians and Households May Differ: A Story of the Same Reduced Form

In this section, I set up the model of household consumption and saving, present the unobserved components (UC) income model used in the literature, emphasize that households and econometricians may use different UC models that imply different information sets, and discuss the potential importance of different UC models, and therefore (income) information sets for consumption dynamics.

In an infinite consumption-savings problem the dynastic household maximizes expected utility from consumption

$$\max E_0[\sum_{t=0}^{\infty} \beta^t U(C_t)], \tag{1}$$

subject to the accumulation (cash-on-hand) constraint,

$$X_{t+1} = R_{t+1}(X_t - C_t) + Y_{t+1}, \tag{2}$$

and the liquidity constraint:

$$C_t \leq X_t \ \ \forall t. \tag{3}$$

Cash on hand in period $t+1$, X_{t+1}, consists of labor income realized in period $t+1$, Y_{t+1}, and resources brought from previous period, accumulated at a possibly stochastic interest rate R_{t+1}. β is the time discount factor, and C_{t+1} is consumption in period $t+1$.

The following modifications to the problem set-up have been introduced in the literature: different parameterizations of the income process;[7] positive probability of zero income instead of the liquidity constraint (Carroll (1992) and Carroll (1997)); taxes, transfers and parameterized means-and asset- tested government programs (Hubbard et al. (1994a) and Hubbard et al. (1994b)); realistic dynamics for the size of a household over the life-cycle (Attanasio et al. (1999)); present-biased preferences (Laibson et al. (1998), Laibson (1997), Angeletos et al. (2001)); and uncertain medical expenses by the elderly (Palumbo (1999)).

If preferences are CRRA, and income is stochastic, the consumption problem cannot be solved analytically, and one needs to use computational methods to obtain the consumption function. Under certain regularity conditions on preferences, interest and the growth rate of income, Deaton (1991) has shown that this model generates buffer stock behavior, whereby a household targets a certain level of wealth to buffer bad income shocks. If shocks to income are unfavorable, households smooth consumption by running down available assets, and gradually rebuild wealth to meet the desired target level. The model is called the buffer stock model;

[7] Deaton (1991) uses i.i.d. income shocks, persistent income, random walk income, and random walk income with a Markov switching growth rate to examine consumption and savings dynamics within a buffer stock model.

it was originally proposed by Deaton (1991) and later refined by Carroll (1992, 1997).[8] The model proved to fit well consumption facts from micro data. Within the model and in the real data, consumption tracks income closely over the life-cycle; consumption is quite sensitive to transitory income shocks; and consumers' low wealth holdings can be the optimal response to exogenous parameters or to institutional constraints rather than the result of poor planning or a lack of foresight.

In this paper I examine the sensitivity of consumption to informational assumptions on the income processes. A popular, intuitively appealing, and empirically justifiable income model is an unobserved components (UC) model, where household income, Y_{t+1}, consists of a random walk permanent component, P_{t+1}, and a transitory component, ϵ^T_{t+1}.[9],[10]

$$Y_{t+1} = P_{t+1}\epsilon^T_{t+1} \tag{4}$$

$$P_{t+1} = G_{t+1}P_t\epsilon^P_{t+1}, \tag{5}$$

where ϵ^P_{t+1} is an innovation to the permanent component, and G_{t+1} is the gross growth rate of income at time $t+1$.

Taking logs, the first difference of income is:

$$\Delta y_{t+1} = g_{t+1} + u^P_{t+1} + \Delta u^T_{t+1}, \tag{6}$$

where y_{t+1} is the log of household income at time $t+1$; g_{t+1} is the log of its gross growth rate at time $t+1$; u^P_{t+1} is the log of ϵ^P_{t+1}; and u^T_{t+1} is the log of ϵ^T_{t+1}. g_{t+1} is composed of the aggregate productivity growth and of the growth in the predictable component of income over

[8] More precisely, the model is called the buffer stock model of savings if it satisfies the impatience condition formulated by Deaton (1991). For convenience, I will later refer to computational consumption models with idiosyncratic income uncertainty as to buffer stock models of savings.

[9] In the context of computational consumption models, this model was first used by Zeldes (1989*b*) and Carroll (1992, 1997).

[10] For some proving evidence that household log-income is a difference stationary process see, e.g., Guiso et al. (2005) and my discussion in Section 5, footnote 35.

the life cycle. After removing g_{t+1}, the growth in income is affected by purely idiosyncratic shocks. Specifically, it is composed of the current value of the permanent shock, u_{t+1}^P; and the first difference in transitory shocks, u_{t+1}^T and u_t^T.

To calibrate the parameters of household income process researchers use micro data, or rely on other studies of household income process like Abowd and Card (1989) or MaCurdy (1982). What are the informational assumptions behind the model of equations (4)–(6)? It is implicitly assumed that information about income and its components is generated exactly by this model, and that both econometricians and households can differentiate between permanent and transitory shocks, usually assumed to be uncorrelated at all leads and lags. Thus, assuming that the growth rate of income and interest rate are non-stochastic, the information set that both households and econometricians hold at time t, is $\Omega_t = \{u_t^P, u_t^T, Y_{t-1}, Y_{t-2}, \ldots, Y_0\}$. Is it innocuous to equate informational sets of econometricians and households? To fix ideas, consider a simple example. Assume a household knows that shocks to its permanent and transitory income are negatively correlated. For example, when the head gets promoted, he expects his bonuses to be cut off. This (negative) correlation helps the household form predictions on its permanent income and adjust consumption appropriately. Econometricians, in turn, do not differentiate between income news known to households, and attribute its larger portion to the permanent component by decomposing orthogonally income news into permanent and transitory. Consequently, econometricians make spurious conclusions about the relative importance of permanent and transitory income. In this case the household's information set is finer than the econometrician's.[11]

To motivate the potential importance of the correct identification of permanent versus transitory component of income, I use some insights from the PIH. As emphasized by Quah (1990), consumption changes implied by the PIH depend crucially on the relative importance of transitory and permanent components of income. It follows that if econometricians observe income news different from the news households observe, they may falsely reject the PIH, even though households behave exactly in accordance with it. This is the main point made by Quah (1990)

[11]Throughout the paper I assume that households perfectly observe distinct income components. Other views on household versus econometrician (income) information have been explored in the literature. Pischke (1995), for example, assumes that household income consists of idiosyncratic and aggregate components and that a household cannot decompose shock to its income into aggregate and idiosyncratic parts. For example, a household differentiates with a lag whether the head's unemployment spell is due to an economy-wide shock, or whether it is the idiosyncratic shock (e.g., employer or individual specific). This assumption enables Pischke to provide microfoundations for the excess sensitivity puzzle in macro data without violating the orthogonality condition of Hall (1978) at the micro level. Wang (2004) assumes that income consists of two, potentially correlated, processes of different persistence. He theoretically shows that a precautionary savings motive strengthens if individual imperfectly observes innovations to each component compared to the case of the perfect knowledge of each component.

that provides one of the solutions to the excess smoothness puzzle. Quah constructs different UC representations of several reduced form models of the aggregate US income, and finds that there always exists an UC model consistent with the relative pattern of variances of consumption and income observed in the aggregate US data, and consistent with the PIH. The intuition behind this result is that the excess smoothness puzzle can be solved if the importance of the permanent component is "reduced." It is possible to suppress the permanent component within an UC model without distortion of the properties of the reduced form process.

I will now present a formal treatment of these ideas in the context of the PIH. If the reduced form income process follows an ARIMA(0,1,q) process, the PIH consumption rule implies the following relation of consumption changes to income news (see, e.g., Deaton(1992)):

$$\Delta C_t = \frac{r}{1+r}\frac{\delta_q(\frac{1}{1+r})}{(1-\frac{1}{1+r})}\epsilon_t = \delta_q(\frac{1}{1+r})\epsilon_t, \tag{7}$$

where $\delta_q(\cdot)$ is the lag polynomial of order q in δ evaluated at $\frac{1}{1+r}$, and ϵ_t is a reduced form income shock. If, for example, $q = 1$ and, consistent with empirical micro data, δ is negative, consumption should change by $1 + \frac{\delta}{1+r}$. δ controls the mean reversion in income, and, along with the standard deviation of income shocks, determines the volatility of consumption changes. If δ is zero, income is a random walk and consumption should change by the full amount of the (permanent) income shock. The closer to –1.0 δ is, the less persistent is the income process, the smaller is the response of consumption to a permanent shock, and the smaller is the volatility of consumption changes for a given volatility of income shocks.

Assume that the reduced form income process, ARIMA$(0, 1, q)$, can be decomposed into permanent IMA$(1, q_P)$ component, and transitory MA component of order q_P, such that $\max(q_P, q_T+1)$ is equal to q, and permanent and transitory shocks are not correlated. It can be shown (see Quah (1990)) that an UC model that agrees with the reduced form ARIMA$(0, 1, q)$ income process implies the following response of consumption changes to transitory and permanent income shocks:[12]

[12]Note that Quah (1990) considers linear difference stationary processes, while equation (6) features log-linear income processes. Campbell and Deaton (1989), however, show in a study of the PIH excess smoothness puzzle that this distinction is of little empirical importance. Furthermore, equation (8), derived using an UC representation of difference stationary linear income processes, serves only as a motivation for the main analysis of this paper. Thus, to avoid notational complications, for now, I interpret ϵ_t^T and ϵ_t^P as transitory and permanent innovations to the level of income within linear income processes. I will be explicit when I switch to log-linear income processes outlined in equations (4)–(6) and commonly used in the literature on household income processes.

$$\Delta C_t = \frac{r}{1+r}\delta_{q_T}(\frac{1}{1+r})\epsilon_t^T + \delta_{q_P}(\frac{1}{1+r})\epsilon_t^P \tag{8}$$

Take $q_P = 0$ and $q_T = 0$, so that the order of auto-covariance of the structural income process is the same as in the example above. In this case the implied consumption change should equal to the sum of $\frac{r}{1+r}$ of the transitory income shock, and the entire permanent income shock. It is obvious that the response of consumption will be stronger if a permanent shock is larger. Similarly, the volatility of consumption changes will be larger if, within a structural income model, the volatility of permanent income shocks dominates the volatility of transitory income shocks. In general, the volatility of consumption changes, as implied by the PIH, depends on the relative importance of the permanent component. The weight of the permanent component in the income series is governed by $\delta_{q_P}(L)$, $\delta_{q_T}(L)$, and the relative variances of ϵ_t^T and ϵ_t^P under the constraint that auto-covariance functions of reduced and structural form processes are identical. Since households have better information on the sequences of permanent and transitory shocks, one may conclude that the "correct" decomposition of income that households observe is the one that leads to the relative variances of consumption and income growth observed in the aggregate data, which is not necessarily the one identified by econometricians.

This intuition underlines the main theme of the paper and can be summarized as follows. The relative dynamics of income components is best known to households and this unique knowledge should be reflected in household consumption choices. Econometricians, in turn, make inferences on income components from the identified models of the income process which may or may not coincide with the model households observe. Ultimately, the importance of the discrepancy in (income) information sets of econometricians and households should be judged by the validity of predictions of household choices made by econometricians. In the next section, I provide some evidence on this importance for the understanding consumption dynamics.

3 The Same Reduced Form But Different Components: Sensitivity of Consumption to Informational Assumptions

In this section, I decompose MaCurdy's (1982) reduced form ARIMA(0,1,1) income model

into permanent and transitory components of different relative sizes. I construct nine decompositions of idiosyncratic household income that differ in the volatility of transitory shocks, and contemporaneous correlation between permanent and transitory shocks. I assume that consumers make their consumption and savings choices in accordance with the buffer stock model, taking into account the knowledge of the joint distribution of permanent and transitory shocks. I examine the effect of different UC decompositions on consumption dynamics in the buffer stock model. Specifically, for different decompositions of the reduced form process, holding other relevant parameters fixed, I simulate economies and estimate the marginal propensity to consume, the excess sensitivity and the excess smoothness at different levels of aggregation of simulated data.

I consider the orthogonal decomposition of income adopted in the literature, along with other potentially valid UC decompositions. Thus, different implications arising from different decompositions may be attributed to differences in information sets held by households and econometricians.[13] To be more precise, econometricians cannot identify correctly the joint distribution of permanent and transitory shocks if the shocks are correlated and household income in first differences is a moving average process. Households, to the contrary, make their consumption decisions having the knowledge on the correctly specified joint distribution of permanent and transitory shocks, be they correlated or not.

Suppose that the reduced form process for log income is an ARIMA(0,1,q) process:

$$\Delta y_t = \delta_q(L)\epsilon_t, \tag{9}$$

where $\delta_q(L)$ is a familiar lag polynomial of order q in L.

Further assume that the structural income process is the sum of a difference stationary permanent component, y_t^P, and a transitory component, a stationary process in log-levels, y_t^T. The reduced and structural forms of observed series should agree in time

[13] One may also imagine two populations of consumers who draw their income realizations from the same process each time period, yet one population cannot differentiate between permanent and transitory news, while another population does differentiate between the shocks. In this case, differences in consumption responses may be attributed to the heterogeneity in the information sets held by households.

$$\Delta y_t = \Delta y_t^P + \Delta y_t^T = A(L)u_t^P + (1-L)B(L)u_t^T, \tag{10}$$

and frequency domains:

$$S_{\Delta y_t}(w) = S_{\Delta y_t^P}(w) + S_{\Delta y_t^T}(w) = |A(e^{-iw})|^2\sigma^2_{u_t^P} + |1-e^{-iw}|^2|B(e^{-iw})|^2\sigma^2_{u_t^T}, \tag{11}$$

where $S_x(w)$ denotes spectral density of series x at frequency w; $A(L)$ and $B(L)$ are the lag polynomials that describe dynamics of the first difference of the permanent component and the level of the stationary component respectively; u_t^P and u_t^T are uncorrelated permanent and transitory innovations, respectively.

As can be readily seen from equation (11), the spectral density of the transitory component vanishes at frequency zero, and the variance of the permanent component is equal to the spectral density of the series at frequency zero, and is determined by estimates of $\delta_q(L)$, and the variance of the innovation from the reduced form process of equation (9).

The auto-covariance function of the reduced form process has $q+1$ non-zero auto-covariances, which is sufficient to estimate q MA coefficients and the variance of the reduced form income shock. An estimable UC model of income may allow at most $q+1$ non-zero parameters, two of which are the variances of structural shocks and the rest determine the dynamics of each unobserved component of income. Without estimation, though, for any known reduced form data generating process one may always construct infinitely many UC representations. By varying the structure of the UC model, one necessarily varies the relative importance of the permanent and transitory components. In the next section, I assume that the true reduced form income process households and econometricians face is ARIMA(0,1,1). Although this process allows two estimable parameters, I may construct infinitely many unobserved components models of income that imply different (income) information sets.

3.1 Procedure for Changing the Relative Importance of Permanent and Transitory Components

For the rest of the paper, assume that log income in differences, after the growth rate g_t has been removed, follows a stationary MA(1) process. This process has empirical support in micro data.[14] The corresponding UC model may be represented as a sum of a random walk permanent component and a transitory white noise process. This particular income process has become the workhorse in simulations of the buffer stock model of savings and for computational models of asset holdings over the life cycle. Following the above notation, the reduced and structural forms of the process for the first differences in income are:

$$\Delta y_t{}^{rf} = (1 + \delta L)\epsilon_t, \tag{12}$$

$$\Delta y_t{}^{sf} = u_t^P + (1 - L)u_t^T, \tag{13}$$

where superscripts rf and sf denote reduced form and structural form respectively.

I will use this process for simulating the buffer stock economy since it is easy enough to deal with computationally, and general enough to allow for decompositions of income into permanent and transitory components of different relative importance.[15]

Since the reduced form has only two pieces of information, the auto-covariances of order zero and one, I can recover (statistically) only two parameters, the variance of permanent shocks and the variance of transitory shocks. To explore the impact of the information structure

[14] MaCurdy (1982), Abowd and Card (1989), and Meghir and Pistaferri (2004) find for different samples and time span of the PSID data that the auto-covariance function of the first differences of log-income is at most of order 2. In Table 5 I show that the auto-covariance function of the first differences in income for my sample is significant up to order one, which is consistent with the reduced form MA(1) model.

[15] Ludvigson and Michaelides (2001) use this process to study "excess smoothness" and "excess sensitivity" puzzles on the aggregated data from a simulated buffer stock model. Michaelides (2001) uses this process to study the same phenomena but for a buffer stock economy of consumers with habit forming preferences. Luengo-Prado (2006) uses this process to study a buffer stock model augmented with durable goods, down payments, and adjustment costs in the market for durable goods. Luengo-Prado and Sørensen (2005) use a generalization of this process to gauge the effects of different layers of uncertainty (idiosyncratic and aggregate) on the marginal propensity to consume in the simulated "state"-level data and in US state-level data. Gomes and Michaelides (2005) and Cocco et al. (2005) calibrate the parameters of this income process to study consumption and portfolio choice over the life cycle.

of income on the consumption process, I allow for a covariance between the permanent and transitory shocks, and then work out the variance of transitory shocks. I match the moments of constructed series to the moments of the reduced form series, thus keeping the stochastic structure of the series intact. I present full details of the procedure in Appendix A. For a specific example, I take the estimated parameters of ARIMA(0,1,1) process from MaCurdy (1982, p. 109): the variance of reduced form innovations is 0.055, and MA parameter is –0.444. The grid of covariances considered in simulations implies the following correlations between structural shocks: –1.0, –0.75, –0.5, –0.25, 0.0, 0.25, 0.5, 0.75, and 1.0. As is clear from the above discussion, the variance of the permanent component is determined by the spectral density of the reduced form series at frequency zero. Thus, for chosen income parameters and for the random walk permanent component, the estimate of the variance of innovations to the permanent component is equal to $S_{\Delta Y_t}(0) = (1+\delta)^2\sigma_\epsilon^2 = 0.0658$. If the covariance between the permanent and transitory innovations is $\sigma_{u^P u^T}$, then the variance of transitory innovations is equal to $-\gamma(1) - \sigma_{u^P u^T}$, where $\gamma(1)$ is the first order auto-covariance of the reduced form process and is equal to $\theta\sigma_\epsilon^2 = -0.444 \times 0.055 = -0.02442$. Thus, for the covariance equal to 0.0135 (and the corresponding correlation between income shocks approximately equal to 1.0), the variance of transitory innovations is 0.0109; for the covariance equal to 0.00, the variance of transitory innovations is 0.0246.

The covariances between transitory and permanent shocks—and the corresponding correlations—"assign" the relative weight to the permanent component. The ranking of correlations in ascending order of this weight is: –1.00, –0.75, –0.50, –0.25, 0.0, 0.25, 0.50, 0.75, 1.0. Thus, the income model with the perfect negative correlation between the permanent and transitory shocks has the smallest permanent component, while the income model with the perfect positive correlation between the permanent and transitory shocks has the largest permanent component.

Correspondingly, I call the models built from these covariances as Model (1)–Model (9) in Table 1, with Model (1) producing the smallest and Model (9)—the largest permanent component. To prove that this is the case I undertake the following exercise. I draw 100 mean-zero, correlated normal transitory and permanent shocks, exponentiate them and calculate permanent, transitory and total income using the income process in equations (4)–(6). I set the initial permanent income to 5.0, and the gross growth rate of income to 1.0 for all periods. For each simulated model, I calculate the ratio of transitory income to permanent income and average the ratio over one hundred periods. I repeat the procedure 5,000 times, and average the ratio over all repetitions. I report the resulting statistic in Table 1. As can be seen from Table 1,

the ratio of transitory income to permanent income is largest for the model with the perfect negative correlation between structural shocks and the largest volatility of transitory shocks. As can be seen from the table, for this reduced form income process, a larger standard deviation of log-transitory shocks implies a relatively smaller permanent component.

3.2 Simulating the Buffer Stock Economy

The solution to the dynamic programming problem in Section 2 is the consumption policy function. Assuming the income process in equations (4)–(6), consumption and cash-on-hand can be expressed in terms of the ratios to permanent income, as in Deaton (1991) and Carroll (1992, 1997). I find the converged policy function that relates consumption to cash-on-hand by iterating the Euler equation:

$$\{C_t^{n+1}(X_t)\}^{-\rho} = R\beta E_t\{C_{t+1}^{n}(X_{t+1})\}^{-\rho}, \tag{14}$$

where ρ is the coefficient of relative risk aversion, and $C^n(X)$ is the consumption function at the n-th iteration.

Rewriting the above equation in terms of ratios to permanent income, and noting that the expectation in equation 14 is the integration over two (possibly correlated) distributions of structural shocks, I obtain:

$$\{c_t^{n+1}(x_t)\}^{-\rho} = R\beta \int_1^\infty \int_1^\infty \{c_{t+1}^{n}[R(x_t - c^n(x_t))/(Gu_{t+1}^P) + u_{t+1}^T]\}^{-\rho}\{Gu_{t+1}^P\}^{-\rho} f(u^P, u^T) du^P du^T, \tag{15}$$

The solution to a dynamic programming problem is the fixed consumption policy function such that $c^{n+1}(x) = c^n(x)$, i.e., the function that returns the same value of consumption for a given value of cash-on-hand at the adjacent iterations (time periods).

Respecting the liquidity constraint, consumption in each period is the minimum between the optimal consumption determined by the above equation, and the cash on hand available in that period.

I assume that the gross interest rate R is non-stochastic and that the joint probability density function of (potentially correlated) transitory and permanent shocks $f(u^P, u^T)$ is time invariant. In addition, shocks are assumed to be jointly log-normal, where the underlying joint normal distribution has a mean vector $(-\sigma^2_{u^P}/2, -\sigma^2_{u^T}/2)'$, and the variance-covariance matrix $\Sigma_{u^P u^T}$:[16]

$$\Sigma_{u^P u^T} = \begin{pmatrix} \sigma^2_{u^P} & \sigma_{u^P u^T} \\ \sigma_{u^T u^P} & \sigma^2_{u^T} \end{pmatrix}$$

To find the converged consumption policy function, I use the 120-point grid for consumption and cash-on-hand, equally spaced between 0 and 10. I calculate the right hand side of equation (15) by Monte Carlo integration, drawing 500 pre-seeded[17] transitory and permanent shocks from the appropriate log-normal distributions. To induce the correlation between the independent normal draws, I use the Cholesky factorization of the variance-covariance matrix $\Sigma_{u^P u^T}$.

I linearly interpolate values of the function between the points of the grid and iterate until $\{\frac{1}{120}\sum_{i=1}^{120}(c^{n+1}(x_i) - c^n(x_i))^2\}^{\frac{1}{2}} < 0.0001$. Upon finding the converged, time-invariant, policy function $c(x_t)$, I simulate the economy populated by 2000 ex ante identical consumers. They are heterogenous ex post due to different history of income draws. Consumers start with zero assets in the beginning of their working life, receive the permanent income normalized to one, receive a draw of a transitory income, save in accordance with their consumption policy rule, and enter into the second period of life with accumulated assets. For each consumer, I create 100 periods of information on consumption choices and income draws. Since consumers are likely to be out of equilibrium in the early periods of life,[18] I keep only the last 50 periods of information.

Since I am interested in the properties of consumption for different decompositions of a given reduced form model of income, I hold all other parameters of the buffer stock model fixed.

[16] The log-normal distribution and this choice of mean and variance generate mean-one transitory and permanent disturbances in levels and a unit root in log income. In the literature on income processes, idiosyncratic income is the residual from the regression of household income on observable characteristics. Therefore, both log-transitory shocks, and log-permanent shocks have zero means. The assumption that log-shocks have means other than zero, used in the simulation exercise of this section and by Carroll (1992) and Carroll (1997) is inconsequential for the results to follow.

[17] This is done to reduce simulation noise when finding the consumption policy function.

[18] In this problem equilibrium occurs at the point when consumer reaches the target level of wealth to permanent income ratio.

Thus, I do not vary the behavioral parameters of the model. Sensitivity of consumption to changes in those may be found elsewhere in the literature. I set the gross growth rate of income to 1.03, the gross interest rate to 1.03, the time discount factor to 0.97, and the coefficient of relative risk aversion to 2.0. I take draws from the joint distribution of log-normal transitory and permanent shocks, the parameters of which are derived from the reduced form ARIMA(0,1,1), as already discussed in detail in the previous subsection. I estimate the excess sensitivity and the excess smoothness using aggregated data. For each time period, aggregate consumption and income are defined as the sums of individual consumption and income over 2000 consumers. I also document the contemporaneous sensitivity of consumption to income by including first differences in log income as a separate regressor. These aggregate statistics are reported in Panel A of Table 2.[19] As can be seen from the table, excess sensitivity is statistically indistinguishable from zero for the models considered. Similar results have been obtained by Ludvigson and Michaelides (2001): aggregation of consumption choices from a standard buffer stock model does not explain the excess smoothness and the excess sensitivity puzzles at the aggregate level. Contemporaneous sensitivity of consumption to income is statistically different from zero for each model, and different from each other. Non-zero MPC is consistent with the PIH, since current income changes contain news about permanent income. What is important, though, is that differences in information sets held by households and econometricians have clear implications for understanding consumption behavior in the aggregate. If econometricians perform a typical orthogonal decomposition of income into transitory and permanent parts, which corresponds to the income Model (5), they may overstate or understate the MPC out of current income. The difference between the true MPC and the MPC predicted by the econometricians depends on the true relative importance of the permanent component, perfectly observed by households but not by econometricians.

I complement my analysis with pooled panel regressions of the growth of (simulated) household consumption on the current and lagged growth of (simulated) household income. These regressions mimic the excess sensitivity regressions on empirical micro data. Results are reported in Panel B of Table 2. The distinctive feature of these regressions, compared to the same regressions on the aggregated data, is that the MPC from lagged income changes is statistically significant. Importantly, the MPC out of current income changes and lagged income changes is larger for income models with relatively more important permanent component.

[19] I report only results for income Models (1), (5), and (9) since the direction of results is linear. Specifically, the MPC is larger for models with a larger size of the permanent component relative to the transitory component, etc. Results for all income models are available upon request.

In Panel C of Table 2, I report the standard deviation of consumption growth and the excess smoothness in simulated micro panel. As can be seen, Quah's (1990) critique of the excess smoothness puzzle at the macro level holds for the household-level buffer stock economy as well. The excess smoothness ranges from 0.36 for the income process with the smallest permanent component to 0.60 for the process with the largest permanent component. The difference between these values is significantly different from zero at any conventional level of statistical significance.

4 The Life Cycle Model of Consumption, and Estimation of the Income Process

4.1 The Model

The simulations in the previous section show that different decompositions of the same reduced form income process lead to sizeable differences in sensitivity of consumption growth to contemporaneous and lagged income growth. Thus, the joint dynamics of consumption and income in real data may help identify parameters of the income process: the variances of permanent and transitory shocks, and correlation between them.[20]

In this section, I present the model used to estimate parameters of the income process and the behavioral parameters. I use a structural life cycle model of consumption and savings, a variation of an infinite horizon buffer stock model of savings of the previous section. I assume that the model households are married couples that maximize expected utility from consumption over the life cycle. The only source of uncertainty in the model is uncertainty over income flows, arising from transitory and permanent income shocks.[21] I assume that all households start working at age 24 and retire at age 65. Households maximize the expected utility from annual consumption flows:

[20] Note that the correlation between permanent and transitory shocks cannot be identified if the reduced form income process is an integrated moving average process of any order and the structural form income process is a random walk plus a moving average transitory component. For the issues of identification of structural form time series processes see, e.g., Harvey (1989) and Morley et al. (2003).

[21] Other poorly insured risks over the life cycle are health shocks. In this paper I do not model medical expenditures and so do not consider health shocks. I purposefully limit my analysis to 24–65 year olds, a subgroup of population for whom these expenses and shocks are relatively less important. I also do not model bequest motives explicitly. Although bequests can be potentially important in reality, introduction of them into the model would complicate the optimization process.

$$E\{\sum_{t=24}^{T} \beta^{t-24} U(C_t, Z_t) + \beta^{T+1} V_{T+1}(X_{T+1})\}.$$

T is set to 65, the last working period age; β is the time discount factor as in the previous section; V_{T+1} is the value function at age 66, equal to the maximized expected utility at age 66 and onwards; C_t is household consumption at age t; Z_t refers to variables that proxy taste shocks at time t, X_{T+1} is the cash on hand at age 66. Utility function is the time separable CRRA utility function of the previous section. Household utility at different points of the life-cycle is affected by a vector of utility shifters, Z_t.[22] Thus, felicity function is expressed as: $U(C_t, Z_t) = \frac{C_t^{1-\rho}}{1-\rho} v(Z_t)$.

As in previous section, I assume that households have access to one instrument for saving and consumption smoothing—a riskless bond with the deterministic gross interest rate R. Cash-on-hand accumulation constraint and the income process are given in equations (2), and (4)–(6) respectively. I assume that households are subject to liquidity constraints so that their total consumption is constrained to be below their total cash-on-hand in each period.

As before, cash-on-hand and consumption can be expressed in terms of the ratios to permanent income, and the state space reduces to one variable, cash-on-hand relative to the permanent income, x_t.[23] As in Gourinchas and Parker (2001), I assume that the consumption function at retirement is linear in cash-on-hand, X_{T+1} and illiquid wealth, H_{T+1}: $C_{T+1} = \kappa_x X_{T+1} + \kappa_h H_{T+1}$, where κ_x is the marginal propensity to consume from liquid assets at retirement, κ_h is the marginal propensity to consume from illiquid assets at retirement. Dividing both sides of this equation by the permanent income at period $T+1$, it becomes $c_{T+1} = \kappa_x x_{T+1} + \kappa_h h_{T+1}$. The age-dependent consumption functions $\{c_t(x_t)_{t=24}^{65}\}$ are found recursively by iterating the Euler equation. The details of the model solution are relegated to Appendix B.

4.2 Estimation by the Method of Simulated Moments

In this section, I describe the method used to estimate the structural parameters of the model. The vector of structural parameters θ consists of the behavioral parameters: β, ρ; the retirement process parameters: κ_x, κ_h; and the parameters of the income process: σ_{u^T}, σ_{u^P} and $\sigma_{u^P u^T}$. I

[22] In the literature, vector Z_t usually contains leisure time of a spouse, the number of adults, and the number of children over the life cycle. I follow Gourinchas and Parker (2001) and use family size for Z_t. I assume that family size affects household marginal utility exogenously and deterministically, and estimate family size adjustment factors from empirical data.

[23] Throughout the paper, big letters refer to the variables in levels, while small letters refer to their values relative to the permanent income.

estimate the model parameters by the method of simulated moments (MSM). Since my model is cast in terms of one state variable, cash-on-hand relative to permanent income, I reformulate the consumption rule at retirement as $c_{T+1} = \kappa_x x_{T+1} + \kappa_0$, where κ_0 is the product of the marginal propensity to consume from illiquid wealth and average illiquid wealth at retirement. As it is hard to benchmark the marginal propensity to consume from illiquid assets from empirical data,[24] I leave κ_0 as a free parameter estimated within the MSM procedure. In total, I estimate seven parameters.

4.2.1 Moments Chosen for Matching

As have been mentioned before, the sensitivity of consumption changes to lagged and contemporaneous income changes may help identify parameters of the income process. In accordance with the results presented in Table 1 and Table 2, the reduced form persistence of income growth helps identify parameters of the structural form income process.[25] Inclusion of persistence into the set of moments to match imposes restrictions on the set of allowed models for idiosyncratic income. For persistence, I use the OLS coefficient $\hat{\psi}$ for the following AR(1) model:

$$\Delta \log y_{it} = \psi \Delta \log y_{it-1} + \xi_{it} \tag{16}$$

Thus, in the matching exercise I need to find the volatility of structural income shocks along with the contemporaneous correlation between them that lead to the closest match to the OLS coefficients and the persistence of income growth estimated from empirical data.

To identify the other four parameters of interest I need at least four other moments. In Table 3 I summarize some recent literature on the estimation of structural models by simulation methods. With the exception of Laibson et al. (2004), the moments used are the age-dependent medians and/or means of consumption and/or wealth, endogenous variables of the model. I follow the literature and, along with the just mentioned moments, match the age-dependent means of log-consumption, or the log-consumption life cycle profile. I construct the consumption

[24] PSID reports housing equity that may qualify for illiquid wealth, yet it does not provide separate records on other important components of illiquid wealth (e.g., pension wealth).

[25] The use of the Beveridge-Nelson decomposition is one way to identify the volatility of the permanent component from reduced form dynamics of the income process. Thus, for any reduced form process of the form $\Delta z_t = \delta_q(L)\epsilon_t$, the persistence of Δz_t, and z_t, is determined by the magnitudes and signs of the coefficients in the polynomial $\delta_q(L)$. For these processes driven by orthogonal innovations, the Beveridge-Nelson decomposition implies that the first difference in the permanent component has the variance $(1 + \sum_{j=1}^{q} \delta_j)^2 \sigma_\epsilon^2$, clearly determined by the persistence of the reduced form processes.

profile from the CEX and PSID data. I match 45 moments in total.

Since the model does not provide a closed form solution for these moments, I simulate the moments and estimate the parameters of the model by matching these simulated moments to the data moments. I estimate the model in two stages. In the first stage I estimate exogenous parameters χ, fixed in the optimization routine; in the second stage I estimate θ. χ consists of the life cycle profile of the gross growth rates of disposable income, $\{G_t\}_{t=25}^{65}$; the life cycle profile of utility shifters, $\{v(Z_t)^{\frac{1}{\rho}}\}_{t=24}^{66}$; the gross interest rate on safe liquid assets, R; the mean and the standard deviation of the distribution of wealth-to-permanent income ratio at age 24, $\{(W/Y^p)_{.,24}, \sigma_{(W/Y^p)_{24}}\}$; the mean and the standard deviation of the distribution of permanent income at age 24, $\{(Y^p)_{.,24}, \sigma_{Y^p_{24}}\}$. I set the gross interest rate on safe liquid assets at 1.0344.[26]

Given the estimates of the first stage parameters, the MSM estimates of the second stage parameters θ are such that the distance between the vector of simulated moments and the vector of empirical moments is as close to zero "as possible." $\hat{\theta}_{MSM}$ is the solution to the minimization of the criterion function

$$(f_{I_{sim}}(\theta;\hat{\chi}) - f_{I_d})' W (f_{I_{sim}}(\theta;\hat{\chi}) - f_{I_d}), \tag{17}$$

where W is a positive definite weighting matrix; f is the 45×1 vector of the second stage moments; $\hat{\chi}$ is the 88×1 vector of the first stage parameters; θ is the 7×1 vector of the second stage parameters; I_{sim} is the number of households used to simulate the model; I_d is the number of households used to construct the data moments.

Given $\hat{\chi}$ and an initial vector of the second stage parameters, θ_0, the algorithm proceeds in following steps:

1. Numerically derive the age-dependent policy functions $\{c_t(x_t;\theta_0,\hat{\chi})\}_{t=24}^{65}$.
2. Given these policy functions, simulate the model economy populated by 5000 households.
3. For these simulated data, calculate the model moments and the MSM criterion function.
4. If convergence is not achieved, update θ using some optimization method. Repeat steps (1)–(3) until convergence is achieved.

It can be shown that a consistent estimate of the variance-covariance matrix of θ_{MSM}, the

[26] This estimate is from Gourinchas and Parker (2001). It is the average real return on Moody's AAA municipal bonds for monthly data spanning 1980-1993.

minimizer of the MSM criterion function, is:[27]

$$V_{\theta_{MSM}} = (F_\theta' W F_\theta)^{-1} F_\theta' W [\Omega_f(\theta_0; \chi_0)(1 + \frac{I_d}{I_{sim}}) + \frac{I_d}{I_1} F_\chi \Omega_\chi F_\chi')] W F_\theta ((F_\theta' W F_\theta)^{-1})', \quad (18)$$

where F_θ is the gradient of the second stage moments with respect to θ; θ_0 is the vector of (unknown) true structural parameters; χ_0 is the vector of (unknown) true first stage parameters; Ω_f is the variance-covariance matrix of the second stage moments, $E[(f_{I_{sim}}(\theta_0; \chi_0) - f_{I_d})(f_{I_{sim}}(\theta_0; \chi_0) - f_{I_d})']$, consistently estimated from the data; F_χ is the gradient of the second stage moments with respect to the first stage parameters χ; Ω_χ is the variance-covariance matrix of the first stage parameters; I_1 is the number of households used to calculate the first stage parameters.

For the weighting matrix W, I use the diagonal matrix with diagonal elements equal to the diagonal elements of the inverse of Ω_f, calculated from the sample data.

Consistent (sample) estimates of the variance of $\hat{\theta}_{MSM}$ are calculated as:

$$var(\hat{\theta}_{MSM}) = diag\{(F_\theta' W F_\theta)^{-1} F_\theta' W [\Omega_f(\frac{1}{I_d} + \frac{1}{I_{sim}}) + \frac{1}{I_1} F_\chi \Omega_\chi F_\chi')] W F_\theta ((F_\theta' W F_\theta)^{-1})'\}) \quad (19)$$

The standard errors of the model parameters are estimated by taking the square root of each element of vector in equation (19). I describe the construction of Ω_f, and Ω_χ in detail in Appendix C.

4.2.2 The Identification Scheme

The identification scheme may be described in the following way. In the absence of uncertainty or in the case of a perfect foresight, both the life cycle and dynastic consumption models predict that the shape of the consumption profile is determined by the structural parameters (the time discount factor and the relative risk aversion parameter) and the interest rate. Thus, the time discount factor and the relative risk aversion parameter may be identified from the life cycle consumption profile, or from the long-run features of consumption data. In the presence of income uncertainty and rational expectations, the life cycle consumption profile is also affected by precautionary motives. Gourinchas and Parker (2001) and Cagetti (2003), guided by these

[27] For derivation of the limiting distribution of the MSM estimators see Duffie and Singleton (1993), Pakes and Pollard (1989), and McFadden (1989).

considerations and treating the volatility of permanent and transitory shocks as given, estimate the relative risk aversion parameter and the time discount factor for different occupations and education groups. Yet, the shape of the consumption profile may be also informative for the identification of the magnitude of income uncertainty over the life cycle. Furthermore, the results from Table 2 suggest that the volatility of individual components of income and the correlation between them determines the joint short-run dynamics of income and consumption.

Summing up, the structure of the income process, the time discount factor and the relative risk aversion parameter can be identified using the long run features of consumption data (contained in the consumption life cycle profile); the short run features of the consumption and income data (determined by the sensitivity of consumption growth to current and lagged income growth); and the long run features of income data (determined by the persistence of income growth).

4.3 Construction of Empirical Moments

In this section, I describe the procedure used to estimate the empirical moments I match and the first stage parameters. I first briefly describe the data sources used.

4.3.1 Data

I obtain consumption information from two data sources, Survey of Consumer Expenditures (CEX), and the Panel Survey of Income Dynamics (PSID). CEX contains detailed information on total expenditures and its components, and the demographics for representative cross sections of the US population. I use the 1980–1998 waves of the CEX. PSID features panel data and is considered to be the best source of income data for the US population. Unlike CEX, PSID limits its coverage of consumer expenditures to consumption of food at home and away from home. Since I am interested in the link between changes of disposable household income and total household consumption, I impute total consumption to the PSID households using information on household food consumption in PSID and CEX, and matched demographics from CEX and PSID; and exploit superior (to CEX) income data from PSID. The PSID data are taken from 1981–1997, 1999, and 2001 waves. I follow the methodology of Blundell et al. (2005) to impute total consumption to the PSID households. Full details on sample selection of CEX and PSID households, and the data sources are described in Appendix D. The imputation procedure is described in Appendix E.

4.3.2 Estimation of Life Cycle Profiles

At each point in time total consumption and disposable income are affected by cohort effects, time (business cycle) effects, life-cycle (age) effects, and idiosyncratic effects. Consumption (and, similarly, income) can be decomposed as follows:

$$\log C_{it} = \alpha_c' c_i + \alpha_f' f_{it} + \alpha_\tau' \tau_t + \alpha_a' Age_{it} + u_{it}, \tag{20}$$

where $\log C_{it}$ is the log-total real household consumption, c_i is the set of cohort dummies (with an omitted category of heads born between 1939-1944), f_{it} is the set of family size dummies (with an omitted category of households with 7 or more members),[28] τ_t is the set of time dummies, Age_{it} is the full set of age dummies (created for households aged between 24–70), and u_{it} is the idiosyncratic effect that consists of the time varying and time-invariant random effects, α_c is the vector of cohort coefficients, α_f is the vector of coefficients on family size dummies, α_τ is the vector of time coefficients, and α_a is the vector of age coefficients. Since the age effects and time effects cannot be estimated simultaneously, I assume that time effects can be captured by regional unemployment, as in Gourinchas and Parker (2001) and French (2005). The equation I estimate is:

$$\log C_{it} = \alpha_c' c_i + \alpha_f' f_{it} + \alpha_u UE_{it} + \alpha_a' Age_{it} + u_{it}, \tag{21}$$

where α_u is the coefficient on regional unemployment, UE_{it}.

I eliminate cohort, family size, and aggregate effects from consumption predicted by equation (21) using the following transformation:

$$\widetilde{\log C_i} = \widehat{\log C_i} + \widehat{\alpha_f}'(f_a - f_i) + \widehat{\alpha_u}(\overline{UE} - UE_i) + \hat{u}_i, \tag{22}$$

where $\widehat{\log C_{it}}$ is the predicted consumption from equation 21, $\widetilde{\log C_i}$ is the transformed consumption; $f_a = \frac{1}{I_a}\sum_{i=1}^{I_a} f_i$, the average family size for a group of I_a people of a certain age, $\overline{UE}$ is the average unemployment over years and individuals, and $\hat{u}_i$ is a household-specific residual

[28] The full set of family size dummies is created for households with family size of 2, 3, 4, 5, 6, and 7 or more members.

at given age.[29]

The unsmoothed life cycle consumption profile is a plot of the age-average of $\widetilde{\log C_i}$ against age. The smoothed profile is constructed from a regression of $\widetilde{\log C_i}$ on the fifth degree polynomial in age and year of birth. Figure 1 plots raw, smoothed and unsmoothed life cycle profiles of consumption. Similarly, I construct the smoothed life-cycle profiles of utility shifters (determined by changes in family size over the life cycle), and household disposable income.

The average smoothed log-consumption profile for ages 24–65 defines 42 empirical data moments I use in the MSM exercise.

4.3.3 Persistence of Income, and the Joint Dynamics of Consumption and Income

The other three moments I am interested in are the persistence in the growth rate of income, and the coefficients from the OLS regression of consumption growth on contemporaneous and lagged income growth. To reconcile empirical data with simulated data, I use $\widetilde{\log C_{it}}$ and $\widetilde{\log Y_{it}}$, log- household consumption and log-household disposable income from empirical data, "net" of cohort, aggregate, and family size effects. I regress the first difference in $\widetilde{\log C_{it}}$ on the current and lagged growth in $\widetilde{\log Y_{it}}$, changes in family size over the life cycle, and a quadratic polynomial in head's age. The results of this regression are presented in Table 4.[30] The estimated relationship between current income changes and consumption changes is very strong: for every 10% increase in income, consumption increases by 1.3%. In accordance with the PIH, if income is a random walk and therefore income changes are permanent, consumption should have changed by 10% instead of the observed 1.3%. The magnitude of the contemporaneous association between consumption and income changes may be taken as evidence that household income contains, apart from a random walk permanent component, a strong mean reverting transitory component. The response of consumption growth to lagged income growth is small and imprecisely estimated. This result is in line with the rational expectations tradition of consumption theory, which predicts that consumption is a martingale, and thus that consumption changes should be orthogonal to any past information, inclusive of the past income changes (e.g., Hall (1978) and Hall and Mishkin (1982)).

[29] I estimate equation (21) by pooled OLS. Thus, the residuals contain household effects, and any household specific variation in consumption. Alternatively, I could have used the fixed effect panel regression to construct profiles. The consumption profile from the fixed effect panel regression is similar to the one presented in Figure 1.

[30] Similar results are obtained from a pooled OLS regression of raw consumption growth on raw current and lagged growth in disposable income, cohort dummies, change in household family size, regional unemployment, quadratic polynomial in head's age, education and race dummies. Results are available upon request.

To benchmark the persistence of idiosyncratic income, I estimate the AR(1) process for income growth by OLS. The OLS estimate of AR(1) coefficient is equal to –0.36 with a standard error of 0.0071. I match the AR(1) coefficient of the reduced form rather than the MA(1) estimate, since an AR(1) process is less time consuming to estimate.[31] This proves to be very important when repeated estimations are performed on the simulated data.

Why are the OLS coefficients in Table 4 that different from those in Table 2? One answer is that the OLS coefficients of Table 2 are functions of parameters for a given reduced form income process, the fixed time discount factor and the fixed risk aversion parameter. In turn, the empirical moments of Table 4 are functions of the very same yet unknown parameters. These unknown parameters and the assumption that the buffer stock model is a valid model of consumption decisions by households give rise to these particular empirical moments. It is the essence of the whole matching exercise to estimate parameters that provide the best match to the moments observed in Table 4, to the persistence of reduced form income process, and the life cycle consumption profile.

5 Univariate Dynamics of Idiosyncratic Household Income

In this section, I present some results on the univariate dynamics of household income in the PSID data. It is important to know whether the income process in equations (4)–(6) is empirically justified. In the next section, I present results on the income process identified from the joint dynamics of income and consumption using the just described matching procedure.

The income measure I consider is the residuals from the cross-sectional regressions of household log-disposable income on education of head, household state of residence, a second degree polynomial in head's age, and head's race. In the literature, it is typically labelled idiosyncratic household income. For the cross-sectional regressions, I use information from the 1980-1997[32] annual family files of the PSID. The sample selection is described in the notes to Table 5. Table 5 presents the auto-covariance function for the growth in household idiosyncratic income. In this table, the auto-covariances and their respective standard errors are pooled over time.

[31] After all, it is well-known that any invertible MA(1) process has an autoregressive representation of the infinite order (see, e.g., Hamilton (1994)). Galbraith and Zinde-Walsh (1994) show that low order auto-regressive approximations of a MA(1) process with a moving average parameter of 0.5 and less in absolute value—as low as order one and up to the third order—perform the best in terms of minimizing the mean squared error. Thus, using the estimated AR(1) process as an approximation to (the infinite order representation of) MA(1) process may be sufficient to benchmark the persistence of household idiosyncratic income growth.

[32] This period corresponds to the time span of consumption data analyzed in the paper.

As can be seen from the table, the auto-covariance function (pooled over time)[33] is statistically significant up to order one, and the first order auto-covariance is negative. This is consistent with the integrated moving average process of MaCurdy (1982) used in Section 3 and analyzed in this paper. Thus, even if household income contains a unit root, it also contains a strong mean-reverting transitory component.

In Table 6 I test the null hypothesis that the auto-covariances of a given order are equal to zero in all time periods. Results of this test indicate that the transitory component may be a moving average process of order one.

Finally, in Table 7, I use the moment suggested by Meghir and Pistaferri (2004) to identify the variance of permanent innovations.[34] It is easy to see that the moment in Table 7 identifies the long-run variance of the structural income process in first differences under the assumption that the auto-covariance function is zero for orders greater than or equal to $q+1$, where q is the moving average order of the transitory component. In the second column, I present results on the volatility of permanent shocks under the assumption that the transitory component is a white noise, while the third column presents results for the assumption that the transitory component is a moving average process of order one. As can be seen, the assumption that the transitory component is a moving average process of order one gives rise to a smaller volatility of permanent shocks. Largely, it can be explained by the fact that, unlike in the case of a white noise transitory component, estimation uses information on the auto-covariances of the second order and those are negative.[35] Both estimates imply a substantial volatility of permanent shocks, considered to be important for consumption decisions over the life cycle.

[33]The full matrix of auto-covariances and the corresponding variance-covariance matrix consists of $\frac{T\times(T+1)}{2}$ (=171) unique elements, where T is the time span of the sample.

[34]See notes to Table 7.

[35]For any reduced form process in first differences, the spectral analog of this moment defines the numerator of the measure of the size of the random walk in Cochrane (1988). More negative auto-covariances of order one and higher orders imply a smaller long-run variance of the process, and therefore a smaller volatility of the permanent component. Importantly, this moment can be used to test the hypothesis that the structural log-income process does not contain a random walk permanent component (see Guiso et al. (2005)). As discussed in this section, the order of the auto-covariance function of the first differences in log-income is at most two. This is consistent both with the stationary log-income process and with the log-income process that contains a permanent random walk component and a stationary, transitory component. Under the null of the absence of the permanent component in the household log-income, the moment should return a zero estimate for the volatility of the permanent component. Intuitively, under the null, the spectral density of the log-income differences at frequency zero (and, consequently, the moment itself) should be zero if income consists solely of the transitory component. The results in Table 7 can be used to reject the null at any plausible level of significance.

6 Discussion of Results

In this section, I present my main results, and discuss potential biasing effects.

6.1 Results

Column (1) of Table 8 contains the main results. They are based on the minimization of the MSM criterion function, using information from the total sample of households with 24-65 year old heads. I estimated three sets of parameters: behavioral parameters (the time discount factor and the relative risk aversion parameter); retirement process parameters; and the income process parameters (the volatility of transitory and permanent shocks, and the contemporaneous covariance between them). I now turn to discussion of results on each set of parameters.

The behavioral parameters are estimated tightly. The time discount factor is practically the same as in Gourinchas and Parker (2001). The estimated relative risk aversion parameter is about 4.0, and is comparable to the estimates in Cagetti (2003) and Nielsen and Vissing-Jorgensen (2005). The retirement process parameter κ_0 is statistically indistinguishable from zero. It is impossible to conclude whether it is the small marginal propensity to consume or the small average holdings of illiquid wealth at retirement that drive this result. If non-zero, this parameter determines the kink of the consumption function at retirement and thus makes it concave, although non-differentiable. The data, however, "prefers" the consumption function at retirement to be linear rather than concave. In economic terms, it may mean that income uncertainty at retirement is not as important as it is at the earlier stages of the life cycle and, therefore, precautionary motives are largely absent at retirement. This interpretation corresponds to the finding of Gourinchas and Parker (2001)—that consumers behave as the PIH consumers in the late stages of the life cycle, and that the precautionary savings motive is very important in the beginning of the life cycle. The estimate of κ_x also confirms this finding of Gourinchas and Parker (2001). The parameter κ_x determines the marginal propensity to consume out of cash-on-hand, or liquid assets, at retirement. For the gross interest rate used in estimation, the marginal propensity to consume out of total and liquid wealth predicted by the PIH is equal to $\frac{r}{1+r}$, or 0.03, and I cannot reject the hypothesis that κ_1 is equal to the PIH value.[36]

The parameters of the income process are precisely estimated and indicate substantial id-

[36]Bequest motives on the part of ageing households may provide yet another reason for the low marginal propensity to consume from liquid wealth at retirement.

iosyncratic income uncertainty over the life cycle. Consistent with the literature on the univariate income dynamics at the household level, the volatility of permanent shocks is smaller than the volatility of transitory shocks.[37] I find that the contemporaneous covariance between transitory and permanent shocks is negative, with the correlation of about –0.4. The null hypothesis of zero covariance between transitory and permanent shocks can be easily rejected.

In economic terms, the sign of the covariance may indicate that unfavorable permanent shocks to disposable household income, like the deterioration of head's health or head's long-term unemployment, are partially offset by increases in the transitory income, like spousal temporary work, or unemployment compensations from the government. It is also likely that this offsetting effect will manifest itself at the annual frequency, the frequency I use for modelling the consumption behavior in the life cycle buffer stock model. Consider another explanation for this finding. Household income derives from multiple sources: wages of wife and head, transfer income of various sorts, (labor part of) business and farm income, (labor part of) income from roomers and boarders, bonuses, overtime and tips.[38] As an example, if a household experiences a negative shock to the head's wages, plausibly assumed to be in the list of permanent shocks, it may compensate its adverse effect by leasing available housing to outsiders.[39]

Column (2) of Table 8 contains results for the case when σ_{u^P}, the standard deviation of permanent shocks, is constrained at the value estimated from the univariate dynamics of household disposable income. I take the conservative, lower estimate of the volatility of permanent shocks from Table 7. The estimates of the volatility of transitory shocks and the covariance between structural shocks are both lower. Importantly, these estimates of the transitory volatility and

[37]Storesletten et al. (2004) estimate an AR(1) process for idiosyncratic log-income and find that it is highly persistent, with an auto-correlation coefficient of about 0.95. Their estimates (p. 705, Table 2) can be used to infer that the (unconditional) standard deviation of persistent ("permanent") shocks ranges from 0.13 to 0.21 while the standard deviation of the white noise transitory shock ranges from 0.24 to 0.56. Carroll and Samwick (1997), for their full sample, estimate the standard deviation of the permanent component to be equal to 0.15, and the standard deviation of the transitory component to be equal to 0.21. Estimates of Gourinchas and Parker (2001) for their total sample are: 0.15—for the standard deviation of permanent shocks and 0.21—for the standard deviation of transitory white noise shocks. The estimates of Blundell et al. (2004) can be used to infer that the (unconditional) standard deviation of permanent shocks is equal to 0.15 while the (unconditional) standard deviation of transitory shocks (to the moving average transitory component) is equal to 0.20. The estimates of Meghir and Pistaferri (2004) for their pooled sample are: 0.18—for the (unconditional) standard deviation of permanent shocks, and 0.21—for the upper bound of the (unconditional) standard deviation of transitory shocks (to the moving average transitory component). The estimates of Cocco et al. (2005) are: 0.11—for the average (across education groups) standard deviation of permanent shocks and 0.28—for the average (across education groups) standard deviation of white noise transitory shocks. These papers assume that the covariance between transitory and permanent shocks is equal to zero. They use data from the PSID with different sample selection criteria and time span. My estimates of the volatility of permanent and transitory shocks fall within the interval of estimates in the just cited literature.

[38]For the list of household income sources, see the PSID documentation for any annual family file.

[39]There are so many houses available for rent in Houston, and, quite plausibly, some of them are rented not because they are simply idle!

covariance imply that the correlation between structural shocks is of practically the same magnitude as that obtained from the unconstrained optimization. Perhaps, this result indicates that while the volatility of permanent innovations can be uniquely identified, the volatility of transitory shocks and the contemporaneous covariance between transitory and permanent shocks are identified jointly.[40] The relative risk aversion parameter is lower than the value in column (1), while the time discount factor is a bit larger. Compared to my main results, the estimate of κ_0 is larger in magnitude, although is still statistically indistinguishable from zero. Evidently, the data moments and the model do not provide enough information for the identification of this parameter. The results in column (3), where I constrain κ_0 at zero, are not appreciably different from the results in column (1).

6.2 Possible Biases

In accordance with the test in Table 6, the order of the auto-covariance of the transitory component is one rather than zero. The latter value has been assumed throughout the paper and has some empirical support (see Table 5). If the transitory component is a moving average process of order one rather than a memoryless, white noise, process, how can it bias my results on the income process? Unfortunately, I cannot evaluate the direction of a bias, if any, in general since the simulation exercise in Table 2 has been done for a white noise transitory component, and since the income process parameters are not solely identified from the income dynamics.[41]

The moments used for matching may be affected by measurement errors in income and consumption. The measurement error in consumption is likely to be averaged out while constructing the consumption profile.[42] The measurement error problem may be more important for other moments identifying structural parameters.

The persistence helps identify the long-run variance of income growth and therefore is an important identifying moment for the volatility of the permanent component of income. If the measurement error in log-income is an i.i.d. process or a short-lived moving average process, the persistence of the income growth estimated from empirical data will be underestimated,

[40] See Appendix A and Morley et al. (2003) for further discussion.

[41] While it is possible to adapt the estimation to the case of a moving average transitory component, the adaptation will necessarily require estimation of an additional, moving average, parameter, and a much longer run time.

[42] The CEX regression in Table 9, I use to impute total consumption, explains only 52% of the variation in household food consumption. I will likely carry over the error of regression to imputed total consumption. If the regression error does not affect age groups disproportionally, the constructed life cycle consumption profile will approximate the true life cycle consumption profile well.

i.e. the AR(1) coefficient from equation (16) will be more negative. In this case, the estimated volatility of permanent shocks will be downward biased. Only in case the measurement error is a random walk, the persistence will be overestimated since the variance of the measurement error will affect the long-run variance of the process. In this case, the volatility of permanent shocks will be estimated with an upward bias.

The volatility of transitory shocks and the covariance between permanent and transitory income shocks are largely identified by the coefficients of the OLS regression of consumption growth on current and lagged income growth. Since income growth may be measured with error, the OLS coefficients are likely to be biased towards zero. In this case, as the results from Table 2 suggest, both the volatility of transitory shocks and the covariance are likely to be pushed away from zero, making the estimated transitory volatility larger and the estimated covariance more negative.

7 Conclusions

I analyze the plausible situation when households have better information about income components than econometricians. In this case, the structure of the income process that econometricians can identify from the univariate dynamics of household income may differ from the true, more complicated, income structure.

I assume that household income can be described by a simple integrated moving average process of order one (the "reduced" form process) and that income is composed of transitory and permanent components. While the income realizations are observed both by households and econometricians, econometricians do not observe the innovations to permanent and transitory income directly. In the literature on household income processes, shocks to the permanent and transitory components are assumed to be uncorrelated. However, the reduced form process can be consistent with many decompositions of income into permanent and transitory components.

I argue that households' unique information about income components should be reflected in their consumption behavior. I use an infinite horizon buffer stock model and estimate the marginal propensity to consume out of shocks to current and lagged income and the excess smoothness for simulated data at different levels of aggregation and different decompositions of the same reduced form income process. I find that different decompositions imply different marginal propensities to consume and excess smoothness.

I further suggest that the empirical marginal propensity to consume out of shocks to current and lagged income should help identify the parameters of the income process, including the correlation between structural income shocks. The latter is not identified from the univariate dynamics of household income.

I estimate a structural life cycle buffer stock model utilizing household consumption and income data from the PSID and the CEX. I find a significant negative contemporaneous correlation between permanent and transitory shocks of about –0.4. The estimates of the time discount factor and the relative risk aversion parameter are precisely estimated and plausible. I find that the relative risk aversion parameter is about 4.0 and the time discount factor is 0.95. The estimates of permanent and transitory volatility, together with the estimates of the relative risk aversion parameter and the time discount factor, imply a substantial precautionary savings motive in household consumption choices over the life cycle.

I showed that consumption dynamics at household and macro levels crucially depends on the structure of the income process. Thus, correct identification of the components of the income process helps specify the consumption function of a life cycle dynamic optimization problem better. This, in turn, can prove to be important for understanding wealth accumulation, portfolio choice, and other life cycle choices of households.

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Figure 1: Household Consumption Profile over the Life Cycle. Total PSID Sample.

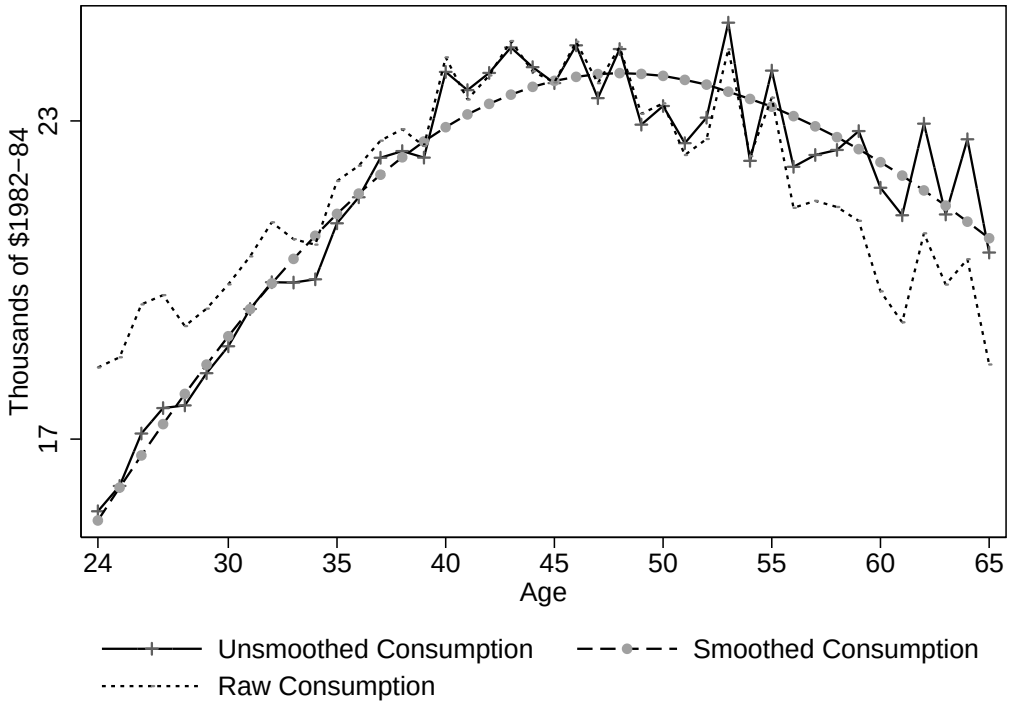

Table 1: Alternative Decompositions of the Reduced Form Income Process and the Relative Importance of Permanent Income

Unobserved Components Income Model*	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ratio of Transitory Income To Permanent Income ($\times 100$)	33.02	32.55	32.16	31.84	31.58	31.37	31.20	31.07	30.97

Notes:
Reduced Form Process for the Growth in Idiosyncratic Income: $\Delta y_{it} = (1 + \theta L)\epsilon_{it}$; θ=–0.444, σ^2_{ϵ}=0.055.
Structural Income Processes: $\Delta y_{k,it} = u^P_{k,it} + \Delta u^T_{k,it}$, $k = 1, \ldots, 9$.

* Parameters for:
Model (1): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.2345$, $\sigma_{u^T u^P} = -0.0306$, $\rho_{u^T u^P} = -1.00$.
Model (2): $\sigma_{u^P} = 0.1304$, $\sigma_{u_T} = 0.2126$, $\sigma_{u^T u^P} = -0.0208$, $\rho_{u^T u^P} = -0.75$.
Model (3): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1922$, $\sigma_{u^T u^P} = -0.0125$, $\rho_{u^T u^P} = -0.50$.
Model (4): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1734$, $\sigma_{u^T u^P} = -0.0057$, $\rho_{u^T u^P} = -0.25$.
Model (5): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1563$, $\sigma_{u^T u^P} = 0.00$, $\rho_{u^T u^P} = 0.00$.
Model (6): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1408$, $\sigma_{u^T u^P} = 0.0046$, $\rho_{u^T u^P} = 0.25$.
Model (7): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1271$, $\sigma_{u^T u^P} = 0.0083$, $\rho_{u^T u^P} = 0.50$.
Model (8): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1148$, $\sigma_{u^T u^P} = 0.0112$, $\rho_{u^T u^P} = 0.75$.
Model (9): $\sigma_{u^P} = 0.1304$, $\sigma_{u^T} = 0.1042$, $\sigma_{u^T u^P} = 0.0136$, $\rho_{u^T u^P} = 1.00$.

For each period t, permanent and transitory income shocks are the respective entries of the 2×1 vector, $exp(chol(\Sigma_{u^T u^P}) \times \epsilon_t))$, where $chol(\Sigma_{u^T u^P})$ is the Cholesky factor of the variance-covariance matrix of structural log-income shocks, and ϵ_t is the 2×1 vector of independent random normal draws. Transitory income $\{\epsilon^T_{it}\}$ is assumed to be a serially uncorrelated sequence of random log-normal draws, and permanent income is defined recursively by $P_{it} = P_{it-1}\epsilon^P_{it}$. Total income in period t is the product of the transitory and permanent income in period t. For each model, I create a sequence of 100 draws of permanent and transitory income. Initial permanent income for all models is set to be equal to 5.0. For each time period and each income model I calculate the ratio of transitory to permanent income. I repeat procedure 5,000 times. Each entry in the table is the averaged ratio over 100 periods and 5,000 repetitions.

Table 2: The Marginal Propensity to Consume, Excess Sensitivity, and Excess Smoothness for the Data from a Simulated (Infinite Horizon) Buffer Stock Model

Simulated Model*	(1)	(5)	(9)
PANEL A			
Aggregate MPC ($\hat{\alpha}_1^{\dagger}$)	0.13	0.38	0.52
	(0.05)	(0.05)	(0.03)
Aggregate Excess ($\hat{\alpha}_2$)	–0.02	–0.007	0.01
Sensitivity	(0.06)	(0.08)	(0.08)
PANEL B			
Panel MPC ($\hat{\beta}_1^{\ddagger}$)	0.02	0.43	0.63
	(0.002)	(0.001)	(0.0005)
Panel Excess ($\hat{\beta}_2$)	0.01	0.15	0.22
Sensitivity	(0.001)	(0.001)	(0.0004)
PANEL C			
Panel St. Dev. of	0.091	0.135	0.152
Consumption[§]	(0.017)	(0.016)	(0.017)
Panel Excess	0.356	0.531	0.595
Smoothness	(0.061)	(0.067)	(0.04)

Notes:
Models are arranged in ascending order of the importance of the permanent component. Columns 1, 5 and 9 utilize the respective models of the income process from Table 1. They correspond to the income models with perfectly negative, zero, and perfectly positive correlation between permanent and transitory shocks respectively.
The following benchmark parameters are preserved in all simulations. $R = 1.03$; $G = 1.03$; β=0.97; ρ=2.0. This combination of parameters satisfies the impatience condition of the buffer stock model and guarantees the model convergence (see Carroll (1997)).
Parameters of MA(1) reduced form process for differences in income are taken from MaCurdy (1982): MA parameter=–0.444; σ_{ϵ}^2=0.055.
[†]Regression Models for Panel A:
$\Delta \log C_t = const + \alpha_1 \Delta \log Y_t + \epsilon_t$;
$\Delta \log C_t = const + \alpha_2 \Delta \log Y_{t-1} + \varepsilon_t$.
$\log C_t = \log(\sum_{i=1}^{N} C_{it})$, $\log Y_t = \log(\sum_{i=1}^{N} Y_{it})$, where N is the number of simulated households set equal to 2000.
[‡]Regression Model for Panel B: $\Delta \log C_{it} = \beta_0 + \beta_1 \Delta \log Y_{it} + \beta_2 \Delta \log Y_{it-1} + \epsilon_{it}$.
[§] Panel Excess Smoothness is defined as $\frac{1}{J}\sum_{j=1}^{J}\{\frac{1}{N}\sum_{i=1}^{N} std_i(\frac{\Delta \log C_{it}^j}{\Delta \log Y_{it}^j})\}$, where N is the number of simulated households set equal to 2000; J is the number of model repetitions set equal to 50; $t = 1, \ldots, T$, where T is the number of usable time periods set equal to 50.
As described in the text, each simulated economy is populated by 2000 ex ante identical consumers and is observed during 100 periods. The first 50 periods of information have been discarded. The results are the averages of the corresponding statistics over 50 repetitions. The average standard errors in parentheses. Standard deviations (over simulations) for the estimates of the coefficients and standard deviations for their standard errors are available upon request.

Table 3: Structural Estimation of Life-Cycle Consumption Models By Simulation Methods

	(C)	(GP)	(LRT)	(F)
Parameters Estimated	CRRA, time disc. factor	CRRA, time disc. factor, retirem. param.	short-term/ long-term time disc. factor	7 param., includ. CRRA and time disc. factor
Data Used	SCF, PSID (wealth) CEX (inc., demogr.)	CEX (inc., cons.) PSID (inc.)	PSID (inc., cons.) SCF (wealth)	PSID (wealth, hours worked, partic. rate, health status)
Unit of Analysis	hhlds aged 25–65, grouped into eight 5-year age cohorts	hhlds aged 25–65	hhlds aged 20–90; no coll. degree	hhlds aged 30–90
Life Cycle Span	stoch.; start life at 25, live randomly; die determ. at 90; use data for 26–65 y.o.	determ. (up to ret.); use data for 25–65 y.o.	stoch.; live randomly; die determ. at 90	stoch.; live randomly; die determ. at 95; use data for 30–70 y.o.
Moments Matched	educ. and cohort-spec. med./means of wealth	mean log cons. by age	mean wealth/inc. ratio at 50–59; fraction of credit card borrowers; mean credit debt/inc. over LC; excess sensit.	med. and mean assets; mean log hours; partic. rate, conditional on health status
Number of Moments	8	40	5	240
Correction of S.E. for the 1st Stage Uncertainty and Sim. noise	No	Yes	Yes	No
Income Process	inc growth—MA(1), different. by educ.	inc growth—MA(1), different. by educ. and occupation	log inc.— persistent AR(1)	log wage— persistent AR(1)
Bequest	Yes	No	Yes	Yes
Model of Retirement	exogenous; i.i.d. earnings after age 65	exogenous; parameterized in value function	exogenous; i.i.d. earnings after age 63	endogenous (focus of the paper)
Number of Simulated Households	10000	20000	5000	5000
Preference Shifters	Yes (leisure of a spouse, family size)	Yes (family size)	Yes (effect. family size)	Yes (family size)

Notes: The table summarizes select recent papers that estimate structural life cycle models of consumption/savings by the method of simulated moments. (C) denotes Cagetti (2003); (GP)—Gourinchas and Parker (2001); (LRT)—Laibson et al. (2004); (F)—French (2005).

Table 4: Sensitivity of Consumption Growth to Current and Lagged Income Growth

Dependent Variable: $\Delta \log C_{it}$

Variable	Coefficient	St. Error
$\Delta \log(Income_{it})$	$.126^{***}$	.016
$\Delta \log(Income_{it-1})$	.016	.017
$\Delta Fam.Size_{it}$	.092	.119
Age_{it}	.004	.006
$Age_{it}^2/100$	$-.006$	.006
Number of Observations		16337

Notes: Consumption and income data are from the 1980–1986 and 1989–1998 surveys of the PSID. Consumption is the imputed total consumption, and income is the taxable income of head and wife, net of federal income taxes, social security, and Medicare taxes. Consumption and income data are "cleaned" of cohort, time, and idiosyncratic family size effects. Family size is the sample average of family size across households of a certain age. *** denotes statistical significance at the 1% level, ** denotes statistical significance at the 5% level, and * denotes statistical significance at the 10% level.

Table 5: Auto-covariances of the First Differences in Log-Idiosyncratic Income

Auto-covariance of Order	Value	St. Error
0	0.0969	0.0079
1	–0.0296	0.0048
2	–0.0044	0.0040
3	–0.0017	0.0037
4	–0.0006	0.0032
5	–0.0022	0.0034

Notes: Income data are residuals from cross-sectional regressions of head's and wife's real disposable income on education of head, household state of residence, second degree polynomial in head's age, and race. The data are drawn from 1980–1997 annual family files of the PSID. The time span corresponds to the time span of consumption data under analysis. I select households with married heads aged between 24 and 65. I drop top-coded observations, and observations for households who report head's labor income equal to zero. In addition, I drop observations with an absolute percentage change in income residual greater than or equal to 300% or with real income below 1000 of 1982–1984 dollars.
A household contributes an observation on income difference if it has stable family composition between year t and year $t-1$. A household is present in the data if at least one income difference is non-missing. Auto-covariances represent the averages of unique elements of the variance-covariance matrix of log income changes corresponding to the theoretical auto-covariances of a given order. Average standard errors in parentheses.

Table 6: Test of the Null Hypothesis of Zero Auto-covariance in All Time Periods

Order:	Statistics	DF	p-value
1	400.51	16	0.00
2	38.79	15	7×10^{-3}
3	15.67	14	0.33
4	16.98	13	0.20

Notes: The test statistic is distributed as χ^2 with degrees of freedom equal to the number of (zero) restrictions (the number of unique auto-covariances of a given order in the estimated variance-covariance matrix). See Appendix A of Abowd and Card (1989) for details of its construction.

Table 7: The Volatility of Permanent Shocks Estimated from the Univariate Dynamics of Idiosyncratic Household Income

Parameter	$q=0$	$q=1$
σ_{u^P}	0.198	0.175
(St. dev. of permanent shock)	(0.004)	(0.004)
Goodness of Fit	22.12	27.75
Degrees of Freedom	14	14
p-value of the model	0.07	0.015
Number of Households (N)		3306
N×T		27546

Notes: For the UC income model $\Delta\tilde{y}_{it} = u^P_{it} + (1-L)\theta_q(L)u^T_{it}$, σ_{u^P} is identified from the following moment condition (equation 5 in Meghir and Pistaferri (2004), p.8): $E[\Delta\tilde{y}_{it}\Sigma^{(1+q)}_{k=-(1+q)}\Delta\tilde{y}_{it+k}]$, where $\Delta\tilde{y}_{it}$ is the the first difference in idiosyncratic household log-income (log-income residual), u^P_{it} is the permanent innovation, u^T_{it} is the transitory innovation, and q is the pre-estimated order of auto-covariance in transitory component of log-income (zero if $q=0$, one if $q=1$). The model is estimated by the optimal minimum distance method (OMD), where the weighting matrix is the inverse of the empirical variance-covariance matrix of the moment. I discard the empirical counterparts of the moment for the first and the last periods, since I do not have enough data to form complete empirical moments for these periods. Income data are residuals from cross-sectional regressions of the head's and wife's real disposable income on the head's education, household state of residence, a second degree polynomial in the head's age, and race. The data are drawn from the 1980–1997 annual family files of the PSID. The time span corresponds to the time span of consumption data under analysis. I select households with married heads aged between 24 and 65. I drop observations with an absolute percentage change in income residual greater than or equal to 300% or with real income below 1000 of 1982–1984 dollars. A household contributes an observation on income difference if it has a stable family composition between year t and year $t-1$. A household is present in the data if at least one income difference is non-missing.

Table 8: MSM Estimates of the Model

Parameter	(1)	(2)	(3)
Behavioral Parameters			
Time Discount Factor, β	0.95	0.97	0.95
	(0.006)	(0.002)	(0.005)
	(0.007)	(0.002)	(0.005)
Relative Risk Aversion (CRRA), ρ	3.69	2.20	3.69
	(0.55)	(0.22)	(0.44)
	(0.70)	(0.22)	(0.56)
Retirement Process Parameters			
κ_0	1.11×10^{-6}	0.06	0.00
	(0.47)	(0.51)	—
	(0.53)	(0.51)	—
κ_x	0.042	0.046	0.042
	(0.028)	(0.025)	(0.005)
	(0.029)	(0.028)	(0.007)
Income Process Parameters			
St. Dev. of Log-Perm. Shock, σ_{u^P}	0.182	0.175	0.182
	(0.023)	—	(0.016)
	(0.028)	—	(0.019)
St. Dev. of Log-Trans. Shock, σ_{u^T}	0.243	0.235	0.242
	(0.029)	(0.001)	(0.020)
	(0.037)	(0.002)	(0.025)
Covariance, $\sigma_{u^P u^T}$	–0.0166	–0.0159	–0.0165
	(0.0041)	(0.0007)	(0.0026)
	(0.0058)	(0.0011)	(0.0037)

Notes: Model is estimated by the method of simulated moments (MSM). Standard errors in parentheses. The first row under the parameter estimates contains standard errors with no correction for the first stage of estimation. The second row under the parameter estimates contains standard errors with correction for the first stage of estimation. The weighting matrix used for estimation is the diagonal matrix, where diagonal elements are the diagonal from the inverse of the variance-covariance matrix of the second stage moments.

Appendix A: Matching the Moments of the Reduced Form and UC Models

As in the text, suppose that the first differences of log income are described by reduced form MA(1). Thus, the unique moments of reduced form (rf) are the first-order auto-covariance and zero order auto-covariance:

$$\gamma^{rf}(0) = (1+\theta^2)\sigma_\epsilon^2$$

$$\gamma^{rf}(1) = \theta\sigma_\epsilon^2.$$

Then, for the structural model (sf) with correlated permanent and transitory components described in the text, I need to match,

$$\gamma^{sf}(0) = \sigma_{u^P}^2 + 2\sigma_{u^T}^2 + 2\sigma_{u^P u^T} = \gamma^{rf}(0)$$

$$\gamma^{sf}(1) = -\sigma_{u^T}^2 - \sigma_{u^P u^T} = \gamma^{rf}(1),$$

subject to the constraint that the spectrum of reduced form series and the spectrum of the permanent component at frequency zero are equal:

$$\sigma_{u^P}^2 = \gamma^{rf}(0) + 2\gamma^{rf}(1) = (1+\theta)^2\sigma_\epsilon^2.$$

The above equation is the identifying condition for the variance of the permanent component. The two preceding equations determine the other two unknowns, $\sigma_{u^P u^T}$ and σ_{u^T}. Since the equations are linearly dependent, it is obvious that there is no unique solution for $\sigma_{u^P u^T}$ and σ_{u^T}. Thus, I choose the grid of covariances between shocks such that they return the correlation that is less than or equal to one in absolute value. This procedure uniquely determines the variance of the transitory innovation from the following condition:

$$\sigma_{u^T}^2 = -\gamma^{rf}(1) - \sigma_{u^P u^T}.$$

Appendix B: Details of Solution of the Life Cycle Consumption Model, and Numerical Procedures

In this appendix, I lay out the details of model solution, and describe the choices made in numerical analysis.

(i) Model Solution.

As mentioned in the text, I assume that the consumption function at age 66 is linear in total wealth: $c_{T+1} = \kappa_x x_{T+1} + \kappa_0$.

The Euler equation that links consumption at age 65 and 66 is: $v(Z_T)U'(c_T) = v(Z_{T+1})\beta R E_T U'(c_{T+1})$, and, therefore, consumption at age 65 is $c_T = (\frac{v(Z_T)}{v(Z_{T+1})})^{\frac{1}{\rho}}(\beta R)^{\frac{-1}{\rho}}(\kappa_x x_{T+1}+\kappa_0)$, or $c_T = (\frac{v(Z_T)}{v(Z_{T+1})})^{\frac{1}{\rho}}(\beta R)^{-\frac{1}{\rho}}(\kappa_x R(x_T - c_T)+\kappa_x+\kappa_0)$. The last equality follows from the assumption that income at age 66 is non-stochastic, and is equal to the permanent income at age 65. For each value of $s_T = (x_T - c_T)$, the last equation uniquely determines c_T. Cash-on-hand x_T is obtained by summing s_T and c_T. The consumption function at age 65, the last working period, $c_T(x_T)$ is obtained by linear interpolation. For ages 24-64, the consumption function is obtained by iterating the Euler equation, $c_t = min\langle x_t, (\frac{v(Z_t)}{v(Z_{t+1})})^{\frac{1}{\rho}}(\beta R)^{-\frac{1}{\rho}}\{E_t U'[c_{t+1}(\frac{R(x_t - c_t)}{G_{t+1}\epsilon^P_{t+1}} + \epsilon^T_{t+1})G_{t+1}\epsilon^P_{t+1}]\}^{-\frac{1}{\rho}}\rangle$. I create the grid of 120 equally spaced points between 0 and 40 in $s_t = x_t - c_t$, and the 250×2 matrix of pre-seeded independent random normal deviates ϵ_1, and ϵ_0. Expectation is taken over potentially correlated shocks ϵ^P, and ϵ^T, permanent and transitory shocks respectively. Correlated log-normal random draws are obtained by taking the elements of the vector $exp[chol(\Sigma_{u^T u^P})\epsilon)]$, where $chol(\Sigma_{u^T u^P})$ is the Cholesky factor of the variance-covariance matrix of log-permanent and log-transitory shocks, and ϵ is the vector of independent random *normal* deviates $(\epsilon_{j0}, \epsilon_{j1})$. I perform this operation for $j = 1, \cdots, 250$ to arrive at the 250×2 matrix of correlated random log-normal deviates. $E_t U'[\cdot]$ is calculated as $\frac{1}{250}\sum_{j=1}^{250}\{c_t(\frac{Rs_t}{G_{t+1}\epsilon^P_j} + \epsilon^T_j)\}^{-\rho}\{G_{t+1}\epsilon^P_j\}^{-\rho}$ for each value of s_t from the grid, and each pair of $(\epsilon^T_j, \epsilon^P_j)$ from the joint distribution of random *log-normal* shocks. x_t is calculated as the sum of s_t and the corresponding choice of consumption, c_t. The consumption function at age t is linearly interpolated, using the points from x_t and c_t. If x_t falls outside the grid, the corresponding consumption is calculated as the value of consumption from the life cycle model without uncertainty. Using the budget constraint and the Euler equation, it is straightforward to show that this value is defined as $C_t = \frac{1-(\beta R)^{\frac{1}{\rho}}R^{-1}}{1-[(\beta R)^{\frac{1}{\rho}}R^{-1}]^{D-t+1}}(A_t + \sum_{i=0}^{D} Y_{t+i}R^{-i})$, where D is the age of death (assumed to be 90), and A_t is the level of liquid assets at age t. Dividing through by P_t, and assuming that income grows deterministically at gross growth rate G_{t+1} from t to D (set to be 1.0 for all time periods), the value of consumption per permanent income is calculated as $c_t = \frac{1-(\beta R)^{\frac{1}{\rho}}R^{-1}}{1-[(\beta R)^{\frac{1}{\rho}}R^{-1}]^{D-t+1}}(a_t + \frac{(1-\frac{G_{t+1}}{R})^{D-t+1}}{1-\frac{G_{t+1}}{R}})$, where a_t is the value of liquid assets per permanent income at age t.

(ii) Initialization of the Model Economy.

I create a matrix of (5000×2) pre-seeded random draws for permanent and transitory shocks, and simulate the economy populated by 5000 households, each with unique income history, using age-dependent consumption functions $\{c_t(x_t)\}_{t=24}^{65}$. In the beginning period of the life cycle, at age 24, each household obtains a draw from the estimated empirical distribution of wealth-to-permanent income ratio at age 24, and a transitory shock. The sum of these draws defines initial cash-on-hand. I use the estimates

of mean and variance of the distribution of initial liquid wealth-to-permanent income ratio to generate the 5000×1 vector of pre-seeded log-normal draws from this distribution. To initialize the permanent income, for each household I take a draw from the estimated distribution of permanent income at age 24. I describe estimation of parameters of both initial distributions in Appendix D. Once the economy is generated, I calculate the (second stage) model moments used in optimization. During optimization search, created sequences of random draws are kept the same.

(iii) Optimization Procedure.

I use the constrained optimization module (CO) in GAUSS to find the model parameters θ. I impose the following bounds on coefficients: $\beta \in [0,1]$, $\rho \in [0,\infty)$, $\kappa_x \in [0,1]$, $\kappa_0 \in [0,\infty)$, $\sigma_{u^T} \in [0,\infty)$, $\sigma_{u^P} \in [0,\infty)$, and $\sigma_{u^T u^P} \in (-\infty,\infty)$. To obtain a "proper" variance-covariance matrix of income shocks, I impose two additional constraints. First, to calculate the Cholesky factorization, the determinant of the covariance matrix should be greater than zero, that is $(\sigma^2_{u^T}\sigma^2_{u^P} - \sigma_{u^T u^P} > 0)$; second, the correlation between shocks should be less than or equal to 1 in absolute value, that is $|\frac{\sigma_{u^T u^P}}{\sigma_{u^T}\sigma_{u^P}}| \leq 1$.

Appendix C: Construction of the Covariance Matrix for the First and Second Stage Moments

The first stage moments used in estimation are gross growth rates of disposable income, $\{G_t\}_{t=25}^{65}$; utility shifters due to changes in family composition over the life, $\{v(z_t)^{1/\rho}\}_{t=24}^{66}$; gross interest rate on safe liquid assets, R; mean and standard deviation of the distribution of wealth-to-permanent income ratio at age 24, $\{(W/Y^p)_{.,24}, \sigma_{(W/Y^p)_{24}}\}$; mean and standard deviation of the distribution of permanent income at age 24, $\{(Y^p)_{.,24}, \sigma_{Y^p_{24}}\}$.

For each individual contributing to the first stage of estimation, the first stage moment vector is:

$$\chi_i = \begin{pmatrix} G_{i,25} \\ G_{i,26} \\ \vdots \\ G_{i,65} \\ v(z_{i,24})^{1/\rho} \\ v(z_{i,25})^{1/\rho} \\ \vdots \\ v(z_{i,66})^{1/\rho} \\ (W/Y^p)_{i,24} \\ \{(W/Y^p)_{i,24} - (W/Y^p)_{.,24}\}^2 \\ Y^p_{i,24} \\ \{Y^p_{i,24} - Y^p_{.,24}\}^2 \end{pmatrix}$$

The estimate of the covariance matrix of the first stage parameters is a symmetric matrix Ω_χ:

$$\Omega_\chi = \begin{pmatrix} \frac{1}{J_{1,1}} \sum_{i=1}^{J_{1,1}} (\chi_{i,1} - \chi_{.,1})(\chi_{i,1} - \chi_{.,1})' & \frac{1}{J_{1,2}} \sum_{i=1}^{J_{1,2}} (\chi_{i,1} - \chi_{.,1})(\chi_{i,2} - \chi_{.,2})' & \cdots & \frac{1}{J_{1,88}} \sum_{i=1}^{J_{1,88}} (\chi_{i,1} - \chi_{.,1})(\chi_{i,88} - \chi_{.,88})' \\ \frac{1}{J_{2,1}} \sum_{i=1}^{J_{2,1}} (\chi_{i,2} - \chi_{.,2})(\chi_{i,1} - \chi_{.,1})' & \frac{1}{J_{2,2}} \sum_{i=1}^{J_{2,2}} (\chi_{i,2} - \chi_{.,2})(\chi_{i,2} - \chi_{.,2})' & \cdots & \frac{1}{J_{2,88}} \sum_{i=1}^{J_{2,88}} (\chi_{i,2} - \chi_{.,2})(\chi_{i,88} - \chi_{.,88})' \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{J_{88,1}} \sum_{i=1}^{J_{88,1}} (\chi_{i,88} - \chi_{.,88})(\chi_{i,1} - \chi_{.,1})' & \frac{1}{J_{88,2}} \sum_{i=1}^{J_{88,2}} (\chi_{i,88} - \chi_{.,88})(\chi_{i,2} - \chi_{.,2})' & \cdots & \frac{1}{J_{88,88}} \sum_{i=1}^{J_{88,88}} (\chi_{i,88} - \chi_{.,88})(\chi_{i,88} - \chi_{.,88})' \end{pmatrix}$$

Typical elements of matrix Ω_χ are: $\frac{1}{J_{1,1}} \sum_{i=1}^{J_{1,1}} (G_{i,25} - G_{.,25})^2$, $\frac{1}{J_{1,2}} \sum_{i=1}^{J_{1,2}} (G_{i,25} - G_{.,25})(G_{i,26} - G_{.,26})$, $\frac{1}{J_{1,88}} \sum_{i=1}^{J_{1,88}} (G_{i,25} - G_{.,25})(Y^p_{i,24} - Y^p_{.,24})^2$, $\frac{1}{J_{88,88}} \sum_{i=1}^{J_{88,88}} ((Y^p_{i,24} - Y^p_{.,24})^2 - \sigma^2_{Y^p_{.,24}})^2$, $\frac{1}{J_{1,42}} \sum_{i=1}^{J_{1,42}} (G_{i,25} - G_{.,25})(v(z_{i,24})^{1/\rho} - v(z_{.,24})^{1/\rho})$.

$J_{1,1}$ denotes the number of households that have data records on $G_{i,25}$; $J_{1,2}$ is the number of households that have a non-missing product between $G_{i,25}$ and $G_{i,26}$; $J_{1,88}$ is the number of households that have a non-missing product between $G_{i,25}$ and $(Y^p_{i,24} - Y^p_{.,24})^2$. The rest of the $J_{k,l}$ terms are defined accordingly.

The 45×1 vector of the second stage moments, θ, consists of the log average consumption profile over the life cycle, $\{\log C_t\}_{t=24}^{65}$, and regression estimates of the response of (imputed) consumption growth to contemporaneous and lagged income growth, $\hat{\beta}_{1,ols}$, $\hat{\beta}_{2,ols}$. The estimate of the covariance matrix of the second stage moments is a symmetric matrix Ω_f:

$$\Omega_f = \begin{pmatrix}
\frac{1}{I_{1,1}} \sum_{i=1}^{I_{1,1}} (\log C_{i,24} - \log C_{.,24})^2 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{1}{I_{2,1}} \sum_{i=1}^{I_{2,1}} (\log C_{i,25} - \log C_{.,25})(\log C_{i,24} - \log C_{.,24}) & \frac{1}{I_{2,2}} \sum_{i=1}^{I_{2,2}} (\log C_{i,25} - \log C_{.,25})^2 & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{1}{I_{42,1}} \sum_{i=1}^{I_{42,1}} (\log C_{i,65} - \log C_{.,65})(\log C_{i,24} - \log C_{.,24}) & \cdots & \cdots & \frac{1}{I_{42,42}} \sum_{i=1}^{I_{42,42}} (\log C_{i,65} - \log C_{.,65})^2 & \cdots & \cdots & \cdots \\
\{(X'X)^{-1} \frac{1}{I_{43,1}} \sum_{i=1}^{I_{43,1}} X_{i,24} \epsilon_{i,24} (\log C_{i,24} - \log C_{.,24})\}_2 & \cdots & \cdots & \cdots & var(\hat{\alpha_1}) & cov(\hat{\alpha_1}\hat{\alpha_2}) & 0 \\
\{(X'X)^{-1} \frac{1}{I_{44,1}} \sum_{i=1}^{I_{44,1}} X_{i,24} \epsilon_{i,24} (\log C_{i,24} - \log C_{.,24})\}_3 & \cdots & \cdots & \cdots & cov(\hat{\alpha_1}\hat{\alpha_2}) & var(\hat{\alpha_2}) & 0 \\
K^{-1} \frac{1}{I_{45,1}} \sum_{i=1}^{I_{45,3}} (\log C_{i,24} - \log C_{.,24}) \Delta \log Y_{i,23} \xi_{i,24} & \cdots & \cdots & \cdots & 0 & 0 & var(\hat{\gamma})
\end{pmatrix}$$

$I_{1,1}$ denotes the number of heads having the data record on $\log C_{i,24}$, $I_{1,42}$, is the number of heads who have a non-missing product between the first and the 42 element of the second stage moment vector, $\log C_{i,24}$ and $\log C_{i,65}$. The rest of the $I_{m,n}$ terms are defined accordingly. $X'X$ in row 43 and 44 is the 6×6 vector, while each cell in row 43 and 44 (besides $\Omega_f[43:44, 43:44]$) is the element of the 6×1 vector. X and $\epsilon_{i,j}$ refer to regression in Table 4. A typical element of X, $X_{i,j} = (1, \Delta \log C_{ij}, \Delta \log C_{ij-1}, \Delta Fam.Size_{ij}, Age_{ij}, Age_{ij}^2/100)'$, where i is a head from the regression sample, and j is this head's age. Subscript 2 beneath each element of row 43 means that I pick the second element of the 6×1 vector, the contribution made by the j-th group towards the second element of the vector $(X'X)^{-1}X'\epsilon$, the difference between pooled OLS estimates and their expected values. Subscript 3 beneath each element of row 44 means that I pick the third element of the 6×1 vector, the contribution made by the j-th group towards the third element of the vector $(X'X)^{-1}X'\epsilon$. In row 45, K is equal to $(\sum_{i=1}^{N} \sum_{j=24}^{65} \Delta \log Y_{i,j-1})$, where N is the number of observations in the pooled OLS regression $\Delta Y_{i,j} = \psi \Delta Y_{i,j-1} + \xi_{i,j}$. I assume that the distributions of $\hat{\psi}$ and $\hat{\alpha_j}$ $(j = 1, 2)$ are independent.

Appendix D: Data Used and Sample Selection

(i) Data from the Survey of Consumer Expenditures (CEX).

I use CEX data on total consumer expenditures and food consumption, available at the NBER website. The data set spans the period 1980–1998. The CEX survey is designed by Bureau of Labor Statistics to construct the CPI at different levels of aggregation. According to the design of the survey, respondents are followed at most five quarters. The survey publishes at most four quarters of information on individual consumption, along with demographic information. The NBER extracts lump quarterly records into one annual record. Thus, each annual file contains cross-sections of households with non-repeated observations across other available years.

Households may enter the survey in the same year but in different quarters. If a household enters the survey, say in the first quarter of 1981, the earliest consumption information it brings will reflect consumption in the fourth quarter of 1980.

Total consumption is defined as household total expenditures less expenditures on education, medical expenses, mortgage interest payments, and taxes on housing. It includes food at home and food away from home, clothing, expenses on personal care items, rent, or imputed rent for homeowners, housing operation expenses, personal business expenses (life insurance and business services), transportation expenses (inclusive of the purchases of vehicles), recreation and charity expenses.

I assume that household data belong to year t if a household starts survey in quarter 1 or 2 of year t, or in quarter 3 or 4 of year $t-1$. This way a household may have as few as six (if it enters the survey in quarter 3 of year $t-1$), and as many as twelve months of information on spending (if it enters the survey in quarter 2 of year t) as of year t. I deflate annual food consumption by the BLS food CPI with the base 100 in 1982–1984. I deflate total consumption by the BLS CPI for all items with the same base period. I use the CPI indices for the last month of the first quarter of respective year.

In the CEX, the head of a household is the person who owns or rents the unit of a household residence. In the PSID, head of a household is male, unless he is permanently disabled (Hill (1992)). These definitions are not directly comparable and it is not clear how to select CEX households that match the PSID criterion of a male-headed household. Thus I keep CEX households "headed" either by married female or male.

I keep households that report consumption in all four quarters of the year, whose heads are not part-time or full-time in school, and who are classified as full income reporters. Although I do not use income information from the CEX, the latter restriction is done for comparability with Gourinchas and Parker (2001), and Cagetti (2003). I keep households whose heads are between 24-70 years old, whose real total consumption is greater than real food consumption, and whose real total consumption and real income are greater than 1,000 of real 1982 dollars. I drop households with family size below 2, whose head does not have education, race, age, or state of the residence records. Finally, I drop households with five largest and five smallest values for real food and real consumption in the data set of merged annual cross-sections. My final CEX sample consists of 2,1854 households with complete demographic data, and data on food and total consumption.

(ii) Data from the Panel Survey of Income Dynamics (PSID).

The Panel Survey of Income Dynamics (PSID) started in 1968, interviewing 4,802 households. Sixty per cent of the households interviewed in 1968 belong to the "core" representative sample, the other portion is known as the low-income SEO sample (Survey of Economic Opportunity Sample). The PSID followed these original households and households initiated by their offsprings over time, collecting a

panel data set on income, wealth, demographic information, food consumption, and housing. I collect PSID data for 1980–1998, the same time span over which I have data on consumption and demographics in the CEX. To have a more representative sample, I drop SEO households and their offsprings from my sample. The PSID has consistently collected only two items of consumption over time: food consumption at home and food consumption away from home (excluding food at work). Since I am interested in *total* household consumption and income dynamics over the life cycle, I use the superior (to the CEX) income data from the PSID, and impute *total* consumption to the PSID households, using demand for food equation estimated on the CEX data. There are some problems with interpretation of timing of household food consumption in the PSID due to timing of a questionnaire. Most of the studies that use PSID food consumption assume that food consumption recorded in survey year t reflects typical weekly food consumption flow in year $t-1$ (obviously, annual food consumption is obtained by multiplying weekly food consumption by 52). In this paper I adopt the same strategy. Over the time span considered, PSID did not collect food consumption data in 1988, and 1989 surveys.[43] Correspondingly, my final sample of analysis lacks food and total consumption data for 1987, and 1988. Food away from home and food at home are deflated by the respective 1982-1994 CPI components taken from the BLS. I drop top-coded observations on food consumption. As with the CEX data, I use the CPI indices for the third month of the first quarter of a given year. Total real food consumption is obtained by summing real food consumption at home and away from home.

From the PSID data, I choose households headed by married males, with heads aged between 24 and 70, and with at least one observation on food consumption during 1980–1998. I drop households whose heads report more than a two year absolute difference in age in adjacent years. If a household passes this criterion, I use the data on age from adjacent years to impute age if record on age is missing in a given year. I fill in missing state of the residence data using records on state of the residence of a household in adjacent years (I use non-missing state of the residence data as far as 4 years backward and forward from a missing record). I group households into three education categories: households with heads who did not complete high school (below 12 years of education), households with heads who completed high school (12 years), and households with heads whose years of education exceed high school (above 12 years). I "allow" households to switch education categories over time if heads move to an upper education category. For example, I keep households whose heads have 12 years of minimum education attained, but attain additional education in subsequent years. I drop households whose heads report years of education in year t greater than years of education in any year after t, *and if*, during the sample span, they switch education categories (that is, if they report minimum education attained below 12 years and maximum education attained equal or above 12 years; or if they report minimum education attained equal to 12 years, and maximum attained above 12 years). If the head of a household has at least one non-missing race record in the sample considered, and has missing race records in any other year, I impute race records to this head. I retain observations for the years when households did not experience significant changes in family composition. Specifically, I keep observations if households did not have any changes in family composition, or had changes in members other than head or wife (e.g., a child leaving a household). I drop observations for households with family size below 2.

The income data used in the paper are combined taxable income of head and wife. I subtract federal income taxes, social security taxes, and Medicare taxes from household income. The published PSID data on income and federal taxes refer to previous calendar year. I use the PSID estimate of federal income taxes for years 1981–1991. Starting from 1992, the PSID discontinued calculation of federal income taxes. I impute federal income taxes to each household for survey years 1992–1997 and 1999[44] using records on combined taxable income of head and wife and published tables with federal tax schedules. I assume that head and wife file tax form jointly, and use the appropriate schedule for each year. I calculated Medicare taxes and social security taxes for head and wife separately, taking their wage income as the base for these taxes. My estimate of disposable after-tax household income is max{0,(Taxable Income of Head and Wife–Federal Income Taxes of Head and Wife–Social Security Taxes of Head and Wife–Medicare Taxes of Head and Wife)}. I deflate disposable after-tax income by the BLS 1982–1984 CPI for all items,

[43] PSID does not have data on food and total consumption in the 1973 survey either.

[44] Starting from 1997, the PSID switched to surveying households every 2 years.

the same deflator I use for total consumption.

After imputing total consumption to the PSID households, my other sample selection criteria are as follows. I drop observations if disposable income or total imputed consumption is below 1000 of real 1982–1984 dollars; if real disposable income, or real imputed total consumption is below real food consumption; if real imputed total consumption, or real food consumption is below the fifth smallest value in the CEX sample; if real imputed total consumption, or real food consumption is above the fifth largest value in the CEX sample; if an absolute percentage change of real disposable income, or real food consumption, or real total imputed consumption is greater than 300%. Finally, since I am interested in the link between income and consumption dynamics, I keep observations with non-missing records on both total imputed consumption and disposable income. The final PSID sample used to construct the life cycle profiles of income and consumption consists of 28,859 worth of income and consumption observations, along with complete information on state of household residence, race of the head, education of the head, family size and age of the head. All the demographic data (with the exclusion of family composition change) pertain to year t of a survey year, while consumption and income data refer to year $t-1$.

(iii) Distributions of Initial Income and Initial Wealth-to-Permanent Income: Estimates from the PSID.

To initialize the model economy, I need an estimate of the initial income distribution, and the distribution for wealth-to-permanent income ratio.

I assume that the wealth-to-permanent income ratio and the initial income are log-normally distributed. I take liquid wealth of 24 and 25 year old heads from the PSID wealth supplements available in 1984, 1989, 1994, 1999, and 2001. "Liquid" wealth is equal to the sum of the monetary value of checking and saving accounts, net value of real estate other than main housing, net value of vehicles, net value of shares, stocks and bonds, minus the credit debt. For each household with a record on liquid wealth and with head aged between 24–25 years old, I estimate the permanent income as the average income during a 4-year span. I use the 1983–1986 data on real disposable income if household with the wealth record is observed in 1984, 1988–1991 income data—if the wealth record is in 1989, 1993–1996 income data—if the wealth record is in 1994, income data in 1996, 1998, and 2000—if the wealth record is in 1999, or 2001. For estimation of the distribution of the wealth-to-permanent income ratio, I keep only households with a positive ratio. I have a cross-section of 275 observations on log wealth-to-permanent income ratio. Since households with records on wealth and income are observed at different points of business cycle, I attempt to eliminate the time (business cycle) effect with the following transformation: $\widetilde{w_{i,t}} = w_{i,t} - (w_{.,t} - w_{.,.})$, where i stands for household, t is any year from the set {1984, 1989, 1994, 1999, 2001}, $w_{.,t}$ is the average of wealth-to-permanent income ratio across observations in year t, $w_{.,.}$ is the average across all households and all years. I use vector of $\widetilde{w_{i,t}}$'s to estimate mean and variance of the distribution of wealth-to-permanent income ratio. The mean is –1.23, and the standard deviation is 1.22.

I estimate the value of the initial permanent income, using the data for households with positive and negative wealth-to-permanent income ratio. I have 298 observations on initial income. I transform the income data in the same manner as the data on wealth-to-permanent income ratio. Mean and variance of these transformed data serve as the estimates for population mean and variance of the log-normal distribution of the initial permanent income. The mean is equal to 5.26, and the standard deviation is 0.41.

Appendix E: Imputation of Total Consumption to the PSID Households

In this appendix, I describe the procedure used to impute total consumption to the PSID households. There are several available imputation methods adopted in the literature. Absent data on total consumption in the PSID, imputation is usually done in order to exploit the panel structure of the PSID, and superior (to the CEX) data on income. Skinner (1987), using the 1972–1973 and 1983 waves of the CEX survey, showed that total CEX consumption tightly relates to several consumption items, also available in the PSID (food at home and away from home, number of vehicles owned, and housing rent). Moreover, he showed that this relationship is stable over time. Several researchers, inspired by this finding, used coefficients from the CEX regression of total consumption on consumption items, also available in the PSID, and household data from the PSID, to impute total consumption to the PSID households. Another way of imputation is to use the data on liquid assets and income in the PSID to construct savings and total consumption (Ziliak (1998), and Zeldes (1989*a*)). Blundell et al. (2005) pioneered a more sophisticated approach that inverts the food demand equation estimated on the CEX data. They relate log-food consumption to log-total expenditures, household demographics, price variables, time dummies, cohort dummies, and expenditures interacted with time dummies and head's education category. I run a slightly modified regression, and use the coefficients from this regression to impute total consumption to the PSID households. I use consumption and demographic data of merged CEX cross-sections for years 1980–1998. The detailed sample selection procedure is described in Appendix D. I assume that business cycle effects are captured by regional unemployment. Regions considered are the U.S. Census regions: Northeast, North Central, South, and West. Results of the food demand equation are presented in Table 9. The elasticity of food consumption with respect to total expenditures is positive, tightly estimated, and is falling over time. This result implies that the share of food consumption in total expenditures falls over time: given improvements in standards of living, and increases in total expenditures over time, it reflects the well-known fact that food is a necessity. The own price elasticity of food consumption, albeit negative, is statistically indistinguishable from zero. There is a significant positive difference between food consumption of those who have more than 12 years of education and those who have not finished high school. The latter group also has larger elasticity of food consumption to changes in total expenditures, and, therefore, spends larger share of their budgets on food. All the demographic variables have expected signs and are statistically significant. Regression explains about 52% of variation in the log-food consumption.

Table 9: (Appendix E) Food Equation Estimated on CEX data

Dependent Variable: Log-Food Consumption. CEX Sample: 1980–1998.

Variable	Coefficient	St. Error
Log-Total Consumption	.548***	.012
Log-Food CPI	−.214	.132
Regional Unemployment	−.008***	.003
Total Consumption×1981	.002	.004
Total Consumption×1982	−.024***	.005
Total Consumption×1983	−.029***	.006
Total Consumption×1984	−.033***	.006
Total Consumption×1985	−.029***	.007
Total Consumption×1986	−.028***	.007
Total Consumption×1987	−.029***	.008
Total Consumption×1988	−.153***	.029
Total Consumption×1989	−.019*	.010
Total Consumption×1990	−.021*	.012
Total Consumption×1991	−.015	.012
Total Consumption×1992	−.016	.013
Total Consumption×1993	−.015	.013
Total Consumption×1994	−.022	.014
Total Consumption×1995	−.021	.014
Total Consumption×1996	−.029**	.015
Total Consumption×1997	−.029*	.016
Total Consumption×1998	−.029*	.016
Born Between 1915–1920	−.022	.025
Born Between 1921–1926	−.032	.027
Born Between 1927–1932	−.046	.031
Born Between 1933–1938	−.037	.037
Born Between 1939–1944	−.019	.044
Born Between 1945–1950	−.024	.049
Born Between 1951–1956	−.017	.056
Born Between 1957–1962	−.026	.063
Born Between 1963–1968	−.018	.069
Born Between 1969–1975	−.024	.078
Family Size	.079***	.002
Finished High School (HS)	−.016	.053
Did Not Finish HS	−.277***	.065
Finished HS× Tot. Cons.	−.006	.010
Did Not Finish HS× Tot. Cons.	.041***	.013
Age	.037***	.002
$Age^2/100$	−.036***	.002
Black	−.166***	.008
Constant	.937	.618
Number of Observations		21854
Adjusted R^2		0.5203

Notes: The sample consists of merged CEX cross-sections for years 1980-1998. Data are accessed from the NBER website. Food consumption is the sum of real food consumption at home and away from home, exclusive of food at work. Total consumption is the sum of real expenditures on all items, exclusive of education, medical expenses, mortgage interest, other interest, and taxes on housing. Regional unemployment is the variable created by interaction of a survey year and unemployment in the region of a household residence at the time of a survey. The sample consists of households with married heads aged between 24 and 70. Heads enrolled part-time or full-time in school are dropped. Sample households are present in all four quarters of a survey, and are complete income reporters. Family size is the average family size during a survey year (across four quarters). Head belongs to a high school category if he/she reports 12 years of education; "Did Not Finish High School" is the indicator variable equal to 1 if head's education is below 12 years; omitted category consists of heads with education above 12 years. Omitted cohort comprises households with heads born between 1910–1914. Price variable is the log of the BLS CPI index for total food consumption with the base in 1982–1984.

*** denotes statistical significance at the 1% level, ** denotes statistical significance at the 5% level, and * denotes statistical significance at the 10% level.