Trade, Diffusion and the Gains from Openness

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Abstract

Building on Eaton and Kortum's (2002) model of Ricardian trade, Alvarez and Lucas (2005) calculate that a small country representing 1% of the world's GDP experiences a gain of 41% as it goes from autarky to *frictionless* trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. This paper presents and then calibrates a model where countries interact through trade and diffusion of ideas, and then quantifies the overall gains from openness and the contribution of trade to these gains. Having the model match the trade data (i.e., the gravity equation) and the observed growth rate is critical for this quantification to be reasonable. It is shown that for this match it is necessary to introduce diffusion and/or knowledge spillovers to the basic model of trade and growth in Eaton and Kortum (2001). The main result of the paper is that, compared to the model without diffusion, the gains from trade are smaller whereas the gains from openness are much larger when diffusion is included in the model.

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1 Introduction

How much does a country gain from its relationship with the rest of the world? Consider for example the recent work by Alvarez and Lucas (2005), who build on Eaton and Kortum's (2002) model of Ricardian trade. According to their quantitative model, a small country like Argentina, which represents approximately 1% of the world's GDP, experiences an income gain of 41% as it goes from autarky to *frictionless* trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. Even if a country were to shut down trade, it could still benefit from foreign ideas through foreign direct investment (FDI), migration, books, journals, the Internet, etc.

The goal of this paper is to construct and calibrate a model where countries interact through trade and diffusion of ideas, and then to quantify the overall gains from openness and the contribution of trade to these gains. The main result is that the gains from trade are smaller than those quantified by Alvarez and Lucas (23% rather than 41% for a country with 1% of the world's GDP) whereas the gains from openness are enormous (290% for a country with 1% fo the world's GDP). Thus, trade contributes with only a small part of the overall gains from openness.

Calculating the gains from trade in a model that allows for trade and diffusion represents a significant departure from the standard practice in the literature, which is to consider trade as the only means through which countries interact. This alternative approach has at least two advantages. First, having both trade and diffusion in the model shows that the gains from trade depend on the way in which trade and diffusion interact. In the model I present here, trade and diffusion are substitutes: if a country cannot import a good then it can in principle adopt a foreign technology to produce the good domestically with a lower efficiency loss. This substitution is clearly important: think about all the trade that does not take place because technology diffusion has allowed many countries to satisfy their own demand for thousands of goods. On the other hand, if a country cannot adopt foreign technologies, it can always import the goods produced abroad with those technologies. Since trade and diffusion are substitutes, then shutting down trade in this model leads to smaller losses than in models with no diffusion such as Eaton and Kortum (2002) and Alvarez and Lucas (2005).

A second advantage from studying diffusion and trade together is that one can compare the gains from trade with the overall gains from openness, and this may provide a way to judge whether the numbers are reasonable. The usual reaction of economists to the calculated gains from trade in quantitative models is that they are "too small." Apparently, economists have a prior belief that these gains are much higher, so there has been a search for mechanisms through which trade can have a larger effect, such as scale effects, intra-industry reallocations or gains from increased variety. But the result of this search has been generally disappointing (see Tybout, 2003). This paper suggests that the reason for this may be that the gains from trade are in fact "small," while economists' priors about large gains may in fact be about the overall gains from openness. More importantly, this strategy may have relevant implications for research and policy regarding how countries integrate with the rest of the world. In particular, the result of this paper that the gains from trade appear to be quite small relative to the overall gains from openness suggests that both research and policy should at least partially redirect their attention from trade to all the other ways through which countries interact. More attention should be devoted, for example, to understanding the importance of FDI and migration in the international exchange of ideas, and to think about policies that countries can follow to speed up the adoption of foreign technologies.

In Eaton and Kortum's (2002) model of Ricardian trade with no diffusion, countries gain from openness through specialization according to comparative advantage. In the model I construct here, countries also gain from diffusion of ideas. Both the gains from trade and the gains from diffusion come from the same basic phenomenon, namely the sharing of the best ideas across countries. Consider, for example, Japan's superior technology for producing automobiles. This technology can be shared through trade by having Japan export automobiles or through diffusion by having other countries produce their own automobiles using Japan's technology. In both cases, thanks to the non-rivalry of ideas emphasized by Romer (1990), sharing ideas leads to an increase in worldwide income.

These gains from sharing the best ideas are the same ones that give rise to aggregate increasing returns to scale in models of quasi-endogenous growth such as Jones (1995) and Kortum (1997). Consider Kortum (1997). In the simplest version of this model, the arrival of new ideas is proportional to the population level and the quality of each idea is drawn from an unchanging distribution. The technology frontier at a certain point in time is the set of best ideas available to produce the given set of goods, and the average productivity of the technology frontier determines the income per capita level. A larger economy has more ideas, more ideas imply that the best available ideas are more productive, and this allows the economy to sustain a higher income level. This entails a scale effect in levels so that income per capita y is increasing with population L, $y = \beta L^{\eta}$, where β and η are positive constants. Jared Diamond's main argument in his book *Guns, Germs and Steel* can be interpreted as saying that this scale effect from sharing ideas is what allowed large Eurasia to attain a superior level of productivity (Diamond, 1997). For our purposes, the relevant implication is that a country can achieve a level of income that is much lower in isolation than sharing ideas in a world of six billion people.

A scale effect of the kind just described is the key element in quasi-endogenous growth models, as it implies that the growth rate is proportional to the growth rate of population, $g = \eta g_L$. This implication allows for a simple calibration, which reveals the magnitude of the gains from openness (in steady state levels). With $g = 1.5\%$ and $g_L = 4.8\%$,[1] the equation $g = \eta g_L$ implies that $\eta = 0.31$, which in turn implies that a country with 1% of the world's population enjoys gains from openness equal to 320% ($100^{0.31} = 4.2$). Using the quasi-endogenous growth model due to Jones (1995), Klenow and Rodríguez-Clare (2005) performed a similar exercise and also found enormous gains from openness. They finished their paper calling for "research documenting... the vehicles of knowledge diffusion," arguing that "trade, joint ventures, FDI, migration of key personnel, and imitation may all play important roles." This paper can be seen as a first step in meeting this challenge.

To explore the role of trade, it is necessary to have a model that is quantitatively consistent with both the observed growth rate and the observed trade volumes. Matching the observed growth rate is essential, since - as shown above - this is what pins down the gains from openness. I build on Eaton and Kortum's (2001) model of trade and growth, which can be seen as an extension of Kortum (1997) to incorporate trade. A key parameter in this model, θ, determines the variability of the distribution of the quality of ideas.[2] If θ is calibrated to match the gravity equation, as is done in Eaton and Kortum (2002), then a puzzle emerges in that the implied growth rate is almost an order of magnitude lower than the one we observe for the

[1] The rate of growth of y is the rate of growth of income per worker after subtracting the contribution from increases in average human capital and in the capital-output ratio (see Jones, 2002, and Klenow and Rodríguez-Clare, 2005). The value for g_L comes from Jones (2002) and corresponds to the rate of growth of researchers. It is significantly higher than the 1.1% rate of growth of population observed in the OECD in the last decades because of an increasing share of the population devoted to research. Doing this exercise with a lower g_L would lead to even larger gains from openness.

[2] In Eaton and Kortum (2001) the quality of ideas is distributed Pareto with parameter θ. Thus, the variance of this distribution *increases* as θ *falls*. I instead follow Alvarez and Lucas (2005), who flip this parameter around and have a higher θ increase the variability of the quality of ideas.

OECD countries in the last decades. Alternatively, if θ is calibrated to match the observed rate of growth in the OECD, then the model generates too much trade, since the pattern of comparative advantage is too strong and dominates the estimated trade costs.

There are (at least) two ways to deal with this puzzle: first, by allowing for diffusion of ideas across countries, and second, by allowing for knowledge spillovers.[3] Consider first how diffusion may allow the model to match both the gravity equation and the growth rate. Intuitively, the excessive volume of trade generated by the high θ needed to match growth of 1.5% per year is dampened when countries can share ideas through diffusion rather than trade. Introducing diffusion into the model leads to a gravity equation with a discontinuous border effect (i.e., trade falls discontinously as trade costs increase from zero) that is not present in Eaton and Kortum (2001, 2002). Estimating θ from this equation leads to $\theta = 0.22$ rather than Eaton and Kortum's $\theta = 0.12$, and this helps to increase the model's implied growth rate from $g = 0.29\%$ to $g = 0.53\%$. But this is still significantly below the observed $g = 1.5\%$.

To increase the model's implied growth rate without affecting its trade implications, I allow for progress and diffusion in "general ideas," which are relevant for all goods and hence affect absolute but not comparative advantage.[4] Analogously to the role played by θ for ideas associated with specific tradable goods, which I will call "specific" ideas, parameter γ determines the variability of the distribution of the quality of general ideas. One can then use $\theta = 0.22$ to match the gravity equation, and $\gamma = 0.41$ so that the model generates $g = 1.5\%$.

Knowledge spillovers in research present an alternative way to build a model that is quantitatively consistent with the observed growth rate and the gravity equation. Such spillovers allow the rate of growth of the stock of ideas to increase above the rate of growth of resources devoted to research (g_L above), and in this way make the model consistent with the observed growth rate even without postulating the existence and diffusion of general ideas. In building the quantitative model with knowledge spillovers I still allow for diffusion of specific ideas, because this improves the model's fit with the trade data. Thus, the models differ only in that in one I introduce progress and diffusion of general ideas, whereas in the other I introduce knowledge spillovers in research.

Having these two models that are quantitatively consistent with observed growth and trade volumes, I can then calculate the gains from openness and the contribution of trade to these

[3] I thank Sam Kortum for suggesting knowledge spillovers as a way to deal with this puzzle.

[4] One can instead consider ideas that affect the productivity of non-tradable goods. The resulting model is identical.

gains. The results in both models are very similar and imply that the gains from openness are large, and the overwhelming majority of these gains come from diffusion of ideas rather than trade. For a country with 1% of the world's GDP, the gains from trade are 23% of autarky income, whereas the gains from openness are between 235% and 292% of the income level in isolation (i.e., with no trade and no diffusion).[5]

This paper is related to the literature on trade and endogenous growth associated with Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991), among others. This group of papers showed that under some conditions trade or international knowledge spillovers would lead a higher growth rate thanks to the exploitation of scale economies in R&D at the global level. This is essentially what Jones (1995) called a "strong scale effect," whereby larger markets exhibit higher growth rates. Jones' empirical analysis showed that such a strong scale effect is not consistent with the data, however, so there has been a shift towards quasi-endogenous growth models, where the growth rate is not affected by scale variables. In this paper I focus on this class of models and explore the *quantitative* implications of openness on steady state income levels.

Another related literature is the one that focuses on international diffusion of technologies and ideas.[6] The closest paper is by Eaton and Kortum (1999), who develop and calibrate a model of technology diffusion and growth among the five leading research economies. These authors then perform a counterfactual analysis to see the implications for the U.S. of detaching itself from sharing ideas with the rest of the world, much as I do to quantify the gains from diffusion. My contribution in this paper is to build a model where one can explore the gains from *both* trade and diffusion.

The rest of the paper is organized as follows. In the next section I lay out the basic model without diffusion or knowledge spillovers, and show that if θ is calibrated to match trade volumes then the implied growth rate is too low. In Section 3 I first introduce diffusion in specific ideas and show that this improves the model's fit with the trade and growth data, but the growth rate is still too low. I then introduce progress and diffusion in general ideas, calibrate the model to match both growth and trade volumes, and quantify the gains from trade and diffusion. Section 4 takes up the model with diffusion in specific ideas and introduces knowledge

[5] These gains from openness of 242% differ from the ones stated above (320%). The reason for this is that the model developed in the paper and its calibration incorporate frictions in the diffusion process or home bias in knowledge spillovers that are not present in the simple numerical exercise considered above.

[6] See Keller (2004) for recent a survey.

spillovers to match the growth rate. The resulting quantitative model is then used to quantify the gains from trade and diffusion. The final section offers some concluding comments and topics for future research.

2 The model without diffusion

In this section I first present a model of trade and growth without diffusion or knowledge spillovers based on Eaton and Kortum (2001).[7] I then calibrate an enriched version of the model and show that the implied growth rate is too low.

2.1 A model of trade and growth

There is a single factor of production, labor, N countries indexed by i, and a continuum of tradable intermediate goods indexed by $u \in [0,1]$. The intermediate goods are used to produce a final consumption good via a CES production function with an elasticity of substitution $\sigma > 0$. The productivity with which individual intermediate goods are produced (i.e., output per unit of the labor) varies across intermediate goods u and across countries, and this gives rise to trade. Let us focus on a single country for now so that we can momentarily leave aside the use of country subscripts. It is convenient to work with the inverse of productivity. To do so, let $x(u)$ be a parameter that determines the cost of producing intermediate good u at time t. In particular, let the cost of producing such a good be given by $x(u)^{\theta}w$, where w is the wage level. Note that the parameter θ, which will be constant across goods and countries, magnifies the variability of the cost parameter x on the actual cost structure across goods and countries. This parameter will be crucial in the analysis that follows.

At any point in time the cost parameters $x(u)$ are the result of previous research efforts in each country. Following Kortum (1997) and Eaton and Kortum (2001), research is modeled as the creation of ideas, although for simplicity here I assume that this is exogenous. In particular, I assume that there is an instantaneous (and constant) rate of arrival ϕ of new ideas per person. In the concluding section I argue that the main results of the paper would not change significantly if research efforts were endogenous.

[7] One difference with Eaton and Kortum (2001) is that I assume perfect competition and an exogenous process for the arrival of ideas, whereas they have Bertrand competition and endogenous innovation. As I explain below, this is just to simplify the presentation: the main results would not be affected by endogeneizing innovation.

Ideas are specific to goods, and the good to which an idea applies is drawn from a uniform distribution in $u \in [0,1]$. Since this interval has unitary mass, then at time t there is a probability $R(t) \equiv \phi L(t)$ of drawing an idea for any particular good, where $L(t)$ is the population level at time t. This implies that the arrival of ideas is a Poisson process with rate function $\phi L(t)$, so the number of ideas that have arrived for a particular good by time t is distributed Poisson with rate $\lambda(t) \equiv \int_0^t R(s)ds$. Again, since the set of goods has unitary mass, then $\lambda(t)$ also represents the total stock of ideas (applying to all goods) at time t. (From here onwards, I will suppress the time index as long as it does not cause confusion.) Assuming that $L(t)$ grows at the constant rate g_L (assumed to be common across countries) then in steady state we must have $\lambda = R/g_L$, so λ also grows at rate g_L.

Ideas for producing a particular intermediate good differ only in terms of a "quality" parameter, and the economy's productivity for intermediate good u is determined by the best idea available for the production of this good. The quality of ideas is *independently* drawn from a distribution of quality which is assumed to be Pareto with parameter one.[8] Letting $x(u)$ be the inverse of the quality of the best idea that has arrived up to time t for good u, then it is easy to show that $x(u)$ is distributed exponentially with parameter λ.[9]

Transportation costs are of the iceberg type, with one unit of a good shipped from country j resulting in $k_{ij} \leq 1$ units arriving in country i. I assume that $k_{ii} = 1$, that $k_{ij} = k_{ji}$, and that the triangular inequality holds (i.e., $k_{ij} \leq k_{ik}k_{kj}$ for all i, j, k).

2.1.1 Equilibrium

Following Alvarez and Lucas (2005), I relabel goods by $x \equiv (x_1, x_2 ... x_n)$ rather than u. The price of good x in country i is then

$$p_i(x) = \min_j \left\{ \left(\frac{w_j}{k_{ij}} \right) x_j^\theta \right\}$$

[8] Eaton and Kortum (2001) assume that the distribution of quality is Pareto with parameter θ, whereas here I assume instead a Pareto distribution with parameter 1, with θ being a parameter that expands the differences in cost across ideas, as in Alvarez and Lucas (2005). The two approaches are equivalent except that the θ here is the inverse of Eaton and Kortum's θ.

[9] Letting q represent the quality of ideas, then $\Pr(Q \leq q) = H(q) = 1 - 1/q$. Letting v be the quality of the best idea that has arrived up to time t, then using $e^x \equiv \sum_{k=0}^{\infty} x^k/k!$ we get $\Pr(V \leq v) = \sum_{k=0}^{\infty} \left(e^{-\lambda}(\lambda)^k/k!\right) H(v)^k = e^{-\lambda/v}$, and hence $x \equiv 1/v \sim \exp(\lambda)$.

Letting $s_i(x) \equiv \min_j \left\{ (w_j/k_{ij})^{1/\theta} x_j \right\}$, then $p_i(x) = s_i(x)^\theta$. From the properties of the exponential distribution it follows that $s_i(x)$ is distributed exponentially with parameter $B^{-1/\theta}\psi_i$,[10] where

$$\psi_i \equiv \sum\nolimits_j \psi_{ij} \text{ and } \psi_{ij} \equiv (w_j/k_{ij})^{-1/\theta} \lambda_j \tag{1}$$

Letting p_{mi} be the price index of the final good, then $p_{mi}^{1-\sigma} = \int p_i(x)^{1-\sigma} dF(x)$ and assuming $1 + \theta(1-\sigma) > 0$,[11] we get

$$p_{mi} = BC_S\psi_i^{-\theta} \tag{2}$$

where $C_S = \Gamma[1 + \theta(1-\sigma)]^{1/(1-\sigma)}$, with $\Gamma()$ being the Gamma function.

To determine wages we introduce the trade-balance conditions. As shown by Eaton and Kortum (2002), the average price charged by any country j in any country i is the same, and hence the share of total income in country i spent on imports from country j, D_{ij}, is equal to the share of goods for which country j is the lowest cost supplier in country i. In turn, this share is equal to the probability that $(w_j/k_{ij})x_j^\theta = \min_l\{(w_l/k_{il})x_l^\theta\}$. From the properties of the exponential distribution, this probability is $D_{ij} \equiv \psi_{ij}/\psi_i$. Given that total income in country i is $L_i w_i$, then the trade balance conditions are simply

$$L_i w_i = \sum\nolimits_j L_j w_j D_{ji} \tag{3}$$

The previous conditions determine a competitive equilibrium. In particular, a competitive equilibrium is a couple of vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mn})$ and $w = (w_1, w_2, ..., w_n)$ such that, given $D_{ij} \equiv \psi_{ij}/\psi_i$ and (1), conditions (2) and (3) are satisfied.

2.1.2 Growth and the gains from trade

I now turn to the implications of the model for growth and the gains from trade. Once we choose a numeraire, wages are constant in steady state since all λ_i are growing at the same rate g_L. The growth rate in real wages is then given by the rate of decline in p_{mi}. But from (2) it is clear that p_{mi} falls at rate θg_L, so the growth rate of real wages or consumption is

$$g = \theta g_L \tag{4}$$

[10] These propertires are: (1) if $x \sim \exp(\lambda)$ and $k > 0$ then $kx \sim \exp(\lambda/k)$; and (2) if x and y are independent, $x \sim \exp(\lambda)$ and $y \sim \exp(\mu)$, then $\min\{x, y\} \sim \exp(\lambda + \mu)$.

[11] The assumption that $1 + \theta(1-\sigma) > 0$ entails $\sigma < 1 + 1/\theta$. In principle, I could explore whether this inequality holds given estimates of σ and given the values of θ that I will discuss in the text below. In practice, however, the empirical value of σ depends on the level of aggregation that we use for inputs, which in turn should be determined by the level at which technologies differ in the way specified in the model. Thus, the restriction $1 + \theta(1-\sigma) > 0$ must be taken as an assumption for now.

The gains from trade are determined by the increase in the real wage, w_i/p_{mi}, as a country goes from autarky to trade. In calculating these gains here and in the following sections I emphasize the case of frictionless trade because this allows for simpler derivations and because this establishes an upper bound for the gains from trade. The result I derive in the following sections that trade contributes with only a small share of the gains from openness would only be strengthened if instead I calculated the gains from trade as those generated by moving from autarky to trade with realistic trade costs. In autarky $\psi_i = w_i^{1/\theta}\lambda_i$, and plugging into (2) and using $\lambda_i = R_i/g_L$ yields $p_{mi}/w_i = BC_S(R_i/g_L)^{-\theta}$. Similarly, with frictionless trade we have $p_m = BC_S\left(\sum_j w_j^{-1/\theta}R_j/g_L\right)^{-\theta}$. With $\phi_i = \phi$ then it is easy to show that there is factor price equalization (i.e., $w_i = w_j$ for all i, j), and hence the gains from trade are

$$GT_i = \left(\frac{\sum L_i}{L_i}\right)^{\theta}$$

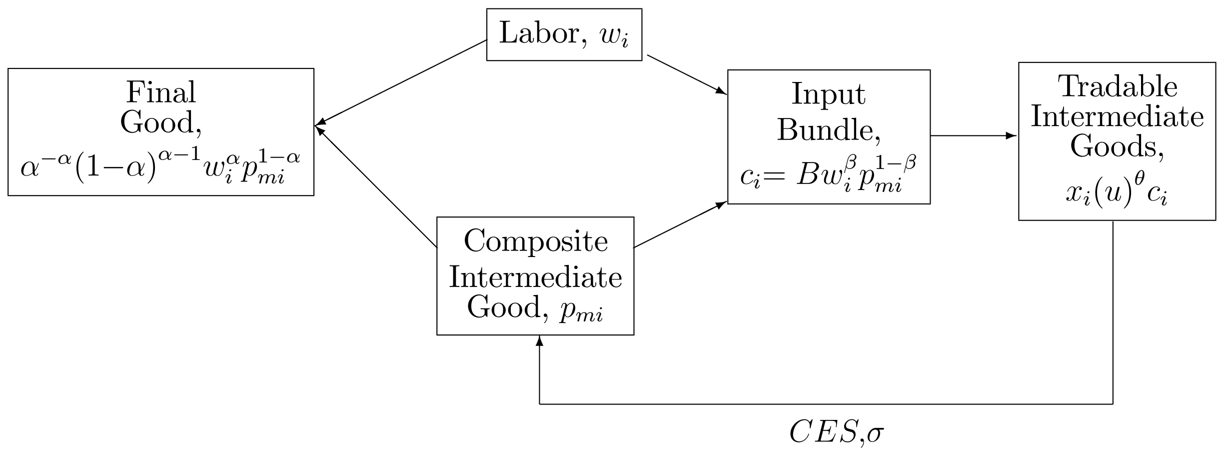
Since they generate a smaller share of the world's best ideas, smaller economies have more to gain from integrating with the rest of the world. Moreover, a high θ leads to higher gains from trade. As explained in the Introduction, the reason for this is that a high θ increases the variability of cost differences across countries and hence leads to a stronger pattern of comparative advantage.

2.2 Towards a quantitative model

I now enrich and calibrate the model to explore its quantitative implications. There are two modifications. First, as in Eaton and Kortum (2002) and Alvarez and Lucas (2005) it is assumed that intermediate goods are used in the production of intermediate goods, thus generating a "multiplier" effect that expands the gains from trade and the growth rate. Second, as in Alvarez and Lucas, it is assumed that production of the consumption good uses labor directly and not only through intermediate goods. Alvarez and Lucas introduce this feature to capture the existence of non-tradables that dampen the gains from trade. In this model, this will also reduce the growth rate, since technological progress is confined to tradable intermediates.

These two modifications are captured formally as follows and illustrated in Figure 1. The intermediate goods are used to produce an "composite intermediate good" with a CES production function with elasticity σ, so that p_{mi} - which above was the price index of the consumption good - is now the price index of this composite good. In turn, the composite good together with labor are used to produce intermediate goods with a Cobb-Douglas production function with

Figure 1: The Production Structure



labor share β. One can think of an "input bundle" produced from labor and the composite intermediate good that is in turn used to produce all the intermediate goods. The cost of the input bundle in country i is then $c_i \equiv Bw_i^{\beta}p_{mi}^{1-\beta}$ where $B \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$, while the cost of intermediate good u in country i is now $x(u)^{\theta}c_i$. Finally, the consumption good is produced from the composite intermediate good and labor with a Cobb-Douglas technology with labor share α. Thus, the price of the consumption good is $p_i = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}w_i^{\alpha}p_{mi}^{1-\alpha}$. Note that if $\beta = 1$ and $\alpha = 0$ then we are back to the model above. The individual intermediate goods are the only tradeable goods.

These modifications do not substantially affect the qualitative results above: the only difference is that now the wage w_i must be substituted by the unit cost of the input bundle, c_i, in the definition of ψ_{ij} in equation (1). But there are important quantitative implications. In particular, the growth rate is now

$$g = \theta\left(\frac{1-\alpha}{\beta}\right)g_L \tag{5}$$

If intermediate goods have a high share in the production of intermediate goods (i.e., high $1-\beta$), so that there is a large multiplier $1/\beta$, then the growth rate will be higher. Similarly, the growth rate increases with the share of intermediate goods in the production of the consumption good (i.e., $1-\alpha$). The term $\left(\frac{1-\alpha}{\beta}\right)$ also affects the gains from trade, which in the case of $\phi_i = \phi$ considered above are now

$$GT_i = \left(\frac{\sum L_i}{L_i}\right)^{\theta(1-\alpha)/\beta} \tag{6}$$

The key parameters of the model are θ, α, β, and g_L. Eaton and Kortum (2002) estimate θ from the gravity equation generated by the model together with bilateral import and price data for the OECD countries. They focus on the way in which θ determines the impact of trade costs on trade volumes. To isolate this aspect of the gravity equation, Eaton and Kortum focus on "normalized trade flows." Let the normalized bilateral imports of country i from country j be D_{ij}/D_{jj}. If there are no trade costs then $\psi_i = \psi_j$, $p_{mi} = p_{mj}$ and hence $D_{ij}/D_{jj} = 1$ for all i, j. With trade costs we have

$$D_{ij}/D_{jj} = \left(\frac{p_{mj}}{p_{mi}k_{ij}}\right)^{-1/\theta}$$

Taking logs, and letting $m_{ij} \equiv \ln(D_{ij}/D_{jj})$ and $d_{ij} = \ln(p_{mj}/p_{mi}k_{ij})$, then

$$m_{ij} = -(1/\theta)d_{ij} \tag{7}$$

Eaton and Kortum (2002) construct m_{ij} from 1990 data on trade and production of manufactures for 19 OECD countries and d_{ij} from data on prices from the UN ICP 1990 benchmark study, which gives retail prices for 50 manufactured products in these countries.[12] A simple method of moments estimation of $1/\theta$ in (7) or an OLS regression with no intercept (as required by theory) yields $\theta = 0.12$.

Alvarez and Lucas (2005) calibrate the parameters α and β to match the fraction of U.S. employment in the non-tradables sector and the share of labor in the total value of tradables produced, respectively. They find $\alpha = 0.75$ and $\beta = 0.5$. For g_L I could use the growth rate of population in the OECD over the last decades, which is $g_L = 1.1\%$. But as Jones (2002) has emphasized, there has been an upward trend in the share of people devoted to R&D in the rich countries over the last decades. According to Jones, the rate of growth of researchers has been 4.8% over the period 1950-1993 in the G-5 countries. Plugging these values in equation (5) together with Eaton and Kortum's $\theta = 0.12$ yields $g = 0.29\%$, which is significantly lower than the observed rate of growth of productivity in the OECD countries, which is close to $g = 1.5\%$. One could, of course, calibrate θ to match the observed growth rate, but this would lead to inconsistent implications for the role of gravity in trade. In particular, bilateral trade volumes would decline too slowly as trade costs increase.

Turning to the gains from trade, these parameters ($\theta = 0.12$, $\alpha = 0.75$, and $\beta = 0.5$) imply from (6) that the gains from frictionless trade for a country with 1% of the world's total

[12] The 19 countries included in the sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States.

population are $100^{0.06} = 1.3$, or 30%.[13] If instead we use the "central value" of θ in Alvarez and Lucas (2005), namely $\theta = 0.15$, then the gains from trade are 41%, as mentioned in the Introduction.

3 Diffusion, trade and growth

In this section I introduce diffusion to construct a model that is quantitatively consistent with the observed growth rate and trade volumes. I start by introducing international diffusion of the ideas mentioned in the previous section, which I call "specific ideas" because they apply to specific tradable goods. The model is then extended to include "general ideas," which apply to all tradable goods and hence do not directly affect trade.

3.1 Specific ideas

As in Eaton and Kortum (2006), diffusion of specific ideas can be seen as a process by which national ideas become "global" ideas, which are available in all countries. Assuming a constant rate of diffusion δ_S, then $\dot{\lambda}_i = \phi_i L_i - \delta_S \lambda_i$: $\phi_i L_i$ is the flow into the "pool" of national ideas and $\delta_S \lambda_i$ is the flow out of this pool and into the pool of global ideas. Hence, in steady state the stock of national ideas in country i is

$$\lambda_i = (\phi_i/(g_L + \delta_S))L_i \tag{8}$$

while the stock of global ideas is

$$\lambda_T = (\delta_S/g_L)\sum \lambda_i \tag{9}$$

For each intermediate good there are n best national ideas (one for each country) plus a best global idea. Letting $x_S(u)$ be the productivity of the best global idea for good u, then $x_S(u)$ is exponentially distributed with parameter λ_T, which grows at g_L.

3.1.1 Equilibrium

Each country can produce a good with either its own best national idea or the best global idea. Thus, each country has $2N$ ways of procuring a good: country i can buy the good produced

[13] Note that if $\phi_i = \phi_j$ for all i, j then with frictionless trade wages are equal across countries, so a country with 1% of the population also has 1% of the world's GDP. For convenience, I used this case to refer to the gains from trade in the Introduction.

in country j (including $j = i$) with country $j's$ best national idea or with the best global idea. Labeling goods by $\widetilde{x} = (x_1, x_2, ..., x_n, x_S)$, the price of good $\widetilde{x}$ in country i is then

$$p_i(\widetilde{x}) = \min\left\{\min_j\left\{\left(\frac{c_j}{k_{ij}}\right)x_j^\theta\right\}, \min_j\left\{\left(\frac{c_j}{k_{ij}}\right)x_S^\theta\right\}\right\}$$

Without loss of generality, for every country i there is a country $\hat{\jmath}(i)$ (which may be country i itself) which has the least cost of delivering goods produced with global technologies.[14] This is defined as $\hat{\jmath}(i) = \arg\min_j\{c_j/k_{ij}\}$. Using this in the expression for $p_i(\widetilde{x})$ above and letting

$$\xi_i(\widetilde{x}) \equiv \min\left\{\min_j\left\{\left(\frac{c_j}{k_{ij}}\right)^{1/\theta}x_j\right\}, \left(\frac{c_{\hat{\jmath}(i)}}{k_{i\hat{\jmath}(i)}}\right)^{1/\theta}x_S\right\}$$

then $p_i(\widetilde{x}) = \xi_i(\widetilde{x})^\theta$. Given the properties of the exponential distribution, ξ_i is distributed exponentially with parameter $\hat{\psi}_i$ where

$$\hat{\psi}_i \equiv \sum\nolimits_j \hat{\psi}_{ij} \quad \text{and} \quad \hat{\psi}_{ij} \equiv \begin{cases} \left(w_j^\beta p_{mj}^{1-\beta}/k_{ij}\right)^{-1/\theta}\lambda_j & if \quad j \neq \hat{\jmath}(i) \\ \left(w_j^\beta p_{mj}^{1-\beta}\right)^{-1/\theta}(\lambda_j + \lambda_T) & if \quad j = \hat{\jmath}(i) \end{cases} \tag{10}$$

Thus,

$$p_{mi} = BC_S\hat{\psi}_i^{-\theta} \tag{11}$$

Wages w are determined by the trade-balance conditions, as in (3), but with $D_{ji} = \hat{\psi}_{ji}/\hat{\psi}_j$. Thus, the competitive equilibrium is now determined by the vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mn})$ and $w = (w_1, w_2, ..., w_n)$ such that, given $D_{ij} = \hat{\psi}_{ij}/\hat{\psi}_i$ and (10), conditions (11) and (3) are satisfied.

3.1.2 Implications for growth and trade

Just as in the previous section, in steady state wages are constant and equations (10) and (11) imply that p_{mi} declines at rate $\widetilde{g} = \theta g_L/\beta$. It follows that the formula for the growth rate is the same as in the model without diffusion (equation (5)). Next I turn to the trade implications of diffusion of specific ideas.

It is convenient to introduce the notion of "global goods," which are goods for which the best idea is a global idea. Formally, a good x is a global good if it satisfies $x_S > x_i$ for all i.

[14] If $c_j/k_{ij} = c_k/k_{ik}$ then country i is indifferent between countries j and k in terms of where to buy goods produced with global technologies. But this indifference would occur only for a measure-zero set of transportation costs. One relevant case is when there are no transportation costs. This is explored later in this paper.

Note that this is not a permanent characteristic of a good u, but just a momentary feature that disappears with the arrival of a national idea that is superior to the best global idea. Global goods will always be produced with the best global idea, but this does not rule out trade in such goods. In fact, if trade costs are sufficiently low, countries with a low research intensity (i.e., a low ϕ_i) would naturally specialize in the production of global goods. One could argue, for example, that the Chinese export miracle of the last decades is due to technology diffusion, so that a large share of the goods currently exported by China are global goods.

As this discussion makes clear, the direction of bilateral trade flows for global goods in equilibrium depends on the distribution of trade costs and research intensities across countries. There are obviously many different configurations that could arise. But there is one case that is particularly relevant and easy to characterize. This is the case in which there is no trade in global goods, so that each country satisfies its own demand for global goods with domestic production using the best global ideas. I refer to this as the NTG condition. Recalling the definition of $\hat{\jmath}(i)$ above as the country that has the least cost of delivering global goods to country i, the NTG condition implies $\hat{\jmath}(i) = i$ for all i.

As shown in the Appendix, one case in which the NTG condition is satisfied entails a common research intensity across countries. Formally, if $\phi_i = \phi_j$ for all i, j and if $x_S(u) = \min_i\{x_i(u)\}$ then there is an equilibrium in which there is no trade in intermediate good u. To gain some intuition for this result, consider the case of no trade costs. With a common research intensity, the absence of trade costs implies that in equilibrium all countries have the same wage and the same unit cost for the input bundle, i.e. $c_i = c_j$ for all i, j. In turn, this implies that in equilibrium one can have every country satisfy its own demand for global goods. If trade costs are positive, then *a fortiori* the NTG condition will be satisfied. The assumption of a common research intensity across countries is only necessary so that the NTG condition is satisfied for *any* structure of trade costs k_{ij}. Alternatively, one could assume a simple structure of trade costs with $k = k_{ij}$ for all i, j, and then find the maximum k (i.e., minimum trade costs) necessary for the NTG condition to be satisfied given any vector of research intensities $(\phi_1, \phi_2, \ldots, \phi_n)$. Clearly such a maximum k exists since the NTG condition is satisfied for k close to zero.

In what follows, I assume that research intensities are sufficiently similar that given the presence of trade costs the NTG condition is satisfied. Although clearly this is not a reasonable characterization for trade in the whole world, it is a reasonable assumption to characterize trade among the richest countries in the world.[15] This is the relevant case to consider in the empirical analysis that I conduct below to estimate the parameters θ and δ_S in the model with diffusion of specific ideas. In a later section I explore an extension to a simple case where the NTG is not satisfied.

Applying the equilibrium definition above to the case with $\hat{\jmath}(i) = i$ for all i we see that now

$$p_{mi} = BC_S \widetilde{\psi}_i^{-\theta} \tag{12}$$

where

$$\widetilde{\psi}_i = \sum_j \widetilde{\psi}_{ij} \text{ and } \widetilde{\psi}_{ij} = \begin{cases} \left(w_j^\beta p_{mj}^{1-\beta}/k_{ij}\right)^{-1/\theta} \lambda_j & if \quad i \neq j \\ \left(w_i^\beta p_{mi}^{1-\beta}\right)^{-1/\theta} (\lambda_i + \lambda_T) & if \quad i = j \end{cases} \tag{13}$$

Since trade shares are given by $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$, then D_{ii} is higher with diffusion than without, implying that country i buys a higher share of goods domestically and trade volumes are lower. Intuitively, trade is now lower because some of the goods that would be imported without diffusion are now produced domestically.

The relationship between normalized import shares and trade costs is now

$$m_{ij} = -\ln\left(1 + (\delta_S/g_L)R_T/R_j\right) - (1/\theta)d_{ij} \tag{14}$$

where $R_T = \sum R_j$. Using the same data on trade volumes and trade costs as Eaton and Kortum (2002), and using population or GDP levels as proxies for aggregate research levels R in (14), I estimated δ_S and θ from this equation using non-linear least squares among 19 OECD countries and $g_L = 0.048$.[16] Both parameters are precisely estimated. With population data the estimate of $1/\theta$ is 4.6 with a s.e. of 0.36 while the estimate of δ_S is 0.008 with a s.e. of 0.002.[17],[18] The results with GDP data are basically the same (same $1/\theta$, slightly lower $\delta_S = 0.007$). Note that $1/\theta = 4.6$ implies $\theta = 0.22$, significantly higher than Eaton and Kortum's $\theta = 0.12$. The higher

[15] Readers interested in the characterization of trade in global goods among rich and poor countries should consult Eaton and Kortum (2006).

[16] See footnote 12 for the list of countries included in this regression.

[17] There are 342 observations, and the R-squared is 0.93.

[18] Equation (14) characterizes normalized bilateral trade flows among rich countries assuming that these are the only countries in the world. One could extend the model easily to include poor countries as follows: assume that rich countries are the only ones doing research, that δ_S is the rate of diffusion among rich countries, and that ideas that have diffused within rich countries diffuse to the rest of the world (RW) at rate δ'_S. Then it is easy to show that bilateral trade among rich countries is still characterized by equation (14) except that the term δ_S/g_L is replaced by $\delta_S/(\delta'_S + g_L)$. The estimate of θ remains unchanged, and the result for $\delta_S = 0.008$ now implies the restriction $\delta_S = 0.017\delta'_S + 0.0008$. Since δ'_S is not large, then the resulting δ_S would not be significantly different than 0.0008.

value of θ helps the model better match the observed growth rate: the implied growth rate is now $g = 0.53\%$. But this is still significantly below the observed $g = 1.5\%$.

Alvarez and Lucas (2005) note that their calibrated model does well in matching the observed import shares for the 59 economies with the largest total GDPs.[19] The import share of country i is $v = [(1-\alpha)/\beta](1 - D_{ii})$. Assuming symmetry (i.e., $L_i = L_j$ and $\phi_i = \phi_j$ for all i, j) and $k_{ij} = k$, then

$$v(n) = \frac{1-\alpha}{\beta} \frac{(n-1)k^{1/\theta}}{1 + n\delta_S/g_L + (n-1)k^{1/\theta}} \tag{15}$$

The expression $v(n)$ is the import share for each of n equal countries under symmetric trade costs. If countries are not equal then there is no closed form solution for import shares, which (in the presence of trade costs) must be computed numerically. But Alvarez and Lucas (AL) show that $v(n)$ closely approximates the import share of a country with a share $1/n$ of the world's GDP in the case of unequal countries. Next I use this approximation to obtain an alternative estimate of δ_S from equation (15). AL assume $\theta = 0.15$, $k = 0.7$ and $\delta_S = 0$ (i.e., no diffusion).[20] Instead, I use $\theta = 0.22$ as estimated above, but maintain $k = 0.7$ and use the same data as AL (and $g_L = 0.048$) to estimate δ_S from equation (15) with non-linear least squares, excluding the four outliers identified by AL, namely Hong Kong, Singapore, Belgium and Malaysia. The result is $\delta_S = 0.005$ (with a s.e. of 0.0008), a bit lower but in the same ballpark as the 0.008 estimated above with bilateral trade data among the OECD countries.

Table 1 shows that both for the whole sample (without outliers) and for the OECD countries (without Belgium) the fit of the model with $\theta = 0.22$ and diffusion ($\delta_S = 0.005$ or $\delta_S = 0.008$) is significantly better than with $\theta = 0.15$ and no diffusion, suggesting that diffusion helps the model better match the trade data.[21]. Figure 2 shows the import-share data (for the whole sample) together with the import shares implied by AL's calibrated model with $\theta = 0.15$ and $\delta_S = 0$, and the model with $\theta = 0.22$ and diffusion under $\delta_S = 0.005$ (low diffusion, LD) and $\delta_S = 0.008$ (high diffusion, HD). Transportation costs are given by $k = 0.7$ in all three cases.

[19] The data is an average for the 1994-2000 period, from the 2002 WDI cdrom, but is available from Fernando Alvarez's home page.

[20] In fact, Alvarez and Lucas use $k = 0.75$ but they also have tariffs of 10%. Since I don't have tariffs in my model, I use the approximation $k = 0.7$.

[21] In Table 1 the whole sample excludes the four outliers Hong Kong, Singapore, Malaysia and Belgium, while the OECD refers to the 1990 members but excluding Belgium.

	$\theta = 0.15$ and $\delta_S = 0$	$\theta = 0.22$ and $\delta_S = 0.005$	$\theta = 0.22$ and $\delta_S = 0.008$
Whole sample	1.86	0.86	1.04
OECD	0.35	0.1	0.13

Table 1: Sum of square residuals for import shares

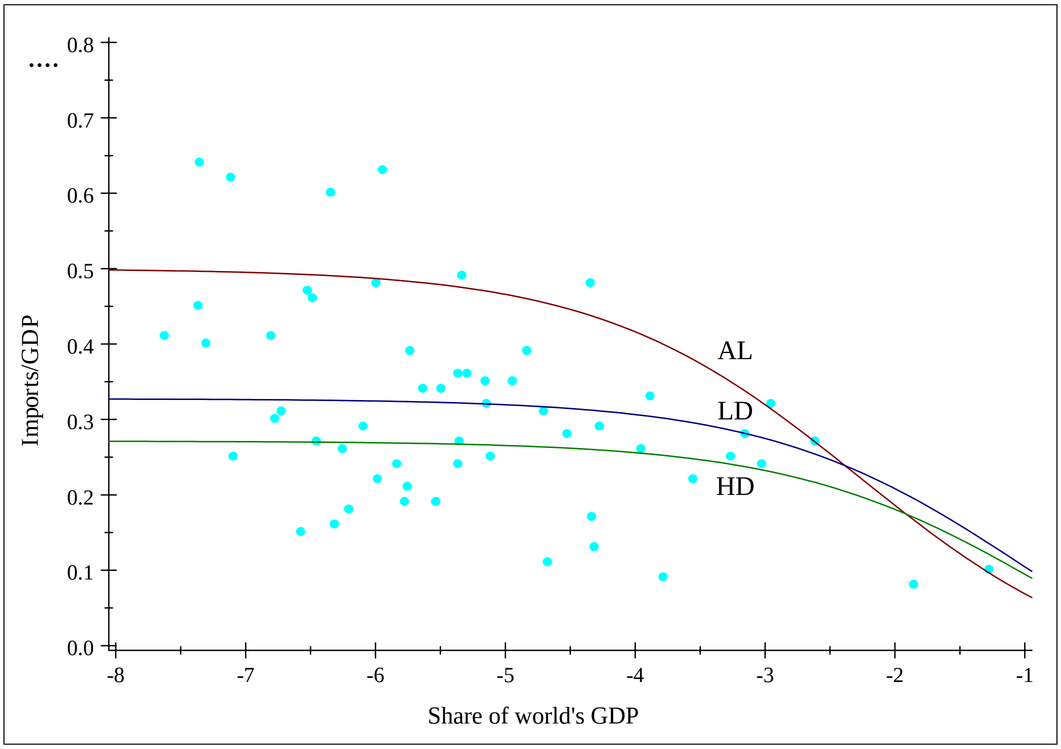


Figure 2: Import shares and size

The estimate of δ_S that I will use in calculating the gains from trade and diffusion below is the one estimated with the OECD bilateral trade data, namely $\delta_S = 0.008$. This is because this estimation relies on a closer link between the model and the data. One concern with this estimate is that it implies a mean lag for diffusion of $1/\delta_S = 125$ years, which may seem too high.[22] If I assumed instead that $\delta_S = 0.1$, so that the mean diffusion lag is 10 years, the implied trade volumes would be too low. One reason why the estimate of the rate of diffusion may be so low is that we are assuming that all observed trade is Ricardian trade, whereas in reality a significant part of this trade (especially among OECD countries) is explained by increasing returns and product differentiation (as in the Krugman model). Moreover, note that diffusion as modeled here occurs simultaneously to *all other* countries, whereas what we observe in practice is sequential diffusion to just one or a few countries. A more realistic model would entail ideas diffusing to one country at a time, and in the mean time there would be trade between the countries that have and the ones that do not have the technology. To see the importance of this for trade volumes, imagine the extreme case where ideas diffuse only to one country. With ten countries and a mean diffusion lag of ten years, the import share with $\theta = 0.22$, $k = 0.7$, and $g_L = 0.048$ would be 41% whereas with complete diffusion (as modeled above) and the same parameters but $\delta_S = 0.008$ the import share would be 40%. Pursuing this line of research to explore the implications of a more realistic process of diffusion on trade flows is an important topic for future research, but does not seem essential for the calculation of the gains from trade and diffusion performed below.

This is a good place to discuss the implication of the NTG condition for the estimation of θ above. Imagine that ideas diffused from country j to country i at an instantaneous rate of δ_S^{ij}. Evidence suggests that countries that are closer together, or have a common language or a common border, have stronger knowledge flows (see Keller, 2004). Thus, δ_S^{ij} is likely to be negatively correlated with trade costs $1/k_{ij}$ or d_{ij}. Since a high δ_S^{ij} implies lower trade flows from j to i then omitting this from the regression above is likely to *underestimate* the impact of trade costs on trade flows, and hence overestimate θ. Intuitively, high trade costs go together with trade flows that are not too low *not* because of a high θ but rather because of little diffusion between such countries. The main result below, namely that the gains from trade are a small part of the overall gains from openness, would only be strengthened if a richer

[22] Comin, Hobijn and Rovito (2006) find that diffusion lags in telephones range from 45 to almost 90 years (p. 12), so the estimated diffusion lag may actually be not that high.

modeling of diffusion led to a lower estimate of θ.

3.2 General ideas

As shown above, introducing diffusion of specific ideas leads to a higher estimate of θ, which in turn increases the model's growth rate from 0.29% to 0.53%. But this is still well below the observed growth rate of 1.5%. In this section I extend the model to capture technological progress in general ideas. This will finally lead to a model that is consistent with both the trade data and the observed growth rate.

In contrast to specific ideas, which affect the productivity of specific tradable goods, general ideas determine the productivity of non-tradable goods or of all tradable goods, and hence do not lead to trade. Examples of these ideas are improvements in telecommunications and finance, or ideas that are applicable to all tradable inputs, such as better management practices.

There are a few equivalent ways of introducing general ideas into the model. A simple one entails assuming that the composite intermediate good is produced in three stages: first, the tradeable intermediate goods are used to produce a (non-tradeable) stage-one input through a CES production function with elasticity of substitution σ, at unit cost c_* (I leave aside the use of country subscripts for the moment); then the stage-one input is used to produce a continuum of (non-tradeable) stage-two inputs indexed by $v \in [0, 1]$ with cost parameters $z(v)$, so that the unit cost of stage-two input v is $z(v)^{\gamma} c_*$; finally, these inputs are used to produce the (non-tradeable) composite intermediate good through a CES production function with elasticity of substitution σ, at unit cost p_m.[23] The tradeable intermediate goods are produced with labor and the composite intermediate good exactly as above, so that their unit cost remains $c = Bw^{\beta} p_m^{1-\beta}$. Note that the parameter γ plays the same role in affecting the cost of inputs v in the second stage as the parameter θ plays in affecting the cost of the tradeable intermediate goods.

The cost parameters $z(v)$ for stage-two inputs are determined by general ideas in a similar (but not exactly the same) way as the cost parameters $x(u)$ for tradable intermediate goods are determined by specific ideas. I will be more specific about this below. For now, I simply postulate that $z(v)$ is distributed exponentially with parameter η_i in country i.

[23] The assumption that the elasticity of substitution is the same in the first and the third stages is made to minimize notation and plays absolutely no role in the results.

3.2.1 Equilibrium

Given the unit cost $z(v)^{\gamma}c_{*i}$ of the second-stage input in country i, the price of the composite input there is implicitly given by

$$p_{mi}^{1-\sigma} = \int_0^1 z(v)^{\gamma(1-\sigma)} c_{*i}^{1-\sigma} dv$$

Since $z(v)$ is distributed exponentially with parameter η_i, then, assuming that $1+\gamma(1-\sigma) > 0$, we have $p_{mi} = c_{*i}C_G\eta_i^{-\gamma}$ where $C_G \equiv \Gamma(1+\gamma(1-\sigma))^{1/(1-\sigma)}$. But it was established before that $c_{*i} = BC_S\widetilde{\psi}_i^{-\theta}$ and hence

$$p_{mi} = BC_S C_G \widetilde{\psi}_i^{-\theta} \eta_i^{-\gamma} \tag{16}$$

Note that, by lowering the cost of the composite intermediate good, general ideas lower the cost of all the tradeable intermediate goods equally; hence, they affect *absolute* and not *comparative* advantage. Hence, the characterization of equilibrium and, in particular, the implications for trade and the gravity equation, remain valid in the presence of general ideas. Formally, the competitive equilibrium satisfying the NTG condition is characterized by the vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mn})$ and $w = (w_1, w_2, ..., w_n)$ such that, given $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ and (13), conditions (16) and (3) are satisfied.

3.3 Implications for growth and trade

The trade implications of the model are as in the previous subsection. I now turn to its growth implications.

It will be shown below that $\dot{\eta}_i/\eta_i = g_L$ for all i, hence equation (16) implies that $\dot{p}_{mi}/p_{mi} = (\theta+\gamma)g_L/\beta$. In turn, this implies the rate of growth is now given by

$$g = (\gamma+\theta)\left(\frac{1-\alpha}{\beta}\right) g_L \tag{17}$$

Given $\alpha = 0.75$, $\beta = 0.5$, and $\theta = 0.22$, a value of γ equal to 0.41 is needed to to match $g = 1.5\%$.

3.3.1 Arrival and diffusion of general ideas

Next I turn to the modelling of the arrival and diffusion of general ideas. Although I could simply model this in exactly the same way as with specific ideas, I choose instead to do so in a slightly more detailed way, the goal being to be able to use the parameters for diffusion estimated by Eaton and Kortum (1999).

In Eaton and Kortum's model of research and diffusion, ideas do not diffuse instantaneously even within a country. A simple set-up that is very close to theirs entails assuming a constant instantaneous rate of diffusion δ_{ji} from country i to country j and a process for the arrival of general ideas equal to our simple set-up for specific ideas. Thus, letting μ_{ji} be the stock of ideas originated in country i that have not diffused to country j, then $\dot{\eta}_i = \sum_j \delta_{ij}\mu_{ij}$. But $\dot{\mu}_{ij} = R_j - \delta_{ij}\mu_{ij}$, so in steady state we must have $\mu_{ij} = R_j/(g_L + \delta_{ij})$. To simplify, I assume that $\delta_{ij} = \delta'_G$ for $i = j$ and $\delta_{ij} = \delta_G$ for all $i \neq j$. In steady state we then have

$$\eta_i = \left[\delta_G(g_L + \delta_G)^{-1}\left(1/r_i - 1\right) + \delta'_G(g_L + \delta'_G)^{-1}\right] R_i/g_L \tag{18}$$

where $r_i \equiv R_i/R_T$ is country $i's$ share of world research. Note that if $\delta'_G \to \infty$ then this collapses to a model that is identical to what we had for specific ideas above. The expression for η_i in equation (18) is in terms of exogenous variables and parameters, namely the research flow in country i, R_i, the worldwide research flow, R_T, the diffusion parameters δ_G and δ'_G, and g_L.

Eaton and Kortum (1999) estimate rates of diffusion within and across countries. Using the average of their estimated rates of "within country" diffusion for Germany, France, U.K., Japan and the U.S., we can set $\delta'_G = 1.5$. They also estimate that the rate of diffusion within countries is 17.7 faster than across countries, hence $\delta_G = 0.085$.

3.4 Gains from trade and diffusion

We now have a model that is consistent with the gravity equation and the observed rate of growth for a group of rich countries. This model entails diffusion of *both* specific and general ideas. With this model it is now possible to perform a simple exercise to understand the contribution of trade to the total gains from openness. More generally, the goal is to quantify the gains from trade, diffusion of specific ideas, and diffusion of general ideas. A simple yet useful way of doing this is by considering the gains from *frictionless* trade. Such gains are much easier to calculate and set an upper bound to the more realistic gains with frictions that we care about. I also calculate the gains from trade and openness for the special case of n identical countries and symmetric trade costs $k_{ij} = k = 0.7$ for all i, j. For these calculations I use the parameters calibrated by Alvarez and Lucas (2005) - $\alpha = 0.75$ and $\beta = 0.5$ - together with the estimated parameters $\theta = 0.22$ and $\delta_S = 0.008$, the calibrated parameter for general ideas $\gamma = 0.41$, the parameters $\delta_G = 0.085$ and $\delta'_G = 1.5$ from Eaton and Kortum (1999), and $g_L = 0.048$. Also, to maintain the NTG condition even in the case of no trade barriers, I assume in this section that countries have a common research intensity (i.e., $\phi_i = \phi$ for all i). The next section considers an extension to the case of varying research intensities.

Just as in Section (2), the gains from trade and diffusion are determined by the effect of these forces on the real wage, which is given by $w_i/p_i = (p_{mi}/w_i)^{-(1-\alpha)}$. To calculate the gains from trade, I compare the real wage under autarky but with diffusion to the case of frictionless trade with diffusion (full openness). Next I compute the real wage in these two settings.

Under autarky (but with diffusion) we have

$$\widetilde{\psi}_i = \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-1/\theta} (\lambda_i + \lambda_T)$$

From (16) we then get

$$p_{mi}/w_i = (BC_S C_G)^{1/\beta} (\lambda_i + \lambda_T)^{-\theta/\beta} \eta_i^{-\gamma/\beta}$$

With frictionless trade and diffusion the NTG condition requires $\phi_i = \phi_j$ for all i, j, an assumption that will be maintained for the rest of this subsection. This implies that in equilibrium we must have the unit cost of the input bundle be the same across countries, or $c_i = c_j$ for all i, j. I hereafter refer to this condition as the equal-cost or EC condition. Incidentally, from (13) this implies that $\widetilde{\psi}_i = \widetilde{\psi}_j = \widetilde{\psi}$ for all i, j and hence from (16) we find

$$w_i/w_j = \left(\eta_i/\eta_j\right)^{\gamma(1-\beta)/\beta} \tag{19}$$

This result implies that countries with a higher stock of general ideas enjoy higher wages. In other words, there is factor price equalization up to differences in the stock of general ideas across countries.

The EC condition implies that under frictionless trade the common ψ is given by

$$\widetilde{\psi} = \left(w_i^{\beta} p_{mi}^{1-\beta}\right)^{-1/\theta} \left(\sum \lambda_i + \lambda_T\right)$$

and hence from (16) we find

$$p_{mi}/w_i = (BC_S C_G)^{1/\beta} \left(\sum \lambda_i + \lambda_T\right)^{-\theta/\beta} \eta_i^{-\gamma/\beta} \tag{20}$$

Combining these results and using (8) and (9), shows that the gains from (frictionless) trade are given by

$$GT_i = \left(\frac{1 + \delta_S / g_L}{r_i + \delta_S / g_L} \right)^{\theta(1-\alpha)/\beta} \tag{21}$$

If $\delta_S = 0$ then $GT_i = r_i^{-\theta(1-\alpha)/\beta}$. But when there is diffusion, GT is smaller than this because the diffusion of specific ideas acts as a substitute for trade in the international exchange of ideas. Intuitively, imports allow a country to benefit from foreign ideas that have not yet diffused, and diffusion allows a country to benefit from global ideas; since ideas "compete" with each other, then shutting down trade leads a country to rely more on diffusion and this attenuates the resulting losses.[24] For concreteness, imagine that $r_i = 1\%$.[25] Then $GT_i = 1.23$, or 23%.

Next I derive the gains from trade and diffusion of specific ideas *together*. These gains are obtained by comparing the real wage with no trade and no diffusion of specific ideas, with the real wage with frictionless trade and diffusion of specific ideas (with $\delta_S = 0.0008$); diffusion of general ideas is maintained in both scenarios. Under autarky and with no diffusion of specific ideas, a country just benefits from the ideas originated there. This implies that

$$\widetilde{\psi}_i = \left(w_i^{\beta} p_{mi}^{1-\beta} \right)^{-1/\theta} (1 + \delta_S / g_L) \lambda_i$$

Following a similar logic as above, this implies that the gains from trade and diffusion of specific ideas together are given by

$$GTS_i = r_i^{-\theta(1-\alpha)/\beta} \tag{22}$$

For $r_i = 1\%$, $GTS_i = 1.66$, or 66% gains.

There are three things to note from these results. First, in the presence of frictionless trade, diffusion of specific ideas does not contribute to the overall gains from openness. This can be verified simply by noting that GTS_i is not affected by δ_S. In particular, GTS_i is still given by (22) if there is no diffusion of specific ideas (i.e., with $\delta_S \to 0$). Intuitively, when trade is not costly and countries have the same research intensity, ideas can be fully shared through trade, so there is no need for diffusion. Second, as explained above, trade and diffusion of specific ideas are *substitutes*. To see this, consider again a country with $r_i = 1\%$. If this country shuts down

[24] This substitutability between trade and diffusion is similar to the substitutability between trade and factor flows in Mundell (1957).

[25] Even though r_i is country $i's$ share of worldwide research, at least for rich countries this is also a good approximation for country $i's$ share of the world's GDP. This approximation becomes perfect as trade costs go to zero and research intensities become equal. This is why in the Introduction I refer to these results as applying to a country with 1% of the world's GDP.

trade but allows diffusion, the losses are 23%; if it allows (frictionless) trade but shuts down diffusion, there are no losses; but if both trade and diffusion are shut down simultaneously, the loses are 66%. A more direct way to verify that trade and diffusion are substitutes is by noting that the gains from trade increase as δ_S falls towards zero, with GT reaching GTS as $\delta_S \to 0$. Finally, taking into account diffusion of specific ideas makes a big difference: with $\theta = 0.15$ and $\delta_S = 0$ as in Alvarez and Lucas (2005), the gains from frictionless trade for a country with $r_i = 1$ would be 41% compared to the 23% gains found here.

I now derive the gains from diffusion of general ideas. These gains are calculated by comparing the real wage for a case where there is no diffusion of general ideas to the case where there is such diffusion while preserving trade and diffusion of specific ideas in both scenarios. Note from (20) that these gains are determined by the ratio of the stock of general ideas under diffusion relative to the case of no diffusion. Letting η_i^O and η_i^I be the stocks of general idea with openness and isolation, respectively, then the gains from diffusion of general ideas are

$$GG_i = \left(\eta_i^O/\eta_i^I\right)^{\gamma(1-\alpha)/\beta} \tag{23}$$

From (18) we get

$$\frac{\eta_i^O}{\eta_i^I} = \frac{\delta_G(g_L+\delta_G)^{-1}\left(1/r_i - 1\right)}{\delta_G'(g_L+\delta_G')^{-1}} + 1 \tag{24}$$

Again, focussing on the case of $r_i = 1\%$, then $GG = 2.36$, or 136%.

The total gains from openness are equal to the gains from trade and diffusion of specific ideas plus the gains from diffusion of general ideas, namely $GO = GTS \cdot GG = 3.92$ or 292%. Thus, the gains from (frictionless) trade (23% of autarky income) can be said to be small compared to these large overall gains from openness.[26]

The previous calculations have been made for the case of $r = 1\%$. Figure 3 shows how these results generalize to different levels of r. The curve GT represents the gains from trade, the curve GTS represents the gains from trade and diffusion of specific ideas, and the curve $GO = GTS \cdot GG$ represents the overall gains from openness. In general, trade contributes a small share of the overall gains from openness, which come mostly from diffusion of general ideas.

Figure 4 shows the relationship between the gains from trade in this model and the gains from trade in the AL model. It only shows the figures for $1/r$ up to 15 so that the relationship

[26] If $\delta_G, \delta_G' \to \infty$ then $GG = r_i^{-\gamma(1-\alpha)/\beta}$ and $GO_i = r_i^{-(\theta+\gamma)(1-\alpha)/\beta}$, so the total gains from openness for a country with 1% of the world's population is 4.3, as in the introduction.

can be better appreciated; for $1/r > 15$ the gains from trade with diffusion are always lower than in the AL model. The figure shows that for low $1/r$ the gains from trade are higher in the model with diffusion. This is because the model with diffusion has a higher level of θ, and this effect dominates for low $1/r$.

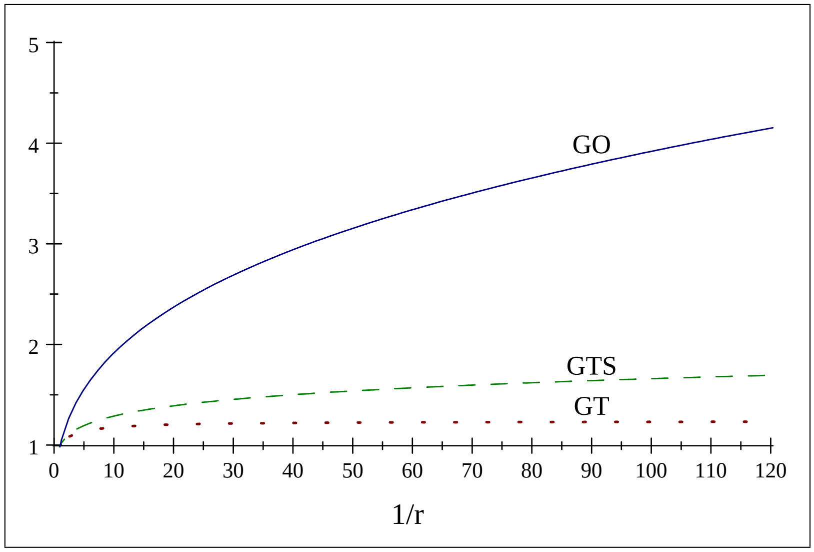


Figure 3: Gains from Trade and Diffusion (GT = Gains from Trade, GTS = Gains from Trade and Diffusion of Specific Ideas, GO = Gains from Openness)

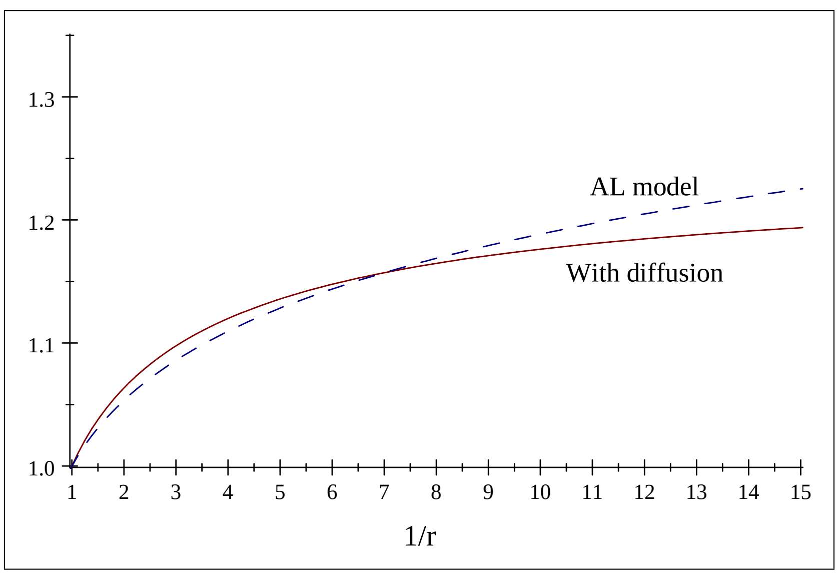


Figure 4: Gains from Trade in the model with diffusion ($\delta_S = 0.008$, $\theta = 0.22$) versus the AL model ($\theta = 0.15$, $\delta_S = 0$)

All the previous calculations for the gains from trade and the gains from openness have been done for the case of frictionless trade. I now make the analogous calculations for the case of n identical countries with $k_{ij} = k$ for all i, j. I use $k = 0.7$, as considered by Alvarez and Lucas (2005). It is worth noting that the resulting gains from openness will take into account both frictions in trade and in the diffusion of ideas (since δ_S, δ_G, δ'_G are all finite).

Since countries are identical, then it is easy to show that the gains from trade are now:

$$GT = \left(\frac{(n-1)k^{1/\theta} + 1 + (\delta_S/g_L)n}{1 + (\delta_S/g_L)n} \right)^{\theta(1-\alpha)/\beta}$$

For $n = 100$ this yields gains of 8.5%. Similarly, the formula for the GTS is now:

$$GTS = \left(\frac{(n-1)k^{1/\theta}\lambda + \lambda + (\delta_S/g_L)n\lambda}{\lambda + (\delta_S/g_L)\lambda} \right)^{\theta(1-\alpha)/\beta}$$

Again, for $n = 100$ this yields gains from trade and diffusion of specific ideas of 46%.

The gains from diffusion of general ideas are not affected by trade frictions, so the gains from openness are 245%. Thus, as intuition would suggest, the result that the gains from trade (8.5% for $n = 100$) are small compared to the overall gains from openness (245% for $n = 100$) remains valid if we consider trade frictions.

3.5 Varying research intensities under frictionless trade

The previous discussion has been confined to the case of common research intensities. I now consider the case of varying research intensities under two special conditions: first, that there are no trade costs (i.e., $k_{ij} = 1$ for all i, j), and second, that cross-country differences in research intensities are not too large relative to the rate of diffusion of specific ideas, in a sense that will be specified shortly. Under these conditions, the equilibrium entails factor price equalization up to differences in stocks of general ideas (η_i), and countries with low research intensities export global goods, so these countries can attain trade balance in spite of the fact that their stock of national ideas per person is relatively low. The drawback of this alternative case is that it has no "gravity" implications, but it is useful for the analysis of gains from trade, since I focus on frictionless trade anyway.

I consider an equilibrium in which all countries produce global goods. Under frictionless trade, this requires that the unit cost of the input bundle be equal across countries, so that there is indifference about where to buy global goods. In what follows I show conditions under which the equilibrium satisfies this property. As in the case of frictionless trade with no trade in global goods analyzed above, the condition $c_i = Bw_i^{\beta} p_{mi}^{1-\beta} = c$ for all i (the EC condition) together with $\widetilde{\psi}_i = \widetilde{\psi}$ implies that relative wages satisfy (19), so that countries with a higher stock of general ideas enjoy higher wages.

From now on I will refer to goods for which the best technology is a non-diffused technology as "national goods." Thus, the whole set of goods is composed of national goods and global goods. Also, note that each national good is associated with the country which owns the best (non-diffused) technology for that good.

The trade-balance conditions are still given by (3). In equilibrium, country j will at least supply the whole world of the national goods associated with j. Using (8) and (9), the share of such goods is given by

$$\lambda_i / (\sum \lambda_j + \lambda_T) = \frac{R_i/R_T}{1 + \delta_S/g_L}$$

Given the absence of trade costs and the EC condition, then this is also the share of each country's total spending that will be allocated to buying national goods from country i. This implies that the shares D_{ji} must satisfy the conditions

$$D_{ji} \geq \frac{R_i/R_T}{1 + \delta_S/g_L} \tag{25}$$

for all i, j. Given the absence of trade frictions, there is no loss of generality in assuming that $D_{ji} = D_i$ for all i, j. Hence, from the trade-balance conditions, condition (25) is equivalent to

$$\frac{w_i L_i}{\sum w_j L_j} \geq \frac{R_i/R_T}{1 + \delta_S/g_L}$$

for all i. Using the result for relative wages above, this can be rewritten as

$$\frac{\phi_i/\phi_T}{\widetilde{\eta}_i/\widetilde{\eta}_T} \leq 1 + \delta_S/g_L \tag{26}$$

where $\widetilde{\eta}_i \equiv \eta_i^{\gamma(1-\beta)/\beta}$, and $\widetilde{\eta}_T \equiv (1/L_T) \sum \widetilde{\eta}_j L_j$.

The inequality (26) ensures that - given the EC condition - every country has some resources left over for producing global goods. Focusing first on the RHS of this inequality, note that the condition is relaxed as the rate of diffusion of specific ideas δ_S increases. This is because a higher δ_S implies that a lower share of goods are national goods. Turning to the LHS of (26), note that given $\widetilde{\eta}_i/\widetilde{\eta}_T$ this condition sets an upper bound on ϕ_i/ϕ_T: if this ratio were too high for some country then that country would not be able to satisfy the world demand for its national goods given the EC condition, and the equilibrium could not take the form that I have postulated here. The same problem occurs if $\widetilde{\eta}_i/\widetilde{\eta}_T$ is too low for some country, because this would imply a low share of world income for that country, and given ϕ_i/ϕ_T this would violate condition (26).[27]

Now, since $\widetilde{\eta}_i$ is a function of $R_i = \phi_i L_i$ and $R_T = \phi_T L_T$, then - given population levels L_i - condition (26) is a condition on the vector $(\phi_1, \phi_2, \ldots, \phi_n)$. Given the calibrated values of γ and β, it can be shown that the LHS of (26) is increasing in ϕ_i/ϕ_T, so this condition effectively requires the $\phi' s$ to be not too different.

To recapitulate, if condition (26) is satisfied, then in equilibrium each country i produces and exports everywhere all its national goods and a share of global goods, and there is factor price equalization up to differences in countries' stocks of general ideas, η_i (i.e., wages satisfy (19)). It is interesting to note that in the case of $\eta_i = \eta_j$ for all i, j (perhaps because of infinite diffusion of general ideas across countries) diffusion allows countries with (not too) different research intensities to replicate an integrated economy. Also, note that the price of the composite intermediate good relative to the wage in each country satisfies (20) so the rate of growth is again given by (17).

It is now possible to explore the gains from trade and openness in this special case of varying research intensities and frictionless trade (under condition (26)). Even though there is trade in global goods, all the other properties of the equilibrium here and in autarky are as in the case considered in the previous section. Thus, the results for GT, GTS, and GG are as given in the previous section. The only difference is that now $r_i = R_i/R_T = (\phi_i/\phi_T)(L_i/L_T)$ is also affected by differences in ϕ_i. So now we can see that countries that have lower research intensities gain more from trade and diffusion. Klenow and Rodríguez-Clare (2005) had a similar decomposition for the total gains from openness in the case of countries with varying research intensities. They noted that this result implies that countries with high research intensities have less to gain from openness, and they referred to this as the "Silicon Valley" effect.

[27] One way to understand this is by looking at the trade balance condition (3): if $\widetilde{\eta}_i$ falls then w_i and hence total income $w_i L_i$ in country i falls. To restore trade balance, this requires a decline in the share of global goods produced by country i, and this makes condition (26) less likely to be satisfied.

4 Knowledge Spillovers

As mentioned in the Introduction, an alternative way to have the model match the observed growth rate, instead of the arrival and diffusion of general ideas, is the existence of knowledge spillovers in the *generation* of ideas. Formally, assume that instead of $\dot{\lambda}_i = R_i$ we have

$$\dot{\lambda}_i = \left(\lambda_i^a \lambda_W^{1-a}\right)^b R_i \tag{27}$$

where $a \in [0,1]$, $b \in [0,1[$ and $\lambda_W = \sum_i \lambda_i$. The parameter b measures the strength of knowledge spillovers, and the parameter a measures the importance of national as opposed to global spillovers. In steady state $g_\lambda = \dot{\lambda}_i/\lambda_i = \dot{\lambda}_W/\lambda_W = g_L/(1-b)$, and the growth rate is now

$$g = \theta\left(\frac{1-\alpha}{\beta}\right)\left(\frac{g_L}{1-b}\right) \tag{28}$$

Given $\theta = 0.22$, $\alpha = 0.75$, $\beta = 0.5$, and $g_L = 0.048$ then we need $b = 0.65$ to match $g = 1.5\%$. Note that here I still assume that there is diffusion of specific ideas, since this proved to be useful to improve the model's match with the trade data. The only change from the model above is that instead of introducing general ideas, I allow for knowledge spillovers to have the model match the observed growth rate.

The gains from knowledge spillovers are easy to calculate for the case of n identical countries. Letting L be the population level in each country, then it can be shown that $\lambda_i = \zeta n^{(1-a)b/(1-b)} L^{1/(1-b)}$ where ζ is some positive constant. There are now three sources of gains from openness: trade, diffusion of (specific) ideas and knowledge spillovers.

Consider first the gains from trade and diffusion of ideas. The only difference with the analysis of the previous subsection is that now $\lambda_T = \delta_S n\lambda/g_\lambda$. Thus, the formula for GT_i in 21 now has to use $g_\lambda = 0.14$ rather than $g_L = 0.048$. In the case of $n = 100$, then $GT = 35\%$ rather than 23%. The gains from trade and diffusion together remain the same as before (as in (22) but with n instead of r_i).

The gains from openness due to knowledge spillovers are associated with the increase in λ from isolation to openness, or $n^{[(1-a)b/(1-b)]\theta(1-\alpha)/\beta}$.[28] These gains depend on the value of a. In

[28] Note that if we shut down knowledge spillovers in one country, but still have diffusion, then the country would have a low λ_i/L_i and hence would specialize in exporting goods produced with global technologies, which would change the trading equilibrium we characterized before. To keep the analysis simple, I am assuming for this exercise that all countries shut down knowledge spillovers simultaneously. This implies that the gains from trade and diffusion of specific ideas remain as in the previous subsections.

the case of $a = 0$, so that knowledge spillovers are global, then the gains from openness are 156%. This is equal to the gains from the diffusion of general ideas when $\delta_G, \delta'_G \to \infty$. The case of $a = 1$ (i.e., no international knowledge spillovers) does not seem reasonable, not only because in reality researchers can clearly use global knowledge, but also because this would generate national size effects that are not consistent with the data (see Rose, 2006).

One way to think about $a > 0$ in (27) is that - controlling for the ratio of national to foreign ideas - researchers are more likely to use national than foreign ideas. In other words, there is a "home bias" in the use of previous ideas. This suggests that data on international patent citations may provide a way to estimate a. Jaffe and Trajtenberg (1999) use such data to explore the importance of geography for the spread of knowledge in research. They find, for example, that controlling for the field of research and other factors, researchers in the U.S. are 71% as likely to cite a British patent as they are to cite a U.S. patent. They estimate this home bias in patent citations among the five major innovating countries, namely the U.S., the U.K, France, Germany and Japan.

A rough mapping between this paper and their findings can be done as follows. Imagine that research is composed of many processes, each of which uses previous ideas. Each process can either rely on national ideas or the worldwide stock of ideas. The parameter a is the probability that a process is restricted to using only national ideas. The home bias in the use of previous knowledge can then be captured by the ratio of national to foreign ideas used in research relative to the ratio of the stocks of national to foreign ideas. For country i the *inverse* of this home bias is given by

$$\tau_i = \left(\frac{a + (1-a)\lambda_i / \sum \lambda_j}{(1-a)\sum_{j \neq i} \lambda_j / \sum \lambda_j} \right) \left(\frac{\sum_{j \neq i} \lambda_j}{\lambda_i} \right) = \frac{a + (1-a)\lambda_i / \sum \lambda_j}{(1-a)\lambda_i / \sum \lambda_j}$$

Note that if $a = 1$ (i.e., no home bias) then $\tau_i = 1$ for all i, and also that τ_i declines as a increases.

Consider first the case of the U.S. Jaffe and Trajtenberg note that this country holds approximately 65% of the total number of patents granted in the period of their analysis (1963 - 1993), and that $\tau_{US} = 0.65$.[29] Plugging these numbers into the equation above and solving for a yields $a = 1/4$. Following the same procedure for the other four countries yields values of a equal to 0.27 for Japan, 0.07 for the U.K., and 0.08 for both France and Germany.

[29] This is an average of the home bias in the U.S. relative to the other four countries. The actual numbers are 71% for U.K., 60% for France, 55% for Germany, and 72% for Japan.

Using $a = 1/4$ together with the other parameters calibrated above implies that for a country with 1% of the world's population the gains from knowledge spillovers are 2.02, or 102%. Since GTS is not affected by knowledge spillovers, the gains from openness are now 235%. This is a bit lower than with diffusion of general ideas. If I use $a = 0.08$ instead, the gains from knowledge spillovers are 2.38, or 138%, which is much closer to the gains from diffusion of general ideas.

5 Conclusion

If the parameter θ is calibrated to the trade data, as in Eaton and Kortum (2002), then the growth rate implied by the Eaton and Kortum's (2001) model of trade and growth turns out to be significantly lower than the one observed in the data. Introducing diffusion of specific ideas leads to an even better fit with the bilateral trade data and a higher estimate of θ. But the implied growth rate is still too low. One of two additional extensions of the model allows the model to match the observed growth rate: first, introducing progress and diffusion in general ideas; and second, introducing knowledge spillovers. In both cases, the resulting quantitative model implies that the gains from openness are large, but trade contributes with only a small share of these gains. Thus, diffusion of ideas is *much* more important than trade in accounting for the gains from openness.

One concern is that the analysis implicitly assumes that the flow of ideas is independent of the volume of trade. Perhaps trade has a much larger role precisely through a positive effect on diffusion. This could happen through several channels, such as flows of ideas arising from the interaction between nationals and foreigners, the competitive pressure from imports inducing domestic firms to engage in faster technology adoption, or complementarities between foreign technologies and foreign inputs.[30] There is a large empirical literature exploring the significance of these and similar mechanisms through which trade may induce productivity growth in domestic firms.[31] This seems like an important topic for future research.

One limitation of the model concerns the way in which diffusion is captured. There are at least three specific tasks ahead. First, as mentioned above, assuming that diffusion entails national ideas becoming global is clearly unrealistic. It seems important to model diffusion

[30] We tend to see countries that are closed to trade also being closed to foreign ideas; think of North Korea. But this empirical association between trade and diffusion does not imply any causal role for trade in accelerating diffusion.

[31] See for instance Bernard and Jensen (1999), and Hallward-Driemeier et. al. (2002), and Tybout (2003).

according to the qualitative and hopefully quantitative features found in the data (see for example Jaffe and Trajtenberg, 1999, and Comin et. al. 2006) and explore the implications of this for growth and trade.

Second, FDI is surely one mechanism through which diffusion takes place. Ramondo (2006) has already shown a way to model FDI within the Eaton and Kortum framework when there is no trade. The next step is to have a model with trade, FDI and (pure) diffusion and quantify the gains from each of these channels separately. Including migration into such a model would be another worthwhile step.

Finally, this paper has ignored the importance of differences across countries in technology adoption. One conjecture that would be interesting to explore is that countries with lower rates of technology adoption have less to gain from diffusion and more to gain from trade, with lower overall gains from openness.

Another limitation of the model is the assumption that research efforts are exogenous. How would the results change if this assumption were relaxed? In the simplest model with no diffusion of specific ideas, as in Eaton and Kortum (2001), trade does not affect countries' research intensity (i.e., the share of the labor force devoted to research). The reason for this is the standard one that although trade expands the market for ideas, it also increases competition, and these two effects exactly balance out. This implies that, to a first approximation, the gains from trade would not be affected by having endogenous research efforts. Something similar happens with the gains from diffusion. To see this, consider the extreme case in which diffusion is instantaneous (no frictions to the international diffusion of ideas, i.e. $\delta_S, \delta_G, \delta_{G'} \to \infty$) and research productivities are equal across countries (i.e., $\phi_i = \phi_j$ all i, j). Shutting down diffusion would imply larger returns to ideas in the home market, but would prevent the exploitation of ideas in foreign markets. As shown in Eaton and Kortum (2006), these two effects exactly cancel out, so diffusion has no effect on innovation. This discussion suggests that the results obtained here with exogenous research efforts would not be affected significantly by the extension to endogenous research. Still, a thorough analysis of this issue seems worthwile, and is left for future research.

Appendix

This Appendix proves that if $\phi_i = \phi_j$ for all i, j then the equilibrium characterized by (12) and (3) with $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ and (13) is consistent with the NTG condition. The following condition is both necessary and sufficient for the NTG condition to hold:

$$c_i \leq c_j/k_{ij} \text{ for all } i, j \tag{29}$$

The following Lemma is sufficient to prove the claim:

Lemma 1 *Assume that* $\phi_i = \phi$ *for all* i*. If* $p_m = (p_{m1}, p_{m2}, ..., p_{mn})$ *and* $w = (w_1, w_2, ..., w_n)$ *satisfy* (12) *and* (3) *with* $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ *and* (13)*, then* (29) *is satisfied.*

The rest of the Appendix proves this Lemma. Letting $\mu \equiv (BC_S)^{-1/\theta}$, then it is easy to show that

$$D_{li} = \begin{cases} \mu c_i^{-1/\theta} p_{ml}^{1/\theta} k_{li}^{1/\theta} \lambda_i & \text{for } l \neq i \\ \mu c_i^{-1/\theta} p_{mi}^{1/\theta} (\lambda_i + \lambda_T) & \text{otherwise} \end{cases}$$

Since ϕ is the same across countries, I can use $\phi = g_L$ without no loss of generality, so $\lambda_i = L_i$ for all i. Then the trade balance conditions are $\sum \lambda_l w_l D_{li} = \lambda_i w_i$, which can be expressed as

$$\sum_{l \neq i} \mu c_i^{-1/\theta} p_{ml}^{1/\theta} k_{li}^{1/\theta} \lambda_i \lambda_l w_l + \mu c_i^{-1/\theta} p_{mi}^{1/\theta} (\lambda_i + \lambda_T) \lambda_i w_i = \lambda_i w_i$$

Dividing by λ_i and multiplying by $c_i^{1/\theta}/\mu$ this becomes

$$c_i^{1/\theta} w_i/\mu = \sum_{l \neq i} p_{ml}^{1/\theta} \lambda_l w_l k_{li}^{1/\theta} + p_{mi}^{1/\theta} (\lambda_i + \lambda_T) w_i$$

Moreover, given (12) then

$$p_{mi}^{-1/\theta}/\mu = \sum_{l \neq i}^{n} c_l^{-1/\theta} k_{il}^{1/\theta} \lambda_l + c_i^{-1/\theta} (\lambda_i + \lambda_T)$$

Letting $b \equiv 1/\theta$, $a_{il} \equiv k_{il}^b$, and $z_i \equiv (p_{mi}/c_i)^b \lambda_i$, and then using $a_{il} = a_{li}$, these two equations can be transformed into:

$$\begin{aligned} 1/\mu &= \sum_{l \neq i} a_{li} z_l (w_l/w_i)(c_l/c_i)^b + z_i(1 + \lambda_T/\lambda_i) \\ 1/\mu &= \sum_{l \neq i}^{n} a_{li} z_l (p_{mi}/p_{ml})^b + z_i(1 + \lambda_T/\lambda_i) \end{aligned}$$

Both of these equations have the same structure, namely

$$1/\sigma = \sum_{l \neq i} a_{li} z_l x_l + z_i(1 + \lambda_T/\lambda_i)$$

This can be seen as a system of n linear equations in n unknowns, x_i for $i = 1, ..., n$, with z_i for $i = 1, ..., n$ as constants. In matrix notation, this is $Ax' = d'$, where x' is the transposed $x = (x_1, x_2, ..., x_n)$ vector and d' is the transposed $d = (d_1, d_2, ..., d_n)$, where $d_i = 1/\mu - z_i(1+\lambda_S/\lambda_i)$, and where $A = A_1 * A_2$, and

$$A_1 = \begin{bmatrix} 0 & a_{21} & \dots & a_{n1} \\ a_{12} & 0 & \dots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & 0 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n \end{bmatrix}$$

This system has a unique solution iff $\det(A) = \det(A_1)\det(A_2) \neq 0$. Given that the $z's$ are non-zero, then $\det(A_2) \neq 0$. On the other hand, the set of values of a_{ij} such that $\det(A_1) = 0$ has dimension $n - 1$. Thus, without loss of generality we can say that A is invertible. This implies that the system $Ax' = d'$ has a unique solution and hence

$$(w_j/w_i)(c_j/c_i)^b = (p_{mi}/p_{mj})^b$$

Recall that $c_i = Bw_i^{\beta} p_{mi}^{1-\beta}$. Plugging this into the previous equation we get

$$\begin{aligned} (w_j/w_i)\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{w_i^{\beta} p_{mi}^{1-\beta}}\right)^b &= (p_{mi}/p_{mj})^b \\ (w_j/w_i)^{1+b\beta} &= (p_{mi}/p_{mj})^{b(2-\beta)} \\ w_j/w_i &= (p_{mi}/p_{mj})^{b(2-\beta)/(1+b\beta)} \end{aligned}$$

Plugging this back above we get

$$c_j/c_i = (w_j/w_i)^{\beta} (p_{mj}/p_{mi})^{1-\beta}$$

and hence

$$c_j/c_i = (p_{mi}/p_{mj})^{\gamma} \tag{*}$$

where

$$\gamma \equiv \beta - 1 + b\beta(2 - \beta)/(1 + b\beta)$$

(Note that this is not the same γ used for general ideas in the main text.) For future reference, note that

$$\begin{aligned}\gamma &= \frac{(\beta-1)(1+b\beta)+b\beta(2-\beta)}{1+b\beta} \\ &= \frac{\beta-1+b\beta^2-b\beta+2b\beta-b\beta^2}{1+b\beta} \\ &= \frac{\beta-1+b\beta}{1+b\beta}\end{aligned}$$

so $\gamma \in]1,1[$.

Now, note that $p_{mi} < p_{mj}/k_{ij}$ is equivalent to $a_{ij}p_{mj}^{-b} < p_{mi}^{-b}$, and

$$\begin{aligned}a_{ij}p_{mj}^{-b}/\mu &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + a_{ij}a_{ji}c_i^{-b}\lambda_i + a_{ij}c_j^{-b}(\lambda_j+\lambda_T) \\ &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + c_i^{-b}\lambda_i + a_{ij}c_j^{-b}(\lambda_j+\lambda_T)\end{aligned}$$

But from

$$p_{mi}^{-b}/\mu = \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l + c_i^{-b}(\lambda_i+\lambda_T)$$

we see that

$$c_i^{-b}\lambda_i = p_{mi}^{-b}/\mu - \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_T$$

Plugging this above we get

$$\begin{aligned}a_{ij}p_{mj}^{-b}/\mu &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + p_{mi}^{-b}/\mu - \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}(\lambda_j+\lambda_T) \\ &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + p_{mi}^{-b}/\mu - \sum_{l\neq i,j}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}\lambda_T\end{aligned}$$

and hence

$$a_{ij}p_{mj}^{-b}/\mu = p_{mi}^{-b}/\mu + \sum_{l\neq i,j}^{n}\left[a_{ij}a_{jl}-a_{il}\right]c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}\lambda_T \tag{30}$$

From the triangular inequality, $k_{il} \geq k_{ij}k_{jl}$ for all i,j,l, we have $a_{il} \geq a_{ij}a_{jl}$. Hence the second term on the RHS of the last line is non-positive. Since $k_{ij} \leq 1$ for all i,j, then this is also the case for the third term. Note also that if $k_{ij} < 1$ then the third term is strictly negative. Thus $a_{ij} p_{mj}^{-b} < p_{mi}^{-b}$ if $\lambda_T = 0$ or

$$c_i^{-b} > a_{ij} c_j^{-b}$$

This implies that

$$\begin{aligned} c_j/k_{ij} &> c_i \Longrightarrow p_{mi} < p_{mj}/k_{ij} \qquad (+) \\ p_{mi} &> p_{mj}/k_{ij} \Longrightarrow c_j/k_{ij} < c_i \end{aligned}$$

Imagine first that $\gamma = 0$. Then c_i/c_j for all i, j, and hence clearly $c_i \leq c_j/k_{ij}$ for all i, j.

Now consider the case $\gamma > 0$. There are two possibilities: $p_{mj} > p_{mi}$ or $p_{mj} < p_{mi}$. In the first case then from (*) we see that

$$c_i/c_j = (p_{mj}/p_{mi})^{\gamma} > 1$$

(thanks to $\gamma > 0$) which implies

$$c_i/k_{ji} > c_i > c_j$$

In the second case, $p_{mj} < p_{mi}$, then from (*) we see that $c_i < c_j < c_j/k_{ij}$. From (+) this implies that $p_{mi} < p_{mj}/k_{ij}$. But from (*) we see that

$$c_j/c_i = (p_{mi}/p_{mj})^{\gamma} < (1/k_{ij})^{\gamma} < 1/k_{ij}$$

where the second inequality follows from $\gamma > 0$ and the last inequality follows from $\gamma < 1$. Thus, we also get $c_i/k_{ij} > c_j$.

Finally, consider the case $\gamma < 0$. The following lemma will prove useful:

Lemma 2 *If* $\gamma < 0$ *and* $p_{mi} < p_{mj}/k_{ij}$ *for any* i, j *then* $c_i < c_j/k_{ij}$ *for any* i, j

Proof. $p_{mi} < p_{mj}/k_{ij}$ for any i, j is equivalent to $p_{mj} < p_{mi}/k_{ji}$ for any i, j. This implies

$$(p_{mj}/p_{mi})^{-\gamma} < (1/k_{ji})^{-\gamma}$$

Thus,

$$\begin{aligned} c_j/c_i &= (p_{mi}/p_{mj})^{\gamma} \\ &= (p_{mj}/p_{mi})^{-\gamma} < (1/k_{ji})^{-\gamma} < 1/k_{ji} \end{aligned}$$

where the last step follows from $1 > -\gamma > 0$. ∎

We now show that if $\gamma < 0$ then $c_i \leq c_j/k_{ij}$ for any i, j. From inequality 30 or $i \neq j$ we have

$$\mu^{-1}\left((k_{ij}/p_{mj})^b - (1/p_{mi})^b\right) < \lambda_T\left((k_{ij}/c_j)^b - (1/c_i)^b\right) \tag{31}$$

Assume that $\lambda_T = 0$. Then,

$$(k_{ij}/p_{mj})^b - (1/p_{mi})^b < 0 \iff p_{mi} < p_{mj}/k_{ij} \quad \text{for any } i, j$$

From Lemma 2 we obtain that $c_i < c_j/k_{ij}$, which is equivalent to $(k_{ij}/c_j)^b - (1/c_i)^b < 0$. Hence, we proved that for $\lambda_T = 0$: $(k_{ij}/p_{mj})^b - (1/p_{mi})^b < 0$ and $(k_{ij}/c_j)^b - (1/c_i)^b < 0$ for any i, j. From continuity there exists ε such that for $\lambda_T \in [0, \varepsilon)$: $(k_{ij}/p_{mj})^b - (1/p_{mi})^b < 0$ and $(k_{ij}/c_j)^b - (1/c_i)^b < 0$ for any $i \neq j$.

Suppose there exists $\lambda_T > 0$ such that $(k_{ij}/p_{mj})^b - (1/p_{mi})^b > 0$ or $(k_{ij}/c_j)^b - (1/c_i)^b > 0$ for some i and j. From continuity and (31) we know that there must be a value of λ_T, λ_T^*, such that $(k_{ij}/p_{mj})^b - (1/p_{mi})^b < 0$ and $(k_{ij}/c_j)^b - (1/c_i)^b = 0$ for some i and j. Thus, we have for some i and j

$$\begin{aligned} p_{mi} &< p_{mj}/k_{ij} \\ c_j/k_{ij} &= c_i \end{aligned}$$

This implies that

$$\begin{aligned} (p_{mi}/p_{mj})^\gamma &= \frac{c_j}{c_i} = k_{ij} < p_{mj}/p_{mi} \implies \\ (p_{mi})^{\gamma+1} &< (p_{mj})^{\gamma+1} \implies p_{mi} < p_{mj} \end{aligned}$$

where the last step follows from the fact that $\gamma > -1$. Since γ is negative then $p_{mi} < p_{mj}$ implies that $c_j/c_i > 1$, which then implies $k_{ij} > 1$, a contradiction. This establishes that λ_T^* does not exist, and hence for any $\lambda_T \geq 0$ we have

$$(k_{ij}/p_{mj})^b - (1/p_{mi})^b < 0, \quad (k_{ij}/c_j)^b - (1/c_i)^b < 0 \quad \text{for any } i \neq j.$$

Q.E.D.

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