The General Equilibrium Incidence of Environmental Mandates

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ABSTRACT

Regulations that restrict pollution by firms also affect decisions about use of labor and capital. They thus affect relative factor prices, total production, and output prices. For non-revenue-raising environmental mandates, what are the general equilibrium impacts on the wage, the return to capital, and relative output prices? Perhaps surprisingly, we cannot find any existing literature that even asks that question, in any model. This paper starts with the standard two-sector tax incidence model and modifies one sector to include pollution as a factor of production that can be a complement or substitute for labor or for capital. We then look not at taxes but at four types of mandates, and for each mandate determine conditions that place more of the burden on labor or on capital. Stricter regulation does not always place less burden on the factor that is a better substitute for pollution. Also, a restriction on the absolute amount of pollution creates scarcity rents, and it thus raises the dirty sector's output price by more than a relative restriction on pollution per unit of output. In some perverse cases that we identify, some of those policies might *reduce* the dirty sector's output price.

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Much literature compares the efficiency properties of environmental policies, generally finding that incentives like taxes or permits are more cost-effective than mandates – at least in the case where firms are heterogeneous and government cannot know how to tailor mandates to each firm. In contrast, the literature on the distributional effects of such policies is limited. Some papers identify the demographic characteristics or locations of households in jurisdictions that are differentially affected by environmental protection, while others look at the burdens on households that buy products made more expensive by environmental protection.[1] All of these papers ignore effects of environmental policies on the wage rate and the return to capital – both of which also affect real incomes. Yet, restrictive command and control (CAC) regulations can simultaneously affect both the product prices and factor prices.

Of course, the public economics literature since Harberger (1962) is replete with general equilibrium studies of the incidence of taxation. A few such papers look at the incidence of environmental taxes, where the question is about how the revenue burden is distributed. No such literature looks at mandates, perhaps implicitly because mandates do not have revenue whose burden can be distributed.[2] Yet CAC mandates clearly interfere with firms' decisions about use of labor, use of capital, the amount to produce, and the price to charge. We therefore find it surprising that we cannot find in the literature any general equilibrium model of the incidence of non-revenue-raising environmental regulations, with simultaneous effects on the uses side of income (product prices) and sources side of income (factor prices).

To begin such a literature, this paper starts with rudimentary models in the style of Harberger (1962), with two competitive sectors and constant returns to scale, but we add the important complication that the "dirty" sector uses three inputs to production: labor, capital, and pollution. Thus, any two of these inputs can be complements or substitutes. The "clean" sector uses only labor and capital, which are in fixed supply but perfectly mobile between sectors. We then solve models representing four types of non-revenue-raising policies: the handout of pollution permits, a restriction on the "absolute"

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[1] Examples of papers in the first category include Becker (2004) or Sieg et al (2005). Examples in the second category include Robison (1985), Metcalf (1999), or West and Williams (2004).

[2] The incidence of a pollution tax is studied by e.g. Bovenberg and Goulder (1997), Chua (2003), and Fullerton and Heutel (2004). The effect of a pollution mandate on factor prices is studied by Das (2004) in a trade model with fixed world output prices. We find both factor prices and output prices in a closed economy for several different kinds of mandates, but certainly results would be different in open economy.

quantity of pollution, a "relative" standard on pollution per unit of output, and a relative standard on pollution per unit of an input (such as capital).

A key to understanding results from these models is that the first two kinds of regulations restrict the amount of pollution, and so create scarcity rents. The rights to these scarcity rents may be fairly obvious: gains accrue to whatever entity is handed the permits or the rights to the restricted quantity of pollution. To cover the higher shadow price of pollution, however, the product price must rise more than in the other two cases. These policies also may reduce the return to either labor or capital, whichever is a relative complement to pollution or whichever is used intensively in the polluting sector. We start with those models and policies because they are fairly easy to understand.

The more interesting results here pertain to the "relative" standards. In one case, the firm can acquire more of the valuable pollution rights only if it produces more output, and so it may raise demands for either or both inputs (relative to the absolute quantity restriction). Production does not shrink as much, and the output price does not rise to cover the cost of paying for scarcity rents. In fact, output price can fall. This policy can easily distribute gains to some kinds of individuals, but it shifts production and imposes costs on others. The restriction on pollution per unit of capital implies that firms acquire pollution rights only by using more capital, which tends to raise the demand for capital and its return. But this policy also raises costs and reduces output, which tends to reduce demand for capital if the sector is capital intensive. Thus the return to capital can rise or fall, even if the dirty sector is capital intensive. Finally, if the return to capital falls and the dirty sector is capital intensive, then the overall cost of production can fall, so that a tighter environmental restriction reduces the price of the dirty good.

Thus, we see some standard principles of tax incidence at play, but some other effects specific to mandates are introduced. Next, Section 1 summarizes the importance of such mandates in actual policymaking and reviews the literature that has studied them. Section 2 then introduces the simplest version of our model, for pollution permits. Section 3 looks at absolute quantity standards, section 4 models a restriction on pollution per unit of output, and section 5 models pollution per unit input. Section 6 concludes.

1. Review of Environmental Mandates and Modeling

Especially in the earlier years of their existence, environmental regulations have most often been command-and-control (CAC) or technology mandates rather than incentives like pollution taxes or tradable permits. Those mandates, though, can take many forms for different industries.[3] The Clean Air Act Amendments (CAAA) set national ambient air quality standards, and jurisdictions that do not meet these standards are forced to create individual implementation plans. These plans often differ greatly from each other. Modeling the Clean Air Act as a single limit on emissions is difficult, except perhaps for the national emissions standards for new facilities under the New Source Performance Standards of the 1970 CAAA. Like the NPDES water standards these air pollution regulations are technology-based, that is, determined by the current state of abatement technology.

Systems of tradable emissions permits are becoming more popular, including the 1990 CAAA's national market for sulfur dioxide (SO_2) emissions, the market in the northeast for nitrogen oxide (NO_X) emissions, the South Coast Air Quality Management District permit system for SO_2 and NO_X emissions in the Los Angeles area launched in January 1994, and the seven states that have established emissions credit programs for NO_X and volatile organic compounds (VOC) since 1989 under the EPA's emissions trading program framework. These tradable emissions permits may achieve the same net level of environmental benefits as technology mandates, and perhaps more cheaply, but they have important distributional differences.

Finally, emissions standards may be relative rather than absolute. A policy may mandate the maximum emissions per unit output, or per unit of some input like a particular chemical or oil. These policies may be seen as more reasonable than an absolute limit per firm, especially when applied to firms of various sizes. A regulator would not expect a large firm to reach the same level of emissions as a small firm. By enacting a relative policy, the regulator can avoid deciding on a specific allocation of allowed emissions levels. Because of the variety of environmental mandates under different state implementation plans, it is difficult to pinpoint what policies have this relative form. In a 1982 survey of regulators administered by Resources for the Future, however, 97% of air pollution regulating agencies and 100% of water pollution regulating agencies said they use limits on emissions per unit of some input, and 70% of

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[3] Historically, the Water Quality Act of 1965 was the first national policy to allow states to determine maximum discharge limits for various pollutants and to allocate (nontradable) permits to polluters to reach that goal. The National Pollution Discharge Emissions System (NPDES) of the Federal Water Pollution Control Act of 1972 imposes effluent standards on water emissions, and it gives polluters the freedom to choose the technology they want to use to achieve those standards.

air and 50% of water agencies said they use limits on emissions per unit output (Russell et al 1986, p. 19). Such large proportions suggest modeling some environmental policies as limits on ratios of pollution to output or to an input.

Current federal environmental regulations are quite complex. The Code of Federal Regulations Title 40 lists hundreds of rules that apply to various emitters and industries, and most states have their own sets of regulations. Some mandates are described in terms of emissions per unit output, such as the VOC standard for automobile refinish coatings that is stated in terms of grams per liter of coating. For new plants that produce sulfuric acid, the emissions standard for SO_2 is 2 kg per metric ton of acid produced.[4] The Texas Commission for Environmental Quality sets standards for municipal hazardous waste generators that are based on the amount of output produced.[5] The state of New York sets limits on fluoride emissions per unit output from aluminum reduction plants.[6] Mandates also take the form of emissions per unit of some input. The federal standard for particulate matter emissions for fossil-fuel-fired steam generators is 43 nanograms per joule of heat input derived from fossil fuel or wood residue.[7] Other standards are stated in terms of emissions per unit heat input in Texas for electric generators and solid fossil-fuel fired steam generators.[8] Finally, even for a particular industry, emissions standards can differ based on the technology employed. Phosphoric acid manufacturing plants, for example, face standards for fluorides and particulate matter that depend on the production technology of the plant.[9] Emissions rates for iron and steel processes in New York depend on the technology.[10] Limits on SO_2 emissions for oil and gas producers in Texas are 25 tons/year *per facility*.[11] Standards per facility also apply in New York for petroleum refineries.[12]

We cannot incorporate all of these different types of mandates in a single model with clear analytical results. We can, however, model a few types of mandates and compare results, to see their differential impacts on the distribution of costs. For

[4] Code of Federal Regulations, Title 40, Chapter 1, §60.82.
[5] Texas Administrative Code, Title 30, Section 1, §335.69.
[6] New York Environmental Conservation Rules and Regulations §209.2.
[7] Code of Federal Regulations, Title 40, Chapter 1, §60.42.
[8] Texas Administrative Code, Title 30, Section 1, §117.105, and Ibid, §112.8.
[9] Code of Federal Regulations, Title 40, Chapter 1, §63.602.
[10] New York Environmental Conservation Rules and Regulations, §216.3.
[11] Texas Administrative Code, Title 30, Section 1, §106.352.
[12] New York Environmental Conservation Rules and Regulations §223.3.

example, we model technology mandates and per facility standards as limits on the amount of pollution per unit capital.

In the economics literature, environmental mandates are typically modeled as limits on the amount of pollution emitted, though most mandates actually take other forms.[13] The most exhaustive theoretical analysis of different types of environmental mandates is in Helfand (1991). Her model contains a single consumption good produced using a "dirty" input that causes pollution and a "clean" one that abates pollution. The various mandates considered are: a limit on emissions, a limit on output, an upper limit on the dirty input, a lower limit on the clean input, and limits on the ratio of emissions to output or the ratio of emissions to either of the inputs. By normalizing all of these types of standards so that they result in the same reduction in emissions, she can compare their effects on output produced, inputs used, and firm profits. For example, she finds that the restriction on output most reduces input and output levels. The restriction on pollution itself yields the highest firm profits. In most cases, however, the signs of these changes depend on the form of the production function. Some counterintuitive results are reached as well. For instance, a standard per unit output may actually increase total emissions; the same result may occur from a standard limiting total output. More recently, Jou (2004) compares absolute emissions standards with emissions/output standards and finds that the former leads to less pollution.[14]

While the Helfand paper provides a number of valuable insights regarding the differences between types of mandates, it does not speak to the question of incidence.[15] In fact, the input supply curves are horizontal, so no policy can have incidence on the sources side, even in a partial equilibrium model. In contrast, our general equilibrium

[13] Exceptions include Hochman and Zilberman (1978), who model standards as limits on emissions per unit output or per unit input. Harford and Karp (1983) compare the two policies and find that a standard per unit output is more efficient than a standard per unit input. Similarly, Thomas (1980) compares the welfare costs of different policies. Fullerton and Metcalf (2001) model a technology restriction as a limit per unit output. Fredriksson et al (2004) model environmental policy as a limit on the energy-capital ratio, citing the Corporate Average Fuel Economy Standards. None of these studies investigate distributional impacts.

[14] Also, Goulder et al (1999) compare efficiency effects of environmental policies in the presence of distortionary taxes. Aidt and Dutta (2004) develop a political economy model of the choice of policy and find that the increasing use of incentives follows from increasingly high environmental goals. See also Keohane et al (1998) for a similar model of policy choice. Montero (2002) compares effects on R&D incentives, while Requate and Unold (2003) compare incentives to adopt abatement technology. Bovenberg et al (2005) look at how the efficiency costs of mandates and taxes are affected by a constraint to avoid adverse industry-distributional effects.

[15] Helfand and House (1995) empirically estimate the costs of different environmental policies for lettuce growers in California's Salinas Valley. They find that mandates reduce farm profits less than do taxes.

model allows for endogenous input prices as well as output prices. Furthermore, the two inputs in Helfand's model are a clean and dirty input. Even as these two input prices change, the implications are unclear for returns to labor and capital. Similarly, while Jou's model solves for the impact of policy on capital investment, the wage rate is set exogenously. Here, we model production as using capital, labor, and pollution. All three inputs have endogenous prices, so we can capture the differential effects of environmental standards on the relative returns to labor and capital. Which factors gain or lose can have a large effect on what policies are chosen (Keohane et al 1998).

2. Tradable Pollution Permits

The distributional effects of a tradable permit policy depend on how those permits are allocated. The permits impose costs by forcing firms to reduce emissions or to buy permits. The mandated overall limit on pollution creates scarcity rents, however, and the distribution of those rents must be considered as part of the incidence.[16] If the permits are grandfathered to the firms, then their owners capture those scarcity rents by not having to pay for those emissions. If permits are auctioned, then the government captures those rents and can use the funds in various ways. In addition to evaluating changes in returns to capital and labor, our model solves for changes in permit-created scarcity rents. All three of these price changes contribute to the sources-side incidence of the policy.

Our model is similar to that in Fullerton and Heutel (2004), where we analyze only a tax on emissions. Here, we model a variety of mandates, but we start with an absolute quantity limit because it is most analogous to the tax on emissions: with many identical firms and no uncertainty, the model of firm decision making is the same whether the firm faces a tax or a permit price per unit of emissions.

Our closed economy consists of two competitive sectors, one that produces a clean good (X) and one that produces a dirty good (Y). Output prices are p_X and p_Y, respectively. The clean sector uses only capital and labor in production (K_X and L_X); the dirty sector uses capital, labor, and pollution (K_Y, L_Y, and Z). As in the Harberger (1962) model, capital and labor are perfectly mobile and are available in fixed total supply. Totally differentiating the resource constraints gives:

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[16] In a partial equilibrium model, Parry (2004) estimates the distribution of scarcity rents created by emissions permits for carbon, SO_X and NO_X.

$$\hat{K}_X \lambda_{KX} + \hat{K}_Y \lambda_{KY} = 0 \tag{1}$$

$$\hat{L}_X \lambda_{LX} + \hat{L}_Y \lambda_{LY} = 0 \tag{2}$$

where a hat over a variable represents a proportional change (e.g. $\hat{K}_X \equiv dK_X / K_X$). The λ_{ij} parameter represents sector j's share of input i (e.g. $\lambda_{KX} \equiv K_X / \overline{K}$, where $\overline{K}$ is the total capital available in the economy).

The clean sector's production decision can be characterized by σ_X, the elasticity of substitution in production between capital and labor:

$$\hat{K}_X - \hat{L}_X = \sigma_X (\hat{w} - \hat{r}) \tag{3}$$

where w and r are the returns to labor and capital, respectively. The choice of inputs in the dirty sector can be modeled using input demand equations for each of the three inputs (capital, labor, and pollution). While economy-wide pollution is set exogenously by the total number of permits, each individual firm chooses its own level of Z based on the market permit price it faces, p_Z. We differentiate the three input demand equations and use the fact that only two of the three are independent to get:

$$\hat{K}_Y = a_{KK} \hat{r} + a_{KL} \hat{w} + a_{KZ} \hat{p}_Z + \hat{Y} \tag{4}$$

$$\hat{L}_Y = a_{LK} \hat{r} + a_{LL} \hat{w} + a_{LZ} \hat{p}_Z + \hat{Y} \tag{5}$$

where a_{ij} is the elasticity of demand for factor i with respect to the price of factor j. Allen (1938) shows that $a_{ij} = \theta_{Yi} e_{ij}$, where θ_{Yi} is the share of production for factor i in sector Y (e.g. $\theta_{YK} \equiv \frac{rK_Y}{p_Y Y}$), and e_{ij} is the Allen elasticity of substitution between inputs i and j.[17] The assumptions of perfect competition and constant returns to scale production yield the following equations:

$$\hat{p}_X + \hat{X} = \theta_{XK} (\hat{r} + \hat{K}_X) + \theta_{XL} (\hat{w} + \hat{L}_X) \tag{6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK} (\hat{r} + \hat{K}_Y) + \theta_{YL} (\hat{w} + \hat{L}_Y) + \theta_{YZ} (\hat{p}_Z + \hat{Z}) \tag{7}$$

$$\hat{X} = \theta_{XK} \hat{K}_X + \theta_{XL} \hat{L}_X \tag{8}$$

[17] See also Mieszkowski (1972) for an example of the same methodology.

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z} \;. \tag{9}$$

In these equations, we suppose an existing permit restriction, so that the initial price p_Z is positive and pollution Z is finite. We then suppose an exogenous decrease in the number of emissions permits, $\hat{Z} < 0$.

Finally, consumer preferences are modeled using σ_u, the elasticity of substitution in consumption between the clean and dirty goods:

$$\hat{X} - \hat{Y} = \sigma_u(\hat{p}_Y - \hat{p}_X) \tag{10}$$

The clean good is chosen as numeraire, so $\hat{p}_X$ is fixed at zero, and we have ten equations for the ten unknown changes: $\hat{K}_X, \hat{K}_Y, \hat{L}_X, \hat{L}_Y, \hat{w}, \hat{r}, \hat{X}, \hat{p}_Y, \hat{Y}, \hat{p}_Z$. Thus, the system can be solved by successive substitution. The steps are omitted but may be requested from the authors. While the model can be used to solve for all ten endogenous variables, we report here only the solutions for $\hat{r}$, $\hat{w}$, $\hat{p}_Z$, and $\hat{p}_Y$. The first three of these determine the sources-side incidence of the policy, and the last determines the uses-side incidence. Because X is produced with no excess profit using only labor and capital, and its output price is fixed by assumption, r and w cannot both move in the same direction. If $\hat{r} = \hat{w} = 0$, the implication is not that factors bear no burdens. Rather, since p_Y may rise, $\hat{r} = \hat{w} = 0$ means that labor and capital bear burdens in proportion to their shares of national income. Hence, a positive value for $\hat{r}$ just means that capital bears less of the burden than does labor.

The general solutions are presented in Table 1, but some are difficult to interpret. In the factor price equations, we can see effects first identified by Mieszkowski (1967). The first term in the curly brackets is his "output effect": assuming the denominator D is positive, the policy $\hat{Z} < 0$ raises the cost of production and thus reduces output in a way that depends on consumer preferences σ_u. Then if Y is capital intensive, $(\gamma_K - \gamma_L) > 0$, the output effect reduces r and raises w. The other terms represent a "substitution effect": they involve the Allen elasticities, e_{KZ} and e_{LZ}, which determine whether labor or capital is a better substitute for pollution. However, the denominator cannot be signed without significant restrictions. Moreover, the equations for p_Z and p_Y seem quite cumbersome. To make the interpretations clearer and to see the importance of various effects, we consider two special cases: equal factor intensities (to isolate the substitution effect), and no substitution in the dirty sector (to see the output effect).

Table 1: Incidence of Absolute Quantity Restrictions

$$\hat{r} = \frac{\theta_{YZ}\theta_{XL}}{D}\{\sigma_u(\gamma_K - \gamma_L) - e_{KZ}\gamma_K(1+\gamma_L) + e_{LZ}\gamma_L(1+\gamma_K)\}\hat{Z},$$

$$\hat{w} = \frac{\theta_{YZ}\theta_{XK}}{D}\{-\sigma_u(\gamma_K - \gamma_L) + e_{KZ}\gamma_K(1+\gamma_L) - e_{LZ}\gamma_L(1+\gamma_K)\}\hat{Z},$$

$$\hat{p}_Z = F^{-1}\{(\frac{1}{D})(G\gamma_L(1+\gamma_K) - F\gamma_K(1+\gamma_L))(\frac{\sigma_X}{A}(C+\beta_L) + \theta_{YL}\theta_{XK}(e_{KL} - e_{KK})) - \frac{\gamma_L(1+\gamma_K)}{A}\}\hat{Z}$$

$$\hat{p}_Y = \{[G\gamma_L(1+\gamma_K) - F\gamma_K(1+\gamma_L)][\frac{\theta_{YZ}}{D}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}) + \frac{\sigma_X(C+\beta_L) + A\theta_{YL}\theta_{XK}(e_{KL} - e_{KK})}{AFD}] - \frac{\theta_{YZ}\gamma_L(1+\gamma_K)}{FA}\}\hat{Z}$$

where: $\gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}} = \frac{L_Y}{L_X} > 0$, $\gamma_K \equiv \frac{\lambda_{KY}}{\lambda_{KX}} = \frac{K_Y}{K_X} > 0$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK} > 0$,

$\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL} > 0$, $A \equiv \gamma_L\beta_K + \gamma_K\beta_L > 0$, $C \equiv \beta_K\theta_{YL} - \beta_L\theta_{YK}$,

$F \equiv \sigma_u(\frac{\gamma_L(1-\theta_{YK}) - \gamma_K\theta_{YL}}{A}) + e_{KZ}$, $G \equiv \sigma_u(\frac{\gamma_K(1-\theta_{YL}) - \gamma_L\theta_{YK}}{A}) + e_{LZ}$, and

$$D \equiv \sigma_u[\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}][G(A - \gamma_L(1+\gamma_K)) - F(A - \gamma_K(1+\gamma_L))] + \sigma_X[G(C+\beta_L) + F(\beta_K - C)] + A\theta_{XL}\theta_{YK}[Fe_{KL} - Ge_{KK}] - A\theta_{YL}\theta_{XK}[Fe_{LL} - Ge_{KL}]$$

2.1 Equal Factor Intensities

The assumption of equal factor intensities means that γ_L and γ_K are equal to each other. Let their common value be γ, and note that this condition implies that $L_Y/L_X = K_Y/K_X$. The output effect then disappears, and the substitution effect simplifies. For this permit policy, we then have:

$$\hat{r} = -\frac{\theta_{YZ}\theta_{XL}}{D}\gamma(1+\gamma)(e_{KZ} - e_{LZ})\hat{Z}$$

$$\hat{w} = \frac{\theta_{YZ}\theta_{XK}}{D}\gamma(1+\gamma)(e_{KZ} - e_{LZ})\hat{Z}$$

$$\hat{p}_Z = \frac{1}{F}\{-\frac{\gamma(1+\gamma)}{D}(e_{KZ} - e_{LZ})[\frac{\sigma_X \beta_L}{A} + \theta_{YL}\theta_{XK}(e_{KL} - e_{KK})] - \frac{1+\gamma}{\theta_{YL} + \theta_{YK} + \gamma}\}\hat{Z}$$

$$\hat{p}_Y = \{-\gamma(1+\gamma)[e_{KZ} - e_{LZ}][\frac{\sigma_X \beta_L - A\theta_{YL}\theta_{XK}(e_{KL} - e_{KK})}{AFD}] - \frac{\theta_{YZ}\gamma(1+\gamma)}{FA}\}\hat{Z}$$

and the denominator D in the general solution reduces to

$$D = \sigma_X[\theta_{YZ}\sigma_u + (\theta_{XL}\gamma + \theta_{YL})e_{LZ} + (\theta_{XK}\gamma + \theta_{YK})e_{KZ}] \\ + A[\theta_{XL}\theta_{YK}(Fe_{KL} - Ge_{KK}) - \theta_{YL}\theta_{XK}(Fe_{LL} - Ge_{KL})]$$

Under some conditions, this D can be signed. We know that σ_u and σ_X must be positive, and that e_{ii} must be negative for all i. From the conditions derived by Allen (1938), at most one of the cross-price elasticities e_{ij} can be negative. Suppose that all three of these cross-price elasticities are positive, that is, suppose that all three of the inputs in the dirty sector are substitutes. Then, the constants F and G are strictly positive, and the entirety of D is positive. In this case, we reach a definitive conclusion about the effect of the regulation on r and w. The signs of these price changes depend on the sign of $(e_{KZ} - e_{LZ})$. When emissions must be reduced, the dirty sector wants to substitute into both labor and capital, but if labor is a better substitute for pollution ($e_{LZ} > e_{KZ}$), then labor is hurt relatively less by the policy (i.e. $\hat{r} < 0$ and $\hat{w} > 0$).

Surprisingly, this simple intuition cannot be applied to the effect of the policy on the permit price. In the equation for $\hat{p}_Z$, the final term inside the curly brackets is unambiguously positive and could be called a "direct effect": it reflects a downward-sloping demand curve for emissions permits, so the leftward shift of the vertical supply curve tends to raise the equilibrium permit price.[18] Then the long first term could be called the "indirect effect," but it need not be positive. The sign of this term depends on the sign of $e_{KZ} - e_{LZ}$, and so it is clearly related to factor substitution. If this whole term is negative, then it offsets part of the direct effect on the permit price. If it is sufficiently negative, then a decrease in the total permit allocation may actually *decrease* the permit price. The conditions under which this counterintuitive effect occurs are, like the general solutions presented earlier, quite cumbersome and difficult to interpret, and hence they

[18] This final ratio is greater than one, so the direct effect tends to raise p_Z by *more* than the reduction in Z, but the whole term in curly brackets is divided by F (which may be more or less than one).

are not presented here. Yet the effect is analogous to the earlier finding that an increase in the pollution tax can lead to an *increase* in emissions.[19]

Yet, unlike the incidence on labor and capital owners, the incidence on permit holders is not determined solely by the change in their factor price. Labor and capital are in fixed total supply and earn net returns determined by w and r, but the supply of permits has just been restricted by the policy ($\hat{Z} < 0$). The total return to permit holders is $p_Z \cdot Z$, and the proportional change in this product is $\hat{p}_Z + \hat{Z}$. Even if the policy raises the price p_Z, then permit holders are still not necessarily better off.[20]

Furthermore, even the uses-side incidence result $(\hat{p}_Y)$ is ambiguous. The final term in the curly brackets of this expression is a direct effect on the cost of production. This positive term is subtracted, indicating that a decrease in the number of permits allotted tends to increase the price of the dirty good relative to that of the clean good. However, the previous term is an "indirect effect" that cannot be signed. It allows for the possibility of another counter-intuitive result: reducing the number of emissions permits may hurt consumers of the clean good more than consumers of the dirty good.

Similarly, even effects on r and w can be counterintuitive when D is negative. This can only occur, as we have seen, when at least one of the cross-price elasticities is negative. In fact, D is negative when the following inequality holds ("Condition 1"):

$$e_{KL} < \frac{-\sigma_X[\theta_{YZ}\sigma_u + (\theta_{XL}\gamma + \theta_{YL})e_{LZ} + (\theta_{XK}\gamma + \theta_{YK})e_{KZ}] - A\theta_{XL}\theta_{YK}(Ge_{KK} + Fe_{LL})}{A\theta_{XL}\theta_{YK}(F+G)}.$$

Since the constant on the right of the inequality is negative, Condition 1 says that e_{KL} is even more negative (K and L are sufficiently complementary). If this condition holds, then the intuitive results regarding the burden of the policy across labor and capital are reversed; if labor is a better substitute for pollution than is capital, then labor bears *more* of the burden of the policy. This can be explained in the following way. When forced to reduce emissions, and labor is the better substitute for emissions, firms tend to increase demand for labor relative to capital. However, the complementarity of labor and capital

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[19] See DeMooij and Bovenberg (1998) or Fullerton and Heutel (2004).

[20] Suppose the expression for $\hat{p}_Z$ is summarized by $\hat{p}_Z = H \cdot \hat{Z}$. Then we have $\hat{p}_Z + \hat{Z} = (H+1)\hat{Z}$. The policy benefits permit holders only if $H <$ -1. Since the direct effect in H is the subtraction of a ratio that exceeds one, then permit holders can lose if the indirect effect is both positive and large (depending on F).

means that the increased demand for labor leads to an increased demand for capital. If this complementarity is large enough, then the latter effect dominates the former, and the dirty sector's demand for capital increases even more than their demand for labor. Hence, the price of capital rises relative to the wage.

2.2 No Substitution in Dirty Sector

To see the impact of factor intensities on the incidence of this policy, we now let the factor intensities of the two sectors differ but assume away any ability of the dirty sector to substitute among its inputs. That is, we assume $e_{ij} = 0$ for all i, j. While this is clearly a restrictive assumption, it allows us to isolate the impact of the factor intensities. Under this assumption the denominator D in Table 1 simplifies to $\theta_{YZ}\sigma_X\sigma_u$, and the substitution effects in $\hat{r}$ and $\hat{w}$ disappear. The incidence results become:

$$\hat{r} = \frac{\theta_{XL}}{\sigma_X}(\gamma_K - \gamma_L)\hat{Z}$$

$$\hat{w} = -\frac{\theta_{XK}}{\sigma_X}(\gamma_K - \gamma_L)\hat{Z}$$

$$\hat{p}_Z = -\frac{1}{\sigma_u[\gamma_L(\theta_{YK} - 1) + \gamma_K\theta_{YL}]}\{\frac{(\gamma_K - \gamma_L)}{\theta_{YZ}}(C + \beta_L) - \gamma_L(1 + \gamma_K)\}\hat{Z}$$

$$\hat{p}_Y = \{(\gamma_K - \gamma_L)[-\frac{1}{\sigma_X}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}) - \frac{C + \beta_L}{AF\theta_{YZ}}] - \frac{\theta_{YZ}\gamma_L(1 + \gamma_K)}{FA}\}\hat{Z}$$

In this case, the sources side incidence includes only an output effect, determined by the sign of $\gamma_K - \gamma_L$. This expression was zero in the first case above, but here it is positive whenever the dirty sector is capital-intensive. Suppose this is the case, and suppose that a policy change reduces the allotment of emissions permits $(\hat{Z} < 0)$. Then the rental rate falls and the wage rises relative to the numeraire; capital bears a disproportionately high burden of the policy. For firms unable to substitute among inputs, a reduction in pollution permits forces the dirty industry to use less labor and capital in equal proportions. If the dirty industry is capital-intensive, then the decreased demand for capital exceeds the decreased demand for labor, and the rental rate falls. The magnitude of this effect is mediated by σ_X, the elasticity of substitution in production of the clean good. If the clean industry can easily substitute between capital and labor, then these effects on input prices become smaller, since the clean sector can more easily accommodate the additional labor or capital.

Again, as before, the effect on the permit price is not as intuitive. The constant in front has indeterminate sign, as does the first term inside the curly brackets. As in the case with equal factor intensities, a perverse result can occur where a decrease in the number of permits leads to an increase in the permit price.[21] The result for the uses-side incidence contains one term that can be signed and one that cannot. The second term in the curly brackets is an unambiguous "direct effect" on output price: a decrease in the allotment of permits increases the price of good Y relative to the numeraire. This direct effect may be offset by the other term in the expression, which cannot be signed.

In other words, the effect of factor intensities on relative input prices follows intuition, but effects on permit and output prices are more complicated. The ambiguities remain even though the assumption in this case eliminates the factor substitution effect.

3. Command and Control Restrictions on Absolute Pollution Quantities

We started with tradable pollution permits above, because the permit market is easy to comprehend with a vertical supply, a downward-sloping demand, and many identical firms that each can buy as many permits as desired at the equilibrium market price p_Z. Then all firms in the dirty industry have symmetric demands for the three inputs (K, L, Z) based on the three input prices (r, w, p_Z).

We next consider briefly the case where *each* firm faces a restriction on its use of Z. Pollution has no market clearing price p_Z, but each firm with a restriction on Z can be said to face a shadow price p_Z. Each firm gets an allocation of permits that are not tradable. In our model with many identical firms, however, the firms cannot gain from trade. With constant returns to scale, each firm's labor and capital can adjust to its allocation of nontradable permits in a way that is equivalent to the transfer of permits to some other firm using that same labor and capital. In other words, firm-specific restrictions on pollution levels in this model yield the same results as we just derived for tradable permits. Equations above can be used for effects on total dirty-industry use of labor and capital and for consequent economy-wide returns to labor and capital.

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[21] The last term is a "direct effect" for permits, as in Table 1. It reflects a standard downward-sloping demand for emissions, where a decrease in the permit allotment increases the permit price.

4. "Performance Standard": Emissions per unit Output

An alternative form of environmental policy is to limit the ratio of emissions to output, a policy we call a "performance standard". With heterogeneous firm sizes, at least some consideration of this ratio seems necessary for a plausible policy. A large producer cannot reasonably be expected to achieve the same limit on emissions as a small firm. Considerations like these are also taken into account in other policies, such as a fixed number of tradable permits that are initially allocated according to market share. If firm-specific emission limits are tied directly to the firm's output level, then the policy may have no absolute limit on total emissions. Instead, total emissions vary with total output in a way that affects incentives and prices.

We consider the same production functions as in the previous section. Total capital and labor are in fixed supply, as in equations (1) and (2). Likewise, production in the clean sector is unchanged, with equations (3), (6), and (8). Consumer preferences in equation (10) remain unchanged. The only change is to incentives facing firms in the dirty sector. The maximization problem for these firms is:

$$\max_{K_Y, L_Y, Z} \quad p_Y Y(K_Y, L_Y, Z) - rK_Y - wL_Y$$

subject to the constraint $Z/Y \leq \delta$. The firms pay no explicit price for the input Z. Instead, their use of that input is limited by their output. The constraint must bind, since the production function is monotone increasing in all inputs.[22] Solving the firms' first order conditions and rearranging terms yields $r = \dfrac{p_Y Y_K}{1-\delta Y_Z}$ and $w = \dfrac{p_Y Y_L}{1-\delta Y_Z}$, where subscripts on Y denote marginal products. The firm does not set the marginal value of an input equal to the input price, as it would without the performance standard, because of the denominator in these two equations. This denominator is less than one, so the marginal value of the factor is set lower than its input price. In other words, the firm wants to proceed further down its factor demand curves, using more labor and capital in order to increase output and qualify for an increase in valuable emission rights.

[22] Suppose that the constraint does not bind at the firm's optimum point of production. Firms could then increase use of Z without changing their use of K or L. Then output and revenue would increase, since the marginal product of Z is positive, with no change in costs. Therefore, the initial point was *not* an optimum, a contradiction.

Totally differentiating the production function and substituting in these equations yields the equation analogous to (9) in the previous model:

$$\hat{Y} = \theta_{YK}(1-\nu)\hat{K}_Y + \theta_{YL}(1-\nu)\hat{L}_Y + \nu\hat{Z}\,, \tag{9'}$$

where $\nu \equiv \delta Y_Z = Y_Z Z/Y$. In the prior model with (9), an increase in a factor would raise output in proportion to its factor share. Now, since the marginal product of each factor is reduced by $(1 - \nu)$, its marginal contribution to output is reduced by $(1 - \nu)$. An increase in emissions Z raises output in proportion to $\nu = \delta Y_Z$, to reflect its marginal product Y_Z and its factor share $\delta = Z/Y$. Emission rights are valuable, of course, but firms do not pay for them through an explicit price. Instead, they pay for emission rights by paying factors more than their marginal products.

The assumptions of perfect competition and free entry/exit lead to a zero profit condition in the previous model. This condition remains under the policy specified here, though it takes a different form. Since costs no longer include the price of emission permits, the final term in equation (7) is dropped. The zero profit condition thus implies

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y)\,. \tag{7'}$$

The constraint may impose a "shadow price" on the factor Z, but since no explicit price is paid for that input, it is not included in the profits equation.[23]

Finally, we must replace equations (4) and (5) with their counterparts under the new policy. Input demand equations can no longer be functions of output and three explicit input prices $(r,\ w,\ p_Z)$. Instead, we write input demand equations as functions of r, w, δ, and Y. Then totally differentiate these equations to get:

$$\hat{K}_Y = b_{KK}\hat{r} + b_{KL}\hat{w} + b_{KZ}\hat{\delta} + \hat{Y}$$

$$\hat{L}_Y = b_{LK}\hat{r} + b_{LL}\hat{w} + b_{LZ}\hat{\delta} + \hat{Y}$$

$$\hat{Z} = b_{ZK}\hat{r} + b_{ZL}\hat{w} + b_{ZZ}\hat{\delta} + \hat{Y}\,.$$

Notice that the b_{ij} variables appear in a form similar to the a_{ij} variables in equations (4) and (5). They both represent input demand elasticities. For example, either a_{KL} or b_{KL} is the percent change in capital for a one percent change in the wage. However, a_{KL} is

[23] This alters the dynamic of firm entry and exit, but since our concern is general equilibrium effects and not the transition periods leading up to them, it does not affect our results.

that response in the first model holding p_Z and Y constant (so Z can change), while b_{KL} is that response in the model holding δ and Y constant (so $Z = \delta Y$ cannot change).

The third equation giving the input demand for Z can be simplified greatly. We know that the constraint binds, so $Z = \delta Y$. Then total differentiation yields:

$$\hat{Z} = \hat{\delta} + \hat{Y}, \qquad (5')$$

which implies that $b_{ZK} = b_{ZL} = 0$, and $b_{ZZ} = 1$. Since only two of the three equations are independent, we subtract the second equation from the first and get

$$\hat{K}_Y - \hat{L}_Y = b_r \hat{r} + b_w \hat{w} + b_\delta \hat{\delta}, \qquad (4')$$

where $b_r = b_{KK} - b_{LK}$, $b_w = b_{KL} - b_{LL}$, and $b_\delta = b_{KZ} - b_{LZ}$. In Appendix A1, we calculate values of the b_{ij} elasticities. Although differences between the a_{ij} and b_{ij} parameters were just described, it helps to think of them analogously. The Appendix shows that b_{KK} and b_{LL} are negative, since increasing the price of a factor decreases its demand, even with the constraint on $\delta = Z/Y$. It also shows that the cross-price values b_{ij} are positive ($i,j = K,L$). This is true whether or not capital and labor are substitutes as defined by the sign of the Allen cross-price elasticity. That is, a higher price of labor means more capital demand. Why is complementarity ruled out in this case? The Allen elasticities are defined for the input demand functions where all three inputs are allowed to vary. Raising the price of labor w may then decrease the demand for capital, if the two inputs are complements, but the firm would be forced to increase its other input, pollution. Here, however, the third input demand equation ($\hat{Z} = \hat{\delta} + \hat{Y}$) indicates that a change in w, with no change in δ or Y, cannot change Z. Only labor and capital can vary, so they must be substitutes.

Thus, in (4'), a higher wage increases the capital/labor ratio ($b_w > 0$), and higher price of capital reduces it ($b_r < 0$). In fact, the Appendix shows that $b_r = -b_w$. Finally, we show in Appendix A1 that $b_\delta = b_{KZ} - b_{LZ}$ has the opposite sign of $e_{KZ} - e_{LZ}$. A tighter regulation means that δ is decreased, and less pollution is allowed per unit output. If capital is a better substitute for pollution than is labor, that is, if $e_{KZ} > e_{LZ}$, then more capital must be used relative to labor ($b_{KZ} < b_{LZ}$, and hence b_δ is negative). If b_δ is positive, then labor is a better substitute than is capital: a tightening of environmental policy means lower δ and lower K/L ratio.

For this model we now have ten equations: (1), (2), (3), (4'), (5'), (6), (7'), (8), (9'), and (10). As before, we set $\hat{p}_X = 0$ and solve for the changes in returns to capital and labor attributable to a small change in the policy variable (δ). The solutions are presented in Table 2. Compared to the general solutions in Table 1, these equations are not as difficult to interpret. Firstly, the denominator D is positive-definite. Secondly, the expressions for $\hat{r}$ and $\hat{w}$ can be decomposed into three terms, each corresponding to a single effect. The second term is the "output effect," as before, and the last term is the "substitution effect". Here, however, the first term is a new effect we call an "output-subsidy effect": since the policy mandates a lower *ratio* of pollution to output, it can be satisfied partially by increasing output (which helps the factor used intensively). We again analyze these effects by focusing on two special cases.

Table 2: Performance Standard (Restriction on Z/Y)

$$\hat{r} = [-\frac{\theta_{XL}\nu}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\nu\sigma_u}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\eta}{D}b_\delta]\hat{\delta}$$

$$\hat{w} = [\frac{\theta_{XK}\nu}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\nu\sigma_u}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\eta}{D}b_\delta]\hat{\delta}$$

$$\hat{p}_Y = \{\frac{1}{D}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(-\nu(1-\sigma_u)(\gamma_K - \gamma_L) + \eta b_\delta) - \frac{\nu}{1-\nu}\}\hat{\delta}$$

where $\eta \equiv (\theta_{YK}\gamma_L + \theta_{YL}\gamma_K + 1)(1-\nu) > 0$ and

$$D \equiv (1-\nu)\sigma_u(\theta_{XL}\theta_{YK} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_L) + \eta[b_w + \sigma_X(\theta_{XL}\gamma_L + \theta_{XK}\gamma_K)] > 0$$

4.1 Equal Factor Intensities

Since the output effect and output-subsidy effect operate through differential factor intensities, the assumption $\gamma_K = \gamma_L = \gamma$ makes them both disappear. Then only the third term for the substitution effect remains in $\hat{r}$ and $\hat{w}$. The solutions reduce to:

$$\hat{r} = \frac{\theta_{XL}\gamma b_\delta}{\sigma_X + \gamma b_w}\hat{\delta}$$

$$\hat{w} = \frac{-\theta_{XK}\gamma b_\delta}{\sigma_X + \gamma b_w}\hat{\delta}$$

$$\hat{p}_Y = \frac{-\nu}{1-\nu}\hat{\delta}.$$

In this case, the factor that is a relative substitute for pollution is burdened less by a strengthening of environmental policy ($\hat{\delta} < 0$). If labor is the better substitute for pollution ($b_\delta > 0$), the coefficient in front of $\hat{\delta}$ in the first expression is positive. Then the return to capital falls, while the return to labor rises. In fact, note that this simple intuition could fail with the emissions permit market. Here, this intuition cannot fail since the sign of the denominator cannot switch.

This case also provides unambiguous results for incidence on the uses side of income. Only the last term remains from the long expression for $\hat{p}_Y$ in Table 2, and it is negative. A tightening of environmental policy increases the price of the dirty good relative to the price of the clean good, hurting consumers of the dirty good.

4.2 No Substitution Effect in Dirty Sector

As we did with the previous policy, we can isolate the effect of factor intensities by assuming away differential substitution in the dirty sector. In the previous case we set all a_{ij} to zero, but here we set only b_δ to zero.[24] Under the policy in question, then, any change in the policy parameter δ has no effect on the relative input demands for capital and labor. Hence, the substitution effect is eliminated. Under these assumptions, the general solutions reduce to:

$$\hat{r} = -\frac{1}{D}\theta_{XL}\nu(1-\sigma_u)(\gamma_K - \gamma_L)\hat{\delta}$$

$$\hat{w} = \frac{1}{D}\theta_{XK}\nu(1-\sigma_u)(\gamma_K - \gamma_L)\hat{\delta}$$

$$\hat{p}_Y = [-(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})\nu(1-\sigma_u)(\gamma_K - \gamma_L)\frac{1}{D} - \frac{\nu}{1-\nu}]\hat{\delta},$$

where $D \equiv (1-\nu)\sigma_u(\theta_{XL}\theta_{YK} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_K) + \eta\sigma_X(\theta_{XL}\gamma_L + \theta_{XK}\gamma_K) > 0$.

In the first two expressions, we combine the "output effect" and the "output-subsidy effect" from the general solutions in Table 2. Suppose that the dirty sector is capital intensive, so that $(\gamma_K - \gamma_L) > 0$. Then, capital is hurt more than labor from a

[24] We cannot set all b_{ij} elasticities to zero, since Appendix 1 shows that some of them are of definite sign.

tightening of policy only if σ_u, the elasticity of substitution in consumption between X and Y, is greater than one. Tightening the policy imposes a burden on the dirty sector only, hurting capital when Y is capital-intensive. This is the "output effect." However, the tighter policy can be accommodated by producing more Y, since that drives down the Z/Y ratio. This is the "output-subsidy effect," helping capital. The usual output effect dominates only when $\sigma_u > 1$. If consumers are *not* highly responsive to relative output prices, then the output-subsidy effect dominates, and tighter environmental policy places *less* burden on the factor that is used intensively in the dirty sector.

The three effects of a tighter performance standard are summarized in Table 3. Each entry shows the sign of that column's effect on that row's price. For example, the box in the first row and first column contains $(\gamma_L - \gamma_K)$. If that term is positive (Y is labor intensive), then a tighter performance standard increases the rental rate. The table shows that the output effect and the output-subsidy effect always work in opposite directions, and always through the relative factor intensity of the two sectors. The substitution effect depends on b_δ, which is equal in sign to $e_{KZ} - e_{LZ}$.

Table 3: Summary of Effects from a Tighter Performance Standard

	Output Effect	Output-Subsidy Effect	Substitution Effect
$\hat{r}$	$(\gamma_L - \gamma_K)$	$(\gamma_K - \gamma_L)$	b_δ
$\hat{w}$	$(\gamma_K - \gamma_L)$	$(\gamma_L - \gamma_K)$	$-b_\delta$

5. "Technology Mandate": Emissions per unit Input

Whereas the previous section examines a limit on emissions per unit output, we now examine a regulation that limits emissions per unit of an input. Such limits are common, as described in our first section above. We have only two clean inputs in our model, so we capture the nature of a limit on emissions per unit input by modeling a limit on emissions per unit of capital. We refer to this policy as a technology mandate, since forcing the adoption of a particular technology in production may effectively fix the emissions/capital ratio. Capital and labor are each in fixed supply and mobile between sectors, so they are perfectly symmetric in this model. Thus, the results for a limit per unit labor can be obtained directly from results below by interchanging every K and L (as well as every w and r).

As with the other two policies considered earlier, the equations that describe the behavior of consumers and of producers of the clean good do not change here. Equations (1), (2), (3), (6), (8), and (10) fall into this category and are applicable to this section. The only aspect of the model that requires revision is the behavior of producers of the dirty good. Consider their maximization problem. As in the previous policy considered, firms pay no explicit price for the pollution input. Instead, they face an exogenous ceiling on their ratio of emissions to capital. Formally, this problem is

$$\max_{K_Y, L_Y, Z} \quad p_Y Y(K_Y, L_Y, Z) - rK_Y - wL_Y$$

subject to the constraint $Z/K_Y \leq \zeta$. A tightening of environmental policy is defined as a decrease in ζ. It is clear that the policy constraint binds: since firms pay no price per unit of pollution, and this input is productive, they will employ as much of it as possible, an amount $Z = \zeta K_Y$. Thus, we use below the fact that $\partial Z/\partial K_Y = \zeta$. The first order conditions for the maximization problem are

$$r = p_Y(Y_K + \zeta Y_Z)$$

$$w = p_Y Y_L.$$

The second of these equations is identical to the first order condition in the original problem where firms face a price for all three inputs and no other constraint: the marginal value of labor is equal to the wage. The first equation differs from the standard condition. For the choice of capital input demanded, the marginal value of capital is *lower* than the rental rate (since ζY_Z is positive). The intuition here is that each unit of capital employed gives value to the firm in two different ways. First, it increases their output directly (since $Y_K > 0$). Second, it allows more pollution, which also increases output. The second term represents this effect, since Y_Z is the marginal product of pollution and $\zeta = \partial Z/\partial K_Y$ is the pollution increase made possible by the increased capital. The value of investing in a marginal unit of capital is composed of these two terms and at the optimum is set equal to the cost of that investment, the rental rate r.

Totally differentiate the production function and substitute in these first order conditions. After dividing through by Y, we have:

$$\hat{Y} = (\theta_{YK} - \nu)\hat{K}_Y + \theta_{YL}\hat{L}_Y + \nu\hat{Z} \tag{9''}$$

The constant v is still equal to $Y_Z Z/Y$, as in the previous section. Also, as before, an increase in any one input does not generally increase output by a proportion equal to its factor share. This condition does hold for labor in (9"), since that input choice is not distorted by the technology mandate. It cannot hold for pollution, however, since no share is "paid" to that input. Also, the constraint distorts the choice of capital. Yet, from (9"), we do see that a one percent increase in all three inputs yields a one percent increase in output, from the assumption of constant returns to scale. The zero profit condition still holds as well, even though firms do not pay for pollution, because entry and exit are still allowed. Thus equation (7') from the prior model also applies to this one.

Finally, the dirty sector's chosen amount of each input (K_Y, L_Y, and Z) depends on input prices, the policy parameter, and output (r, w, ζ, and Y). We totally differentiate these input demand equations to get:

$$\hat{K}_Y = c_{KK}\hat{r} + c_{KL}\hat{w} + c_{KZ}\hat{\zeta} + \hat{Y}$$

$$\hat{L}_Y = c_{LK}\hat{r} + c_{LL}\hat{w} + c_{LZ}\hat{\zeta} + \hat{Y}$$

$$\hat{Z} = c_{ZK}\hat{r} + c_{ZL}\hat{w} + c_{ZZ}\hat{\zeta} + \hat{Y}.$$

The elasticity of demand for input i with respect to price j is defined here as c_{ij} (but this response depends on the nature of the constraint, so the c_{ij} elasticities are not the same as the a_{ij} or b_{ij} elasticities). Only two of these equations are independent of each other, so we subtract each of the bottom two equations from the top one to get two equations to use in our solution. The first of these equations is

$$\hat{K}_Y - \hat{L}_Y = c_r\hat{r} + c_w\hat{w} + c_\zeta\hat{\zeta}, \qquad (4")$$

where $c_r \equiv c_{KK} - c_{LK}$, $c_w \equiv c_{KL} - c_{LL}$, and $c_\zeta \equiv c_{KZ} - c_{LZ}$. The second resulting equation can be simplified using the policy constraint $Z/K_Y = \zeta$, since total differentiation gives:

$$\hat{K}_Y - \hat{Z} = -\hat{\zeta}. \qquad (5")$$

Substituting this into the equations above implies that $c_{KK} - c_{ZK} = 0$, $c_{KL} - c_{ZL} = 0$, and $c_{KZ} - c_{ZZ} = -1$. These relationships are verified in Appendix A2.

Also in that Appendix, we evaluate the elasticities of input demand. An important condition for their signs relates to the relative complementarity of capital and pollution. Let Condition 2 be defined as: $e_{KZ} > (e_{KK} + e_{ZZ})/2$. The right hand side of this inequality must be negative since all own-price elasticities are negative. This condition always holds, then, whenever capital and pollution are substitutes ($e_{KZ} > 0$). It also holds when capital and pollution are not "too" complementary. The Appendix shows that Condition 2 implies $c_r < 0$ and $c_w > 0$. That is, an increase in the capital rental rate must reduce the ratio K_Y/L_Y demanded, and an increase in the wage rate must increase that ratio. The ratio of Z to K_Y is fixed, and so producers really have only two inputs between which they can substitute; once they choose K_Y and L_Y, then Z is given by the constraint. With only two inputs K_Y and L_Y, they must be substitutes.

Now consider the case when Condition 2 fails (so that $c_r > 0$ and $c_w < 0$). Then an increase in r *raises* the desired K_Y/L_Y ratio (and an increase in w raises relative labor demand). This result is highly counter-intuitive, but it can be explained by noting that capital and pollution are highly complementary in this case [$e_{KZ} < (e_{KK} + e_{ZZ})/2 < 0$]. Then a higher r means that firms want less K and less Z. Wanting less Z reduces the pressure of the constraint ($Z/K_Y \leq \zeta$), which reduces the shadow price on Z (i.e., the right to emit is not so valuable). The reduced shadow price on Z by itself would mean more demand for Z and more K_Y, since they are complements. If they are *sufficiently* complementary, then the result is a net increase in capital relative to labor.

Table 4: Technology Mandate (Restriction on Z/K)

$$\hat{r} = \frac{\theta_{XL}}{D}[-(1-\sigma_u)\nu(\gamma_K - \gamma_L) + [(\gamma_L(1+\gamma_K) + \theta_{YL}(\gamma_K - \gamma_L)]c_\zeta]\hat{\zeta}$$

$$\hat{w} = \frac{\theta_{XK}}{D}[(1-\sigma_u)\nu(\gamma_K - \gamma_L) - [(\gamma_L(1+\gamma_K) + \theta_{YL}(\gamma_K - \gamma_L)]c_\zeta]\hat{\zeta}$$

$$\hat{p}_Y = \{\frac{(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})}{D}[(1-\sigma_u)\nu(\gamma_L - \gamma_K) + [(\gamma_L(1+\gamma_K) + \theta_{YL}(\gamma_K - \gamma_L)]c_\zeta] - \nu\}\hat{\zeta}$$

where
$$D \equiv \sigma_X(\theta_{XK}\gamma_K + \theta_{XL}\gamma_L + 1) + \sigma_u(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_L) + (\theta_{XL}c_r - \theta_{XK}c_w)(\gamma_L(-1-\gamma_K) + \theta_{YK}(\gamma_L - \gamma_K))$$

The system of equations containing (1), (2), (3), (4"), (5"), (6), (7'), (8), (9"), and (10) are ten equations in ten unknowns, once we set $\hat{p}_X = 0$. In Table 4, these equations are solved for the proportional change in each price as a function of an exogenous change in ζ. As in the previous policy, the expressions for $\hat{r}$ and $\hat{w}$ contain terms involving factor intensities and substitution elasticities (in this case c_ζ). In these general solutions, however, we cannot separately identify an output effect, a capital-subsidy effect, and a substitution effect. Instead these effects are discussed in two special cases. Unlike in Table 2, the denominator D here cannot be signed.

5.1 Equal Factor Intensities

As before, the assumption $\gamma_K = \gamma_L = \gamma$ makes the output effect disappear. Then only the substitution effect remains in $\hat{r}$ and $\hat{w}$. The solutions simplify to:

$$\hat{r} = \frac{\theta_{XL}\gamma c_\zeta}{\sigma_X + \gamma(c_w\theta_{XK} - c_r\theta_{XL})}\hat{\zeta}$$

$$\hat{w} = \frac{-\theta_{XK}\gamma c_\zeta}{\sigma_X + \gamma(c_w\theta_{XK} - c_r\theta_{XL})}\hat{\zeta}$$

$$\hat{p}_Y = -\nu\hat{\zeta} \ .$$

These equations are strikingly similar to their counterparts for the previous policy (but the c_{ij} elasticities are not the same as the b_{ij} elasticities). Suppose that Condition 2 holds, so that $c_r < 0$ and $c_w > 0$ (a higher rental rate decreases the K/L ratio employed by the dirty sector, and a higher w increases it). Then the denominator is positive in both expressions. The effect on factor prices then depends completely on the sign of c_ζ. A tighter environmental policy ($\hat{\zeta} < 0$) increases the return to capital relative to the wage if and only if $c_\zeta < 0$ (which means $c_{KZ} < c_{LZ}$, so lower ζ raises the desired K/L ratio). Without differences in factor intensities, the policy change induces producers in the dirty sector to demand relatively more capital than labor, which raises the equilibrium r relative to w. While this intuition is simple enough, the conditions for the sign of c_ζ are not. Details are provided in Appendix A2, but we provide intuition here for two offsetting effects. First, the "capital-subsidy effect" is that firms can help satisfy the newly reduced $\zeta = Z/K_Y$ by raising K_Y (making c_ζ more likely negative). Second, if e_{LZ} is high enough, then a "substitution effect" of the reduced Z is to raise L_Y (making c_ζ more likely positive).

On the uses side, incidence is unambiguous. A tighter environmental policy must increase the price of the dirty good relative to the price of the clean good – due to the direct effect of the policy on the cost of production in the Y sector only.

5.2 No Substitution Effect in Dirty Sector

Here we assume that $c_r = c_w = c_\zeta = 0$, or that no change in any input price or mandate has any effect on the ratio of K/L demanded by the dirty sector. This is not quite as strong as saying that the dirty sector cannot substitute at all, since we do not assume that all of the c_{ij} elasticities are zero.[25] Instead, our assumption simply eliminates the effects of substitution between labor and capital, and it thus allows us to consider only the effect of relative factor intensities. The solutions in this case reduce to:

$$\hat{r} = -\frac{\theta_{XL}}{D}(1-\sigma_u)\nu(\gamma_K - \gamma_L)\hat{\zeta}$$

$$\hat{w} = \frac{\theta_{XK}}{D}(1-\sigma_u)\nu(\gamma_K - \gamma_L)\hat{\zeta}$$

$$\hat{p}_Y = \{-\frac{\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}}{D}(1-\sigma_u)\nu(\gamma_K - \gamma_L) - \nu\}\hat{\zeta}\ ,$$

where the denominator $D = \sigma_X(\theta_{XK}\gamma_K + \theta_{XL}\gamma_L + 1) + \sigma_u(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_L) > 0$.

Just as in Section 4.2, then, the effect on relative factor prices depends on whether the elasticity of substitution in consumption, σ_u, is greater than one or less than one. Somewhat surprisingly, when the dirty sector is capital intensive, a tighter environmental policy can *raise* r (whenever $\sigma_u < 1$). For intuition, consider two effects of the reduction in $\zeta = Z/K_Y$. The "capital-subsidy effect" is that the tighter mandate can be met partially by using more K_Y. Since we have assumed in this section that the policy change has no effect on the ratio of K/L demanded, this capital-subsidy effect also increases labor demand. If Y is capital intensive, then this increases the ratio r/w. Second, the usual "output effect" is that the tighter mandate applies only to production of Y, which tends to raise the equilibrium price of Y and reduce demand for Y (and the use of both inputs K_Y and L_Y). Under the continuing assumption that the dirty sector is capital intensive, this output effect reduces overall demand for capital and thus reduces r/w. These effects

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[25] In fact, all c_{ij} cannot be zero, since we showed earlier that $c_{KZ} - c_{ZZ} = -1$.

work in opposite directions, and they exactly offset when $\sigma_u = 1$. In our simple model, $\sigma_u < 1$ means that the output effect is dominated, and thus r/w rises.

The effect on output price also depends on whether σ_u is greater than or less than one. The sign of the last term $(-v)$ is definitely negative; this "direct effect" means that a lower ζ raises the cost of production (and thus raises the breakeven price p_Y). The long first term is an indirect effect. Since $\gamma_K - \gamma_L$ has the same sign as $(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})$, this term has the opposite sign of $(1 - \sigma_u)$. When σ_u is smaller than one, then a tighter mandate must increase the price of good Y. When σ_u is large, however, the two effects offset. If the indirect effect dominates the direct effect, then a tighter mandate *decreases* the price of the dirty good.[26]

6. Conclusion

Just like taxes, regulations that restrict emissions affect producer decisions about use of labor and capital, and they thus affect relative factor prices, total production, and output prices. Existing models analyze the distribution of burdens from taxes, but this paper points out that non-revenue raising restrictions also have burdens on the sources side of income through changes in factor prices as well as burdens on the uses side through changes in output prices. Our model is based on the standard two-sector tax incidence model, but with two important modifications. First, we allow one sector to include pollution as a factor of production that can be a complement or substitute for labor or for capital. Second, we look not at taxes but at four types of mandates.

The model in this paper could be extended in any of the many ways that the Harberger model has been extended, for example to consider increasing returns to scale, imperfect competition, international trade, or capital mobility. Future research could consider capital formation, endogenous technology, and uncertainty. With no existing research on this topic at all, however, we thought that this simple model was a good place to start. And even in this simple model, we get some interesting results. First, a mandate may hurt consumers of the clean good more than consumers of the dirty good. Second, we show how a mandate may disproportionately burden either the factor that is a better substitute for pollution or the factor that is a relative complement to pollution. Third,

[26] In the $\hat{p}_Y$ equation, for a large indirect effect, suppose σ_u and $(\gamma_K - \gamma_L)$ are large. The sector is highly capital intensive. The output effect dominates the capital-subsidy effect, so the tighter mandate means less demand for capital. Thus r falls. As seen in the $\hat{r}$ equation, large σ_u and $(\gamma_K - \gamma_L)$ mean r falls a lot. The dirty sector is highly capital intensive, so its cost of production and p_Y fall.

restrictions on the absolute level of emissions differ from restrictions on emissions per unit output or per unit of an input. For example, a restriction on pollution per unit of output has not only an "output effect" that burdens any factor used intensively in production, but also an "output-subsidy effect" that encourages output to help satisfy the mandated ratio. Similarly, a restriction on pollution per unit capital creates a "capital-subsidy effect" that increases demand for capital and thus raises the rental rate.

An implication is that researchers need to be careful about the nature of an environmental restriction before concluding that it injures the factor used intensively or the factor that is a better substitute for pollution. Those usual effects can be completely offset by other effects we identify in this paper.

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Appendix A1: Finding the Substitution Elasticities b_{ij}

The b_{ij} elasticities are evaluated from the production function in the dirty sector in a manner analogous to Allen (1938, p. 505-508). We are solving for the derivatives of input demands with respect to changes in either input prices or δ, the policy parameter. These input demand equations come from the firm's cost minimization problem, where the total quantity to be produced is exogenous. First consider a small change in the price of capital, dr. If we differentiate the production function with respect to r we get

$$Y_K \frac{dK_Y}{dr} + Y_L \frac{dL_Y}{dr} + Y_Z \frac{dZ}{dr} = \frac{dY}{dr} = 0 ,$$

where the last equation comes from the fact that total output demanded is exogenous and not a function of the rental rate.

The first order condition of the minimization problem with respect to the choice of K_Y is $\frac{p_Y}{1-\delta Y_Z} Y_K = r$. Differentiate this equation with respect to r, multiply through by $\frac{1-\delta Y_Z}{p_Y}$, and collect terms to get

$$\frac{Y_K}{p_Y}\frac{dp_Y}{dr} + [Y_{KK} + Y_K Y_{ZK}\xi]\frac{dK_Y}{dr} + [Y_{KL} + Y_K Y_{ZL}\xi]\frac{dL_Y}{dr} + [Y_{KZ} + Y_K Y_{ZK}\xi]\frac{dZ}{dr} = \frac{1-\delta Y_Z}{p_Y},$$

where $\xi \equiv \frac{\delta}{1-\delta Y_Z}$. Similarly, differentiate the next first order condition, $\frac{p_Y}{1-\delta Y_Z} Y_L = w$, with respect to r and rearrange to get

$$\frac{Y_L}{p_Y}\frac{dp_Y}{dr} + [Y_{LK} + Y_L Y_{ZK}\xi]\frac{dK_Y}{dr} + [Y_{LL} + Y_L Y_{ZL}\xi]\frac{dL_Y}{dr} + [Y_{LZ} + Y_L Y_{ZK}\xi]\frac{dZ}{dr} = 0.$$

Note that the right hand side of this equation is zero, since a change in r has no effect on w, which is exogenous to this input demand system. Finally, the policy constraint binds, so $Z = \delta Y$. Since Y and δ are both exogenous variables in the input demand system, a change in r has no effect on their values. Hence, differentiating this equation with respect to r yields $\frac{dZ}{dr} = 0$.

Writing these four equations in matrix form allows use of Cramer's rule to evaluate the derivatives. This equation is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK} + Y_K Y_{ZK}\xi & Y_{Kl} + Y_K Y_{ZL}\xi & Y_{KZ} + Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK} + Y_L Y_{ZK}\xi & Y_{LL} + Y_L Y_{ZL}\xi & Y_{LZ} + Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dr} \\ dK_Y/dr \\ dL_Y/dr \\ dZ/dr \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1-\delta Y_Z}{p_Y} \\ 0 \\ 0 \end{bmatrix}$$

Follow the notation of Allen (1938) and use F to denote the determinant of the bordered Hessian of the production function, and use F_{ij} to denote the cofactor of element i,j of that matrix. The determinant of the matrix of coefficients in the above equation simplifies to F_{ZZ} (the terms with ξ all cancel each other out). With an odd number of inputs, the assumption of constant returns to scale (linear homogeneity) implies that $F <$ 0 and $F_{ZZ} > 0$. Using Cramer's rule, we solve for the derivatives of interest:

$$\frac{dK_Y}{dr} = \frac{-(Y_L)^2(1-\delta Y_Z)}{p_Y F_{ZZ}} < 0 \ , \ \frac{dL_Y}{dr} = \frac{Y_L Y_K (1-\delta Y_Z)}{p_Y F_{ZZ}} > 0 \, .$$

These sign indicate that $b_{KK} < 0$ and $b_{LK} > 0$, as we now show. The term $1 - \delta Y_Z$ is strictly positive for the following reason. The policy parameter $\delta = Z/Y$ is the inverse of *average* output per unit of Z. It is multiplied by Y_Z, the *marginal* output per unit of Z. Since production is constant returns to scale, the average output must exceed the marginal output, and hence $\delta Y_Z < 1$. Furthermore, both first derivatives of Y are positive, and $F_{ZZ} < 0$ as mentioned before. Thus $b_{KK} < 0$ and $b_{LK} > 0$.

We take the production function, the first order conditions for the cost minimization problem, and the binding constraint, and then we differentiate all, this time with respect to w. Writing these four equations in matrix form yields a similar system of equations:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK} + Y_K Y_{ZK}\xi & Y_{Kl} + Y_K Y_{ZL}\xi & Y_{KZ} + Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK} + Y_L Y_{ZK}\xi & Y_{LL} + Y_L Y_{ZL}\xi & Y_{LZ} + Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dw} \\ dK_Y/dw \\ dL_Y/dw \\ dZ/dw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1-\delta Y_Z}{p_Y} \\ 0 \end{bmatrix}.$$

The matrix of coefficients is the same as for dr above; the only difference is in which element of the vector of constants is nonzero. Here it is the element corresponding to the differentiation of the first order condition for labor input, since w is changing. Solving this system yields

$$\frac{dK_Y}{dw} = \frac{Y_K Y_L (1-\delta Y_Z)}{p_Y F_{ZZ}} > 0 \ , \ \frac{dL_Y}{dw} = \frac{-(Y_K)^2(1-\delta Y_Z)}{p_Y F_{ZZ}} < 0 \, .$$

These solutions can be used to evaluate the input demand elasticities.

$$b_r = b_{KK} - b_{LK} = \frac{r}{K_Y}\frac{dK_Y}{dr} - \frac{r}{L_Y}\frac{dL_Y}{dr} = \frac{r(1-\delta Y_Z)Y_L}{p_Y F_{ZZ}}\left(-\frac{Y_L}{K_Y} - \frac{Y_K}{L_Y}\right) < 0$$

$$b_w = b_{KL} - b_{LL} = \frac{w}{K_Y}\frac{dK_Y}{dw} - \frac{w}{L_Y}\frac{dL_Y}{dw} = \frac{w(1-\delta Y_Z)Y_K}{p_Y F_{ZZ}}\left(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}\right) > 0 \, .$$

We can substitute in the first order conditions $p_Y Y_K = r(1 - \delta Y_Z)$ and $p_Y Y_L = w(1 - \delta Y_Z)$ to simplify these expressions.

$$b_r = -\frac{Y_K Y_L}{F_{ZZ}}(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}) \ , \ b_w = \frac{Y_K Y_L}{F_{ZZ}}(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}) .$$

This substitution demonstrates that $b_r = -b_w$.

Lastly, we want to find the derivatives of factor demands with respect to a change in the policy parameter δ. Again, differentiate the production function and the first order conditions, here with respect to δ. The policy constraint $(Z = \delta Y)$ differentiated with respect to δ yields $dZ/d\delta = Y$. The matrix form of this system of equations is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK} + Y_K Y_{ZK}\xi & Y_{Kl} + Y_K Y_{ZL}\xi & Y_{KZ} + Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK} + Y_L Y_{ZK}\xi & Y_{LL} + Y_L Y_{ZL}\xi & Y_{LZ} + Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{d\delta} \\ dK_Y/d\delta \\ dL_Y/d\delta \\ dZ/d\delta \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{Y_Z Y_K}{1-\delta Y_Z} \\ -\frac{Y_Z Y_L}{1-\delta Y_Z} \\ Y \end{bmatrix}.$$

Again the matrix of coefficients is the same, with determinant F_{ZZ}. Solving for the derivatives of interest yields:

$$\frac{dK_Y}{d\delta} = Y\frac{F_{KZ}}{F_{ZZ}} \ , \ \frac{dL_Y}{d\delta} = Y\frac{F_{LZ}}{F_{ZZ}} ,$$

where again F_{ij} denotes the cofactor of element i,j in the bordered Hessian of the production function. These cofactors are not immediately interpretable, but they are an integral part of the definition of the Allen elasticities. They are defined as:

$e_{ij} \equiv \frac{p_Y Y}{i_Y j_Y} \cdot \frac{F_{ij}}{F}$, where i_Y is the quantity of input i used. With these definitions we can calculate the remaining input demand elasticities:

$$b_\delta = b_{KZ} - b_{LZ} = \frac{\delta}{K_Y}(Y\frac{F_{KZ}}{F_{ZZ}}) - \frac{\delta}{L_Y}(Y\frac{F_{LZ}}{F_{ZZ}}) = \frac{\delta Z}{p_Y}\frac{F}{F_{ZZ}}(e_{KZ} - e_{LZ}) ,$$

where e_{ij} is the Allen elasticity of substitution between inputs i and j. Since $F/F_{ZZ} < 0$, the sign of b_δ is opposite the sign of $e_{KZ} - e_{LZ}$; if capital is a better substitute for pollution than is labor, then b_δ is negative.

Appendix A2: Finding the Substitution Elasticities c_{ij}

We calculate these elasticities using a method similar to the one in Appendix A1. First, consider the effect of small changes in the capital rental rate. If we differentiate the production function with respect to r we get, as before:

$$Y_K \frac{dK_Y}{dr} + Y_L \frac{dL_Y}{dr} + Y_Z \frac{dZ}{dr} = \frac{dY}{dr} = 0$$

The first order condition from the maximization problem with respect to capital is $r = p_Y(Y_K + \zeta Y_Z)$. Differentiate this with respect to r, divide through by p_Y, and rearrange terms to get:

$$\frac{Y_K + \zeta Y_Z}{p_Y}\frac{dp_Y}{dr} + [Y_{KK} + \zeta Y_{ZK}]\frac{dK_Y}{dr} + [Y_{KL} + \zeta Y_{ZL}]\frac{dL_Y}{dr} + [Y_{KZ} + \zeta Y_{ZK}]\frac{dZ}{dr} = \frac{1}{p_Y}.$$

The first order condition for labor is $w = p_Y Y_L$. Differentiating this equation by r and similarly rearranging yields

$$\frac{Y_L}{p_Y}\frac{dp_Y}{dr} + Y_{LK}\frac{dK_Y}{dr} + Y_{LL}\frac{dL_Y}{dr} + Y_{LZ}\frac{dZ}{dr} = 0.$$

Finally, differentiate the policy constraint $Z = \zeta K_Y$ by r to obtain

$$\frac{dZ}{dr} = \zeta \frac{dK_Y}{dr}.$$

Combining these four equations into matrix form allows us to solve for any of the derivatives. This matrix equation is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K + \zeta Y_Z & Y_{KK} + \zeta Y_{ZK} & Y_{Kl} + \zeta Y_{ZL} & Y_{KZ} + \zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dr} \\ dK_Y/dr \\ dL_Y/dr \\ dZ/dr \end{bmatrix} = \begin{bmatrix} 0 \\ 1/p_Y \\ 0 \\ 0 \end{bmatrix}.$$

We solve for these derivatives using Cramer's Rule, where the denominator is the determinant of the matrix of coefficients. Call this denominator D. Solving along the bottom row, and using known properties of determinants, we get:

$$D = \zeta(F_{KZ} + \zeta(-F_{KK})) - (F_{ZZ} + \zeta(-F_{KZ})) = -\zeta^2 F_{KK} - F_{ZZ} + 2\zeta F_{KZ},$$

where the F_{ij} notation is from Allen (1938), just as in the previous section. We can solve for this denominator in terms of the Allen elasticities using their definitions:

$$D = -\zeta^2 \frac{Fe_{KK}K_Y^{\ 2}}{p_Y Y} - \frac{Fe_{ZZ}Z^2}{p_Y Y} + 2\zeta \frac{Fe_{KZ}K_Y Z}{p_Y Y}.$$

And, since $\zeta = Z/K_Y$,

$$D = \frac{FZ^2}{p_Y Y}(-e_{KK} - e_{ZZ} + 2e_{KZ}).$$

We can sign the denominator with information about these three Allen elasticities. The ratio in the front of this expression is negative, since $F < 0$ and all of the other constants are positive. The own-price elasticities e_{KK} and e_{ZZ} must be negative. Hence, D is negative if and only if e_{KZ} is not too negative:

$$\text{Condition 2:} \quad e_{KZ} > \frac{e_{KK} + e_{ZZ}}{2}.$$

Since the right hand side of this inequality is strictly negative, a sufficient condition for D to be negative is capital and pollution are substitutes in production ($e_{KZ} > 0$). However, D is still negative if K and Z are not too complementary.

We now use Cramer's Rule to solve for the derivatives.

$$\frac{dK_Y}{dr} = \frac{1}{D}\frac{Y_L^{\ 2}}{p_Y} \quad , \quad \frac{dL_Y}{dr} = -\frac{1}{D}\frac{Y_L(Y_K + \zeta Y_Z)}{p_Y}.$$

When $D < 0$, then $dK_Y/dr < 0$ and $dL_Y/dr > 0$. We can also use Cramer's rule to solve for dZ/dr, but differentiation of the policy constraint provides it as a function of dK_Y/dr.

Now, we solve for the elasticities c_{KK} and c_{LK}, and the difference (which is defined as c_r in the text):

$$c_r \equiv c_{KK} - c_{LK} \equiv \frac{r}{K_Y}\frac{dK_Y}{dr} - \frac{r}{L_Y}\frac{dL_Y}{dr} = \frac{r}{K_Y}\frac{Y_L^{\ 2}}{Dp_Y} + \frac{r}{L_Y}\frac{Y_L(Y_K + \zeta Y_Z)}{Dp_Y} = \frac{rY_L}{Dp_Y}\left(\frac{Y_L}{K_Y} + \frac{Y_K + \zeta Y_Z}{L_Y}\right)$$

The sign of c_r is thus equal to the sign of D.

The same method is used to solve for the derivatives with respect to w and ζ. Differentiating the four equations with respect to w yields:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K + \zeta Y_Z & Y_{KK} + \zeta Y_{ZK} & Y_{Kl} + \zeta Y_{ZL} & Y_{KZ} + \zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dw} \\ dK_Y/dw \\ dL_Y/dw \\ dZ/dw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/p_Y \\ 0 \end{bmatrix}.$$

The denominator again is D. Solving for the derivatives gives:

$$\frac{dK_Y}{dw} = -\frac{1}{D}\frac{Y_L(Y_K + \zeta Y_Z)}{p_Y}, \quad \frac{dL_Y}{dw} = \frac{1}{D}\frac{Y_K(Y_K + \zeta Y_Z)}{p_Y}.$$

So if $D < 0$, then $dK_Y/dw > 0$ and $dL_Y/dw < 0$. This gives us an expression for c_w:

$$c_w \equiv c_{LK} - c_{LL} = \frac{w}{K_Y}\frac{-Y_L(Y_K + \zeta Y_Z)}{Dp_Y} - \frac{r}{L_Y}\frac{Y_K(Y_K + \zeta Y_Z)}{Dp_Y} = -\frac{w(Y_K + \zeta Y_Z)}{Dp_Y}\left(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}\right)$$

The sign of c_w is the opposite of the sign of D.

Finally, we differentiate the four equations with respect to ζ to generate:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K + \zeta Y_Z & Y_{KK} + \zeta Y_{ZK} & Y_{Kl} + \zeta Y_{ZL} & Y_{KZ} + \zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{d\zeta} \\ dK_Y/d\zeta \\ dL_Y/d\zeta \\ dZ/d\zeta \end{bmatrix} = \begin{bmatrix} 0 \\ -Y_Z \\ 0 \\ -K_Y \end{bmatrix}.$$

The difference on the right hand side comes from the fact that, when differentiating with respect to ζ, the term ζ can no longer be treated as a constant. For example, the policy constraint $Z = \zeta K_Y$ when differentiated yields $\frac{dZ}{d\zeta} = K_Y + \zeta\frac{dK_Y}{d\zeta}$, the bottom row of the matrix equation.

The denominator is the same as in earlier cases. Solving for the derivatives gives:

$$\frac{dK_Y}{d\zeta} = \frac{1}{D}(-Y_L{}^2 Y_Z - K_Y(F_{KZ} - \zeta F_{KK})),$$

$$\frac{dL_Y}{d\zeta} = \frac{1}{D}(Y_L Y_Z(Y_K + \zeta Y_Z) + K_Y(F_{LZ} - \zeta F_{KL})).$$

The first derivative above consists of two offsetting terms whenever capital and pollution are substitutes, since $D < 0$, $F_{KZ} < 0$, and $F_{KK} > 0$. Therefore, when policy is tightened and ζ falls, then demand for capital may fall or rise. The sign of the derivative of labor demand with respect to ζ is also ambiguous. It depends on both D and the relative magnitude of F_{KZ} and F_{LZ}, or e_{KZ} and e_{LZ}.

Solving for the elasticity $c_\zeta \equiv c_{ZK} - c_{ZL} \equiv \frac{\zeta}{K_Y}\frac{dK_Y}{d\zeta} - \frac{\zeta}{L_Y}\frac{dL_Y}{d\zeta}$, we get:

$$c_\zeta = \frac{\zeta}{K_Y}\frac{-Y_L{}^2 Y_Z - K_Y(F_{KZ} - \zeta F_{KK})}{D} - \frac{\zeta}{L_Y}\frac{Y_L Y_Z(Y_K + \zeta Y_Z) + K_Y(-F_{LZ} - \zeta F_{KL})}{D}$$

$$= \frac{\zeta}{D}(-Y_L Y_Z(\frac{Y_L}{K_Y} + \frac{Y_K + \zeta Y_Z}{L_Y}) + \frac{FK_Y Z}{p_Y Y}(-e_{KZ} + e_{LZ} - e_{KL}))$$

The sign of this term depends on the relationship between the three Allen cross-price elasticities, but it is complicated. The text explains two offsetting effects than can make c_ζ positive or negative.

Finally, note that the equations relating the c_{ij} elasticities from the text, $c_{KK} - c_{ZK} = 0$, $c_{KL} - c_{ZL} = 0$, and $c_{KZ} - c_{ZZ} = -1$, can be verified using the derivations of the appropriate elasticities.