Australian Government
The Treasury

Australian Government Balance Sheet Management

Wilson Au-Yeung, Jason McDonald and Amanda Sayegh

Paper prepared for the
Annual East Asian Seminar on Economics
in the Philippines, 23-25 June.

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② ABSTRACT

Since almost eliminating net debt, the Australian Government's attention has turned to the financing of broader balance sheet liabilities, such as public sector superannuation. Australia will be developing a significant financial asset portfolio in the 'Future Fund' to smooth the financing of expenses through time. This raises the significant policy question of how best to manage the government balance sheet to reduce risk.

This paper provides a framework for optimal balance sheet management. The major conclusions are that:

- fiscal sustainability depends on both the expected path of future taxation and the risks around that path;
- optimal balance sheet management requires knowledge of how risks affect the balance sheet (and therefore volatility in tax rates); and
- the government's financial investment strategy should reduce the risk to government finances from macroeconomic shocks that permanently affect the budget.

Based on this framework, we find that a Future Fund portfolio that included (amongst other potential investments) domestic nominal securities and equities of selected countries would reduce overall balance sheet risk.

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1. INTRODUCTION

The Australian Government has taken steps over the past decade to improve the sustainability of government finances. In particular, the Government has significantly reduced its debt liabilities, avoiding many of the risks associated with high debt levels.

Having achieved a low level of debt, the Government's attention has turned to the financing of broader balance sheet liabilities, such as superannuation. The Government has announced that it will establish a Future Fund to finance public sector superannuation liabilities.[1] This will assist in relieving future generations of some of the financing burden associated with intergenerational fiscal pressures.

The creation of the Future Fund raises the significant policy question of how best to structure the Government's balance sheet to reduce overall financial risk. This paper sets out a framework for optimal government balance sheet management and presents some preliminary estimates of the types of financial assets and liabilities that would reduce overall financial risk.[2]

1 These liabilities relate to public sector employees only, not broader social insurance obligations found in many OECD countries.

2 The paper does not discuss the appropriate size of government expenditure, the level or composition of taxation necessary to fund it or the optimal size of a net asset portfolio (Future Fund) through time.

2. THE AUSTRALIAN GOVERNMENT BALANCE SHEET

The Australian Government's *Charter of Budget Honesty Act 1998* (the Charter) highlights the need for governments to manage balance sheet risks. The purpose of the Charter is to improve fiscal policy outcomes by requiring the fiscal strategy to be based on principles of sound fiscal management and by facilitating public scrutiny of fiscal policy and performance.

The Charter facilitates optimal balance sheet management in two ways. First, the Charter requires governments to make regular financial reports that comply with external reporting standards, including the Australian Bureau of Statistics (ABS) Government Finance Statistics (GFS) and Australian Accounting Standards (AAS). This means the government balance sheet is comparable across entities and jurisdictions. Second, the Charter requires the 'prudent' management of financial risks, including those relating to the broader government balance sheet (such as risks relating to the tax base). By requiring transparent presentation of the balance sheet and effective management of financial risks, the Charter allows the community to hold the government accountable for its financial performance.

2.1 The balance sheet

The Australian Government general government sector has published a balance sheet in the budget papers since 1999-2000 consistent with international reporting standards.[3] The balance sheet reported in the 2003-04 Final Budget Outlook is reproduced below.

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3 The Charter requires a balance sheet to be published as part of the budget papers (usually produced in May), the mid-year economic and fiscal outlook (by the end of January or six months after the last budget) and at the final budget outcome (up to 3 months after the end of the financial year). The Charter requires the balance sheet to be on both the ABS GFS and AAS basis. However, the primary budget statements (and therefore all references in this paper) are on a GFS basis.

Table 1: Australian Government Balance Sheet: 2003-04

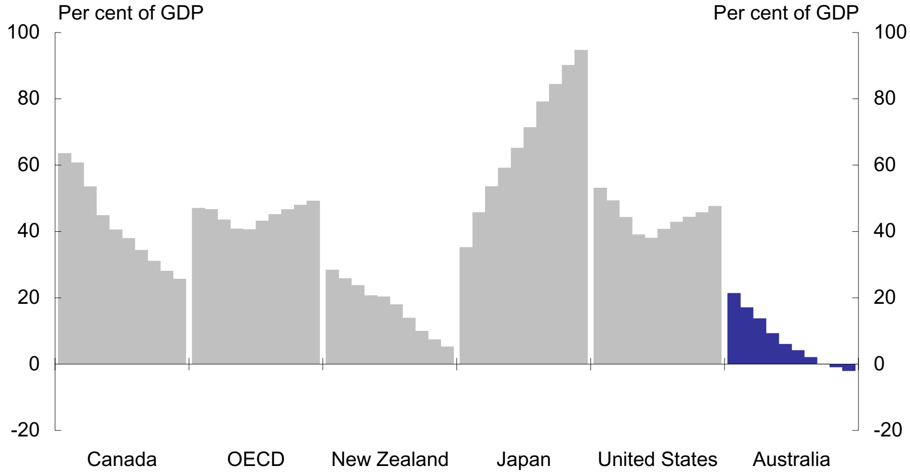
	2003-04 Estimate at 2004-05 Budget $m	2003-04 Outcome $m
Assets		
Financial assets		
Cash and deposits	1,607	1,591
Advances paid	19,027	18,060
Investments, loans and placements	19,668	24,188
Other non-equity assets	17,541	16,671
Equity(a)	47,061	49,560
Total financial assets	*104,905*	*110,070*
Non-financial assets		
Land	4,576	5,196
Buildings (excluding heritage)	13,417	14,152
Plant, equipment and infrastructure(b)	7,951	8,122
Inventories	4,237	4,832
Heritage and cultural assets(b)	4,949	6,442
Other non-financial assets	1,714	1,747
Total non-financial assets	*36,844*	*40,491*
Total assets	**141,749**	**150,560**
Liabilities		
Deposits held	325	364
Advances received	0	0
Government securities	60,555	60,650
Loans	5,271	5,979
Other borrowing	175	267
Superannuation liability	87,869	88,090
Other employee entitlements and provisions	8,426	8,541
Other non-equity liabilities	22,672	24,474
Total liabilities	**185,294**	**188,364**
Net worth(c)	**-43,545**	**-37,803**
Net debt(d)	26,024	23,421

(a) The 2003-04 equity and net worth outcomes include the Telstra shareholding valued at the closing share price on 30 June 2004.
(b) Heritage and cultural assets were previously included in plant, equipment and infrastructure.
(c) Net worth is calculated as total assets minus total liabilities.
(d) Net debt equals the sum of deposits held, advances received, government securities, loans and other borrowing, minus the sum of cash and deposits, advances paid and investments, loans and placements.
Source: Final Budget Outcome 2003-04, Australian Government.

The major assets on the Government's balance sheet are: financial equity, mainly reflecting the government's remaining share in Australia's major telecommunication company Telstra ($50 billion); non-equity assets, mainly taxes owed but not yet received by the Government ($17 billion); and deposits at the Reserve Bank ($15 billion). The major liabilities are superannuation liabilities ($88 billion) and gross debt issuance ($61 billion).

There are two notable aspects to the Australian Government balance sheet. First, the government has almost completely reduced net debt to zero — net debt has fallen from $96 billion (18.2 per cent of GDP) in 1996-97 to $23 billion (2.9 per cent of GDP) in 2003-04. This is in stark contrast with the net debt positions in nearly all other OECD countries (Chart 1).

Chart 1: General Government net debt in selected countries (1997 to 2006)



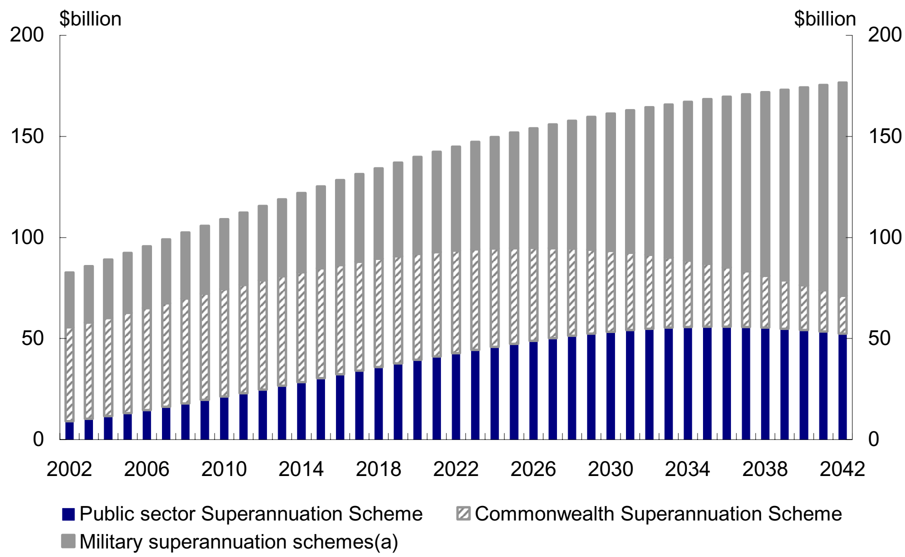
Source: Budget Strategy and Outlook, 2005-06, Budget Paper No. 1, Australian Treasury.

The reduction in net debt reflects fiscal surpluses and asset sales over a number of years. It also reflects that these surpluses have been invested in debt assets. Following the *Review of the Commonwealth Government Securities Market 2002*, the Government decided to maintain the domestic bond market to facilitate interest rate risk management by the private sector. The Government therefore maintains a stock of around $50 billion of mainly long dated securities, while investing the proceeds of debt issuance in term deposits at the Reserve Bank of Australia.

The other notable feature about the balance sheet is that the Government's most significant financial liability is public sector superannuation, estimated to grow to $100 billion (9.8 per cent of GDP) by 2007-08 (Commonwealth of Australia, 31:2004). While this liability is expected to increase further in the future, a significant portion reflects liabilities to past government employees. The Australian Government closed the main public sector superannuation fund to new members from 1 July 2005. This means the government will pay the superannuation liability for new public servants employed after this date as they accrue, rather than growing the superannuation liability further. Also, in 2004-05 the government paid $4.6 billion to Telstra and Australia Post to extinguish remanent superannuation liabilities from the corporatisation of these firms a decade or so ago.

Despite these policies, the existing superannuation liability is expected to remain sizeable, reaching $140 billion in 2020 (7.1 per cent of GDP), largely due to growth in the superannuation schemes for Military and Defence employees.[4]

Chart 2: Public sector superannuation liability (2002 to 2042)



(a) Includes the Military Superannuation and Benefits Scheme and the Defence Force Retirement and Death Benefits Scheme.
Source: Department of Finance and Administration and the Australian Government Actuary.

In response to these financial management challenges, the government has announced the creation of a 'Future Fund' with the aim of offsetting the Government's unfunded superannuation liabilities by 2020. The fund will assist in increasing the Government's net worth and increasing national savings. Contributions will be made to the fund whenever the budget is in surplus. That is, rather than realised budget surpluses being used to retire debt or build up term deposits at the Reserve Bank as currently occurs, they will be invested in the fund.

The fund will be established using accumulated cash reserves currently on term deposit with the Reserve Bank. Additional contributions from realised surpluses and the reinvestment of returns on the fund's assets will be needed to meet the Government's target.

4 Since the liability depends on the final salaries of public sector employees, there are risks around this estimate. Revaluations of the liability are regularly reported in the budget papers.

2.2 Contingent risks

The government balance sheet provides important information on the financial performance of the government from period to period. An increasing net worth means that a government is reducing rather than increasing net liabilities on future generations. However, there are many rights and obligations of government that are excluded from the balance sheet, mainly because of valuation problems. The most significant item missing from the balance sheet is the 'primary asset' of the government — the power to tax.[5] While this power is limited by such factors as the constitution, international tax competition, the size and growth of the economy, the effects of tax rate and base changes on economic efficiency and equity — the taxing power provides strong assurance of the government's ability to meet its liabilities.

On the other side of the ledger, certain explicit government obligations that do not meet accounting standards tests for the recognition of liabilities are also not recorded.[6] Under the international IMF GFS framework, only obligations payable *in any event* are on balance sheet, whereas those that occur only on *uncertain events* (even if they are probable) are not (International Monetary Fund, 2001:34). Unless presented carefully, this can lead to mis-understanding of the underlying economic value of specific assets and liabilities on a government's balance sheet. For example, the Australian Government departs from the GFS framework by recording provisions against expected defaults on student loans in the balance sheet (Commonwealth of Australia 2004:27).[7]

Probably the largest contingent liabilities not recorded on the balance sheet relate to future pensions and health costs. However, these obligations to fund future expenses have an impact on the economy today, as well as on fiscal sustainability. So the Charter also requires the government to produce an inter-generational report every five years, which essentially captures those obligations not recorded on the balance sheet. The last report from 2002-03

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5 Problems also exist in valuing substantial heritage assets on the balance sheet, such as Parliament House and the Australian War Memorial.

6 This paper is concerned only with explicit financial risks, defined as rights or obligations on government established by law or contract. Implicit financial risks provide a different set of policy problems, such as policy issues, which are beyond the scope of this paper.

7 The Swedish government budgets by appropriating the anticipated loss from guarantees for individual risks, ensuring equivalence between traditional outlays and financial instruments that transfer risk to the government (Hagelin and Thor, 2003). Similarly, the governments of the United States and the Netherlands explicitly appropriate the subsidy component of concessional loans and loan guarantees (Schick, 2002:90). The Australian National Audit Office has valued the potential exposure from other selected financial instruments containing contingent risks — such as financial guarantees — at $115 billion (ANAO, 2002-03).

projected spending associated with an aging population to require taxes to rise 5.0 per cent of GDP by 2041-42, or $87 billion (Commonwealth of Australia, 1:2002a).

These conceptual and measurement problems mean the government balance sheet is not directly comparable with similar private sector financial statements.[8] Government balance sheet management therefore requires a different framework for determining whether investment strategies are optimal. In particular, contingent assets and liabilities are likely to have a significant influence on how best to structure the government balance sheet to reduce risk and improve fiscal sustainability.

3. GOVERNMENT BALANCE SHEET MANAGEMENT

The government balance sheet is a measure of the government's financial position at a point in time. Government balance sheet management is concerned with how the balance sheet is expected to move through time. Managing the risks affecting the government balance sheet can assist in avoiding, or at least ameliorating, sharp changes in the financial position flowing from macroeconomic shocks. A government's balance sheet can be significantly affected by contingent risks affecting certain obligations.

Despite sound monetary and fiscal policies, as well as high domestic savings rates, many Asian economies suffered serious recessions in the late 1990's. These recessions were compounded, if not caused, by the crystalisation of contingent liabilities, particularly around commitments to support exchange rates and banking systems. Public injections into the banking system after the Asian crisis more than doubled the size of government debt to GDP in Korea and Thailand (Wheeler, 2004:105).[9] In emerging countries more generally over the last 15 years, bail outs to public enterprises and banking systems have contributed more to the build up of government debt than recurrent deficits (Kharas and Mishra, 2001). Indeed, the deterioration in the debt positions of emerging countries since the 1990's has been largely attributed to interest rate and exchange rate movements and the recognition of off-balance sheet and contingent liabilities (International Monetary Fund, 2003:117).

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8 Indeed, the *National Commission of Audit* (Commonwealth of Australia, 1996) recommended that the term 'balance sheet' be replaced with 'Statement of Assets and Liabilities' to avoid misleading comparisons with the private sector.

9 Korea's government debt to GDP ratio went from 10.5 per cent to 26.5 per cent after the costs of bank recapitalisation, while Thailand went from 14.6 per cent to 46.6 per cent.

For developed nations, managing balance sheet risks may not be as important in averting crises. However, balance sheet management can be used to improve the fiscal sustainability of government through time (and ultimately, avoid financial crises).[10] Since governments can rely on taxation to finance themselves, the concept of fiscal sustainability must relate in some way to the expected path of taxation. Fiscal sustainability not only requires that governments are likely to remain solvent — in the sense that the anticipated path of taxation is reasonable — but that the volatility (or risks) around that path are not significant. The International Monetary Fund is incorporating country risk analyses into their fiscal sustainability assessments for some countries (for example, Barnhill and Kopits, 2003). Indeed, the relationship between taxation and balance sheet assets and liabilities is central to the economics literature on balance sheet management.

3.1 A framework for analysis

3.1.1 Inter-temporal budget constraint

An important conceptual tool for analyzing government balance sheet management is the inter-temporal budget constraint. This budget constraint requires that at any date the sum of net worth and the net present value of taxation be equal to the net present value of government spending.

In this way, the inter-temporal budget constraint relates the government balance sheet in any period to the contingent asset and liabilities that can affect the balance sheet.[11] If current period government spending is higher than current period taxation, the government can issue debt (or some other liability). However, this simply means taxes need to be higher sometime in the future. In this framework, debt (and other liabilities) passed onto the future are effectively 'congealed taxation'. The inter-temporal budget constraint requires taxes to rise from their current levels to finance future anticipated expenses.

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10 There are other potential objectives of debt management policy, such as attempting to ameliorate the effects of incomplete or imperfect markets (eg improving market efficiency through improved risk sharing). However, alternative objectives have a less secure conceptual basis and some implementation problems (see Missale, 1997).

11 In a series of excellent papers (from Bradbury *et al*, 1999 and Grimes, 2001, onwards) the New Zealand Treasury has used the inter-temporal budget constraint to derive the concept of 'comprehensive net worth' for balance sheet management purposes.

3.1.2 Tax smoothing

Once the limit to future taxation and spending is identified, the optimal path of taxation needs to be found. Barro (1979) uses the standard public finance assumption that the excess burden of taxation rises by more than any rise in the tax rate — a doubling of tax rates has more than twice as many costs. These costs are the loss in overall welfare caused by tax rates distorting people's consumption choices. They can also be interpreted (somewhat imprecisely) as the waste in terms of administration and compliance costs that higher taxes cause.

Given anticipated government expenditure, these costs are minimised through time if tax (defined as a proportion of GDP) is constant, with temporary macroeconomic shocks leading to deficit financing and surpluses. That is, for a given financing requirement, a constant tax rate through time will impose a smaller cost on the economy than would a low tax rate in one year and a high tax rate in the next. An important implication of tax smoothing is that it is *anticipated* future tax rises, rather than simply current tax rates, which distort economic behaviour. For example, if tax rates are expected to rise significantly in the future, investment (and therefore growth) is likely to be discouraged. This standard result on optimal long run fiscal policy accords with the Australian Government's commitment of 'no increase in the overall tax burden from 1996-97 levels'.

This result depends critically on assumptions of the excess burden of taxation.[12] In the absence of these costs, there may be no role for Government to smooth taxes through time, since individuals could adjust their own portfolios to account for the uncertainty in future tax liabilities. It is the presence of such costs which gives government balance sheet management its power.

3.1.3 Balance sheet risk

For a macroeconomic shock that temporarily reduces economic growth, the government could resort to deficit financing by selling financial assets or issuing debt. However, if an unanticipated shock lead to a permanent change in the resources available to government (for example, a fall in the present value of taxation revenue), the government would need to adjust fiscal policy because deficit financing would not be sustainable. Alternatively, governments could attempt to structure their financial portfolios to hedge against such risks.

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12 If the loss function is linear, then there is no need to minimise the variance in tax rates (Hansen, 2003:9).

Bohn (1990) extends the tax smoothing result to incorporate such uncertainty, by imposing a budget constraint across all anticipated states of the world, as well as across time. Bohn shows that the government can reduce the expectation that tax rates will change, by holding and issuing specific financial instruments.[13] In particular, an effective budget hedge would see the government's financial returns vary negatively with tax revenue during a macroeconomic shock. For example, the government's balance sheet is protected somewhat if it issues debt where repayments fall with economic growth and tax revenue.

This framework suggests that the optimal portfolio for a country depends on the structure of the economy. If an economy is susceptible to supply side shocks, where inflation and growth move in opposite directions, then nominal debt issuance performs such a role. For example, the real value of government debt falls if an oil shock causes recession and inflation. Alternatively, if an economy is subject to demand side shocks, where inflation and growth are positively correlated, then inflation-indexed and variable interest rate debt are better hedges.

3.2 Formal presentation of the model

The intuition expressed above is set out more formally below and draws on the model developed by Bohn (1990).

Individuals are assumed to be infinitely lived and risk neutral, and maximise the expected utility derived from all future consumption:

$$U_t = E_t \sum_{j \geq 0} \rho^j c_{t+j} \qquad (1)$$

where ρ is a discount factor, and c_{t+j} is consumption in period t + j.

Individuals receive a stream of endowments Y_{t+j} and pay taxes on endowments at a rate τ_t. As taxes are distortionary there is an excess burden of taxation denoted by a convex loss function $h(\tau_t)$. Individuals are also able to trade a given set of assets, so that the individual budget constraint is given by

$$c_t + \sum_k p_{t,k} A_{t,k} = Y_t[1 - \tau_t - h(\tau_t)] + \sum_k (p_{t,k} + f_{t,k}) A_{t-1,k} \qquad (2)$$

13 Bohn (1990) assumes risk neutral individuals, so the costs relate to the expectation that taxes will change. For risk averse individuals, the uncertainty that the government will raise taxes at times of low financial returns (i.e. high marginal utility of consumption) is an additional cost (see the Appendix in Hansen, 2003).

where $A_{t,k}$ is the quantity of asset *k* held at the end of period t; $p_{t,k}$ is the price of asset *k* (denoted in terms of consumption goods); and $f_{t+j,k}$ is the stream of cash flows derived from holding asset *k*. Individual optimisation implies that expected returns across assets are equal, that is: $E_t(1+r_{t+1,k}) = 1/\rho$ for all k, where $r_{t+1,k} = \left(p_{t+1,k} + f_{t+1,k}\right)/\,p_{t,k} - 1$. This assumption is non-trivial, particularly so when we introduce equities into our analysis.

The government can use tax revenues $\tau_t Y_t$ and issue debt, $B_{t,k}$, to finance government expenditure, G_t (which we treat as exogenous in this model), and to meet outstanding debt obligations. The government budget constraint is given by:

$$\tau_t Y_t + \sum_k p_{t,k} B_{t,k} = G_t + \sum_k \left(p_{t,k} + f_{t,k}\right) B_{t-1,k} \tag{3}$$

The government can choose the type of debt instrument, *k*, and may be a net lender or net borrower in any security, as such B_t should be interpreted as the government's net liabilities.

Following Bohn, we recast the objective function in terms of government policy by substituting (2) and (3) into (1), which gives[14]

$$U_t = E_t \sum_{j \geq 0} \rho^j \left\{ Y_{t+j} \left[1 - h\left(\tau_{t+j}\right) \right] \right\} \tag{4}$$

The government chooses an optimal tax rate and debt portfolio to maximise individual utility (4) subject to its own budget constraint (3). In effect, the government's objective is to choose the structure of taxes and debt that minimise the expected present value of the excess burden of $h(\tau_t)$. The first-order conditions are[15]

$$\left.\begin{array}{ll} E_t\left[h'\left(\tau_{t+1}\right)\right] = h'\left(\tau_t\right) & \text{for } k = 0 \\ \rho E_t\left[h'\left(\tau_{t+1}\right)\left(1 + r_{t+1,k}\right)\right] = h'\left(\tau_t\right) & \text{for } k > 0 \end{array}\right\} \tag{5}$$

where k = 0 is the risk free asset. That is, optimality requires that the expected marginal excess burden of taxation is constant through time.

14 As in Bohn (1990) we drop exogenous terms for simplicity, as they are irrelevant for deriving the first order conditions for optimality.

15 See Appendix A for derivation of the first order conditions.

As in Bohn, we assume a quadratic excess burden, so that the deadweight loss of a tax rate, τ_t, is $h(\tau_t) = (h/2)\tau_t^2$. It follows then from the first order conditions that an optimal policy requires

$$Cov_t\left(\hat{\tau}_{t+1}, \hat{r}_{t+1,k}\right) = 0 \tag{6}$$

where $\hat{\tau}_{t+1} = \tau_{t+1} - E\tau_{t+1}$ is the innovation in the tax rate, and $\hat{r}_{t+1,k}$ is the innovation in the return to asset *k*. These innovations reflect the unanticipated components of changes in tax rates or returns.

Equation (6) implies zero conditional covariance between taxes and returns on available securities. That is, if the covariance between innovations in the tax rate and returns, for a specific debt, is negative then the government could improve tax-smoothing by issuing more of this form of debt. The converse is also true: if the covariance is positive then the government could improve tax smoothing by purchasing more of this form of debt. This is the principal conclusion of Bohn — the government should smooth tax rates across different states of the world, as well as over time.

3.3 The optimal structure of the Government's balance sheet

To estimate the government's optimal portfolio an expression for the innovation in tax rates is required. The innovation in the tax rate determined by the government's budget constraint is:

$$\hat{\tau}_{t+1} = \left(1-\psi\right)e^{-\bar{y}}\left[\sum_k \hat{r}_{t+1,k}d_{t,k} + \sum_{j\geq 0}\rho^j \hat{g}_{t+1+j}\right] - \tau_t \sum_{j\geq 0}\psi^j \hat{y}_{t+1+j} \tag{7}$$

where y_t is the growth rate of real output and $\bar{y}$ is its mean. The term $\sum_{j\geq 0}\psi^j \hat{y}_{t+1+j}$ is the present value of innovations in future growth rates of real output, where ψ is the discount factor and $\hat{y}_{t+1+j} = E_{t+1}y_{t+1+j} - E_t y_{t+1+j}$. That is, it captures unexpected permanent changes in output and therefore in the government's ability to raise tax revenues at a constant tax rate. Similarly, $\sum_{j\geq 0}\rho^j \hat{g}_{t+1+j}$ is the present value of innovations in government spending relative to output, where $\hat{g}_{t+1+j} = (E_{t+1}G_{t+1+j} - E_t G_{t+1+j})/Y_t$. The ratio of security *k* debt to output is denoted by $d_{t,k}$.

The intuition behind equation (7) is that the present value of tax revenues must cover initial debt plus the present value of government spending. That is, tax rates will need to adjust whenever there are *unexpected* changes in the value of government debt, government spending or output growth. For a government that is already optimally managing the balance sheet, the current tax rate already incorporates anticipated obligations.

Substituting the above into equation (6) gives the optimality condition for each government security:

$$\sum_{l} \operatorname{cov}_t\left(\hat{r}_{t+1,k}, \hat{r}_{t+1,l}\right) d_{t,l} + \operatorname{cov}_t\left(\hat{r}_{t+1,k}, \sum_{j \geq 0} \rho^j \hat{g}_{t+1+j}\right) - w_t \operatorname{cov}_t\left(\hat{r}_{t+1,k}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}\right) = 0 \quad (8)$$

where $w_t = \left[\frac{e^{\bar{y}}}{(1-\psi)}\right]\tau_t$ is a weighting factor.[16]

That is, the government can smooth taxes to offset unexpected shocks in the present value of government spending and output through the issuance or purchase of state contingent securities.

As the paper focuses on shocks that affect the present value of output growth, we assume that the covariance between innovations in the present value of government spending and returns on assets (the second term in equation (8)), is zero. The following equation provides us with a solution to the government's optimal portfolio:[17]

$$\mathbf{d}_t = w_t \sum\nolimits_r^{-1} \cdot \sum\nolimits_{y,r} \quad (9)$$

where $\sum_r$ is the variance–covariance matrix of returns (assumed to be non-singular) and $\sum_{y,r}$ is the covariance vector matrix between returns and the present value of unexpected innovations in real output growth.

Methodology

In order to solve equation (9) and evaluate the optimality of various portfolios of government assets, we need to calculate innovations in returns and the present value of future rates of growth in real output.

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16 For the purposes of the empirical analysis, we assume a discount factor of 0.98 (which equates to a 2 per cent per quarter discount), an average tax rate of 24 per cent and an average real growth rate of 0.75 per cent per quarter. The value of the weighting factor does not affect any of the qualitative conclusions.

17 For derivation of equation (9) see Appendix B.

We limit our analysis to a bivarate comparison. For our first analysis we are interested in the optimal share of long-term domestic debt and long-term foreign debt. We then extend this analysis to consider alternative asset classes, such as equites.

The real return on long-term domestic debt, $r_{t+1,d}$, is influenced by the domestic nominal long term interest rate l_{t+1}, changes in the current long-term market interest rate (which is used as an approximation of the capital gain component), and domestic inflation, π_{t+1}[18] Foreign long-term debt returns, $r_{t+1,f}$, will in addition be influenced by the change in the exchange rate Δs_{t+1}. Innovations in returns are therefore given by:

$$\hat{r}_{t+1,d} = -\left(\ell_{t+1,d} - E_t \ell_{t+1,d}\right) - \left(\pi_{t+1} - E_t \pi_{t+1}\right)$$
$$\hat{r}_{t+1,f} = -\left(\ell_{t+1,f} - E_t \ell_{t+1,f}\right) - \left(\pi_{t+1} - E_t \pi_{t+1}\right) + \left(\Delta s_{t+1} - E_t \Delta s_{t+1}\right)$$

To calculate these innovations in real returns, vector autoregressions (VARs) are used to formulate expectations for the inflation rate, the percentage change in the exchange rate, the long-term domestic interest rate and the long-term foreign interest rate. Following Hawkesby and Wright (1997) expectations are formed for each variable (y_{t+1}, π_{t+1}, $l_{t+1,d}$, Δs_{t+1}, $l_{t+1,f}$) by regressing that variable on a constant and one lag of the variable, together with one lag of all other variables.[19] As expectations at time t depend only on information available up to time t, we must run a new VAR for each time period.

The same method is used to calculate innovations in the growth of real output: $\hat{y}_{t+1+j} = E_{t+1} y_{t+1+j} - E_t y_{t+1+j}$. Expectations for real output growth need to be formed at each time period for rates of growth in all future time periods. That is an expectation is formed at time t, for ($y_{t+1}, y_{t+2}, \ldots y_{t+n}$) and at time t+1 for ($y_{t+1}, y_{t+2}, \ldots y_{t+n}$) given the additional information. The differences in expectations are then discounted at a rate, ψ (assumed to be 0.98).[20] This process is repeated for each time period to derive a time series for innovations in the present value of output.

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18 The proxy used for capital gains may lead to an understatement of this component in the innovation of real returns since it does not take into account the time to maturity. The longer the time to maturity the larger will be the capital gain (or loss) associated with changes in interest rates.

19 The lag specification was chosen based on the lag length that minimised the Akaike and Schwarz information criteria as suggested by Stock and Watson (2001) among others. The estimated model does not capture the full range of variables that could be expected to determine output and inflation. For a more complete model of the Australian economy see Dungey and Pagan (2000).

20 We also estimated the results using a lower discount factor, which did not change our broad conclusions. However, the case for investing abroad was slightly weaker under this scenario.

The methodology used to derive innovations in equity returns and output is the same as that outlined above, with innovations in equity returns given by:

$$\hat{r}_{t+1,e} = \left(p_{t+1,f} - E_t p_{t+1,f} \right) - \left(\pi_{t+1} - E_t \pi_{t+1} \right) + \left(\Delta s_{t+1} - E_t \Delta s_{t+1} \right)$$

where $\hat{p}_{t+1} = \left(p^*_{t+1} - E_t p^*_{t+1} \right)$ is the unanticipated component of capital gains. Capital gains are calculated using accumulation share indices for each country, which incorporate both share price growth and dividend growth.

3.3.1 Data

All data are quarterly data for the post float period 1983:4 — 2004:3. Long-term interest rates are the long-term government bond yields converted into quarterly returns. We take the first difference in bond yields, as we cannot reject non-stationarity over the sample period (based on the Augmented Dickey-Fuller and Phillips-Perron tests). Expectations for bond yields are then derived from the summing the expectations of the first difference. Equity returns are derived using accumulation indices when these are available and for the periods where they are not, capital weighted share indices are used (this effectively assumes that dividend growth is constant over this period). The exchange rate data is the log difference of the spot exchange rate expressed as the Australian dollar price of foreign currency. Inflation is estimated by taking the log differences of the GDP deflator. Growth rates in output are the log differences of real GDP.

3.3.2 Results

Table 2 shows the variance-covariance matrix for innovations series using domestic and foreign debt. The results show that innovations in domestic returns and foreign returns vary negatively with innovations in output. This suggests that it is optimal for the Australian Government to purchase securities denominated in both domestic and foreign currency. These results are consistent with the findings of Hawkesby and Wright (1997) and Missale (1999).

Table 2: Variance-covariance matrix for innovations series: debt securities

	US	Japan	Germany
Variance-covariance matrix			
Var (r_d)	0.32	0.37	0.37
Var (r_f)	10.11	41.81	36.95
Cov (r_d,rf)	-0.21	0.28	0.11
Cov (r_d,y)	-0.12	-0.10	-0.04
Cov (r_f,y)	-0.02	-0.38	-0.26
Optimal portfolios			
Domestic	-4.59	-3.17	-1.22
Foreign	-0.12	-0.09	-0.08

Note: The above results are for pair-wise comparisons between Australia and the reported country. This means that a separate VAR is calculated for each country including five variables: output, inflation, nominal domestic bond yield, nominal foreign bond yield, and the percentage change in the exchange rate.

We also report the optimal portfolio of domestic and foreign debt as a ratio to quarterly GDP, calculated by solving equation (9). The results show that it is optimal for the government to invest a relatively larger amount in domestic rather than foreign debt. This is largely driven by the volatility in the exchange rate, which acts as a 'penalty' on foreign investment. Volatility in the exchange rate (and therefore in foreign returns) is not necessarily bad, provided innovations in the exchange vary negatively with innovations in output. While this is the case for Japan and Germany, our results show a positive covariance between innovations in the exchange rate and output for the US.[21]

The above results can be disaggregated into the various elements that make up innovations (or unexpected changes) in returns (see Appendix C). Doing so reveals that there is a positive covariance between innovations in output and inflation, which is a key driver of our results. This implies that periods of unexpectedly low inflation (and therefore high returns) have tended to occur during periods of unexpectedly low output. This may largely result from the early 1990s recession, where inflation and domestic interest rates fell substantially.

Turning now to equities, a priori, we might expect that it would be unlikely that domestic equity investment would provide an effective hedge against macroeconomic shocks, given the high correlation between company profits and output. This is confirmed by our results, which show that the covariance between innovations in domestic equities and output is positive (table 3).

21 The volatility in the exchange may be partly driven by the method used to derive exchange rate innovations. Meese and Rogoff (1983) have shown that models used to explain exchange rate movements over short intervals, generally perform worse than a simple random walk.

Table 3: Variance-covariance matrix for innovations series: equities

	Australia	US	Japan	Germany
Variance-covariance matrix				
Var (r_d)	0.31	0.32	0.34	0.33
Var (r_e)	206.98	60.30	91.40	204.61
Cov (r_d,r_e)	0.56	0.35	0.50	-0.10
Cov (r_d,y)	-0.04	-0.07	-0.08	-0.05
Cov (r_e,y)	0.21	-0.10	-0.18	-0.25
Optimal portfolios				
Domestic debt	-1.52	-2.49	-2.85	-1.75
Equities	0.02	-0.01	-0.01	-0.02

Note: The above results are for pair-wise comparisons between domestic debt securities and equites in the reported country. This means that a separate VAR is calculated for each country including five variables: output, inflation, nominal domestic bond yield, equity prices (to proxy capital gains), and the percentage change in the exchange rate.

In contrast, the covariance between innovations in foreign equities and output is negative, suggesting that an optimal portfolio would include some investment in foreign equities for the countries considered.

4. Discussion

4.1 Policy implications

The key conclusion from the debt management literature is that the optimal financial investment strategy depends on the types of shocks affecting the macro economy. That is, the optimal way to structure debt (short or long, nominal or price-indexed, domestic or foreign currency) or invest in financial assets is an empirical question. Hansen (2003) (following Missale (1997)) summarised the main results from this literature:

(a) buy (short-sell) assets whose returns have a positive (negative) correlation to public spending and negative (positive) correlation to the tax base; and

(b) issue

- nominal debt for government spending and productivity shocks;
- price-indexed debt for monetary and real demand shocks causing inflation;
- foreign currency debt when output and inflation shocks are correlated internationally;
- maturity structure of debt to match structure of planned fiscal surpluses; and
- short maturity debt when positive correlation between output and real interest rates.

Our results suggest that the Australian economy has been subject to more demand shocks than supply side shocks over the sample period. Within the tax smoothing framework, this implies that the government should purchase domestic nominal bonds. During high output periods in Australia, inflation and tax revenues would increase offset by lower real returns on domestic nominal bonds, such as government (including State government debt) or high grade corporate debt. However, during low output periods, inflation and tax revenues will fall and be offset by higher real returns to domestic nominal bonds. These results support the government's policy of reducing net debt — both by reducing the size of gross debt issuance and investing in debt assets. The results also suggest that balance sheet risk could be further reduced by issuing price-indexed bonds (such as Treasury indexed bonds), rather than nominal bonds.

Our results also suggest that investing in foreign equities is likely to reduce overall balance sheet risk. In effect, the government already has a significant stake in Australian equities because of the tax revenue earned from domestic capital income (not to mention the presumably highly correlated flow on effects through taxation of domestic labour income). Auerbach (2004) notes that, even though the US government does not hold much equity directly, it has significant exposure to variations in stock prices through its claims to future tax revenues. Indeed, Auerbach argues that the US government's 'implicit equity position' is larger than the stock market itself, consistent with the fact that revenues from all sources are responsive to stock market returns.

There are some significant qualifications to our results that require further investigative effort. First, we have not constrained the size of the optimal portfolio necessary to reduce balance sheet risk. For example, the optimal portfolio suggests that 2 per cent of quarterly GDP should be invested in German shares. This is clearly unrealistic. However, in the absence of obvious constraints on the portfolio at the time of writing this paper, we decided to report the unconstrained case only. Further work to determine the optimal constrained portfolio within government policy needs to be undertaken. Second, we have ignored the relationship between innovations in output and government spending. That is, government expenditure is also likely to be linked to macroeconomic shocks and potentially able to be offset by government financial investment policy. It is likely that the effects of a macroeconomic shock on spending reinforce the impact on taxation. However, Bohn (1990) suggests that this impact is likely to be small and insignificant. Again, further work on this relationship is likely to improve any policy recommendations from this analysis.

4.2 Potential criticisms

4.2.1 Policy endogeneity

There are some important criticisms of the tax smoothing approach that can affect our policy conclusions. First, there is the potential problem of *policy endogeneity*. If the government's improved financial asset performance encourages greater government spending then the independence between government spending and taxing is violated. Similarly, there may be risks that if the government takes a controlling interest in a domestic company, some sections of the community may expect increased assistance for that company. In such circumstances, the optimal balance sheet strategy for government might be to avoid accruing a financial asset portfolio altogether and simply eliminate all risk, balancing the budget through the cycle (see Pinfield, 1998). However, another solution that maintains the benefits of optimal balance sheet management is to restrict the degree of controlling interest a government investment fund can maintain in specific domestic companies. Further, it is unlikely that the Future Fund will increase the incentive of future governments to spend more on public sector superannuation expenses. The bulk of the super liability relates to past liabilities that are clearly defined. Finally, the government reports its underlying cash surplus exclusive of fund earnings so that they cannot be used for recurrent expenditure.

Moral hazard is a particularly severe form of policy endogeneity that appears to have limited the use of some financial instruments to manage government balance sheets. Traditionally, the economics literature on optimal debt management has focused on 'state-contingent' debt. As early as 1941, inflation-indexed bonds were seen as a means for removing the incentive of governments to inflate the economy and reduce the real value of their obligations (Bach and Musgrave, 1941). However, in the real world there is little evidence of state-contingent debt instruments being issued by governments.[22] This may primarily be due to some state-contingent debt instruments being subject to moral hazard problems (sometimes referred to as 'time inconsistency') if governments can affect the states (Calvo and Guidotti, 1990; Bohn, 1990). For example, bond returns that fall when an index of government expenditure rises would hedge the balance sheet against economic downturns. However, governments would

22 Real world examples include Mexico issuing bonds tied to oil prices and Costa Rica, Bulgaria and Bosnia issuing bonds containing an element of indexation to GDP (Borensztein and Mauro, 2002). Even in Australia, the pool of inflation indexed bonds is relatively small, with outstanding Treasury Indexed Bonds around 10 per cent (or $6.4 billion) of total Commonwealth debt (Commonwealth of Australia, 2002b).

also have an incentive to increase expenditure. This risk would then be priced in the value of such bonds, making them unattractive even for well-intentioned governments to issue.

More recently, the literature has focused on hedging the balance sheet by optimal design of the maturity and denomination of conventional debt securities. For example, a shorter average debt maturity increases the exposure to short term interest rate rises. Long maturities can avoid exposure to 'roll-over' risk (Barro, 1995). It is not necessary, as our results show, to issue state contingent debt to offset specific balance sheet risks. Investing in a broad and diverse range of financial assets effectively eliminates moral hazard type problems.

4.2.2 Agency costs

Second, there are *agency costs* associated with government management of financial assets. In the tax smoothing model, the government is assumed to maximize the welfare of all individuals in the community. However, in practice the incentives of government and the agents used by government may not be so aligned. This can lead to poor investment decisions. The solution to the agency cost problem is to ensure that the governance of the Future Fund is clear and transparent and investments are made on a commercial basis within the investment guidelines set by government. Indeed, applying best corporate practice would allow individual government financial entities to set their own strategic asset allocation, after taking into account the nature of their liabilities (Grimes, 2001). This is likely to improve governance, accountability and entity performance. In the case of the Future Fund, this would involve directing the Fund to invest in assets of a similar risk to the government's superannuation liabilities.

In dealing with agency costs, the government imposes constraints on the optimal portfolio. The significance of these constraints has been highlighted by the results of Fowlie and Wright (1997) for New Zealand. They found that the optimal financial portfolio incorporated foreign currency denominated debt when taxes are included, but only domestic debt when taxes are excluded. This means that a narrow focus only on balance sheet assets can lead to financial investments that increase the chances of volatile tax changes.

However, this does not mean that financial investment decisions should not be decentralized (down to an agency level) or linked to narrow portfolio benchmarks (such as matching financial assets to future superannuation liabilities). Rather, a single central agency needs to be aware of how individual elements of the balance sheet interact with each other during macroeconomic shocks. Some commentators see the centralisation of broader balance sheet risk management with debt management as a 'logical step' (Currie and Velandia-Rubiano, 2002). For example, the Swedish Debt Management Office advises government on the costs of contingent liabilities and the government debt portfolio (Hörngren, 2003a).[23] Such a structure allows natural hedges in the balance sheet to be identified, reducing the need (and costs) from individual agencies hedging. Alternatively, such a balance sheet perspective allows for large cumulative risks to be identified and brought to the attention of government (Wheeler, 2004:67).

4.2.3 Imperfect capital markets

The model discussed in this paper assumes that capital markets are not perfect; or at least that certain restrictions exist that stop governments from using financial instruments to perfectly hedge balance sheet risk. While governments can use some existing financial instruments to reduce balance sheet risk, certain types of risks are still likely to remain unhedged. In particular, incomplete capital markets may mean governments are unable to hedge against certain types of risks (such as catastrophic risk). There may be no private sector substitutes for government bonds (Arrow and Lind 1970, Stiglitz 1983). If capital markets are incomplete, there may be gains from governments issuing standardised products which can outweigh benefits from state-contingent products (Missale, 1997). Alternatively, governments may have other policy objectives, such as maintaining some debt to allow the development of important financial products (Comley and Turvey, 2004). There may also be other reasons for investing domestically. For example, informational, governance or tax advantages may lead to a home country bias for equities (for a review, see Karolyi and Stulz, 2003). Such constraints can limit the ability of government's movement towards the optimal portfolio outlined in this paper.

Bohn (1995) suggests that if capital markets are imperfect, the appropriate policy response is to invest in financial assets to the fullest extent possible using available securities. For the remnant unhedged risks, the government should consider building and maintaining a positive balance of net worth as self-insurance.[24]

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23 Other countries where debt managers are integrated with contingent liability management include New Zealand, South Africa and Colombia (Wheeler, 2004:24).

24 Hansen (2003:11) notes that building a 'precautionary balance' is worthwhile if and only if the unhedged risks would otherwise result in a negative correlation between tax rates and consumption.

5. Conclusion

The economics literature relating to balance sheet management suggests that the government's financial portfolio should be structured to reduce the budget impacts of macroeconomic shocks. More specifically, an optimally structured balance sheet can reduce the risk that a major macroeconomic shock will see large changes in tax rates. This not only reduces the distortions caused by volatile tax rates, but increases the flexibility of governments to respond to unexpected fiscal pressures. A government that invests well has less need to significantly raise taxes or cut spending to finance itself. In most countries, this has meant structuring the debt portfolio so that liabilities do not become overly burdensome during recessions. However, Australia is amongst a small number of countries determining how best to structure a financial portfolio that includes positive net financial assets.[25]

This paper has shown that it is not only the budget position that is important for sustainability, but how the financial assets and liabilities of government are allocated. Our results support the Future Fund investing in a broad range of financial assets that includes nominal domestic debt and equities from selected countries. Indeed, by investing optimally the government is likely to reduce risks on the budget and improve growth prospects.

25 Other OECD countries with significant financial asset funds (including pension funds) are New Zealand, Norway, Ireland, Finland Denmark (see Comley and McKissack, 2005).

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APPENDIX A: DERIVATION OF FIRST ORDER CONDITIONS

The government chooses taxes and debt to maximise the individual objective function (A1) subject to its budget constraint (A2).

$$\begin{aligned} U_t &= E_t \sum_{j \geq 0} \rho^j \left\{ Y_{t+j} \left[1 - h\left(\tau_{t+j}\right) \right] \right\} \\ &= E_t Y_t \left[1 - h\left(\tau_t\right) \right] + \rho E_t \left\{ Y_{t+1} \left[1 - h\left(\tau_{t+1}\right) \right] \right\} + \ldots \end{aligned} \tag{A1}$$

$$\tau_t Y_t + \sum_k p_{t,k} B_{t,k} = G_t + \sum_k \left(p_{t,k} + f_{t,k} \right) B_{t-1,k} \tag{A2}$$

From the budget constraint, we can solve for τ_t , τ_{t+1} , etc, and substitute into the objective function, U_t, which can then be maximised with respect to $B_{t,k}$, $B_{t+1,k}$, etc.

The first order condition with respect to $B_{t,k}$ is:

$$\frac{\partial U_t}{\partial B_{t,k}} = E_t \left\{ Y_t \left[-h'\left(\tau_t\right) \frac{\partial \tau_t}{\partial B_{t,k}} \right] \right\} + \rho E_t \left\{ Y_{t+1} \left[h'\left(\tau_{t+1}\right) \frac{\partial \tau_{t+1}}{\partial B_{t,k}} \right] \right\} = 0 \tag{A3}$$

Now, from the constraints:

$$\frac{\partial \tau_t}{\partial B_{t,k}} = -\frac{p_{t,k}}{Y_t} \qquad \text{and,} \qquad \frac{\partial \tau_{t+1}}{\partial B_{t,k}} = \frac{\left(p_{t+1,k} + f_{t+1.k} \right)}{Y_{t+1}} \tag{A4}$$

Substituting into (A3) gives:

$$E_t \left\{ -h'\left(\tau_t\right) p_{t,k} \right\} + \rho E_t \left\{ h'\left(\tau_{t+1}\right) \left(p_{t+1,k} + f_{t+1,k} \right) \right\} = 0$$

Since $p_{t,k}$ is known at time t, and τ_t is chosen at time t, equation (A4) can be written as:

$$h'\left(\tau_t\right) = \rho E_t \left\{ \frac{h'\left(\tau_{t+1}\right)\left(p_{t+1,k} + f_{t+1,k} \right)}{p_{t,k}} \right\}$$

Recalling that $r_{t+1,k} = (p_{t+1,k} + f_{t+1,k})/p_{t,k} - 1$, and noting also that the condition that expected returns must be equal implies that for the risk free asset (defined as k = 0), $r \equiv 1/\rho - 1$, then the above expression will yield the first-order condition obtained in equation (5) of section 3.

APPENDIX B: DERIVATION OF EQUATION (9)

The optimality condition for each Government security k (k=1,…,K) given in equation (8) is:

$$\sum_l \text{cov}_t\left(\hat{r}_{t+1,k},\hat{r}_{t+1,l}\right)d_{t,l}+\text{cov}_t\left(\hat{r}_{t+1,k},\sum_{j\geq 0}p^j\hat{g}_{t+1+j}\right)-w_t\,\text{cov}_t\left(\hat{r}_{t+1,k},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)=0 \qquad \text{(B1)}$$

As discussed in Section 3 we assume that the second term is equal to zero and so the above can be reduced to:

$$\sum_l \text{cov}_t\left(\hat{r}_{t+1,k},\hat{r}_{t+1,l}\right)d_{t,l}-w_t\,\text{cov}_t\left(\hat{r}_{t+1,k},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)=0 \qquad \text{(B2)}$$

$$\begin{pmatrix}\text{cov}_t\left(\hat{r}_{t+1,1},\hat{r}_{t+1,1}\right) & \cdots & \text{cov}_t\left(\hat{r}_{t+1,1},\hat{r}_{t+1,l}\right)\\ \vdots & \ddots & \vdots\\ \text{cov}_t\left(\hat{r}_{t+1,K},\hat{r}_{t+1,1}\right) & \cdots & \text{cov}_t\left(\hat{r}_{t+1,K},\hat{r}_{t+1,l}\right)\end{pmatrix}\begin{pmatrix}d_{t,1}\\ \vdots\\ d_{t,l}\end{pmatrix}=w_t\begin{pmatrix}\text{cov}_t\left(\hat{r}_{t+1,1},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\\ \vdots\\ \text{cov}_t\left(\hat{r}_{t+1,K},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\end{pmatrix} \qquad \text{(B3)}$$

Rearranging this gives:

$$\begin{pmatrix}d_{t,1}\\ \vdots\\ d_{t,l}\end{pmatrix}=w_t\begin{pmatrix}\text{cov}_t\left(\hat{r}_{t+1,1},\hat{r}_{t+1,1}\right) & \cdots & \text{cov}_t\left(\hat{r}_{t+1,1},\hat{r}_{t+1,l}\right)\\ \vdots & \ddots & \vdots\\ \text{cov}_t\left(\hat{r}_{t+1,K},\hat{r}_{t+1,1}\right) & \cdots & \text{cov}_t\left(\hat{r}_{t+1,K},\hat{r}_{t+1,l}\right)\end{pmatrix}^{-1}\begin{pmatrix}\text{cov}_t\left(\hat{r}_{t+1,1},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\\ \vdots\\ \text{cov}_t\left(\hat{r}_{t+1,K},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\end{pmatrix} \qquad \text{(B4)}$$

This can be simplified with the following notation:

$$\mathbf{d}=w_t\sum\nolimits_r^{-1}\cdot\sum\nolimits_{y,r} \qquad \text{(B5)}$$

In our first estimations, we restrict ourselves to the analysis of domestic and foreign currency debt.

$$\begin{pmatrix}d_d\\ d_f\end{pmatrix}=\frac{w_t}{\Delta}\begin{pmatrix}\text{cov}\left(\hat{r}_{t+1,2},\hat{r}_{t+1,2}\right) & -\text{cov}\left(\hat{r}_{t+1,2},\hat{r}_{t+1,1}\right)\\ -\text{cov}\left(\hat{r}_{t+1,1},\hat{r}_{t+1,2}\right) & \text{cov}\left(\hat{r}_{t+1,1},\hat{r}_{t+1,1}\right)\end{pmatrix}\begin{pmatrix}\text{cov}\left(\hat{r}_{t+1,1},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\\ \text{cov}\left(\hat{r}_{t+1,2},\sum_{j\geq 0}\psi^j\hat{y}_{t+1+j}\right)\end{pmatrix} \qquad \text{(B6)}$$

where

$$\Delta = \begin{vmatrix} \text{cov}\left(\hat{r}_{t+1,1}, \hat{r}_{t+1,1}\right) & \text{cov}\left(\hat{r}_{t+1,2}, \hat{r}_{t+1,1}\right) \\ \text{cov}\left(\hat{r}_{t+1,1}, \hat{r}_{t+1,2}\right) & \text{cov}\left(\hat{r}_{t+1,2}, \hat{r}_{t+1,2}\right) \end{vmatrix}$$

We recall that the domestic and foreign innovation of returns is given by the following equations

$$\begin{aligned} \hat{r}_{t+1,d} &= -\left(\ell_{t+1} - E_t \ell_{t+1}\right) - \left(\pi_{t+1} - E_t \pi_{t+1}\right) \\ \hat{r}_{t+1,f} &= -\left(\ell^*_{t+1} - E_t \ell^*_{t+1}\right) - \left(\pi_{t+1} - E_t \pi_{t+1}\right) + \left(\Delta s_{t+1} - E_t \Delta s_{t+1}\right) \end{aligned}$$

and define $\Delta s = \Delta s_{t+1} - E_t \Delta s_{t+1}$, $\pi = \pi_{t+1} - E_t \pi_{t+1}$, $l = l_{t+1} - E_t l_{t+1}$, $y = \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}$ and denote $Cov(x, y) = c(x, y)$ and $Var(x) = v(x)$. Expanding equation (B6) we get:

$$\begin{pmatrix} d_d \\ d_f \end{pmatrix} = \frac{w_t}{\Delta} \begin{pmatrix} v\left(\Delta s - l^* - \pi\right) c\left(-l - \pi, y\right) - c\left(\Delta s - l^* - \pi, -l - \pi\right) c\left(\Delta s - l^* - \pi, y\right) \\ v\left(-l - \pi\right) c\left(\Delta s - l^* - \pi, y\right) - c\left(-l - \pi, \Delta s - l^* - \pi\right) c\left(-l - \pi, y\right) \end{pmatrix}$$

APPENDIX C: DISAGGREGATED VARIANCE-COVARIANCE MATRIX OF INNOVATIONS

Table C1: Variance-covariance matrix: US and domestic currency debt securities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.873						
π	0.108	0.330					
I_d	0.013	-0.014	0.020				
Δs	0.089	0.635	-0.106	10.895			
I_e	0.004	-0.006	0.010	-0.078	0.008		
r_e	-0.022	0.311	-0.101	10.338	-0.080	10.106	
r_d	-0.120	-0.317	-0.006	-0.529	-0.003	-0.210	0.322

Table C2: Variance-covariance matrix: Japan and domestic currency debt securities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.510						
π	0.072	0.362					
I_d	0.028	-0.006	0.022				
Δs	-0.326	0.181	-0.109	41.692			
I_e	-0.016	-0.010	0.004	-0.063	0.006		
r_e	-0.381	-0.171	-0.107	41.574	-0.060	41.805	
r_d	-0.100	-0.356	-0.017	-0.072	0.006	0.279	0.373

Table C3: Variance-covariance matrix: Germany and domestic currency debt securities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.298						
π	0.009	0.365					
I_d	0.029	-0.010	0.022				
Δs	-0.259	0.529	-0.258	37.293			
I_e	-0.003	0.014	0.007	-0.155	0.008		
r_e	-0.264	0.150	-0.255	36.918	-0.177	36.946	
r_d	-0.038	-0.355	-0.012	-0.271	-0.021	0.106	0.367

Table C4: Variance-covariance matrix: domestic debt and domestic equities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.873						
π	0.108	0.309					
I_d	0.013	-0.014	0.021				
Δs	0.089	0.635	-0.106	10.895			
I_e	0.004	-0.006	0.010	-0.078	0.008		
r_e	-0.022	0.311	-0.101	10.338	-0.080	10.320	
r_d	-0.120	-0.317	-0.006	-0.529	-0.003	-0.210	0.314

Table C5: Variance-covariance matrix: domestic debt and US equities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.298						
π	0.046	0.329					
I_d	0.019	-0.016	0.020				
Δs	-0.104	0.750	-0.103	10.882			
I_e	0.052	-0.680	-0.006	-4.939	59.111		
r_e	-0.098	-0.259	-0.093	5.193	54.851	60.304	
r_d	-0.065	-0.313	-0.004	-0.647	0.686	0.352	0.317

Table C6: Variance-covariance matrix: domestic debt and Japanese equities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.319						
π	0.057	0.337					
I_d	0.023	-0.011	0.022				
Δs	-0.410	-0.113	-0.136	40.406			
I_e	0.290	-0.160	0.231	-8.798	67.711		
r_e	-0.178	-0.609	0.106	31.720	59.072	91.402	
r_d	-0.080	-0.326	-0.011	0.250	-0.072	0.504	0.337

Table C7: Variance-covariance matrix: domestic debt and German equities

	Y	π	I_d	Δs	I_e	r_e	r_d
Y	0.328						
π	0.014	0.323					
I_d	0.033	-0.010	0.022				
Δs	-0.540	0.809	-0.235	31.201			
I_e	0.303	-0.074	-0.087	-37.678	249.916		
r_e	-0.252	0.413	-0.312	-7.286	212.312	204.613	
r_d	-0.047	-0.313	-0.013	-0.574	0.160	-0.101	0.326