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Rethinking Optimal Currency Areas*

V. V. Chari

University of Minnesota
and Federal Reserve Bank of Minneapolis

Alessandro Dovis

Pennsylvania State University

Patrick J. Kehoe

University of Minnesota,
University College London,
and Federal Reserve Bank of Minneapolis

ABSTRACT

The traditional Mundellian criterion, which implicitly assumes commitment, is that countries with similar shocks should form unions. Without commitment a new criterion emerges: countries with dissimilar credibility shocks, namely those that exacerbate time inconsistency problems, should form unions. Critical to this new criterion is that monetary policy is benevolent in that it takes into account the interests of all the countries in the union. With dissimilar credibility shocks, benevolent unions help overcome the time inconsistency problems faced by individual countries. Existing unions can mitigate their time inconsistency problems by admitting new members even when such members have more severe time inconsistency problems because policy adjusts as the composition of the union changes.

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We analyze currency unions in which monetary policy is chosen taking into account the interests of all the countries in the union. We show that even when such a *benevolent union* lacks the ability to commit, it can help overcome the time inconsistency problems faced by individual countries under flexible exchange rates. This insight leads to a new criterion for optimal currency areas with respect to both the shocks that countries face and the observable aggregates. Under this new criterion, it can be optimal for countries with very dissimilar shocks to form unions.

The traditional criterion for forming a union is that countries with similar shocks have the most to gain from forming a union. This criterion stems from the classic analyses of Friedman (1953) and Mundell (1961).¹ The idea is that in a union, by definition, monetary policy cannot be tailored to each country's shocks. This inability to tailor policy is the main cost of forming a union and implies the Mundellian conclusion that countries with similar shocks have the most to gain forming a union and will do so if the benefits from increased trade outweigh the costs. These analyses implicitly assume that the monetary authority can commit to its policies.

We revisit these analyses in a standard sticky price model. With commitment, we show that monetary policy should respond only to a subset of shocks, which we label *Mundellian shocks* and should not respond to other shocks, labeled *credibility shocks*. In our model, the Mundellian shocks correspond to productivity shocks and credibility shocks correspond to markup shocks. These markup shocks affect the degree of monopoly power and therefore the extent of distortions. We show that with commitment, countries with highly positively correlated productivity shocks have the most to gain from forming a union. This conclusion is similar to the Mundellian conclusion in that countries with similar shocks have the most to gain from forming a union, but only with respect to a subset of shocks.

The main focus of our paper is on the desirability of forming a union in environments without commitment. We show that the inability to tailor monetary policy to country-specific shocks, which is the main cost under commitment, can be a benefit without commitment. This insight leads to our first result: groups of countries for which the correlation of markup

¹Dellas and Tavlas (2009) provide a comprehensive discussion of the contributions of other authors to developing these arguments.

shocks is not too positive have the most to gain from forming a union. This finding implies that, in contrast to the standard Mundellian criterion, very dissimilar countries can gain by forming a union. In particular, our criterion is that a group of countries without commitment should form a union if their markup shocks are sufficiently dissimilar in that the volatility of idiosyncratic markup shocks is sufficiently large relative to that of productivity shocks. When expressed in terms of observables rather than shocks, this result implies that forming a union is optimal if the volatility of output is sufficiently large relative to that of real exchange rates.

Since policy in the union takes into account the interests of all member countries, such policy endogenously responds to the composition of the union. This endogenous response yields our second result: an existing union may find it desirable to admit new members whose economies are, on average, more distorted than those of the current members. The endogeneity of this response is also central to our third result: when countries are allowed to form unions freely, the stable outcome is a hierarchical structure in which members of lower-ranked unions would like to join higher-ranked unions, but such higher-ranked unions find it optimal not to admit them.

We obtain our results in a sticky price monetary model that is related to those of Obstfeld and Rogoff (1995), Gali and Monacelli (2005), and especially Farhi and Werning (2013). The economy consists of a continuum of ex ante identical countries, each of which uses labor to produce traded and nontraded goods. The production of nontraded goods is subject to both *productivity shocks* and *markup shocks*, which affect the elasticity of substitution between the varieties of nontraded goods. To keep the analysis simple, we purposefully abstract from the standard sources of gains from a monetary union, namely the reduction in transactions costs in trade. Doing so allows us to focus on the gains from solving credibility problems.

We introduce frictions in the model to capture the idea underlying the Friedman and Mundell analyses and to allow for lack of commitment to affect outcomes by introducing a time inconsistency problem. To capture the Friedman-Mundell idea that under commitment countries gain by having flexible exchange rates, we assume that nontraded goods prices are sticky. Flexible exchange rates allow the relative prices of nontraded to traded goods to

adjust to shocks, even though no individual nontraded goods price can adjust.

We introduce a time inconsistency problem by assuming that nontraded goods are produced by monopolistically competitive firms. Monopoly power implies that the nontraded goods prices carry a markup over expected marginal cost, thereby inducing distortions. These distortions give the monetary authority an incentive to engineer a surprise inflation so as to diminish the effective markup and increase the production of nontraded goods. In order to ensure that an equilibrium exists, surprise inflation must have costs as well as benefits. We introduce costs by assuming that purchases of traded goods must be made with previously acquired money. (See the work of Svensson (1985), Nicolini (1998), and Albanesi, Chari, and Christiano (2003) for a similar device.)

With commitment we show that the best outcome is that all countries adopt flexible exchange rates. In this outcome, exchange rates and monetary policy respond only to productivity shocks and not to markup shocks. Monetary policy responds to productivity shocks in order to ensure that marginal rates of substitution in consumption and production are made as closely as possible. For example, when the idiosyncratic productivity of nontraded goods in a country is high, efficiency requires reducing the relative price of nontraded goods. Since nontraded goods prices are sticky, this relative price reduction requires an increase in the price of traded goods—a devaluation of the exchange rate. Such an efficient adjustment to idiosyncratic productivity shocks is not possible in a union because exchange rates are fixed. Thus, if productivity shocks are not highly correlated, forming a union is costly.

In contrast to productivity shocks, monetary policy should not respond to markup shocks because such responsiveness does not affect distortions and simply induces undesirable fluctuations in inflation. Distortions are not affected because any attempt to do so is defeated by monopolists who simply alter their prices in anticipation of such an attempt. Hence, the union's inability to respond to idiosyncratic markup shocks is not costly. Together, these results imply that under commitment, countries with highly correlated productivity shocks have the most to gain from forming a union and that markup shocks are irrelevant.

The more interesting analysis is what happens when countries have credibility issues. We model lack of commitment by considering a monetary authority that sets its policy in a Markovian fashion. To understand the nature of the time inconsistency problem, consider

what happens under flexible exchange rates after a high markup shock is realized. Since markups are high, the economy is highly distorted and the monetary authority is strongly tempted to generate a surprise inflation. Monopolists anticipate that the monetary authority will generate high inflation and, upon seeing a high markup shock, simply increase their prices. In equilibrium, the increase in the temptation results only in higher inflation. Such inflation reduces the consumption of traded goods, since these goods must be purchased with previously acquired money. Thus, lack of commitment leads to more volatility in both inflation and the consumption of traded goods than under commitment.

To understand the forces determining the desirability of forming a union, consider how monetary policy is set in such a union. Because this authority maximizes the welfare of the union as a whole, it reacts only to unionwide variations in markup shocks. Since monetary policy does not react to the idiosyncratic markup shocks in any given country, traded goods consumption is less volatile and this lower volatility, by itself, raises welfare. Since monetary policy does not react to productivity shocks, however, the economy does not adjust efficiently to productivity shocks. This lack of adjustment, by itself, lowers welfare. These forces imply our criterion for forming a union: a group of countries should form a union if the volatility of idiosyncratic markup shocks is large relative to that of productivity shocks.

Using our model, we express our criterion in terms of observables rather than shocks. Doing so allows us to relate our work to a large empirical literature that has asked whether certain countries are good candidates for forming a union by looking at the behavior of macroeconomic aggregates such as the idiosyncratic components of output and real exchange rates. (For a recent survey, see the work of Silva and Tenreyro (2010).) Building on the Mundellian conclusion, the standard view in the literature is that countries are poor candidates for forming a monetary union if the variances of the idiosyncratic components of output and real exchange rates are large.

Viewed through the lens of our model, this standard view can be misleading: even when the variances of the idiosyncratic components of output and real exchange rates are both high, forming a union may be desirable. In particular, without commitment the optimal currency area criterion is simple: form a union if and only if the volatility of output relative to that of the real exchange rate is sufficiently high.

We turn to the analysis of the optimal configuration of unions when countries are asymmetric. This analysis highlights the role of the endogenous response of policy to the composition of the union. We imagine one group of countries, called the *North*, has already formed a union and is choosing the number of countries from the *South* to let in. We assume that the South is more distorted than the North in that their markup shocks are both larger on average and more variable than those in the North. We show that if the correlation between the markup shocks in the North and the South is not too positive, the North will admit some Southern countries. The key idea here is that admitting some Southerners into the union may be beneficial for the North because of the resulting changes in monetary policy. When the distortions are imperfectly correlated, the benevolent monetary authority's policy decisions become less sensitive to fluctuations in the aggregate distortions in the North.

We use our model to ask what configurations of unions are stable in the sense that no group of countries can deviate and profitably form their own union. We show that all countries rank unions the same way. This common ranking implies that the stable configuration of unions has a hierarchical form in that every country would like to join any union above its current one in the hierarchy. Higher-ranked unions choose not to admit new members because doing so will alter their policies in a way that lowers welfare.

Thus far we have considered unions in which policy endogenously responds to the interests of all members. An alternative literature considers a very different type of union in which policy responds only to the interests of one of its members. (See the work of Friedman (1973), Alesina and Barro (2002), Alesina, Barro, and Tenreyro (2003), Clerc, Dellas, and Loisel (2011), and Monacelli (2003).) This type of union can be thought of as one in which small countries, called *clients*, adopt the currencies of large stable countries, called *anchors*. The key assumption of this work on *anchor-client unions* is that the anchors decide their policy without regard to the interests of the clients. We briefly analyze anchor-client unions in our model and find a result similar to that in the Mundellian analysis: clients should adopt anchors whose productivity shocks are most similar to their own, and the correlation of markup shocks between anchor and client is irrelevant. Of course, if many clients joined one anchor, then the key assumption becomes less tenable and our analysis with endogenous policy becomes relevant.

Other Related Literature

The idea that delegating policy to other agents can help solve time inconsistency problems dates back at least to the work of Rogoff (1985). Anchor-client unions are a vivid example of this type of delegation: the client simply delegates policy to the anchor. Forming benevolent unions can also be interpreted as a type of delegation. The key difference between our work and Rogoff's is that in our work the objective function of the delegated agent is endogenously pinned down by the composition of the union rather than being exogenously given.

Aguiar et al. (2014) analyze the configuration of unions when policy endogenously responds to the composition of the union. In their model, high debt countries gain by forming unions with a mixture of high and low debt countries, but the low debt countries lose by doing so. In contrast, in our study of the optimal configuration of unions, we require that all countries must gain from forming a union.

Devereux and Engel (2003) adopt an alternative pricing system in which prices are set in the currency of the importing country, referred to as local currency pricing. They show that with such pricing, the Mundellian gains to flexible exchange rates disappear. Their paper can be interpreted as an argument for forming a union if monetary authorities can commit to their policies. Our argument for forming a union, in contrast, depends critically on how forming a union can help improve credibility.

Our model abstracts from gains to monetary unions arising from the need to overcome the externalities that are generated in some models when countries pursue independent monetary policy. For example, in the work of Cooley and Quadrini (2003), raising the nominal interest rate reduces the demand for foreign goods and thereby reduces the terms of trade. This channel is similar to that in the optimal tariff literature. Cooper and Kempf (2001 and 2004) and Fuchs and Lippi (2006) emphasize the role of free-rider problems when regions pursue independent monetary policies. Finally, we have abstracted from externalities arising from the interactions of monetary and fiscal policies in unions. See, for example, the work of Canzoneri, Cumby, and Diba (2006) and Beetsma and Uhlig (1999). We have abstracted from all of these issues in order to focus on the role that currency unions can play in overcoming credibility concerns.

In our model, we assume that countries that form a union cannot leave it. With our shock structure, we need only assume that countries that form a union cannot leave it until the end of the current period. For analyses with endogenous exit, see the work of Fuchs and Lippi (2006) and Alvarez and Dixit (2014).

1. A Monetary Economy

Our monetary economy builds on the work of Obstfeld and Rogoff (1995), Gali and Monacelli (2005), and especially Farhi and Werning (2013). The economy consists of a continuum of countries, each of which produces traded and nontraded goods and in which consumers use currency to purchase goods. The traded goods sector in each country is perfectly competitive. The nontraded goods sector consists of a continuum of firms, each of which produces a differentiated product. The production function for each of the nontraded goods producers is subject to both aggregate and country-specific shocks to *productivity* and to the elasticity of substitution between the varieties of nontraded goods. We refer to these latter shocks as *markup shocks*. Traded goods have flexible prices and are bought with cash, whereas nontraded goods have sticky prices and are bought with credit. We have purposefully chosen the ingredients of our model so that it captures key forces and is otherwise as simple as possible.

A. Environment

In each period t , an i.i.d. aggregate shock $z_t = (z_{1t}, z_{2t}) \in Z$ is drawn, and each of a continuum of countries draws a vector of idiosyncratic shocks $v_t = (v_{1t}, v_{2t}) \in V$ which are i.i.d. both over time and across countries. The probability of aggregate shocks is $f(z_{1t}, z_{2t}) = f^1(z_{1t})f^2(z_{2t})$, and the probability of the idiosyncratic shocks is $g(v_{1t}, v_{2t}) = g^1(v_{1t})g^2(v_{2t})$. Here Z and V are finite sets. We let $s_t = (s_{1t}, s_{2t})$ with $s_{it} = (z_{it}, v_{it})$ and let $h(s_t) = h^1(s_{1t})h^2(s_{2t})$ with $h^i(s_{it}) = f^i(z_{it})g^i(v_{it})$. These aggregate and idiosyncratic shocks are to the nontraded goods sector and affect the elasticity of substitution between goods in this sector denoted $\theta(s_{1t})$ and referred to as markup shocks and the productivity in this sector denoted $A(s_{2t})$. We let s^t denote the history of these shocks and $h_t(s^t)$ the corresponding probability, and we use similar notation for any components of these shocks. We will use the

notation

$$E_v(\theta|z_1) = \sum_{v_1} g^1(v_1)\theta(z_1, v_1) \text{ and } E_v(A|z_2) = \sum_{v_2} g^2(v_2)A(z_2, v_2)$$

to denote the means of θ and A conditional on the aggregate shocks and use similar notation for other random variables.

The timing of events within a period is the following: first the markup shocks are realized, then the sticky price firms make their decisions, then the productivity shocks are realized, and then the monetary authority chooses its policy, and then consumers and flexible price firms make their decisions.

In all that follows, we will identify a country by its history of idiosyncratic shocks $v^t = (v_0, \dots, v_t)$. This identification assumes a type of symmetry in that all countries with the same history of idiosyncratic shocks receive the same allocations.

Production of Traded and Nontraded Goods

Consider first the production of traded and nontraded goods. The production function for traded goods in a given country is simply $Y_T(s^t) = L_T(s^t)$ where $Y_T(s^t)$ is the output of traded goods and $L_T(s^t)$ are the inputs of labor in the traded goods sector. The problem of traded goods firms is then to solve

$$(1) \quad \max_{L_T(s^t)} P_T(s^t)L_T(s^t) - W(s^t)L_T(s^t)$$

so that in equilibrium

$$(2) \quad P_T(s^t) = W(s^t).$$

The nontraded good in any given country is produced by a competitive final consumption firm using $j \in [0, 1]$ intermediates according to

$$(3) \quad Y_N(s^t) = \left[\int y_N(j, s^t)^{\theta(s_{1t})} dj \right]^{1/\theta(s_{1t})}.$$

This firm maximizes

$$P_N(s^{t-1}, s_{1t})Y_N(s^t) - \int P_N(j, s^{t-1}, s_{1t})y_N(j, s^t)dj,$$

where the notation makes clear that, consistent with our timing assumption, the prices of nontraded goods cannot vary with s_{2t} . The demand for an intermediate of type j is thus given by

$$(4) \quad y_N(j, s^t) = \left(\frac{P_N(s^{t-1}, s_{1t})}{P_N(j, s^{t-1}, s_{1t})} \right)^{\frac{1}{1-\theta(s_{1t})}} Y_N(s^t).$$

The intermediate goods are produced by monopolistic competitive firms using a linear technology:

$$(5) \quad y_N(j, s^t) = A(s_{2t})L_N(j, s^t).$$

The problem of an intermediate good firm of type j is to choose $P = P(j, s^{t-1}, s_{1t})$ to solve

$$(6) \quad \max_P \sum_{s_{2t}} Q(s^t) [P - W(s^t)] \left(\frac{P_N(s^t)}{P} \right)^{\frac{1}{1-\theta(s_{1t})}} Y_N(s^t)$$

subject to (5) where $Q(s^t)$ is the nominal stochastic discount factor. Throughout we will assume that $\theta(s_{1t}) \in (0, 1)$, so that the induced demands are elastic and that the optimal price for the monopolist is finite. The solution to this problem gives that for all intermediate goods producers j ,

$$(7) \quad P_N(j, s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \frac{\sum_{s_{2t}} Q(s^t) Y_N(s^t) \frac{W(s^t)}{A(s_{2t})}}{\sum_{s_{2t}} Q(s^t) Y_N(s^t)},$$

where $1/\theta(s_{1t})$ is the markup in period t . Since this price does not depend on j , we can write $P_N(j, s^{t-1}, s_{1t}) = P_N(s^{t-1}, s_{1t})$. This result implies that the labor hired by each intermediate goods firm is the same, so that $L_N(j, s^t)$ can be written as $L_N(s^t)$ and the final output of

nontraded goods is simply

$$(8) \quad Y_N(s^t) = A(s_{2t})L_N(s^t).$$

Consumers and the Government

The consumers in any given country have preferences given by

$$(9) \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t h_t(s^t) U(C_T(s^t), C_N(s^t), L(s^t)),$$

where $C_T(s^t)$ is the consumption of traded goods, $C_N(s^t)$ is the consumption of the nontraded good, and $L(s^t)$ is labor supply. In most of our analysis we will specialize preferences to be

$$(10) \quad U(C_T, C_N, L) = \alpha \log C_T + (1 - \alpha) \log C_N - bL.$$

The critical feature of these preferences is their quasi-linearity in labor, which allows us to obtain useful aggregation results along the lines of Lagos and Wright (2005).

Consumers are subject to a cash-in-advance constraint that requires them to buy traded goods at t using domestic money brought in from period $t - 1$, namely $M(s^{t-1})$, so that

$$(11) \quad P_T(s^t)C_T(s^t) \leq M(s^{t-1}).$$

The budget constraint of the consumer is given by

$$(12) \quad \begin{aligned} & P_T(s^t)C_T(s^t) + P_N(s^{t-1}, s_{1t})C_N(s^t) + M(s^t) + B(s^t) \\ & \leq W(s^t)L(s^t) + M(s^{t-1}) + (1 + r(s^{t-1}))B(s^{t-1}) + T(s^t) + \Pi(s^t), \end{aligned}$$

where $T(s^t)$ are nominal transfers, $\Pi(s^t)$ are the profits from the nontraded goods firms, $r(s^t)$ is the nominal interest rate in the domestic currency, and $B(s^t)$ are nominal bonds.

Under our preferences (10) and our shock structure, countries have no incentive to borrow and lend to each other so that in equilibrium $B(s^t) = 0$. The first order conditions

for the consumer are summarized by

$$(13) \quad \frac{U_N(s^t)}{P_N(s^{t-1}, s_{1t})} = -\frac{U_L(s^t)}{W(s^t)}$$

$$(14) \quad \frac{U_T(s^t)}{P_T(s^t)} = -\frac{U_L(s^t)}{W(s^t)} + \phi(s^t)$$

$$(15) \quad -\frac{U_L(s^t)}{W(s^t)} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{U_T(s^{t+1})}{P_T(s^{t+1})}$$

$$(16) \quad \frac{1}{1+r(s^t)} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1}, s_{1t})}{U_N(s^t)},$$

where $\phi(s^t) \geq 0$ is the (normalized) multiplier on the cash-in-advance constraint. Notice also that the nominal stochastic discount factor for the country is

$$(17) \quad Q(s^{t+1}) = \beta h(s^{t+1}|s^t) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1}, s_{1t})}{U_N(s^t)},$$

where $Q(s^t)$ is the price of a state-contingent claim to local currency units at s^t in units of local currency at s^t . This is the relevant price that firms use to discount profits in (6). The monetary authority's budget constraint is simply that newly created money is transferred to consumers in a lump-sum fashion:

$$(18) \quad T(s^t) = M(s^t) - M(s^{t-1}).$$

In this economy, policies can be described as a sequence of interest rates, money supplies, and transfers that satisfy (16) and (18). In terms of what follows, either we can let the monetary authority choose a nominal interest rate policy and let nominal transfers and money growth be endogenously determined, or we can let the monetary authority choose money growth rates and let interest rates and transfers be endogenously determined.

An *equilibrium with flexible exchange rates* is a set of allocations, prices, and policy such that given initial conditions M_{-1}, B_{-1} , i) the decisions of consumers are optimal, ii) the

decisions of firms are optimal, iii) the labor market clears in each country,

$$(19) \quad L_N(s^t) + L_T(s^t) = L(s^t),$$

iv) the traded and nontraded goods markets clear,

$$(20) \quad C_T(s^t) = Y_T(s^t), \quad C_N(s^t) = Y_N(s^t),$$

v) the monetary authority's budget constraint (18) holds.

So far we have expressed each country's prices in units of its own currency. Since the law of one price holds for traded goods, we can write the (multilateral) nominal exchange rate between a particular country and all others as

$$(21) \quad e(s^t) = \frac{P_T(s^t)}{\int_{v^t} P_T(z^t, v^t) g^t(v^t) dv^t},$$

where $g^t(v^t) = g(v_0) \dots g(v_t)$ and the term on the denominator is the simple average over all countries, where countries are identified by the history of their idiosyncratic shocks v^t . Because of the law of large numbers, the exchange rate for a given country does not depend on the realization of idiosyncratic shocks for any countries other than the given country.

We model a monetary union as the restriction that the nominal exchange rate $e(s^t) = 1$ for all s^t , but otherwise we let the rest of monetary policy differ across countries. Using (21) this restriction implies that if one country has a history $s^t = (z^t, v^t)$ and another has history $\tilde{s}^t = (z^t, \tilde{v}^t)$, then for any s^t and \tilde{s}^t ,

$$(22) \quad P_T(s^t) = P_T(\tilde{s}^t).$$

An *equilibrium in a monetary union* is defined analogously to an equilibrium with flexible exchange rates with the added restriction that $e(s^t) = 1$ for all s^t .

2. Optimal Policy with Commitment

We turn now to analyzing optimal policy under flexible exchange rates and in a monetary union. We will show that the lack of monetary independence in a monetary union

imposes a loss on member countries and leads to our modified version of Mundell's optimal currency area criterion: countries should form a union only if the idiosyncratic component of productivity shocks is small enough.

We start by defining the *Ramsey problem* for a country under *flexible exchange rates*. The problem is to choose allocations, prices, and policy given initial conditions $\{M_{-1}, B_{-1}\}$ to maximize consumer utility (9) subject to the consumer and firm first order conditions and the resource constraints.

In a monetary union the price for traded goods cannot vary with idiosyncratic shocks. The *Ramsey problem* in a *monetary union* can thus be written as choosing allocations, prices, and policy to maximize an equally weighted sum of the utilities over all countries subject to the consumer and firm first order conditions and the resource constraints and the additional constraint (22). Note that because countries are ex ante identical, the objective functions in the two Ramsey problems are the same.

Since the Ramsey problem under flexible exchange rates is a more relaxed version of the Ramsey problem in a monetary union, we have the following result.

Proposition 1. The Ramsey problem under flexible exchange rates leads to higher welfare than the Ramsey problem in a monetary union.

We turn now to characterizing the Ramsey allocations. We will show that the distortions from monopoly can be captured by a single constraint on the Ramsey problem. To obtain this constraint, substitute for $W(s^t)$ and $Q(s^t)$ from the consumer first order conditions into (7) to get the *markup condition*:

$$(23) \quad \sum_{s_{2t}} h(s^t | s^{t-1}, s_{1t}) C_N(s^t) \left[U_N(s^t) + \frac{1}{\theta(s_{1t})} \frac{U_L(s^t)}{A(s_t)} \right] = 0.$$

Thus, the Ramsey problem under flexible exchange rates reduces to a sequence of static problems of choosing allocations to maximize expected utility in period t subject to the resource constraints and the markup condition (23).

The Ramsey problem in a union reduces to a similar sequence of static problems with the additional constraint that arises from fixed exchange rates. Combining (13), (14), and (22) and comparing two histories $s^t = (z^t, v^{t-1}, v_{1t}, v_{2t})$ and $\tilde{s}^t = (z^t, v^{t-1}, v_{1t}, \tilde{v}_{2t})$ gives the

union constraint:

$$(24) \quad \frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)} \text{ for all } v_{2t}, \tilde{v}_{2t}.$$

We turn now to comparing the Ramsey allocations and prices under flexible exchange rates with those in a monetary union for our preferences (10). (In all that follows, we assume this specification for preferences.) The consumption of traded goods in both regimes is the same and is given by $C_T = \alpha/b$. The consumption of nontraded goods under flexible exchange rates and a union are given by

$$(25) \quad C_N^{flex}(s) = \frac{(1-\alpha)}{b} \theta(s_1) A(s_2) \text{ and } C_N^{union}(s) = \frac{1-\alpha}{b} \frac{\theta(s_1)}{E_v(1/A|z)}.$$

From the characterization of the allocations, it follows that the expected value of labor supply is equal across regimes so that the difference in utility in the regimes is that due to the differences in the consumption of nontraded goods. It then follows that whenever the idiosyncratic component of productivity shocks has strictly positive variance, the utility under flexible exchange rates is greater than it is in a monetary union.

Our first main result is that the variability of the idiosyncratic component of productivity shocks plays a key role in determining the costs of forming a union and that markup shocks are irrelevant in determining these costs.

Proposition 2. The utility difference between the two regimes is given by

$$(26) \quad (1-\alpha) E_z \left[\log E_v \left(\frac{1}{A} | z \right) - E_v \left(\log \frac{1}{A} | z \right) \right] > 0.$$

Clearly, (26) is strictly positive since the log function is a concave function. We find it useful to consider the simple case in which $A(v_2, z_2) = A_v(v_2)A_z(z_2)$ and $A_v(v_2)$ is log normal with mean μ_v and variance σ_v^2 . Here the utility difference (26) reduces to $(1-\alpha)\sigma_v^2/2$ so that the losses in forming the union are increasing in the volatility of the idiosyncratic productivity shocks. Note that markup shocks play no role in determining the utility difference between the two regimes.

One way to gain intuition for Proposition 2 is to recall the classic argument of Friedman (1953) that flexible exchange rate systems are desirable because changes in the exchange rate can be used to mimic the price changes that would have occurred if prices in the economy were flexible rather than sticky. Friedman’s argument applies directly to this environment. To apply this argument, consider a flexible price version of our economy in which monopolists set prices after the productivity shock is realized rather than before. With our preferences (10), the flexible price allocations under the Friedman rule are also the Ramsey allocations for the sticky price economy. (For a similar result in different models, see the work of Correia, Nicolini, and Teles (2008) and Devereux and Engel (2003).) That is, it is indeed desirable to run the flexible exchange rate system to reproduce the flexible price allocations under the Friedman rule. To implement these allocations, the relative price of nontraded to traded goods must move with the productivity shock. Since doing so is not feasible in a monetary union, welfare is lower.

To see how flexible exchange rates allow the relative price of traded to nontraded goods to move with country-specific productivity shocks, note that the price of traded goods under flexible exchange rates and in a union are given by

$$(27) \quad p_T^{flex}(s_{2t}) = \kappa A(s_{2t}) \text{ and } p_T^{union}(z_{2t}) = \frac{\kappa}{E_v(1/A(z_{2t}, v_{2t}))},$$

where κ is a number sufficiently small so that the cash-in-advance constraint is not binding in any state and we have normalized all prices by the relevant beginning-of-period money stock. These prices imply that the resulting exchange rate

$$e(s) = \frac{A(z_2, v_2)}{E_v(A|z_2)}$$

depreciates whenever the country-specific component of productivity is high.

Proposition 2 implies an optimal currency area criterion expressed in terms of shocks: under commitment countries are good candidates for forming a union if their productivity shocks move closely together while the comovement of markup shocks is irrelevant. This criterion represents a refinement of the standard Mundellian criterion. Here the source of the shocks is critical; some shocks are important, whereas others are irrelevant even though they

contribute to aggregate fluctuations.

In empirical work, the optimal currency area criterion is expressed in terms of observables instead of shocks. As we argue below, our refinement implies a very different optimal currency area criterion in terms of observables than does the traditional criterion.

3. Optimal Policy without Commitment

Consider now the same environment except that the monetary authorities cannot commit. We model this lack of commitment as having these authorities choose policies in a Markovian fashion.

Recall that in the environment with commitment, markup shocks play no role in determining the costs or benefits of forming a monetary union. In contrast, in the environment without commitment, markup shocks play a critical role in determining these costs and benefits. In particular, the more variable are markup shocks, the larger are the gains from forming a monetary union. It turns out that the productivity shocks play similar roles with and without commitment. To focus on the role of markup shocks, we assume for most of what follows that productivity is constant across countries and time. Under this assumption, there are only first stage shocks and, hence, for simplicity we write (z_1, v_1) as (z, v) .

The timing is the same as before. The first stage—the *sticky price stage*—occurs at the beginning of the period after the markup shocks associated with (z, v) have been realized. At this stage, the sticky price firms make their decisions. At the next stage—the *policy stage*—monetary policy is set. Then at the *consumer stage*, the consumer and the flexible price firms make their decisions.

A. Flexible Exchange Rates

Under flexible exchange rates, the allocations and prices in any given country are not affected by the decisions of consumers, firms, and the monetary authorities in other countries. This observation allows us to focus on a particular country in isolation. We begin by describing the state variables for the sticky and flexible price firms, the consumers, and the monetary authority. We normalize all nominal variables by the beginning-of-period aggregate stock of money M_{-1} in the given country. With this normalization the normalized aggregate money stock is 1.

Here we set up the equilibrium recursively, which is easiest to do so by working backward from the end of a period. When flexible price firms and consumers choose their decisions, they are confronted with an aggregate state $x_H = (s, p_N, \mu)$ where $p_N = P_N/\bar{M}_{-1}$ denotes the normalized nontraded goods price and μ the growth rate of money. The *flexible price firm state* is simply x_H , and the flexible price firm solves a problem similar to (1). The solution to this problem implies that

$$(28) \quad p_T(x_H) = w(x_H).$$

Next consider the consumer's problem. Each consumer has, in addition to the aggregate state x_H , an individual state $m_H = M_H/M_{-1}$ consisting of that consumer's normalized level of money, so that the *consumer's state* is (m_H, x_H) . The consumer's problem is

$$(29) \quad V(m_H, x_H) = \max_{C_T, C_N, L, m'_H} U(C_T, C_N, L) + \beta \sum_s h(s') V(m'_H, x'_H)$$

subject to the cash-in-advance constraint in normalized form:

$$p_T(x_H)C_T \leq m_H$$

and the budget constraint in normalized form:

$$(30) \quad p_T(x_H)C_T + p_N C_N + \mu m'_H \leq m_H + w(x_H)L + [\mu - 1] + \pi(x_H),$$

where the term $\mu m'_H = (M'_H/M)(M/M_{-1})$ and $x'_H = (s', p_N(s'), \mu(s', p_N(s')))$. This problem defines the consumer decision rules. We denote the consumer decision rule for the consumption of the traded good C_T as $C_T(m_H, x_H)$ and use similar notation for other consumer choices. Note that these consumer decision rules depend explicitly on the current nontraded goods price p_N , the choice of the monetary authority μ and implicitly on the future rules for nontraded goods prices $p_N(s')$, and the rule governing monetary policy $\mu(s', p_N(s'))$.

The monetary authority chooses its policies after the sticky price firms have chosen their prices, so that the *monetary authority state* is $x_G = (s, p_N)$. The problem of the

monetary authority is to choose μ to maximize consumer welfare, that is, to solve

$$(31) \quad W(x_G) = \max_{\mu} V(1, (x_G, \mu)).$$

In this problem the monetary authority implicitly takes as given the decision rules of private agents and that, in equilibrium, the normalized money balances $m_H = 1$. Note that the monetary authority also takes as given the decision rules of the monetary authority in the future $\mu(x'_G)$ as well as the decision rules of firms.

Finally, the *sticky price firm state* at the beginning of the period is simply the exogenous shock s . Taking as given the prices set by other sticky price producers $p_N(s)$, an individual sticky price firm solves

$$(32) \quad \max_{p_N} [p_N - w(x_H)] \left(\frac{p_N(s)}{p_N} \right)^{\frac{1}{1-\theta(s_1)}} C_N(m_H, x_H),$$

where $m_H = 1$ and $x_H = (s, p_N(s), \mu(s, p_N(s)))$ is the consumer state that this firm forecasts. In equilibrium the resulting pricing rule for nontraded goods is

$$(33) \quad p_N(s) = \frac{1}{\theta(s)} \frac{w(x_H)}{A}.$$

A *Markov equilibrium under flexible exchange rates* consists of a pricing rule for nontraded goods $p_N(s)$, a profit rule $\pi(x_H)$, the monetary authority's policy rule $\mu(x_G)$ and value function $W(x_G)$, consumer decision rules $C_N(m_H, x_H)$, $C_T(m_H, x_H)$, $L(m_H, x_H)$, $m'_H(m_H, x_H)$ and value function $V(m_H, x_H)$, a wage rate rule $w(x_H)$, and a price rule for traded goods $p_T(x_H)$, such that i) the sticky price firms and the flexible price firms maximize profits, ii) the monetary authority maximizes consumer welfare, iii) the consumer maximizes welfare, iv) markets clear in that $C_N(1, x_H) = AL_N(x_H)$, $C_T(1, x_H) = L_T(x_H)$, $L(1, x_H) = L_N(x_H) + L_T(x_H)$, and the money market clears in that $m'_H(1, x_H) = 1$.

To characterize this equilibrium, note that maximization by the consumer and the flexible firm, together with market clearing, implies that

$$(34) \quad L = C_T + \frac{C_N}{A}$$

$$(35) \quad \frac{U_N}{p_N} = -\frac{U_L}{p_T}$$

$$(36) \quad \frac{U_T}{p_T} \geq -\frac{U_L}{p_T}$$

$$(37) \quad p_T C_T \leq 1,$$

where if the cash-in-advance constraint (37) is a strict inequality, then (36) holds as an equality, and

$$(38) \quad -\mu \frac{U_L}{p_T} = \beta \sum_{s'} h(s') \frac{U_T(1, x'_H)}{p_T(x'_H)}.$$

Here $U_T(1, x'_H)$ is evaluated using the consumer decision rules evaluated at $m'_H = 1$ and $x'_H = (s', p_N(s'), \mu(s'), p_N(s'))$. Note that in (37) and (38) we have used that money market clearing implies that $m_H = 1$ and $m'_H = 1$.

Now consider the problem faced by a monetary authority in choosing its policy. We find it convenient to write the problem in primal form in the sense that we think of this authority as directly choosing prices and allocations subject to the first order conditions of firms and consumers and the resource constraints. Under flexible exchange rates, the policy rule $\mu(x_G)$ is part of a Markov equilibrium if it solves the *primal Markov* problem

$$(39) \quad W^{flex}(x_G) = \max_{p_T, C_T, C_N, L, \mu} U(C_T, C_N, L) + \beta \sum_s h(s') W^{flex}(x'_G)$$

subject to (34)–(38). Note that the primal Markov problem takes the future decision rule for monetary policy as given and obtains a current decision rule. In equilibrium, the current and future decision rules must coincide.

Next we show that the monetary authority's problem is static. To see this result, note that x'_G and x'_H are determined by future decision makers and are independent of the current money growth rate choice. Therefore, the continuation value $W^{flex}(x'_G)$ and the right side of (38), namely the expected marginal utility of one unit of normalized money tomorrow, are also independent of the current money growth rate choice. Since μ appears only in (38), we can use this constraint to eliminate μ as a choice variable and solve the *static primal Markov*

problem of maximizing current period utility $U(C_T, C_N, L)$ subject to (34)–(37). We can think of this problem as determining the *best response of the monetary authority* $p_T^{flex}(s, p_N)$ to a given choice of nontraded goods price by the nontraded goods firms, and then given this best response, we can determine C_T, C_N , and L from the constraints.

Consider now the problem of the sticky price producers. Substituting for the wage rate from (28) in the pricing rule for nontraded goods (33) gives $p_N(s) = p_T(x_H)/\theta(s)A$ where $x_H = (s, p_N(s), \mu(s, p_N(s)))$. Hence, in any equilibrium, the price of traded goods only varies with s , and the equilibrium *outcome* which can be written as $\bar{p}_T(s)$ at state s must be a fixed point of

$$(40) \quad \bar{p}_T(s) = p_T^{flex} \left(s, \frac{\bar{p}_T(s)}{\theta(s)A} \right).$$

Once we have the fixed point $\bar{p}_T(s)$, the rest of the equilibrium *outcomes* $\bar{C}_T(s)$ and $\bar{C}_N(s)$ are given from the constraints on the monetary authority's problem. (Here the bars help distinguish outcomes, which vary only with shocks, from decision rules which also vary with the endogenous state variables.)

To understand the trade-offs confronting the monetary authority in the primal Markov problem, note that, for a given price of nontraded goods p_N , raising the price of traded goods has a marginal benefit because it reduces the markup distortion by moving the marginal rate of substitution closer to the marginal rate of transformation. If the cash-in-advance constraint is not binding, then raising this price has no cost and the solution is to set $p_N = p_T/A$ so that the marginal rate of substitution between traded and nontraded goods equals the marginal rate of transformation between these goods. Such an outcome cannot be an equilibrium because sticky price producers forecasting this policy response will set the price of nontraded goods at a markup over wages, or equivalently, over the price of traded goods, so that $p_N = p_T/\theta A$. Thus, the cash-in-advance constraint must be binding in equilibrium, and raising the price of traded goods has a marginal cost because it reduces the consumption of traded goods. At the Markov equilibrium, the benefits and costs from raising the price of traded goods are equated.

Using our preferences (10) and the result that the cash-in-advance constraint binds in

a Markov equilibrium, the static primal Markov problem can be written as follows. Given p_N and θ , choose (C_N, C_T, p_T) to solve

$$(41) \quad \max \alpha \log C_T + (1 - \alpha) \log C_N - b [C_T + C_N/A]$$

subject to

$$(42) \quad C_T = \frac{1}{p_T} \quad \text{and} \quad C_N = \frac{1 - \alpha}{b} \frac{p_T}{p_N}.$$

The best response of the monetary authority $p_T^{flex}(s, p_N)$ clearly depends only on p_N and is given by

$$(43) \quad p_T = F\left(\frac{1}{Ap_N}\right)$$

for a quadratic function F defined in the Appendix. Since $p_N = p_T/\theta A$, the equilibrium outcome $\bar{p}_T(s)$ solves the fixed point problem $\bar{p}_T(s) = F(\theta(s)/\bar{p}_T(s))$, and given $\bar{p}_T(s)$ the equilibrium outcomes $\bar{C}_T(s)$ and $\bar{C}_N(s)$ are given from the constraints (42).

Here we need to bound the markups from above to guarantee that a Markov equilibrium exists. Briefly, if the gains to the monetary authority of inflating in order to reduce the distortion ex post are sufficiently high, then no matter what the level of the price of nontraded goods p_N , the monetary authority will always have an incentive to raise the price of traded goods and no equilibrium exists. Mechanically, in this case (43) does not have a positive fixed point. It turns out that an equilibrium exists only if the markups $1/\theta(s)$ are not too large in that they satisfy

$$(44) \quad \frac{1}{\theta(s)} < \frac{1 - \alpha}{1 - 2\alpha} \quad \text{for all } s.$$

For the rest of the paper, we will assume without further mention that the bounds in (44) hold.

Solving the fixed point problem for $\bar{p}_T(s)$, we then have the following lemma.

Lemma 1. The allocations in the Markov equilibrium with flexible exchange rates

are given by

$$(45) \quad C_T^{flex}(s) = \frac{\alpha}{b} - \frac{1-\alpha}{b}(1-\theta(s)) \quad \text{and} \quad C_N^{flex}(s) = \frac{1-\alpha}{b}\theta(s)A$$

and $L(s) = C_T(s) + C_N(s)/A$.

Now consider comparing the commitment outcomes to the no commitment outcomes under flexible exchange rates when productivity is constant in both. From (25) and (45) we see that the consumption of nontraded goods is identical. In contrast, the consumption of traded goods differs: under commitment it is α/b , and under no commitment it is given in (45).

We now show that the time inconsistency problem worsens when markup shocks become more volatile. Inspecting the allocations under commitment and no commitment gives that the expected difference in utilities with and without commitment is

$$(46) \quad \alpha [\log \alpha - E \log(\alpha - (1-\alpha)(1-\theta(s)))] .$$

Since the log function is a concave function, we have that a mean-preserving spread in θ increases this expected difference.

Proposition 3. Under flexible exchange rates, a mean-preserving spread in θ worsens the time inconsistency problem in that the differences in welfare between the commitment and the no commitment outcomes increase.

Here a mean-preserving spread in θ makes the consumption of traded goods more volatile and, because preferences are concave, lowers utility. Briefly, as markups become more volatile, the prices set by the monopolists become more volatile. The monetary authority reacts to this higher volatility by making inflation and, hence, the traded goods prices more volatile. Since the cash-in-advance constraint is binding, this increase in volatility in prices increases the volatility of traded goods. In equilibrium, the attempt by the monetary authority to undo the monopoly distortions is frustrated, and all the monetary authority accomplishes is an increase in the volatility of traded goods consumption. In short, an increase in the volatility of either aggregate or idiosyncratic markup shocks exacerbates the time inconsistency problem.

B. Monetary Union

To set up the equilibrium in the monetary union recursively, we follow the same procedure as we did with flexible exchange rates: we define the state that confronts each decision maker and then define policies and decision rules as functions of the state. Here the natural normalization for all nominal variables is the beginning-of-period aggregate money stock for the union as a whole. To construct such a unionwide money stock, we start with an arbitrary measure Λ over country-level money stocks M_{-1} and define the *unionwide money stock* as

$$\bar{M}_{-1} = \int M_{-1} d\Lambda(M_{-1}).$$

Here the state variables at a given stage in the period for a decision maker consist of a complete description of the relevant states of such decision makers in the union, that is, a measure over all such states.

Consider, for example, a sticky price firm in a given country. The country-specific state x_F consists of the money stock of that country relative to the unionwide money stock, $m = M_{-1}/\bar{M}_{-1}$, together with the idiosyncratic shock v . The aggregate state at this stage of the period is $S_F = (z, \lambda_F)$ where z is the aggregate shock and λ_F is a measure over the states of all the sticky price firms in the rest of the union. Thus, the *sticky price firm state* is (x_F, S_F) , and the sticky price firm's normalized decision rule is $p_N(x_F, S_F)$. At the time the monetary authority chooses its policy, each country's state is given by $x_G = (m, p_N, v)$ and the *monetary authority's state* is $S_G = (z, \lambda_G)$ where λ_G is a measure over the states x_G in all countries. The consumer's state and the flexible price firms' state are defined in a similar fashion.

We define a Markov equilibrium in a nearly identical fashion to that under flexible exchange rates. After following steps very similar to those we followed under flexible exchange rates, we can set up the primal Markov problem in a monetary union. Since countries are indexed by x_G , we think of this authority as choosing a money growth rate for each country, $\mu(x_G, S_G)$, and we use similar notation for other variables. These money growth rates imply a unionwide money growth rate $\gamma(S_G)$. Since in a union the price of traded goods does not vary with the country, we think of the monetary authority in each aggregate

state as choosing a common price p_T for all countries. Thus, suppressing for a moment the dependence of current allocations on the aggregate state S_G , the primal Markov problem is to choose $x = (p_T, C_T(x_G), C_N(x_G), L(x_G), \mu(x_G))$ to solve

$$(47) \quad W^{union}(S_G) = \max_x \int U(C_T(x_G), C_N(x_G), L(x_G)) d\lambda_G + \beta \sum_s h(s') W^{union}(x'_G, S'_G)$$

subject to

$$(48) \quad \frac{U_N(x_G)}{p_N} = -\frac{U_L(x_G)}{p_T}$$

$$(49) \quad \frac{U_T(x_G)}{p_T} \geq -\frac{U_L(x_G)}{p_T}$$

$$(50) \quad p_T C_T(x_G) \leq m,$$

where if (50) is a strict inequality, then (49) holds as an equality, and

$$(51) \quad \gamma \frac{-U_L(x_G)}{p_T} = \beta \sum_{s'} h(s') \frac{U_T(m', x'_H, S'_H)}{p_T(x'_H, S'_H)}$$

$$(52) \quad L(x_G) = C_T(x_G) + \frac{C_N(x_G)}{A}$$

for all $x_G = (m, p_N, v)$. These constraints capture the market clearing conditions and first order conditions for all the consumers in the union.

Under our preferences, this problem can be simplified because it turns out that the Markov equilibrium has a degenerate distribution for money holdings across countries.

Lemma 2. In any Markov equilibrium in a monetary union, given any initial distribution of money at the beginning of the period, the end-of-period money holdings are concentrated on a single point.

The proof of this lemma has two ideas.

For the first idea, note that the consumer first order condition (51) implies that the marginal cost of earning one unit of money today must be equated to the expected marginal utility that money provides when used to purchase traded goods tomorrow. Since preferences are quasi-linear in labor and nominal wages are equal across countries in the union, this first

order condition implies that the expected marginal utility from one unit of money tomorrow must also be equal across countries. If, however, consumers have differing levels of money balances at the end of the period and the cash-in-advance constraint binds in at least one state in the next period, then these consumers have different expected marginal utility from one unit of money tomorrow. This argument yields a contradiction.

The second idea is that combining the incentives of the monetary authority to correct monopoly distortions, together with the incentives of the monopolists to set their prices at a markup over expected marginal costs, implies that the cash-in-advance constraint is always binding for reasons similar to those under flexible exchange rates. Combining these two ideas gives Lemma 2.

Lemma 2 implies that regardless of the distribution of money holdings in period 0, the distribution of money holdings in all future periods is degenerate. In keeping with our assumption that all countries are ex ante identical, we assume that the initial distribution is also degenerate. Then the normalized level of money balances m is one in each country in all periods. (Of course, the absolute level of money balances will typically be changing over time.) Thus, as in the flexible exchange rate case, we can drop m from the individual state, and thus λ_G is a distribution only over (p_N, v) . Since S'_G, x'_H, S'_H are determined by agents in the future and are independent of the choice of current policy, the continuation value and the right side of (51) are also independent of the current money growth rate choice. Since γ appears only in (51), we can use this constraint to eliminate γ as a choice variable and drop this condition also.

Here, the beginning-of-period aggregate state is a distribution of nontraded good prices $\{p_N(s)\}$ and an aggregate shock z . Given this state, the primal Markov problem becomes

$$(53) \quad \max_{C_T(s), C_N(s), p_T} \sum_v g(v) \left[\alpha \log C_T(s) + (1 - \alpha) \log C_N(s) - b \left(C_T(s) + \frac{C_N(s)}{A} \right) \right]$$

subject to

$$C_T(s) = \frac{1}{p_T} \text{ and } C_N(s) = \frac{1 - \alpha}{b} \frac{p_T}{p_N(s)} \text{ for each } s = (z, v),$$

where we have imposed the result that the cash-in-advance constraint is binding. Let the maximized value of this problem be denoted $U(\{p_N(z, v)\}, z)$. Because policy in the monetary union is chosen to maximize an equally weighted sum of utility of all countries, the weights $g(v)$ in the summation in (53) represent the fraction of all countries with idiosyncratic realization v . Since this fraction also represents the probability that an individual country will experience an idiosyncratic realization v , the maximized value $U(\{p_N(z, v)\}, z)$ is also the expected utility for any individual country.

Solving this problem gives the best response of the monetary authority to any given $\{p_N(s)\}$ and z , which can be written as $p_T = p_T^{Union}(\{p_N(s)\}, z)$. It turns out that this best response only depends on a simple summary statistic of the distribution of nontraded goods prices, namely $E(1/p_N(s)|z)$, the conditional mean of the inverse of these prices. We can then write the best response as

$$(54) \quad p_T^{Union}(\{p_N(s)\}, z) = F\left(E\left(\frac{1}{Ap_N(s)}|z\right)\right)$$

for the same quadratic function F defined under flexible exchange rates in (43). In equilibrium, the price of traded goods must satisfy the following fixed point equation:

$$(55) \quad \bar{p}_T(z) = F\left(E\left(\frac{\theta(s)}{\bar{p}_T(z)}|z\right)\right).$$

Using this value, it is easy to solve for the rest of the allocations from the constraints.

Lemma 3. The allocations in the Markov equilibrium in a monetary union satisfy

$$(56) \quad C_T^{union}(s) = \frac{\alpha}{b} - \frac{1-\alpha}{b}(1 - E_v(\theta|z)) \quad \text{and} \quad C_N^{union}(s) = \frac{1-\alpha}{b}\theta(s)A$$

and $L(s) = C_T(s) + C_N(s)/A$ where $s = (z, v)$.

Note that here the normalized price of traded goods is given by $p_T^{union} = 1/C_T^{union}(s)$ and hence depends on the average of the markup shocks in the union.

The analog of Proposition 3 applies here: a mean-preserving spread in the aggregate component of θ worsens the time inconsistency problem in that the differences in welfare between the commitment and the no commitment outcomes increase. Here, however, a mean-

preserving spread to the idiosyncratic component of θ has no effect on the time inconsistency problem because in a union, policy does not react to idiosyncratic shocks.

C. Comparing Welfare

We now compare welfare under flexible exchange rates with that under a monetary union. We show that with only markup shocks, forming a union is beneficial and these benefits are increasing in the variability of idiosyncratic shocks. We then introduce productivity shocks and show that if the idiosyncratic volatility of productivity shocks is sufficiently small relative to that of markup shocks, then a monetary union is preferred to flexible exchange rates.

Only Markup Shocks

Comparing the allocations (45) with those in (56), we see that the allocations under flexible exchange rates differ from those in a monetary union only with respect to the consumption of the traded good and the labor needed to produce it. Using the expressions for tradable and nontradable consumption under the two regimes in the objective function and simplifying, we see that the difference in value for a given initial aggregate state z between the welfare in a union and that under flexible exchange rates is

$$(57) \quad K(E[\theta|z]) - E[K(\theta)|z],$$

where the function $K(\theta) = \alpha \log((1 - \alpha)(\theta - 1) + \alpha)$. Since the function $K(\theta)$ is strictly concave in θ , the welfare difference between the regimes is nonnegative and is strictly positive whenever there is variability in the idiosyncratic shock ν .

Proposition 4. With only markup shocks, the ex ante utility in the Markov equilibrium for a monetary union is strictly higher than the ex ante utility in the Markov equilibrium with flexible exchange rates.

The idea behind this proposition is that because of concavity of preferences over traded consumption goods, the ex ante welfare associated with the Markov equilibrium in a monetary union is higher than that under flexible exchange rates. Clearly, a mean-preserving spread in the idiosyncratic component of θ increases the gains from forming a union.

Interestingly, inflation rates are not only less volatile but also lower on average in a

union than they are under flexible exchange rates. To see this result, consider the inflation rates in the tradable and nontradable sectors from state s at one date to state s' at the next. Under flexible exchange rates, these inflation rates are given by

$$\pi_T^{flexible}(s, s') = G(\theta(s)) \text{ and } \pi_N^{flexible}(s, s') = \frac{\theta(s)}{\theta(s')}G(\theta(s)),$$

and in the union they are given by

$$\pi_T^{union}(s, s') = G(E(\theta|z)) \text{ and } \pi_N^{union}(s, s') = \frac{\theta(s)}{\theta(s')}G(E(\theta|z)),$$

where $G(\theta) = \beta\alpha / [(1 - \alpha)\theta - (1 - 2\alpha)]$. The convexity of G implies that in a monetary union, inflation not only is less volatile than it is under flexible exchange rates but also is lower on average. This lower and less volatile inflation rate is beneficial because it results in distortions in the consumption in the tradable good that are on average lower and less volatile.

Both Shocks

When we allow for both markup shocks and productivity shocks, we have two competing forces. Forming a union has *commitment benefits*: doing so effectively commits the country to not react to the idiosyncratic component of its markup shocks. But forming a union also has *Mundellian losses*: doing so also prevents the country from reacting to the idiosyncratic component of its productivity shocks. Our main result is a new optimal currency area criterion.

Proposition 5. When the volatility of markup shocks is sufficiently high relative to that of productivity shocks, the commitment benefits are higher than the Mundellian losses and forming a union is preferable to flexible exchange rates. In contrast, when the reverse is true, flexible exchange rates are preferred to a union.

The first part of this result immediately follows from Proposition 4 and continuity of the equilibrium values in the parameters of the model. Except for one subtlety, the proof of the second part mimics the argument with commitment.

It is useful to develop a simple approximation that allows us to determine how large

markup shocks must be relative to productivity shocks for a union to be beneficial. The approximation is needed because when productivity shocks are stochastic, the Markov equilibrium does not have a closed-form solution. We approximate the gains to forming a union using a first order log-linear approximation of the decision rules in both regimes, adjusting the mean as in the work of Kim and Kim (2003). In particular, we make sure that our approximation is exact for the case in which we have a closed-form solution for prices, allocation, and welfare (see Lemma 3 and Proposition 4). Under this approximation, the welfare gains of forming a monetary union are given by

$$(58) \quad u_\theta \sigma_\theta^2 - u_A \sigma_A^2,$$

where u_θ and u_A are positive constants given by $u_\theta = \alpha\phi^2/(1-\phi)^2/2$ and

$$u_A = [(1-\alpha)(1-(\phi-1)^2) - \alpha\phi^2]/2$$

and σ_θ^2 and σ_A^2 are the idiosyncratic variances of $\log \theta$ and $\log A$. Here ϕ is given by $\phi \equiv -F'/(Fp_NA) > 0$ where F is defined in (43) and ϕ is evaluated at the deterministic steady state.² Here ϕ measures the elasticity of the monetary authority's best response to a change in price by the monopolists. Specifically, it is the percentage increase in the price of traded goods by the monetary authority for a 1 percentage increase in the price of nontraded goods by the monopolists.

Consider now the welfare gains that result from forming a union. The first term in (58) represents the commitment gains of a monetary union: entering a union allows the country to avoid reacting to idiosyncratic markup shocks, which simply add unwanted volatility to the consumption of traded goods. The second term in (58) represents the standard Mundellian losses associated with the inability to respond to productivity shocks. Thus, there is a cutoff $r = u_\theta/u_A$ such that forming a union is preferable to staying with flexible exchange rates if and only if $\sigma_A^2/\sigma_\theta^2 < r$.

We complement this expression with Figure 1, which gives the exact solution for the

²The deterministic steady state is given by $p_N = p_T/\theta A$ and $p_T = b/[\alpha - (1-\alpha)(1-\theta)]$.

value of utility in a Markov equilibrium under the two regimes as we vary the relative volatility of the idiosyncratic component of the productivity shock in the nontradable sector.³ The figure illustrates that there is a cutoff level on the relative variances of these shocks, $\sigma_A^2/\sigma_\theta^2$, such that it is preferable to form the union if and only if these relative variances are below this level.

4. Criteria in Terms of Macroeconomic Aggregates

So far we have stated our criterion in terms of properties of the stochastic processes for productivity and markups. A large empirical literature has examined whether certain countries are good candidates for forming a union by looking at the behavior of simple functions of standard macroeconomic aggregates such as the idiosyncratic components of output and real exchange rates. The standard view in the literature is that countries are poor candidates for forming a monetary union if the variances of the idiosyncratic components of output and real exchange rates are large. (See, for example, the work of Alesina, Barro, and Tenreyro (2003) and the references therein.)

In this section we argue that through the lens of our model, this standard view can be misleading: both with and without commitment, even when the variances of the idiosyncratic components of output and real exchange rates are both high, it may be desirable to form a union. The key reason our model gives a different prediction from the standard view is that our model implies that the desirability of forming a union depends critically on the source of the shocks, even if these shocks induce similar volatilities in real exchange rates and outputs, whereas the standard view makes no such distinction.

For example, if under commitment a group of countries have large idiosyncratic movements in real exchange rates and output, these countries are good candidates for forming a union if these movements are driven mostly by markup shocks and poor candidates if they are driven by productivity shocks. Thus, one subtlety is that we need a criterion that is based on observables but can differentiate between these two scenarios. The added subtlety is that the map between observables and shocks is itself a function of the stand we take on commitment:

³We parameterize the model by considering a simple case with no aggregate shocks: $\theta(\nu_1) \in \{1.1, 1.2\}$ and $A(\nu_2) \in \{1 - \varepsilon, 1 + \varepsilon\}$ where $g^1(\nu_1)$ and $g^2(\nu_2)$ are uniform and we vary $\varepsilon \geq 0$.

under commitment, policy in countries is not reacting to markup shocks, whereas under no commitment, it is.

To translate our criterion on shocks into a criterion on macroeconomic aggregates, we use our model to express output and real exchange rates as functions of shocks and use these functions to rewrite our criterion in terms of observables. We begin by relating output and real exchange rates to the consumption and prices of traded and nontraded goods. To do so, note that we can write output and real exchange rates relative to their world averages in log-deviation form as

$$(59) \quad \log y(s) = \alpha \log C_T(s) + (1 - \alpha) \log C_N(s) - E_v [\alpha \log C_T(z, v) + (1 - \alpha) C_N(z, v)]$$

$$(60) \quad \log q(s) = (1 - \alpha) \log p_N(s)/p_T(s) - (1 - \alpha) E_v [\log p_N(z, v)/p_T(z, v)],$$

where the second equation is derived in the Appendix. In order to make clear the role of the idiosyncratic components of the shocks, we assume in what follows that the productivity shocks and the markup shocks can be expressed as multiplicative functions of an aggregate component and an idiosyncratic component and that the two components are independent of each other. Specifically, we assume that $A(z, v) = A_z(z) A_v(v)$ and $\theta(z, v) = \theta_z(z) \theta_v(v)$.

Next, suppose that the countries are contemplating forming a union with commitment and currently are in a flexible exchange rate regime pursuing the Ramsey policies. Here $p_N/p_T = 1/(A(s)\theta(s))$, $C_T = \alpha/b$, and C_N is given by (25). Since shocks have a multiplicative form, we have that

$$(61) \quad \log y(s) = (1 - \alpha) [\log \theta_v(v) + \log A_v(v)]$$

$$(62) \quad \log q(s) = -(1 - \alpha) [\log \theta_v(v) + \log A_v(v)].$$

Thus, given any observed volatility of the idiosyncratic components of output and real exchange rates, Proposition 2 makes clear that large welfare losses are associated with forming a union only if most of the volatility in these variables is arising from the productivity shocks. Clearly, since only the sum of the idiosyncratic shocks enters these two expressions, we cannot disentangle the separate roles of each shock from output and real exchange rates alone.

Interestingly, under commitment we can use a simple statistic to infer the volatility of

productivity shocks: the volatility of the growth rate of the nominal exchange rate. To see why, note that in log-deviation form we have

$$(63) \quad \log e(s')/e(s) = [\log P_T(s') - E_v \log P_T(z', v')] - [\log P_T(s) - E_v \log P_T(z, v)].$$

Under a regime of flexible exchange rates in which countries are pursuing Ramsey policies, we can rewrite (63) as

$$(64) \quad \log e(s')/e(s) = \log A_v(v').$$

Hence, under commitment, the Mundellian costs associated with moving from a regime of flexible exchange rates to a union are proportional to the idiosyncratic variance of the country's nominal exchange rates before it enters the union.

Next, suppose that the countries are contemplating forming a union without commitment and currently are in a flexible exchange rate regime pursuing Markov policies. As we show in the Appendix, without commitment the nominal exchange rate is no longer particularly useful. Instead, we use a log-linear approximation of the Markov outcomes for output and real exchange rates. Using (59) and (60), we obtain

$$(65) \quad \log y(v) = \left[1 - \alpha \left(\frac{1 - 2\phi}{1 - \phi} \right) \right] \log \theta_v(v) + (1 - 2\alpha)\phi \log A_v(v)$$

$$(66) \quad \log q(v) = -(1 - \alpha) \log \theta_v(v) - (1 - \alpha)\phi \log A_v(v),$$

where ϕ is the elasticity of the monetary authority's best response discussed above. Clearly, these two expressions can be solved to express the variances of the idiosyncratic shocks in terms of the variances of the endogenous variables. Doing so gives that the welfare gains of a monetary union relative to flexible exchange rates are given by

$$w_y \text{var}(y) - w_q \text{var}(q),$$

where w_y and w_q are positive constants defined in the Appendix. This expression for welfare gains implies that without commitment there is a simple cutoff rule in terms of the standard

macroeconomic aggregates: forming a union is optimal if and only if the relative volatility of output to real exchange rates is sufficiently high in that

$$(67) \quad \text{var}(y) / \text{var}(q) > w_q / w_y.$$

Here, of course, these volatilities must be calculated from a regime of flexible exchange rates in which countries are following their Markov policies.

The criterion developed in (67) is novel and stands in contrast to all of those developed in the literature on optimal currency unions. We note that we have derived this criterion from first principles using a simple sticky price model in the spirit of Obstfeld and Rogoff (1995) and Gali and Monacelli (2005).

5. The Optimal Configuration of Unions

So far we have assumed that all countries are symmetric and studied their incentives to form a monetary union rather than stay under a regime of flexible exchange rates. Here we introduce asymmetry by assuming that one group of countries (the *North*) is less distorted than another group of countries (the *South*) in that the South's distortions are both larger on average and more variable than those in the North. We imagine that the countries in the North have already formed a union and are choosing the number of countries from the South to let in.

Our main result is that if the distortions in the South are not perfectly correlated with those in the North, then as long as the average distortions in the South are not too large, the North will find it optimal to admit some Southerners into their union. The key idea here is that even if each country in the South has a worse time inconsistency problem than each country in the North, admitting some Southerners into the union may be beneficial for the North because the imperfect correlation of distortions leads monetary policy to be less sensitive to fluctuations in the aggregate distortions in the North.

In terms of comparative statics, we find that the North will admit fewer countries from the South the greater are the South's mean distortions, the greater is the variance of these distortions, and the greater is the correlation of their distortions with those in the North. We end with a brief analysis of a stable configuration of unions.

More formally, we imagine there are two groups of countries, North, N , and South, S , with a measure \bar{m}^N of Northern countries and a measure of Southern countries \bar{m}^S . Here we focus on an economy with only markup shocks, and we let the markup shocks in the North be $\theta^N(s_t)$ and the markup shocks in the South be $\theta^S(s_t)$. These shocks are realized at the beginning of the period (and, as before, we drop the subscript 1 denoting the beginning of the period for simplicity). Throughout we assume that the Southern countries are *more distorted* than the North in that

$$(68) \quad E\theta^S \leq E\theta^N \text{ and } \text{var}(\theta^S) \geq \text{var}(\theta^N).$$

To interpret this condition, note that with only markup shocks we can combine the first order condition of the monopolist with that of private agents to see that

$$-\frac{U_L}{U_N} = A\theta(s_t) < A$$

so that $1 - \theta$ is the *wedge* between the marginal rate of substitution between labor and nontraded goods and the corresponding marginal rate of transformation. Thus, our condition that Southern countries are more distorted implies that the South has wedges that are both larger on average and more volatile than those in the North. We will assume that (68) holds in all of our comparisons below.

We turn to asking whether a union of Northern countries should admit Southern countries. The Northern countries understand that if they let in a measure m^S of Southern countries, then the policy followed in the mixed union will be one that maximizes a weighted average of the utility of the Northern and Southern countries where the weights are proportional to group size in that

$$(69) \quad \lambda^N = \frac{\bar{m}^N}{\bar{m}^N + m^S}, \quad \lambda^S = \frac{m^S}{\bar{m}^N + m^S},$$

so that the resulting vector $\lambda = (\lambda^N, \lambda^S)$ satisfies $\lambda^i \in [0, 1]$ and $\lambda^N + \lambda^S = 1$. For now, we assume that the Southern countries are originally under flexible exchange rates and will form the union only if they receive higher utility in the union than under flexible exchange rates.

To determine the size of the union, we begin by solving for the Markov equilibrium and the welfare of the Northern and Southern countries for any given composition of the union. We then ask what composition maximizes the welfare of the Northern countries given that the Southern countries that join the union must be made better off by doing so.

Given a particular composition of the union (λ^N, λ^S) , we solve for the allocations and welfare for the Northern and Southern countries. We start by solving for the policy in the mixed union for some given composition. To do so, note that here the distribution of money holdings is degenerate since the analog of Lemma 2 applies and the cash-in-advance constraint binds in both the North and the South. Thus, the problem for the union in the aggregate state z given $\{p_N^N(s), p_N^S(s)\}$, is $U(\{p_N^N(s), p_N^S(s)\}, z) =$

$$\max_{C_T^i(s), C_N^i(s), p_T} \sum_{i=N,S} \lambda^i \sum_v g(v) \left[\alpha \log C_T^i(s) + (1 - \alpha) \log C_N^i(s) - b \left(C_T^i(s) + \frac{C_N^i(s)}{A^i} \right) \right]$$

subject to

$$C_T^i(s) = \frac{1}{p_T} \text{ and } C_N^i(s) = \frac{1 - \alpha}{b} \frac{p_T}{p_N^i(s)} \text{ for each } s = (z, v).$$

Notice that, as remarked earlier, the summation over v in the objective function is not because there is any uncertainty to be realized but rather because it simply represents the utilitarian welfare weights. The resulting allocations are summarized in the next lemma.

Lemma 4. With constant productivity A^N, A^S , the allocations in the Markov equilibrium in a monetary union with composition λ are

$$(70) \quad C_T^i(s, \lambda) = \frac{\alpha}{b} - \frac{1 - \alpha}{b} \left(1 - \sum_{i=N,S} \lambda^i E_v(\theta^i | z) \right), \quad C_N^i(s, \lambda) = \frac{1 - \alpha}{b} \theta^i(s) A^i$$

and $L^i(s, \lambda) = C_T^i(s, \lambda) + C_N^i(s, \lambda)/A^i$ for $i = N, S$ where $s = (z, v)$.

We use the allocations in this lemma to construct the expected welfare of both Southern and Northern countries for a given composition:

$$(71) \quad W^i(\lambda) = \alpha E \log C_T^i(s, \lambda) + (1 - \alpha) E \log C_N^i(s, \lambda) - b E L^i(s, \lambda).$$

Note that the allocations imply that Northern and Southern countries rank different compositions the same way: if the North prefers composition $\hat{\lambda}$ to λ , then so does the South. The reason is simply that the North and the South have the same stochastic process for traded goods consumption and have stochastic processes for nontraded goods consumption that are independent of the composition of the union.

We then turn to asking what is the optimal measure of Southern countries that the North finds optimal to admit to the union. Formally, this problem is to solve

$$(72) \quad \max_{\lambda} W^N(\lambda)$$

subject to the feasibility constraint

$$(73) \quad \lambda^N \geq \frac{\bar{m}^N}{\bar{m}^N + \bar{m}^S}$$

and the participation constraint of Southern countries $W^S(\lambda) \geq W_{flex}^S$ where W_{flex}^S is defined from the allocations under flexible exchange rates given in Lemma 1. We will assume that \bar{m}^S is sufficiently large compared with \bar{m}^N so that the feasibility constraint (73) does not bind. It is straightforward to prove that if the Southern countries are more distorted in the sense of (68), then they always prefer joining the union with the North to staying on their own. Hence, we drop the participation constraints in all that follows.

We begin by supposing that the mean distortions in the North and the South are the same, in that $E(\theta^N) = E(\theta^S)$. Clearly from (70) and (71), the optimal composition of the union simply maximizes the expected utility from the consumption of traded goods. Defining the random variable $x(s, \lambda) = \sum_{i=N,S} \lambda^i E_v(\theta^i | z)$, the problem reduces to one of maximizing the expected value of $\log(\alpha - (1 - \alpha)(1 - x))$. Up to a second order approximation, the optimal composition of the union minimizes the variance of x and thus solves

$$\min_{\lambda^N, \lambda^S} (\lambda^N)^2 \sigma_N^2 + (\lambda^S)^2 \sigma_S^2 + 2\lambda^N \lambda^S \rho \sigma_N \sigma_S$$

subject to $\lambda^N + \lambda^S = 1$. Solving this problem yields the following proposition.

Proposition 6. Assume $E(\theta^N) = E(\theta^S)$ and (68) holds. If $\rho < \sigma_N/\sigma_S$, then

$$(74) \quad \lambda^S = \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_S^2 + \sigma_N^2 - 2\rho\sigma_N\sigma_S},$$

and if not, $\lambda^S = 0$. Furthermore, the measure of Southerners admitted to the union decreases with the volatility of the markup shocks σ_S in the South and with the correlation ρ of the Southern shocks with the Northern shocks.

This proposition shows that when mean distortions in the North and the South are the same, then the North will always admit some Southern countries if the correlation between the distortions is not too positive. Since the outcomes are continuous in the parameters, this proposition implies that even when the mean distortions are somewhat larger in the South, the North will admit some Southern countries if the correlation between distortions is not too positive.

To get a sense of the magnitudes involved, assume that the Southern and Northern shocks are uncorrelated so that $\rho = 0$. Then if the variances of these shocks are equal, it is optimal to have a union in which half of its members are Southern, whereas if the variance of the Southern shocks are double those in the North, this fraction is one-third.

Now we ask what configurations of unions will form in this model. We will focus on configurations that are *stable* in the sense that there is no deviation by a group of countries to form their own union that makes all of the members of the deviating group weakly better off and at least one type of them strictly better off.⁴ Clearly, if two unions have the same composition, then they will pursue identical policies and hence their exchange rates will be fixed. Thus, we can consider any pair of unions with the same composition as constituting one union.

In developing our analysis, we will use the result that all countries rank unions with different compositions in the same way. Hence, our economy has a hierarchy of unions: a

⁴More formally, let $\{m_i\}_{i=1}^I$ with $m_i = (m_i^N, m_i^S)$ with $\sum_i m_i^N = \bar{m}^N$ and $\sum_i m_i^S = \bar{m}^S$ and $m_i^N + m_i^S > 0$ for each i be a *partition* of the union, and let $V_i = (V^N(m_i), V^S(m_i))$ be the associated welfare. A configuration $\{m_i\}_{i=1}^I$ is *stable* if there exists a deviating group of countries $\{\hat{m}_i\}_{i=1}^I$ with $\hat{m}_i \leq m_i$ such that $V^N(\sum \hat{m}_i) \geq V_i^N$ for all i such that $\hat{m}_i^N > 0$ and $V^S(\sum \hat{m}_i) \geq V_i^S$ for all i such that $\hat{m}_i^S > 0$, where at least one of these previous inequalities is strict.

most preferred union, a second most preferred union, and so on. When there are two types of countries, say North and South as above, then there is a unique stable configuration of unions: a preferred union, which is a mixed North-South union with the mixture chosen as above, say at $\hat{\lambda}$, and a less preferred union. If there are sufficiently many Southern countries so that the feasibility constraint (73) holds, then the less preferred union consists purely of Southern countries. If this constraint is violated, then there are not enough Southerners to reach this optimal mix when all the Northerners are included in the preferred union and the less preferred union consists purely of Northern countries. In either case, since the mixed union maximizes the welfare of both types of countries, neither type has an incentive to defect. We summarize this discussion as follows.

Proposition 7. Under (68) and (73), a mixed North-South union with λ chosen to solve (72) and a pure Southern union consisting of the remainder Southern countries is the unique stable configuration of unions. The mixed union has higher welfare for both countries than the pure Southern union.

We briefly consider a more general case with three groups of countries: North, Middle, and South. Let these groups be ranked in a pecking order in that mean distortions and volatilities are increasing from South to North, whereas the volatilities are increasing from North to South.

The stable configuration of unions has a simple hierarchy form and can be constructed as follows. In the highest-ranked union, the weights $\lambda_1 = (\lambda_1^N, \lambda_1^M, \lambda_1^S)$ maximize $W^N(\lambda)$, whereas in the second-ranked union, the weights $\lambda_2 = (0, \lambda_2^M, \lambda_2^S)$ maximize $W^M(\lambda)$ subject to the restriction that $\lambda^N = 0$. In the third-ranked union, the weights are $\lambda_3 = (0, 0, 1)$. It is straightforward to construct the masses of countries (m_i^k) in each of these groups. In the construction we assume that the measure of countries is such that \bar{m}^M/\bar{m}^N and \bar{m}^S/\bar{m}^M are sufficiently large so that the configuration we construct is feasible. In the Appendix we discuss a more general case.

Proposition 8. If \bar{m}^M/\bar{m}^N and \bar{m}^S/\bar{m}^M are sufficiently large, the configuration $\lambda_1, \lambda_2, \lambda_3$ given above is the unique stable configuration of unions. Furthermore, at the stable configuration, $W^i(\lambda_1) > W^i(\lambda_2) > W^i(\lambda_3)$ for $i = N, M, S$.

6. Anchor-Client Union

So far we have considered environments in which the defining feature of a monetary union is that monetary decisions are made jointly by members of the union. We turn now to analyzing a very different kind of monetary union that we call an *anchor-client union*. The anchor chooses monetary policy solely to maximize the welfare of its residents, and the client maintains a fixed exchange rate with the anchor. Such a union is nearly identical to one in which the client country *dollarizes*.⁵

A key distinction between this institutional arrangement and our benevolent union setup is that here there is no connection between the composition of the union and the policy followed by it. In contrast, when we considered a Northern union that admitted a positive measure of Southern countries, the union's policy endogenously changed as the composition of the union changed. This lack of endogenous feedback implies that in an anchor-client union, the correlation of the markup shocks between the anchor and client is irrelevant.

Proposition 9. The ranking of potential anchors is independent of the correlation of the markup shocks of the client and the potential anchor.

We then turn to the characterization of the ideal anchor for a given client. The answer is immediate: the ideal anchor is the country that follows the policies that the client country would follow if it had commitment. Obviously, when the client adopts the policy of such an ideal anchor, it achieves its own Ramsey welfare level and cannot do better. The anchor that achieves the Ramsey outcomes for the client is one that has productivity shocks that are identical to those of the client and either has commitment or follows Markov policies and has no distortions, in that $\theta^i \equiv 1$.

Next, suppose that such an ideal anchor is not available but instead there are I potential anchors, all of which either have commitment (or follow Markov policies and have $\theta^i \equiv 1$). Let $\{\theta(s_{1t}), A(s_{2t})\}$ denote the stochastic processes of the client, and let $\{\theta^i(s_{1t}), A^i(s_{2t})\}$ for $i = 1, \dots, I$ denote the processes for the potential anchors. Within this class, the best anchor is the one with a stochastic process for productivity shocks that is closest to that of the client, in the sense made precise in the following proposition.

⁵The only distinction is that in an anchor-client union, the client gets to keep the seignorage; under dollarization, it does not.

Proposition 10. Consider a given client country with stochastic process $\{\theta(s_{1t}), A(s_{2t})\}$. The optimal anchor country i^* from a set of potential anchors that have commitment is the one that solves

$$(75) \quad \min_i \log \left(E \left[\frac{A^i(s_{2t})}{A(s_{2t})} \right] \right) - E \left[\log \frac{A^i(s_{2t})}{A(s_{2t})} \right],$$

which in the log-normal case minimizes the variance of the ratio $A^i(s_{2t})/A(s_{2t})$.

Notice that (75) holds for general specifications of the stochastic processes for the client and the anchor. If we assume that the processes for productivity shocks have the form $A_i(s_{2t}) = A_z(z_{2t})A_{vi}(v_{2t})$ and $A(s_{2t}) = A_z(z_{2t})A_v(v_{2t})$ so that the anchors and the client have a common aggregate component to productivity shocks, then the optimal anchor i^* solves

$$\min_i \log (E[A_{vi}(v_{2t})]) - E[\log A_{vi}(s_{2t})],$$

which in the log-normal case implies that it is optimal to pick the anchor with the lowest variance of idiosyncratic shocks.

We turn now to the optimal choice of an anchor when the anchor follows Markov policies and the set of potential anchors does not include one with no distortions. For simplicity, assume that the set of potential anchors all have the same mean distortions $E\theta^i$ but have different variances. To focus on the commitment gains, we also suppose that there are only markup shocks. Using a second order approximation for welfare gives the following proposition.

Proposition 11. Consider a given client country with stochastic process $\{\theta(s_{1t})\}$. The optimal anchor country i^* from the set of potential anchors with $E\theta^i$ at the same level is the one that has the smallest variance $var(\theta^i)$ for its distortions.

In sum, in an anchor-client union, the lack of feedback between the composition of the union and the policies pursued by the union makes the selection of the best anchor simple: find a country with small and stable distortions that has highly correlated productivity shocks. In contrast, the crux of our analysis of benevolent unions hinges on this endogenous feedback.

7. Conclusion

The key theme in the existing literature on currency unions is that countries with similar shocks should form a union. This theme is pervasive not only in the original contributions of Friedman (1953) and Mundell (1961) on benevolent optimal currency areas that builds on it, but also in the work on anchor-client unions by Friedman (1973), Alesina and Barro (2002), Alesina, Barro, and Tenreyro (2003), and Clerc, Dellas, and Loisel (2011).

Our contribution is to show that when countries suffer from credibility problems, forming a benevolent union may be more desirable the more dissimilar they are with respect to the shocks that exacerbate these credibility problems. We have demonstrated this simple idea using a simple sticky price model in the spirit of Obstfeld and Rogoff's (1995) now classic contribution. We have also translated our criterion over shocks into an operational one over observables. We find that, in contrast to the literature, it is not the absolute volatility of output and real exchange rates that matters, but rather their relative volatility.

We have analyzed both benevolent unions and anchor-client unions. In practice, the determining feature of whether a given union is better described as a benevolent union or an anchor-client union is whether the policy responds to the needs of the union as a whole or to the needs of one country in the union. There is a growing consensus that in the European Monetary Union, the policy is best described as responding to the needs of the union as a whole. For example, Mihov argues that "estimation of monetary policy reaction functions finds that the European Central Bank is closer to an aggregate of the central banks in Germany, France, and Italy than to the Bundesbank alone" (Mihov (2001), p. 370).

For similar view, see Alesina et al. (2001), Alesina et al. (2003), and Nechio (2011). In this sense, the European Monetary Union appears to be a benevolent union rather than an anchor-client union. Examples of anchor-client union, include one-time colonies of Britain and France and countries that have dollarized, such as Ecuador and El Salvador.

In our simple model, there is a sharp distinction between Mundellian shocks and credibility shocks in that each shock is of one type or the other. In a more general model, each shock will have some Mundellian elements and some credibility elements, and our criterion will change appropriately.

We have derived these results using pen and paper in a very simple model in which it

is easy to understand the strategic interplay between price-setting agents and an optimizing monetary authority. A useful extension would be to build a richer, quantitatively relevant version of this model and use it to assess the benefits and costs of forming a union in practice.

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8. Appendix

Here we provide some omitted derivations and proofs.

A. Derivation of the Ramsey Outcome

The equilibrium allocations both under flexible exchange rates and in a monetary union satisfy the markup condition (23) and the resource constraints. Consider a relaxed version of the Ramsey problem: choose allocations to maximize utility subject to the markup condition and the resource constraints. Clearly, the consumption of traded goods is given by

$$(76) \quad C_T(s^t) = \frac{\alpha}{b}.$$

Letting $\eta(s^{t-1}, s_{1t})$ be the multiplier associated with (23), and dividing the first order condition for $C_N(s^t)$ by that for $L(s^t)$ gives

$$(77) \quad C_N(s^t) = \frac{1}{b} \frac{A(s_{2t})(1 - \alpha)}{1 + \eta(s^{t-1}, s_{1t})\theta(s_{1t})}.$$

Then, substituting this expression for $C_N(s^t)$ into (23) and solving for $\eta(s^{t-1}, s_{1t})$, we get

$$(78) \quad 1 + \eta(s^{t-1}, s_{1t})\theta(s_{1t}) = \frac{1}{\theta(s_{1t})},$$

which when substituted back into (77) gives that the expression for nontraded consumption in (25) and labor is clearly given by

$$(79) \quad L(s^t) = C_T(s^t) + \frac{C_N(s^t)}{A(s_{2t})}.$$

We next show that this allocation can be implemented as a competitive equilibrium and therefore solves the original Ramsey problem. We construct prices so that the multiplier on the cash-in-advance constraint is zero in all states. To do so, we construct the prices so that the cash-in-advance constraint holds with equality at the highest level of productivity of the nontraded goods and is a strict inequality at all other shocks. (A moment's reflection makes clear that there is a one-dimensional degree of indeterminacy in the price level. Here we have resolved this indeterminacy in one particular way, but we could support the same allocations with prices such that the cash-in-advance constraint holds as a strict inequality at all shocks.)

For all t, s^t , recursively construct prices normalized by the beginning-of-the-period money holdings, $p_T(s^t) = P_T(s^t)/M(s^{t-1})$ and $p_N(s^{t-1}, s_{1t}) = P_N(s^{t-1}, s_{1t})/M(s^{t-1})$ and the money growth rate as

$$(80) \quad p_N(s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \min_{s_2} \left\{ \frac{b}{\alpha} \frac{1}{A(s_{2t})} \right\} = \frac{1}{\theta(s_{1t})} \frac{b}{\alpha} \frac{1}{\max A(s_{2t})}$$

$$(81) \quad p_T(s^t) = A(s_{2t})\theta(s_{1t})p_N(s^{t-1}, s_{1t})$$

$$(82) \quad \frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{A(s_{2t})}{A(s_{2t+1})}.$$

Let $W(s^t) = P_T(s^t)$, and let the nominal interest rates $\{r(s^t)\}$ and state prices $\{Q(s^t)\}$ be given by (16) and (17) at these allocations and prices. We claim that our constructed allocations, prices, and money supplies are a competitive equilibrium outcome. First notice that the necessary conditions for consumers' optimality are satisfied: since $W(s^t) = P_T(s^t)$ and (76) holds, then (14) holds; combining (81), (80), and (77) and using $W(s^t) = P_T(s^t)$ gives (13); next (82), (76), (77), (81), and (80) imply (15); (16) and (17) hold by construction; finally, notice that (11) is satisfied by substituting (81) and (76) in the cash-in-advance constraint. The constructed prices satisfy (7) because the allocations satisfy (23). Finally, market clearing follows from the feasibility of the allocations.

We now turn to the Ramsey problem for a monetary union. We begin by showing that in a union, nontraded goods consumption cannot vary with idiosyncratic productivity shocks. To see how this arises, note that the restriction that the price of traded goods is equal in all countries, (22), when combined with the consumer first order condition (14) and

our preferences, implies that

$$(83) \quad \frac{C_N(s^t) b}{1 - \alpha} = \frac{P_T(z^t)}{P_N(s^{t-1}, s_{1t})},$$

which in turn implies that in the union $C_N(s^t)$ cannot vary with v_{2t} , that is,

$$(84) \quad C_N(s^t) = C_N(s^{t-1}, s_{1t}, z_{2t}) \quad \text{for all } v_{2t}.$$

Consider the following relaxed problem:

$$\max_{\{C_T(s^t), C_N(s^t), L(s^t)\}} \sum_t \sum_{s^t} \beta^t h(s^t) [\alpha \log(C_T(s^t)) + (1 - \alpha) \log(C_N(s^t)) - bL(s^t)]$$

subject to the resource constraints (23) and (84). The first order condition for $C_T(s^t)$ gives

$$(85) \quad C_T(s^t) = \frac{\alpha}{b}.$$

After substituting the restriction on nontraded goods consumption into the objective function, the first order condition for the consumption of nontraded goods can be written as

$$(86) \quad C_N(s^{t-1}, s_{1t}, z_{2t}) = \frac{(1 - \alpha) \theta(s_{1t})}{(1 + \eta(s^{t-1}, s_{1t})) b X(z_{2t})},$$

where $\eta(s^{t-1}, s_{1t})$ is the multiplier on (23) where $X(z_2) = \sum_{\nu_2} g^2(\nu_2)/A(s_2)$. Substituting back into (23), we can solve for the multiplier

$$(1 + \eta(s^{t-1}, s_{1t})) = \sum_{s_2} h^2(s_{2t}) \frac{1}{A(s_{2t}) X(z_{2t})}.$$

Substituting this expression for $\eta(s^{t-1}, s_{1t})$ into (86) gives

$$(87) \quad C_N(s^{t-1}, s_{1t}, z_{2t}) = \theta(s_{1t}) \frac{1 - \alpha}{b} \frac{1}{X(z_{2t}) \sum_{\tilde{s}_2} h^2(\tilde{s}_2) / (A(\tilde{s}_2) X(\tilde{z}_2))},$$

and obviously

$$(88) \quad L(s^t) = C_T(s^t) + \frac{C_N(s^t)}{A(s_{2t})}.$$

We now show that the allocations in (85), (87)–(88) can be implemented as a competitive equilibrium under a monetary union. Here also there is a one-dimensional degree of indeterminacy in the price level, and we resolve it by having the cash-in-advance constraint hold with equality in the aggregate state at which the $X(z_2) = \sum_{\nu_2} g^2(\nu_2)/A(s_2)$ is at its lowest value (but with a multiplier of zero) and hold as an inequality at all other values.

For all t, s^t , construct prices normalized by the beginning-of-the-period money holdings, $p_T(s^t) = P_T(s^t)/M(s^{t-1})$ and $p_N(s^{t-1}, s_{1t}) = P_N(s^{t-1}, s_{1t})/M(s^{t-1})$ and the money

growth rate as follows:

$$(89) \quad p_N(s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \frac{b}{\alpha} \min_{z_2} \{X(z_2)\} \sum_{s_2} h^2(s_2) \frac{1/A(s_{2t})}{X(z_{2t})}$$

$$(90) \quad p_T(s^t) = A(s_{2t})\theta(s_{1t})p_N(s^{t-1}, s_{1t}) = \frac{b \min_{z_2} \{X(z_{2t})\}}{\alpha X(z_{2t})}$$

$$(91) \quad \frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) X(z_{2t+1})/X(z_{2t}).$$

We also let $W(s^t) = P_T(s^t)$ and let the nominal interest rates $\{r(s^t)\}$ and state prices $\{Q(s^t)\}$ be given by (16) and (17) at these allocations and prices.

We claim that our constructed allocations, prices, and policies are a competitive equilibrium outcome in a monetary union. First notice that the sufficient conditions for consumers' optimality are satisfied. Here $W(s^t) = P_T(s^t)$ and (85) gives (14); combining (90), (89), and (87) and using $W(s^t) = P_T(s^t)$ gives (13); (91), (85), (87), (90), and (89) imply (15); finally, notice that (11) is satisfied by substituting (90) and (85) in the cash-in-advance constraint. The constructed prices satisfy (7) because the allocations satisfy (23). Finally, market clearing follows from the feasibility of the allocations.

B. Markov Equilibrium Outcomes under Flexible Exchange Rates and Lemma 1

We start with the characterization of the Markov equilibrium under flexible exchange rates. Here we derive the outcomes allowing for both productivity shocks and markup shocks, since we will use this more general formulation in Proposition 5.

Allowing for productivity to be stochastic makes the analysis a bit more subtle than the main case of the text when productivity is constant. The reason is that in the Markov equilibrium, the cash-in-advance constraint may be slack when the realization of productivity shocks is sufficiently below its average level. To see why, suppose the realized productivity is sufficiently low that it is possible to completely offset the monopoly distortion and not have the cash-in-advance constraint bind. This occurs when at $p_T = p_N A$, the cash-in-advance constraint is slack so that $C_T = \alpha/b < 1/p_T = 1/Ap_N$. Hence, when $Ap_N \leq b/\alpha$, this is the outcome; otherwise it is the typical case in which the cash-in-advance constraint binds. Of course, for such an outcome to be part of an equilibrium, such a setting for p_N must be optimal for the monopolists. From (7) it is clear that the monopolists set their prices, in part, based roughly on the average productivity shock. When the realization of productivity is sufficiently low relative to this average, then p_N can be such that $Ap_N \leq b/\alpha$ and this scenario can occur. Of course, when productivity is constant, $p_N = p_T/\theta A$ and it cannot.

We formalize this logic in the following lemma.

Lemma A1. In the Markov equilibrium outcome with flexible exchange rates, the price of traded goods $\bar{p}_T(s^t)$ only depends on the current shock s_t and if $Ap_N \leq b/\alpha$ satisfies $\bar{p}_T(s_t) = A(s_{2t})\bar{p}_N(s_{1t})$ and otherwise satisfies

$$(92) \quad \bar{p}_T(s_t) = \frac{\bar{p}_N(s_{1t})A(s_{2t})}{2(1-\alpha)} \left[(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha) \frac{1}{A(s_{2t})} \frac{b}{\bar{p}_N(s_{1t})}} \right],$$

and the normalized nontraded good price $\bar{p}_N(s^{t-1}, s_{1t})$ only depends on s_{1t} and solves

$$(93) \quad \bar{p}_N(s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} h^2(s_{2t}) \frac{\bar{p}_T(s_t)}{A(s_{2t})}.$$

Furthermore, $C_T(s_t) = \min\{1/\bar{p}_T(s_t), \alpha/b\}$, and $C_N(s_t) = (1 - \alpha)\bar{p}_T(s_t)/b\bar{p}_N(s_{1t})$. Finally, the money growth rate is $\mu(s_t) = \beta\alpha\bar{p}_T(s_t)/b$ and the inflation rate in sector $i = T, N$, defined as $\pi_i(z_{t-1}, z_t) = P_i(s_t)/P_i(s_{t-1})$, is $\pi_i(s_{t-1}, s_t) = \mu(s_{t-1})\bar{p}_i(z_t)/\bar{p}_i(z_{t-1})$.

Proof. Suppose first that the realization of A is such that the cash-in-advance constraint is binding. Then using an argument similar to that in the text, the primal problem for the monetary authority is (41). From the first order conditions to that problem, it is easy to show that the optimal price for traded goods satisfies

$$(94) \quad \frac{1 - 2\alpha}{p_T/p_N} = (1 - \alpha) \frac{1}{A(s_2)} + \frac{b}{p_N} \frac{1}{(p_T/p_N)^2}.$$

If this constraint is slack, the optimal price clearly satisfies $p_T = Ap_N$. Thus, the monetary authority's best response has two parts: if $Ap_N \leq b/\alpha$ then $p_T(p_N, s) = A(s_2)p_N$; otherwise it equals the p_T that solves (94), namely

$$(95) \quad p_T(p_N, s) = \frac{p_N A(s_2)}{2(1 - \alpha)} \left[(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4(1 - \alpha) \frac{1}{A(s_2)} \frac{b}{p_N}} \right],$$

where the right-hand side of this equation defines the function F in the text. Substituting into the pricing rule for nontraded goods (33) for the wage rate from (28) gives

$$(96) \quad p_N(s_1) = \frac{1}{\theta(s_1)} \sum_{s_2} h^2(s_2) \frac{p_T(p_N(s_1), s)}{A(s_2)}.$$

The equilibrium outcome for nontraded goods is a fixed point of these equations, and hence, combining the two-part best response of the monetary authority $p_T(p_N, s)$ and the pricing rule for nontraded goods $p_N(s_1)$ in (96) gives

$$(97) \quad 1 = \frac{1}{\theta(s_1)} \sum_{s_2} h^2(s_2) \max \left\{ \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4(1 - \alpha) \frac{1}{A(s_2)} \frac{b}{p_N(s_1)}}}{2(1 - \alpha)}, 1 \right\},$$

which implicitly defines $p_N(s_1)$. Using such a $p_N(s_1)$, we then have that (95) implies the equilibrium outcome $p_T(s)$. The other relevant equilibrium objects can be recovered by substituting for $p_N(s_1)$ and $p_T(s)$ into the constraints (42). *Q.E.D.*

If A is not stochastic, the cash-in-advance constraint always binds and we can solve

$$(98) \quad 1 = \frac{1}{\theta(s_1)} \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4(1 - \alpha) \frac{1}{A} \frac{b}{p_N(s_1)}}}{2(1 - \alpha)}$$

and (95) to get the expressions for prices and then use the constraints on the primal problem to construct the consumption and labor allocations of Lemma 1.

C. Proof of Lemma 2

We start by showing that with our preferences, the primal Markov problem can be split into a static part and a dynamic part. The static part is to solve

$$(99) \quad \max_{p_T, C_T(x_G), C_N(x_G)} \int \left[U \left(C_T(x_G), C_N(x_G), C_T(x_G) + \frac{C_N(x_G)}{A(x_G)} \right) \right] d\lambda_G$$

subject to

$$(100) \quad C_N(x_G) = \frac{1 - \alpha}{b} \frac{p_T}{p_N(x_G)}$$

$$(101) \quad C_T(x_G) = \min \left\{ \frac{m(x_G)}{p_T}, \frac{\alpha}{b} \right\}.$$

For any given p_T , the dynamic part is to solve

$$(102) \quad \max_{\mu(x_G)} \sum_s h(s') W^{union}(S'_G)$$

$$(103) \quad \gamma \frac{b}{p_T} = \beta \sum_{s'} h(s') \frac{\alpha}{p_T(x'_H, S'_H) C_T(m\mu(x_G)/\gamma, x'_H, S'_H)}$$

$$\gamma = \int [\mu(x_G, S_G) m] d\lambda_G.$$

We can separate these problems because the value of the dynamic part is independent of p_T . To see why, note that the aggregate growth rate of money is homogeneous of degree 1 in $\mu(x_G)$, while the value $W^{union}(S'_G)$ and the right-hand side of the constraint (103) are homogeneous of degree 0 in $\mu(x_G)$. Hence the value in (102) does not depend on p_T .

We prove a preliminary lemma that immediately implies Lemma 2.

Lemma A2. i) Under our preferences (10), if at the end of any period there is a nondegenerate money holding distribution, then the cash-in-advance constraint in the next period has a zero multiplier for all m and all z , and ii) in any Markov equilibrium, the multiplier on the cash-in-advance is binding for at least one level of aggregate shocks z and for a positive of measure of relative money holdings m in the support of λ_m .

Proof of part i. Suppose by way of contradiction that the end-of-period money holding distribution across countries is not degenerate so that there are two countries, say country 1 and country 2, whose consumers have money holdings at the beginning of the next period that satisfy $m_1 < m_2$ and the cash-in-advance constraint in the next period binds for country 1 for some realization of the shocks.

From (101) we see that the value of consumption of the traded good, $p_T C_{Ti} = \min[m_i, \alpha p_T / b]$ for $i = 1, 2$ does not vary with the idiosyncratic shock. It follows that

$p_T C_{T1} \leq p_T C_{T2}$ with strict inequality for at least one aggregate state. It follows that

$$(104) \quad \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_1, x_{H1}, S_H)} > \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_2, x_{H2}, S_H)}.$$

But the first order condition for money holdings from period $t - 1$ to t implies that for both countries $i = 1, 2$,

$$(105) \quad \frac{b}{\alpha p_T} = \beta \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_i, x_{Hi}, S_H)},$$

where p_T is the price of traded goods in period $t - 1$. Clearly, (104) contradicts (105).

Proof of part ii. Suppose by way of contradiction that the cash-in-advance constraint is slack for all countries for all realizations of the aggregate shock. Consider the static problem (99). The first order conditions with respect to p_T evaluated with the equilibrium rule imply that

$$(106) \quad 1 = \int \frac{p_T(z, \lambda_G)}{A(x_G) p_N(x_F, S_F)} d\lambda_G(x_G).$$

Now the sticky price first order condition evaluated in equilibrium is

$$(107) \quad p_N(x_F, S_F) = \frac{1}{\theta(s_1)} \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2)},$$

which since $1/\theta(s_1) > 1$ for all s_1 implies that

$$(108) \quad \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2) p_N(x_F, S_F)} < 1.$$

Integrating (108) over the state x_F with respect to the measure λ_F implies that

$$(109) \quad \int \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2) p_N(x_F, S_F)} d\lambda_F(x_F) = \int \frac{p_T(z, \lambda_G)}{A(x_G) p_N(x_G)} d\lambda_G(x_G) < 1,$$

where in the first equality we have used the property that the marginal measure of λ_G over x_F is λ_F . The inequality in (109) contradicts (106). *Q.E.D.*

Note that the intuition for the second part of the lemma is similar to why the cash-in-advance constraint must be binding under flexible exchange rates. If the cash-in-advance constraint were slack in all states, then the monetary authority would eliminate the monopoly distortion on average in the sense of (106). But the monopolists always set their price as a markup over the average value of the price of traded goods in the sense of (108). These two conditions are incompatible if the markup is always positive. Thus, in equilibrium the cash-in-advance constraint must bind for enough countries so that the benefits of raising the price of traded goods to correct the monopoly distortion just balance the costs of lowering the consumption of traded goods.

Combining parts i) and ii) of Lemma A2 immediately implies Lemma 2.

D. Markov Equilibrium Outcome for a Monetary Union and Lemma 3

It turns out that it is particularly simple to characterize the Markov equilibrium with fixed exchange rates when the cash-in-advance constraint always holds with equality. It follows from the proof of Lemma 2 that a sufficient condition for this result to be true is that productivity shocks in the nontraded goods sector have no aggregate component or more generally that the fluctuations in this aggregate component are not too large.

Lemma A3. Assume that all agents begin with the same initial holdings of money, (10) holds, and the cash-in-advance constraint holds with equality in all states. Then the Markov equilibrium outcome in a monetary union is such that the prices and consumption of nontraded and traded goods can be written as $p_N(s_{1t})$, $C_N(s_{1t}, z_{2t})$, $p_T(z_t)$, and $C_T(z_t)$ and solve

$$(110) \quad p_N(s_{1t}) = \frac{1}{\theta(s_{1t})} \sum h^2(s_{2t}) \frac{p_T(z_t)}{A(s_{2t})},$$

where

$$(111) \quad p_T(z_t) = \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4 \sum_v g(v) \frac{(1-\alpha)}{A(z_{2t}, v_2)} \frac{b}{p_N(z_{1t}, v_1)}}}{\sum_v g(v) \frac{2(1-\alpha)}{A(z_{2t}, v_2)} \frac{1}{p_N(z_{1t}, v_1)}}.$$

Furthermore, $C_T(z_t) = 1/p_T(z_t)$ and $C_N(s_{1t}, z_{2t}) = (1 - \alpha)p_T(z_t)/bp_N(s_{1t})$. Finally, the aggregate money growth rate is $\gamma(z_t) = \beta\alpha p_T(z_t)/b$, and the inflation rate in sector $i = T, N$, defined as $\pi_i(z_{t-1}, z_t) = P_i(z_t)/P_i(z_{t-1})$, is $\pi_i(z_{t-1}, z_t) = \gamma(z_{t-1})p_i(z_t)/p_i(z_{t-1})$.

Proof. Under our assumptions, the problem for the unionwide monetary authority is (53). The solution to the problem above satisfies

$$0 = \frac{1 - 2\alpha}{p_T} + b \left(\frac{1}{p_T} \right)^2 - \sum_v g(v) \frac{(1 - \alpha)}{A(z_2, v_2)} \frac{1}{p_N(z_1, v_1)}.$$

Solving this expression gives the monetary authority's best response:

$$(112) \quad p_T(z, \{p_N(z_1, v_1)\}) = \frac{(1 - 2\alpha) + \sqrt{(1 - 2\alpha)^2 + 4 \sum_v g(v) \frac{(1-\alpha)}{A(z_2, v_2)} \frac{b}{p_N(z_1, v_1)}}}{\sum_v g(v) \frac{2(1-\alpha)}{A(z_2, v_2)} \frac{1}{p_N(z_1, v_1)}},$$

where in the text we write the right-hand side at $F(E(1/Ap_N))$.

In equilibrium we must impose that the sticky price firm's first order condition (110) is satisfied. Hence, (110) and (112) give a system of equations in $\bar{p}_N(z_1, \nu_1)$ and $\bar{p}_T(z)$ that can be solved, yielding the price of the nontraded and traded goods on the equilibrium path. Finally, $C_T(s)$ and $C_N(s)$ can be recovered using (110) and (111) in (100), (101) with a cash-in-advance constraint holding with equality.

When A is not stochastic, we can solve for the equilibrium outcomes obtaining the expressions in Lemma 3.

E. Proof of the Second Part of Proposition 5

The proof has two parts. The easy part mimics the logic with the commitment case in that for any given price of nontraded goods, under flexible exchange rates the monetary authority is better able to adjust the price of traded goods to idiosyncratic shocks. The more subtle part shows that, in equilibrium, the price of nontraded goods that confronts the monetary authority under flexible exchange rates is actually lower than it is under a monetary union. A lower price of nontraded goods means that the economy is less distorted in terms of monopoly power, and this feature tends to reinforce the benefits of flexible exchange rates.

We begin with the more subtle part by showing that, in equilibrium, the price of nontraded goods that confronts the monetary authority under flexible exchange rates is lower than it is under a monetary union, that is, $p_N^{flex} < p_N^{union}$. Combining the expressions for p_T and p_N from Lemma A1, namely (92) and (93), and assuming the cash-in-advance constraint binds in all states, p_N^{flex} , is defined by

$$\sum_z f(z) \sum_v g(v) H \left(\frac{1}{A(z, v)} \frac{1}{p_N^{flex}} \right) = B(\theta),$$

where the function H is defined by

$$H \left(\frac{1}{A(s)} \frac{1}{p_N} \right) \equiv \left[(1 - 2\alpha)^2 + 4(1 - \alpha) \frac{1}{A(s)} \frac{b}{p_N} \right]^{1/2}$$

and $B(\theta) \equiv 2(1 - \alpha)\theta - (1 - 2\alpha)$. For the union, a similar analysis gives that $p_N^{union}(z)$ solves

$$\sum_z f(z) H \left(\sum_v g(v) \frac{1}{A(z, v)} \frac{1}{p_N^{union}} \right) = B(\theta)$$

for the same functions H and B . Since the function H is concave in $1/A$ for a given p_N , $p_N^{flex} < p_N^{union}$.

For the rest of the proof, note first that for the same p_N , the value of the Markov primal problem under flexible exchange rates is greater than that under a union, $U^{flex}(p_N) \geq U^{union}(p_N)$, simply because the problem under flexible exchange rates is less constrained. Note next that the $U^{flex}(p_N)$ is decreasing in p_N . Intuitively, the higher is p_N , the higher are the implied distortions for the traded good. Then since $p_N^{flex} < p_N^{union}$, we have that

$$U^{flex}(p_N^{flex}) > U^{flex}(p_N^{union}) \geq U^{union}(p_N^{union}).$$

F. Approximation for the Markov Equilibrium

Here we derive a log-linear approximation for prices and quantities of traded and nontraded goods under both regimes. We also compare welfare using a variant of the method in the work of Kim and Kim (2003).

Flexible Exchange Rates

The equilibrium conditions in Lemma A1 can be log-linearized and expressed as

$$(113) \quad \log p_T(s) = \mu_{p_T}^{flex} - \frac{\phi}{1-\phi} \log \theta(s) + \phi \log A(s)$$

$$(114) \quad \log p_N(s) = \mu_{p_N}^{flex} - \frac{1}{1-\phi} \log \theta(s)$$

$$(115) \quad \log C_T(s) = \mu_{C_T}^{flex} + \frac{\phi}{1-\phi} \log \theta(s) - \phi \log A(s)$$

$$(116) \quad \log C_N(s) = \mu_{C_N}^{flex} + \log \theta(s) + \phi \log A(s),$$

where we have written (92) as

$$\log p_T(s) = \log F \left(\frac{1}{p_N(s_1)A(s_2)} \right)$$

and we have evaluated $\phi \equiv -F' / (Fp_NA) > 0$ at the deterministic steady state. The constants $\mu_{p_T}^{flex}$, $\mu_{p_N}^{flex}$, $\mu_{C_T}^{flex}$, $\mu_{C_N}^{flex}$ allow us to make the Kim and Kim (2003) correction that allows for the mean of these variables in the stationary distribution under flexible exchange rates to differ from their value in the deterministic steady state.

Monetary Union

The equilibrium conditions in Lemma A3 can be log-linearized and expressed as

$$(117) \quad \log p_T(z) = \mu_{p_T}^{union} - \frac{\phi}{1-\phi} \log \theta_z(z) + \phi \log A_z(z)$$

$$(118) \quad \log p_N(s) = \mu_{p_N}^{union} - \frac{1}{1-\phi} \log \theta(s)$$

$$(119) \quad \log C_T(z) = \mu_{C_T}^{union} + \frac{\phi}{1-\phi} \log \theta_z(z) - \phi \log A_z(z)$$

$$(120) \quad \log C_N(s) = \mu_{C_N}^{union} + \theta(s) + \phi \log A_z(z),$$

where we have written (111) as

$$\log p_T(z) = \log F \left(\sum_{v_1} g^1(v_1) \frac{1}{P_N(z_1, v_1)} \sum_{v_2} \frac{h^2(v_2)}{A(z_2, v_2)} \right)$$

and ϕ is the elasticity of F evaluated at the deterministic steady state and the constants here $\mu_{p_T}^{union}$, $\mu_{p_N}^{union}$, $\mu_{C_T}^{union}$, $\mu_{C_N}^{union}$ allow us to make the Kim and Kim (2003) correction.

Welfare Differences

To compute an approximation for the welfare difference, first notice that the markup condition (23) implies that the average level of labor in nontraded goods in both regimes

satisfies

$$(121) \quad EL_N = EC_N/A = (1 - \alpha)E\theta/b.$$

This condition captures the fact that in equilibrium, labor is distorted by the average wedge $E(1 - \theta)$. Since we are measuring the welfare difference as the difference in expected values in the two regimes, we need only to derive expressions for C_T , L_T , and C_N . To compare welfare across regimes where the mean of the endogenous variables is different, we follow a procedure similar to that in the work of Kim and Kim (2003). In particular, we assume log-normality of shocks and endogenous variables. We pin down the average for the stochastic economy by setting $\mu_{C_N}^{flex}$, $\mu_{C_T}^{flex}$ and $\mu_{C_N}^{union}$ and $\mu_{C_T}^{union}$ so that (121) is satisfied in both regimes:

$$(122) \quad E \frac{C_N^{flex}(s)}{A(s)} = \frac{1 - \alpha}{b} E(\theta)$$

$$(123) \quad E \frac{C_N^{union}(s)}{A(s)} = \frac{1 - \alpha}{b} E(\theta)$$

and by imposing that the expected consumption of traded goods is constant across regimes:

$$(124) \quad EC_T^{flex} = EC_T^{union}.$$

The last assumption is motivated by the fact that in the case that we can solve in closed form (no productivity shocks), we find that (124) holds (see Lemma 1 and Lemma 3). Clearly, from (124) we can only derive $\mu_{C_T}^{flex}$ relative to $\mu_{C_T}^{union}$, but this is enough for our comparison. Moreover, we will assume that $\theta(s) = \theta_z(z)\theta_v(v)$ where θ_i is log-normal ($\mu_{\theta_i}, \sigma_{\theta_i}^2$) for $i = z, v$ and $A(s) = A_z(z)A_v(v)$ where A_i is log-normal ($\mu_{A_i}, \sigma_{A_i}^2$) for $i = z, v$.

Under flexible exchange rates, substituting (116) into (122), and using the properties of log-normals to solve for $\mu_{C_N}^{flex}$ gives

$$(125) \quad \mu_{C_N}^{flex} = \log\left(\frac{1 - \alpha}{b}\right) + (1 - \phi) \sum_{i=z,v} \mu_{A_i} - \sum_{i=z,v} \frac{(\phi - 1)^2 \sigma_{A_i}^2}{2}.$$

In a monetary union, substituting (120) into (123) and using the properties of log-normals to solve for $\mu_{C_N}^u$ gives

$$(126) \quad \mu_{C_N}^{union} = \mu_{C_N}^{flex} + \phi\mu_{A_v} - \frac{1 - (\phi - 1)^2}{2} \sigma_{A_v}^2.$$

Finally, we set $\mu_{C_T}^{flex}$ and $\mu_{C_T}^{union}$ so that $EC_T^{union} = EC_T^{flex}$. Using (115) and (119) into (124) we obtain

$$(127) \quad \mu_{C_T}^{union} = \mu_{C_T}^{flex} + \frac{\phi}{1 - \phi} \mu_{\theta_v} + \frac{1}{2} \left(\frac{\phi}{1 - \phi}\right)^2 \sigma_{\theta_v}^2 - \phi\mu_{A_v} + \frac{\phi^2 \sigma_{A_v}^2}{2}.$$

We now turn to comparing expected utility in the two regimes. By construction the

labor component of utility is constant in the two regimes. Therefore, the difference in welfare is given by comparing expected utility from consumption. Using (115), (116), (119), (120), (125), (126), and (127) we have

$$\begin{aligned}
& E \left[\alpha \log C_T^{union} (s) + (1 - \alpha) \log C_N^{union} (s) \right] - E \left[\alpha \log C_T^{flex} (s) + (1 - \alpha) \log C_N^{flex} (s) \right] = \\
&= \alpha \left[\mu_{C_T}^{union} - \mu_{C_T}^{flex} - \frac{\phi}{1 - \phi} \mu_{\theta v} + \phi \mu_{A_v} \right] + (1 - \alpha) \left[\mu_{C_N}^{union} - \mu_{C_N}^{flex} - \phi \mu_{A_v} \right] \\
&= \frac{\sigma_{\theta v}^2}{2} \alpha \left(\frac{\phi}{1 - \phi} \right)^2 - \frac{\sigma_{A_v}^2}{2} \{ (1 - \alpha) [1 - (\phi - 1)^2] - \alpha \phi^2 \} \\
&= \sigma_{\theta v}^2 u_\theta - \sigma_{A_v}^2 u_A,
\end{aligned}$$

which is (58) in the text.

The constants used in (67) are given by $w_y = u_\theta [1 + (1 - 2\alpha)/(A_0^2 - (1 - 2\alpha)^2)]/A_0 + u_A A_0^2 / (\phi^2 (A_0^2 - (1 - 2\alpha)^2))$ and $w_q = u_\theta A_0^2 / [(1 - \alpha)\phi^2 (A_0^2 - (1 - 2\alpha)^2)] + u_A (1 - 2\alpha)^2 / (A_0^2 - (1 - 2\alpha)^2)$, $A_0 = 1 - \alpha(1 - 2\phi)/(1 - \phi)$ and u_θ and u_A are defined in (58).

G. Derivation of Expressions for the Real and Nominal Exchange Rates

We start by deriving our expression for real exchange rates (60). To do so, start with the definition of the multilateral real exchange rate of a country with idiosyncratic shock history v^t , namely

$$q(s^t) = \frac{e(s^t) P_T(s^t)^\alpha P_N(s^t)^{1-\alpha}}{\int e(s^t) P_T(s^t)^\alpha P_N(s^t)^{1-\alpha} dv^t},$$

where $P_T(s^t)^\alpha P_N(s^t)^{1-\alpha}$ is the consumer price index for a country with idiosyncratic shock history v^t . Hence,

$$q(s^t) = \frac{(P_N(s^t)/P_T(s^t))^{1-\alpha}}{\int (P_N(s^t)/P_T(s^t))^{1-\alpha} dv^t} = \frac{(p_N(s_t)/p_T(s_t))^{1-\alpha}}{\int (p_N(s_t)/p_T(s_t))^{1-\alpha} dv_t},$$

where the first equality follows from using our expression for the multilateral nominal exchange (21). The second equality, which gives (60), follows by definition of the normalized prices. Note that the second equality implies that the real exchange rate depend only on the current shocks.

We now derive equation (64). Under the Ramsey policy, letting $\bar{p}_T(z^t) = \int_{v^t} P_T(z^t, v^t) g^t(v^t) dv^t / \bar{M}(z^{t-1})$ we can write the growth in nominal exchange rate as

$$\frac{e(s^{t+1})}{e(s^t)} = \frac{p_T(s^{t+1})}{p_T(s^t)} \frac{\bar{p}_T(z^t)}{\bar{p}_T(z^{t+1})} \frac{M(s^t)}{M(s^{t-1})} \frac{\bar{M}(z^{t-1})}{\bar{M}(z^t)}$$

which using (82) to express the money growth rate we have

$$\frac{e(s^{t+1})}{e(s^t)} = \frac{p_T(s^{t+1})}{p_T(s^t)} \frac{\bar{p}_T(z^t)}{\bar{p}_T(z^{t+1})} \frac{A(s_t) E(1/A(s_{t+1}))}{A(z_t) E(1/A(z_{t+1}))}.$$

Using $p_T(s^t) = bA(s_t)/(\alpha \max A(s_t))$ and $\bar{p}_T(z^t) = bA(z_t)/(\alpha \max A(z_t))$ at t and $t + 1$ gives

$$\frac{e(s^{t+1})}{e(s^t)} = \frac{A(s_{t+1})/\max A(s_{t+1})}{A(z_{t+1})/\max A(z_{t+1})} \frac{\max A(s_t)E(1/A(s_t))}{\max A(z_t)E(1/A(z_t))}.$$

With $A(s) = A_z(z)A_v(v)$ and $E_v A_v(v) = 1$, we have $E(1/A(s_t)) = E(1/A(z_t))$. Since the support of A is constant, this reduces to (64) in the text.

We turn now to showing that without commitment, the growth rate of nominal rates is not particularly informative about the desirability of forming a union. To do so, suppose that the countries are contemplating forming a union without commitment and currently are in a flexible exchange rate regime pursuing Markov policies. Here the volatility of the growth rate of nominal exchange rates,

$$(128) \quad \log e(s')/e(s) = \frac{\phi}{1-\phi} \log \theta_v(v') + \phi \log A(v'),$$

is not informative at all about the desirability of forming a union. The key intuition for this difference in informativeness is that under commitment, movements in the nominal exchange rate are driven by a country's desire to efficiently respond to productivity shocks. Hence, if nominal exchange rates are volatile under commitment, it is very costly to form a union. In contrast, without commitment, the high volatility of the nominal exchange rate might just be due to a country struggling with a time inconsistency problem arising from volatile markup shocks. Hence, even if nominal exchange rates are volatile without commitment, it might be very beneficial to form a union (or not if the movement is coming from productivity shocks).

H. Proof of Lemma 4

For a given state $(z, \{p_N^i(z, v)\})$, the Markov primal problem reduces to

$$\max_{p_T^i} \sum_{i=N,S} \lambda^i \sum_v g(v) \left[-\alpha \log p_T + (1-\alpha) \log \frac{1-\alpha}{b} \frac{p_T}{p_N^i(s_1)} - b \left(\frac{1}{p_T} + \frac{1-\alpha}{b} \frac{p_T}{p_N^i(z, v)} \frac{1}{A^i} \right) \right].$$

The first order condition for this problem, namely,

$$0 = (1-2\alpha) \frac{1}{p_T} + b \frac{1}{p_T^2} - (1-\alpha) \sum_{i=N,S} \lambda^i \sum_v g(v) \frac{1}{p_N^i(z, v) A^i},$$

defines the best response of the monetary authority in state $(z, \{p_N^i(z, v)\})$. From the sticky price first order condition we have

$$p_N^i(z, v) = \theta^i(s_1) \frac{p_T(z)}{A^i}.$$

The equilibrium is a fixed point of these two equations: combining them and using the result that the cash-in-advance constraint is binding so $C_T^i(z) = 1/p_T(z)$, we can solve for $C_T^i(s)$ obtaining (70).

I. Proof of Proposition 6

We argued in the text that if $E(\theta^N) = E(\theta^S)$, the problem (72) reduces to one of maximizing the expected value of $W(x(s, \lambda)) = \log(\alpha - (1 - \alpha)(1 - x(s, \lambda)))$ where $x(s, \lambda) = (1 - \lambda)\theta^N(s) + \lambda\theta^S(s)$. Up to a second order approximation around $\bar{x} = E(x(s, \lambda))$, we have

$$\begin{aligned} EW(x(s, \lambda)) &= W(\bar{x}) + E[W'(\bar{x})(x(s, \lambda) - \bar{x})] + \frac{1}{2}E[W''(\bar{x})(x(s, \lambda) - \bar{x})^2] \\ (129) \qquad &= [\log(c + d\bar{x}) - (1 - \alpha)\bar{x}] - \frac{1}{2} \frac{d^2}{(c + d\bar{x})^2} \text{var}(x(s, \lambda)), \end{aligned}$$

where $c = (2\alpha - 1)$ and $d = (1 - \alpha)$. Since $E(\theta^N) = E(\theta^S)$, then the mean of x is independent of λ . Hence, maximizing the expression (129) is equivalent to choosing λ to minimize the variance of $x(s, \lambda)$, that is, to solve

$$\min_{\lambda \in [0, 1]} (1 - \lambda)^2 \sigma_N^2 + \lambda^2 \sigma_S^2 + 2\lambda^N \lambda^S \rho \sigma_N \sigma_S,$$

where $\lambda = \lambda^S$, σ_i^2 is the variance of $\theta^i(s)$ and ρ is the correlation of $\theta^N(s)$ and $\theta^S(s)$. Taking the first order condition gives

$$0 = -(1 - \lambda) \sigma_N^2 + \lambda \sigma_S^2 + \rho \sigma_N \sigma_S - 2\lambda \rho \sigma_N \sigma_S,$$

whereas the second order condition is

$$\sigma_N^2 + \sigma_S^2 - 2\rho \sigma_N \sigma_S = \sigma_N^2 - \rho \sigma_N \sigma_S + \sigma_S^2 - \rho \sigma_N \sigma_S > 0.$$

When the solution is interior, we have

$$(130) \quad \lambda = \frac{\sigma_N^2 - \rho \sigma_N \sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho \sigma_N \sigma_S} = \frac{\sigma_N^2 - \rho \sigma_N \sigma_S}{\sigma_N^2 - \rho \sigma_N \sigma_S + \sigma_S^2 - \rho \sigma_N \sigma_S}.$$

Clearly, if $\rho < \sigma_N/\sigma_S$ the second order condition is satisfied. Also both the numerator and the denominator in (130) are positive and the solution is interior. If $\rho > \sigma_N/\sigma_S$, the second order condition is violated and, by inspection, the solution is $\lambda = 0$.

To obtain our comparative statics results when $\rho < \sigma_N/\sigma_S$, we differentiate (130) with respect to σ_S to obtain after some algebra:

$$\frac{\partial \lambda}{\partial \sigma_S} \propto \rho (\sigma_N^2 + \sigma_S^2) - 2\sigma_S \sigma_N.$$

Using $\rho \sigma_S < \sigma_N$ and $\sigma_S > \sigma_N$, simple algebra gives that this derivative is negative. Next, differentiating (130) with respect to ρ , we obtain after some algebra:

$$\frac{\partial \lambda}{\partial \rho} \propto [\sigma_N^2 - \sigma_S^2] \leq 0,$$

since $\sigma_S > \sigma_N$. *Q.E.D.*

J. Remarks on Proposition 8

Consider the general case with arbitrary measures of $\bar{m}^N, \bar{m}^M, \bar{m}^S$ and let (m_i^N, m_i^M, m_i^S) for $i = 1, 2, 3$ be the composition of the three unions. Suppose first that

$$(131) \quad \lambda_1^M \bar{m}^N \leq \bar{m}^M \text{ and } \lambda_1^S \bar{m}^N \leq \bar{m}^S,$$

then $m_1^N = \bar{m}^N$. Here there are sufficiently many Middle and Southern countries to achieve the optimal mixture in the most preferred union even when all of the Northerners are in this union. Now if

$$(132) \quad \lambda_2^S (\bar{m}^M - \lambda_1^M \bar{m}^N) \leq \bar{m}^S - \lambda_1^S \bar{m}^N$$

holds, then there enough Southern countries left over to achieve the optimal mixture in the second most preferred union even when all of the remaining Southerners are in this union. Under (131) and (132), the construction in Proposition 8 is feasible. Clearly, a sufficient condition for (131) (132) to hold is that \bar{m}^M/\bar{m}^N and \bar{m}^S/\bar{m}^M are sufficiently large.

Now if the measures are such that (131) holds but (132) fails, then the second most preferred union has all the Southerners that are left over from the first union in that $m_2^S = \bar{m}^S - \lambda_1^S \bar{m}^N$ and

$$m_2^M = \frac{\lambda_2^M}{\lambda_2^S} (\bar{m}^S - \lambda_1^S \bar{m}^N),$$

whereas the third most preferred union consists solely of middle countries.

Many more cases work similarly. In all of them, the most preferred union has the weights as constructed in the text, and that union has all the members of at least one of the three groups, that is the group that is, in the relevant sense, the most scarce. The second most preferred union has the optimal mixture subject to the constraint that it contain no members of this most scarce group. This union has the remaining members of the second most scarce group. The third union is composed solely of the members of one group that is the most abundant. Notice that if \bar{m}^N is sufficiently large relative to both \bar{m}^M and \bar{m}^S and the Middle and Southern countries are not too distorted in that λ_1^M and λ_1^S are both positive, then the least preferred group is composed solely of Northern countries.

K. Proofs of Propositions 9, 10, and 11

To prove all three propositions, we first characterize the equilibrium outcomes for a client given that the anchor follows an arbitrary policy. The anchor's policy matters for the client only to the extent that it influences the stochastic process for the price of traded goods denoted $p_T^i(s)$ where i denotes the anchor and prices are normalized by the anchor's money supply.

Following logic very similar to that in the proof of the first part of Lemma 2, it follows that we can set the money supply of the client equal to that of the anchor, $M(s^t) = M^i(s^t)$ for all $t \geq 0$. When the anchor is not following the Friedman rule, this is a necessary condition (as it was in Lemma 2), whereas if the anchor is following the Friedman rule, it is without loss of generality. Thus, the price of traded goods normalized by the anchor's money supply equals the price of traded goods normalized by the client's money supply.

The prices and allocations in the client country are then given by

$$(133) \quad \begin{aligned} p_N(s) &= \frac{1}{\theta(s)} E(p_T^i(s)/A(s)) \\ C_N(s) &= \frac{1-\alpha}{b} \frac{p_T^i(s)}{p_N(s)} = \frac{1-\alpha}{b} \theta(s) \frac{p_T^i(s)}{E(p_T^i(s)/A(s))} \\ C_T(s) &= \min\{1/p_T^i(s), \alpha/b\}, \end{aligned}$$

and, ignoring constants, the utility is given by

$$(134) \quad \begin{aligned} &\left[\alpha E \log \left(\min \left\{ \frac{1}{p_T^i(s)}, \frac{\alpha}{b} \right\} \right) + (1-\alpha) \left[E \log p_T^i(s) - \log E_A \left(\frac{p_T^i(s)}{A(s)} \right) \right] \right] \\ &+ (1-\alpha) E \log \theta(s) - b E \min \left\{ \frac{1}{p_T^i(s)}, \frac{\alpha}{b} \right\} - (1-\alpha) E \theta(s). \end{aligned}$$

Proof of Proposition 9. Clearly the value of (134) does not depend on the correlation of the markup shocks of client and the anchor.

Proof of Proposition 10. Under commitment, the cash-in-advance constraint does not bind for either the anchor or the client, so that $C_T(s) = \alpha/b$ and the price of traded goods has the form $p_T^i(s) = \kappa A^i(s)$ for a constant κ chosen so that the cash-in-advance constraint never binds. Substituting into (134) and ignoring terms that do not vary with the anchor gives that maximizing the welfare of the client is equivalent to maximizing

$$\log \left(E \left[\frac{A^i(s)}{A(s)} \right] \right) - E \left[\log \frac{A^i(s)}{A(s)} \right].$$

Proof of Proposition 11. Here the price of traded goods for the anchor is the Markov equilibrium price

$$p_T^i(s) = \frac{b}{c_0 + (1-\alpha)\theta^i(s)},$$

where $c_0 = 2\alpha - 1$ and we have used Lemma 1 and $p_T^i(s) = 1/C_T^i(s)$. Substituting this price into (134) and suppressing constants gives

$$\alpha E \log (c_0 + (1-\alpha)\theta^i(s)) - E [c_0 + (1-\alpha)\theta^i(s)].$$

Taking a second order approximation of this expression gives that the utility of the client is

$$\alpha \log (c_0 + (1-\alpha) E\theta^i) - [c_0 + (1-\alpha) E\theta^i] - \frac{(1-\alpha)}{[c_0 + (1-\alpha) E\theta^i]^2} \text{var}(\theta^i).$$

Thus, if the set of potential anchors have the same expected distortions $E\theta^i$, then the best anchor is the one that minimizes $\text{var}(\theta^i)$.

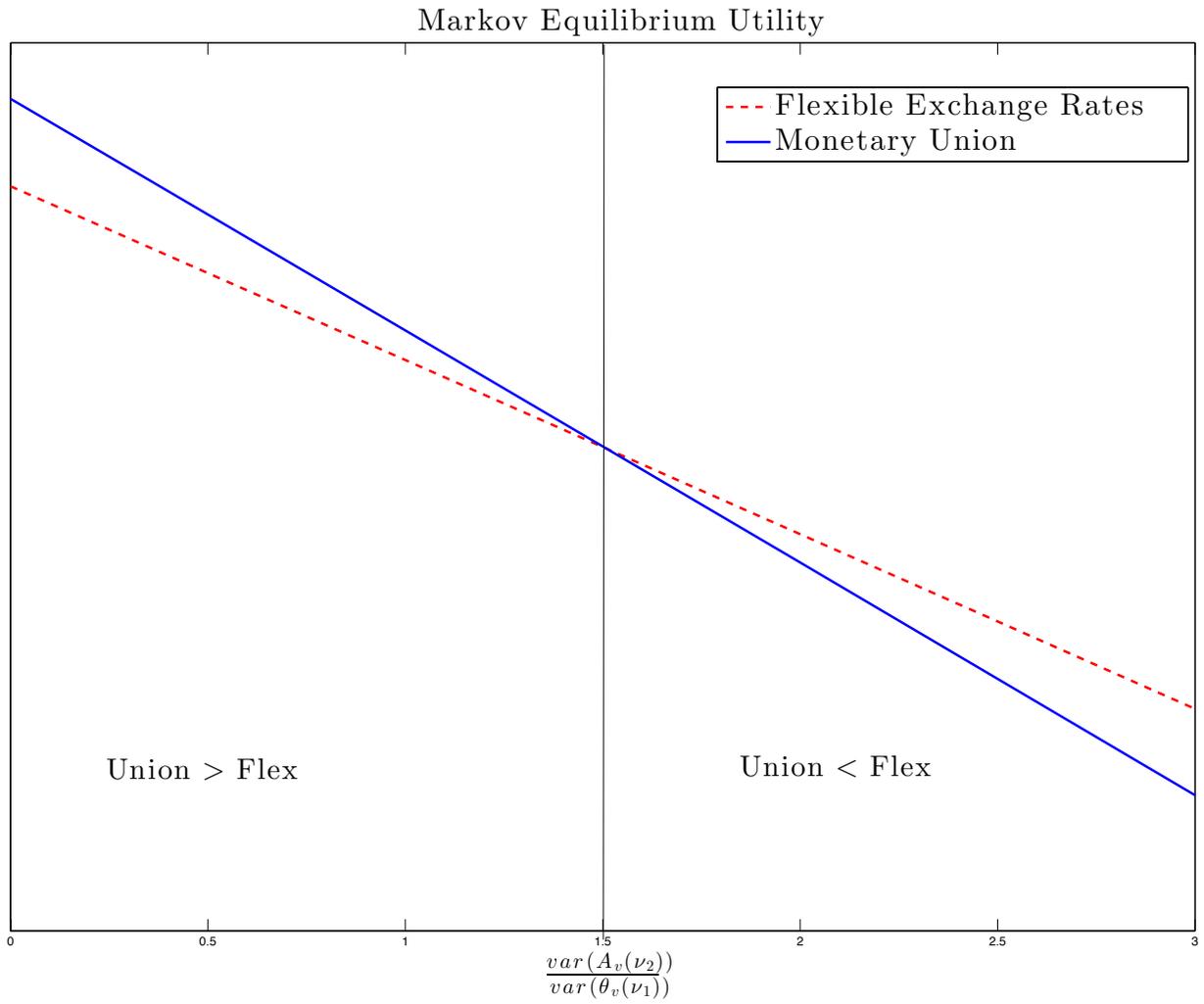


Figure 1. Markov equilibrium utility versus the relative idiosyncratic variances of the shocks