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ON THE UNCERTAINTY OF FUTURE PREFERENCES

BY H. S. WITSENHAUSEN

An often overlooked difficulty in long range planning is the likelihood that future preferences be incompatible with the initial ones, due to changes of a political, psychological and sociological character. Under such conditions, it is not clear what optimization means. The usual procedure is to assume that the current preferences will remain in effect permanently. This amounts to the approximation of assumed certainty-equivalence for the problem in which optimality is defined in terms of future, not yet exactly known, preferences. Models of opinion evolution can be used to improve this approximation.

1. INTRODUCTION

The application of optimization theory requires in most cases the consideration of uncertainty. On one hand, realistic models of most systems are necessarily stochastic, on the other hand there often is considerable uncertainty as to whether the chosen model provides a sufficiently accurate representation of the relations that exist in reality. This uncertainty applies to the coefficients of the model as well as to its structure.

However there is yet another factor that introduces uncertainty in dynamic optimization when large spans of time are involved. This is the uncertainty of future preferences, which seems to have been largely ignored.

When a model has been selected, the problem is to find an optimal policy (feedback law). Within the model, uncertainty can be thought of as the choice by "nature" of an element of a set Ω. The possible policies form a set Γ and the model will define for each ω in Ω and γ in Γ the resulting history of the system (say the sequence of state and control variables). This may be thought of as a function $h = S(\omega, \gamma)$. Now, if we accept the axioms of the Von Neumann–Morgenstern utility theory, the preferences of the decision maker among the possible histories, hence among the policies, can be represented by a utility function which assigns to policy γ a number

$$V(\gamma) = E_P\{U(h)\} = E_P\{U(S(\omega, \gamma))\}$$

where P is the, possibly subjective, prior probability distribution on Ω and U is the utility function. The determination of U and P from interrogation of the decision maker is not an easy matter but it has been the object of much research [2] and will be taken for granted here. What is done in essence is to formalize the *mood* of the decision maker.

However, moods change. The energy policy of the United States is not judged in 1973 by the same criteria as would have been considered appropriate in 1953. The weight given to environmental considerations has definitely increased. It was not possible to foresee with certainty in 1953 the extent of this evolution nor can the future evolution be predicted clearly today. However the 1953 decision maker might (and we argue that he should) have perceived the *possibility* of changes in preference and this observation applies to any long range planning. Taking the initial preference relation as absolute and permanent can lead to commitments which will be harshly judged in a later climate.

This difficulty has long been perceived [3] but in careful treatments of dynamic utility theory [1] it is only mentioned to be assumed away.

2. The Problem

Consider the case of a discrete time system operating over $t = 0, 1, \ldots, T$. A decision making (policy review) committee meets at each period. At period t its preferences are described by the pair (P_t, U_t). At best this sequence is a stochastic process of known characteristics. For instance (P_t, U_t) may be a known function of the mood m_t at time t and some social scientist may have proposed a stochastic model for the evolution of m_t, which model we accept.

A few remarks are in order here to avoid confusions: (i) each pair (P_t, U_t) applies to the entire history, part of which will be past history at time t and part future. (ii) P_t denotes the subjective *prior* probability as judged at time t. Of course the data available at time t will lead to an updating of this prior distribution into a posterior one by Bayes formula. However the very same data will lead to different posterior distributions for different P_t. (iii) it is not true in general that after some time the influence of the prior distribution will be washed out in the Bayesian updating for mild variations in P_t. Indeed, P_t is a joint distribution for the choices of nature at the initial, past and future times and some of these choices have not even affected the system yet at time t, and may be nearly independent of the earlier choices. (iv) a variable prior distribution is included only because it could occur in reality. For all that follows one could restrict himself to the case of fixed, constant P_t and variable U_t, all the difficulties would persist.

The utility function U_t may have, for instance, the form of an expected initial worth,

$$V_t(\gamma) = E_{P_t}\left\{\sum_{\tau=0}^{T}\left(\exp - \sum_{k=1}^{\tau} \lambda_{t,k}\right) f_{t,\tau}(x_\tau, u_\tau)\right\}$$

where x_τ, u_τ are the stochastic processes depending on γ describing the state and control variables. Here $\lambda_{t,\tau}$ is the opinion at time t of the decision making committee as to the proper rate of discounting utility between times $\tau - 1$ and τ. For those financial problems where this discounting would be given by an actual interest rate, there will be uncertainty only as to the future evolution of this rate, i.e., for $\tau > t$. In general, however, $\lambda_{t,\tau}$ is a subjective quantity and even the past values may be revised as opinions change.

It is not generally meaningful to compare the values of $V_t(\gamma)$ for the same γ and two different t, because the origin and scale of each utility evaluation is arbitrary. However ratios such as $[V_t(\gamma) - V_t(\gamma_0)]/[V_t(\gamma_1) - V_t(\gamma_0)]$ are comparable for different t and fixed $\gamma, \gamma_0, \gamma_1$.

If a stochastic model for the evolution of (P_t, U_t) is accepted, then for each γ, $V_t(\gamma)$ becomes a random process. No total ordering of such processes is agreed upon and therefore it is not clear how a policy is to be selected. If there is no stochastic model for the evolution of the committee's opinion, this conclusion is only reinforced.

One could select any number of arbitrary procedures in order to arrive at a decision. For instance one may select the policy γ_t to be used in period t, as a function of the information available as to mood evolution, by a "superpolicy". This information will consist at least of the past realizations of $(P_{t'}, U_{t'})$, $t' \leq t$ and in addition there may be sociological observables. The information about behavior of the controlled system itself is already taken care of by the fact that γ is a feedback policy. The superpolicy might for instance be selected so as to maximize the expectation of $\sum_{t=0}^{T} V_t(\gamma)$, with respect to the stochastic model accepted for evolution of opinion. In this way, one obtains a stochastic control problem for an enlarged system with a definite criterion. However the selection of the sum of the V_t is completely arbitrary and unsupported and depends on scaling at different times.

Rather than trying to find some dogmatic recipe, it may be more enlightening to describe what could actually occur.

3. The Assumed Permanence Procedure

What may be happening in practice is that no overall preference ordering is ever defined. Instead, a certain decision making procedure is followed, without possibility of even speaking of optimality. The most likely procedure is to assume at each time period that the current opinion is definitive and will no longer change. Then at time 0, an (ε)-optimal solution $\gamma^\circ = (\gamma_0^\circ, \gamma_1^\circ, \ldots, \gamma_T^\circ)$ is found for the stochastic-control problem defined with (P_0, U_0) and γ_0° is used. At time t, one optimizes the future policy $(\gamma_t^t, \gamma_{t+1}^t, \ldots, \gamma_T^t)$ so as to optimize the evaluation under (P_t, U_t) of the entire history of the system given the decisions made in the past. Again γ_t^t is used and the procedure repeated at $t + 1$. For simplicity it is assumed that all past decisions and data are always available, that is, the classical information pattern prevails.

The great practical advantage of assumed permanence is that no modeling of opinion evolution is required.

The disadvantage is that early commitments may be made which are costly to reverse if opinion changes so as to make them look disastrous. In practice the decision maker may well reject an "optimal" solution and prefer one which is inferior in the present preference ordering, because this other solution offers more flexibility for later adjustment. This is nothing but a, perhaps intuitive, recognition of the problem we are discussing.

4. The Hindsight Criterion

From the point of view of stochastic control theory the assumed permanence procedure is a suboptimal superpolicy, namely the certainty-equivalence policy for the problem in which

(a) the criterion is $V_T(\gamma)$

(b) the "best estimate" of (P_T, U_T) at time t is (P_t, U_t).

In this view, it would be better to choose a superpolicy maximizing the expectation of $V_T(\gamma)$, which is possible in principle if a stochastic model for opinion evolution is available. The result would be to formally vindicate the intuitive feeling (of non-optimality of assumed permanence) mentioned above. One may call this the hindsight criterion.

Does it make sense to base everything upon $V_T(\gamma)$? One can only outline what kind of philosophy would justify it. Clearly, the committee is trying to do the "right thing" which is assumed to exist in some Platonic universe but of which only imperfect knowledge is available. This knowledge is assumed to increase with time, due to the wisdom derived from experience. Of course that is an optimistic assumption, which historians would accept only at best as a slight trend under vast fluctuations.

If T is infinite, there may be difficulty in estimating $V_\infty(\gamma)$, a limit that might not exist. It is more natural to fall back on a limited look-ahead procedure, namely to select γ_t at time t as the initial portion of a policy γ optimizing the expectation of $V_{t+\Delta}(\gamma)$. Then again no overall preference ordering results, and the assumed permanence procedure is included as a special case for $\Delta = 0$. The choice of Δ represents a compromise between shortsightedness and the extreme uncertainty of the far future.

More generally Δ could depend on t (it might be the time to the next election). Or the criterion used could be a combination of the evaluations at several future time periods which may be indicated if there are periodic phenomena in the model of opinion evolution.

5. Conclusions

Optimization theory cannot entirely ignore the problem of opinion evolution, since this phenomenon is material in at least some of the problems to which the theory is to be applied. What is required, is at least a rough model of opinion evolution. Establishing such a model is outside the realm of control theory. In its simplest form, the utility function would contain parameters whose evolution is described by a system of stochastic difference equations.

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