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INTRODUCTION TO SELECTED PAPERS FROM THE SECOND NBER STOCHASTIC CONTROL CONFERENCE

BY GREGORY C. CHOW AND MICHAEL ATHANS*

This paper introduces the selected papers from the Second NBER Stochastic Control Conference which are published in the January 1974 issue of the Annals of Economic and Social Measurement. *The conference was held at the University of Chicago, from June 7 to June 9, 1973. Some 85 economists and control scientists attended. The papers are divided into three groups: topics in stochastic control theory; methods for computing optimal control solutions; and studies of economic problems. This paper also suggests areas of future research and cooperation among economists and control scientists.*

A second Stochastic Control Conference was held at the University of Chicago from June 7 to June 9, 1973, under the joint sponsorship of the NBER Conference on the Computer in Economic and Social Research and the Graduate School of Business of the University of Chicago. While attending the first NBER Stochastic Control Conference at Princeton University in May 1972, Robert L. Graves and Dov Pekelman of the Graduate School of Business, University of Chicago, suggested that the second conference be held at their institution. This welcome suggestion was carried out and Dov Pekelman was appointed Conference Chairman.[1] Michael Athans and Gregory C. Chow served as Program Co-chairmen, respectively to plan sessions of presentations by control scientists and by economists. Over 85 people attended the conference, and some 28 papers were presented. The conference program is exhibited in the Appendix.

During the period of these two conferences, the interests among economists in the subject of optimal stochastic control experienced a remarkable growth. About 55 economists received announcements of the conference early in 1973, and 35 of them responded by submitting papers for presentation. Only 18 of these papers were included in the final program, after a difficult and somewhat painful selection process. The surge of interest among economists would appear to be a natural outcome of developments in several related areas of research: the advancement of econometric methods for the estimation of systems of dynamic economic relationships together with techniques for analyzing the dynamic properties of such stochastic systems: the growing interest in quantitative economic policy as implemented by the use of econometric models: the study of optimization over time in both micro and macroeconomics, and the evolution of dynamic economics in general: and the parallel development of modern control theory which, as some of the papers in this volume will illustrate, is similar (though not identical) in concepts and techniques to the above three areas of research of the economists. Optimal stochastic control has now become an important part of economics.

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[1] Jean Howard and Anna Tremblay served as administrative assistants for the conference and their help has been greatly appreciated.

The papers contributed by the control scientists were the result of a limited call for papers communicated by M. Athans to about 75 members of the control community. The topics were narrowed down to contributions that were as tutorial as possible, dealing with estimation methods, Kalman filtering techniques, stochastic control, and adaptive control methods. By design then, some of the papers presented by control theorists did not report brand new research, but rather provided an overview of existing methods in a language that may be the most familiar to economists.

The control community as a whole has great interest in economic problems for two main reasons.

(a) There are several economic areas that existing control theory and algorithms are directly applicable.

(b) Economic systems do present theoretical and algorithmic challenges to the control theorists: these challenges are not of the type naturally associated with engineering systems.

Since introductory material on the subject of stochastic control in economics is already covered in the October 1972 issue of the *Annals* (Volume I, No. 4), which was devoted to selected papers from the first conference, there is no need to go over the same ground here. For this issue, we have divided the papers into three groups. The first group covers several topics in modern control theory, some of the papers in the form of a survey, which are deemed to be relevant to quantitative, stochastic economics. The second group deals with methods for computing optimal control solutions or approximations thereof. The third group consists mainly of studies of economic problems applying optimal control techniques, including studies of both macroeconomic policy and microeconomic problems. This division of the subject matter is bound to be somewhat arbitrary. For example, the papers in the first group are not entirely theoretical in nature and may contain economic applications. Also, empirical applications are presented in the papers of the second group, and discussion of methods cannot be avoided in the substantive studies of the third group. Each group of papers is fairly self-contained, and the reader may choose to study them in any order that he or she pleases. Let us suggest, however, that one not overlook the interesting paper by James Pierce of the Board of Governors of the Federal Reserve System which immediately follows this introduction. Pierce describes how optimal control techniques are being applied as an aid to the making of important decisions at the Federal Reserve Board. It not only motivates the subject, but also suggests some interesting problems for further research.

In the first group, the paper by R. K. Mehra, "Identification in Control and Econometrics: Similarities and Differences," deals with the formulation of dynamic stochastic models and the associated statistical estimation problems as they are treated by control scientists and economists. It serves to communicate to either profession the approach of the other discipline to these problems and may hopefully open up possibilities for cooperative research in these problems. The paper by Athans suggests possible applications of Kalman filtering in the estimation of parameters in a system of econometric equations. It illustrates the importance of Kalman filtering in econometrics, as does another paper by A. H. Harris, "A Bayesian Approach to Estimation of Time-Varying Regression Coefficients."

which was presented at this conference but has already appeared in the October 1973 issue of the *Annals* (Volume 2, No. 4). For other applications of Kalman filtering to the estimation of time-varying coefficients in regressions, the reader may refer to the above special issue of the *Annals*. While the applications presented there are concerned mainly with a regression equation or a set of regression equations, the paper by Athans studies applications in the context of a system of econometric equations.

The third paper of the group, "Adaptive Dual Control Methods," by Edison Tse is an exposition of a method of adaptive control recently developed by the author for systems with unknown states and/or parameters. Control is to serve the dual purpose of improving the dynamic performance of the system and of gaining knowledge about the unknown states or parameters for the purpose of future control. The approximate solution provided by the author is one of many now in existence in the control literature, and probably the most sophisticated one with respect to explicit formulation, dealing with the subject. The paper by Harold J. Kushner on stochastic stability provides several definitions of stability for stochastic systems which may be useful, and suggests techniques for ascertaining the stability of a stochastic system. Economists have long had an interest in the stability of deterministic systems. It is therefore important to extend the discussion to stochastic systems. The paper by H. S. Witsenhausen, "On the Uncertainty of Future Preferences," studies the interesting problem of hedging against undesirable consequences of one's own action which was taken according to a previous set of preferences that have since been changed. In this paper Witsenhausen formalizes for the first time how the fact that future decision makers may use different objective functions, or different tradeoff parameters, may have to be taken into account as far as present decisions are concerned. The paper is purely formative and no answers are provided.

The paper by Masanao Aoki considers the problem of choosing a variable p_t (price) for controlling another variable x_t (excess demand) when the latter is assumed to be a linear function of p_t plus white noise, the parameters of the linear function being unknown. One solution is by applying a stochastic approximation scheme. A second is by applying the Bayesian method to a one-period optimization problem to make the expected value of x equal to 0, and a third by minimizing the expected value of x^2. A fourth solution is a Bayesian solution to the multiperiod decision problem of minimizing the expected value of the sum of squares of the x's plus the squared deviation of the terminal stock from a specified level. It is shown that all four of these (price) adjustment mechanisms are the same up to $0(1/t)$ with probability 1. The summary paper by D. L. Kleinman deals with the problem of modelling a human decision maker. This approach, which models the human as a Kalman filter cascaded with a least squares controller (with a few additional twists such as time delays, multiplicative noises), has been very useful during the past few years in predicting human behavior (e.g. pilots, gunners, etc.) and the theoretical results correlate very well with experimental results. These engineering techniques may turn out to be useful in economic systems when one wishes to obtain a mathematical model of a human decision maker or a decision agency (such as the FRB). If the modelling is possible, one may then study via simulation the performance of the existing human decision mechanism with respect to alternate strategies.

In the second group of papers on computations, the paper by Ray C. Fair applies several standard maximization algorithms to the solution of deterministic optimal control problems in discrete time and discusses possible extensions to the stochastic case. The standard algorithms can be applied once the value of the objective function can be computed for any given set of values of the control variables for the finite time interval of interest. Fair's results indicate that it is feasible to solve problems of moderate size by these algorithms. However, there was considerable discussion on whether or not this is the most effective way of gaining insight into the structure and properties of the stochastic system. Another method for controlling a nonlinear system, under the assumption of a quadratic welfare function, is proposed by Robert S. Holbrook. If Newton's method is to be applied to maximize the welfare function with respect to the control variables, after the dynamic system is used to eliminate the other variables, the second derivatives of the welfare function with respect to the control variables would be required. By the chain rule of differentiation, this would require the second derivatives of the state variables with respect to the control variables. For a linear system, these second derivatives vanish, and only the first derivatives are required in the optimization by Newton's method. Rather than following Newton's method faithfully for the case of a nonlinear system, the author has suggested essentially that one linearize the system and to omit the second derivatives of the state variables with respect to the control variables in the iterations. The speed of convergence of such a procedure remains unknown. Holbrook applies this method to the Michigan Quarterly Econometric model of the U.S. economy. A third method for controlling a large nonlinear econometric system is presented by J. Phillip Cooper and Stanley Fisher. It first generates observations from the nonlinear stochastic model along some tentative paths for the control variables, fits linear distributed lag equations explaining the target variables by the control variables to the data so generated, and then obtains optimal feedback control rules using the linear equations and an appropriate welfare function. A possible advantage of fitting the linear distributed lag equations to the data generated by stochastic Monte Carlo simulation of a nonlinear model, rather than simply to the original economic data, lies in the possibility that the nonlinear model, if specified correctly, may contain more information about the dynamics of the economy than the original data. A distinguishing feature of the Cooper–Fischer approach, as compared with the approaches of Fair and Holbrook, is that the solution is in the form of feedback control equations. These equations are used by Cooper and Fischer, together with the St. Louis model of the U.S. economy, to ascertain the dynamic performance of the economy under control by stochastic simulations.

The study by Gordon C. Rausser and John W. Freebairn compares the results of six approximate adaptive control solutions to the setting of import quotas for beef in the United States, incorporating consumers' welfare, producers' welfare, and the behavior of the level of the import quota in the objective function. The six solutions are: (1) certainty equivalence, (2) stochastic control which treats the parameters as uncertain but ignores the possibility of learning from additional observations, (3) and (4) being respectively the open-loop versions of (1) and (2), which allow learning to take place passively but not in the control design, (5) the approximate adaptive control method suggested by Elizabeth MacRae (*Annals*, October 1972) which employs only approximate updating equations for the means and covariance matrix of the unknown parameters in the control design, and (6) M-Measurement feedback control, which is an approximation by assuming that additional observations will effect the information for optimal decisions for no more than M future periods.

Among the studies of economic problems, three are concerned with macroeconomic policies, besides the contribution by James L Pierce to which we have already referred. The first study, by Robert S. Pindyck and Steven M. Roberts, is concerned with the choice of the control variable, unborrowed reserves, by the Federal Reserve Board in order that two intermediate target variables, money supply and the rate of interest, will follow closely their assigned paths which, for the purpose of this study, are taken as given. A linearized version of a monthly money market model constructed at the Federal Reserve Board is used for this purpose. Calculations of the trade-off between the performance of the stock of money and the rate of interest are presented, both for the deterministic version of the model and for the stochastic version including additive random disturbances. The second paper is a progress report, by Jeremy Bray, on research undertaken at Queen Mary and Imperial Colleges, London, to study optimal control for the U.K. economy. An econometric model of the U.K. economy is reported and simulation runs representing the informal control methods actually used by U.K. governments are presented. The informal methods consist mainly of choosing control variables to steer the economy to a full employment equilibrium growth path within two years. Evaluation of the parameters of the quadratic social welfare function is discussed. The stage is therefore set for comparing the performance of the economy subject to control by these informal methods and the performance under optimal control, but the optimal control calculations are yet to be performed.

The third paper, by Christopher A. Sims, suggests that, when one computes an optimal control solution using a finite time horizon, there may be a danger that the time horizon used is not long enough, in the sense that, if it were extended longer, the results would be very different. As a prime example, when a very small cost is attached to the variations in the instrument, one may not realize that his finite-horizon solution will eventually lead to explosive variations in the instruments. Sims provides the solution to a very simple optimal control problem subject to the constraint that the instrument be stable. He recommends using appropriate terminal conditions to avoid the possible pitfalls of a finite-horizon solution.[2]

A microeconomic problem of the firm choosing the rate of dollar spending on an R & D project is studied by Morton I. Kamien and Nancy L. Schwartz. The firm is assumed to maximize the discounted value of all cash flows associated with the project. On the revenue side is the reward from completion of the project times the probability that the project will be completed at the specified time interval. On the cost side is the rate of expenditure for the project while it is still incomplete. Both would be relevant only if no rival will have succeeded in completing its R & D

[2] It should be noted, however, that Sims relies partly on Fourier transform methods as described in a book of P. Whittle dated 1963, whereas Whittle himself, in a later article, "A View of Stochastic Control Theory," *Journal of Royal Statistical Society*, series B, Vol. 132 (1969), has conceded that the Fourier transform methods are outdated and superceded by the methods in the time domain which can deal with non-stationary situations and are computationally simpler.

project by that time. Under stated assumptions, an optimal non-null expenditure plan is shown to have planned spending increased through time: a necessary condition for the existence of a non-null optimal policy is stated. The effects of increasing the probability of a rival completing its project by a given time on the firm's optimal expenditure plan is investigated.

The study by Michael Rothschild, which is only abstracted in this volume, deals with optimal Bayesian search rules for a consumer who wishes to buy some good but does not know the distribution of its price among different stores. Under the assumption that the unknown distribution of prices is multinominal, the author specifies the optimal policy by a functional equation which is derived by backward induction as in dynamic programming. It is shown that, using the optimal strategy, search terminates after a finite number of times, and that the number of searches decreases as cost increases. If the searcher's prior distribution is a Dirichlet (the natural conjugate prior for the multinomial) then it is shown that search terminates if and only if the observed price is less than or equal to some reservation price (which, however, changes as the searcher's information changes) and that, as the perceived or expected distribution of prices becomes more dispersed, the intensity of search increases. Thus, under reasonable assumptions, optimal rules for search from an unknown distribution of prices have the same qualitative properties as in the case of a known distribution of prices.

In the study by D. L. Birto and M. D. Intriligator, a model of the armaments race between two countries is formulated. This model incorporates a previous model of Brito to explain the choice between consumption and defense expenditures by each of two countries in a gaming situation, and a previous model of Intriligator to study dynamic strategies during a missile war concerning the rate of firing missiles and the choice of target (enemy missiles or otherwise) by each of two countries. It also incorporates the lag between the time a missile is launched and the time it hits its target and the uncertainty concerning whether a given missile site is empty. During a missile war, the military authority of each country is assumed to maximize an objective function with both countries' stocks of missiles and numbers of casualties as arguments. The paper derives some properties of the optimal strategies and provides sufficient conditions for the existence and stability of an equilibrium level of missiles in each country.

There is no question that future research will continue in the directions as exemplified by the papers of this volume. As the papers of the first conference laid the ground work, and as the papers of this volume have reported on progress in both theory and application of stochastic control, it is hoped that, once the methodological barrier is removed, future research will delve deeper into substantive economic problems by involving the active participation of the many economists interested in microeconomic dynamics and macroeconomic policies.

We also feel that cooperation between economists and control theorists will indeed continue, and we predict that there may be several interdisciplinary groups within the next few years. Such continued interaction will certainly motivate the control theorist to tackle relevant theoretical areas that arise in economic systems. However, in the short run, these two past workshops have demonstrated that although mathematical economists are well versed in the basic methodology of stochastic control, as well as in the application of the theory, nonetheless there exist certain "gaps" that once filled would provide additional impetus for collaboration, and perhaps save time in rediscovering results and algorithms that are already obtained by one discipline or the other.

Following the workshop. M. Athans conducted an informal survey of the control theorists who participated in this workshop to find out their impressions. Without exception all of them agreed that it was a useful meeting. They also pointed out that there are many *fundamental* concepts in modern control theory that are not fully appreciated by mathematical economists.

The key notion of the state variable description seems to be misinterpreted. There are deep structural results implicit in the state variable representation, such as controllability, observability, identifiability and so on. These fundamental system concepts are of more than theoretical interest. They are crucial in stochastic control problems and govern the "good" or "bad" behavior of the control systems over the infinite horizon. They are also crucial in parameter estimation as indicated in the paper by R. K. Mehra.

The second observation regarding stochastic control deals with the possible over-reliance upon Monte Carlo simulations. These tend to hide some very fundamental problems about the utilization of future expected information, as remarked in the paper by Tse. In the adaptive control problem, there is a tremendous difference on both the theoretical and algorithmic level in how one sets up the mathematics to deal with future measurements.

Finally, from the discussions held at the workshop, it is evident that mathematical economists are also concerned with sequential dynamic team and game problems. Certain of these issues have also been considered in the control literature (Nash equilibria, Stackelberg strategies, pareto-optimality, as well as the dynamic extension of the Radner--Marschak theories). There are tremendous differences between the deterministic and stochastic versions of these problems, and the *certainty equivalence principle* (or the separation theorem in control jargon) seldom holds. More cooperation in this class of problem between economists and control scientists will certainly be very beneficial.

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Appendix

Program of the Second NBER Stochastic Control Conference

University of Chicago, June 7–9, 1973

Thursday, June 7

2.00–4.30 *First Session, Chairman: M. Athans (MIT)*

1. M. Athans (MIT) "The Importance of Kalman Filtering Methods for Economic Systems"
2. A. H. Sarris (MIT and NBER) "A Bayesian Approach to Estimation of Time-Varying Regression Coefficients"
3. R. K. Mehra (Harvard) "Identification in Control and Econometric System : Similarities and Differences"

4. H. J. Kushner (Brown) "Some Basic Ideas in Stochastic Stability"
5. J. Bray (Battelle Institute) "Predictive Control of a Stochastic Model of the U.K. Economy: Simulating Present Policy Making Practice by the U.K. Government"

6.00–7.00 Reception by the Graduate School of Business and Department of Economics

Friday, June 8

9.30–12.00 *Second Session. Chairman: G. C. Chow (Princeton)*

1. R. S. Holbrook (Michigan) "A Practical Method for Controlling a Large Nonlinear Stochastic System"
2. R. C. Fair (Princeton) "On the Solution of Optimal Control Problems as Maximization Problems"
3. J. P. Cooper and S. Fischer (Chicago) "A Method for Stochastic Control of Large Nonlinear Econometric Models"
4. R. S. Pindyck (MIT) and S. M. Roberts (FRB) "Optimal Policies for Monetary Control"
5. G. C. Rausser (Chicago) and J. W. Freebairn (Australian National University) "A Comparison of Approximate Adaptive Control Solutions to the U.S. Beef Trade Policy Problem"
6. D. Kendrick and J. Majors (Texas) "Stochastic Control in Macroeconomic Models: An Approximation"

2.00–4.30 *Third Session. Chairman: R. K. Mehra (Harvard)*

1. E. Tse (Systems Control) "Dual Adaptive Control Methods"
2. D. G. Lainiotis and T. N. Upadhyay (Texas) "Structure Identification and Adaptive Control Application to Economic Stabilization"
3. M. Aoki (UCLA) "On Some Price Adjustment Schemes"
4. H. W. Witsenhausen (Bell Labs) "On the Uncertainty of Future Preferences"
5. D. L. Kleinman (Systems Control) "Modelling Human Decision Making via Modern Control Theory"

6.00 Dinner

7.30 James L. Pierce (FRB) "Quantitative Analysis for Decisions at the Federal Reserve Board"

Saturday, June 9

9.00–12.00 *Fourth Session, Chairman: M. Nerlove (Chicago)*

1. M. Rothschild (Princeton) "Searching for the Lowest Price When the Distribution of Prices is Unknown"
2. P. von zur Muehlen (FRB) "Price Adjustment in Atomistic Competition"
3. R. M. Cyert and M. H. DeGroot (Carnegie-Mellon) "Sequential Strategies in Duopoly and Dual Controls"
4. F. Kydland and E. C. Prescott (Carnegie-Mellon) "Optimal Stabilization: A New Approach"
5. T. Takayama and G. Judge (U. of Ill.) "An Analysis of Optimal Control Formulations of Temporal–Spatial Price Equilibrium Models—Continuous and Discrete, Deterministic and Stochastic"

1.30–4.00 *Fifth Session, Chairman: A. Zellner (Chicago)*

1. C. A. Sims (Minnesota) "Optimal Stable Policies for Unstable Instruments"
2. J. B. Taylor (Columbia) "A Criterion for Multiperiod Controls in Economic Models with Unknown Parameters"
3. E. Burmeister, J. Jackson and S. A. Ross (Pennsylvania) "The Computational Welfare Evaluation of Simple and Optimal Decision Rules"
4. R. H. Day (Wisconsin) "Behavioral Control of Economic Systems"
5. M. I. Kamien and N. L. Schwartz (Northwestern) "Risky R and D with Rivalry"
6. D. L. Brito (Ohio State) and M. D. Intriligator (UCLA) "Uncertainty and the Stability of the Armaments Race"