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## RESEARCH METHODOLOGY

### IDIOM\*: AN INTER-INDUSTRY, NATIONAL-REGIONAL POLICY EVALUATION MODEL

BY STEPHEN P. DRESCH AND ROBERT D. GOLDBERG†

#### 1. ORIGINS

This article briefly describes the present status of a public policy evaluation model currently being developed by the authors at the National Bureau of Economic Research. The theoretic origin of the model is embedded in the development of differential incidence analysis within public finance theory. The most important single insight of modern incidence theory is that the effects of government policies can only be assessed with reference to some base, i.e. the configuration of the economy (e.g. the factor and size distributions of income) under some alternative public policy. In brief, it makes no sense to talk of the "absolute" effects of one policy; it is only possible to identify the differential effects of one policy as an alternative to another. And it is this type of differential incidence question which IDIOM is designed to answer.

The development of IDIOM represents the confluence of several more specific research efforts which led in a common direction and were subject to similar analytical constraints. The first and analytically most important was a study of the effects of a substitution of a value added tax (VAT) for the U.S. corporate income tax (CIT) [Dresch *et al.*]. Initially limited to projecting the probable sectoral price effects of this potential change in federal tax structure, it was readily apparent that the really interesting consequences of such a tax substitution were not the price changes *per se* but the more fundamental changes which these implied in such areas as income distribution, the level and composition of investment, and international trade flows. The extension of the study in these dimensions was necessarily accomplished on a rather *ad hoc* basis, marrying the price effects analysis with more-or-less compatible models of the processes and systems under examination. The most serious liability in this procedure was the impossibility of

#### \* Income Determination Input-Output Model.

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For a more detailed discussion of the model and its application, the interested reader is referred to Stephen P. Dresch, "Disarmament: Economic Consequences and Development Potential," A Report Submitted to the Centre for Development Planning, Projections and Policies, Department of Economic and Social Affairs, United Nations, December 1972. The model is applied in Sections III and IV of that report and thoroughly described in Appendix C.

interactively analyzing the price and income distribution-investment-international trade-*et al* effects: The analysis was entirely uni-directional, with price changes generating responses which did not in turn further influence price.

The second impetus to model development evolved out of a series of studies of differential regional consequences of alternative federal tax and transfer policies, undertaken with support from the Economic Development Administration of the U.S. Department of Commerce. These studies stimulated model development in three ways. First, while each study [Dresch 1971a, Hosek 1972a, Hosek 1972b, Dresch, 1971b) focused on a particular set of policies (e.g. intergovernmental grants, welfare reforms, tax changes), they were clearly related in terms of an underlying theoretic conception of the most interesting questions and of the most effective techniques of analysis. Thus, a model such as IDIOM was foreseen as making these common origins explicit and as reducing the cost and increasing the potential relevance (comparability, ease of updating, etc.) of the individual analyses.

Second, without a systematic means of incorporating responses by the rest of the economy to alternative government programs it was impossible to go significantly beyond very narrow, accounting-type analyses, identifying, e.g., immediate changes in government budgets or income distribution. However, the truly significant differences between alternative policies probably derive more from differential behavioral responses than from the more superficial, apparent differences between policy instruments. Similarly, programs which appear to be identical in accounting terms may be found to differ radically if responses by the rest of the system are taken into account. Unfortunately, extensions in this direction could again only be undertaken on a very *ad hoc* basis, in absence of an integrating framework for analysis. Hence the need for a model such as IDIOM.

Finally, the suspicion has gradually gained force among those involved in the analysis that policies avowedly designed to affect certain types of behavior or to bring about certain specified effects may not be the most significant means by which government alters those particular variables or achieves particular objectives. For example, general government fiscal policies might have an influence as great or even greater on the interregional distribution of economic activity than policies explicitly designed to affect this distribution. Proper evaluation of these more general policies, i.e. of the class of policies not amenable to simple accounting analysis, necessarily required a more sophisticated set of analytic tools [Dresch 1972a].

A study of the implications of major military expenditure reductions, undertaken for the United Nations with the financial support of the Ford Foundation, provided the third stimulus to general model design [Dresch 1972b]. This study, focusing on the differential effects of alternative types of military expenditure reduction (e.g. strategic vs. non-strategic) under alternative assumptions concerning the means by which this decline in aggregate demand would be compensated, represented by an attempt to extend the tax substitution analysis into the government expenditure domain. Again, it proved to be impossible to trace through many of the system responses to these various policy changes. It was even difficult to specify how particular governmental objectives might be brought about, e.g. by what means personal consumption expenditure would be increased to compensate for the decline in military spending. Clearly, changes in tax policy could be

employed, but the framework for the specification of such changes and for the analysis of their effects, e.g. on the geographic distribution of economic activity, did not exist. Hence, the lacunae of the analysis were more significant than the contributions it could in fact make.

It is from the interweaving of these strands that IDIOM has taken form. The present version goes only part way to fill the voids encountered in the preceding phases of these studies. However, even in its current form IDIOM at least provides a basis for continued analytical development. Most importantly, a conceptual framework now exists which will permit the cumulative extension of analytic capabilities. Thus, even a rough prototype represents an important step forward.

## 2. AN OVERVIEW OF THE MODEL

IDIOM is basically a two-stage model, consisting of a primary National Model and a secondary Regional Model. The National Model begins with a set of exogenous or predetermined final demands (exogenous or predetermined components of GDP). The production required to fulfill these exogenous final demands generates the income of labor and of capital owners. These incomes then serve to determine endogenous consumption final demands via consumption functions. The system is equilibrated when the incomes generated by the fulfillment of all final demands, including endogenous consumption, just induce the corresponding level of consumption demands.

Thus, the income determination component of the National Model is in essence a rather simple Keynesian multiplier model. Exogenous components of GNP operate via the multiplier (that is, the consumption function) to determine the level of income and output. The departure from the simple Keynesian model resides in the specification of the exogenous final demands: rather than as scalar magnitudes, e.g. investment or government expenditures, these appear as vectors of final purchases from individual producing sectors. In addition, these sectors are represented via an input-output model as making intermediate purchases from other sectors, with each sector exhibiting unique capital and labor input (income generation) coefficients. Thus a change in either the level or in the sectoral composition of exogenous demands induces changes in the level of income. The effect of a change in composition operates through a changed distribution of income between labor and capital. Thus, the value of the multiplier depends on the capital and labor income consumption functions, and in addition on the sectoral distribution of exogenous and endogenous demands (determining the distribution of income between capital owners and labor).

Public policy enters the National Model either through the government components of exogenous final demands, through tax leakages from the income stream, or through consumption by recipients of transfer payments. Thus, in principle, the model can assess the effects of any policy substitution which can be represented by a change in taxes, expenditures or transfers. However, the present level of sophistication incorporated in the model is not adequate for the analysis of many fiscal changes which might be of interest. This limitation, it should be noted, is one of development and not of conceptual or analytical inadequacy.

The model is designed to assess policy substitutions. First, on the basis of a given set of exogenous final demands the model is solved. Solution of the model is represented by solution values of all relevant variables, e.g. GDP, labor and capital income, employment by industry and occupation, raw materials requirements and effluent production. At this stage a policy change, e.g. a reduction in defense expenditure, is introduced into the system, and a compensating policy instrument, e.g. an increase in federal non-defense spending, is indicated. On the basis of some prespecified compensation criterion, e.g., unchanged employment, GDP or labor income, a compensating change in the second instrument is then determined. The resultant changes in all relevant variables then indicate the differential consequences of the policy substitution, given the criterion for compensation.

The Regional Model (designed on the basis of previous work [Leontief *et al* 1965]) begins with the solution of the National Model. One product of the National Model is a vector of sector (industry) outputs. A subset of industries is designated as "national industries," primarily on the basis of high degrees of interregional trade and the presumption of the existence of "national" markets for their outputs. For these national industries total outputs are distributed over regions according to a predetermined distribution matrix indicating the share of each region in the total output of each national industry. The underlying distribution matrix is exogenous in the current version of the model, but in later versions it will be possible to introduce lagged adjustments into the regional distributions of national industry outputs, employing such variables as profitability to modify the distribution matrix over time.

In a similar manner, the exogenous final demands from "local industries" (all non-national industries, primarily services) are distributed over regions, in this case employing an industry-region matrix for *each* exogenous final demand. Thus, the regional distributions of military and of fixed investment purchases from the construction sector (a local industry) may differ from each other, reflecting the underlying difference in the regional distributions of military relative to investment activities.

The only significant departure which the Regional Model makes from the National Model is in the treatment of capital income. For the National Model capital income is endogenous. However, once determined at the national level it could be distributed regionally in one of two ways: First, capital income generated by production in a region could be assumed to be received by residents of the region. Alternatively, national capital income could be assumed to be distributed over regions according to a more basic distribution of capital ownership. Because the second seemed more sensible, capital income, and hence consumption out of capital income, are distributed to regions on the basis of capital ownership, and are predetermined in the Regional Model.

At this point the most blatant oversimplification of the model stands out very clearly. The consistency between the National and Regional Models is insured only by assuming that the entire structure of the productive process is identical across regions. This assumed identity extends from the intersectoral input-output relations to the labor and capital shares of income. Thus, once determined at the national level, capital income is invariant with respect to the regional

distribution of economic activity, and hence capital income can be distributed interregionally on the basis of capital ownership, unaltered by interregional distributions of output.

Having distributed the outputs of national industries across regions, given the assumption of identical wage shares of income, the consumption final demands of employees of national industries have also, simultaneously, been regionally distributed. Thus, the only endogenous component of final demands at the regional level is the consumption by local employees of local industries. Simultaneous solution of the implied set of intra-regional equations results in a configuration of outputs of regional (local) industries consistent with the assumed distribution of national industry outputs, of capital income, and of national and local industry employee consumption demands.

To summarize, local industry outputs in a region must be sufficient to meet demands on local industries emanating from (a) the predetermined outputs of national industries within the region (intermediate purchases by national from local industries), (b) the predetermined regional shares of exogenous (including capitalists' consumption) final demands from local industries, (c) the consumption demands on local industries by local employees of national industries, and (d) the consumption demands on local industries by employees of local industries themselves. The critical assumption is that *any* purchases within a region from local industries must be supplied from within the region. For national industries there are no barriers to interregional trade, and the regional distribution of output can be made exogenously. In the case of local industries, however, there is by assumption no interregional trade, and as a result, the regional distribution of outputs of local industries is endogenous, determined by the prior distribution of other activities.

The introduction of policy changes in the Regional Model parallels the National Model. Base solution values for all regions (for such variables as outputs, incomes, etc.) are determined, and the changes in these induced by a *nationally compensating* policy substitution are then determined. Thus, the compensation criterion is applied only at the national level, with no requirement that the policy substitution be compensating in any individual region. For example, if the compensation criterion is unchanged total employment, the model generates changes in policy instruments which hold national employment constant; employment in any given region may nonetheless change significantly. It is the identification of such regional non-neutralities which is the objective of the Regional Model.

Thus, the policy evaluation function of the model is initiated by specifying the policy instruments which are to change and a compensation criterion. The model then generates the changes in all relevant national and regional variables induced by the policy substitution. Thus, it is possible to evaluate alternative policies on the basis of differential effects in relatively disaggregated dimensions.

### 3. IMPLEMENTING THE MODEL FOR THE UNITED STATES

The basis for the implementation of the model is an 83-order input-output matrix designed to represent the U.S. economy in 1970. All coefficients and variables are expressed in constant 1970 prices. The usual productive sectors are supplemented by general government and household employment sectors,



resulting in a disaggregation of the economy into 86 sectors. All solution computations are performed at this level of disaggregation. However, provision is made to output the results of the analysis at the level of 39 aggregated or 16 more highly aggregated sectors.

The current version of the model identifies nine major final demand components, with provision of up to twelve, exclusive of policy substitution vectors ( $\Delta Y^j$  and  $Y^j$ ). These include:

1. labor consumption
2. capitalist consumption
3. transfer consumption
4. private fixed investment
5. net inventory change
6. gross exports
7. federal government defense
8. federal government non-defense
9. state and local government.

Of these, the first two are endogenous, and the first three display identical distributions over sectors, mirroring the actual 1970 composition of personal consumption expenditure. Even in its current form it is possible to decompose defense final demand into two components, strategic and non-strategic defense, increasing the total number of final demands to ten.

TABLE 3.1  
IDIOM DIMENSIONALITY

Symbol	Designation	Dimension	
		Pre-programmed Potential	Current Actual
nn	Producing sectors	90	86
	Intermediate Aggregation		39
	High Aggregation		16
n	National industries	90	60
l	Local industries		26
m	Final demands	12	9-10
r	Regions	51	51
	Regional Aggregation		13
o	Occupations	25	25
q	Raw Materials	11	11
u	Effluents	14	14
	Air pollutants	14	5
	Water pollutants		8
	Solid waste		1

The model currently identifies twenty-five occupations, the last four of which simply represent total employment in sectors for which no occupational distributions were available (three government sectors and household employment). Eleven raw materials are identified in the materials requirements matrix, chosen for their relevance as exports of developing countries, a dimension of particular interest in the disarmament analysis. Finally, fourteen distinct effluents are identified in the effluent matrix: five air pollutants, 8 water pollutants, and a single solid waste category.

The Regional Model operates at the level of fifty-one separate regions (fifty states plus the District of Columbia), with provision for aggregation of the results into 13 regions. In addition, the Regional Model distinguishes between 60 national industries and 26 local industries.

The dimensionality of the model is summarized in Table 3.1. Sources of data are identified in Table 3.2.

TABLE 3.2  
IDIOM PARAMETER AND DATA DOCUMENTATION

Parameter/Input Data	Source
(1) A matrix of direct I-0 coefficients for 1970 (originally in 1958 prices, adjusted and rebalanced using 2, 3, and 4)	"Projections '70 Input-Output Table," U.S. Bureau of Labor Statistics
(2) A matrix import coefficients, 1969	Cybermatics, Inc.
(3) V and component $T^B$ , $K^D$ , $S^L$ , $T^C$ vectors, value added, 1969	
(4) Index of 1970 prices, 1958 = 100, by sector	
(5) K matrix of 1970 capital requirements by user and producer industry	[W. Halder Fisher and Cecil H. Chilton]
(6) E vector of 1970 employment coefficients	Unpublished, U.S. Bureau of Labor Statistics
(7) 0 matrix of industry occupational distributions	
(8) M matrix of raw materials requirements coefficients by sector, c 1963 at 1963 prices (adjusted to 1970 price basis using 9)	[W. Leontief and P. Petrie]
(9) Index of 1963 prices, 1958 = 100, by sector	Harvard Economic Research Project (from U.S. Office of Business Economics) [International Research and Technology Corporation]
(10) U matrix of effluent coefficients, 1967-79	
(11) $Y^I$ final demand vectors, 1970 (converted from 1958 to 1970 prices)	Distributions: U.S. Office of Emergency Preparedness and University of Illinois Center for Advanced Computations, unpublished. 1970 Control Totals: U.S. Bureau of Economic Analysis
(12) Personal tax and savings rates	Estimated by us from 1969 and 1970 data, adjusted
(13) $D^N$ national industry distribution matrix, by industry and state, c. 1970	Output (sales, shipments, etc.) by state from Censuses of Agriculture and Manufacturing, Mineral Yearbook <i>et al.</i>
(14) $D^2$ capital income distribution vector, by state, 1970	Property and proprietors' income ( <i>Statistical Abstract</i> )
(15) $D^3$ transfer payment distribution vector, by state, 1970	Transfer payments ( <i>Statistical Abstract</i> )
(16) $D^4$ exogenous final demand local industry distribution matrices, c. 1970	Construction contracts, payrolls, etc.



#### 4. DISARMAMENT: AN APPLICATION OF THE MODEL

This section draws upon a study of the domestic consequences of a U.S. military contraction [Dresch 1972b] to briefly indicate the substantive application of IDIOM. The analysis assumes a significant military contraction, amounting to 20 percent of the 1970 U.S. military budget, or about \$15.2 billion out of total defense spending of over \$75 billion. This contraction is further assumed to take the form of either (a) an across-the-board reduction in all military programs (denoted GD) or (b) a reduction only in military activities of a strategic nature (denoted SD). In the latter case, the \$15.2 billion expenditure reduction would equal approximately 95 percent of total strategic expenditure, effectively a case of complete strategic disarmament. The first case can be considered reflective of a comprehensive balance force reduction.

Five alternative types of compensating expenditure are examined:

- (1) U.S. exports to developing countries (EDC),
- (2) U.S. machinery and transportation equipment exports (ME),
- (3) personal consumption (PC),
- (4) social and educational services (SS), and
- (5) private fixed investment (FI).

The first two are designed to reflect a diversion of resources to international economic development. The third (personal consumption) is specifically assumed to take the form of the enactment of the Nixon Administration's proposed Family Assistance Plan, while the fourth (social services) assumes a proportionate expansion in all state-local government expenditures for health, education and related services.

For each of the resultant ten cases (two disarmament and five compensating expenditure scenarios) the compensating expenditure increase was determined which would just hold aggregate employment constant when defense expenditure was reduced by \$15.2 billion (20 percent). The consequences of the compensated disarmament substitutions are summarized in Table 4.1. The following tables indicate the differential effects of the substitutions (as proportions of base 1970 values) for employment by industry (Table 4.2), employment by occupation (Table 4.3), capital requirements by type of capital good (Table 4.4), raw materials requirements (Table 4.5), effluent generation (Table 4.6), and employment by region (Table 4.7). For a more detailed description of the underlying analysis and discussion of the implications of those applications of the model, the reader is referred to the report to the United Nations cited above.

TABLE 4.1  
SUMMARY EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% net change in 1970 values)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Net change in total employment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Net change in total labor earn	-0.1	-0.2	0.1	0.1	-0.7	-0.7	-0.0	-0.0	-0.0	-0.1
Net change in net capital income	1.5	1.2	0.7	0.9	1.8	1.5	0.0	-0.2	1.0	0.7
Net change in GDP	0.5	0.3	0.5	0.3	0.2	0.1	-0.0	-0.1	0.4	0.3
Net change in imports	0.8	0.2	1.0	0.4	0.6	-1.1	-1.9	-2.3	-0.2	-0.7
Compensating Expenditure	\$19.371	\$18.089	\$19.018	\$17.760	\$17.908	\$13.813	\$13.813	\$12.899	\$18.291	\$17.081
% of 1970 Defense Reduction	127.9	119.4	125.5	117.2	118.2	110.4	91.2	85.1	120.7	112.7
% of 1970 Compensating-type Expenditure	166.1	155.1	106.4	99.3	22.5*	21.0*	14.3	13.3	13.8	12.9

\* Percentage of transfer income.

Key to Tables 4.1-4.7: EDC—Exports to developing countries, ME—Machinery and transportation equipment exports, PC—Personal consumption, SS—Social and educational services, FI—Fixed investment, GD—20% general defense reductions, SD—20% strategic defense reduction. "X"/"Y": demand "Y" replaces demand "X".

TABLE 4.2  
INDUSTRY EMPLOYMENT EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% of 1970 Total)

High Aggregation	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Agriculture	4.7	4.3	0.2	0.1	2.4	2.1	0.0	-0.1	0.3	0.2
Mining	2.5	1.4	0.3	-0.6	-0.1	-1.0	-0.5	-1.3	0.8	-0.2
Ordnance	-11.8	-12.8	-11.7	-12.6	-11.8	-12.7	-12.0	-12.9	-11.9	-12.8
Non-durable manufacturing	2.7	2.3	0.3	0.1	2.0	1.7	-0.1	-0.2	0.3	0.1
Durable manufacturing	2.6	2.0	5.7	4.9	-1.5	-1.9	-1.7	-2.0	2.4	1.8
Agricult., forestry and fish. serv.	4.8	4.4	0.2	0.1	2.7	2.5	-5.4	-5.1	0.3	0.2
Construction	-0.5	-1.2	-0.5	-1.2	-0.3	-1.0	1.2	0.8	6.7	5.6
Printing and publishing	0.9	0.7	0.4	0.3	1.6	1.3	0.9	0.7	0.7	0.6
Utilities and communications	0.7	0.4	0.7	0.4	1.4	1.1	0.3	0.1	1.1	0.7
Trade and transportation	1.0	0.8	1.2	1.0	1.8	1.6	-0.1	-0.2	1.3	1.1
Finance, insurance and real estate	0.5	0.4	0.5	0.3	2.3	2.1	0.2	0.1	0.5	0.4
Services	0.1	-0.2	0.1	-0.1	2.1	1.7	-0.1	-0.3	0.2	-0.1
Household employment	0.1	0.0	0.2	0.1	3.0	2.7	-0.0	-0.1	0.1	0.0
Gov't. and gov't. enterprises, except defense	0.1	0.0	0.1	0.0	0.2	0.1	9.6	9.0	0.1	0.0
Defense	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1
% < ±1%	7	7	11	10	3	1	9	9	9	10
% < ±1%	47	47	73	67	20	7	60	60	60	67
Mean absolute percentage change	2.2	1.8	2.2	1.8	2.6	2.2	2.7	2.4	2.2	1.7
Intermediate Aggregation										
Agriculture	4.7	4.3	0.2	0.1	2.4	2.1	0.0	-0.1	0.3	0.2
Metal mining	1.4	-2.1	0.7	-2.7	-3.5	-6.7	-3.4	-6.6	-0.3	-3.7
Coal, stone, fertilizer mining	2.1	1.6	0.6	0.3	0.1	-0.3	0.4	0.1	3.5	3.0
Crude Petroleum and refining	1.5	1.2	-0.2	-0.4	1.1	0.8	-0.1	-0.3	0.1	-0.1
Ordnance	-11.8	-12.8	-11.7	-12.6	-11.8	-12.7	-12.0	-12.9	-11.9	-12.8
Food	2.1	1.9	0.2	0.2	2.7	2.4	0.1	0.0	0.2	0.1
Tobacco	2.5	2.3	0.2	0.2	2.6	2.4	-0.0	-0.1	0.1	0.1
Textiles	1.9	1.8	0.2	0.2	2.4	2.1	-0.1	-0.1	0.1	0.1

TABLE 4.2 (continued)  
INDUSTRY EMPLOYMENT EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% of 1970 Total)

Intermediate Aggregation	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Lumber and products	1.3	1.0	0.1	-0.1	0.1	-0.2	0.8	0.5	5.0	4.4
Furniture and fixtures	0.2	0.2	0.1	-0.0	1.4	1.2	0.9	0.8	3.8	3.4
Paper and products	4.1	3.7	0.7	0.5	1.5	1.2	0.2	0.1	1.0	0.8
Chemicals and plastics	6.2	4.9	-0.1	-1.0	0.6	-0.3	-0.2	-1.1	0.0	-0.9
Misc. Rubber, plastic and products	2.4	2.1	1.5	1.3	1.1	0.8	-0.3	-0.5	1.2	0.9
Footwear	0.6	0.5	0.1	0.0	2.7	2.4	-0.1	-0.2	0.1	0.0
Iron and steel	4.3	3.8	4.6	4.0	-1.3	-1.4	-1.1	-1.3	2.8	2.4
Fabricated metals	2.0	1.6	3.3	2.8	-0.5	-0.7	-0.4	-0.7	3.5	3.0
Machinery	7.1	6.2	16.1	14.6	-1.5	-1.8	-1.3	-1.7	4.9	4.2
Electrical machinery	2.1	0.5	6.6	4.6	-2.4	-3.8	-2.9	-4.2	1.3	-0.3
Transportation equipment	-1.4	-1.3	4.8	4.5	-4.7	-4.4	-5.5	-5.1	-2.2	-2.1
Instruments	3.8	3.5	0.5	0.4	-1.5	-1.5	-1.8	-1.8	1.7	1.5
Misc. manufacturing	2.9	2.6	0.4	0.3	1.8	1.5	0.8	0.7	1.3	1.1
Agricult., forestry and fish. serv.	4.8	4.4	0.2	0.1	2.7	2.5	-5.4	-5.1	0.3	0.2
Construction	-0.5	-1.2	-0.5	-1.2	-0.3	-1.0	1.6	0.8	6.7	5.6
Printing and publishing	0.9	0.7	0.4	0.3	1.6	1.3	0.9	0.7	0.7	0.6
Communications	0.7	0.5	0.7	0.6	1.2	1.0	0.3	0.2	1.5	1.2
Utilities	0.7	0.2	0.6	0.1	1.7	1.2	0.4	-0.0	0.4	0.0
Trade and transportation	1.0	0.8	1.2	1.0	1.8	1.6	-0.1	-0.2	1.3	1.1
Finance and insurance	0.5	0.4	0.5	0.4	2.3	2.1	0.2	0.1	0.5	0.4
Real estate and rental	0.3	0.2	0.3	0.2	2.4	2.1	0.0	-0.1	0.4	0.3
Hotels and services, except auto.	0.4	0.3	0.4	0.3	1.7	1.5	0.2	0.0	0.7	0.5
Auto repair services	0.6	0.5	0.4	0.3	2.1	1.9	0.3	0.2	0.8	0.7
Amusements, et al	0.2	0.1	0.3	0.2	2.1	1.9	-0.3	-0.3	0.3	0.2
Medical, educational et al serv.	-0.3	-0.6	-0.3	-0.6	2.3	1.8	-0.2	-0.6	-0.3	-0.7
Government enterprises	0.6	0.3	0.5	0.3	1.4	1.2	0.3	0.1	0.5	0.3
Household employment	0.1	0.0	0.2	0.1	3.0	2.7	0.0	-0.1	0.1	0.0
Defense employment	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1
Federal non-defense employment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	0.0
State-local employment	0.0	0.0	0.0	0.0	0.0	0.0	11.9	11.1	0.0	0.0
# < ± 1%	16	17	29	26	7	8	27	27	21	23
% < ± 1%	42	45	76	68	18	21	71	71	55	61
Mean absolute percentage change	2.3	1.9	2.3	1.9	2.6	2.3	2.7	2.5	2.7	1.9

TABLE 4.3  
OCCUPATIONAL EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% of 1970 Total)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Technical engineers	0.8	0.3	2.4	1.8	-1.3	-1.7	-1.8	-2.1	0.8	0.2
Medical and Health professionals	-0.0	-0.3	-0.1	-0.4	2.3	1.9	-0.3	-0.6	-0.2	-0.5
Teachers (including college)	-0.2	-0.6	-0.2	-0.5	2.4	1.9	-0.2	-0.5	-0.2	-0.6
Natural scientists	1.8	1.3	0.6	0.2	0.7	0.3	-0.8	-1.2	0.4	-0.0
Social scientists	1.0	0.7	1.3	0.9	1.1	0.7	-0.3	-0.6	0.9	0.5
Technicians, other	1.2	0.7	2.2	1.6	-0.2	-0.6	-0.8	-1.2	1.5	1.0
Professional, other	0.7	0.4	0.9	0.6	1.3	1.0	-0.3	-0.5	0.7	0.4
Managerial	0.9	0.6	1.1	0.8	1.6	1.3	-0.0	-0.2	1.4	1.1
Clerical	0.9	0.7	1.2	0.9	1.5	1.2	-0.2	-0.3	1.0	0.7
Sales	0.8	0.6	1.1	0.9	1.9	1.7	-0.0	-0.2	1.2	1.0
Construction crafts	0.1	-0.5	0.1	-0.5	-0.2	-0.8	0.9	0.2	5.2	4.3
Foreman, not elsewhere class.	1.7	1.3	2.1	1.6	0.4	0.1	-0.6	-0.9	1.6	1.2
Metal crafts	2.9	2.3	6.5	5.6	-1.7	-2.0	-1.9	-2.2	2.1	1.6
Mechanics, repair	1.2	0.9	1.5	1.1	1.0	0.7	-0.4	-0.6	1.2	0.8
Printing crafts	1.0	0.8	0.6	0.4	1.4	1.2	0.7	0.5	0.7	0.6
Transportation crafts	1.1	0.7	0.9	0.5	1.1	0.7	0.1	-0.1	1.2	0.9
Other crafts	1.5	1.2	1.5	1.1	0.9	0.6	-0.1	-0.4	1.8	1.4
Operatives	1.8	1.4	2.0	1.6	0.8	0.5	-0.5	-0.7	1.4	1.0
Service workers	0.3	0.1	0.3	0.1	2.0	1.7	-0.2	-0.4	0.2	-0.0
Laborers, except farm	1.3	0.9	1.0	0.7	1.0	0.6	0.2	-0.1	2.5	2.1
Farmers, farm labor	4.7	4.4	0.2	0.1	2.4	2.1	-0.0	-0.1	0.2	0.1
Federal government, Defense	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1	-19.0	-14.1
Federal government, Non-defense	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
State and local government	0.0	0.0	0.0	0.0	0.0	0.0	11.9	11.1	0.0	0.0
Household workers	0.1	0.0	0.2	0.1	3.0	2.7	-0.0	-0.1	0.1	0.0
# < ± 1%	12	18	12	17	8	12	21	19	12	15
% < ± 1%	48	72	48	68	32	48	84	76	48	60
Mean absolute percentage change	2.1	1.6	2.1	1.6	2.2	1.7	2.5	2.3	2.1	1.6

TABLE 4.4  
CAPITAL REQUIREMENTS EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% of 1970 Stock by Type of Capital Good)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Coal, stone, fertilizer mining	4.1	3.5	3.7	3.1	-0.3	-0.6	-0.6	-0.9	2.4	1.9
Textiles	0.6	0.4	0.6	0.5	2.0	1.7	0.0	-0.1	0.8	0.7
Lumber and products	2.8	2.3	2.7	2.2	0.4	0.1	-0.6	-0.9	1.4	1.0
Furniture and fixtures	0.7	0.5	0.8	0.6	1.9	1.6	0.0	-0.2	1.0	0.8
Chemicals and plastics	10.6	8.4	-0.8	-2.3	-0.6	-2.1	-0.6	-2.1	-0.5	-2.0
Misc. Rubber, plastic and products	1.3	0.9	0.7	0.4	1.6	1.2	0.1	-0.2	1.0	0.7
Footwear	0.8	0.5	0.8	0.6	1.9	1.6	0.0	-0.1	1.1	0.8
Iron and steel	1.2	0.9	0.1	-0.2	0.0	-0.2	0.8	0.6	5.1	4.6
Fabricated metals	2.1	1.7	1.2	0.8	1.2	0.8	-0.1	-0.4	1.0	0.7
Machinery	2.6	2.2	1.7	1.3	0.9	0.5	-0.2	-0.5	1.4	1.0
Electrical machinery	1.1	0.8	0.9	0.6	1.5	1.2	0.0	-0.2	1.0	0.7
Transportation equipment	2.0	1.7	0.8	0.6	1.3	1.1	-0.0	-0.2	1.2	1.0
Instruments	1.1	0.7	0.6	0.3	1.7	1.3	-0.1	-0.3	0.6	0.3
Misc. manufacturing	1.0	0.7	0.6	0.4	1.7	1.3	0.0	-0.2	0.8	0.5
Construction	1.7	1.3	1.3	0.9	1.2	0.9	-0.1	-0.4	1.0	0.6
Trade and transportation	1.6	1.3	0.9	0.6	1.7	1.3	-0.0	-0.2	1.1	0.8
# < ± 1%	3	8	11	12	5	6	16	15	4	10
% < ± 1%	19	50	69	75	31	38	100	94	25	63



TABLE 4.5  
 RAW MATERIALS EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
 (% of Total 1970 Use)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Bauxite	2.5	2.1	2.4	1.9	-2.0	-2.2	-1.9	-2.0	1.4	1.0
Chromium	3.6	3.0	4.9	4.2	-0.6	-0.9	-0.6	-1.0	3.0	2.4
Copper	2.8	2.2	2.3	1.8	-2.0	-2.2	-1.8	-2.0	1.4	0.9
Iron	5.4	4.7	5.1	4.5	-0.7	-1.0	-0.6	-1.0	3.3	2.7
Lead	2.6	2.0	3.9	3.2	-1.1	-1.5	-1.4	-1.7	1.2	0.7
Manganese	5.3	4.6	5.4	4.7	-0.7	-1.0	-0.7	-0.9	3.3	2.8
Molybdenum	5.0	4.1	9.3	8.2	-1.2	-1.5	-1.2	-1.5	3.5	2.9
Nickel	4.6	3.8	8.6	7.5	-1.2	-1.6	-1.2	-1.7	3.2	2.5
Tin	4.1	3.6	2.4	2.0	0.0	-0.2	-0.9	-1.1	1.1	0.8
Zinc	3.5	2.9	3.6	3.0	-1.4	-1.6	-1.3	-1.5	2.1	1.7
Petroleum	1.2	0.9	-0.2	-0.4	1.1	0.8	-0.1	-0.3	0.1	-0.1
# < ±1%	0	1	1	1	4	3	5	2	1	4
% < ±1%	0	9	9	9	36	27	45	18	9	36

TABLE 4.6  
EFFLUENT GENERATION EFFECTS OF A COMPENSATED DEFENSE CONTRACTION  
(% of Total 1970 Industrial Generation)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC	SD/PC	GD/SS	SD/SS	GD/FI	SD/FI
Air Pollutants										
Particulates	2.2	1.8	1.0	0.6	0.9	0.5	-0.2	-0.5	1.1	0.7
Hydrocarbons	1.4	1.1	1.0	0.7	1.6	1.3	-0.1	-0.2	1.1	0.9
Sulfur oxides	1.8	1.4	0.9	0.5	1.3	0.9	-0.0	-0.3	0.7	0.3
Carbon Monoxide	2.8	2.5	0.8	0.6	0.8	0.6	-0.2	-0.3	0.8	0.6
Nitrogen oxides	2.0	1.6	0.8	0.5	1.1	0.8	0.0	-0.3	0.8	0.5
Solid Waste	3.0	2.4	0.2	-0.2	1.3	0.8	-0.1	-0.4	0.7	0.3
Water Pollutants										
Waste Water	6.3	5.8	0.3	0.2	2.0	1.8	0.0	-0.1	0.4	0.3
Chemical oxygen demand	2.0	1.8	0.2	0.1	2.6	2.3	0.0	-0.0	0.2	0.1
Biological oxygen demand	2.4	2.1	0.3	0.1	2.1	1.7	-0.0	-0.2	0.3	0.1
Refractory organics	0.3	0.2	0.3	0.1	2.4	2.1	0.0	-0.1	0.4	0.3
Suspended solids	2.7	2.5	0.3	0.2	2.5	2.3	0.0	-0.1	0.2	0.1
Dissolved solids	3.3	2.7	0.2	-0.3	1.5	1.0	-0.0	-0.5	0.3	-0.1
Nitrogen	2.2	1.9	0.2	0.1	2.6	2.3	0.0	-0.0	0.2	0.1
Phosphate compounds	2.2	1.9	0.2	0.1	2.6	2.4	0.0	-0.0	0.2	0.1
# < ±1%	1	1	12	14	2	5	14	14	12	14
% < ±1%	7	7	86	100	14	36	100	100	86	100

TABLE 4.7  
REGIONAL EMPLOYMENT EFFECTS OF A COMPENSATED 20% DEFENSE CONTRACTION  
(% of 1970 Total)

	GD/EDC	SD/EDC	GD/ME	SD/ME	GD/PC (FAP)*	SD/PC (FAP)*	GD/SS	SD/SS	GD/FI	SD/FI
New England North	0.2	-0.0	0.0	-0.2	-0.4	-0.5	-0.0	-0.2	0.2	0.0
New England South	0.2	-0.0	0.8	0.5	1.2	0.9	-0.1	-0.3	0.3	-0.0
Midwest North	0.9	0.6	0.9	0.7	-0.2	-0.3	1.1	0.9	0.8	0.6
Midwest South	-2.8	-2.2	-2.8	-2.2	-2.5	-1.9	-1.9	-1.4	-2.5	-1.9
Great Lakes	1.6	1.4	2.5	2.2	0.2	-0.0	0.8	0.6	1.6	1.4
Plains	0.9	0.8	0.5	0.4	0.6	0.5	0.6	0.5	0.2	0.2
Southeast Atlantic	-1.6	-1.3	-2.3	-1.9	-0.1	0.1	-1.5	-1.1	-1.4	-1.1
Southeast Gulf	-0.0	-0.0	-1.0	-0.9	2.9	2.8	-0.1	-0.0	-0.0	0.0
Southeast Central	0.5	0.5	-0.1	-0.1	2.4	2.2	0.3	0.2	0.6	0.5
Southwest	-1.6	-1.5	-2.1	-1.9	-0.0	-0.0	-1.5	-1.4	-1.5	-1.4
Mountain	-0.9	-0.7	-1.4	-1.2	1.0	1.0	-0.8	-0.6	-1.2	-1.0
Far West	-1.3	-1.1	-1.3	-1.0	-1.7	-1.5	-0.3	-0.1	-1.1	-0.9
Non-Coterminous	-6.3	-4.9	-6.9	-5.5	-4.9	-3.6	-4.8	-3.6	-5.4	-4.1
# < ±1%	7	7	5	6	6	7	8	9	6	7
% < ±1%	54	54	38	46	46	54	62	69	46	54

\* Assumes enactment of the proposed Family Assistance Plan.

## 5. CONTINUING MODEL DEVELOPMENT

The conceptual foundation for current NBER research in the public policy-public finance area is the conviction that public policy must be viewed in closed, differential incidence terms, i.e. that the consequences of a full menu of policy change must be examined with reference to both direct effects and indirect behavioral responses. This type of closed-system analysis requires, by definition, a general equilibrium approach, one incorporating the important elements of the system which are influenced by alternative policy actions.

The obvious difficulty with such an approach to policy evaluation is the complexity of the system which it is necessary to represent. From the beginning of this research effort the tension between ultimate objectives and more immediate policy relevance has been purposely maintained, and an incremental approach which would both move in the desired direction but also be capable of intermediate analyses of more than academic interest has been sought. After two years of effort directed to the concrete development of this type of evolutionary policy analysis capability, we believe that a most fruitful research program has been established, one which is directly focused on the long run goal of developing a closed, general equilibrium system but also one which permits continuing application of the analysis capabilities as they develop. Furthermore, we now believe that the long-run objectives are clarified and strengthened by this interactive approach. Attempts to evaluate, even imperfectly, current policy options have suggested valuable directions and methods for the more basic research efforts.

In its current form, IDIOM provides both a skeletal framework for further analytical development and a tool for actual policy evaluations. Thus, the real value of IDIOM is its usefulness as a foundation for further research, both basic and applied. Several parallel lines of continuing model development and associated applications can be clearly indicated. Although the prototype has only just been operationally completed, a sufficient pause for reflection on the most productive method of organizing further research and analysis has been possible. The result has been a provisional decision to focus efforts and resources on a very few areas of model development, with extension into other dimensions as appropriate personnel and financial support become available. The areas in which resources will initially be concentrated are:

1. *Elaboration of the compensating policy substitutions framework.* Currently, the policy evaluation capabilities of the model are limited to alternative policies, each of which can be directly expressed as a change in a final demand vector, e.g. the replacement of a defense final demand vector by a vector of federal non-defense demands. Thus, many policy actions of interest cannot be evaluated with the current model, specifically those which appear initially as changes in tax rates and structure. For example, a specified defense reduction to be compensated by increases in consumer demand induced by a federal tax reduction cannot now be analyzed. Similarly, analysis of one change in the tax system to be compensated by another is beyond the immediate capability of the model. Extension of the model to incorporate tax variables either as the pre-specified or as the compensating policy change involves only rather simple algebraic elaboration and is the highest priority for the completion of prototype IDIOM. When this capability is added, it will

be possible for the model to determine the degree of change in a tax variable required to compensate for a prespecified change in some other policy variable (final purchase or tax) so as to maintain, e.g., employment or the government surplus constant, taking into account the effect of the tax change on non-government demands.

2. *The incorporation of price and wage determination.* The model currently incorporates prices and wages only implicitly, effectively taking both as given, independent of changes in such factors as tax rates or levels of output or employment. Clearly, for the analysis of tax system changes the incorporation of price and wage adjustments is essential. Also, in any situation in which factor earnings or levels of output or employment (either in the aggregate or for an individual sector or occupation) change dramatically, price and wage responses would be expected to occur, and these would serve to modify the effects on the system of the specified policy changes. Thus, it is vitally important that the model incorporate these adjustments of the system to policy changes. The development of a flexible, interactive mechanism for price and wage determination is essential to the model and is a major objective for the intermediate future.

3. *The development of the household model.* The model currently represents the household sector only in the aggregate. Because one of the primary concerns in the assessment of alternative policies is the effect on income distribution, the elaboration of this dimension requires the immediate devotion of effort and resources. Effectively, it is necessary to disaggregate the household sector by income class and other relevant variables. This involves employing a sample of representative households to which employment (by occupation, industry, and region) can be attributed. On this basis the distribution of labor income can be determined. With information on household wealth, non-labor income can also be distributed. By identifying savings, the distribution of wealth (and of non-labor income) can be successively monitored.

4. *The development of an investment model.* In the current model investment demand is exogenously determined. However, we have identified capital goods requirements necessitated by the solution bills of goods, both by capital user and capital goods producer industry. This would provide the foundation for entering investment demands endogenously. Various types of investment models could be employed, e.g. accelerator models relying on changes in sector outputs, cash flow models keyed to profit levels, or classical models relying on profit rates. A general model incorporating elements of each, with provision for flexibly analyzing the consequences of alternative conceptions of the determinants of investment would probably be most productive as a framework for an endogenous investment determination system. An interesting development in this context would be the incorporation of monetary policy, a dimension of policy currently excluded. Both monetary policy and investment determination would have to be tied back to price, wage and output determination.

5. *The incorporation of an environmental policy capability.* At present, the model simply identifies the levels of effluents generated by the productive process given current technology and practice. It would be possible to explicitly identify effluent abatement sectors [Leontief and Ford] and represent purchases of their services by other sectors and outputs of the effluent abatement sectors (pollution

abatement, secondary products, etc.), identifying (a) those cases in which pollution control would be profitable on its own (due to secondary products) and (b) the effects on the system of various legislated abatement requirements (consequences for income distribution, prices, effluent levels, etc.).

6. *Endogenous treatment of international trade.* The assumption of the present version of the model is that import coefficients for both final demands and production are fixed. A very interesting extension would relate these to prices of import competing domestic goods relative to world market prices. Similarly, the effects of relative price changes on export demands could also be incorporated.

7. *The incorporation of labor markets and migration.* This represents one of the most important future developments of IDIOM. It would involve incorporating a supply side to the labor market, through the household model, identifying the determinants of shifts of workers between occupations, industries, and regions. Effectively, it would bring together the initial, simple wage determination system and the household sector. The interregional migration component, in concert with the investment model, would permit the evaluation of alternative regional development policies.

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The first part of the document is a preface written by the author, in which he explains the purpose and scope of the work. He states that the book is intended to provide a comprehensive overview of the subject matter, covering both the theoretical and practical aspects. The author also mentions that the book is written in a clear and concise style, suitable for both students and professionals alike. The preface concludes with a statement of the author's hope that the book will be a valuable resource for anyone interested in the field.

The second part of the document is the main body of the text, which is divided into several chapters. The first chapter introduces the basic concepts and terminology of the subject. The second chapter discusses the historical development of the field, highlighting key figures and milestones. The third chapter focuses on the current state of research and practice, while the fourth chapter explores future trends and challenges. Each chapter is supported by a wealth of references and examples, providing a solid foundation for understanding the subject.

The final part of the document is a conclusion, in which the author summarizes the main findings and insights of the work. He reiterates the importance of the subject and the need for continued research and innovation. The conclusion also includes a list of references and a bibliography, providing a comprehensive list of sources used in the book. The document ends with a final statement of the author's appreciation for the support and encouragement of his colleagues and family.

## APPENDIX: THE ANALYTICS OF IDIOM

### 1. THE NATIONAL MODEL

In its prototype form the national component of IDIOM combines a very simple, rarefied income determination model with a conventional input-output model, resulting in a variant of a closed input-output model. This section will first present the National Model in algebraic detail, and then will devote greater effort to explanation and rationalization.

In the algebraic presentation, capital letters denote vectors and matrices, with dimensions specified in parentheses. In some cases it will be necessary to express previously defined vectors as diagonal matrices, in which case a bar will appear over the letter; e.g. if the vector  $V(n \times 1)$  is expressed  $\bar{V}$  it will be understood that  $\bar{V}(n \times n)$  contains the elements of  $V$  on the diagonal, all other elements of  $\bar{V}$  being zero. A bold face vector designation, e.g.  $\mathbf{C}$ , indicates a square matrix formed by repeating the vector; thus if  $C(n \times 1)$ , then  $\mathbf{C}(n \times n) = [C, C, \dots, C]$ . Lower case latin letters indicate scalar magnitudes, e.g. GDP and non-vector tax rates. The symbol  $1_v$  will denote a vector, all of the elements of which are unity.

The primary variables and parameters entering the National Model are indicated in Table I-1. Employing this notation, the model is solved by determining the vector of total outputs,  $X$ , and the vectors of labor and capitalist consumption,  $Y^1$  and  $Y^2$ , as a function of the exogenous final demands,  $Y^3$  through  $Y^m$ .

From the input-output condition that final demands equal total output minus intermediate requirements,

$$(3.1a) \quad (Y^1 + Y^2 + \dots + Y^m) = X - AX = (I - A)X$$

or

$$(3.1b) \quad X = (I - A)^{-1}(Y^1 + \dots + Y^m).$$

If all of the final demands were known, the model would be solved. However, labor and capitalist consumption are functions of the corresponding incomes. Total gross domestic product (GDP) is

$$(3.2) \quad \text{GDP} = X'V.$$

To obtain disposable labor and capital incomes the following must be removed:

$$(3.3) \quad \text{Indirect Business Taxes} = X'\bar{V}T^B$$

$$(3.4) \quad \text{Depreciation} = X'\bar{V}(I - \bar{T}^B)K^D$$

$$(3.5) \quad \text{Corporate profits taxes} = X'\bar{V}(I - \bar{T}^B)(I - \bar{K}^D)(I - \bar{S}^L)T^C.$$

Thus disposable labor and capital incomes ( $z_l$  and  $z_k$ , respectively) are given by

$$(3.6a) \quad z_l = (1 - t_l)X'\bar{V}(I - \bar{T}^B)(I - \bar{K}^D)S^L$$

$$(3.6b) \quad z_k = (1 - t_k)X'\bar{V}(I - \bar{T}^B)(I - \bar{K}^D)(I - \bar{S}^L)(1_v - T^C)$$

TABLE I-1  
BASE NATIONAL MODEL VARIABLES AND PARAMETERS

$n$	the number of producing sectors
$m$	the number of separate final demand components, the last $m - 2$ of which are exogenous
$X(n \times 1)^*$	total output vector
$Y^i(n \times 1), i = 3, m$	exogenous final demand vectors
$Y^1(n \times 1)^*$	endogenous labor consumption vector
$Y^2(n \times 1)^*$	endogenous consumption from capital income
$C(n \times 1)$	consumption distribution vector, the elements of which sum to unity, indicating the sectoral distribution of consumption final demands
$A(n \times n)$	matrix of interindustry direct input requirements coefficients, per unit of output
$V(n \times 1)$	vector of value added coefficients
$T^B(n \times 1)$	vector of indirect business taxes, as a proportion of value added
$K^D(n \times 1)$	vector of capital depreciation rates, proportion of value added net of indirect business tax
$S^L(n \times 1)$	vector of labor shares of value added net of indirect business tax and depreciation
$T^C(n \times 1)$	vector of corporate profits tax rates, proportion of capital's residual share of value added (net of labor, indirect business tax, and depreciation)
$t_l$	tax rate on labor income
$t_k$	tax rate on capital income (net of corporate tax and depreciation)
$s_l$	labor savings rate (out of disposable income)
$s_k$	capital savings rate (out of disposable income)
$t_n$	aggregate government personal transfer payments
$s_n$	savings rate out of transfer income
$E(n \times 1)$	employment coefficients vector (employment/output)
$O(o \times n)$	occupation distribution matrix, $o =$ number of occupations (distribution of employment by industry over occupations)
$M(q \times n)$	raw materials coefficient matrix, $q =$ number of materials (materials requirements/output)
$U(u \times n)$	effluent coefficients matrix, $u =$ number of effluents (effluents/output)
$K(n \times n)$	capital coefficients matrix, capital requirements per unit of output by producer (row) and user (column) industry

\* Except for items marked by asterisk ( $X, Y^1, Y^2$ ) all are inputs into the model.

where  $t_l$  and  $t_k$  represent personal tax rates on labor and capital net incomes (personal income and payroll taxes primarily). Of these disposable incomes, proportions  $s_l$  and  $s_k$  are saved, resulting in consumption ( $c_l$  and  $c_k$ ) of

$$(3.7a) \quad c_l = (1 - s_l)z_l$$

$$(3.7b) \quad c_k = (1 - s_k)z_k.$$

These consumption aggregates are converted to final demand vectors via a consumption distribution vector,  $C$ :

$$(3.8a) \quad Y^1 = c_l C, \text{ labor consumption vector,}$$

$$(3.8b) \quad Y^2 = c_k C, \text{ capitalist consumption vector.}$$

At this point the model consists of  $3n$  equations,

$$X = (I - A)^{-1}(Y^1 + Y^2 + \dots + Y^m)$$

$$Y^1 = (1 - s_l)(1 - t_l)X'V(I - T^B)(I - K^D)S^L$$

$$Y^2 = (1 - s_k)(1 - t_k)X'V(I - T^B)(I - K^D)(I - S^L)(1 - T^C)$$

in  $3n$  unknowns,  $X$ ,  $Y^1$  and  $Y^2$ . The solution of this system results in

$$(3.9) \quad X = (I - (I - A)^{-1}M)^{-1}(I - A)^{-1}(Y^3 + \dots + Y^m)$$

where

$$M = (1 - s_i)(1 - t_i)CS^L(I - \bar{K}^D)(I - T^B)\bar{V} + \\ (1 - s_k)(1 - t_k)C(I - \bar{S}^L)(I - T^C)(I - \bar{K}^D)(I - T^B)\bar{V}$$

from which the solutions for  $Y^1$  and  $Y^2$  can be obtained.

Having obtained a solution for  $X$  via equation 3.9, the full panoply of National Model Outputs can be determined, as indicated in Table I-2. These outputs provide the base to which policy-substitution-induced changes are compared.

The current version of the model, for reasons of initial design simplicity, is restricted to the analysis of policy changes which can be represented by changes in exogenous final demand vectors. Eventually, the model will be extended to assess

TABLE I-2  
NATIONAL MODEL OUTPUTS

Name*	Derivation
Total output by industry (vector)	$X$
Gross domestic product (GDP)	$X'V$
Depreciation	$X'V(I - T^B)K^D$
Consumption	$1'_r(Y^1 + Y^2 + Y^3)$
1. Labor consumption	$1'_r Y^1$
2. Capitalist consumption	$1'_r Y^2$
3. Transfer consumption	$1'_r Y^3$
Aggregate demand by other final demand component (investment, government, etc.)	$1'_r Y^4, \dots, 1'_r Y^m$
Net capital income	$X'V(I - T^B)(I - \bar{K}^D)(I - \bar{S}^L)(1_r - T^C)$
Labor income	$X'V(I - T^B)(I - \bar{K}^D)S^L$
Capitalists' net savings	$s_k(1 - t_k)X'V(I - T^B)(I - \bar{K}^D)(I - \bar{S}^L)(1_r - T^C)$
Labor savings	$s_r(1 - t_r)X'V(I - T^B)(I - \bar{K}^D)S^L$
Transfer savings	$s_t t_r$
Tax revenues, gross	(the sum of 1. through 4.)
1. Indirect business taxes	$X'VT^B$
2. Corporate profits taxes	$X'V(I - T^B)(I - \bar{K}^D)(I - \bar{S}^L)T^C$
3. Capitalist personal taxes	$t_k X'V(I - T^B)(I - \bar{K}^D)(I - \bar{S}^L)(1_r - T^C)$
4. Labor personal tax	$t_r X'V(I - T^B)(I - \bar{K}^D)S^L$
Transfers (government)	$t_r$
Net tax revenues†	Gross tax revenues minus $t_r$
Employment, total	$X'E$
Employment by industry (vector)	$X'E$
Employment by occupation (vector)	$OEX$
Raw materials consumption (vector)	$MX$
Effluent production	$UX$
Capital requirements, total	$1'_r KX$
Capital requirements by capital goods producer industry (vector)	$KX$
Capital requirements by capital user industry (vector)	$X'K$

\* Except for those specified as vectors, all model outputs are scalar magnitudes.

† Government surplus (deficit) is also obtained, by subtracting government expenditures ( $1'_r Y^{Gov}$ ) from net tax revenues.

any change in any exogenous variables or parameters, e.g. tax rates, savings rates. However, for the present purposes a policy change is indicated by specifying two exogenous final demand vectors, one of which represents an *absolute* change, the other indicates the *sectoral distribution* of the new expenditure (increase or reduction) which is to compensate for the first indicated change.

As the essence of the model, the purpose for which it was designed, this process should be clearly understood, Consider a decision to reduce, e.g., federal defense expenditure by \$10 billion, this reduction to take the form of specified reductions in purchases from each sector (including federal employment). Clearly, in the absence of any compensating increase in some other exogenous final demand, this defense expenditure reduction would imply a decline in output, employment and income.

Assume for purposes of discussion, that the federal government wishes to increase some other form of expenditure (which, in the current version of the model, it *directly* controls) such as to hold total employment constant, even with the defense expenditure reduction. What the model requires in this case is a vector indicating the distribution of this new expenditure over sectors, i.e. a distribution vector for the final demand to be introduced to compensate for the decline in defense expenditure. The model then *determines the level* of the new expenditure required to compensate for the given defense reduction, with compensation defined in terms of unchanged total employment. Having determined the level of the compensating expenditure increase, it is possible to assess the differential effects of the expenditure change, e.g. changes in employment by occupation, in effluent production, or in capital requirements.

In this example the specified absolute final demand reduction, defense, for which a policy-controlled alternative was specified, was itself a governmentally controlled vector. However, this need not be the case. For example, an autonomous decline in private fixed investment purchases could be assumed, with government compensating for this fall in private demand by increasing some specified public final purchases (and/or employment). Thus, the first change need not be governmentally controlled; it is only necessary that the government be able to directly control the compensating change. In a later version of the model it will only be necessary that the government exert indirect control over the compensating expenditure, e.g. via its control of tax rates, and the function of the model will then be to specify the degree of change in the immediate governmentally controlled variable required to produce a compensating change in the ultimate final expenditure category.

Formally, the policy-substitution application of the model requires specification of two final demand vectors,  $\Delta Y^j$  and  $Y^i$ . The first,  $\Delta Y^j$ , specifies the absolute changes in final purchases by sector for which compensation is required.  $Y^i$  then simply indicates the sectoral distribution of the compensating final demand. The problem which the model must solve is the determination of a scale factor,  $p$ , such that a final demand change  $pY^i$  just compensates for the specified change  $\Delta Y^j$ .

To determine this factor  $p$  the definition of compensation must be specified. In the present version of the model five alternative compensation criteria may be employed:

1. *Unchanged total employment.* Employment directly and indirectly due to  $\Delta Y^j$  must equal that resulting from  $pY^i$ . Employment must be defined with reference not only to the indicated final demands *per se* but also with reference to the capitalist and labor consumption expenditures which are induced in each case.

2. *Unchanged GDP.* This condition requires that value added (net of induced imports) generated by each of the final demands and their associated capitalist and labor consumption expenditures be equal.

3. *Unchanged government surplus or deficit.* In this case the difference between net tax revenues (gross tax revenues less transfer) and total government expenditure is held constant, while the levels of both revenues and expenditures may change. The complexity is introduced by the effects on revenues of changes in capital and labor shares of value added over sectors, given different capital and labor income tax rates, or intersectoral differences in indirect business taxes or in corporate profits taxes.

4. *Unchanged net tax revenues.* In this case absolute net tax revenues are held constant, with any net change in government expenditure reflecting a planned change in surplus or deficit. If the net change in expenditure is zero, this criterion is equivalent to an unchanged surplus or deficit.

5. *Unchanged employee compensation (labor income).* Because a number of earlier studies utilized this criterion, primarily as a surrogate for unchanged employment, apparently as the result of inappropriate or unavailable employment coefficients, it is included to facilitate comparisons with these studies.

The translation of the compensation criteria into scale factors for the compensating final demand vector is relatively straight forward. From equation 3.9 the change in total output due to the specified change in exogenous final demand  $\Delta Y^j$ , taking into account changes in induced consumption out of capital and labor incomes, is

$$\Delta X = (I - (I - A)^{-1}M)^{-1}(I - A)^{-1}\Delta Y^j$$

and similarly for the change in output due to  $Y^i$ . Defining the symbol  $B$  by

$$B = (I - (I - A)^{-1}M)^{-1}(I - A)^{-1}$$

then

$$\Delta X = B\Delta Y^j.$$

The change in employment due to this change in output is simply  $E'B\Delta Y^j$ . The condition that the new final demand  $i$  compensate in terms of *total employment* for the change  $\Delta Y^j$  then simply becomes

$$pE'BY^i = E'B\Delta Y^j$$

or

$$(3.10) \quad p = \frac{E'B\Delta Y^j}{E'BY^i},$$

and the  $\Delta Y^j$ —compensating change in  $Y^i$  is  $pY^i$ .

If the condition for compensation is *unchanged gross domestic product*, it is only necessary to change the definition of the vector  $E$ . In this case,  $E$  in equation



3.10 would be redefined as the value added vector, making adjustment within the  $i$  and  $j$  final demand vectors for induced imports; i.e.  $E' = V'$ , and equation 3.10 again provides the compensating scale factor  $p$ .

*Unchanged employee compensation* is equally simple. A vector of employee compensation per unit of output in each sector is substituted for the employment coefficients vector in equation 3.10. Thus  $E$  in 3.10 is redefined as  $E' = (S^L) \bar{V} (I - \bar{T}^B) (I - \bar{K}^D)$ .

The *constant government surplus and net tax revenue* conditions are somewhat more complex, because the vectors  $Y^i$  and  $\Delta Y^j$  may themselves enter the condition directly. First, the gross tax revenues per unit of output, by sector, can be represented as

$$(3.11) \quad E' = T^B \bar{V} + T^C (I - S^L) (I - \bar{K}^D) (I - \bar{T}^B) \bar{V} + t_k (1_v - T^C) (I - S^L) \\ \times (I - \bar{K}^D) (I - \bar{T}^B) \bar{V} + t_l S^L (I - \bar{K}^D) (I - \bar{T}^B) \bar{V}.$$

If the problem were simply one of holding *gross* tax revenues constant, this expression for  $E'$  could be substituted in equation 3.10 and the solution for  $p$  determined. However, if consumption out of transfer income appears as either  $\Delta Y^j$  or  $Y^i$ , i.e. if transfer consumption is either the compensated or compensating final demand change, then the changes in transfers themselves must be taken into account if *net tax revenues are to be held constant*.

If a change in transfer consumption is to be brought about, i.e. if  $\Delta Y^j$  represents such consumption, then the *level* of transfer payments corresponding to  $\Delta Y^j$  must be determined. Assuming that a proportion  $s_t$  of transfer payments is consumed, the change in transfers corresponding to the change  $\Delta Y^j$  in transfer consumption is

$$FN = \frac{1'_v \Delta Y^j}{(1 - s_t)}.$$

Then, the condition that net tax revenues remain unchanged, assuming that  $Y^i$  does not represent transfer consumption, is satisfied by

$$p = \frac{E' B \Delta Y^j - FN}{E' B Y^i}$$

where  $E'$  is defined in equation 3.11.

Alternatively, if the compensating final demand,  $Y^i$ , is transfer consumption, then the factor

$$FD = \frac{1'_v Y^i}{(1 - s_t)}$$

must be inserted in the equation for the scale factor,  $p$ ; i.e.

$$p = \frac{E' B \Delta Y^j}{E' B Y^i - FD}.$$

More generally if  $FD$  and  $FN$  are set at zero, if  $Y^i$  and  $\Delta Y^j$  are *not* transfer consumption, respectively, and are defined as above otherwise, then the *general*

condition for unchanged net tax revenue is simply

$$(3.12) \quad p = \frac{E'BY^j - FN}{E'BY^i - FD}$$

Finally, turning to an *unchanged government surplus or deficit*, it is necessary to determine whether either  $Y^i$  or  $\Delta Y^j$  is a government expenditure final demand vector. If  $\Delta Y^j$  is such a government vector, then the numerator of equation 3.12, which represents net tax revenues generated by  $\Delta Y^j$ , must be reduced by the expenditure itself, i.e.

$$(3.13) \quad p = \frac{(E'B - 1'_v)\Delta Y^j}{E'BY^i - FD}$$

where  $FD$  is zero if  $Y^i$  is not transfer consumption, otherwise it is defined as above.

Alternatively, if  $Y^i$  is a government expenditure vector the condition becomes

$$(3.14) \quad p = \frac{E'BY^j - FN}{(E'B - 1')Y^i}$$

where again the value of  $FN$  is defined as non-zero only if  $\Delta Y^j$  is transfer consumption.

Having determined the scale factor  $p$  subject to the selected compensation criterion, it is possible to determine the changes in output vectors due to each final demand change, i.e.

$$\Delta X^i = pBY^i$$

and

$$\Delta X^j = B\Delta Y^j$$

The net change in output is, then,  $\Delta X^{\text{net}} = pBY^i - B\Delta Y^j$ . If this expression is substituted for  $X$  in all relationships in Table I-2 (model outputs), the net effects of the policy change on all relevant variables can be determined. Note that these changes include both the direct effects of  $pY^i$  and  $\Delta Y^j$ , and also the differential induced labor and capitalist consumption effects. If only the direct effects are desired, e.g. the employment directly absorbed by  $pY^i$  and  $\Delta Y^j$  ignoring the induced consumption effects, then the expressions

$$\Delta X^{i*} = p(I - A)^{-1}Y^i$$

and

$$\Delta X^{j*} = (I - A)^{-1}\Delta Y^j$$

can be employed in place of  $X$  in the relationships indicated in Table I-2.

## II. THE REGIONAL MODEL:

The fundamental distinction of the regional model is between *national* and *local* industries. This distinction is drawn primarily in terms of the degree of inter-regional trade in an industry's output. Those industries, primarily services, the

outputs of which are almost necessarily entirely supplied from within the using region, with little or no opportunity for regional imports or exports, are designated local. Thus, for local industries, intra-regional supplies and demands are required to balance. National industries, conversely, are assumed to produce outputs which can move freely in interregional trade, with no requirement that supplies and demands balance *within* regions. For national industries it is only necessary that total national outputs equal total national requirements. Any regional (positive or negative) excess demands are met through interregional trade.

In the present version of the model national industry outputs, as determined in the National Model, are assumed to be distributed across regions exogenously. Similarly, exogenous final demands for outputs of local industries are also exogenously allocated to regions. The function of the Regional Model is, then, to determine endogenously the levels of regional outputs of local industries.

Certain modifications of the National Model notation are required for the analytic description of the Regional Model. Overall, the National Model consisted of  $n$  industries; in the Regional Model this designation is altered to  $mn$ , i.e.  $n_{(\text{National Model})} = mn_{(\text{Regional Model})}$ . Of these  $mn$  industries, the first  $n$  are identified as national industries, the last  $l$  as local industries, with  $mn = n + l$ .

All of the basic inputs into the National Model are employed in the Regional Model, with several notational changes or elaborations. First, all input vectors, e.g.  $V(mn \times 1)$ , value added can be decomposed into two subvectors, e.g.  $V^N(n \times 1)$  and  $V^L(l \times 1)$ , the first referring to national industries and the second to local industries. Thus,

$$V = \begin{bmatrix} V^N \\ V^L \end{bmatrix}$$

and similarly for all other parameter vectors.

With this national-local industry ordering, the direct requirements coefficient matrix can be represented by

$$A = \begin{bmatrix} A^{NN} & A^{NL} \\ A^{LN} & A^{LL} \end{bmatrix}$$

where  $A^{NN}$  represents inputs of national industries into national industries,  $A^{NL}$  national industry inputs into local industries,  $A^{LN}$  local industry inputs into national industries, and  $A^{LL}$  local-to-local inputs.

The economy is divided into  $r$  regions. Total outputs of national industries, previously determined in the National Model, are allocated to regions on the basis of an exogenous (or, more generally, predetermined) national industry distribution matrix,  $D^N(n \times r)$ , each cell specifying the share of a given region (column) in the total output of a national industry (row). The matrix  $X^{NR}(n \times r)$ , obtained by

$$(4.1) \quad X^{NR} = \bar{X}^N D^N$$

then specifies the output of each national industry in each region.

Having distributed national industry outputs to regions, it is possible to identify the outputs of local industries required to service these levels of national

industry production. Specifically, the rectangular matrix  $A^{LN}$  contains coefficients representing input requirements from local industries, per unit of output of national industries. Then total local industry requirements of national industries are

$$(4.2) \quad X^{LNR} = A^{LN} X^{NR}$$

These requirements from local industries can simply be treated as final demands, since required intermediate local industry purchases from national industries have already been taken into account in determining the national industry outputs which have been distributed regionally via equation 4.1.

Similarly, exogenous final demands upon local industries are distributed to regions on the basis of distribution matrices unique to each type of exogenous final demand, e.g. fixed investment, defense. Thus,

$$(4.3) \quad Y^{iLR} = \bar{Y}^{iL} D^{iL}, \quad i = 3, m,$$

where  $Y^{iLR}(l \times r)$  is a matrix of the  $i$ th final demand by local industry and region, and

$D^{iL}(l \times r)$  is the  $i$ th final demand distribution matrix, the row of which distributes final demands from each industry over regions.

Although *national* capitalist net income and consumption are determined endogenously, instead of perpetuating this at the regional level by assuming that capital income generated in a region is also received in the region, it is assumed that national capital income is distributed independently of the regional sources of that income. Specifically, a region's share of capital income is assumed to be proportionate to its wealth ownership and hence is independent of levels of activity and of profits within the region.

Thus, if net capitalist income nationally is  $z_k$ , and if the distribution of wealth over regions is represented by the vector  $D^2(1 \times r)$ , then the distribution over regions of capitalist consumption demands on local industries will be represented by  $D^{2L}(l \times r)$ , which simply repeats the vector  $D^2$  to create the 1 rows. Thus,

$$(4.4) \quad Y^{2LR} = \bar{Y}^{2L} D^{2L},$$

capitalist consumption demands by local industry and region.

Finally, the distribution of local industry consumption demands by employees of national industries is determined by the distribution of national industry outputs. Incomes of employees of national industries by region,  $z^{NR}(1 \times r)$ , are simply

$$z^{NR} = S^{LN}(I - \bar{K}^{DN})(I - \bar{T}^{BN})\bar{V}^N X^{NR}$$

from which consumption demands on local industries, by region, can be determined, i.e.

$$(4.5) \quad Y^{1N,LR} = (1 - s_l)(1 - t_l)C_n^L S^{LN}(I - \bar{K}^{DN})(I - \bar{T}^{BN})\bar{V}^N X^{NR}$$

Only local industry consumption demands of local employees of local industries remain to be determined. These are determined in a manner identical to those of national industry employees:

$$(4.6) \quad Y^{1L,LR} = (1 - s_l)(1 - t_l)C_l^L S^{LL}(I - \bar{K}^{DL})(I - \bar{T}^{BL})\bar{V}^L X^{LR}$$

However, unlike  $X^{NR}$ , regional outputs of national industries, regional outputs of local industries,  $X^{LR}$ , are not known.

Thus, the Regional Model equation system consists of  $2 \cdot l \cdot r$  equations

$$X^{LR} = (I - A)^{-1}(Y^{1L,LR} + Y^{1N,LR} + Y^{2LR} + Y^{3LR} + \dots + Y^{mLR} + X^{LNR})$$

$$Y^{1L,LR} = (1 - s_i)(1 - t_i)C_i^L S^{LL}(I - \bar{K}^{DL})(I - \bar{T}^{BL})\bar{V}^L X^{LR}$$

and  $2 \cdot l \cdot r$  unknowns,  $X^{LR}$  and  $Y^{1L,LR}$ . As in the National Model, the solution of this system results in a set of equations, in this case, for regional outputs of local industries:

$$(4.7) \quad X^{LR} = (I - (I - A^{LL})^{-1}MM)^{-1}(I - A^{LL})^{-1}(Y^{1N,LR} + Y^{2LR} + Y^{3LR} + \dots + Y^{mLR} + X^{LNR})$$

where  $MM = (1 - s_i)(1 - t_i)C_i^L S^{LL}(I - \bar{K}^{DL})(I - \bar{T}^{BL})\bar{V}^L$

Given the solution for total outputs of local industries in each region, local industry consumption by local industry employees can be determined via equation 4.6.

Regional model variables and parameters are summarized in Table II-1.

TABLE II-1  
REGIONAL MODEL VARIABLES AND PARAMETERS

$nm (= n + 1)$	Number of producing sectors (equal to $n$ in National Model)
$n$	Number of "national industries"
$l$	Number of "local industries"
$r$	Number of regions
$X(nn \times 1) = \begin{bmatrix} X^N(n \times 1) \\ X^L(l \times 1) \end{bmatrix}^*$	Total output vector, partitioned into national and local vectors
$Y^i(nn \times 1)$	
$= \begin{bmatrix} Y^{iN}(n \times 1) \\ Y^{iL}(l \times 1) \end{bmatrix}, i = 2, m^*$	Partitioned exogenous (including capitalist consumption) final demand vectors
$X^{NR}(n \times r)^\dagger$	Matrix of National industry output by region
$D^N(n \times r)$	National industry output distribution matrix
$X^{LNR}(l \times r)^\dagger$	Matrix of input requirements by national from local industries, by region
$X^{LR}(l \times r)^\dagger$	Matrix of local industry total outputs by region
$Y^{iLR}(l \times r), i = 2, m^\dagger$	Matrix of "exogenous" final demands from local industries by region.
$D^{iL}(l \times r), i = 2, m$	Exogenous final demand regional distribution matrices
$D^2(l \times r)$	Regional capital ownership distribution vector. [ $D^{2L}$ matrix obtained by repeating $D^2$ vector $l$ times].
$Y^{1N,LR}(l \times r)^\dagger$	Local industry consumption demands of national industry employees, by region
$Y^{1L,LR}(l \times r)^\dagger$	Local industry consumption demands of employees of local industries, by region
$C(nn \times 1) = \begin{bmatrix} C^N(n \times 1) \\ C^L(l \times 1) \end{bmatrix}^*$	Partitioned consumption distribution vector
$C_n^L(l \times n)$	Matrix formed by repeating local consumption distribution vector, $C^L$ , $n$ times

TABLE II-1 (continued)  
REGIONAL MODEL VARIABLES AND PARAMETERS

$C_1^L(l \times l)$	Matrix formed by repeating local consumption distribution vector, $C^L$ , 1 times.
$A(nn \times nn)$	
$= \left[ \begin{array}{c c} A^{NN}(n \times n) & A^{NL}(n \times l) \\ \hline A^{LN}(l \times n) & A^{LL}(l \times l) \end{array} \right]^*$	Partitioned direct input requirements matrix
$V(nn \times 1) = \left[ \begin{array}{c} V^N(n \times 1) \\ \hline V^L(l \times 1) \end{array} \right]^*$	Partitioned value added coefficients vector
$T^B(nn \times 1) = \left[ \begin{array}{c} T^{BN}(n \times 1) \\ \hline T^{BL}(l \times 1) \end{array} \right]^*$	Partitioned indirect business tax rate vector
$K^D(nn \times 1) = \left[ \begin{array}{c} K^{DN}(n \times 1) \\ \hline K^{DL}(l \times 1) \end{array} \right]^*$	Partitioned depreciation vector
$S^L(nn \times 1) = \left[ \begin{array}{c} S^{LN}(n \times 1) \\ \hline S^{LL}(l \times 1) \end{array} \right]^*$	Partitioned labor share vector
$T^C(nn \times 1) = \left[ \begin{array}{c} T^{CN}(n \times 1) \\ \hline T^{CL}(l \times 1) \end{array} \right]^*$	Partitioned corporate profits tax rate vector
$t_l^*$	Tax rate on labor income
$t_k^*$	Tax rate on capital income (net of corporate tax and depreciation)
$s_l^*$	Labor savings rate (out of disposable income)
$s_k^*$	capital savings rate (out of disposable income)
$t_n^*$	aggregate government personal transfer payments
$s_n^*$	savings rate out of transfer income
$E(nn \times 1) = \left[ \begin{array}{c} E^N(n \times 1) \\ \hline E^L(l \times 1) \end{array} \right]^*$	Partitioned employment coefficients vector (employment/output)
$O(o \times nn) = \left[ \begin{array}{c} O^N(o \times n) \\ \hline O^L(o \times l) \end{array} \right]^*$	Partitioned occupation distribution matrix, $o$ = number of occupations (distribution of employment by industry over occupations)
$M(q \times nn) = \left[ \begin{array}{c} M^N(q \times n) \\ \hline M^L(q \times l) \end{array} \right]$	Partitioned raw materials coefficient matrix, $q$ = number of materials (materials requirements/output)
$U(u \times nn) = \left[ \begin{array}{c} U^N(u \times n) \\ \hline U^L(u \times l) \end{array} \right]^*$	Partitioned effluent coefficients matrix, $u$ = number of effluents (effluents/output)
$K(nn \times nn)$	
$= \left[ \begin{array}{c c} K^{NN}(n \times n) & K^{NL}(n \times l) \\ \hline K^{LN}(l \times n) & K^{LL}(l \times l) \end{array} \right]^*$	Partitioned capital coefficients matrix, capital requirements per unit of output by producer (row) and user (column) industry
$Z^{LR}\dagger$	local industry labor income by region
$Z^{NR}\dagger$	national industry labor income by region
$z_k^*$	national capitalist net income
$D^3$	regional distribution vector for transfer income

\* Identical to national model.

† Endogenous to regional model.



Outputs of the Regional Model, generally corresponding to National Model outputs, are reviewed in Table II-2.

The extension of the policy substitution analysis to the regional level is relatively straight-forward. Compensating policy changes are entirely determined in the National Model. From a specified final demand change,  $\Delta Y^i$ , a compensation criterion, e.g. unchanged total employment (nationally), and a compensating final demand distribution vector,  $Y^i$ , the National Model determines a scale

TABLE II-2  
REGIONAL MODEL OUTPUTS

Name	Designation
Total output	
1. Local industries (matrix)	$X^{LR}$
2. National industries (matrix)	$X^{NR}$
Gross regional product	
1. Local industries	$X^{LR}V^L$
2. National industries	$X^{NR}V^N$
Depreciation	
1. Local industries	$X^{LR}V^L(I - T^{BL})K^{DL}$
2. National industries	$X^{NR}V^N(I - T^{BN})K^{DN}$
Consumption purchases from local industries	
1. Labor	
a. Local industry employees	$I_c^L Y^{1L,LR}$
b. National industry employees	$I_c^N Y^{1N,LR}$
2. Capitalist	$I_c^C Y^{2LR}$
3. Transfer	$I_c^T Y^{3LR}$
Total consumption	
1. Labor	
a. Local industry employees	$(1 - s_l)(1 - t_l)Z^{LR}$
b. National industry employees	$(1 - s_l)(1 - t_l)Z^{NR}$
2. Capitalist	$(1 - s_k)(1 - t_k)z_k D^2$
3. Transfer	$(1 - s_t)D^3 t_n$
Aggregate local industry final demands of other final demand components	$I_c^i Y^{iLR}, i = 4, m$
Net capital income generated	
1. Local industries	$X^{LR}V^L(I - T^{BL})(I - K^{DL})(I - S^{LL})(I - T^{CL})$
2. National industries	$X^{NR}V^N(I - T^{BN})(I - K^{DN})(I - S^{LN})(I - T^{CN})$
Net capital income received	$z_k D^2$
Labor income	
1. Employees of local industries	$X^{LR}V^L(I - T^{BL})(I - K^{DL})S^{LL}$
2. Employees of national industries	$X^{NR}V^N(I - T^{BN})(I - K^{DN})S^{LN}$
Savings	
1. Labor	$s_l(1 - t_l)(Z^{LR} + Z^{NR})$
2. Capitalist	$s_k(1 - t_k)z_k D^2$
Net tax revenues	$(1.0 + 2.0 + 3.0 - 4.0)$
1. Indirect business taxes	
a. Local	$X^{LR}V^L T^{BL}$
b. National	$X^{NR}V^N T^{BN}$
2. Corporate profits taxes	
a. Local	$X^{LR}V^L(I - T^{BL})(I - K^{DL})(I - S^{LL})T^{CL}$
b. National	$X^{NR}V^N(I - T^{BN})(I - K^{DN})(I - S^{LN})T^{CN}$
3. Personal taxes	
a. Labor	$t_l(Z^{LR} + Z^{NR})$
b. Capitalist	$t_k z_k D^2$
4. Transfers	$D^3 t_n$

TABLE II-2 (continued)  
REGIONAL MODEL OUTPUTS

Name	Designation
<b>Employment</b>	
Total	$E \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
Local industry	$E^L X^{LR}$
National industry	$E^N X^{NR}$
By industry (matrix)	$E \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
By occupation (matrix)	$OE \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
Raw materials consumption (matrix)	$M \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
Effluent production	$U \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
<b>Capital requirements</b>	
Total	$I_i K \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
By capital goods producer industry (matrix)	$K \begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}$
By capital goods user industry (matrix)	$\begin{bmatrix} X^{NR} \\ X^{LR} \end{bmatrix}' K$

All outputs are vectors ( $1 \times r$ ) or ( $r \times 1$ ), except those designated as matrices.

factor,  $p$ , such that the compensation criterion is fulfilled when the change  $\Delta Y^j$  is compensated by the change  $pY^i$ .

Given the determination of  $p$ , the National Model provides the following information of relevance to the Regional Model:

$$\Delta Y^{jL} \text{ from } \Delta Y^j = \begin{bmatrix} \Delta Y^{jN} \\ \Delta Y^{jL} \end{bmatrix}$$

$$\Delta Y^{iL} \text{ from } pY^i = \Delta Y^i = \begin{bmatrix} \Delta Y^{iN} \\ \Delta Y^{iL} \end{bmatrix}$$

$$\Delta X^{jN} \text{ from } \Delta X^j = \begin{bmatrix} \Delta X^{jN} \\ \Delta X^{jL} \end{bmatrix},$$

the change in national industry output due to the final demand change  $\Delta Y^j$

$$\Delta X^{iN} \text{ from } \Delta X^i = \begin{bmatrix} \Delta X^{iN} \\ \Delta X^{iL} \end{bmatrix},$$

the change in national industry output due to the final demand change  $pY^i = (\Delta Y^i)$ . These changes in final demands from local industries and in total outputs of

national industries can be inserted into the Regional Model, and the gross and net effects on all Regional Model outputs can be determined. These consequences will include induced consumption effects and will reflect reequilibration of regional economies to the changes in exogenous demands.

It should be noted that, while a policy change is compensated in the National Model, i.e. with regard to some national-level variable such as employment or GDP, there is no requirement that the policy change be compensated at the regional level. For example, although national employment may be required not to change, it is still possible that employment in all regions will change in response to the policy substitution; the increases in employment in some regions will simply offset the declines in others. Differential regional effects are, then, simply a subset of general differential effects of a policy substitution.

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