This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: Studies in Income and Wealth, Volume 14

Volume Author/Editor: Conference on Research in Income and Wealth

Volume Publisher: NBER

Volume ISBN: 0-870-14169-4

Volume URL: http://www.nber.org/books/unkn51-2

Publication Date: 1951

Chapter Title: Asset Identities in Economic Models

Chapter Author: K. E. Boulding

Chapter URL: http://www.nber.org/chapters/c9722

Chapter pages in book: (p. 229 - 258)

Part VII

Asset Identities in Economic Models

K. E. BOULDING, *University of Michigan*

IN THE VARIOUS 'MODELS' OR DETERMINATE SYSTEMS of economic variables which are such an important feature of macroeconomic analysis, identities play an important, perhaps even an essential role. These identities are obtained by equating two different breakdowns of a single aggregate or by equating a breakdown of an aggregate to the aggregate itself. A simple example is the 'savings equals investment' identity, which in its most significant form is based on the breakdown of total output, Y, into the part that is consumed, C, and the part that is not consumed and is therefore accumulated, A. Since $Y \equiv C + A$,

$$A \equiv Y - C, \tag{1}$$

where A is 'investment' and $Y - C$ is 'savings', or income minus consumption. Irving Fisher's equation of exchange likewise results from the division of a single aggregate, total payments, into two products: price multiplied by the quantity of the exchangeables for which payment is made and the total quantity of money multiplied by its average velocity of circulation.

Naturally, an identity must be true. It may or may not be interesting, and there is no sense in formulating uninteresting identities. The interest of an identity depends upon whether its components have enough homogeneity, independence, and connectedness to be related in a set of functional equations sufficient to determine them. Thus the savings-equals-investment identity is of some interest because its components can be related in functions that make some sense, at least at the level of first approximation. In the simplest possible model of a 'Keynesian' system we assume a consumption function,

$$C = F_c(Y) \tag{2}$$

and an investment function,

$$A = F_a(Y) \tag{3}$$

or, if we like, assume A to be given by exogenous factors: these two equations together with the identity (1) are sufficient to determine the three unknowns, Y, A, and C. The value of such a model of course depends entirely upon whether the functional relations it assumes are reasonably stable. Their stability in turn depends upon whether the relations rest upon some stable attributes of human behavior and whether the variables are homogeneous enough to justify the neglect of their parts and structure.

Little has been done with identities involving assets, yet these are actually more fundamental, and frequently more illuminating, than the income identities commonly used. Income quantities, such as output, consumption, savings, expenditures, and receipts, are essentially *changes* per given period (gross or net) in asset quantities. The income identities, there-

fore, are obtained by the differentiation of the asset identities with respect to time.

The most fundamental asset identity, the balance sheet identity, is based upon a twofold division of the total value of resources controlled by a firm or other social organism: into a classified list of the resources controlled (the assets side) and into the distribution of this total value among the various types of claimant to it (the liabilities side). The breakdown can be as fine or as coarse as the nature of the problem requires. For the purpose of constructing economic models a very convenient breakdown rests upon the assumption that all accounts in the system can be classified under three heads — businesses, households, and government. For any business, then, we can classify the items in its balance sheet into assets and liabilities. Any item in the balance sheet can be classified under one or another of these headings although, as in all taxonomic structures, there may be doubtful cases which must be resolved in more or less arbitrary fashion. Bank deposits, for instance, are strictly classified under k_{bb} and government money under k_{gb}, but for many purposes we may wish to classify them under m_b or perhaps as a separate item.

BUSINESS BALANCE SHEET

Assets		*Liabilities*	
Money stock	m_b	Debts to businesses	k_{bb}'
Value of physical capital	q_b	Debts to households	k_{bh}'
Debts from businesses	k_{bb}	Debts to government	k_{bg}'
Debts from households	k_{hb}	Net worth	g_b
Debts from government	k_{gb}		

The balance sheet identity for a single business then reads:

$$m_b + q_b + k_{bb} + k_{hb} + k_{gb} \equiv k_{bb}' + k_{bh}' + k_{bg}' + g_b \qquad (4)$$

The balance sheets of all businesses can now be added. If we use capital letters to represent the aggregate quantities, M_b ($=\Sigma m_b$) is the total money stock of all businesses, Q_b ($=\Sigma q_b$) the total value of real capital held by businesses, and so on. The aggregate balance sheet identity may then be written:

$$M_b + Q_b + K_{bb} + K_{hb} + K_{gb} \equiv K_{bb}' + K_{bh}' + K_{bg}' + G_b \qquad (5)$$

When the balance sheets of all businesses in a closed society are added, however, the sum of all debts from businesses to other businesses, K_{bb}, is obviously the same quantity as the sum of all debts to businesses from other businesses. K_{bb}', as each inter-business debt appears twice in the aggregate of balance sheets — as a liability in one balance sheet and as an asset in another. We can therefore rearrange and rewrite identity (5) as the *aggregate business net worth identity:*

$$G_b \equiv M_b + Q_b + (K_{hb} - K_{bh}') + (K_{gb} - K_{bg}') \qquad (6)$$

Highly significant income identities can be derived by differentiating (6) with respect to time. If by dG_b, dM_b, etc. we mean the changes in the quantities G_b, M_b, etc. in a given period, we have:

$$dG_b \equiv dM_b + dQ_b + dK_{hb} - dK_{bh}' + dK_{gb} - dK_{bg}' \qquad (7)$$

Each quantity in this identity has economic significance; dG_b, the total change in business net worths, is closely related to what the Department of Commerce calls 'undistributed profits'. However, because of a certain ambiguity in this term, which is used also to designate a certain part of the total net worth of business in the balance sheet statement, 'business savings' seems preferable. It represents the net addition to net worth, i.e., the part of profits (gross additions) that has not been distributed in interest or dividends. Identity (7) may therefore be called the *aggregate business savings identity*.

dM_b is the increase in the money stock held by businesses. For the purposes of this exposition bank deposits and government currency are assumed to be included under the money category and correspondingly excluded from the debt categories to which they would otherwise belong. This item may be divided still further into the balance of payments of businesses with households, X_{bh}, the balance of payments of businesses with government, X_{bg}, and the portion of net additions to the money stock that remains in the accounts of businesses, dM_b'. The balance of payments of businesses with households is the excess of money receipts of businesses from households over the money expenditures of businesses to households. Neglecting government and changes in the money stock for the moment, we can visualize the money stock of society as a shifting cargo, now shifting into business balance sheets as households spend more toward businesses than businesses are spending toward households, now shifting toward household balance sheets when the reverse takes place. The positive balance of payments of businesses of course equals the negative balance of payments of households. We would ordinarily expect X_{bh} to fluctuate between positive and negative magnitudes, being positive when money surges into business balances, negative when it surges out into household balances. The longer the period the more these alternate positive and negative values will cancel, and the smaller this item will be in relation to the other magnitudes of the economy: in the long run, that is to say, X_{bh} approaches zero.

The balance of payment of businesses with government, X_{bg}, is likewise the excess of money receipts of businesses from government over money expenditures by businesses to government. The first item consists of payments for goods and services or securities purchased from businesses by government, plus subsidy or other transfer payments; the second con-

sists of tax payments and payments for goods, services, or securities purchased from government by businesses. Because of the peculiar power of government to create money, there is no necessity for this item to approach zero in the long run, though under conservative canons of public finance its long run value is presumably small. War inflation is likely to render it perennially positive. dM_b' represents mainly the increase in bank deposits. For the purposes of our model we include banks under government and regard bank deposits as part of the public debt. For some purposes it is desirable to set up a separate account for the banking system.

From identity (7) an identity for total profits can immediately be derived. Total profits, V, must equal business distributions out of profits in net business taxes, T_b, and in dividends and interest, D plus business savings, dG_b. We have therefore:

$$V \equiv T_b + D + dG_b \equiv dQ_b + (D + X_{bh} + dK_{hb} - dK_{bh}') + (X_{bg} + T_b + dK_{gb} - dK_{bg}') + dM_b' \qquad (8)$$

A similar identity can now be constructed for total wages. First, we construct a balance sheet for a household. Again representing the sum

HOUSEHOLD BALANCE SHEET

Assets		*Liabilities*	
Money stock	m_h	Debts to businesses	k_{hb}'
Value of physical capital	q_h	Debts to households	k_{hh}'
Debts from businesses	k_{bh}	Debts to government	k_{hg}'
Debts from households	k_{hh}	Net worth	g_h
Debts from government	k_{gh}		
Net worths of businesses	g_{bh}		

of the various items in all household accounts by capital letters, we find on adding the balance sheets of all households that $K_{hh} = K_{hh}'$, and we have a *household net worth identity:*

$$G_h \equiv M_h + Q_h + K_{bh} + K_{gh} - K_{hb}' - K_{hg}' + G_b \qquad (9)$$

As will be observed, in the household balance sheet we included an item, g_{bh}, on the assets side, representing the part of the net worth of businesses that is owned by the household. The entire net worth of businesses must theoretically be allocated among households, as a business is a fictitious 'person'. The actual allocation of this net worth may, of course, be somewhat arbitrary. Nonprofit institutions such as universities and churches present some difficulties: they may either be regarded as 'households' in themselves or their assets may be allocated to the individual households or persons benefiting from their existence. The sum of all these allocations must equal the sum of business net worths: i.e., $G_{bh} \equiv G_b$.

The household net worth identity also may be differentiated, the prefix

d being used as before to indicate the change during a fixed period in the variable so modified:

$$dG_h \equiv dM_h + dQ_h + dK_{bh} + dK_{gh} - dK_{hb}' - dK_{hg}' + dG_b \quad (10)$$

This may be called the *household savings identity*, as dG_h, the increase in household net worths, is the amount saved by households, which is the same thing as total savings. It may be objected that as household savings, on this definition, included business savings, which are not directly under the control of households, it is improper to say that the increase in household net worths is the amount 'saved' by households. As far as business savings are reflected in household balance sheets, however, households will rightly regard such savings as income, i.e., additions to net worth, and may, if they wish, offset them by consumption. Here we have a problem of the *form* in which an increase in net worth manifests itself — whether, for instance, in liquid or in nonliquid form — and the effects of the form on individuals. But at the level of approximation of this paper such complications may be neglected. The household savings identity then identifies household savings with the increase in the money stock of households plus the increase in the physical capital of households plus the increase in the net debts of business and of government to households plus business savings. If identities (7) and (10) are combined we have:

$$dG_h \equiv dM_h + dM_b + dQ_h + dQ_b + dK_g \quad (11)$$

where dK_g is the increase in net government debt to both households and businesses. When both business and household balance sheets are added, all inter-business and inter-household debts cancel: $dK_{hb} \equiv dK_{hb}'$, $dK_{bh} = dK_{bh}'$, and we are left with the identity that household savings equal the increase in the money stock of society plus the increase in the value of total physical capital plus the increase in net government debt.

Household savings consist also of household income minus household consumption, C_h, minus household taxes, T_h. Household income consists of wages, W, business distributions, D, and business savings, dG_b. We have therefore:

$$dG_h \equiv W + D + dG_b - C_h - T_h \quad (12)$$

The increase in the money stock of households may likewise be analyzed into three parts: first, the positive balance of payments of households with business, i.e., the excess of household receipts from business over household expenditures to business, which is exactly the same as the negative balance of business payments, $-X_{bh}$. The second part of dM_h is the positive balance of payments of households with government, X_{hg}; the third part is the new money that finds its way into the balances of households, dM_h'. We have therefore:

$$dM_h \equiv X_{hg} - X_{bh} + dM_h' \quad (13)$$

Combining identities (10), (12), and (13) we get an identity for *total wages:*

$$W \equiv (C_h + dQ_h) - (D + X_{bh} + dK_{hb} - dK_{bh}') + (X_{hg} + T_h + dK_{gh} - dK_{hg}') + dM_h' \qquad (14)$$

It will be observed that

$$D + X_{bh} + dK_{hb} - dK_{bh}' \equiv T \qquad (15)$$

occurs in both the total profits and the total wages identity. It may be called the *transfer factor.* The items

$$(X_{bg} + T_b + dK_{gb} - dK_{bg}') \equiv V_g \qquad (16a)$$

and

$$(X_{hg} + T_h + dK_{gh} - dK_{hg}') \equiv W_g \qquad (16b)$$

represent government contributions to total profits and to total wages respectively. Equations (8) and (14) can then be written in simple form:

$$V \equiv dQ_b + T + V_g + dM_b' \qquad (17a)$$

$$W \equiv (C_h + dQ_h) - T + W_g + dM_h' \qquad (17b)$$

Adding these two identities we obtain a familiar identity for *national income:*

$$Y = V + W = dQ_b + (C_h + dQ_h) + (V_g + W_g) + (dM_b' + dM_h') \qquad (18)$$

dQ_b is 'business accumulation' or 'investment'. $C_h + dQ_h$, total household purchases or absorption, corresponds to the Keynesian 'consumption'. The third item, the net government contribution to national income, and the fourth item, the increase in the money stock, will be analyzed later.

Suppose for the moment that we neglect the third and fourth items. The identities then indicate that the *distribution* of national income between wage and nonwage income is related to the *composition* of national income by business investment and household absorption through the transfer factor, T, which added to business 'investment' yields total profits and subtracted from household absorption yields total wages. Figure 1 illustrates the concept: national income is divided first into business investment, AC, and household absorption, CB, and second into total profits,

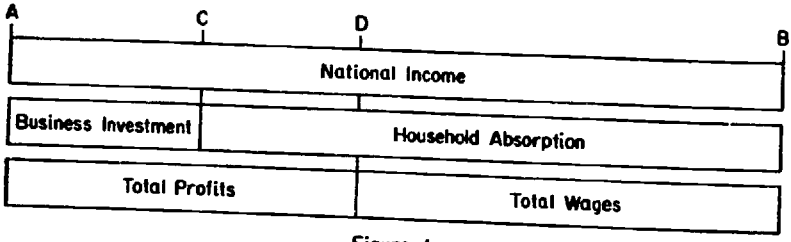

Figure 1

AD, and total wages, *DB*. *CD* is the transfer factor. The concept of a transfer factor would, of course, be meaningless unless it could be shown that it is related to certain aspects of human behavior. But identity (15) analyzes the transfer factor into four items, each of which is a rough 'parameter of behavior' and can, therefore, be profitably used in economic models. The first item, *D*, depends upon the dividend policy of corporate businesses and, in the case of unincorporated businesses, on entrepreneurial withdrawals. In the short run we can regard the interest and contractual rent items in *D* as constant, determined by the structure of debt and financial contracts, so that any fluctuation in *D* can be attributed to dividend policy. Unfortunately it is not at all clear what determines dividend policies, and in the absence of much detailed study there must be some doubt concerning what should be put into a 'dividends function' in an economic model. But there is little doubt that past profits are a major item in it, and also perhaps the liquidity position of businesses. There are, however, certain institutional, conventional, even fashion-determined elements in dividend policy that may undermine the stability of any dividends-function postulated.

The second item, X_{bh}, is the positive balance of payments of businesses toward households or, what is the same thing, the negative balance of payments of households toward businesses. The excess of household payments to businesses over business payments to households is, as already noted, likely to fluctuate between positive and negative values. In the short run, however, it may be an important contribution to (or subtraction from) profits: a 'surge' of several billion dollars into or out of business balances is not impossible in short periods. This is one of the big gaps in our statistical information, and a continuous series showing the distribution of liquid assets between business and household accounts would be instructive. The chief determinants of this item are the relative liquidity preferences of households and businesses. If both are trying to decumulate or to accumulate money at the same rate, or with the same degree of intensity, neither will succeed. But if the liquidity preference of one declines faster, or increases more slowly, than the other, liquid assets will 'surge' into the accounts of the one with the relatively weaker liquidity preference. Parameters for liquidity preference can be set up in terms of relative velocities of circulation: in simple models, however, the balance of payments item itself may be used as an irregularly fluctuating, exogenous variable, or it may be related in the short run to such variables as dividend or wage payments. Thus an increase in wage or dividend payments may well cause an initial surge of the money stock into household and out of business accounts. But in the long run the balance of payments item approaches zero and may be neglected.

dK_{hb} is the increase in household indebtedness to business, i.e., mainly in consumer credit, book or instalment. Since it represents the equity of businesses in household goods (automobiles, household equipment, furniture, clothing, etc. held by households), it can increase continuously with household capital. But it is likely to fluctuate considerably, and there is nothing to prevent its being negative. Like the balance of payments, it may be expected to swing between positive and negative values if the average volume of consumer credit is constant: a secular rise in total consumer credit will, of course, make the positive values of dK_{hb} predominate over the negative values. However, an increase in consumer credit that exceeds the secular trend is almost certain to be followed by a decline, i.e., by a shift from a positive to a negative value for dK_{hb}. To that extent a rapid increase in consumer credit is almost certain to set up cyclical movements in the transfer factor. The problem of a 'consumer credit function' in economic models presents great difficulties. Income is probably the main determining factor, yet the relation is certainly not linear. Consumer credit per household reaches a maximum at middle or high-middle income levels. Other factors, such as household liquidity, may also be important, and factors on the business side influencing the willingness to grant consumer credit. The structure of household capital itself is also important, as consumer credit is closely related to household accumulation, dQ_h, and distortions in the age distribution of household capital are likely to lead to fluctuations in replacements and additions, as in the case of business capital.

The fourth item in the transfer factor, dK_{bh}', is the increase in debts due households from businesses. These debts include a rather heterogeneous aggregate of unpaid wage claims and accrued but unpaid interest or rent, but consist chiefly of business securities held by households. We thus get the paradoxical proposition that the sale of bonds by businesses to households actually diminishes total profits, the other variables being constant. Again there is considerable question regarding the variables to be included in a 'securities function': naturally it is likely to be related to both business investment and dividend policy. Certain special problems also related to equity financing and to the creation of new businesses are postponed at this stage.

What has emerged from the analysis of asset identities, then, is the outline of a macroeconomic theory of the distribution of national income into labor and nonlabor income. The absence of such a theory has been a great weakness of the Keynesian, indeed of all, economics. The identities clearly show that the distribution structure is not a result of the productive process but is in the main attributable to investment and financial

processes and decisions. Investment itself is the chief determinant of profits, and is in turn determined in part by expected profits. It would not be surprising if under these conditions the economic system was markedly unstable. The identities indicate also that the distributional pattern is largely independent of the money wage bargain, except as far as the wage bargain affects indirectly the significant determinants, such as dividend or investment policy. From the identities we get some important clues about the future of the distributional shares — the question that so greatly interested the classical economists and has been so important in Marxian economics but seems to have dropped into the background recently. From the business savings identity, (7), it is clear that there cannot be any business savings in the stationary state, unless the national debt perpetually increases, for investment, dQ_b, will cease, and we can hardly expect a permanent rate of increase in the money stock or in consumer credit. The disappearance of business savings, however, does not necessarily involve the disappearance of profits, for as long as businesses are willing to distribute profits, profits will return to them to distribute. This is the 'widow's cruse' effect foreshadowed, for instance, by Keynes in his *Treatise on Money* (Book 1, p. 139). Hence there is a curious indeterminancy in the distributional pattern, and our models give results more akin to the economics of J. S. Mill than to that of J. B. Clark. The apparent distributional determinism that resulted from the marginal productivity theory (which, by allying the laws of distribution with those of production, apparently removed distribution from the sphere of human influence) may be shown to be due to an illegitimate extension of microeconomic principles to the macroeconomic field. The marginal productivity theory is a theory of the demand for input from a particular enterprise: it cannot be generalized to the economy at large.

We now return to consider in more detail the contribution of government to output as a whole and to the distributional shares. Consider first equation (16a). The balance of payments of businesses with government, X_{bg}, can be analyzed further into payments for net purchases of government from businesses, i.e., government expenditure for goods and services bought from businesses, E_{gb}, minus net business taxes, i.e., taxes minus subsidies, T_b, minus business payments to government for securities, S_b.

$$X_{bg} \equiv E_{bg} - T_b - S_b \tag{19a}$$

Similarly for households, if E_{hg} is the net payment by government to households for goods and services, T_h is net taxes, paid out of household accounts, and S_h is net payments of households to government for securities,

$$X_{hg} \equiv E_{hg} - T_h - S_h \tag{19b}$$

We have therefore, from equations (16a and b), further identities for the contribution of government to profits and to wages:

$$V_g \equiv E_{bg} - S_b + dK_{gb} - dK_{bg}' \tag{20a}$$

$$W_g \equiv E_{hg} - S_h + dK_{gh} - dK_{hg}' \tag{20b}$$

The total contribution of government to national income, Y_g, is given by:

$$Y_g \equiv V_g + W_g = E_{bh} + E_{hg} \tag{21}$$

as the total net payment to government for securities $(S_b + S_h)$ must equal the net increase in government securities held $(dK_{gb} - dK_{bg}' + dK_{gh} - dK_{hg}')$, if we neglect income arising from changes in the price of securities already in private balance sheets. That is to say, the government contribution to national income equals the total government absorption of goods and services from both households and businesses.

For many purposes the significant variable is not 'total' but 'available' income, i.e., income after the deduction of taxes. If V' represents available profits and W' available wages, we have:

$$V' \equiv dQ_b + T + V_g' + dM_b' \tag{22a}$$

$$W' \equiv C_h + dQ_h - T + V_g' + dM_h' \tag{22b}$$

where

$$V_g' \equiv E_{bg} - S_b - T_b + (dK_{gb} - dK_{bg}') = X_{bg} + (dK_{gb} - dK_{bg}') \tag{23a}$$

$$W_g' \equiv E_{hg} - S_h - T_h + (dK_{gh} - dK_{hg}') = X_{hg} + (dK_{gh} - dK_{hg}') \tag{23b}$$

The two balance of payments factors for businesses, X_{bh} and X_{bg}, and for households, X_{bh} and X_{hg}, are likely to be closely related. If liquidity preferences are stable, a shift in the distribution of the total balance of payments of government (the cash deficit or surplus) between business and household accounts — occasioned, for instance, by a shift in the distribution of total taxes between business and household taxes — will be offset by a corresponding shift in the balance of payments between businesses and households. Thus suppose there is an increase in business taxes and a corresponding decrease in household taxes. The initial effect is to shift money out of business into household accounts, as more money in taxes is taken out of business accounts and less is taken out of household accounts. If, however, the liquidity preferences of business and households are unchanged, this shift in money stocks will be offset immediately by a shift in the business-households balance: in order to recoup their depleted money stocks, businesses will spend less toward households, and households will spend more toward businesses as they find themselves with larger money stocks in consequence of the decline in household taxes.

We may expect, therefore, that the quantities

$$X_b = X_{bh} + X_{bg} \text{ and } X_h = -X_{bh} + X_{hg} \tag{24}$$

will be fairly stable in the absence of changes in attitudes toward money. If then we take X_{bh} out of the transfer factor and write $T' = T - X_{bh}$, we can rewrite the available profits and wages identities as follows:

$$V' \equiv dQ_b + T' + (X_b + dM_b') + (dK_{gb} - dK_{bg}') \tag{25a}$$

$$W' \equiv (C_h + dQ_h) - T' + (X_h + dM_h') + (dK_{gh} - dK_{hg}') \tag{25b}$$

dM_b' and dM_h' need some comment. As government creation of money is taken care of in the factors X_b and X_h, $dM_b' + dM_h'$ represent the private creation of money. In a commodity-money economy this would represent simply the distribution between households and businesses of the total amount of money-commodity, e.g., gold, produced. In a bank-money economy the problem is more complicated: if banks are included in private businesses, bank deposits should strictly be regarded as debts. Since we include banks with government, we put bank deposits under government debt. To take account of these complications adequately would require a four-part model, i.e., adding the banking system to the three-part model already constructed. Such a model is rather beyond the scope of this paper, and as the general principles can be indicated without it the dM_b' and dM_h' factors will be neglected in what follows. It may be assumed roughly that the nongovernment increase in money stocks will be distributed in the same proportions as the general money stock.

An interesting conclusion with respect to the incidence of taxation follows from equations (25a) and (25b): if the investment, consumption, liquidity, and debt behavior patterns are stable, available profits and wages are quite independent of the distribution of the total tax burden between business and household taxes. That is, if these other factors are constant, a shift from household toward business taxes will be exactly compensated by a rise in profits before taxes and a fall in wages before taxes. In practice, a shift in the tax structure is likely to affect business and household decisions somewhat, and hence contribute to changing available incomes. This change, however, is a result of the effect on private decisions, not of the tax directly. The history of the last few years indicates that profits after taxes are markedly stable despite changes in taxes.

It seems to follow also that the available national income itself is independent of the tax load. This is the very agreeable 'widow's cruse' theory of taxation, that an increase in taxes always creates an equal increase in national income with which to pay them, and hence leaves income after taxes unchanged! But this proposition is subject to many qualifications.

It assumes first that the budget deficit does not change, so that the tax increase actually represents an increase in government absorption of product: it implies also that private absorption does not change, and that unemployed resources are available, so that the increase in government absorption produces an equal rise in the product. This condition can be fulfilled only at low levels of employment.

An indefinite variety of models can be constructed from these identities by assuming various types of relation among their components. Merely by way of illustration, one of the simplest models is based on the assumption that household absorption, $H = dQ_h + C_k$, business accumulation $B = dQ_b$, and the transfer factor, T, are functions of the relative distribution between wages and profits, neglecting the government variables: Then we have three equations: $H = F_h\left(\frac{W}{Y}\right), B = F_b\left(\frac{V}{Y}\right), T = F_t\left(\frac{W}{Y}\right)$; and three identities: $Y \equiv W + V \equiv H + B$ and $T \equiv V - B$ (or $T \equiv H - W$). The six equations determine the six unknowns: H, B, T, W, V, and Y. This model is susceptible to simple graphic analysis, as in Figure 2. The base line WV shows the relative distribution, W representing 100 percent wages and V 100 percent profits. Intermediate points such as K represent a proportion WK going to profits and KV going to wages. CC_o and I_oI are the 'consumption' (household absorption) and 'investment' (business accumulation) functions. Investment is measured from WV downward, so that at any point, K, on WV, KC_k is consumption. KI_k is investment, and therefore C_kI_k is consumption plus investment, i.e., national income. YY_mY is the national income curve — in this figure exhibiting maximum at Y_m. We now divide the line C_kI_k at the point S_k, where $\frac{C_kS_k}{S_kI_k} = \frac{KV}{WK}$. Then C_kS_k is the absolute amount of wages and S_kI_k the absolute amount of profits when the ratio of wages or profits to national income is given by the point K. The locus of S_k is the dash line I_oC_o which, measured from the base line WV, is the $V - B$ curve: $KS_k = S_kI_k - I_kK = V - B$ $(= KC_k - C_kS_k = H - W)$. We now postulate a T curve, T_oET, showing the value of the transfer factor T at each relative distribution. T is likely to rise with an increasing proportion of profits, as dividends will be larger: consumer credit also may be larger. The point of equilibrium is where the T curve cuts the $V - B$ curve at E. When the T curve is above the $V - B$ curve, decisions will be made at any given relative distribution that will raise profits above the given proportion; when the T curve is below the $V - B$ curve, decisions will be made that will lower profits below the given proportion. This is shown by the arrows. The equilibrium E is obviously stable; the equilibrium at E' would be unstable. The possibility of a very high 'shiftability' of equilibrium in this

model is all too apparent; i.e., a slight change in the functions involved may bring about a large change in the position of equilibrium because of the similarity in the slopes of the curves T_oT and I_oC_o. Indeed, a relatively slight shift upward of the curve T_oT might produce a situation in which there was no equilibrium, and profits would increase indefinitely at the expense of wages until some sort of subsistence level was reached: this is the 'Marxian' case. On the other hand, a shift downward in the line T_oT until it lay entirely below I_oC_o would produce a situation in which wages increase indefinitely at the expense of profits until the economy broke down in unemployment. There is a suggestion of this kind of collapse in the debacle of 1929-32.

In this model it is interesting to note that a rise in the investment function (represented in Fig. 2 by a shift in the line I_oI downward) will increase the proportion going to profits in equilibrium, as the line I_oC_o is also pulled downward and E moves to the right. Similarly, a rise in the consumption function pulls the line I_oC_o upward and moves the equilibrium toward a higher proportion of wages.

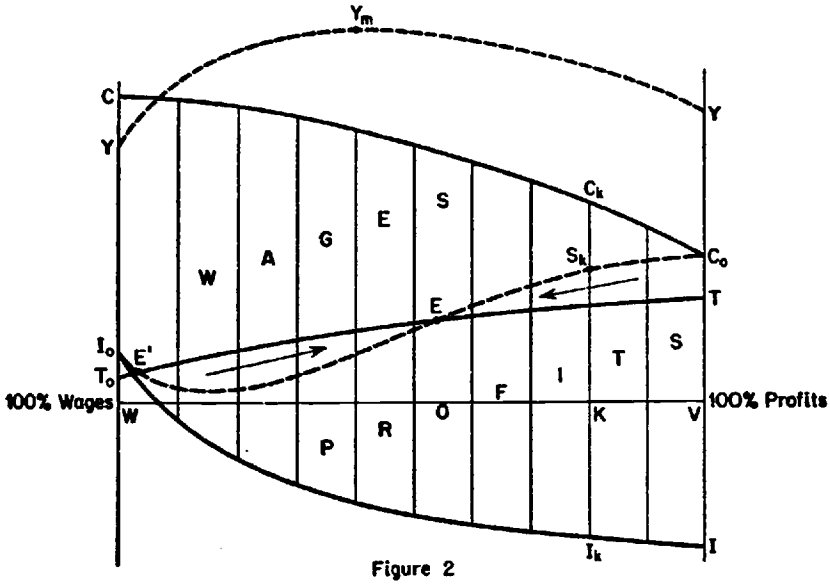

Figure 2

Other possible models will come readily to the reader's mind. Thus the more general model involving the equations $H = F_h(W,V)$; $B = F_b(W,V)$; $T = V - B = F_t(W,V)$; and $H + B \equiv W + V$ has many interesting properties, and exhibits in general the same kind of 'shiftability' as the simpler model.

The approach through general equilibrium models and comparative statics is not, of course, the only method of approaching macroeconomic

relationships. The movements of these variables can be explored directly in a true dynamic approach by means of postulating difference equations connecting consecutive time-values of the variables. Thus if we assume that the magnitude of each component of an identity is determined by certain magnitudes of the identity in the preceding period (or even in many preceding periods), the time-course of the variables can be traced out and if the system yields an equilibrium, the equilibrium position will be successively approximated, regardless from what values we start. This method has the advantage also that it is susceptible to graphic analysis, no matter how complex the fundamental identity, as long as the difference equations relate only two, or at most three, variables.

The method is illustrated for a very simple case in Figures 3-5. We suppose that the transfer factor is constant, that there is a business investment function $B = F_b(V)$ (Fig. 3), and a household absorption function $H = F_h(V + W)$ (Fig. 4). Then the identities $T \equiv V - B \equiv H - W$ give us four equations to solve for the four unknowns, H, B, V, and W. We interpret the investment and household absorption functions as difference equations, so that we should strictly write $B_{t+1} = F_b V_t$ and $H_{t+1} = F_h(V_t + W_t)$. The graphic solution of the equilibrium position is not difficult: in Figure 3, OS is measured downward from the origin equal to the transfer factor T (assumed constant), and a 45° line drawn from S to cut the investment curve in B_e; V_eB_e is the equilibrium level of investment ($B = V - T$). Now in Figure 4 we draw OR downward from the origin equal to V_eB_e in Figure 3, and draw a 45° line from R to cut the household absorption curve in H_e; this gives the equilibrium value of national income.

Now suppose that we start with any arbitrary magnitudes of the four variables represented by the line $A_oB_oC_oD_o$ in Figure 5: $B = A_oB_o$, $H = B_oD_o$, $V = A_oC_o$, $W = C_oD_o$, and $T = B_oC_o$; national income, $(Y) = A_oD_o$. We then make OV_o in Figure 3 equal A_oC_o in Figure 5, and find the corresponding value of B, V_oB_1 in Figure 3; on a new line in Figure 5 we draw $A_1B_1 = V_oB_1$. Similarly, we draw OY_o in Figure 4 equal to A_oD_o in Figure 5, get the value of H, Y_oH_1, and draw B_1D_1 in Figure 5 equal to Y_oH_1 in Figure 4. This gives us all the variables for time t_1. Repeating the process we get lines $A_2B_2C_2D_2$, $A_3B_3C_3D_3$, and so on, each line being derived from the one above it. In this case it is evident that we are approaching a stable equilibrium rather rapidly: with other functions, of course, the difference equations might lead to an explosive solution. This method of attack can be employed even when the fundamental identity is too complicated to allow a graphic solution of the equilibrium position. The possibility of statistical attack by this method needs further exploration.

Neither the identities nor the models mentioned so far have included

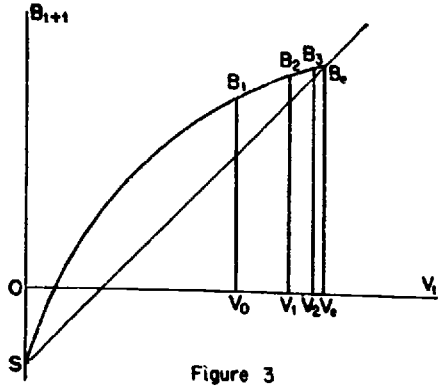

Figure 3

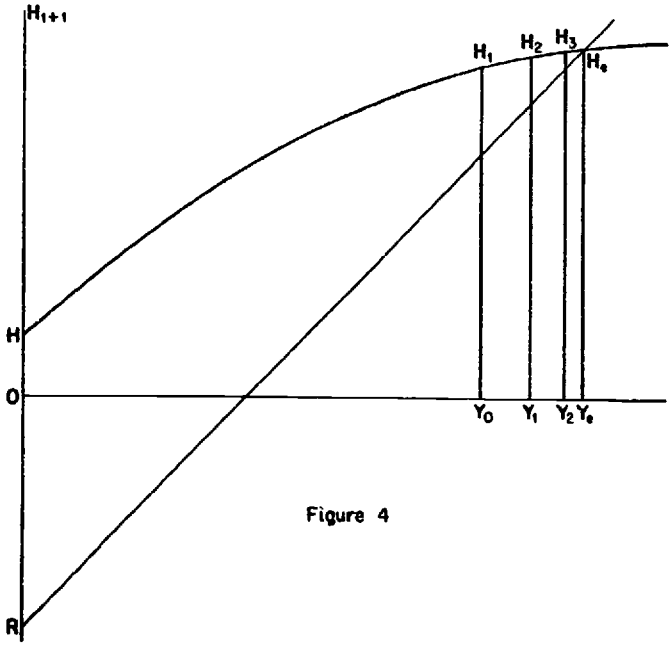

Figure 4

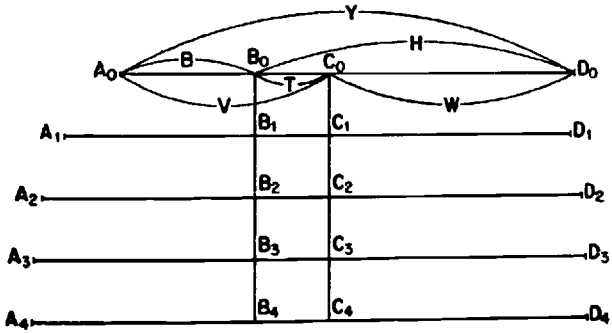

Figure 5

prices explicitly, though a price structure is implicit in the valuation of real assets. The 'real capital' items Q_b and Q_k are value items, and can be divided into some index of physical quantity of assets multiplied by some index of their prices. Likewise, the investment items dQ_b and dQ_k can be divided into two parts, one of which represents the increase in physical assets at constant prices, and the other the rise in the dollar value of existing assets as a result of a rise in their prices. From the viewpoint of national income the latter portion may be regarded as 'spurious': from the viewpoint of individual or sectional group incomes, however, it is not entirely spurious as it represents a redistribution of income, those who hold the assets that are rising in price benefiting at the expense of those whose assets are not rising. Models of the Keynesian type have never, to my knowledge, succeeded in incorporating prices and price levels as explicit variables of the system. This is a great weakness, as it means in effect that the models are valuable only at low levels of employment: as the system approaches capacity and price changes begin to be important, the functional relations assumed in the models break down and become too unstable to use. The models do not give an explicit picture of how the system behaves as it begins to approach capacity output, and in particular do not treat the mechanism of price-wage inflation explicitly. Nor is there any analysis of the impact of 'exogenous' changes in prices or money wages on the other variables in the system. I do not propose in the remaining space of this paper to eradicate this defect or even to attack a problem that has hardly even been stated: the effect of monopolistic and imperfectly competitive market conditions on aggregate economic activities. Nevertheless, there are indications that asset-analysis will throw a good deal of light on this problem.

In 'A Liquidity Preference Theory of Market Prices' (*Economica*, May 1944) I have shown that market price in a competitive market can be expressed in terms of money stocks, commodity stocks, or some other 'priced' exchangeable, and two 'parameters of behavior' reflecting the psychological attitude toward these assets (preferred asset ratios). Thus if p_a is the market price of any exchangeable, M the stock of money held by the marketers, A the stock of the exchangeable, and if r_m, the preferred liquidity ratio, is the desired proportion of money to total assets and r_a is the desired proportion of the value of the exchangeable to the value of total assets, then

$$p_a = \frac{Mr_a}{Ar_m} \tag{26}$$

For, if T is the total value of all assets held by the marketers, by definition $r_m = \frac{M}{T}$ and $r_a = \frac{p_a A}{T}$; eliminating T between these identities gives

us (26). The preferred asset ratios of the market are complex averages of the preferred asset ratios of the individuals in the market, and changes in them reflect a change in the 'state of mind' of the market as a whole. The liquidity preference theory of interest is a special case of this formula where the 'exchangeable' is fixed-interest securities.

The inference may be drawn from this identity that if the price structure is to enter explicitly into the macroeconomic models stocks of assets also must enter as given. There are also very convincing reasons for including asset stocks in investment and consumption functions, as Lawrence Klein shows in his paper. Consumption is consumption of assets, hence the larger the stock of assets the higher may the rate of their consumption be expected to be. Indeed, it is not wholly preposterous to assume that each asset consumes itself at its own rate, irrespective of use, and that consumption therefore is primarily a function of the composition of the asset stock. Similarly, investment, being itself an addition to real assets, should depend largely upon the size and composition of the existing stock. But the difficulties of analysis here seem to lie in the fact that the composition of the stock may be much more important than its over-all magnitude, and it is not at all easy to see offhand which breakdowns are significant. Asset quantities, however, are significant not merely because asset identities are the basis for significant income identities but also because they must enter explicitly into the models. The task of constructing such models is beyond the scope of this paper.

COMMENT

Ta-Chung Liu, *International Monetary Fund* and *The Johns Hopkins University*

Mr. Boulding's paper is interesting and enlightening. But since some characteristics of economic identities have not been given enough attention, some of his conclusions are rather misleading. One such characteristic is that an identity does not necessarily imply a cause and effect relation. By looking at the identities alone, one can seldom reach any conclusion about the forces determining the magnitudes of the variables involved. Secondly, any number of economic identities may be relevant to a given problem. By including different identities or by eliminating different sets of variables

from the analysis by different ways of substitution through the identities, surprisingly different (or even directly opposite) conclusions can be obtained for essentially the same problem.[1] The important point is, of course, to include the identities and variables that have the most bearing on the problem.

Some of Boulding's conclusions about income distribution are rather startling; if they were true, they would be significant. They seem, however, to be the results of misinterpreting identities as cause and effect relations and of improper selection of identities and variables. For example, he says:

"What has emerged from the analysis of asset identities, then, is the outline of a macroeconomic theory of the distribution of national income into labor and nonlabor income. . . . The identities clearly show that the distribution structure is not a result of the productive process but is in the main attributable to investment and financial processes and decisions. . . . The identities indicate also that the distributional pattern is largely independent of the money wage bargain, except as far as the wage bargain affects indirectly the significant determinants, such as dividend or investment policy."

Boulding's 'significant' determinants are embodied in his 'transfer factor' T (see his equation 15) among the components of which is D, dividend payments. Boulding therefore reaches the strange conclusion that dividend policies that merely decide where total profits should be kept have more to do with the relative proportion of total profits and the total wage bill in national income than the wage bargain. Common sense tells us this cannot be true. The truth is, of course, that Boulding's identities (15), (17a), and (17b) do not furnish him with sufficient basis to draw any conclusion on income distribution.

After stating his conclusions quite generally, Boulding went on to formulate a simple self-determined model which tends to confirm his theory.[2] A slight modification of his model would lead to a directly opposite conclusion.

1) Retain his H and B functions and the identities $Y = W + V$ and $Y = H + B$.

2) Omit the variable T (therefore, his T function and T identity).

3) Introduce the new variables N, number of wage earners employed, and R, the average wage rate. Treat the latter as exogenous and determined by the wage bargain.

4) Introduce the new function $N = F_n\ (H + B)$ and the new identity $W = NR$, which would seem to be more relevant to the problem of income distribution than those omitted.

[1] See the example worked out below where a slight modification of Mr. Boulding's model leads to a directly opposite conclusion.

[2] Incidentally, his model contains none of the true asset identities he believed to be "actually more fundamental, and frequently more illuminating, than the income identities commonly used".

In the new model every variable (and therefore the proportion of W and V in national income) is determined by the exogenous variable, the wage rate decided upon by the wage bargain, together with the given functions and identities. Now, what has happened to T, Boulding's 'determinant' of income distribution? T, of course, still equals $V - B$, an identity omitted from the new model. Since both V and B are determined by the wage bargain, we have reached the equally strange but directly opposite conclusion that, among other things, dividend policies, i.e., D, a component of T, are determined by the wage bargain.

The above is merely a caution against careless use of identities; it should not be allowed to obscure the contribution Boulding makes in deriving the asset identities.

A. G. Hart, *Columbia University*

Boulding's paper is extremely stimulating, and I hope will be followed up in later wealth discussions. My impression is, though, that it tries to make a preliminary and incomplete analysis bear more weight than it is fit for. A good test would be to take the concrete evidence on balances of payments among sectors offered by the Moneyflows Study, and see how far his paper's argument will help in its analysis.

The extent to which his paper involves bricks without straw is evident in connection with Figure 2. Consider its CC_o curve, which shows household absorption of output as a function of the proportion of income paid for personal services. Is it plausible to draw this curve with a negative slope throughout, or will it not have a maximum? Will not the 100 percent wages point show wages 100 percent of a rather small total? This curve must presumably be taken to express interaction between producers and consumers. Or consider the I_oI curve, expressing investment. If investment is not very small at '100 percent profits', it must be because employers count on more than VC_o of household absorption in the not too distant future. To draw this curve implies some specific period under scrutiny, and some specific expectation pattern about later periods.

Another point at which the paper seems to prove too much is in the widow's cruse analysis.[1] Roughly, this says that business is as profitable

[1] "Widow's curse" in the mimeograph copy distributed at the meeting! Discussion at lunch of the difficulty of literary allusions brought out fears that we may soon have to face students and readers who not only are ignorant of Scripture but haven't heard of the indispensable one-hoss shay or of Crusoe. *Curricular suggestion:* After we get done introducing "mathematics for economists", the next step should be a course in "literature for economists and their secretaries".

as it acts — profits varying with expenditures of profits on investment and on owners' consumption. When we start looking for limits to this business psychology view of distribution, we soon hit on the fact that if a given firm gets more optimistic about its profits and spends accordingly, the benefits accrue to other firms, and this firm finds profits below expectations; so that optimism must be limited by punishments to the most optimistic. Going beyond this, we hit next on the fact that greater optimism about profits must reflect either expectations of physical volume expansion (with full employment limits), of favorable shifts in price relationships (with limits from forces that resist inflation), or of cost reducing innovation. Next we must realize that this mechanism has its special leakages, notably through taxes on stockholders.

As Boulding points out at the outset, the interest of an identity depends on whether it can be tied into a set of functional equations in the same variables, with a structure that will determine the variables. If these equations are motivated rather than merely mechanical, they will include comparisons between expected and realized results for the recent past. I infer that there is no use going further with Boulding's identities without setting up explicitly dynamic models of this sort.

Morris A. Copeland, *Cornell University*

Boulding's paper is one of three considered in a session the objective of which was to explore the significance of national balance sheet information for economic analysis. It stands in marked contrast to the other two. Both Klein and Brill are concerned with statistical determinations of assets and obligations and transaction volumes, with analyzing and drawing conclusions from these determinations. Boulding's analysis runs in terms of quantitative concepts that have similar names to those employed by Brill and Klein but he avoids any commitment as to what his concepts mean in a precise empirical sense.

Alvin Johnson once said of this kind of thing that it would be open to no objection if the perpetrator would only delete from his exercise in logic the words that suggest the exercise applies to any actual economy. I have some sympathy with Johnson's stricture, but it has not received much approval. It goes too far. If Boulding has deliberately preferred not to tell us just what his variables mean empirically — and there can be little doubt his preference here was deliberate — he at least maintains a vicarious contact with reality: he relates his analysis to the analyses of other economists, e.g., Keynes, who were more concerned to push model analysis in the

direction of establishing it on an empirical base. Indeed Boulding goes further; he uses many terms that suggest current statistical series. But the closest he comes to committing himself to a precise empirical specification of his meaning is to tell us that "dG_b . . . is closely related to what the Department of Commerce calls 'undistributed profits' ".

Most economists have aspired to make economics a science. If we are to get forward toward this objective we would do well to take a cue from other fields of inquiry that have gone further, e.g., physics, and seek to define our basic quantitative concepts operationally and empirically, i.e., in terms of the method of determining the quantities statistically. I think this Conference has been making a major contribution in this direction, and I hope it will continue to do so.

One thing that more attention to the empirical significance of concepts would bring out is the distinction between variables that are on an accrual and imputation basis and those that are on a cash or moneyflows basis. Among the variables in Boulding's models, I take it, income and profit are accrual and imputation concepts, while the increments in cash balances and debts are facts on a cash basis. In my study of moneyflows it proved necessary to draw a very sharp contrast between the savings and investment account and the account that reports the money various economic sectors advance or return through financial channels and the money they obtain through these channels. I do not know how far it is possible to draw valid conclusions from hybrid models that scramble these two accounts by using both accrual and imputation and cash basis variables. As a minimum precaution it would seem wise to recognize the hybrid nature of the models.

Empirical model analysis is still in a somewhat immature stage. (1) It is doubtful whether any behavioristic equation is beyond the point where we can say that various forms give almost equally plausible fits. (2) Even definitional equations can assume various forms, for we can define their terms empirically in various ways. (3) But we do know that a small change in parameters can give a model either damped or explosive properties. (4) And we know a very great variety of somewhat plausible models is possible. Boulding admits, even insists, on the third and fourth of these points. In view of these points and since he does not seek to establish that no plausible model leading to a contrary conclusion is possible it seems brash of Boulding to imply he has here demonstrated that "the distribution structure is not a result of a productive process". I would not with Johnson urge that such a conclusion be strictly confined to its symbolic form. I would urge that such conclusions be properly qualified to make clear that their demonstrated validity applies to particular hypothetical, empirically undefined models, not to any actual economy.

Roland N. McKean, *Rand Corporation**

Boulding and Klein deal with the most urgent phase of drawing up a national balance sheet — the formulation of significant hypotheses balance-sheet data would help us test. Exploration of such uses for the data is a prerequisite for settling the problems about valuation, consolidation, and general form of the balance sheet. These papers concentrate on utilizing this tool to help predict aggregates such as the distribution of income and total spending — one of the more promising uses of the balance sheet, since adapting it for other purposes would often require prohibitive detail.

Both give convincing reasons supporting the common sense view that predictions about (income) flows are likely to be wide of the mark unless we take initial (asset) stocks into consideration. Boulding's concluding argument is sufficiently important to warrant restatement. The simplified Keynesian system does not include a theory of price-wage determination. Such a model gives relatively little assistance to policy except at very low levels of employment. If the functions are in money terms, they do not appear to be stable. If they are in real terms, the magnitude of any shift in current dollars, e.g., government investment of $3 billion, is indeterminate until we know the impact on prices. Omission of asset stocks from the theory may have led Keynesians to neglect price levels. In any case, introducing asset stocks into these models forcibly directs attention to the gap in the theory.

Of course, this gap may be unimportant. We must test theories by comparing their implications with observations, not by appraising the 'reality' of their assumptions.[1] Klein has found little evidence that the changing real value of liquid assets has significantly affected spending. He tends to believe that "the negative results obtained so far leave the burden of proof on the shoulders of the advocates of monetary policy who rely on large effects of certain wealth variables in spending and saving decisions". On the other hand, tests have often neglected the possible influence of liquidity upon the postponement of investment projects, producing variable lags between changes in interest rates and in outlays; upon risky undertakings by new, small, or innovating firms; upon expenditures at different stages of the cycle; and upon investment or consumption decisions quite apart from any effects on the pure interest rate or on any *observable* rates. These "certain wealth variables" will probably be especially important during a

* The writer is on leave of absence from Vanderbilt University. These comments were written while he was at Vanderbilt.

[1] This is Milton Friedman's position, summarized in G. J. Stigler, *Five Lectures on Economic Problems* (Macmillan, 1950), pp. 23-4.

state of perpetual mobilization, which may set off a cumulative decline of liquidity desire.

Boulding's chief contribution is an intriguing exploration of asset identities.[2] Comments on a few minor matters may help some readers. On the balance sheet for households, K_{bh} designates an asset (debts from business to households), while K_{hb}' refers to a liability (debts from households to business). On the balance sheet for business, however, these same obligations appear as K_{hb} and K_{bh}'. The subscripts tell the story, and the 'prime' symbol is superfluous. Thus dK_{bh} and dK_{hb}' become, if we wish, dK_{bh}' and dK_{hb}. In equation (14) Boulding evidently interchanges these symbols in this manner. It would be clearer, in my opinion, if the 'prime' were omitted.

Another small difficulty is that "aggregate business savings" is in fact the aggregate change in business net worth, presumably including the proceeds of new common stock flotations and the equities of new firms. Also, in view of the general emphasis on the balance sheet, it is a little surprising to find only increments and flows in most of his essay. Stocks of assets practically disappear after the first few pages. However, initial stocks would no doubt appear in the functions determining the flow variables.

Much more important, this ingenious approach throws light on both the insight and the confusion which identities can provide. Readers will perceive the provocative and promising nature of this approach without any outside comment. I shall restrict my remarks largely to reemphasizing the treacherous aspects of manipulating identities.

Confusion of Identities with Equilibrium Conditions

In formulating hypotheses, the cautious manipulation of identities may be a fruitful source of insight. In the final exposition of a model, the introduction of identities is usually a source of confusion. In identities, the actual (observable, measurable) magnitudes are always equal and determine nothing. In economic models, the scheduled (normal, intended) magnitudes are not always equal, but define the equilibrium solution, with the assumption of appropriate stability conditions. For example, quantities purchased and sold are always equal, and determine nothing about price. The quantities demanded and offered according to the demand and supply schedules are not always equal, but determine the equilibrium price. Professor Marschak clarified several disturbing paradoxes by substituting equilibrium conditions for identities in pre-Keynesian models.[3] In balance

[2] I would prefer balance-sheet identities, those involving balance-sheet items.

[3] 'Identity and Stability in Economics: A Survey', *Econometrica*, X (Jan. 1942), pp. 61-74

of payments problems, as in other forms of double-entry bookkeeping, actual debits always equal actual credits. In the equation of exchange, actual cash balances available are identical with actual cash balances held (in other words, measurable MV always equals measurable PT). But in all cases the equalities relevant to prediction are between some sort of normal or scheduled values.

Identities may provide a convenient framework for analysis, explanation, or further probing. They may help explain the process of moving toward equilibrium. For instance, at disequilibrium levels of income, the identity between actual savings and actual investment helps demonstrate the following propositions: (1) We are off the investment or consumption functions (or both). (2) We have some abnormal (unintended) saving, dissaving, investment, or disinvestment. (3) The *behavior* of dissatisfied savers or investors getting back to the functions moves us from the disequilibrium position.

In a predictive model, nonetheless, the clearest way to complete an ordinary system, e.g., a demand and a supply curve, is to use an equality stating that at equilibrium the scheduled magnitudes are equal. In Boulding's system of equations, we sacrifice clarity by making $Y \equiv W + V \equiv H + B$ and $T \equiv V - B$. Why should we use definitional identities stating that certain observed quantities can never be unequal? Wouldn't it be better to substitute equilibrium conditions stating that $Y = W + V = H + B$ and $T = V - B$?[4] In other words, at equilibrium these magnitudes as indicated by the schedules will be equal. When explaining the system graphically, one automatically resorts to equilibrium conditions instead of identities. "The point of *equilibrium* is where the T curve cuts the $V - B$ curve, at E" (italics mine).

Confusion of Measurable Quantities and Functional Relationships

Early in the paper, Professor Boulding makes the following important statement: "The interest of an identity depends on whether its components have enough homogeneity, independence, and connectedness to be related in a set of functional equations sufficient to determine them." To prevent misunderstanding, we should carefully distinguish between these components or observable values and the scheduled quantities that make up the functions or curves. And we should scrupulously avoid saying that any observable quantity is a determinant of any other item. While the observable quantity is a component of the identity, only the functional relationships are determinants of the equilibrium values.

Even Boulding, who writes with exceptional care and clarity, occasion-

[4] See John Lintner, 'The Theory of Money and Prices', *The New Economics*, ed. Seymour Harris (Knopf, 1947), pp. 530-1.

ally slips into misleading statements about causal or determining factors. In his paper, aside from the roles of government and new bank deposits, profits are identical with business investment plus a transfer factor, and wages are identical with household purchases minus the same transfer factor. National income, moreover, is identical with wages plus profits, on the one hand, and with business investment plus household purchases, on the other. "The identities then indicate that the *distribution* of national income between wage and nonwage income is related to the *composition* of national income by business investment and household absorption through the transfer factor. . . ." This transfer factor is the sum of dividend distribution, the balance of payments of households with business, the increase of debts from households to business, and the increase in debts from business to households (a minus item).

He logically suggests relating these components to underlying determinants, deriving a dividends function, a consumer credit function, a securities function, and so on. Nevertheless, unless readers learned their lesson well from the savings-investment controversy, several asides may give them difficulty. "Investment itself is the chief determinant of profits, and is in turn determined in part by expected profits." This seems exactly like saying that the volume of saving itself is the main determinant of income, and is itself determined in part by expected income. A much clearer statement appears later in connection with the system of functional relationships. "In this model it is interesting to note that a rise in the investment function . . . will increase the proportion going to profits in equilibrium."[5]

In another place, "We thus get the paradoxical proposition that the sale of bonds by businesses to households actually diminishes total profits, the other variables being constant". This is true in the same sense that larger savings increase national income, other variables remaining constant. Such plain arithmetic becomes paradoxical only if we lapse into thinking that changes in the volume of corporate bonds do in fact produce corresponding changes in total profits. Actually, the identities do not give any clue to either the immediate or equilibrium effect of an increased propensity to float or to purchase bonds. To illustrate with a familiar model, decreased consumption and unintended inventories might initially match increased savings, leaving national income unchanged. Similarly, the altered balance of payments of households with business might initially offset the increased indebtedness of business to households, leaving the transfer factor and total profits unchanged. At equilibrium, a rise in the savings curve, with the investment curve unchanged, would lower national income, and the

[5] This result is hardly surprising, though, since in setting up the model, we assumed that investment, household purchases, and the transfer factor were certain "functions of the proportionate distribution between wages and profits".

effect of a rise in the bond function on profits would depend upon the nature of the functions.

The two following quotations also illustrate the point (italics mine). "The identities indicate also that the distributional pattern is largely independent of the money wage bargain, *except as far as the wage bargain affects indirectly the significant determinants,* such as dividend or investment policy"; and, one might add, the consumption function, the transfer-factor functions, and so on. Again, "An interesting conclusion with respect to the incidence of taxation follows from equations (25a) and (25b): *if the investment, consumption, liquidity, and debt behavior patterns are stable,* profits and available wages are quite independent of the distribution of the total tax burden between business and household taxes". These statements are undeniably correct. If all the equations in a determinate system are held stable, hydrogen bombs will not budge the equilibrium solution. The meaningful issue is not whether the wage and tax bargains appear in the identities, but whether they affect the functional relationships.

Furthermore, it is the system of functional relationships, not the identities, that may cast doubt upon the usefulness of marginal productivity theory. Even with that proviso, is there necessarily a contradiction between an aggregative theory of distribution and marginal productivity theory? Perhaps, like the theories of relative prices and aggregate income, they pertain to different problems and complement each other. Synthesis, if desirable and feasible, might modify both until they amounted to the same thing without disclosing any fundamental conflict.

Reply

I have read Professor McKean's comments with great interest, and at many points am willing to cry *"peccavi"!* He is quite right, of course, in saying that in an equilibrium system it is the *functions* that are the determining factors, not the values of the mutually determined variables. Nevertheless, when we are considering the dynamics of such a system the emphasis is upon *decisions,* not upon abstract functional relationships. What the functions represent, in terms of human behavior, is a relation between decisions and the data on which they are based. To take one case he cites, the relation of investment to profits. In an equilibrium system these are mutually determined through an investment-profits function of some kind, in conjunction with the other functions and identities of the system. In a simple period analysis, however, we can say that the investment decisions of one 'decision day' are determined by the profits of the preceding period, and largely

determine the profits of the next period. We cannot, of course, reduce the immensely complex dynamic structure of social life to any simple 'this week, next week' kind of period analysis without a pretty heroic abstraction: nevertheless, it helps to illustrate the process by which the equations of equilibrium are 'solved'.

I confess I have perhaps not properly understood the objection to the use of identities in the equilibrium systems themselves: surely what the addition of identities to the functional relationships gives us is a determinate system! Without the identities the system does not have enough equalities to determine it, and they must be present in the system either implicitly or explicitly: it is surely better to make them explicit.

With Mr. Hart's comment I find myself in substantial accord. The next step beyond static models is, of course, dynamic, expectational models. The problem of relating the decisions of one set of people to the disappointments of others, which has been hovering in the background of a great deal of modern economics, also needs to be made much more explicit. It seems to me clear, however, that the links between decisions and disappointments are precisely the macroeconomic identities I have been developing.

Mr. Liu's model illustrates extremely well what should *not* be done with identities! If the models are to be significant the various variables must represent quantities or sums of quantities that are in some way in the power of decision makers to vary. Mr. Liu's 'average wage rate' is not such a quantity, as it must refer to real wages, which are not normally within the power of the wage bargainers to decide. His model might very well become significant, however, if wage bargains (like the General Motors contract) were generally conducted in terms of real wages. This would introduce an element of over-determinateness into the existing system, and something would have to 'give'. What would 'give' of course is the price level, and the profit and wage identities would thereby be adjusted. But I am not especially concerned to defend the realism of my particular 'models', which are intended merely to illustrate a method: the search for more significant 'parameters' is a continuing task of both economic and statistical analysis.

In regard to Mr. Copeland's remarks, surely all statistical series are so arbitrary that there should be no difficulty in fitting any reasonable set of concepts to them. Both analytical and empirical concepts involve the splitting of a heterogeneous universe into heterogeneous parts. Exactly where the split is made is in considerable degree a matter of aesthetics: there will always be difficult, and unimportant, marginal cases of the lines of definition. What is important, however, is that the relevant universe should be completely divided, that the conceptual framework should not leave any portion of the relevant universe outside it. This is the significance of starting with the universal identities derived by splitting the universe in different

ways. All my system involves is a twofold split of the heterogeneous universe 'national income' first into that which is taken off the hands of businesses by households and that which is not, and second into that which accrues to 'labor' and that which does not. In either case the line may be rather arbitrary, as indeed, the distinction between households and businesses is itself arbitrary. But the arbitrariness of a distinction does not detract from its importance, for few important distinctions are clear, and the arbitrariness is in no way removed by making the distinction depend on conformity to statistical rituals.

I must, I think, plead not guilty to the accusation that my models are 'hybrid' with respect to accrual and cash basis variables, except as far as all accounts are hybrid in this respect. I have not had an opportunity to see the moneyflows manuscript; surely, however, where moneyflows exercise their influence on the income structure is in the changes in money stocks in various accounts, as I have indicated. I see nothing 'hybrid' in this, any more than a balance sheet is hybrid because it contains cash, inventories, and equipment.