Volume Title: The Smoothing of Time Series

Volume Author/Editor: Frederick R. Macaulay

Volume Publisher: NBER

Volume ISBN: 0-87014-018-3

Volume URL: http://www.nber.org/books/maca31-1

Publication Date: 1931

Chapter Title: The Smoothing of Economic Time Series, Curve Fitting and Graduation

Chapter Author: Frederick R. Macaulay

Chapter URL: http://www.nber.org/chapters/c9361

Chapter pages in book: (p. 31 - 42)

## CHAPTER I

### THE SMOOTHING OF ECONOMIC TIME SERIES. CURVE FITTING AND GRADUATION.

The statistical problem of fitting a mathematical curve to economic data has many of the characteristics of a problem in the adjustment of physical observations. Historically, the first problem of this kind arose in the adjustment of observations made on one variable. After a large number of observations had been made on one variable it was desired to obtain from those observations the most probable value of the variable, in other words, the most plausible estimate of the true value of the variable which could be made from the observations. The arithmetic mean of the observations was early proposed as a solution of this problem.[1]

The arithmetic mean was used before any attempt was made to develop a mathematical foundation for its use. An attempt at a mathematical foundation came with the theory of least squares. This theory claimed that the distribution of errors tends to be such that if a value of the variable be

[1] It is not necessary for this discussion to go into the question of when the arithmetic mean is the best value to use and when some other type of average would seem preferable.

taken which will make the sum of the squares of the deviations of the observations from this value a minimum, this value will be the most plausible estimate of the true value which could be made on the basis of the given observations. The arithmetic average of the observations is the value of the variable which makes the sum of the squares of the deviations a minimum.

The theory of least squares was extended to cover not only the problem of the most probable value of one variable but also the problem of the most probable relationship between two or more variables. If we think of measurements on one variable being charted as vertical deviations from a horizontal straight line, we see that the true value (which is free from errors of observation) may be represented by a horizontal straight line. We are attempting a numerical solution of the equation $Y = K$, where $K$ is the unknown true value. When we approach the problem of the relations between two variables, we may think of our observations being charted with the values of one variable charted horizontally, and the values of the other variable charted vertically. If the relationship between the two variables is a straight line relationship, the problem is to find the best values for $A$ and $B$ in the equation $Y = A + BX$.

An example of a problem in two variables would be to discover a relation between altitude and barometric pressure. The investigator might first take a large number of observations on barometric pressure at different altitudes. For the purposes of this problem, let us assume that the altitudes have been accurately determined—in other words, let us assume that they are free from "error." Altitude is to be the "independent variable." The investigator may now draw points on a chart. Each point will represent a measurement of barometric pressure at a particular altitude. Altitudes are shown as horizontal distances measured from the left-hand edge of the chart and barometric pressures as vertical distances measured from the base of the chart. The problem is to draw a curve in among the points in such a manner that the ordinates (or vertical distances from the base) of the curve shall represent the most probable barometric pressures corresponding to the altitudes represented by the abscissas (horizontal distances from the left-hand edge). In other words, if we go along the base line 3 inches, which represents 3,000 feet altitude, the height of the curve at this point is to represent the most probable barometric pressure at 3,000 feet altitude.[1] The problem is not a problem of what is the most probable value of one variable

[1] Of course in the above problem, the barometric pressure readings are affected not only by *errors* but what is more important by actual variations in pressure which are not caused by altitude. We neglect the latter consideration in the illustration.

but what are the most probable values of one variable associated with definite values of the other variable. What is the most probable barometric pressure associated with any particular altitude?

In the solution of such a problem there are two parts. The first and generally the more important part is to decide what is the nature of the function relating the two variables to one another. The theory of least squares will give us little aid in making this decision. It may tell us which of two curves[1] gives the better fit; but it cannot draw our attention to such facts as that the curve giving the better fit may be grossly empirical, may not tie in with other known phenomena, and may become absurd immediately outside the range of the data, while the curve giving the poorer fit to the particular set of observations may be subject to no such theoretical drawbacks. The second part of the problem (if the curve be represented by an equation) is concerned with finding the "best" numerical values of the unknown constants. The theory of least squares states that these "best" numerical values of the constants will be such values as will make the sum of the squares of the vertical deviations of the observations from the fitted curve a minimum—in other words make the sum of the

[1] Such curves may or may not be simple mathematical equations covering the whole range of the data. For example, the laws of gases do not necessarily apply to liquids and solids.

squares of the deviations of the observational values from the corresponding theoretical values (on the curve) a minimum.[1]

Fitting a mathematical curve to economic time series is a statistical problem. It differs from an ordinary problem in the adjustment of observations in that the primary object of the fitting is not usually to eliminate errors of observation. The population of a country might conceivably be known exactly year by year for a long period and the statistician might wish to fit a curve to the whole series. Such a curve would not be fitted in order to discover what was the true population at any particular date. In other words, it would not be fitted to eliminate *errors*. It would probably be fitted for the entirely different purpose of discovering a law of population growth which would give a picture of what the population would have been if, throughout the period, it had been affected only by continuously acting and fundamental forces, excluding not only small and erratic influences but also unusual or disturbing influences, such as wars, famines, etc. Assuming some such law

[1] The method of least squares is, in practice, not so useful as the above might lead one to expect. Only if the equations relating the variables to one another are of a linear or parabolic type is the application of the method both direct and simple. With other types of equation, computation is, when not practically impossible, generally extremely laborious. Even the method of moments is not of universal application.

of population growth, the investigator might be interested in seeing how closely he could describe his particular population data by some simple growth curve, because he would be interested in speculating to what extent the continuously acting and typical forces were the controlling forces in the particular case he was examining.[1]

Trend is the outstanding characteristic of population curves. Seasonal fluctuations are negligible. Non-seasonal cyclical fluctuations are relatively small. On the other hand, there are time series in which little or no trend is apparent but which contain pronounced seasonal or cyclical movements. Monthly rainfall in the City of Seattle would be an example of a series having a pronounced "seasonal" fluctuation. Now the student of weather might wish to fit a curve to the monthly data for rainfall in the City of Seattle in such a manner as to eliminate seasonal and minor erratic fluctuations, but leave everything else. Even if the resulting curve showed no regular long time trend or regular and definite mathematical cycles, it might show considerably different amounts of rainfall at different periods.

There are economic time series which show both

[1] The investigator, of course, would not propose that his fitted population curve should, for all purposes, be substituted for the raw data. Graduations of, or curves fitted to, economic data can generally be used to replace the data for specific purposes only. For example, a graduation of monthly Call Money Rates has no

definite trend and pronounced and definite cycles. The production of eggs in the United States is an example of such a series. Annual production of eggs increases with the growth of the country, but there is, of course, a very pronounced 12-months or "seasonal" cycle. Of the series smoothed in the study of interest rates and security prices, Call Money Rates, Time Money Rates, Commercial Paper Rates and Railroad Bond Yields seem to be of a type showing no definite mathematical trend. At least, it would not seem feasible to propose a *law* of trend, such as has been proposed for the growth of population. The series for Railroad Stock Prices is of a type which shows evidences of a rather definite long time trend, though the cyclical fluctuations are extremely large. Bank Clearings and Pig Iron Production show smaller cyclical fluctuations and still more definite trend.

During most of the period covered by the interest rate study, Call Money Rates, Time Money Rates and Commercial Paper Rates show rather pronounced seasonal movements. Such seasonal movements as exist in the case of Bond Yields and Stock Prices are so small as to be practically negligible. It seems extremely improbable that any of the series contain definitely recurring periodic movements other than seasonal fluctuations.

such general replacement relation to the original data as a mortality curve has to its raw data.

The type of smooth curve which might be expected to appear in any particular time series if the series were unaffected by the minor or temporary factors which give rise to seasonal and erratic fluctuations is not necessarily representable throughout its length by any single simple mathematical equation. Its smoothness may be traceable to the fact that the ordinate of each point on the curve is correlated with the ordinate of the immediately preceding point. The curve may in this respect be similar to a curve resulting from the cumulation of a mere chance series. If twelve coins be thrown time after time and if the number of heads in each throw be charted as they occur, the resulting graph will not be a smooth curve nor will it tend to show any particular trend or cyclical fluctuations. Each throw stands by itself, uncorrelated with the preceding throws. However, if the series of throws are cumulated and the results charted, the graph will show considerable smoothness and both trend and cycles. The trend will most probably be representable by the straight line $y = 6x$. The cyclical appearance will be pronounced but the "cycles" will be irregular and not representable by any single simple mathematical equation. If, instead of cumulating the number of heads in each throw, the excess of the number of heads over 6 thrown each time be cumulated, the trend will vanish but the cycles will remain. The smoothness of such cumulative curves is inherent in their very nature. While not only the possible but the probable size of those larger movements which may be termed cyclical is greater in the cumulative than in the non-cumulative curve, both the possible and the probable size of those smaller movements which may be termed erratic is less. In the non-cumulative distribution of heads over 6 in number, the possible size of a "cyclical" movement is 12 (from −6 to +6). The possible "erratic movement" or change in size from one value to the next is also 12 (from +6 to −6 or from −6 to +6). If ten throws (12 coins thrown each time) be *cumulated* the possible size of a "cyclical" or major movement is increased from 12 to 60, while the possible "erratic movement" or *change* in size from one value to the next has decreased from 12 to 6 (+6 or −6). A similar though much less sensational story is told if *probable* rather than *possible* movements are considered. Cumulation smooths a series and introduces movements suggestive of trends and cycles.

Many economic time series seem to be of a type somewhat analogous to such cumulated chance series. Some economic series suggest chance series which have been cumulated twice. For example, each observation on a population series is not only highly correlated with the immediately preceding observation, but the first differences are highly correlated with the preceding first differences. The first differences are the resultant of three factors, births, deaths and migration. Births and deaths are functions of the size of the population and hence highly correlated with the same items for the preceding year. The excess of births over deaths and the amount of migration are correlated with similar items in the preceding year, though the correlation will not necessarily be so high as in the case of births or deaths alone. The commonest type of economic time series suggests a cumulated chance series on which has been superposed another but non-cumulated chance series and a more or less regular and unchanging seasonal fluctuation.

How should such series be smoothed? What sort of procedure would seem adapted to eliminate all seasonal and erratic fluctuations leaving a reasonable picture of the cyclical fluctuations and any underlying trend? While such a series as Railroad Stock Prices shows some evidence of an underlying trend which might be rationally described by some appropriate mathematical equation, its cyclical movements do not suggest any such possibility. Such series as Call Money Rates, Time Money Rates, Commercial Paper Rates and Railroad Bond Yields do not suggest any possibility of rationally describing either their long time trends or cyclical fluctuations by mathematical equations. To fit a straight line, a parabola, a compound interest curve, a logistic curve, or any other trend curve to such a series would seem highly empirical, not to say absurd.

Any attempt to fit a *single* mathematical equation to the whole of one of such series would seem quite as absurd if the equation were designed to describe periodic movements, as if it were designed to describe trend movements. The indiscriminate use of harmonic analysis to describe series which are not provably harmonic in their origin is indefensible. The position of each point on a fitted harmonic curve is affected by the position of each datum point. If short cycles appear in the first two thirds of the data, the harmonic curve will introduce short cycles in the last third even if they do not appear in the data. Any part of a smooth curve assumed to underlie the data curve is a smooth curve only because it is an outgrowth of the immediately preceding portion of such smooth curve. It is no more affected by distant points in the past than the wobbling track left by the rear wheel of a bicycle ridden by an inexperienced rider is affected by the position of the track 100 yards back.

Any part of the track of the rear wheel will be a comparatively smooth curve because of the manner in which it is made. It is an outgrowth of an immediately preceding portion of the track, but this does not mean that it is in any important sense affected by the track 100 yards back. A skillful rider could approach a large scale mathematical curve and ride upon it. Of course, he would not be able to do so instantly. He would have to come out of the particular curve on which he was traveling and gradually approach the mathematical curve on which he wished to travel. The attempt to represent the entire bicycle track by a single mathematical equation, whether designed to describe trend, "cyclical" movements or both, would be plainly quixotic. Harmonic analysis should not be used to describe data unless there are reasons for believing that cycles of constant period are inherent in the very nature of those data. Of course, the analysis itself may offer strong evidence that this is so, but evidently this could hardly be the case with any such bicycle "data." Attempts by various investigators to discover rigidly mathematical cycles (other than seasonal fluctuations) in economic data of the type presented in the study of interest rates and security prices have not so far been sufficiently convincing to make one feel that the above "bicycle" illustration is either far-fetched or illegitimate.