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Chapter Title: Validity of Observed Relations

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mated by different methods. It is seen from the table that marginal combined cost and the marginal cost resulting from unit increments in average weight are approximately the same whether estimated directly or by summation.

Behavior of 'reflated' cost

In establishing the functional relation of cost to output, the prices paid for materials and labor were held constant during the period of analysis. If, however, such statistical functions are to be useful for cost forecasting, as guides to price policy, and in determining whether the cost incurred in any period differs from the general pattern of behavior, prices of input factors appropriate to the period must be substituted for the 'deflated' or stabilized prices used in the analysis. Fortunately, such a computation is relatively easy since if the cost of any group of elements for a given set of prices is known, the physical quantities of the factors can be determined. The magnitude of the elements of cost appropriate for another set of prices can then be found by multiplying the quantities by the appropriate prices.

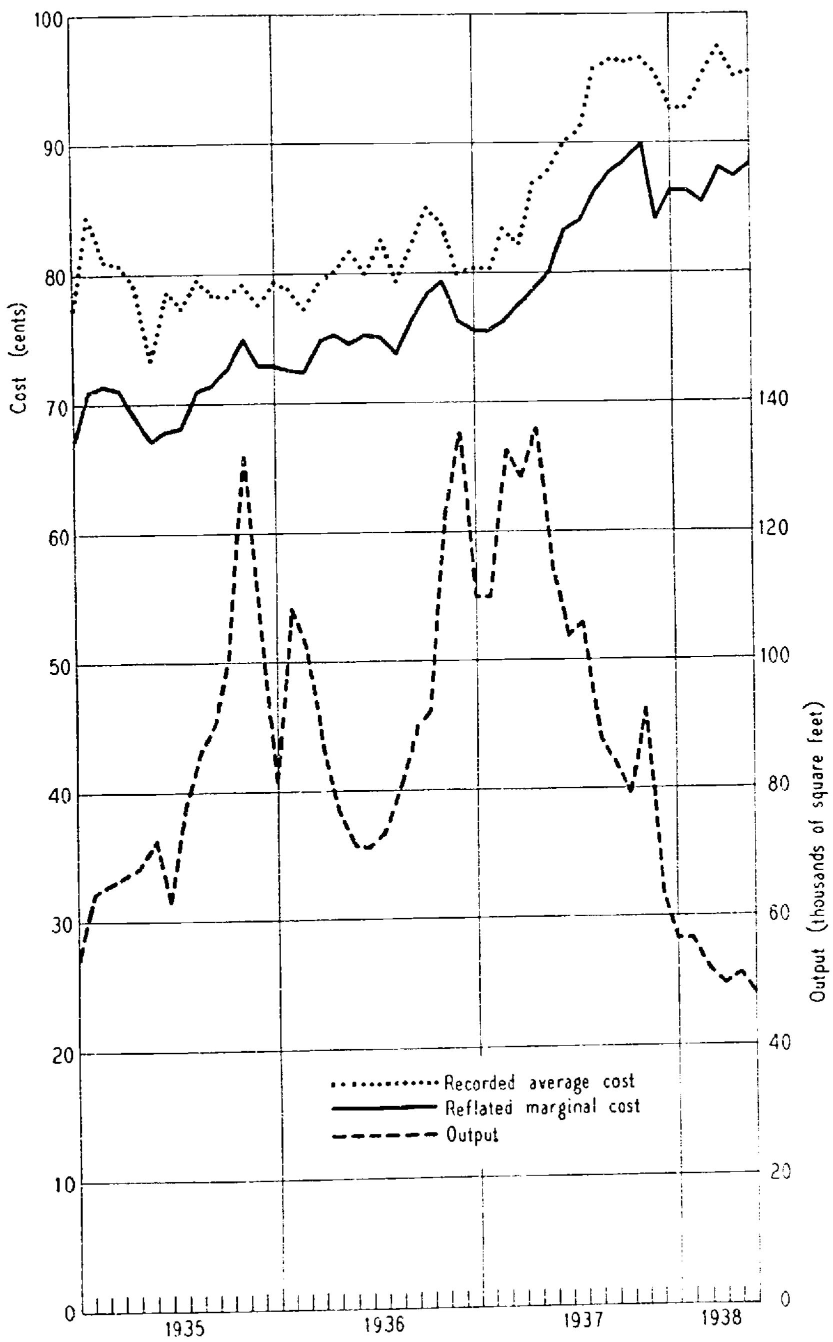
Chart 9 shows marginal cost 'reflated' to reflect the prices actually existing in the period. The rough similarity between the fluctuations of 'reflated' marginal cost and those of recorded average cost arises from the predominant importance of leather cost in both. The departures from similarity, attributable mainly to fluctuations in output (also shown in this chart) reflect the inverse relation between output and the proportion of fixed cost to recorded average cost.

7 Validity of Observed Relations

Some potential sources of error that might influence the statistical results have already been discussed briefly. In order to appraise the validity of the statistical findings, we now examine in more detail their limitations, which may be attributable either to inadequacies inherent in the data or to the technique of analysis. The following considerations may conceivably have an important bearing upon the reliability of the findings of this investigation: (1) The sample may be inadequate, the observations not being representative, particularly for high output. (2) Certain cost elements that bear some relation to output were omitted, for example, allocated general firm overhead. (3) The rectification procedure may have errors and shortcomings, such as improper allocation of cost to time periods, elimination of price changes that may have resulted from variation in the plant's output rate, and the impossibility of eliminating non-random errors in the data. (4) Sufficient account may not have been taken of all operating conditions that influence cost; specifically, the rejection of certain independent variables in the multiple regression analysis that exert an appreciable influence on the behavior of cost may not have been justifiable. (5) The regression function may have been incorrectly specified, a possibility that arises when there is a large scatter in the observations so that there is some doubt concerning which

CHART 9

Fluctuations of Recorded Average Cost, Reflated Marginal Cost, and Output



function fits the data best. The first, third, and fifth of these limitations are of particular interest in this study.

Since the firm belonged to a declining industry, the sample was to a certain extent not representative of industrial conditions in general. Subnormal activity prevailed in the industry during the period studied, January 1935 to June 1938. Subnormal activity refers not only to a low level of output, but also to the aggregate of phenomena correlated with or directly due to a *prolonged* period of low output, including pessimistic expectations concerning future developments. This situation presents, however, certain advantages for statistical cost analysis since under such circumstances it is possible to avoid the difficulties arising from secular growth in scale of plant. Moreover, since in fact numerous industries do experience a secular decline in the demand for their products, the sample may not be too unusual.

Price corrections and allocations of recorded cost to accounting periods, the latter being admittedly arbitrary, are relatively large adjustments that are likely to be attended with inaccuracy. Even price changes must be corrected for approximately, and the method of cost allocation may conceal a tendency for the marginal cost of repairs, cement, and supplies to rise. These elements of cost, however, constitute only 4.22 per cent of combined cost, so that the possible error is negligible.

There is some uncertainty whether regression functions of a linear form describe the observations as well as a curvilinear function form. The theoretical considerations underlying the alternative forms of the short-run total cost curve were examined in Section 1. Two cases of cost behavior were examined: (1) that described by a curvilinear total cost curve, which rises first at a decreasing rate and eventually at an increasing rate as output increases, and (2) that in which the total cost curve is linear to the point of physical capacity, when it begins to bend upwards. Preliminary graphic multiple regression analysis of rectified observations of total combined cost supported the hypothesis of linearity. However, the strong preference of economic theorists for the curvilinear hypothesis made it advisable to test the hypothesis of linearity by methods more rigorous than graphic analysis.

Objective tests of the linearity of the total cost functions are especially desirable since approximate linearity of the curve is no positive assurance that the corresponding marginal cost is constant. Even though the discrepancy between the linear and curvilinear total cost curve is barely perceptible upon visual examination, the smallest degree of curvature in the total cost curve means that the marginal cost curve is curvilinear rather than constant. For this reason it is advisable that the cost functions be examined not only in total form but that the marginal and average cost functions be determined independently of the total cost function. The greater the confidence one has in the shape of the subsidiary marginal and average cost functions, determined independently, the more confidence one can have in the total cost curve consistent with them. In view of the importance of the distinction between the linear and the curvilinear total cost specification, careful attention must be devoted to any evidence that is of aid in choosing between the two. The remainder of this section is consequently concerned with statistical tests that may afford some useful guidance in making this decision:

(1) Monthly total cost observations were classified into arrays corresponding to sub-groupings of output in order to obtain two independent estimates of the variance: (a) the variance of the observations about the means of the array; (b) the variance of the means of the array about the linear partial regression of cost on output. A linear functional relation may be regarded as an adequate representation of the cost-output regression if the variance of the array means about the regression line is not significantly greater than the variance within arrays. If the former variance is unusually greater than the latter, however, one is led to suspect that a curvilinear regression function is preferable.

(2) Residuals from the linear multiple regression surface were computed and classified into ten groups according to output. The variance of residuals about their group means was then compared with the variance of the group means about the general mean. By testing residuals in this form it was possible to avoid the difficulty arising in the preceding test from the slope of the partial regression curve.

(3) The relation of 'incremental' cost[40] to rate of output and to other operating conditions was examined. If incremental cost is not significantly related to output, this is additional evidence that marginal cost is constant and consequently that the total cost function is linear.

(4) Similarly, the relation of average cost (derived directly from the accounting records) to output yields further information concerning the shape of the total cost curve. If the average cost curve does not rise as output increases over the observed range, the existence of the rising phase of the cubic total cost function following a point of inflection is not substantiated.

(5) A cubic cost-output regression function was fitted to the data by multiple regression analysis and the significance of its squared and cubed terms tested by Student's t-test.

Since there is some question concerning the validity of arbitrary, rule-of-thumb applications of statistical tests of significance to data derived from time series, it is necessary first to consider briefly the rationale of these tests. Suppose the residuals from the multiple regression of total

[40] Incremental cost is determined indirectly from the accounting records and is obtained by dividing the difference in total cost for two adjacent months by the corresponding difference in output. Although an approximation to marginal cost, incremental cost is to be carefully distinguished from it.

combined cost are classified into arrays according to values of output, one of the independent variables. Even if these residuals were selected at random from a homogeneous, normal universe, some variation among the means of the arrays would be expected. Fisher's z-test enables one to decide, on the basis of a probability distribution, whether such variation is unusually large, when the hypothesis is that the original observations of cost, output, and weight were selected at random from a trivariate universe in which the regression is linear and the distribution of the arrays of the dependent variable normal and homoscedastic. The power or effectiveness of such a test, as a test of linearity, depends on the fact that curvilinearity of regression leads to unusually large or significant measures of dispersion about the means of arrays. Hence, the test is a good one if other hypotheses differ only in form of the regression function. This is the case here, since mere absence of linearity may be considered the alternative hypothesis.

The assumptions of random sampling, etc. on which the z-test is based are admittedly not satisfied by the data. In this study, however, these assumptions are not so unrealistic as they would be in most studies involving economic time series. Because of rectification to eliminate dynamic influences, evidence of parallel cyclical fluctuations in the form of positive serial correlation among residuals is absent. Indeed, the sign of the serial correlation coefficient is negative. Moreover, it was not even necessary to use time as a catch-all independent variable. Although the various rectification procedures almost certainly improved the data for the purposes of this investigation, the need for such procedures introduces some inexactness in the z-test, even if it were otherwise strictly applicable. The inadequacy of the technique of rectification has probably introduced sources of error additional to and more important than the errors tested; and since the rectification was not part of the least squares fitting process, its influence could not be allowed for by adjusting the number of degrees of freedom.

Despite these misgivings concerning the correspondence of the data with the specifications required for the application of analysis of variance tests, it seemed desirable to test the significance of the relations established by all available methods.

Analysis of variance of total cost observations from linear partial regression [41]

In order to test the linearity of the relation of total cost to output the cost observations were classified according to output rate into ten groups. The variance of the observations about the mean of each group was then compared with the variance of these group means from the corre-

[41] The calculations necessary for the analysis of variance were made by Phyllis van Dyk. John H. Smith made helpful suggestions in their interpretation.

sponding points on the cost regression. Since the cost-output function under examination is a partial regression representing the influence of only one of the two independent variables, it was necessary to correct the total cost observations for the influence of the other variable, average weight per square foot. This was accomplished by expressing the total cost observations as deviations from total cost estimated from the partial regression of cost on average weight, and then adding these deviations to the observed mean of total cost.[42]

The corrected total cost observations having been classified according to output, the mean output and the mean total cost were computed for each group. Comparison of the variance within groups and the variance among the several means of total cost and the total cost values estimated for corresponding means of output revealed a significantly greater intra-group variance. The value of z is found to be -1.1307, when n_1 is the degrees of freedom among the group means and n_2 the degrees of freedom within groups. To substantiate a hypothesis of curvilinear regression, a high positive value of z is required. The 5 per cent point for z when $n_1 = 7$ and $n_2 = 33$ is 0.4164. If a positive value for z higher than this had been determined, curvilinearity would be indicated. The value of z actually found, however, was negative and large (absolutely), -1.1307. Since the mode of z is zero, this value lies below the mode and farther away than would occur frequently by chance. In fact, the magnitude of z that would occur in random sampling once in a thousand is only slightly larger (absolutely), about -1.26. Therefore, the indication of linearity is unusually strong. The conclusions drawn from this test, when compared with those resulting from the analysis of variance of residuals from the linear *multiple* regression, afford interesting evidence of the distortion of the intra-group variance caused by the use of adjusted residuals from steeply sloping partial regressions.

Fisher's z-test, therefore, shows that the variance within arrays is much smaller than would be expected on the basis of random sampling according to the specifications described above, i.e., the sample regression is more nearly linear than one would expect it to be even if the observations on cost, output, and average weight were selected at random from a universe in which the regression is linear.

Analysis of residuals about linear multiple regression surface

Since the large negative value of z in the preceding test seemed to be attributable primarily to intra-group variance caused by the slope of the partial regression line, an alternative form of the z-test was applied.

42 This procedure is not completely satisfactory since it assumes the correctness of the linear regression of cost on average weight, which was determined by least squares fitting. The linearity of this function was strongly indicated by the graphic correlation analysis but it was not subjected to a more objective test.

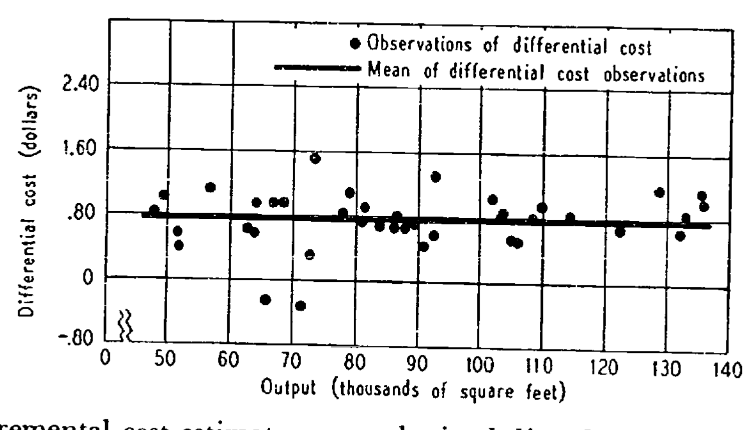
Residuals were computed from the multiple regression surface instead of from the partial regression curve. These residuals were classified by output into ten groups. Their intra-group variance was then compared with the variance among the groups. In this case the value of z is — .03089, using n_1 for the degrees of freedom among groups and n_2 for the degrees of freedom within groups. In contrast to the preceding test, this value of z is not notably below the average value for random samples from a universe of the type specified. The 5 per cent point is 0.4164. The existence of curvilinearity of regression would be expected to cause unusually large variance within the arrays of the residuals about the multiple regression surface. However, since the observed variance within groups is not significantly unusual, this z-test also apparently indicates that the distribution of the observations is consistent with a hypothesis of linearity.

Analysis of incremental cost

The analysis of incremental cost was designed, first, to provide additional evidence concerning the hypothesis of the linearity of the total cost function; second, to explore an alternative method of studying the behavior of marginal cost, and third, to test by an independent estimate the magnitude of marginal cost found by differentiating a fitted total cost function.[43]

CHART 10

Incremental Cost at Various Levels



Incremental cost estimates were obtained directly from the cost observations by expressing the difference between the adjusted [44] total cost

[43] Since small changes in the shape of the total cost function cause relatively large fluctuations in marginal cost, the distribution of incremental cost determined independently constitutes an additional test of the linearity of the total cost function.

[44] In computing these first differences, cost was corrected for errors in accounting allocation, for rate changes, and time lags, but was not adjusted to remove the estimated effect of average weight. Since one objective of this analysis was to test an alternative short-cut method of estimating marginal cost, it would have been inconsistent to use a correction that presupposed a multiple correlation analysis of total cost.

for each month and that for the preceding month as a ratio to the corresponding month to month first difference in output. This ratio represents the observed average increment in cost for the range of the specific increases in output and is designated, as noted above, as incremental cost. The behavior of incremental cost at various levels of output can be seen in Chart 10. On the assumption that the magnitude of incremental cost is unrelated to the level of output, the arithmetic mean of the incremental cost observations was computed and found to be \$.767.[45] The wide scatter of the incremental cost observations, together with the assumption that it is independent of the output rate, restricts the confidence to be placed in this value of incremental cost as an estimate of marginal cost. Nevertheless, such an estimate is very close to the magnitude of marginal cost derived from the total cost function by differentiation (\$.77).[46]

There are, however, certain essential differences in the nature of these two cost estimates. First, average incremental cost has reference to finite and sometimes large increments in output, whereas marginal cost, estimated by differentiating a fitted total cost function, is relevant for very small (theoretically infinitesimal) changes in output. Second, incremental cost was derived from scattered observations subject to much random error while marginal cost, estimated from a continuous function, is not influenced by random variation. Third, the total cost observations used in computing incremental cost were not corrected to remove the estimated average influence of average weight, whereas this distortion was removed in estimating marginal cost.

The hypothesis that the data in total cost form show no evidence that the magnitude of marginal cost is related either to output or to the other independent variables was tested by analyzing the effect on incremental cost of various independent variables, including some not used in the least squares analysis of the total cost observations. Since the primary objective was to ascertain the existence of a relation, rather than to determine its precise nature, the functional relation between incremental cost and output was examined by means of the analysis of variance. This validating device serves as a more rigorous test of the findings concerning constancy of marginal cost, being more objective than visual examination of the incremental cost observations. The reliance to be placed in this test is limited, it should be remembered, by the magnitude of the random variation in incremental cost and the difference between these observations and marginal cost in its more precise sense.

45 The standard error of the mean was \$.055, the standard deviation \$.342, and the coefficient of variation, 44.6 per cent. In the calculation of these estimates one observation was omitted because it showed a change in output of only .4 per cent of the mean output, which made the associated incremental cost unreliable. All other changes were greater than 1.25 per cent.

46 The ratio of the difference between marginal cost and average incremental cost to the standard error of the difference was .0438.

The procedure was to group incremental cost observations (omitting one case because of its unusualness) according to the corresponding values of the particular independent variable involved, and to test the existence of a relation between incremental cost and each variable by applying Fisher's z-test to determine the significance of the ratio of the variance within groups and the variance among groups. If the value of this ratio is found to be not significant, there is on this ground no reason to reject the hypothesis that no relation exists between incremental cost and the particular independent variable considered.

Since the method of grouping may influence the results, tests were applied for two groupings of each independent variable. The observations were first divided into three or four classes, then redivided into ten equal groups and retested in order to ensure that the use of too broad classifications had not obscured the relations.

Both tests for the existence of correlation between incremental cost and output demonstrated that no relation significant in a statistical sense existed. The value of Fisher's z is only slightly larger than its average value in random samples from an uncorrelated universe and might easily have occurred by chance. The value of z is 0.0726 when $n_1 = 9$ and $n_2 = 30$, while the 5 per cent point is 0.3925. This result, as well as the results obtained for the other independent variables, applies to the ten-group classification.

The investigation of the relation of incremental cost to average weight of belting revealed the same general situation. The value of z is −0.1465, the negative value indicating that this magnitude of z is below the average value expected in random samples from an uncorrelated universe.

The lack of a relation between total cost and direction and magnitude of change in output indicated by graphic analysis was substantiated by statistical tests using the analysis of variance. The magnitude of z is −0.7721 when $n_1 = 9$ and $n_2 = 30$, a negative value so far below the average value in random samples from an uncorrelated universe that it would be exceeded in more than 95 in a 100 cases by pure chance. The analysis of variance test for incremental cost and absolute magnitude of change in output does not show the existence of any significant correlation.[47]

In general, the analysis of incremental cost substantiates the findings of the total cost analysis, both in indicating the lack of a relation between marginal cost and output and in providing a subsidiary estimate of the marginal cost almost identical with the marginal cost derived from the total cost equation. The form of the incremental cost observations, however, together with their great chance variability, restricts their reliability as a basis for the validation of relations established by the analysis of

[47] The value of z is 0.1930 when $n_1 = 9$ and $n_2 = 30$, to be compared with the 5 per cent point, 0.3925.

total cost observation and limits their usefulness in estimating marginal cost directly. Although direct analysis of first differences of cost and output is a more economical way to estimate marginal cost than correlation analysis of total cost, it is distinctly less reliable.

Analysis of average cost

The distribution of observations of average cost affords some additional information concerning the shape of the total cost function. A total cost curve of the conventional form, represented by a cubic parabola, leads to a U-shaped average cost curve. The scatter diagram of adjusted observations of average cost in the lower panel of Chart 4 does not suggest this sort of distribution. On the contrary, the scatter conforms closely to the average cost curve derived from the linear total cost curve. This curve, of course, differs from that which would have been found by fitting a curve to recorded average cost. The deviation to be minimized by the least squares fitting would differ for total and average cost observations, since the correlation of total cost and output was not perfect. The curve of average cost derived from the total cost curve nevertheless appears to describe with reasonable accuracy the behavior of recorded average cost. To determine the degree of this correspondence, the correlation coefficient was computed between recorded average cost and average cost derived from the equation

$$_aX_c = .770 + \frac{2.974}{X_2}$$

(this equation was derived from the partial regression equation of total combined cost on output, after allowance for the influence of average weight). This coefficient was found to be 0.866; the multiple correlation coefficient for total cost is 0.998.

These four types of statistical test indicated that the cost and output data for the leather belt shop, for the range of output observed, are consistent with the hypothesis that the total cost function is linear. They showed, moreover, that a different approach to the determination of marginal cost yields substantially the same result as that obtained mathematically from the total cost function, and that the function for average cost obtained from the function fitted to the total cost observations explains most of the variation in recorded average cost.

Analysis of fit of cubic function

To aid in discriminating more specifically between a cubic and a linear functional form, a fifth test was applied. A third degree regression function of the general form

$$_tX_c = b_1 + b_2X_2 + b_3X_3 + b_4X_2^2 + b_5X_2^3$$

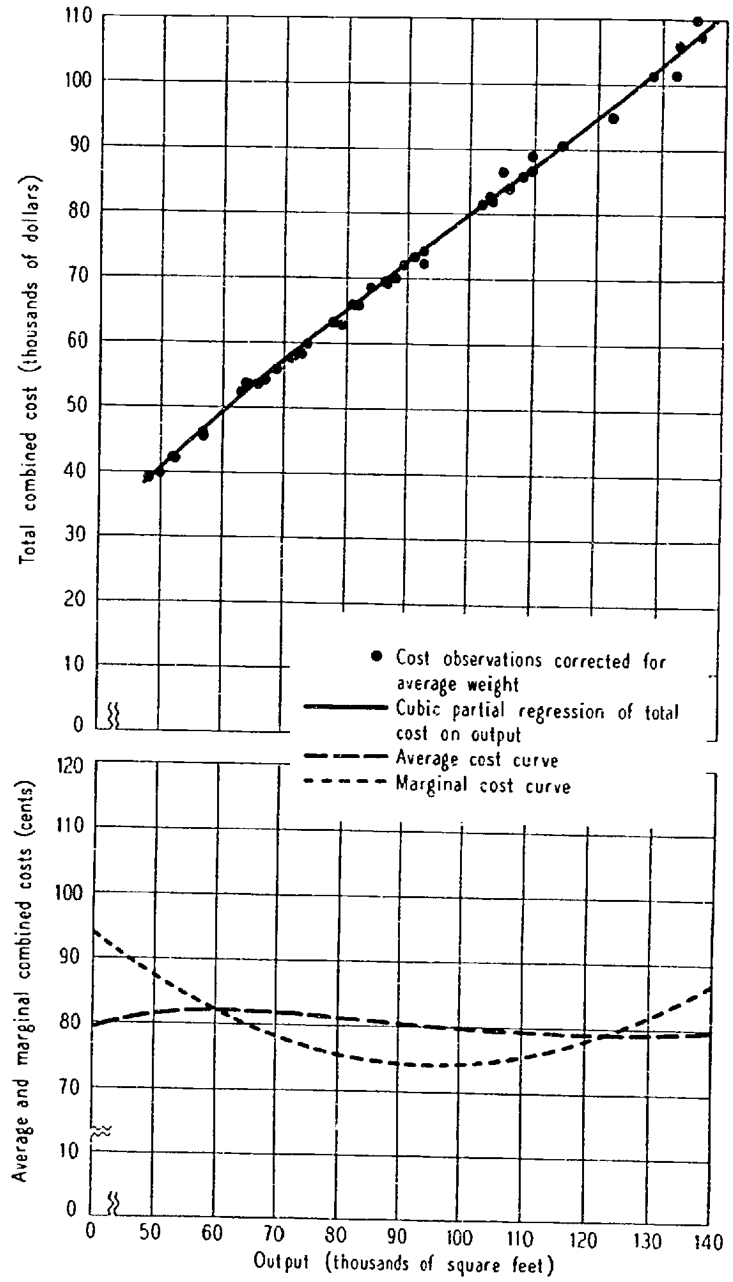
was fitted by least squares multiple regression analysis and the significance of various regression coefficients was examined by applying Student's t-test. The mathematical analysis yielded the following partial regression equation of cost on output:

$$_{t}X_{c} = -12.995 + 1.330\,X_2 - 0.0062\,X_2^2 + 0.000022\,X_2^3$$

The behavior of this regression function is illustrated graphically in Chart 11, in which the derived marginal and average cost curves are also

CHART 11

Partial Regressions of Total, Average, and Marginal Combined Cost on Output for Third Degree Regression Function



shown together with the partial regression of cost on the other independent variable, average weight. Neither the marginal nor the average cost function exhibits great variability. At the extreme of the output range, the parabolic marginal cost curve lies about 14 per cent above the level of constant marginal cost ($.77). At intermediate levels of output it lies a maximum of 4 per cent below. Because the cubic total cost curve has a negative intercept, the average cost curve behaves illogically even within the observed range. In the range between 45,000 and 65,000 square feet, the average cost curve rises; beyond this it falls until it is intercepted by the marginal cost curve, whereafter it rises slightly.

To test the suitability of the cubic function it was necessary to determine whether the regression coefficients of the higher-order terms are small enough to be attributable to errors of sampling.[48] Student's t-test was, therefore, applied by computing for the squared and cubed terms the ratios of the beta coefficients to their respective standard errors. These ratios are -2.01 for the squared term and 1.94 for the cubed term. Interpolating for $n = 38$ in a table of the distribution of t, it is found that the 5 per cent point is 2.025. However, the entries given in this table to determine the criteria of significance are based upon the sum of the tails of the t-distribution—a procedure that does not seem justifiable since the sign of the regression coefficient is specified in the theoretical model. Taking into account only one tail of the distribution, the higher order terms in the equation seem even more significant, since they lie near the 2½ per cent point.

In order to interpret the results of this test properly it is necessary to consider briefly its nature and determine whether the data comply with the statistical specifications implied in this type of test. The t-test applied to a cubic function tests the significance of the fit by setting up the null hypothesis that the universe value of the squared or cubed term is zero. This hypothesis is then examined by determining the probability of finding regression coefficients as great as those observed by random sampling from a universe of cost, output, and average weight in which the regressions are linear and the arrays of cost with respect to the independent variables are normal and homoscedastic. If the probability of obtaining coefficients as great as those observed in random sampling from such a universe is higher than some arbitrarily established level (say 5 per cent), the hypothesis that the cubed term in the true function is zero is not disproved. If the probability is small, 5 per cent or less, the cubed term may be regarded as significant and the hypothesis of linearity rejected. This test is effective in discriminating between the hypotheses of linear and cubic cost behavior, provided the data conform to the requirements of the test and that the two types of behavior seem equally

48 The numerical values of the regression coefficients and the standard errors for the multiple regression equation are presented in Table 10.

realistic in view of the technical methods of production encountered in the manufacturing process analyzed.

TABLE 10

Summary of Statistical Constants for Multiple Regression of Total Combined Cost on Weight, Output, Output Squared, and Output Cubed

	CONSTANT TERM	COEFFICIENT OF X_2	COEFFICIENT OF X_2^2	COEFFICIENT OF X_2^3	COEFFICIENT OF X_3
Regression coefficients	−75.7730	1.3304	−0.0062	0.000022	69.7622
Regression coefficients in standard deviation units		1.737	−1.5011	0.7810	0.0632
Standard error of regression coefficients in standard deviation units		0.3545	0.7470	0.4019	0.0091
Ratio of regression coefficient to standard error in standard deviation units		4.90	2.010	1.913	6.945

The data, however, fail to meet the sampling specifications in several significant respects. The usual limitations inherent in time series are present, though they seem to be less serious for these data than for most, but there are other reasons for not expecting a normal, homoscedastic distribution of residuals. The magnitude of the rectification adjustments, which are unlikely to lead to a random distribution of errors, dwarfed the tiny residuals that are the basis for the t-test. These correction procedures, necessarily approximative, in themselves constitute sources of variation greater than those included in the test specifications. Moreover, it was not possible to include the rectification devices in the test by adjusting the degrees of freedom. If it had been possible to study the residuals in the original data, or to take account of rectification adjustments in the test, entirely different conclusions might have been reached.

In order to approximate a static competitive model, it was assumed that the prices of the factor inputs are independent of the output level of the individual firm. However, the activity in any firm is likely to be closely related to the activity in the industry, and it is not reasonable to assume that input prices are independent of the operating level in the industry. If this close association exists, it is to be expected that rising factor prices accompany high levels of activity in the firm; and falling factor prices, low levels of activity. Moreover, with expanding industry demand, recourse may be had to factors that are inferior in quality, while when industry demand is low, superior factors will be retained by the firm. These two considerations are especially relevant in explaining the cubic shape of the total cost function. If, for high outputs cost increases more than in proportion and for low outputs less than in proportion to output, the associated total cost function has a cubic shape. In the case under consideration the observations apparently responsible for the particular shape of the cubic function, i.e., the costs associated with high and low outputs, were likely to be inadequately rectified for price and quality changes, the bias being in the direction that would create a cubic total cost function.

The four observations lying above the fitted line at the highest output levels were in December 1936, and March, April, and May, 1937 (Chart 11). Thus they occurred at a cyclical peak, when defects in rectification would be expected to overstate deflated cost. The six observations lying below the fitted line at the lowest recorded outputs were depression months—January to June 1938—a period in which the rectification devices may not have accounted adequately for price and quality fluctuations.

The negative intercept of the total cost curve, and the consequent illogical behavior of the average cost curve within the range of observations, cast some further doubt on the validity of the cubic function. Moreover, as pointed out in Section 1 and discussed in Section 2, there are indications that the technical structure of the production process does not correspond to that assumed in the cubic model.

In view of all these considerations, the curvature within the observed range does not seem to substantiate decisively the hypothesis that the total cost function is curvilinear.

8 Conclusions

The statistical evidence presented in Section 7 gives some support to the conclusion that marginal cost is constant within the range of output examined in this study. The findings of such an investigation as this that are most significant for economic theory can be presented adequately by considering solely the behavior of marginal cost; for, if the course of the marginal cost function is known, the shape of the total cost function is apparent. (Supplementary information is needed to determine the magnitude of fixed cost and the behavior of average cost.) Some caution must be observed, however, in comparing the marginal cost function of a model firm under static competitive conditions with marginal cost function derived by statistical methods from empirical data. The observations that are the basis of the statistical estimate may not have been adequately purged of the influence of extraneous variables by the sampling, rectification, and correlation analysis procedures. To the extent that dynamic factors are present in the cost and output observations the empirical curves will not be a precise counterpart of the curves described in theory. It appears likely, however, that the most important dynamic influences were eliminated in the data adjustments.

On the assumption that our statistical techniques have successfully isolated the static marginal cost curve, it is desirable to attempt to ac-