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6 Findings

Examination of the process of manufacturing leather belting in the light of the theoretical considerations presented in Section 1, together with the statistical distribution of the cost observations, led to the specification of a linear total combined cost function in the multiple regression analysis. Support for the hypothesis of linearity of total cost behavior, based on non-statistical considerations, is analyzed in more detail in Section 8, while the statistical aspects of the problem of specification are examined in Section 7.[32]

The findings for combined cost, direct, and overhead cost, and the elements of cost are presented in this section in both graphic and tabular form. The total, average, and marginal aspects of cost behavior are shown for the following independent variables: (1) output, measured in square feet of single-ply equivalent belting (with the effect of average weight upon cost and upon output allowed for); (2) average weight (with the effect of square feet of output allowed for); (3) both output in square feet and average weight of belting.

The regression equations showing this cost behavior and certain other statistical constants are summarized in Table 1. To indicate how closely the various aspects of output used as independent variables are associated, particular attention was paid to the coefficients of multiple correlation and determination.

Analysis of combined cost behavior

The total, average, and marginal combined cost curves derived from the least squares partial regression equation of total combined cost on output (after the effect of average weight has been allowed for) are shown graphically in Chart 4.[33] The total cost curve, which rises at a constant rate, yields a hyperbolic average cost curve, which falls continually within the range of observations. The derived marginal cost curve is a horizontal straight line, lying below the average cost curve. From this it is seen that the cost of producing an additional square foot of single-ply

32 Because the evidence supporting a linear hypothesis may not be conclusive, a third degree function was also fitted to the observations of total combined cost. Discussion of the problem of specification is postponed to Section 7 where the results of the fitting of a cubic function are shown.

33 The partial regression equation for total combined cost (${}_tX_c$) and output (X_2) (allowing for the influence of average weight of belting) is:

$${}_tX_c = 2.974 + 0.770\,X_2$$

The equation for average cost, derived from the above equation, becomes:

$${}_aX_c = 0.770 + \frac{2.974}{X_2}$$

Marginal cost is estimated from the first derivative of the total combined cost function with respect to output. In this instance, marginal combined cost (${}_mX_c$) is constant at \$0.77. The reliability of this estimate of marginal cost is indicated by the standard error of the coefficient of net regression, \$0.0063. However, this error formula is posited upon conditions of sampling and of distribution that are not entirely met by time series data of the type encountered.

TABLE 1

Summary of Findings of Correlation Analysis for Combined, Overhead, and Direct Cost

STATISTICAL CONSTANTS AND EQUATIONS	COMBINED COST	OVERHEAD COST	DIRECT COST
Multiple regression equation	${}_tX_c = -60{,}178 + .770\ X_2 + 70{,}181.30\ X_3$	${}_tX_o = 2{,}157.78 + .0108\ X_2 - 4.828\ X_4$	${}_tX_d = -61{,}636.9 + .760\ X_2 + 69{,}324.130\ X_3$
Standard error of estimate	$\bar{S}_{1.23} = 1{,}050.0$	$\bar{S}_{1.24} = 164.4$	$\bar{S}_{1.23} = 985.2$
Coefficient of multiple correlation	$\bar{R}_{1.23} = .998$	$\bar{R}_{1.24} = .842$	$\bar{R}_{1.23} = .999$
Coefficient of multiple determination	$\bar{R}^2_{1.23} = .997$	$\bar{R}^2_{1.24} = .709$	$\bar{R}^2_{1.23} = .997$
Partial regression equation for output (X_2)	${}_tX_c = 2{,}973.75 + .770\ X_2$	${}_tX_o = 2{,}108.72 + .0108\ X_2$	${}_tX_d = 662.54 + .760\ X_2$
Partial regression coefficient for output (X_2)	$b_{12.3} = .770 \pm .0063$	$b_{12.4} = .0108 \pm .00018$	$b_{12.3} = .760 \pm .0060$
Partial regression equation for average weight (X_3)	${}_tX_c = 7{,}394.2 + 70{,}181.30\ X_3$		${}_tX_d = 5{,}059.80 + 69{,}324.13\ X_3$
Partial regression coefficient for average weight (X_3)	$b_{13.2} = 70{,}181.30 \pm 643.4$		$b_{13.2} = 69{,}324.15 \pm 603.7$
Partial regression equation for change in output (X_4)		${}_tX_c = 3{,}107.69 - 4.828\ X_4$	
Partial regression coefficient for change in output (X_4)		$b_{14.2} = 4.828 \pm 1.760$	

${}_tX_c$ = combined cost in dollars
${}_tX_o$ = overhead cost in dollars
${}_tX_d$ = direct cost in dollars

X_2 = output in square feet of single-ply equivalent belting
X_3 = average weight in pounds per square foot
X_4 = change in output in square feet from preceding month

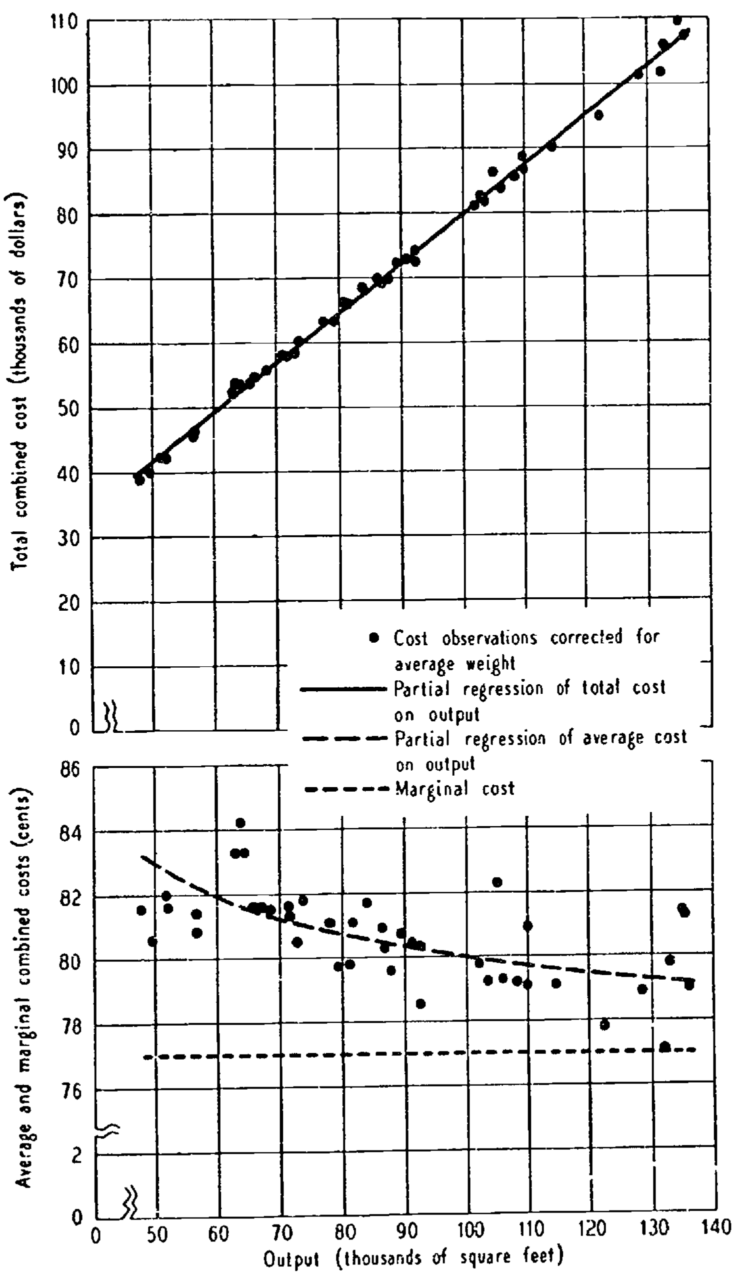
equivalent belting remains unchanged ($0.77) regardless of the rate of output.[34]

The statistical findings concerning the partial regression of combined cost are shown graphically in Chart 5 for the two methods of analysis, graphic and least squares. The lower panel of this chart shows the net

[34] The information in the chart and in the regression equations is also presented in Table 2 as a schedule showing the cost associated with different levels of output.

CHART 4

Partial Regressions of Total, Average, and Marginal Combined Cost on Output



relation of cost to average weight of belting when the influence of output measured in square feet has been removed.[35]

TABLE 2

Estimates of Combined Cost in Dollars
for Different Levels of Output Measured in Square Feet

OUTPUT	TOTAL COST	AVERAGE COST	COST OF INCREMENT IN OUTPUT
40,000	33,770	.844	
60,000	49,168	.819	15,398
80,000	64,566	.807	15,398
100,000	79,964	.800	15,398
120,000	95,362	.795	15,398
140,000	110,760	.791	15,398

If both average weight of belting and number of square feet are introduced as independent variables, the cost function can be expressed in the form of a multiple regression equation which shows the value of total combined cost associated with various combinations of output and average weight. This equation, determined by the method of least squares, is:

$$_tX_c = -60.1780 + 0.770\,X_2 + 70.181\,X_3$$

By making use of this equation, it is possible to arrive at estimates of total combined cost that would be expected for different combinations of the variables, output and average weight. An example of the use of the equation for this purpose is shown in Table 3, in which a number of estimates have been drawn up.[36] The standard error of estimate of cost in this equation was found to be $1,050. This figure must be interpreted subject to the limitations of time series data mentioned above. If variations from the equation are random, an estimate of cost based on the

TABLE 3

Estimated Total and Average Combined Cost in Dollars
for Various Combinations of Output in Square Feet
and Average Weight in Pounds per Square Foot

	AVERAGE WEIGHT					
	.86		.89		.92	
OUTPUT	Total	Average	Total	Average	Total	Average
40,000	30,974	.774	33,079	.827	35,185	.880
60,000	46,372	.773	48,477	.808	50,583	.843
80,000	61,770	.772	63,875	.798	65,981	.825
100,000	77,168	.772	79,273	.793	81,379	.814
120,000	92,566	.771	94,671	.789	96,777	.806
140,000	107,964	.771	110,069	.786	112,175	.801

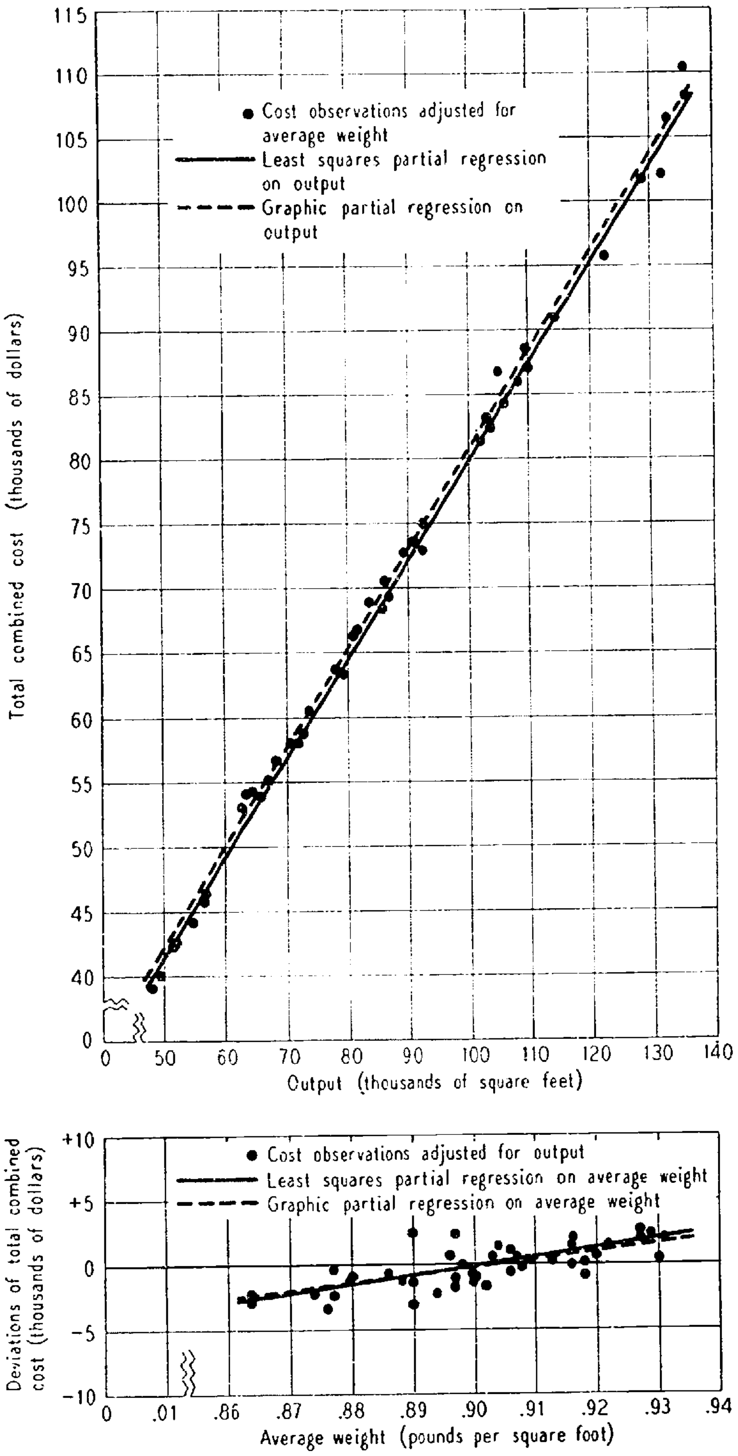
35 The net relation of cost to average weight was established more precisely in the partial regression equation derived by least squares regression analysis:

$$^tX_c = 7.394 + 70.18\,X_3$$

36 For forecasting purposes, all estimates obtained from such a regression equation would have to be modified to take into account changes in wage rates, prices of materials, and tax rates. This type of adjustment is discussed below.

CHART 5

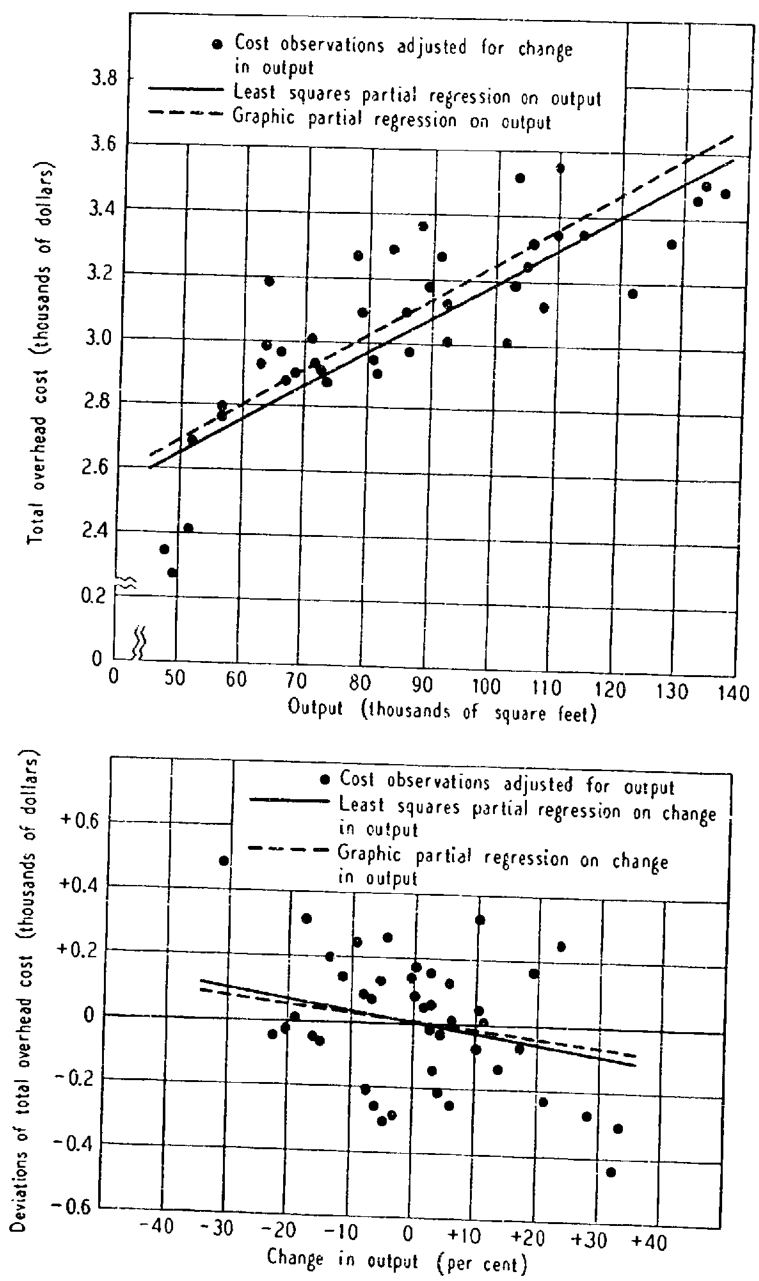
Partial Regressions of Total Combined Cost on Output and Average Weight



regression equation would, in approximately two cases out of three, lie within a range of plus or minus $1,050 of the actual cost for the period. The coefficient of multiple correlation (i.e., the simple correlation between actual cost and cost estimated from the net regressions) was 0.998,

CHART 6

Partial Regressions of Total Overhead Cost on Output and Change in Output from Preceding Period



and the coefficient of multiple determination, 0.997. If the sample were random, a coefficient of multiple determination of 0.997 would indicate that almost the entire variance in cost was accounted for by the variance of the independent variables.

TABLE 4

Estimates of Overhead Cost in Dollars for Different Levels of Output in Square Feet

OUTPUT	TOTAL OVERHEAD COST	AVERAGE OVERHEAD COST	COST OF INCREMENT IN OUTPUT
40,000	2,541	.064	216
60,000	2,757	.046	216
80,000	2,973	.037	216
100,000	3,189	.032	216
120,000	3,405	.028	216
140,000	3,621	.026	216

Analysis of cost components

The two components of the combined cost of the leather belt shop, direct cost and overhead cost, were analyzed in the same way. The findings for overhead cost ($_tX_0$) are presented first. The two independent variables considered are output measured in square feet and the percentage change in output from the preceding month (X_4). The relation of overhead cost to output (after allowing for the effects of the percentage change in output from the preceding month) is illustrated in the upper panel of Chart 6 by net regression lines derived by both graphic and least squares multiple correlation.[37] The independent effect of the magnitude and direction of change from the output of the preceding month (when the effects of the current rate of output are allowed for) is shown by the net

TABLE 5

Estimates of Overhead Cost in Dollars for Various Changes in the Level of Output in Square Feet from Preceding Month

PERCENTAGE CHANGE IN OUTPUT	TOTAL OVERHEAD COST
−30	3,252
−15	3,180
0	3,108
+15	3,036
+30	2,864

37 The meaning of these regression curves can be illustrated by computing the total, average, and marginal overhead cost at different levels of output. The magnitudes of these costs for a hypothetical set of output rates are shown in Table 4. In addition, the total overhead cost associated with various percentage changes in output from the preceding month can easily be calculated (Table 5).

regression line in the lower panel of Chart 6.[38] From this chart it is clear that the relation of overhead cost to the two independent variables is linear, although considerable dispersion of the individual observations about the regression lines is evident.

Partial regression curves for total direct cost $({}_tX_d)$, showing its relation to output and to average weight, determined by both graphic and least squares analysis, are presented in Chart 7.[39] A numerical illustration of the meaning of these regression curves is presented in Tables 6 and 7.

TABLE 6

Estimates of Direct Cost in Dollars for Different Levels of Output in Square Feet

OUTPUT	TOTAL DIRECT COST	AVERAGE DIRECT COST*	COST OF INCREMENT IN OUTPUT
40,000	30,396	.760	15,198
60,000	45,594	.760	15,198
80,000	60,791	.760	15,198
100,000	75,989	.760	15,198
120,000	91,187	.760	15,198
140,000	106,385	.760	15,198

* The intercept of the total direct cost curve on the cost axis is so small that there is no perceptible variation in average cost.

TABLE 7

Estimates of Direct Cost in Dollars for Different Average Weights in Pounds of Belting

AVERAGE WEIGHT	TOTAL DIRECT COST	COST OF INCREMENT IN WEIGHT
.86	64,675	
.88	66,062	1,387
.90	67,448	1,386
.92	68,834	1,386
.94	70,221	1,387

[38] The partial regression equation of overhead cost $({}_tX_o)$ on output (X_2) is:

$${}_tX_o = 2.109 + 0.001\ X_2$$

The partial regression equation for magnitude and direction of change in output (X_4) is:

$${}_tX_o = 3.108 - 0.004\ X_4$$

The multiple regression equation for the overhead cost of the belt shop, when both X_2 and X_4 are variable, is:

$${}_tX_o = 2.158 + 0.001\ X_2 - 0.005\ X_4$$

[39] The equation relating cost to output (X_3 allowed for) is:

$${}_tX_d = 0.663 + 0.760\ X_2$$

When the effect of output measured by surface area is allowed for, the independent relation of cost to average weight is:

$${}_tX_d = 5.06 + 69.32\ X_3$$

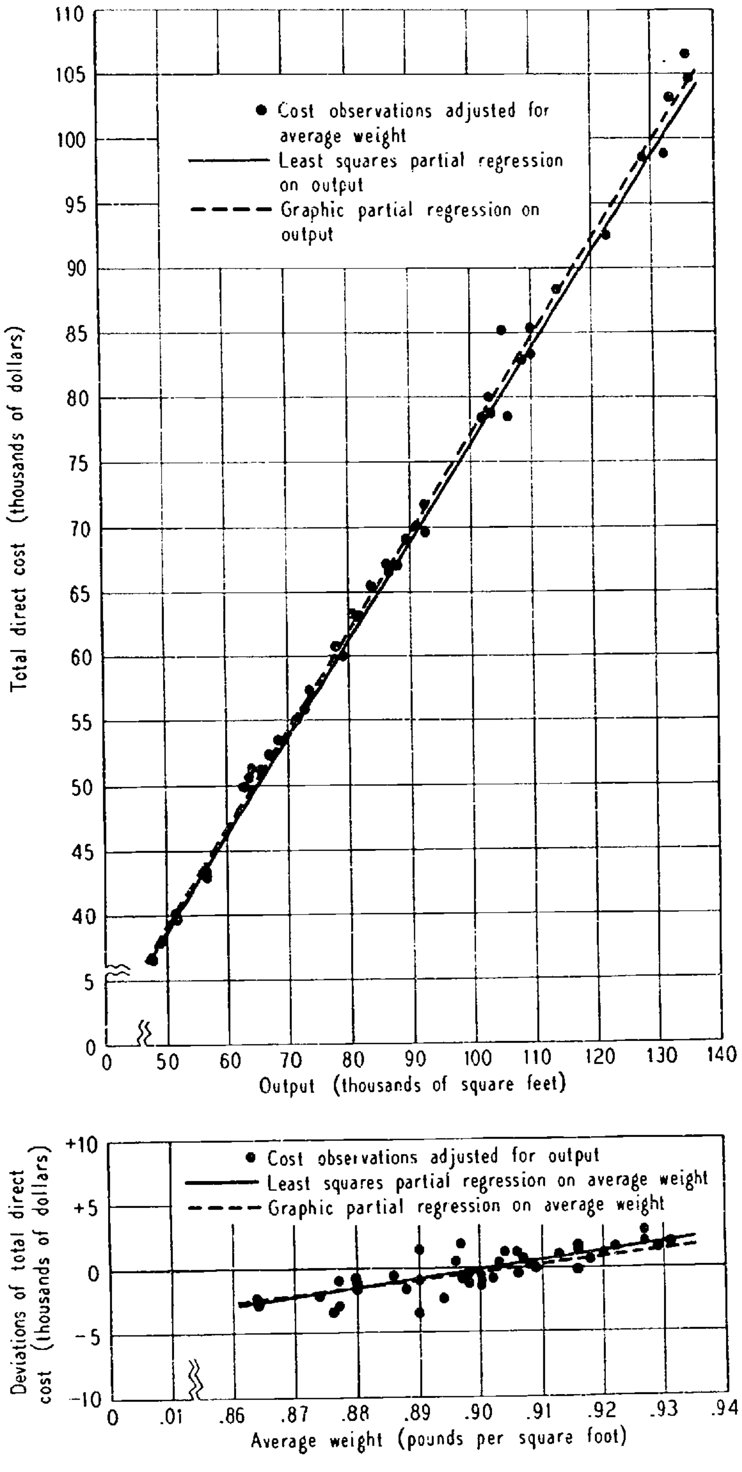
The combined effect of output and weight upon total direct cost is expressed by the multiple regression equation:

$${}_tX_d = -61.639 + 0.760\ X_2 + 69.324\ X_3$$

The standard error of estimate for this equation is approximately $985, the coefficient of multiple correlation, 0.999, and the coefficient of multiple determination, 0.997.

CHART 7

Partial Regressions of Total Direct Cost on Output and Average Weight



Analysis of cost elements

The elements of cost making up the cost components, direct and overhead cost, were examined separately by graphic methods to determine their behavior at different levels of leather belt shop activity. The relations established by this graphic analysis are summarized in Table 8. The various regression lines are illustrated in Chart 8.

TABLE 8

Regression Equations for Elements of Cost on Output Determined Graphically

COST ELEMENT	REGRESSION EQUATION
Direct	
Cement	${}_1X_d = -.06 + .0282\ X_2$
Direct labor	${}_2X_d = .99 + .0538\ X_2$
Leather	${}_3X_d = .90 + .675\ X_2$
Overhead	
Fixed charges *	${}_1X_0 = .905 + .00143\ X_2$
Indirect labor	${}_2X_0 = .249 + .00207\ X_2$
Repairs	${}_3X_0 = .0516 + .00120\ X_2$
Supplies	${}_4X_0 = .098 + .00380\ X_2$

* Taxes, depreciation, insurance, power and water.

Particular interest attaches in this study to the behavior of marginal cost, defined above as the cost attributable to an increment in output of one square foot of single-ply belting. Estimates of this marginal combined cost derived from the regression equation for total combined cost, and

TABLE 9

Estimates of Marginal Cost in Dollars by Alternative Methods

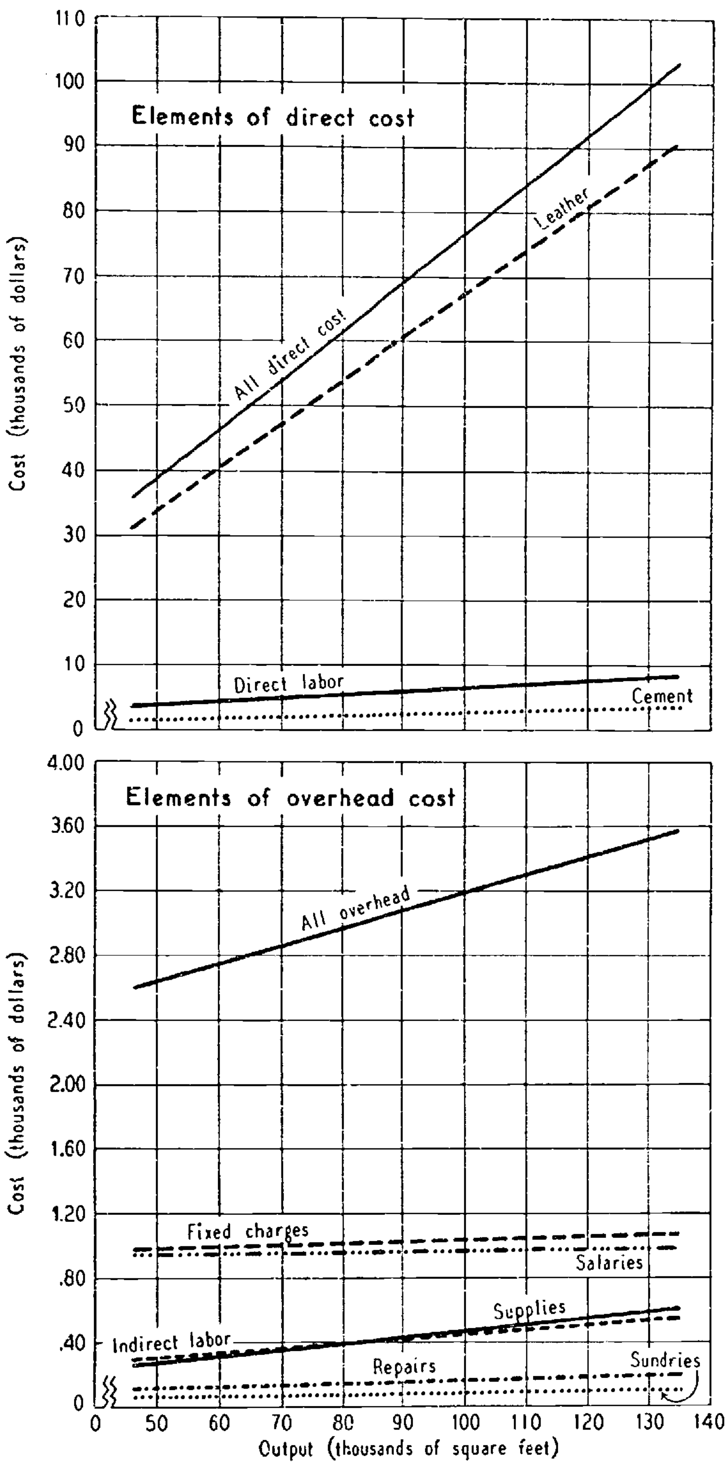
COST ELEMENT	MARGINAL COST OF A SQUARE FOOT	MARGINAL COST OF A POUND*
Cement	0.28	.0251
Direct labor	.054	.0485
Leather	.675	.6063
Sum of elements of direct cost	.757	.6809
Direct cost (estimated independently)	.760	.6826
Fixed charges	.001	.0009
Indirect labor	.002	.0018
Repairs	.001	.0009
Supplies	.004	.0036
Sum of elements of overhead cost	.008	.0072
Overhead cost (estimated independently)	.011	.0099
Sum of all elements of direct and overhead cost	.765	.6881
Combined cost (estimated independently)	.770	.6916

* The marginal cost of a pound was obtained by multiplying the marginal cost of a square foot by a constant representing the number of pounds per square foot. This alternative index of output is not to be confused with the variable, average weight per square foot, included in the analysis.

from the regression equation for the individual elements of cost, are presented in Table 9. In addition, a comparison is made of the analogous marginal cost that results from a unit increase in average weight esti-

CHART 8

Regressions of Elements of Combined Cost on Output



mated by different methods. It is seen from the table that marginal combined cost and the marginal cost resulting from unit increments in average weight are approximately the same whether estimated directly or by summation.

Behavior of 'reflated' cost

In establishing the functional relation of cost to output, the prices paid for materials and labor were held constant during the period of analysis. If, however, such statistical functions are to be useful for cost forecasting, as guides to price policy, and in determining whether the cost incurred in any period differs from the general pattern of behavior, prices of input factors appropriate to the period must be substituted for the 'deflated' or stabilized prices used in the analysis. Fortunately, such a computation is relatively easy since if the cost of any group of elements for a given set of prices is known, the physical quantities of the factors can be determined. The magnitude of the elements of cost appropriate for another set of prices can then be found by multiplying the quantities by the appropriate prices.

Chart 9 shows marginal cost 'reflated' to reflect the prices actually existing in the period. The rough similarity between the fluctuations of 'reflated' marginal cost and those of recorded average cost arises from the predominant importance of leather cost in both. The departures from similarity, attributable mainly to fluctuations in output (also shown in this chart) reflect the inverse relation between output and the proportion of fixed cost to recorded average cost.

7 Validity of Observed Relations

Some potential sources of error that might influence the statistical results have already been discussed briefly. In order to appraise the validity of the statistical findings, we now examine in more detail their limitations, which may be attributable either to inadequacies inherent in the data or to the technique of analysis. The following considerations may conceivably have an important bearing upon the reliability of the findings of this investigation: (1) The sample may be inadequate, the observations not being representative, particularly for high output. (2) Certain cost elements that bear some relation to output were omitted, for example, allocated general firm overhead. (3) The rectification procedure may have errors and shortcomings, such as improper allocation of cost to time periods, elimination of price changes that may have resulted from variation in the plant's output rate, and the impossibility of eliminating non-random errors in the data. (4) Sufficient account may not have been taken of all operating conditions that influence cost; specifically, the rejection of certain independent variables in the multiple regres-