of operations was measured by output and also by labor input. Since this analysis showed no significant partial regression of overhead on the activity of the belt shop, the absence of marginal general overhead cost was indicated. However, in the measurement of combined cost, certain elements of overhead cost as well as all elements of direct cost were included.

Although combined cost was of central theoretical and practical significance, it was found desirable to study also the behavior of individual constituents of cost. Combined cost was first broken down into two components, 'direct' and 'overhead' cost. The terms 'direct' and 'overhead' cost refer to accounting classifications and should not be confused with the economic categories of fixed and variable cost. The distinction between direct and overhead cost is based upon the ease of identifying cost with the particular units of output that give rise to it, whereas the distinction between fixed and variable cost depends upon whether cost varies with changes in output. In the plant studied none of the overhead cost was completely fixed.

The components of cost were broken down still further into their elements. Since the forces affecting cost vary in their impact upon different elements, separate correction of each element was necessary to remove irrelevant variations caused by changes in the prices of input factors and lags in recording. Furthermore, separate analysis of the behavior of the elements provides a basis for setting cost standards and flexible budgets in sufficient detail to be managerially useful. This detailed knowledge also makes possible more specific and exact allowances for changes in factor prices or minor alterations in the technique of production. Individual analyses were carried out, first, for direct cost and its elements: direct labor, leather, and cement; and second, for overhead cost and its elements: indirect labor, supplies, repairs, depreciation, taxes, insurance, salaries, sundries, dies and rings, heat, light and power, and water.

## 4 Rectification of Data

Rectification of the data is designed to eliminate influences that tend to obscure the true relation between cost and output. Since the influences on cost (apart from output) affect the various elements of combined cost differently, composite correction is unlikely to be accurate. For this reason it was found desirable to use specialized rectification devices for the various elements of cost.[9] There are two sources of distortion that necessitate rectification: (1) the time lag between the recording of cost and

[9] This procedure is substantiated by the experience of Ehrke and Schneider in their statistical analysis of cost in a cement mill. Their correction for price changes in the factors was first undertaken by using the *Grosshandels Preisindex*. Finding this unsatisfactory they constructed a special index for the prices of labor, limestone, clay, coal and coke. See Kurt Ehrke, *Die Uhererzeugung in der Zementindustrie* (Jena: Gustav Fischer, 1933), §2, Die Verbesserung der Daten, in the statistical part.

of the associated output, (2) variations in the prices of raw materials, labor, and other factors of production.

*Time lag*

Rectification of the time lag between the recording of cost and of the output causing it ordinarily involves two steps: (1) the determination of the proportion of cost recorded in a period other than that in which the corresponding output is recorded; (2) the determination of the length of the recording time lag. Sometimes these magnitudes are readily found, but it is usually necessary to have recourse to estimates based on technical considerations and engineering opinions. These estimates can be supplemented by statistical analysis designed to test objectively the correctness of the engineering calculations.

The recording of the cost of machinery repairs, supplies, and cement is subject to a time lag sufficient to warrant attention.[10]

The amount of machinery repairs seems to bear a fairly definite relation to output. Information from operating executives indicated that in any given accounting period about one-fifth of machinery-repair expenditures were for minor replacements necessitated by current production, which, therefore, are likely to be recorded in the same period. The larger fraction, although to some extent attributable to the mere passage of time, was caused primarily by output and could be allocated to the production activity of approximately three months earlier. Since the influence of output on repair cost is cumulative and somewhat fortuitous, however, there are wide fluctuations in machinery repairs from period to period.[11] Accordingly, the corrected series of machinery-repair data is composed of one-fifth of the current figure and four-fifths of the cost three months later.[12]

The cost of supplies, in contrast to machinery repairs, is recorded in advance of the output for which they are destined, since usually supplies are not entirely used up within the period in which they are purchased.

[10] During the observation period machinery repairs, supplies, and cement together constitute 4.22 per cent of combined cost.

[11] A method of obtaining a corrected series by means of moving averages was suggested but not used, because the fluctuations followed a pattern that would lead to considerable distortion of the values in parts of the series.

[12] Roy W. Jastram has pointed out to us that this rectification procedure may conceal to some extent any tendency to increasing marginal repair cost. We used a constant ratio for distributing repair cost between the current month and the third month preceding, although the ratio may be expected to vary with the ratio that current output bears to earlier output. The nature of repair expenditure makes it impossible to allocate it accurately. If there is a tendency to concentrate repairs in slack periods repair cost for high rates of output would be understated. The approximation used in this study is admittedly somewhat arbitrary, but seems better than no reallocation. Repair cost, however, constituted so small a percentage of combined cost (0.23) that significant alteration of the form of the marginal cost function by an elaboration of the method seems unlikely.

Analysis of the elements of the cost of supplies, supplemented by opinions of executives, showed that an average lag of one accounting period existed for approximately two-fifths of the recorded expenditure. Consequently, three-fifths of the book figure was combined with two-fifths of the cost in the preceding period to give a set of corrected data from which, it is believed, the recording error was largely removed.[13] The distortion caused by the lag in recording cement cost was recognized but could not be removed. The estimate of the cost that was used was the sum of the value of the inventory of the preceding month and the difference between the value of the inventory at the end of a given month and the total of the invoices for cement purchased in that month. An error in the allocation of this cost arises when invoices for cement consumed in one month are received in the following month. Its magnitude is indicated by the great variation in cement cost expressed in terms of cents per pound of finished product, a quantity that should remain fairly constant because actual cement cost may be expected to be approximately proportional to output. An attempt to offset the error by relating the output of each period to the cement cost of the preceding period yielded less satisfactory results than the use of corresponding months, probably because merely a fraction of the expenditure is incorrectly recorded. Since there seemed to be no way of segregating the wrongly allocated portion, rectification was abandoned.[14]

Part of the irrelevant variation in leather cost attributable to the lag in recording was removed by using the quantity of material charged out of the cutting room into production rather than the quantity charged into the cutting room. Since the cutting department constitutes a reservoir storing widely fluctuating amounts, the quantity of materials entering the cutting room is more remotely related to output than the quantity supplied by it.[15]

*Changes in wage rates and material prices*

To obtain empirical cost functions analogous to the static theoretical functions described above it is necessary to hold the prices of input factors constant at some base level. Two assumptions are implied in this formulation: (1) that substitution among the input factors did not take place as a result of changes in their relative prices; (2) that changes in

[13] This admittedly crude rectification was necessitated by the lack of records of supplies charged to production or of monthly inventories of supplies. The resulting error is unlikely to affect the findings greatly since purchases of supplies averaged only 0.61 per cent of combined cost for the period of analysis.

[14] The degree of possible error is indicated by the fact that cement cost is only 3.62 per cent of direct cost.

[15] There seemed to be no possibility that cutting room work was a function of the output rate of the leather belt shop. Consequently, this treatment of leather cost did not distort the findings concerning the relation of cost to output.

the output rate of the enterprise exerted no influence on the prices paid for its factors.

Examination of the technique of production indicates that the first assumption is justified for the period studied. Whether the second assumption represents the actual circumstances depends upon the conditions under which the output of the particular enterprise fluctuates. Professor Viner has distinguished three kinds of change in competitors' output that affect the relation between the firm's rate of output and the prices of factors: (1) if the change in the firm's output is accompanied by offsetting changes in competitors' output, the industry's demand for input factors remains unchanged; (2) if there is no change in competitor's output, the industry's demands for input factors is increased only by the firm's increase; (3) if the changes in the firm's output are paralleled by changes in competitors' output, substantial increases in the industry's input demand accompany the firm's increased output. The third type of expansion seems most probable in a mature production goods industry of the type under consideration. However, the influence of changes in industry output on factor prices depends on the extent to which factors are specialized to the industry. Since the leather belting industry accounts for merely a small part of the total demand for the principal input factors, it is unlikely that its expansion would be sufficient to induce variations in input prices.

Whether or no it is correct to assume that the firm's rate of output is of negligible influence on factor prices, the only practical approach to the determination of the firm's cost-output functions is to exclude entirely the effects of industry adjustments. Although influences attributable to changes in the output of the industry may have been represented in the observations, these could hardly be disentangled and purged by multiple correlation procedures. It appears preferable to investigate the effect of factor price changes on marginal cost by a method explained in Section 6 where the analysis of cost components is discussed.

For each accounting period variations in total cost arising from changes in the prices paid for factors were partly eliminated either by deflating the costs affected in order to render them comparable with costs at base year prices or by substituting for the prices actually paid an average monthly price for the years studied. The second procedure was applied when more precise rectification seemed unpractical or inadvisable.[16]

Direct and indirect labor, salaries, and cement were corrected by deflating the recorded expenditures to correspond to rates and prices of

[16] The second procedure would be equivalent to ignoring the stabilized items in the correlation analysis, which is justifiable only if no relation exists between them and the independent variables. Since there was no proof of this independence, the charges were included at a uniform figure in the original computations.

the base period. Leather cost, depreciation, insurance, and taxes, on the other hand, were rectified by charging them to production at an average rate for the entire period.

### a) DEFLATION

For direct and indirect labor and salaries an index was constructed from records of actual wage rate and salary changes from the rate existing on January 1, 1935. Since all wage and salary modifications were general, with the exception of a few salary adjustments resulting from reorganization of the executive personnel, the computation was relatively simple.

Of the three constituents of cement cost — film, solution, and liquid cement — only the second and third varied sufficiently during the period studied to necessitate correction. The indexes used were weighted arithmetic averages of price relatives, with January 1935 as the base period and with the proportions of the average 1935 value for each element as weights.[17] The relatives were computed from monthly average prices for each element.

### b) STABILIZATION

The cost of leather for each accounting period was computed on the basis of a uniform price per pound. Inventory price variations of leather reflect fluctuations in the price of hides, changes in the operating cost of processing departments preceding the belt shop, and alterations in the proportions of different qualities of leather going into output. Qualitative differences in this raw material apparently exerted no significant influence on the cost relations under consideration. The most appropriate method of rectification seemed to be the use of an average leather price for the period.[18]

Depreciation was held constant at the average monthly depreciation charge for the period. This procedure, by arbitrarily preventing depreciation from affecting the position or shape of the marginal cost function, may impair, to some extent, the validity of the findings. Ideally, use-depreciation should be separated from time-depreciation, since only

[17] Constant weights were considered satisfactory on *a priori* grounds as well as on the basis of empirical evidence which supported the belief that the physical proportions were kept fairly constant by the technical requirements of production. In addition, the relative importance of the cost elements thus rectified did not seem to justify construction of a more refined index.

[18] Since the firm operates its own curriery, it might be thought that, if higher currying cost and consequent higher leather cost arose from increases in the level of operations in the curriery, and if the rate of operations of the curriery were determined by the activity of the belt shop, such variations in leather cost should not be eliminated from the analysis. In the present instance, a large proportion of the leather used did not come from the firm's own curriery. Furthermore, because of storage facilities, the activity of the curriery was not highly correlated with that of the belt shop. Even if it had been considered desirable to include leather cost variations, however, our findings would remain unaffected, since an analysis of curriery cost showed marginal cost to be constant.

that part of depreciation which arises from the actual operations of a plant is relevant in determining the cost occasioned by different levels of operation.[19] The shape of the marginal cost function depends upon whether use-depreciation is a linear, increasing, or decreasing function of intensity of utilization. This relation as well as the magnitude of use-depreciation depends upon maintenance standards and upon the effects of uninterrupted high speed utilization upon the deterioration of equipment. Depreciation caused by physical deterioration due to the passage of time and by losses in value as a result of technological progress or changes in product specification (obsolescence) affects merely the height of the intercept of the total cost function on the cost axis, not the shape of the function itself.

Unfortunately, from the accounting records, in which depreciation was charged on a 'straight-line' basis, i.e., as a linear function of time, time- and use-depreciation could not be differentiated. The month to month differences in depreciation that did occur in the records arose from arbitrary annual changes in the depreciation rate made to correct for past errors and to adjust for past or expected profits.[20] The stabilization of the depreciation rate at its average monthly value was intended to remove these accidental and irrelevant variations. This procedure understates marginal cost only if, apart from time-depreciation, there occur significant losses of value arising from use after maintenance expenditures have been incurred.

The costs of taxes and insurance were likewise stabilized at their average values for the entire period, again on the hypothesis that their variations are unrelated to the quantity of goods produced each month. The state excise tax, which constitutes merely a small portion of the tax bill, is alone proportional to output. Most of the change in monthly totals is a result of small changes in the annual tax rate, for which refined correction would not be worth while. Insurance cost also varied primarily

19 Use-depreciation may be defined as the loss in value of productive assets not offset by maintenance that is in excess of time-depreciation. Use-depreciation may be zero or negative as well as positive. It will be zero if the loss in value occasioned by physical deterioration due to the passage of time is not increased by more intense use. For example, an automobile body die, which will be rendered obsolete in one year by a planned change in design, may have zero use-depreciation if no conceivable rate of production could diminish its efficiency or hasten its scrapping. This will be true even if physical deterioration results from use, provided its effectiveness and scrap value is unaffected. Use-depreciation will be negative if the loss in value is greater when the equipment is idle than when it is in use. Since the loss in value attributable to use may be reduced or completely balanced by maintenance, use-depreciation represents only the loss in value not restored or avoided by maintenance. To the extent that productive assets are fully maintained in the sense that no residual loss in value results from use, use-depreciation, as we have defined it, may be neglected in estimating marginal cost.

20 Such adjustments amount to a rough, arbitrary assignment of use-depreciation which is unsuited to our purposes.

because of changes in annual rates which were again unrelated to monthly changes in output or other operating conditions.[21]

*Unrectified errors*

Several elements of cost were left wholly or partly unrectified, even though their magnitudes were influenced by some irrelevant variation.

The small relative importance of the cost of dies and rings and the difficulty involved in rectification justified the omission of any correction for this cost. Fluctuations in the cost of supplies arising from price changes were ignored both because of the minor importance of the cost and because of the labor involved in correcting for the great diversity of products recorded in the supplies account.

The book figures for water, heat, light, and power were also used. The water, heat, and light data did not appear to need correction, and only a small part of the variation in power cost could be considered irrelevant. It might have been desirable to remove the fluctuations in the cost of power caused by changes in temperature and number of hours of daylight in different periods, but the complexity of any suitable corrective device indicated that attempts at rectification would not be worth the trouble.[22]

## 5 Methods of Analysis

*Selection of Technique*

Multiple regression analysis seemed most suitable for investigating the relation of the rectified cost to output and the other operating variables.[23] This approach yields measures of: (1) The relation of cost to each independent variable that influences its behavior after the effects of the other variables have been allowed for,[24] a relation displayed in the form

[21] The minimum coverage was so high and the production cycle so short and uniform that changes in inventory arising from changes in the rate of output did not affect the amount of insurance carried.

[22] Since electricity is produced by the company as a joint product with needed heat and steam, the amount of electricity used may not be closely related to changes in output. Allocations to the various plants are based upon engineering estimates which take account not only of the number of lighting units and the rated power consumption of each machine, but also of the utilization of power plant by-products.

[23] The sample was too small for cross tabulation on a multiple basis in order to reflect the influence of various operating conditions; moreover, well defined measures of any existing relations could not be determined. Confluence analysis did not seem necessary, for reasons discussed later; nor were the factors sufficiently numerous or intercorrelated to justify factor analysis.

[24] The precise meaning of the partial correlation coefficient, such as $Y_{12.3}$, should be pointed out. It measures the closeness of the relation between combined cost ($X_1$) and output ($X_2$) after allowing for the effects of average weight ($X_3$). It shows the correlation between cost and output (as measured by the type of function used) excluding the portion due to the co-variation of cost with weight and of output with weight (as measured by the form of the relation used). Thus it measures the correlation between cost and output which is incremental to any correlation between