

This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: The Relation of Cost to Output for a Leather Belt Shop

Volume Author/Editor: Joel Dean

Volume Publisher: NBER

Volume ISBN: 0-87014-447-2

Volume URL: <http://www.nber.org/books/dean41-1>

Publication Date: 1941

Chapter Title: Theories concerning Static Short-Run Cost Functions

Chapter Author: Joel Dean

Chapter URL: <http://www.nber.org/chapters/c9251>

Chapter pages in book: (p. 4 - 8)

and average cost under a given set of operating conditions (adjustable budgets), (2) the additional cost that must be incurred if output is increased by a small amount. Nor is managerial interest in a statistical analysis of cost behavior confined to its immediate usefulness for forecasting purposes; the techniques used here have a wide applicability in the control of costs and in price policy.

Rigorous empirical investigations of cost designed to compare statistical results with the cost behavior prescribed by economic theory have not been numerous. This is to be attributed to the difficulties of obtaining confidential cost data and of finding firms that meet the requirements specified in the underlying cost theory, i.e., single product, unchanging technical methods, unchanging equipment, etc. Although each industry and enterprise offers its own peculiar problems, the case study reported in this paper illustrates the results that can be obtained by statistical analysis of accounting records as well as the problems encountered in determining these results. It is hoped that this description and illustration of analytical methods that have proved valuable in several similar investigations will stimulate research in an area that has both scientific and practical importance.

1 Theories concerning Static Short-Run Cost Functions

In the hope that this paper may reach some non-economist readers, we first summarize the fundamentals of short-run cost theory to clarify the basis of later discussion. Underlying the whole discussion of cost theory is the notion of a cost function — a function that shows the relation between the magnitude of cost and of output. The existence of such a function is postulated upon the following assumptions: (1) there is a fixed body of plant and equipment; (2) the prices of input factors such as wage rates and raw material prices remain constant; (3) no changes occur in the skill of the workers, managerial efficiency, or in the technical methods of production. It is the shape of this cost function that is of primary interest in both the theoretical and statistical parts of this paper.

Money expenses of production depend upon the prices and quantities of the factors of production used. Since prices are assumed to remain unchanged, the shape of the cost function will be determined by the physical quantities of the factors used up at different levels of operation. And since these quantities are functionally related to output, their relation to cost can be represented by a cost-output function. Thus the underlying determinant of cost behavior is the pattern of change in the factor ingredients as output varies.

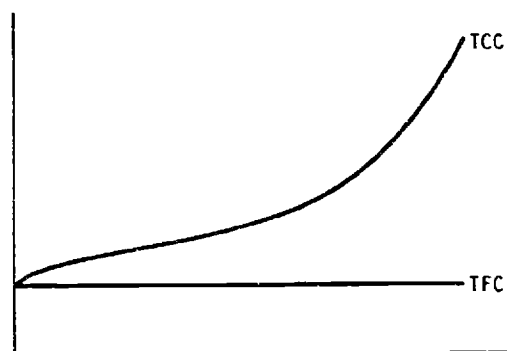
This pattern of change will be determined by the technical conditions of production. In general, when there exist fixed productive facilities to which variable resources are applied, the law of diminishing returns

is assumed to govern the behavior of returns. The mere presence of fixed equipment is, however, not sufficient to ensure the operation of this law unless in addition the productive services that are forthcoming are available only at a fixed rate. The essential question is, therefore, not the physical fixity of the plant but the invariability of the rate of flow of services from the plant.

On the basis of the nature of the fixed productive facilities two technical situations can be distinguished: (1) the services of the fixed productive facilities are available only at a fixed rate, (2) their services can be drawn off at varying rates by bringing these facilities into operation piecemeal. The behavior of cost differs radically as between the two situations. In the first, the law of diminishing returns is fully applicable: marginal returns increase at first and then diminish. The corresponding type of cost function possesses the greatest generality and has usually been considered typical by economists. The behavior of total or combined³ cost, on the above assumption, is shown in Chart 1. Combined cost

CHART 1

Cubic Total Cost Curve



TCC = Total combined cost
TFC = Total fixed cost

is, for analytical purposes, divided into fixed cost and variable cost. Fixed costs, which arise mainly from the firm's investment in fixed plant and equipment, are defined as those expenses whose monthly total is constant irrespective of the output rate. Total fixed cost is shown by the line TFC in Chart 1. Total variable cost, on the other hand, depends on the rate of output and is measured by the vertical distance between the curve of total combined cost, TCC, and of total fixed cost, TFC. In this case

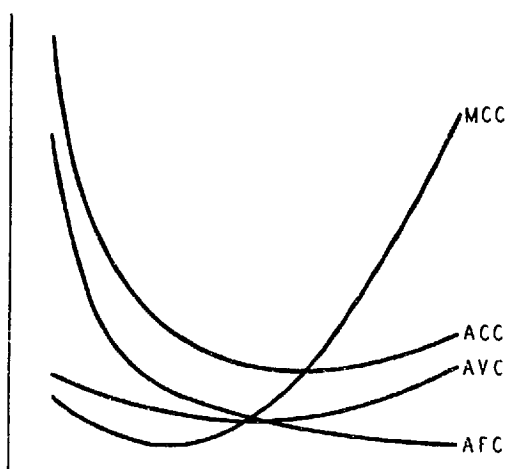
³ Throughout the subsequent discussion combined cost will be used in preference to total cost to refer to the aggregate of the components: overhead and direct cost. In the theoretical discussion combined cost includes all costs incurred, while in the empirical analysis it is less inclusive because of the omission of some cost elements. Total cost will instead refer to the form in which cost is stated—accumulated expenditure for the four-week accounting period—as contrasted with average cost per unit and with marginal cost. Thus combined cost may take several forms: total combined cost, average combined cost, and marginal combined cost.

the total combined cost curve rises first at a decreasing rate and eventually at an increasing rate as output increases.

Fixed cost per unit declines steadily as output increases, the magnitude of average fixed cost being shown by the rectangular hyperbola AFC in Chart 2. Variable costs per unit will be falling at first and then rising as output increases. The curve AVC in Chart 2 describes the postulated behavior of total variable cost. The curve of average combined cost, ACC, derived from the total combined cost curve, is of the same general U-shape as average variable cost, first declining and then rising.

CHART 2

Average and Marginal Cost for
Cubic Total Cost Curve



MCC = Marginal combined cost
ACC = Average combined cost
AVC = Average variable cost
AFC = Average fixed cost

Strictly speaking, the marginal cost curve shows the rate of increase of total combined cost, but approximately it can be taken to indicate the additional cost that must be incurred if output is increased by one unit. The behavior of the marginal cost function is shown by the curve MCC in Chart 2. It has a falling phase in the low range of output and rises as marginal returns begin to diminish. The prominence of marginal cost in economic literature explains our emphasis upon the behavior of this particular function rather than on total or average functions.⁴

The total combined cost function associated with this shape of marginal cost curve can be represented by a great many types of function.

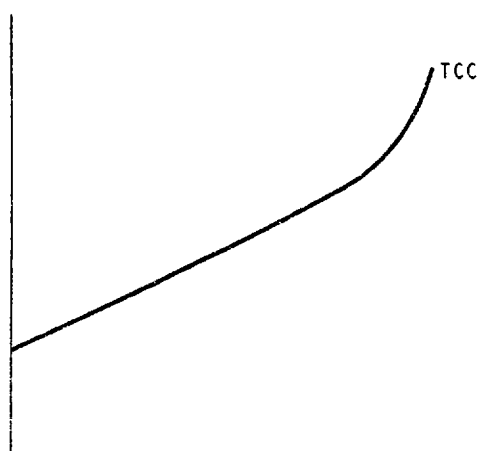
⁴The central position accorded marginal cost follows from a simple principle of maximization. Upon the assumption that the entrepreneur is desirous only to maximize the difference between total revenue and total combined cost, the requisite output is that at which marginal cost and marginal revenue are equal. For the sake of simplicity, the possibility of rivals' reactions are not taken into account.

Since the simplest functional form that possesses the necessary characteristics is probably a third degree parabola or cubic, it is convenient to refer to the cost behavior relevant to this case by specifying the cubic total combined cost function.

The second type of cost behavior mentioned above must now be considered. If the fixed equipment and machinery involved in a process can be broken down into small units, so that each segment can be combined with variable productive services, it may be possible to avoid diminishing returns until all fixed equipment has been brought into operation.⁵ In other words, if each segment is equally efficient, new segments can be introduced whenever returns to the equipment already in use begin to diminish. This means that rising marginal cost can be postponed until all fixed productive facilities have been brought into use. Therefore, over the range of output less than capacity (when 'capacity' means that all segments are in operation) marginal cost will be constant.⁶ The resulting total combined cost curve is illustrated in Chart 3.

CHART 3

Linear Total Cost Curve



TCC = Total combined cost

⁵ There are at least two basic means of segmentation: (1) the use of a series of nearly identical production units, such as machines, so that the rate of output can be altered by varying the number of machines operated; (2) the introduction of flexibility into the time the plant is operated by varying (a) the number of days in a work week, (b) the length of working day, (c) the number of shifts in order to vary the rate of output.

⁶ Professor Viner has pointed out that if the overhead cost of the segments that are intermittently used were allocated to the portion of output they make possible, such an allocation would influence the marginal cost estimates that could be derived from accounting records. The level of marginal cost would be higher, but if each segment were of equal efficiency and its overhead cost were distributed evenly over the block of output to which it contributes, average first differences of cost (incremental cost) would still be constant. If, on the other hand, its cost were allocated to the first unit that required its introduction into the process of production, the marginal cost curve would be discontinuous.

The curve (TCC) is, over most of the output range, a positively sloping straight line whose intercept on the cost axis represents the total fixed cost. At some high level of operation it is assumed that the total combined cost curve ceases to be linear and bends upward. Marginal and average variable cost coincide until the level of output that utilizes all segments is attained; at this output, the two curves diverge. The average combined cost curve lies above the marginal cost curve over the low ranges of output and is eventually intersected by the marginal cost curve at its minimum point.

Two alternative models of short-run cost functions have been considered: in one the total cost curve is a cubic curve; in the other the curve is linear until some extreme level of output is reached. The statistical analysis of cost data that follows is designed to indicate which type of theoretically postulated behavior is consistent with the cost behavior of the plant studied during the period of observation. Before discussing our methods and findings, we examine certain sources of divergence between these theoretical cost functions and their empirical counterparts.

2 Sources of Divergence of Empirical from Theoretical Static Cost Functions

In attempting to determine empirical cost functions that are the strict counterparts of those specified in the static theory of cost, several difficulties were encountered. First, it was not possible to include all costs in combined cost. The difficulty of allocating the cost attributable to jointly produced articles necessitated the omission of certain elements. Furthermore, the cost accounting information available may not be sufficiently accurate to represent faithfully the costs actually incurred. Second, the idealized conditions of production visualized in theory will not be fulfilled in any concrete situation. Not only is it sometimes impossible, for technical reasons, to make the continuous adjustment of the variable factors hypothesized, but also managerial inertia or other rigidities may be sufficient to hinder adjustment to changed conditions. It is perhaps to be expected that theoretical curves intended to be generally descriptive in a qualitative sense of all cost functions will neglect, for the sake of simplicity, the rigidities that may be peculiar to given industries or firms. Rigidities, however small, will be a continuous source of divergence whose influence cannot, practicably, be removed by statistical methods. This divergence is accentuated by the degree of the entrepreneur's knowledge concerning market and technical conditions and his ability or willingness to adjust operations in order to attain for any output the minimum cost combination of factors, the basic assumption on which the theoretical model is drawn up. Nevertheless, an empirical