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# Indexation: Optimality Conditions for Revenue from Inflation and Social Gain

Inflation has always been an appealing method of financing flows of government expenditures in excess of current receipts (explicit taxes plus the net increase in government bonds outstanding). That is, inflation represents an additional form of taxation, which, among other effects, imposes well-known welfare losses. Indexation of a part of the money supply (on the assumption that indexation of currency and demand deposits is not feasible or not desirable) is frequently recommended as one way to eliminate these losses. This proposition usually appears in the context of discussing general indexation from the standpoint of its effects on equity and/or stability. However, indexation is usually restricted to subsets of markets where the manifestations of inflation are particularly visible. Therefore, an alternative line of research might be fruitful, entailing an analysis of the actual effects of indexation when applied to only a particular set of markets while other, related activities are not indexed.

The purpose of this paper is to discuss the conditions under which full indexation of time deposits and other money will be optimal from (1) the point of view of maximizing government revenue from inflation and (2) the social point of view, given a steady and perfectly anticipated rate of inflation. This discussion covers neither the indexation of other financial assets, such as treasury bills, which do not possess characteristics of money

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to a significant degree, nor the case of a growing economy, although the latter can easily be incorporated through a simple extension of the results in this paper.

The argument is developed as follows. First we review the mechanism of adjustment to an exogenous change in the growth rate of nominal cash balances ($\rho$). Then we attempt to determine how the government should set this rate in order to maximize revenue, and how the rate must be changed if indexation of money other than currency is allowed. The last section discusses the optimum rate of indexation from the social point of view.

Here are the main conclusions. In general, full indexation of a money does not lead to the social optimum.[1] Indexation induces increased holding of the indexed money and a reduction in nonindexed moneys. Holdings of money are characterized by a divergence between private and social costs; with downward sloping demand schedules, the social gain from indexation shows decreasing returns for each percentage point increase in the rate of indexation. The social gain will be maximized at a positive rate of indexation smaller than the expected rate of inflation. Full indexation of a money will be socially optimal only in the particular case where the partial elasticities of substitution between indexed and nonindexed moneys tend toward zero.

The government collects inflationary revenue from issuing currency. When a component of money is indexed the optimum rate of inflation (from the point of view of government revenue only) necessarily falls. But general policy implications cannot be derived from this statement. Maximization of revenue entails political risks since such a policy may lead to hyperinflation. With the usual practice of inflating at less than the revenue-maximizing rate, the introduction of indexation increases the rate of inflation required to produce a given (below maximum) revenue. A consistent government, therefore, will only allow indexation of a substitute for currency along with a stabilization program in which total revenue from the inflation tax is planned to fall.

## [I] GOVERNMENT REVENUE FROM INFLATION

Let us start by briefly reviewing the mechanism of adjustment to a once-and-for-all change in the rate of monetary growth. At any point in time, government revenue from money creation is equal to the increase in the real value of its noninterest-bearing liabilities:

$$(1) \quad R_t = c\frac{dM}{dt}\,\frac{1}{P} = c\rho m_t$$

where $M$ is nominal cash balances, $P$, the price level, $C$, the nominal stock of currency, $m$, $M/P$, $c$, $C/M$, $\rho$, $\dot{M}/M$, $\pi$, $\dot{P}/P$, and a dot over a variable denotes a time derivative.

Writing (1) as

$$R_t = c\rho m_t = c\dot{m}_t + cm_t\pi_t$$

emphasizes the nature of the adjustment process. An increase in the growth rate of nominal balances from $\rho_0$ to $\rho_1$ induces a stock adjustment and a flow adjustment. Given a higher expected rate of inflation,[2] people will want to hold a smaller stock of real money balances. Money holders can restore equilibrium by moving out of money and into goods. The excess demand for goods raises the price level and reduces real balances until the yields on money and other assets are equal again, or, alternatively, until the marginal utility of holding money and goods are again equal, at a higher rate of inflation.

If only time deposits are indexed, an increase in the expected rate of inflation will increase the relative attractiveness of time deposits vis-à-vis nonindexed moneys. People will shift from these moneys into time deposits up to the point where the marginal utility of holding the latter equals the marginal utility of holding the former. After all intramoney adjustments, substitution between money and goods will raise the price level as described before.

People will also adjust their flow demand for nominal balances upon a change in the expected rate of inflation. In order to maintain the new desired stock of real money balances, people must add to them nominal balances at a rate of $\dot{M}$ per unit of time. This amounts to a decrease in real disposable income of the private sector (after the inflation tax) and turns out to be equal to the proceeds of the inflation tax collected by the money-issuing institutions.[3]

## [II] OPTIMALITY FROM THE STANDPOINT OF GOVERNMENT

We assume the stock of nonmonetary wealth and the rate of return on this stock to be given in our stationary economy. Defining $T$ and $D$ as time and demand deposits, a total demand for money can be derived, through the solution to the system

(2) $C/P = f(\underset{(-)}{\pi}, \underset{(-)}{i_T}, \underset{(+)}{M/P})$

(3) $D/P = g(\underset{(-)}{\pi}, \underset{(-)}{i_T}, \underset{(+)}{M/P})$

(4) $T/P = h(\pi, i_T, M/P)$
$\qquad\qquad (-)\ (+)\ (+)$

(5) $M/P = C/P + D/P + T/P$

yielding an equation of the form

$$M/P = m(\pi, i_T)$$
$$(-)\ (+)$$

Assume also that the prohibition of interest payments on demand deposits is effective and completely enforced. Under these conditions, the ratio of currency to total money held is a decreasing function of the rate of return on time deposits alone.

We are now able to turn to the discussion of the optimal rate of inflation from the point of view of government revenue. In the absence of indexation, and for a given rate of interest on time deposits, the effect of a change in the rate of monetary growth can be decomposed into two parts:

(6) $$\left.\frac{dR}{d\rho}\right| = cm + c\rho\left(\frac{dm}{d\pi}\frac{d\pi}{d\rho}\right) = cm + c\rho m'\frac{d\pi}{d\rho} = c(m + \rho m')$$

The first term on the right-hand side of (6) shows the change in revenue stemming from the change in the rate of the tax ($\rho$), given the tax base ($cm$). The second term shows the change in revenue due to the adjustment of the tax base to the change in the tax rate. The first term is always positive, while the second is progressively negative with the rate of monetary growth. $R$, therefore, has a maximum for

$$c\rho m' + cm = 0$$

where the maximum revenue rate is given by

$$\rho^* = -\frac{m}{m'} > 0$$

With nominal balances growing at a rate of $\rho^*$, the government is able to maintain a steady deficit of

$$G^* = |\, c\rho^* m(\rho^*)\, | = -c\frac{m^2}{m'} > 0$$

$G^*$ can be interpreted as (1) the maximum sustainable proceeds of the inflation tax, (2) the flow of rents accruing to the government and stemming from its monopoly of currency issue, or (3) the flow of seigniorage profits collected by the government from the steady debasing of nominal currency issued.

$G^*$ will no longer be the maximum sustainable deficit with indexation of time deposits, however. With the rate of indexation defined as

$$\gamma = \theta\rho$$

$c$ becomes dependent on the rate of inflation. Therefore,

$$\frac{dR}{d\rho} = \frac{d}{d\rho}\{\rho\, c(\gamma)m\} = \rho m \frac{dc}{d\gamma}\frac{d\gamma}{d\rho} + \rho c \frac{dm}{d\rho} + cm$$

The maximum $R$ under indexation will be reached with the rate of inflation equal to

$$\rho_\gamma = -\frac{cm}{mc'\theta + cm'} > 0$$

Note that for $\theta = 0$ (no indexation), $\rho_\gamma = -m/m' = \rho^*$ as before. For $\theta > 0$

$$\rho_\gamma = -\frac{m}{m' + (mc'\theta)/c} > 0$$

Therefore, $\rho_\gamma < \rho^*$ and the revenue-maximizing rate of inflation is lower under indexation. Notice, however, that after indexation the actual rate of inflation will be lower only where the government was previously expanding the monetary base at a rate of $\rho^*$. The opposite will necessarily occur if the government was inflating at a rate of $\rho_0 < \rho^*$ prior to the introduction of indexation and it had a target revenue below the maximum revenue. To keep revenue constant under conditions of indexation, the government will have to increase the rate of monetary growth to compensate for the reduction in the real value of the proceeds of the inflationary tax. Thus, there is a possibility of more inflation under indexation, since the discipline of the loss of revenue cannot be imposed on the government.

## [III] THE SOCIAL OPTIMUM

The government deficit $G^*$ is not optimal from the social point of view. The private cost of holding money, $\pi + r$, exceeds the social marginal cost of producing money, which is negligible. This induces a below-optimum desired stock of real money balances and causes a welfare loss.

The welfare loss stems from the substitution of real resources for money in effecting transactions. That is, to produce a flow of monetary services with a suboptimal stock of real balances requires a more intensive use of real resources, and this entails a positive opportunity cost. The source of the loss is the foregone income which is equal to the marginal product of these resources.

Treating the rate of inflation as a tax on real money balances allows us the use of standard public finance tools to find the optimal social rate of indexation of time deposits. We include the real rate of interest as a component of the opportunity cost of holding money and, to find the socially optimal rate of indexation, we assume the real rate of interest to be constant.

Currency and demand deposit holdings are taxed at a rate of $\pi + r$ per unit of time. Indexation makes the rate of tax on time deposits equal to $\pi + r - \gamma$, where $\gamma$, as before, is the rate of indexation of time deposits. Define $\pi + r \equiv T_k$ and $\pi + r - \gamma \equiv T_j$. For a given expected rate of inflation, $T_k$ is constant and $T_j$ varies at the same rate as $\gamma$. A change in $T_j$ changes welfare by

$$(7)\quad \Delta W = \int_{T_j=T_k}^{T_j} T_j \frac{\partial T}{\partial T_j}\, dT_j + \int_{T_j=T_k}^{T_j} T_k \left( \frac{\partial D}{\partial T_j} + \frac{\partial C}{\partial T_j} \right) dT_j$$

where $T$, $D$, and $C$ retain their previous meaning. The first term on the right-hand side of (7) shows the change in welfare due to the introduction of indexation (measured under the demand curve for time deposits), while the second shows the change in welfare due to shifts in the schedules for demand deposits and currency which occur because indexation reduces the cost of holding time deposits. Underlying the use of this measure of social welfare change is the convention proposed by Harberger.[4] A change $dT_j$ causes changes in the equilibrium level of activity $X_k$. When demand price and marginal cost diverge in $X_k$ by the amount of $T_k$, each successive change $dX_k$ entails a social gain equal to $T_k\, dX_k$. The summation of all successive changes $dX_k$ gives the total contribution of the change in welfare in activity $k$ to the total welfare change $\Delta W$. On the other hand, for all related activities where demand price and marginal cost are equal, any change $dX_k$ produces no net contribution to $\Delta W$.[5]

Let us return to expression (7). It can be simplified if we assume linear money-demand schedules which satisfy the summing condition. Under this specification the change in welfare is given by

$$\Delta W = -(1/2)(T_j - T_k)^2\alpha + T_j(T_j - T_k)\alpha + T_k(T_j - T_k)\beta$$

where $\alpha$ is the slope of the demand schedule for time deposits with respect to $T_j$ and $\beta = \Sigma_k \beta_k$ is the sum of the partial terms $\partial D/\partial T_j$ and $\partial C/\partial T_j$.

The optimum rate of indexation $\gamma^*$ is determined through the condition $d(\Delta W)/dT_j = 0$. A reduction in $T_j$ increases the holdings of time deposits and reduces the holdings of currency and demand deposits. But in all three markets demand price exceeds social marginal cost. Therefore, the gain in welfare due to a small change in the rate of indexation ($dT_j$) is equal to $T_j dT$; but this gain is partially offset by welfare losses in the markets for currency ($T_k\, dC$) and demand deposits ($T_k\, dD$), because in these markets demand price exceeds marginal cost, and a reduction in the cost of holding time deposits causes a decrease in the holdings of both $C$ and $D$. And with a downward sloping demand schedule for time deposits, welfare gain increases (but at a decreasing rate) with the rate of indexation.

The optimum rate of taxation on time deposits is

$$T_j^* = -\left(\frac{\beta}{\alpha}\right) T_k$$

But $T_j^* = (\pi + r - \gamma)^*$ and $T_k = \pi + r$. Therefore

$$\gamma^* = \left(\frac{\alpha\beta}{\alpha}\right)(\pi + r)$$

which is necessarily smaller than $\pi + r$ since $\beta$ and $\alpha$ have opposite signs.

Expression (7) admits two limiting cases. For $\beta = 0$ the optimum rate of indexation is equal to the opportunity cost of holding time deposits. In the case of $\beta = |\alpha|$, the optimum policy is no indexation at all, since in this case the marginal losses and gains for a minute change in indexation are equal.

A neat interpretation can be given to expression (7). Noting that

$$\frac{\alpha+\beta}{\alpha} = \frac{\partial M/\partial T_j}{\partial T/\partial T_j} = \frac{M}{T} = \frac{\Delta M}{\Delta T}$$

$$(\Delta T)\gamma^* = (\Delta M)(\pi + r) = (\Delta T + \Delta D + \Delta C)(\pi + r)$$

and

$$(8) \quad \Delta T(\pi + r - \gamma^*) = -(\Delta D + \Delta C)(\pi + r)$$

The term on the left-hand side of (8) shows the marginal gain from one percentage point of indexation, while the right-hand side of (8) shows the marginal loss. The social welfare optimum requires that rate of indexation at which marginal gains and losses balance. Figure 1 depicts this proposition. For simplicity we only draw diagrams for currency and time deposits. We also assume that the terms $\partial I/\partial T_j$ and $\partial C/\partial T_j$ are constant. Different assumptions for these terms can be introduced without changing the argument.

In Figure 1, $OA$ and $O'A'$ are the optimum quantities of time deposits and currency. $OE$ and $O'E'$ are the actual quantities, given $(\pi + r) > 0$. Therefore, the original welfare loss is given by the triangles $ADE$ and $A'D'E'$.

Now let indexation be introduced a step at a time on time deposits. Let the first of these steps be equal to $CF$. Holdings of time deposits increase by $EH$, while holdings of currency fall by $E'E''$. Social welfare increases by $EHGD$ minus $E'E''D''D'$. We can repeat the exercise and increase indexation by $FI$ ($=DF$). Time deposit holdings increase by $HK$ ($=EH$) and currency holdings decrease by $E''D'''$ ($=E'E''$). But while the area $E''E'''D'''D''$ is equal to $E'E''D''D'$ (that is, the loss increases at a constant rate for each percentage point in indexation), the social gain increases at a declining rate (in this case, by $HKJG < EHGD$). Therefore, there is one rate of indexation at which the positive contribution to welfare exactly equals the negative contribution, and this is the optimum rate of indexation.

Therefore, since the private cost of holding currency and demand deposits exceeds the cost of producing these moneys, decreasing the divergence between private and social costs for only time deposits, via

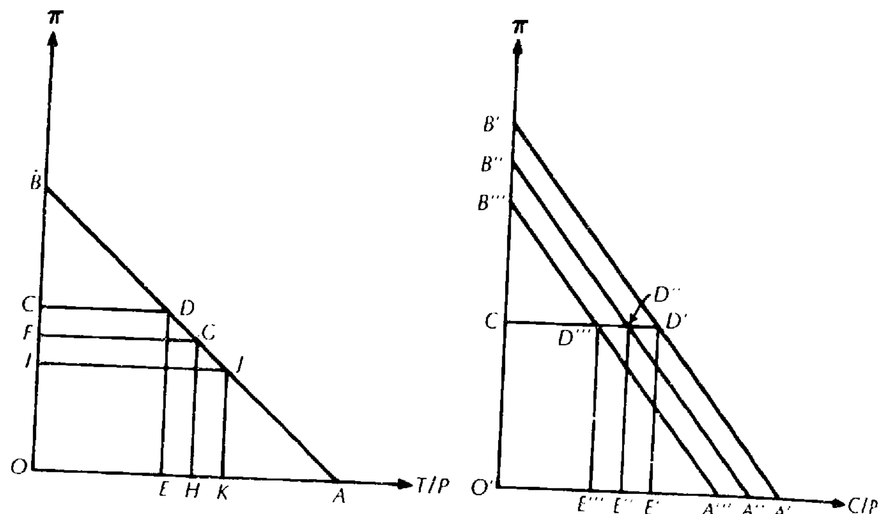


**Figure 1 Welfare Implications of Indexation**

indexation, leads to an overuse of time deposits vis-à-vis nonindexed moneys. Since the latter are "taxed" in full by the rate of inflation and by a positive $r$, standard public finance theory teaches us that a second-best will be obtained by also taxing the holdings of time deposits. The tax rate—i.e., the postindexation opportunity cost of holding time deposits—will depend only on the own-price elasticity of demand for time deposits and on the elasticities of substitution between time deposits and nonindexed moneys.

## NOTES

1. Full indexation means that the nominal value of the indexed asset, at the end of the base period, will be accrued by $\pi$ percent, where $\pi$ is the rate of inflation observed during the base period.
2. Recall that in a steady state the actual and the expected rates of inflation are equal.
3. We ignore government gains (losses) at the point in time the rate of monetary growth is changed.
4. Harberger, A. C., "Three Basic Postulates for Applied Welfare Economics: An Interpretative Essay," *Journal of Economic Literature* 9 (September 1971): 785–797.
5. Harberger, A. C., "The Measurement of Waste," *American Economic Review, Papers and Proceedings* 54 (May 1964): 58–76.