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CHAPTER 16

Investment in Goods in Process

Goods in process comprise the commodities individual manufacturers have begun to fabricate but that are not yet ready for delivery—about 20 percent of manufacturers' holdings (Ch. 7). As argued in Chapter 8, as far as they are held in continuous process industries, that is, in industries in which it is impossible or inconvenient to keep surplus stocks of semifabricated goods over and above the quantity necessary to sustain a certain rate of activity, stocks of goods in process move up and down with the rate of production. And I presented estimates that make it seem likely that these conditions, or a reasonably close approximation to them, characterize a substantial proportion of all goods in process. In such industries stocks of goods in process cannot lag behind production. On the contrary, they are likely to lead. The lead, however, cannot exceed an interval equal to a production period; that is, it cannot exceed the time elapsing between the moment work is begun upon a prospective unit of output in a manufacturing establishment and the time it is ready for delivery. Finally, I estimated that the average length of the production period in manufacturing establishments is unlikely to be less than 15 days and is probably from 20 to 25 days.

Concerning the action of goods in process held in other industries I am less certain. However, since in establishments that combine several stages of production each stage is, so to speak, a 'continuous' industry, at least part of the goods in process must behave like the stocks of continuous industries. Moreover, it is not necessary for the stocks 'between stages' to move inversely during any part of a production cycle. There is only a possibility that they may do so. Hence it seems likely that the behavior of goods in process sketched above is characteristic of a wider range of manufacturing than 'continuous' establishments alone.

Our present concern, however, is with investment in goods in process, that is, with the absolute rate of change in such stocks per time unit, and their relation to business cycles. Now it turns out that the relations between the rate of production and goods in process in continuous industries exist also between the absolute rates of growth in production and in goods in process. The rate of investment in goods in process, we shall find, moves up and down with the rate of increase in output. It will not lag behind the rate of output, and if it leads, as it is likely to do, the lead will not be longer than a production period. If these relations can be established, we can use the pattern of the rate of change in manufacturing production (Ch. 15, Sec. 3) to determine the cyclical pattern of investment in goods in process.

1 Theoretical Relations between Rates of Increase in Output and Investment in Goods in Process

Although we are interested in the relation between output and goods in process, it is more convenient to present the argument in terms of the relation between goods in process and input. Since output must lag behind input by a production period, the relation between stocks and output can be stated by a verbal alteration of the conclusions.

To simplify the argument we assume that a unit of prospective output grows in value at the same absolute rate in dollars from the time work is begun until it is ready for delivery, that is, from the time the unit becomes a part of the stock of goods in process until it becomes a part of the stock of finished goods. Let us denote the number of units put into work continuously on successive days by the symbols a_1, a_2, a_3, and so on; and let us assume that a unit of input is worth x dollars after one day of processing, $2x$ after two days, and so on; finally let us assume that the production period is four days. Let p = the value of goods in process.

Since the value of goods in process on any given day equals the sum of the values attained by the units of input on which work began on the given day and the three preceding days, we may write the value of goods in process at the end of the fourth day after input begins as:

$$1)\quad p_4 = \frac{a_4x}{2} + \frac{a_3x}{2} + a_3x + \frac{a_2x}{2} + 2a_2x + \frac{a_1x}{2} + 3a_1x$$

This expression tells us that the units put into process on the first day, a_1, have, on the average, received three and one-half days' processing by the end of the fourth day, $\frac{a_1x}{2} + 3a_1x$. Goods put into process on the second day have acquired two and one-half days' processing on the average, $\frac{a_2x}{2} + 2a_2x$, by the end of the fourth day, and so on. The total value of goods in process at the end of the fourth day or, in general, any day, is the sum of values acquired by goods put into process during the given day and the number of preceding days that together make up a production period.

Equating x to unity, this expression simplifies to:

$$2)\quad p_4 = \frac{a_4}{2} + \frac{a_3}{2} + a_3 + \frac{a_2}{2} + 2a_2 + \frac{a_1}{2} + 3a_1$$

In the same way, the value of goods in process at the end of the following day is:

$$3)\quad p_5 = \frac{a_5}{2} + \frac{a_4}{2} + a_4 + \frac{a_3}{2} + 2a_3 + \frac{a_2}{2} + 3a_2$$

During the fifth day goods in process have grown by the difference between p_5 and p_4:

$$4)\quad p_5 - p_4 = \frac{a_5}{2} + a_4 + a_3 + a_2 - (3a_1 + \frac{a_1}{2})$$

This expression tells us that between the fourth and fifth days (or any two days) goods in process decline by an amount proportionate to 3.5 times the number of units put into process on the first day of the sequence of five. These are the units that were put into work earliest and in our setup have 3.5 days' processing at the end of the fourth day. At the end of the fifth day all these units have moved into the finished goods category and disappeared from goods in process. On the other hand, goods in process grow by an amount proportionate to the sum of the number of units of goods put into process on the second, third, and fourth days plus half the number put into process on the fifth day. That is, the batches put into process on the second, third, and fourth days have each been given one more day's processing and the goods put into process on the fifth day have, on the average, been given one-half day's processing. The difference between the goods in process at the end of any two days is, of course, simply the difference between the amounts by which goods in process grow and decline in the course of one day.

The fourth equation may be rewritten in more convenient form:

$$5)\quad p_5 - p_4 = a_4 + a_3 + a_2 - 3a_1 + \frac{a_5 - a_1}{2}$$

As long as input increases at an accelerating rate, goods in process also will continue to increase at an accelerating rate. Consider the growth of goods in process on any two successive days during a period when input has been growing at an increasing rate. Let us call these two days, Day 5 and Day 6. The growth of goods in process during these two days may be expressed as:

$$6)\quad \text{Day } 5\text{: } p_5 - p_4 = a_4 + a_3 + a_2 - 3a_1 + \frac{a_5 - a_1}{2}$$

$$7)\quad \text{Day } 6\text{: } p_6 - p_5 = a_5 + a_4 + a_3 - 3a_2 + \frac{a_6 - a_2}{2}$$

The rate of growth of goods in process is increasing if the difference between these two expressions, $p_6 + p_4 - 2p_5$, is positive. If the rate of growth of input is increasing this must be so. First, the sum (in equation 7) of $a_5 + a_4 + a_3$ must, in that case, exceed $3a_2$ by more than the sum (in equation 6) of $a_4 + a_3 + a_2$ exceeds $3a_1$. The difference between the two is $(a_5 - a_2) - 3(a_2 - a_1)$. The difference between a_5 and a_2 is the sum of the following differences: $(a_5 - a_4) + (a_4 - a_3) + (a_3 - a_2)$. And as long as input grows at an increasing rate, each of these three differences is larger than the difference, $a_2 - a_1$. Hence the sum of the three exceeds $3(a_2 - a_1)$. Under the same conditions, secondly, input on Day 6 exceeds input on Day 2 more than input on Day 5 exceeds input on Day 1. That is, $(a_6 - a_2) > (a_5 - a_1)$. This must be true since a_6 exceeds a_5 by more than a_2 exceeds a_1. Therefore $(p_6 - p_5) > (p_5 - p_4)$ and, in general, the rate of investment in goods in process will increase as long as the rate of input is accelerating.[1]

Since this is true, cyclical turns in the rate of investment in goods in process cannot lead turns in the rate of increase in input. Indeed, unless the rate of increase in input falls sufficiently rapidly after it reaches a peak, investment in goods in process will continue to rise for some time longer. This may be seen if we suppose that the rate of increase in input reaches a peak on Day 6. Now compare the growth of goods in process during Day 6 and Day 7. The equations are written in the manner of equation 4.

8) Day 6: $p_6 - p_5 = \frac{a_6}{2} + a_5 + a_4 + a_3 - \frac{7a_2}{2}$

9) Day 7: $p_7 - p_6 = \frac{a_7}{2} + a_6 + a_5 + a_4 - \frac{7a_3}{2}$

The rate of investment in goods in process will be rising if the difference between equations 8 and 9 is positive.

10) $p_7 + p_5 - 2p_6 =$

$$\frac{a_7 - a_6}{2} + (a_6 - a_5) + (a_5 - a_4) + (a_4 - a_3) - \frac{7a_3 - 7a_2}{2}$$

If input has been growing at an increasing rate through Day 6, as we assume, this expression is likely to be positive. For in that event, each of the three differences $(a_6 - a_5)$, $(a_5 - a_4)$, and $(a_4 - a_3)$, is larger than the difference $a_3 - a_2$. Hence the sum of the three larger differences is likely to be larger (it is not necessarily larger) than three and one-half times the smaller difference $a_3 - a_2$. In addition, the increase of investment in goods in process between the two days is bolstered by half the difference of input on Days 6 and 7, which will be positive, except in the unusual case in which input begins to fall immediately after it has reached a peak in its rate of growth. Equation 10 makes it clear also that the less rapid the rate of acceleration of input before the peak in the rate of growth and the more rapid the deceleration after the peak, the

[1] Since investment in goods in process tends to lag behind the peaks and troughs of the rate of growth of input (see the following text), this statement should bear the qualifying clause: *except in the immediate vicinity of a trough in the rate of increase in input.*

shorter will be the lag of investment in goods in process behind the rate of growth of input. On the other hand, the more rapid the acceleration before the peak in the rate of growth and the slower the deceleration after it, the longer will be the lag.

Example 5 illustrates these points. In Part A the rate of acceleration of input before the peak in its rate of growth is exactly the same absolutely as the rate of deceleration after the peak. The peak of growth of goods in process, which comes on Day 8, lags two days behind the peak of growth of input, which comes on Day 6. Part B shows the effect of an acceleration of the growth of input before the peak in the rate of growth that is less rapid than the rate of deceleration after the peak. Goods in process grow most rapidly on Day 7, this time only one day after the peak in the growth of input. Part C reverses these conditions; as a result, the peak in the growth of goods in process lags three days behind the peak in the growth of output.

The cyclical turns in the rate of investment in goods in process, therefore, tend to lag behind those in the rate of growth of input, and the length of the lag varies with the rate of acceleration of input growth before its peak and the rate of deceleration after its peak. Whatever the conditions, however, the lag of investment by any single manufacturing establishment cannot be longer than one production period. We may be assured of this by comparing the factors on which the growth of goods in process depends as they appear three and four days after the peak in the growth of input. As will be remembered, I assume that the production period is four days. If we assume that the peak of input growth was reached on Day 6, we need to examine the growth of goods in process on Days 9 and 10.

11) Day 9: $p_9 - p_8 = a_8 + a_7 + a_6 - 3a_5 + \frac{a_9 - a_5}{2}$

12) Day 10: $p_{10} - p_9 = a_9 + a_8 + a_7 - 3a_6 + \frac{a_{10} - a_6}{2}$

Investment on Day 10 cannot exceed that on Day 9 unless one of two conditions is met: either $a_9 + a_8 + a_7$ must exceed $3a_6$ by more than $a_8 + a_7 + a_6$ exceeds $3a_5$ or else $a_{10} - a_6$ must exceed $a_9 - a_5$. The first condition, however, will be satisfied only if

EXAMPLE 5

Rates of Increase in Input and in Goods in Process

DAY (1)	INPUT (2)	INCREASE OF INPUT (3)	GOODS IN PROCESS[a] (4)	INCREASE OF GOODS IN PROCESS[b] (5)
		PART A		
1	900			
2	910	10		
3	930	20		
4	960	30	7300	
5	1000	40	7450	150
6	1050	50	7680	230
7	1090	40	7980	300
8	1120	30	8320	340
9	1140	20	8650	330
10	1150	10	8900	250
		PART B		
1	815			
2	860	45		
3	906	46		
4	953	47	6838	
5	1001	48	7205	367
6	1050	49	7580	375
7	1090	40	7958	378
8	1120	30	8323.5	365.5
9	1140	20	8650	326.5
10	1150	10	8900	250
		PART C		
1	900			
2	910	10		
3	930	20		
4	960	30	7300	
5	1000	40	7450	150
6	1050	50	7680	230
7	1099	49	7984.5	304.5
8	1147	48	8347	362.5
9	1194	47	8740	393
10	1240	46	9125	385

[a] Calculated by equation 2.
[b] Calculated from col. 4, or by equation 9.

$(a_9 - a_6) > 3(a_6 - a_5)$. But $(a_9 - a_6) = (a_9 - a_8) + (a_8 - a_7) + (a_7 - a_6)$. No one of these differences can exceed $a_6 - a_5$ since input had reached a maximum rate of growth between Days 5 and 6 (by hypothesis). Hence $a_9 - a_6$ cannot exceed $3(a_6 - a_5)$. For the same reason $a_{10} - a_6$ cannot exceed $a_9 - a_5$.

We may conclude, therefore, that investment in goods in process will not lag behind turns in the rate of increase in input by more than one production period. These results are easily transformed into statements about the relation between investment in goods in process and increases in the rate of production. Since the rate of production lags behind the rate of input by one production period, investment in goods in process will not lag behind the rate of increase in output; it is likely to lead, but not by more than one production period.[2] Since the average production period in manufacturing establishments is about three weeks, or perhaps a little longer (Ch. 8), this period defines the maximum lead of investment in goods in process relative to the rate of growth of production.

This conclusion is subject to two qualifications. First, it applies strictly only to goods in process in continuous industries and to goods 'within stages' in other industries. These categories of stocks account for most, but not all, goods in process. Goods 'between stages' in discontinuous industries may or may not act in the fashion described above. When they do not, the effect is probably to cause goods in process in the aggregate to respond to changes in activity somewhat more tardily than they otherwise would. Hence investment in goods in process as a whole is likely to lead the rate of growth of production by less than investment in continuous industries does. It may even lag by a short interval. It seems best, therefore, to say merely that investment in goods in process and the rate of growth of output turn at nearly the same time.

The second qualification is that the relation described above applies directly only to the connection between goods in process and production in individual establishments. It will be strictly true for manufacturing as a whole only if total output weighted by value added in manufacturing turns at the same time as output weighted by the value of goods in process. This is, no doubt, approximately true, but is unlikely to be exactly true (Ch. 8).

2 *Pattern of Investment*

If the conclusions of the preceding section are valid, we may gauge the cyclical timing of investment in goods in process from the be-

[2] These conclusions are independent of the rate at which goods in process grow in value during the production period. The equations and examples in the text make the simplifying assumption that the growth in value is 1 per unit of input per day. As may be seen from the symmetry of the expressions compared in the argument above, however, any figure may be substituted, and may differ from day to day during the production period without affecting the conclusions.

havior of the rates of change in manufacturing production studied in Chapter 15, Section 3. On this basis we may conclude that the rate of liquidation of goods in process increases during the first part of business contractions. The maximum rate of liquidation occurs near the middle of contractions, sometimes earlier, sometimes later, but usually well before the end of the phase.

For expansions there is no simple rule. The rates of change in output suggest that investment in goods in process is usually high at the beginning of expansions. Thereafter the rate of growth of output falls. In some cycles the decline continues to the end of the phase; in others there is a renewed spurt. In some expansions, therefore, investment in goods in process reaches its maximum long before the peak in business and falls toward the end of the phase. In others, after a period of decline, investment turns up again and rises until very near the peak of business.[3]

From these inferences about the timing of investment in goods in process, we may judge its influence on business cycles. In the early stages of expansion and contraction, investment in goods in process intensifies the cyclical movement. But later in contractions it moves against the cyclical tide and the declining rate of liquidation helps revive output and income. In some expansions the second part of the phase is characterized by a lower rate of accumulation which helps bring on recession. In other expansions, however, investment in goods in process revives in the second half of the phase and thus helps sustain the expansion until near the peak in business.

[3] I say 'very near' because the cycle stage used above in measuring the rate of change in output does not permit one to determine timing precisely in the immediate neighborhood of the business cycle reference dates.