Volume Title: Explorations in Economic Research, Volume 4, number 5

Volume Author/Editor: NBER

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/lint77-1

Publication Date: December 1977

Chapter Title: An Essay on Human Wealth

Chapter Author: Lee A. Lillard

Chapter URL: http://www.nber.org/chapters/c9108

Chapter pages in book: (p. 702 - 752)

4

LEE A. LILLARD
The Rand Corporation

AN ESSAY ON HUMAN WEALTH

ABSTRACT: In this paper, I explore both theoretical and empirical aspects of human wealth, which is defined as the discounted present value of an individual's lifetime earnings net of investments in human capital. On the assumption that individuals maximize their human wealth, optimal lifetime investment patterns and the resulting patterns of earnings are developed. Qualitative theoretical predictions are verified using data on a cohort of men on whom earnings information is available over most of their working life. Empirical age-earnings profiles are made to depend on years of schooling, indexes of ability, and family background. Explicit account is taken of permanent but unobserved individual earnings difference. I find that one of the primary predictions of theory is a life-cycle pattern of investments which decline over time and which yield compensating returns later. Both tend to produce individual earnings profiles which are concave and which rise more rapidly for those with larger early investments. These attributes are roughly confirmed by the data considered here. Both more able and more highly schooled individuals, who are presumably investing more, are compensated by more rapidly rising earnings and higher earnings late in the life cycle. I observe substantial variation in human wealth but less inequality there than in earnings within

NOTE: I wish to thank William Haley, James J. Heckman, Sherwin Rosen, James P. Smith, and Finis Welch for their comments on earlier drafts. I am grateful to the reading committee, Robert Michael, Terrence Wales, and Yoram Weiss, for helpful comments and to the NBER Board of Directors reading committee, Lloyd Reynolds, Robert Lampman and Clay La Force. Many valuable ideas were synthesized from discussions in a study group on "Life Cycle Aspects of Earnings and Labor Supply," sponsored by the Committee on Work and Personality in the Middle Years of the Social Science Research Council.

This research was conducted while I was a research associate at the NBER and it was funded by a grant to the NBER from the National Science Foundation (GS-31334) and by a contract from the U.S. Department of Labor (L-73-135). The opinions expressed here are mine and do not necessarily reflect the views of the National Science Foundation or the Department of Labor.

narrow age groups. The coefficient of variation in human wealth is approximately 43 percent compared to 75 percent in earnings and 60 percent within age groups. The direction of inequality is unambiguous. The dominant factor in human wealth inequality is the individual variance component, representing individual unobserved differences. Only 10 to 12 percent of variation in human wealth is due to variation in measured schooling, ability, and background variables. I also find a positive effect of measured ability on human wealth. While ability has a negligible, or even slightly negative, effect on the earnings of young men, the effect becomes positive and larger as the men become older.

INTRODUCTION

Throughout the history of economics there has been a persistent interest in the determinants of earnings and wealth differences among individuals. Until recently, the most successful work in the area was basically empirical and involved devising alternative measures of the dispersion of economic well-being or quantifying the movements in observed inequality among different subgroups or over time periods. The principal intellectual obstacle was the absence of an adequate theoretical framework with which issues relating to the distribution of human earnings and wealth could be analyzed. Interest in the concept of human wealth was rekindled in the early 1960s by Schultz's 1960 presidential address to the American Economic Association and the publication of Gary Becker's *Human Capital*. Ben-Porath (1967) contributed a substantial theoretical innovation by developing a simple but rigorous model of optimal lifetime investment in human capital. The theoretical insights of Becker and Ben-Porath have inspired a host of additional theoretical and empirical studies based on the notion of human wealth maximization and optimal age-earnings profiles.

In spite of the large volume of research on human capital now available,[1] the wealth concept has not been emphasized. Most of the research has been concentrated on the characterization of age-earnings profiles and not on wealth levels inherent in different profiles. Most previous studies considered earnings differences among members of a population at a point in time (a cross section), although comparisons have been made among demographic groups over time. In this study, I emphasize the theoretical and empirical consequences for earnings analysis of taking the lifetime rather than the single-period view. I emphasize the need to use lifetime earnings data to study a lifetime decision problem. Attempts by researchers to incorporate the life-cycle notion include calculation of inequality within narrow age groups and of the present value of cross-sectional lifetime profiles of earnings (for example, see Houthakker 1959 and Wilkinson 1966).[2] Recently available longitudinal earnings data, however, allow estimation of ex post individual human wealth and thus permit a comparison of human wealth inequality with shorter-period earnings inequality.

A primary prediction of all life-cycle human capital investment models is a declining investment profile. This tends to produce concave earnings profiles. The larger early investments are, the more rapidly individual earnings rise. For questions related to the distribution of wealth, the crucial issue is the extent to which differences in earnings patterns are "compensated" in present value: There is no presumption in the human capital model that each individual's maximum wealth should be the same, i.e., that there should be no inequality in human wealth.

Ex post lifetime earnings patterns and human wealth are estimated for a group of men born between 1917 and 1925 (a birth cohort) for whom earnings data are available at several points during their lives. The relationship of lifetime earnings patterns and human wealth to schooling levels, several dimensions of measured ability, and family background is explored. By using longitudinal data incorporating repeated observation of each individual, the variation in permanent earnings differences among individuals can be estimated after adjusting for differences in schooling, ability, and background. The permanent differences among individuals due to unmeasured sources have important implications for variation, and thus inequality, in human wealth.

The primary conclusion to be drawn from this study, and from other related studies I have made, is that within narrow life-cycle ranges, variation among individuals in human wealth, although considerable, is substantially more equally distributed than earnings within age groups, as measured by the coefficient of variation or Gini coefficient. The contributions to variation in human wealth of schooling, measured cognitive ability, and a limited set of background variables are about the same as their contributions to variation in earnings within age groups—roughly 10 to 12 percent. The remaining inequality in human wealth is due to unmeasured factors that create individual differences in earnings which persist over a lifetime.

The empirical life-cycle earnings patterns are largely consistent with the qualitative predictions of the theoretical model. The effect of schooling and ability on the life-cycle pattern of earnings is represented by their interaction with age and with each other. Prior to age thirty, both the more educated and the more able have slightly lower earnings, possibly because of higher levels of job-training investment which in turn causes future earnings to rise more rapidly. Correspondingly, earnings are greater after age thirty. The ability effect on the life-cycle pattern of earnings is found to be due largely to mathematical ability. Given mathematical ability, indexes of general knowledge, mechanical dexterity, and physical dexterity affect earnings additively. Also, the impact on earnings of schooling increases with ability and the impact of ability on earnings increases with years of schooling. While the effect of schooling is larger than the effect of ability at any age, the contribution of schooling to human wealth is much more sensitive to discounting because schooling is associated with a period of forgone earnings. Consequently, measured ability has a positive effect on human wealth that persists even at discount rates sufficiently high to make the return to schooling negative.

I also consider the determinants of schooling and of various types of ability. Partly for theoretical reasons and partly because of the nature of the data used, the empirical model posits a recursive relationship between lifetime earnings, years of schooling attained, and the various ability indexes. Ability and background are determinants of years of schooling, and background is a determinant of ability. These relationships are explored fully in the text, and the overall explanatory power of the predetermined variables is found to be weak.

[I] A MODEL OF HUMAN WEALTH MAXIMIZATION

In this section I develop a simple model of human wealth maximization through investment in human capital. The primary decisions individuals face are how much of their current stock of human capital to allocate to producing additional human capital (via their personal production function), how much to spend on purchased inputs at each point of time, and at what age to stop specializing in the production of human capital (i.e., when to end formal schooling). In the model, newly produced human capital yields returns in future time periods, future earnings are discounted at the market interest rate, and the stock of human capital continuously deteriorates. Wealth-maximizing decisions are influenced by the initial endowment of human capital, the rate at which additional human capital deteriorates, the rental rate of human capital, the price of inputs, and the market rate of interest for borrowing and lending. Each individual chooses an optimal schooling level and lifetime pattern of investment. These decisions then determine a lifetime pattern of earnings, net of investment, with greatest present value. The individual then maximizes his intertemporal utility function subject to this wealth constraint.

It is obvious from the formulation of the model that there will be inequality in human wealth among individuals to the extent that they differ in endowments, constraints, subsidies, and abilities. Individuals will also differ in their corresponding lifetime earnings patterns. The focus of this section is on the sources and consequences of these individual differences.

The basic model, first formulated by Ben-Porath (1967), has become a popular vehicle for detailed refinements of optimal life cycle investment in human capital. Versions of this model, considered by Haley (1973), Johnson (1974), Rosen (1973), and Wallace and Ihnen (1974), have been used to explore implications about lifetime earnings patterns but rarely to investigate the corresponding implications for human wealth inequality. The model represented here differs from previous efforts in several respects. As just mentioned, it is explicitly focused on human wealth as well as the underlying earnings patterns. It develops a comparative statics analysis of human wealth with respect to changes in endowments, constraints, and abilities. It introduces an alternative specification of the market for funds which allows consumption, but not investment, loans, thus permitting a closed form solution for the schooling decision. Otherwise, the new specification leaves virtually all of the previous qualitative predictions of the original Ben-Porath model intact.

The Formal Model

My basic objective here is to derive a set of fairly robust qualitative predictions about life-cycle earnings and levels of human wealth. Individuals are assumed to maximize human wealth:[3]

$$(1) \quad HW = \int_{t=0}^{N} e^{-rt} \left[R \cdot E(t) - R \cdot K(t) - P \cdot D(t)\right] dt$$

subject to the budget constraint

$$(2) \quad R \cdot E(t) - R \cdot K(t) - P \cdot D(t) - g^2(t) = 0$$

and constraints on the rate of change of the capital stock[4]

$$(3) \quad \dot{E}(t) = Q[K(t), D(t)] - \delta E(t)$$

The symbols are defined in Table 1.

The budget constraint implies that direct investments, including both purchased inputs ($P \cdot D$) and forgone earnings ($R \cdot K$), must be financed out of current earnings capacity ($R \cdot E$) and are thus constrained by current earnings.[5] That is, there is no capital market available to finance purchased inputs, while there is a "perfect" capital market available to finance consumption expenditures. These two capital markets are perfectly separable in the sense that funds borrowed cannot be transferred from one purpose to another. Earnings capacity represents total earnings obtainable by individuals at a point in time if they were to allocate all their human capital stock to the labor market. In contrast, net earnings at time t are obtained after making the optimal level of investment, i.e., net of forgone earnings and purchased inputs. The constraint on the rate of change of the stock of human capital is that the change equals the gross production of new human capital via the individual production function less deterioration.

This maximization problem with its constraints can be represented by the maximization of the Lagrangian function:

$$(4) \quad L(t) = e^{-rt} \left[R \cdot E(t) - R \cdot K(t) - P \cdot D(t)\right] + \lambda_1(t) \cdot \{\dot{E}(t) - Q[K(t), D(t)]\} + \lambda_2(t) \{R[E(t) - K(t)] - P \cdot D(t) - g^2(t)\}$$

TABLE 1 List of Variables and Definitions

Variable	Description
Endogenous	
HW	Maximum human wealth
$E(t)$	Human capital stock
$\dot{E}(t)$	dE/dt
$\ddot{E}(t)$	$d(dE/dt)/dt$
$NY(t)$	Earnings net of purchased inputs
$K(t)$	Human capital allocated to producing more human capital
$D(t)$	Purchased investment inputs
$Q(t)$	Produced human capital
$\lambda_1(t)$	Shadow price of net additions to human capital
$\lambda_2(t)$	Lagrange multiplier for constraint
$g(t)$	Slack variable
t^*	Age at which specialization ends
$I(t)$	Total dollar investments, $RK(t) + PD(t)$
Exogenous	
R	Rental rate of human capital
P	Price of purchased investment inputs
E_0	Initial stock of human capital
N	Age of full retirement, end of horizon
t	Age, point in life cycle
r	Constant market rate of interest
δ	Constant rate of deterioration of human capital
β, β_1, β_2	Human capital production parameters: $0 < \beta$; $0 < \beta_2 < 1$; and $0 < (\beta_1 + \beta_2) < 1$

with respect to the decision variables $K(t)$ and $D(t)$.[6] The first-order conditions for a maximum require[7]

$$(5) \quad \frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

where $x = [E, K, D, \lambda_1, \lambda_2, g]$ and $\dot{x} = [\dot{E}, \dot{K}, \dot{D}, \dot{\lambda}_1, \dot{\lambda}_2, \dot{g}]$; that is

$$(6a) \quad Re^{-rt} + R\lambda_2 + \delta\lambda_1 - \dot{\lambda}_1 = 0$$

$$(6b) \quad Re^{-rt} + R\lambda_2 + \lambda_1 \frac{\partial Q(K, D)}{\partial K} = 0$$

$$(6c) \quad Pe^{-rt} + P\lambda_2 + \lambda_1 \frac{\partial Q(K, D)}{\partial D} = 0$$

$$(6d) \quad \dot{E} + \delta E - Q(K, D) = 0$$

$$(6e) \quad R(E - K) - PD - g^2 = 0$$

$$(6f) \quad -2g\lambda_2 = 0$$

Assuming L is a convex function, these conditions must be satisfied at each point in the life cycle for maximization of human wealth. A detailed study of these equations reveals the nature of optimal behavior.

To derive precise implications from the model a particular form must be specified for the human capital production function. The Cobb-Douglas function is used because (1) it guarantees that L will be convex and that interior solutions for K and D ($K \geqslant 0$ and $D \geqslant 0$) will exist at every point in the life cycle, thus simplifying the solution; and (2) the results are directly comparable to the original work of Ben-Porath (1967):

$$(7) \quad Q(K, D) = \beta K^{\beta_1} D^{\beta_2}$$

The production process is constrained by $0 < \beta_1 + \beta_2 \leqslant 1$.[8]

To make the model more realistic, the efficiency of producing human capital, β, is allowed to differ over the life cycle.[9] For example, the time spent specializing in human capital production is assumed to be the time spent in formal schooling. Since schooling is a publicly subsidized activity, efficiency may be greater during this period. The parameter β may be interpreted as representing fixed exogenous inputs into production about which the individual has no choice, i.e., $\beta = \alpha L_1^{\alpha_1} L_2^{\alpha_2}$. The level of, say, L_2 as determined by public or parental policy may be larger for individuals taking formal schooling or training. An equivalent formulation would be for an exogenous source to supplement direct purchases by a proportional amount during the schooling period $(0, t^*)$.

The solution to this problem contains two phases, Phase I, in which constraint (3) is effective and therefore $g = 0$; and II, in which constraint (3) is not effective and therefore $\lambda_2 = 0$. One or the other must be zero at all times. In this model, Phase I corresponds to the period in which the individual specializes in the production of human capital by allocating all available resources to that end. Phase I must occur continuously at the beginning of the life cycle.[10] This is the period when desired investment exceeds available resources and is constrained by earning capacity. Phase II represents the remainder of the life cycle, when net earnings are positive. Gross investment is always positive in Phase II because of the assumed Cobb-Douglas form of the production function.

In both phases, equations 6b and 6c imply

$$(8) \quad RK/\beta_1 = PD/\beta_2$$

or

$$D = (\beta_2 R/\beta_1 P) K$$

which represents the usual contract curve of efficient production points in which ratios of marginal products to factor prices are equalized. A general statement of the characteristics of the optimal earnings profiles in phases I and II follows.

Phase I: Specialization in the Production of Human Capital

In Phase I, $g = 0$; hence, substituting (8) into (6e) yields

(9) $K = [\beta_1/(\beta_1 + \beta_2)]\,E$

and

$$D = [\beta_2 R/(\beta_1 + \beta_2)P]E$$

Expenditures on direct inputs (PD) and forgone earnings (RK) are constant proportions of earning capacity (RE) during the period of specialization. Since PD must be financed by current earnings, i.e., $RE - RK = PD$, observed earnings are positive as the student works to finance his expenditures, but net earnings are zero: $NY = RE - RK - PD = 0$.

Substituting (9) into (6d) yields the differential equation for the growth of the capital stock

(10) $\dot{E} + \delta E = UE^{1-\Delta}$

where

$$U = \beta[\beta_1/(\beta_1 + \beta_2)]^{\beta_1}\,[R\beta_2/P(\beta_1 + \beta_2)]^{\beta_2} = \beta[\beta_1/(1 - \Delta)]^{1-\Delta}\,[R\beta_2/P\beta_1]^{\beta_2}$$

Equation 10 is a simple linear first-order difference equation in E^{Δ}, its solution is

(11) $E^{\Delta} = (U/\delta) + C_1 e^{-\Delta\delta t}$

Using the initial endowment of human capital, the path of earning capacity becomes

(12) $E = [(1 - e^{-\Delta\delta t})U/\delta + E_0^{\Delta}\,e^{-\Delta\delta t}]^{1/\Delta}$

The path of E in Phase I is strictly convex since[11]

(13) $\dot{E} = \delta D^{1-\Delta}\,[U/\delta - E^{\Delta}] > 0$

and

(14) $\ddot{E} = \delta E^{-\Delta}\,\dot{E}[(\beta_1 + \beta_2)U/\delta - E^{\Delta}] > 0$

Phase II: Positive Net Earnings

Phase II is characterized by monotonically declining investment[12] (to zero at N) and concave earnings profiles. In this phase, (6f) implies that $\lambda_2 = 0$; so constraint 3 is not effective and $RE > RK + PD$. The first step in analyzing equations 6 is to determine the shadow price of net additions to the stock of human capital (λ_1), using the transversality condition that the shadow value of human capital accumulation approaches and becomes zero at the end of life. Equation 6a becomes $\dot{\lambda}_1 - \delta\lambda = Re^{-rt}$ with solution

(15) $\lambda_1 = -Re^{-rt}\,(1 - e^{(r+\delta)(t-N)})/(r+\delta)$

Substituting (8) and (15) into (6b) and (6c) yields the optimal paths of K and D:

(16) $K = \beta_1 U_2\,(1 - e^{(r+\delta)(t-N)})^{1/\Delta}/(r+\delta)$

$D = (\beta_2 K/\beta_1 P)K$

where $U_2 = U^{1/\Delta}\,[(1-\Delta)/(r+\delta)]^{(1-\Delta)/\Delta}$ and correspondingly

(17) $I = [R(\beta_1+\beta_2)/\beta_1]K = R(\beta_1+\beta_2)U_2\,(1 - e^{(r+\delta)(t-N)})^{1/\Delta}/(r+\delta)$

Clearly, for this model, investment declines monotonically with age to zero at the end of life: $I(N) = 0$ and $\dot{I} < 0$. Investment initially declines at an increasing rate ($\ddot{I} < 0$), is concave in the region $t < N - \ln(1/\Delta)/(r+\delta)$, has an inflection point, and then declines at a decreasing rate ($\ddot{I} > 0$) and is convex thereafter. The convex region is longer for low rates of depreciation and interest, and as returns to scale ($\beta_1 + \beta_2$) increase.[13]

By substituting (8) and (16) into (6d), we can solve for the optimal path of the stock of human capital (E) or earning capacity (RE), which is the solution to the differential equation

(18) $\dot{E} + \delta E = U_2\,(1 - e^{(r+\delta)(t-N)})^{m}$

where $m = (1-\Delta)/\Delta$. The solution is of the form

(19) $E = C_2 e^{-\delta t} + U_2 e^{-\delta t} \int e^{\delta t}\,(1 - e^{(r+\delta)(t-N)})^{m}\,dt$

The equation can be solved in general by substituting for the infinite binomial expansion[14] and integrating each term of the series to yield

(20) $$E = C_2 e^{-\delta t} + U \sum_{i=0}^{\infty} \frac{(-1)^i \binom{m}{i}}{\delta + i(r+\delta)}\, e^{i(r+\delta)(t-N)}$$

To solve for the unknown constant C, we must specify the stock of human capital at the beginning of Phase II, i.e., at t^*. The full solution is obtained by substituting $t = t^*$ into equation 12 as the initial condition defining C, substituting the resulting $E(t^*)$ into the left-hand side of equation 20, and substituting $t = t^*$ into the right-hand side. As a result

(21) $$E = E(t^*)e^{-\delta(t-t^*)} + U_2 \sum_{i=0}^{\infty} \frac{(-1)^i \binom{m}{i}}{\delta + i(r+\delta)}\,(e^{i(r+\delta)(t-N)} - e^{\delta(t^*-t)}\,e^{i(r+\delta)(t^*-N)})$$

where, from equation 12,

$$(22)\quad E(t^*) = [U(1 - e^{-\delta\Delta t^*})/\delta + E_0^{\Delta}\, e^{-\delta\Delta t^*}]^{1/\Delta}$$

The stock of human capital at any point in time is the stock at the end of specialization[15] less its depreciation during the post-specialization period plus augmentations during the period from t^* to t less their depreciation. Special cases which are more amenable to analysis are presented below.

Presumably, the stock of human capital continues to rise upon entering Phase II if initial depreciation (δE) is not larger than gross production (Q). Since δE grows with $\dot{E} > 0$, and Q declines because $\dot{Q} < 0$, earning capacity grows at a decreasing rate, $\ddot{E} = -\delta\dot{E} + \dot{Q} < 0$, i.e., it is concave up to its peak. Earning capacity must peak during the period, since δE rises as Q approaches zero at N; hence, the two must cross before N. After the peak, $\dot{E}$ becomes negative; therefore, $\ddot{E} = -\delta\dot{E} + \dot{Q}$ is initially negative and likely to remain negative; hence, E is concave.

Earnings Net of Investment

Net earnings, defined as $NY = R(E - K) - PD$, are zero during the period of specialization and become positive in Phase II. Net earnings jump from zero in Phase I to the positive value $RE(t^*) - I(t^*)$ at the saltus point,[16] $t = t^*$, since the level of earning capacity $RE(t^*)$ attained with $\beta = \beta'$ is greater than desired investment with $\beta = \beta''$ in Phase II.[17]

One of the primary implications of this human capital investment model is the prediction of a concave earnings path. Since $N\dot{Y} = R\dot{E} - \dot{I}$ and $\dot{I} < 0$ for every $t > t^*$, earnings net of investment necessarily peak after earning capacity peaks. Furthermore, the growth of net earnings is always greater than the growth in earning capacity and its decline less than the decline in earning capacity when it declines. Net earnings are guaranteed to be concave, $N\ddot{Y} = R\ddot{E} - \ddot{I} < 0$, in the region $t > N - \ln(1/\Delta)/(r + \delta)$ where $\ddot{I} > 0$, since earning capacity is concave. At some initial earlier age it is possible that $\ddot{I} > 0$ will dominate $R\ddot{E}$, making net earnings initially convex.

Net earnings, from equations 21 and 17, are[18]

$$(23)\quad NY = RE(t^*)e^{-\delta(t-t^*)} - RU_2 \sum_{i=0}^{\infty} \frac{(-1)^i \binom{m}{i}}{\delta + i(r+\delta)} e^{\delta(t^*-t)} e^{i(r+\delta)(t^*-N)}$$

$$+ RU_2 \sum_{i=0}^{\infty} (-1)^i \left[\frac{\binom{m}{i}}{\delta + i(r+\delta)} - \frac{\beta_1 + \beta_2}{r+\delta} \binom{m+1}{i} \right] e^{i(r+\delta)(t-N)}$$

where each U_2 is defined with $\beta = \beta''$, but $E(t^*)$ is based on $\beta = \beta'$.

Human Wealth

While lifetime paths of earnings, earning potential, and investment are interesting and important, the primary focus is clearly the level of human wealth attained by the individual. Human wealth is the maximum attainable present value of net earnings, $HW = \int e^{-rt} NY\, dt$, where the limits of integration are from t^* to N, and is obtained by integrating the product of equation 23 and the discounting factor from t^* to N.

$$(24) \quad HW = \frac{RE(t^*)}{r+\delta} e^{-rt^*}(1 - e^{-(r+\delta)(N-t^*)})$$

$$- \frac{RU_2}{r+\delta} e^{-rt^*}(1 - e^{-(r+\delta)(N-t^*)}) \sum_{i=0}^{\infty} \frac{(-1)^i \binom{m}{i}}{\delta + 1(r+\delta)} e^{i(r+\delta)(t^*-N)}$$

$$+ RU_2 \sum_{i=0}^{\infty} \frac{(-1)^i}{-r + i(r+\delta)} \left[\frac{\binom{m}{i}}{\delta + i(r+\delta)} - \frac{\beta_1 + \beta_2}{r+\delta} \binom{m+1}{i} \right] e^{-rN}(e^{[-r+i(r+\delta)](t^*-N)} - 1)$$

Clearly, human wealth is a function of the stock of earning capacity at the end of the specialization period, $E(t^*)$, as well as of the length of the specialization period itself and of all the other parameters of the model such as deterioration rate, production efficiency, and retirement age. It is important to note that a change in any parameter which affects first-period investment will also affect t^* and correspondingly $E(t^*)$. Partial effects on human wealth are analyzed below.

Implicit Solution for t^*, the End of Specialization

Consider the conditions for the existence of Phase I. Since the optimal level of gross investment declines monotonically to zero at N, the period of specialization must occur continuously at the beginning of the life cycle or not at all. Figure 1 gives a geometric representation of the results derived in this section and illustrates the location of t^*.

The necessary and sufficient condition for the existence of Phase I is that

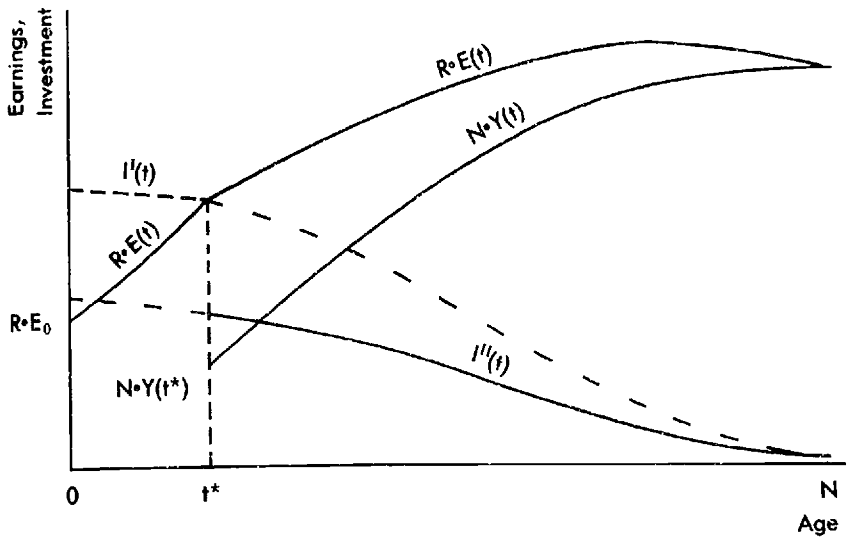
$$(25) \quad RE_0 < RK_0 + PD_0 = I_0$$

where K_0, D_0, and I_0 represent unconstrained desired input and investment levels at time $t = 0$. Desired investment is constrained by earning capacity if, substituting from (8) and (16),

$$(26) \quad E_0^A < (\beta_1 + \beta_2)U(1 - e^{-(r+\delta)N})/(r+\delta)$$

where U is defined by equation 11 with $\beta = \beta'$.[19] The effect of allowing $\beta' > \beta''$ instead of $\beta' = \beta''$ is to increase the likelihood that Phase I exists by raising the cutoff value of E_0^A.

FIGURE 1 Earnings and Investment Paths and Their Relationship to One Another



NOTE: For explanation of symbols, see accompanying text and Table 1.

This existence condition for Phase I implies the sufficient condition for convexity of E in Phase I, as noted earlier. That is, (26) implies

$$(27)\quad E_0^{\Delta} < (\beta_1 + \beta_2)U/\delta < U/\delta$$

since $0 < (\beta_1 + \beta_2) < 1$ and $(1 - e^{-(r+\delta)N})/(r + \delta) < 1/\delta$ for every $N > 0$, $0 < (r + \delta) < 1$, $0 < r < 1$, and $0 < \delta < 1$.

The exact solution for the optimal age at which to end specialization, t^* (when it is positive), is implicitly contained in the equation representing the equalization of earning capacity in Phase I and desired investment from Phase I as implied by constraint 3. That is, equate (17) to (12), each with $\beta = \beta'$ and $t = t^*$. This equation can be solved for E_0^{Δ} to obtain

$$(28)\quad E_0^{\Delta} = (1 - \Delta)/(r + \delta) - (Ue^{\Delta\delta t^*}/\delta) + (U/\delta) - [(1 - \Delta)Ue^{-(r+\delta)N}e^{[r+\delta(1+\Delta)]t^*}/(r + \delta)]$$

This expression cannot in general be solved for t^*, but the direction of partial effects can be ascertained by implicit differentiation of (28). The partials are presented in the following subsection.

The Recursive Relationships between t^* and Phase II Endogenous Optimal Paths

It is important to specify carefully the recursive nature of the relationship between t^*, represented in equation 28, and Phase II levels of earning capacity and net earnings. The system is recursive in the sense that some parameters affect earnings and earning capacity only directly through Phase II behavior; some operate only indirectly, through their effect on Phase I, i.e., through t^* and $E(t^*)$; and others do both. For instance, E_0 and β' affect only t^* and the stock of earning capacity at t^* but have no effect on Phase II investment; they thus contribute additively to earning capacity and net earnings. β'' affects only the Phase II level of investment and through it affects earning capacity and net earnings, with no effect on t^* or $E(t^*)$. All other parameters affect t^*, $E(t^*)$, and $I(t)$, and thus affect earning capacity and net earnings in several ways. For changes in those parameters (x) that affect t^*, there is the implicit relationship represented in equation 28 that specifies the accompanying indirect effect $\partial t^*/\partial x$. This effect on $E(t^*)$ will depend on which parameter is varied. As the model is specified,[20] equation 28 represents an implicit relationship between parameters in equations 21, 22, and 25 that must be satisfied at all times. These relationships and corresponding comparative statics represent the analytics of a closed system. The comparative statics presented in the next subsection are based on these relationships.

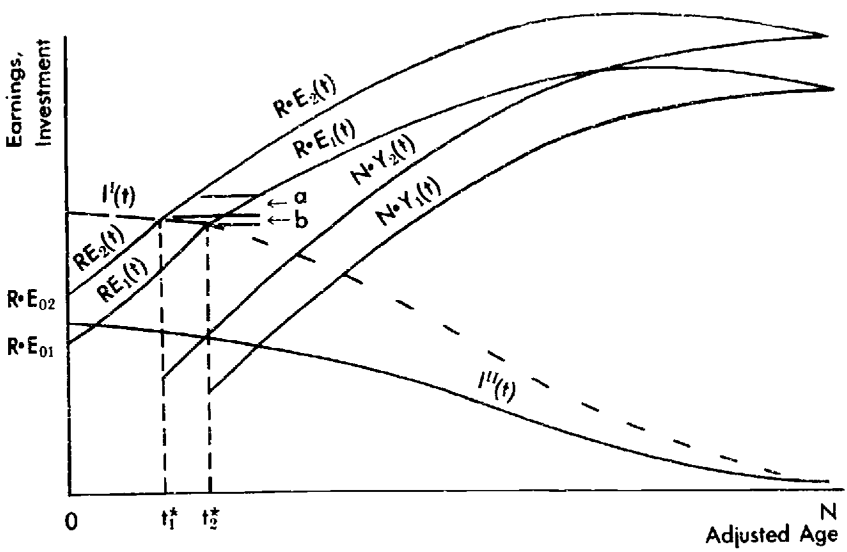
Comparative Statics Effects of Parameters

While the solutions for investment and earnings paths and even human wealth are analytically complex, they illustrate the basic properties of the optimal solution. Some simplified and limiting special cases are presented in the appendix. Consider the comparative statics properties of these basic equations.

Initial Endowment of Human Capital, E_0 The level of investment in Phase II is not a function of E_0; hence, the effect of a change in E_0 is simply a parallel shift in earning capacity and earnings functions. If Phase I does not exist, the shift is dollar for dollar. If Phase I does exist, $t^* > 0$, the length of Phase I is shortened, and the earning capacity upon entering Phase II is greater.[21] The specialization period is shortened because a greater initial endowment alleviates the constraint on desired investment earlier; or alternatively, the likelihood that Phase I exists decreases. Other than a parallel upward shift and an earlier start, the net earnings path in Phase II is unaffected. The net result is that an individual with a larger initial stock of human capital and all other characteristics the same will have higher earnings at every point in the life cycle and will begin earning sooner. Human wealth is clearly increased.

It is interesting and instructive to decompose the upward parallel shift in earning capacity into its two components, the one owing directly to the

FIGURE 2 Changes in Income and Investment Paths Due to an Increase in the Initial Stock of Human Capital



NOTE:

$a = E(t^*) \dfrac{\partial t^*}{\partial E_0} dE_0,$

$b = \dfrac{\partial E(t^*)}{\partial t^*}$

$\dfrac{\partial t^*}{\partial E_0} dE_0$

For explanation of other symbols, see accompanying text and Table 1.

greater initial stock of human capital at the new t_2^* and the other, to the resulting increase in working life (see Figure 2). Since the shift in RE is obviously parallel, we need to analyze only the increment at the initial t_1^*. Note that for very small changes in E_0

$$(29)\quad \frac{\partial E}{\partial E_0} = \frac{\partial E(t^*)}{\partial t^*}\frac{\partial t^*}{\partial E_0} - \dot{E}(t^*)\frac{\partial t^*}{\partial E_0}$$

The first term on the right represents the change in human capital stock at t^* due to a change in t^* caused by a change in E_0. Since desired investment, represented by $I(t)$ in equation 17 with $\beta = \beta'$, is unchanged by E_0 and since $E(t^*)$ is defined by reaching that constraint, $\partial E(t^*)/\partial t^* = \dot{I}(t^*) < 0$. This change is proportional to $\partial t^*/\partial E < 0$; hence, the net result is an increase. The second term, $-\dot{E}(t^*)$ $(\partial t^*/\partial E)$, represents the extra growth in earning capacity as measured by equation 18, with $\beta = \beta''$ allowed by the additional increment of $\partial t^*/\partial E_0$ to the length of working life. Both terms then are separate aspects of the increased earning capacity.

Production Parameters β, β_1, and β_2 In the Cobb-Douglas production function, β_1 represents the productivity of human capital; β_2, the productivity of purchased inputs; and β, the overall efficiency. The ratio β_2/β_1, the relative proportion of forgone earnings and purchased inputs in total investment, remains constant over the entire life cycle.[22] The overall importance of β_1 and β_2 is represented by their sum, $\beta_1 + \beta_2 = 1 - \Delta$, which measures the returns to scale in the production of human capital, and is constrained to be in the interval (0, 1).[23]

A larger β' affects both the growth of earning capacity in Phase I and the length of the phase, but not Phase II investment. Thus, it affects Phase II earning capacity and net earnings only indirectly, through t^* and $E(t^*)$. A larger β' lengthens the period of specialization and increases the likelihood of its existence. An increase in β' lowers the marginal cost of producing a unit of human capital at each age in Phase I while the marginal benefit, λ, in equation 13 remains unchanged. The net result is that the larger the increase in β', the longer the period of time over which the marginal benefit exceeds the marginal cost.[24]

As is evident from equations 10 and 12, as β' increases, the rate of growth of earning capacity rises and thus the stock of human capital at the end of Phase I, $E(t^*)$, rises. The change in $E(t^*)$ causes a parallel shift in the earning capacity and net earnings functions over Phase II.

In Phase II, altering β'' shifts the investment path proportionally to $(\beta'')^{1/\Delta}$. While β'' both increases the rate of growth of net earnings and lowers the initial value by shortening the "jump" in the level of investment from Phase I to Phase II at t^*, the jump in net earnings is obviously due to the difference between β' and β''. An increase in either type of ability obviously increases human wealth.

Changes in E_0 and β' for a Given t^* Of special interest is the case in which a change in characteristics leaves the period of specialization, t^*, unchanged. For example, if only β' and E_0 are allowed to change, any combination of the two that satisfies equation 23 will yield the same value of t^*.[25] It was shown earlier that a rise in either E_0 or β' tends to raise earnings. When both increase in proportion to maintain t^*, the entire earnings profile, NY, as well as earning capacity rise over the entire life cycle. Consider the changes in two parts. First, the result of increasing E_0 is to increase investment only in the period of specialization and correspondingly to increase earnings everywhere. Furthermore, the period of specialization is shortened. Secondly, increasing β' by enough to bring t^* back up to its original position will raise the productivity of investment, complementing the effect of E_0; that is, persons with the same t^* but differing in E_0 and β will have earnings profiles which do not intersect.

Rate of Deterioration of Human Capital (δ) and the Market Rate of Interest (r) Both δ and r affect the optimal investment path similarly. It is the sum of the two which determines the postspecialization investment level. Both tend to dampen the desired level of investment at all ages and thus have a negative effect on earning capacity, net earnings, and human wealth.

The effects do differ, however, in an important way. In Phase I, investment is obviously dampened by an increased deterioration rate since less human capital is available for investment. The rate of interest, however, has no effect on investment in that phase (from equations 9 and 12). The result is that the rate of interest unambiguously shortens the specialization period,[26] while the effect of the deterioration rate on specialization is ambiguous. An increase in either the interest rate or the deterioration rate has a negative effect on earning capacity and human wealth.

Age of Retirement The age of retirement (N) or the length of the life cycle enters primarily through the shadow value of human capital accumulation. Shadow value declines monotonically with age and approaches zero at the end of the life cycle; i.e., $\lambda(N) = 0$. Correspondingly, optimal gross investment declines monotonically with age to zero at retirement. Since optimal investment depends only on the time remaining to recoup the benefits of investment ($N - t$), a change in N simply shifts the investment path horizontally (see equations 14 and 16). Since a later retirement increases the desired level of investment in both phases I and II proportionately, the period of specialization is lengthened,[27] and earning capacity and earnings are enhanced at every age (see equation 15). A longer life cycle obviously enhances earning capacity, postspecialization earnings, and human wealth.

The Rental Rate of Human Capital (R) *and the Price of Purchased Inputs* (P) The effects of R and P are similar in some respects to those of β_1 and β_2. Since invested human capital is effectively purchased at price R through forgone earnings, equation 8 illustrates that the relative proportions of purchased inputs (D) and forgone earnings (K) depend upon their relative prices. If the price of direct inputs rises, the individual has an incentive to substitute human capital for direct inputs in production. Since the price of a factor of production has risen, the optimal investment level will fall everywhere in Phase II.[28] However, R also affects the return on the production of an additional unit of human capital. The shadow value of a unit of human capital at any age is effectively the discounted rental rate on that unit net of its deterioration. The effect of an increase in R is to raise the optimal level of Phase II investment.[29]

[III] DATA, EMPIRICAL MODEL, AND PROCEDURES

This section provides the bridge from theory to empirical analysis. The preceding section was focused on the analytic determinants of optimal human capital investment, the optimal lifetime pattern of earnings, and the corresponding maximum level of human wealth. In this section I introduce a data set and an empirical model from which we can begin to quantify some of the theoretical concepts and verify some of the theoretical predictions. In subsequent sections I present estimates of the determinants of the level of human wealth; the underlying lifetime pattern of earnings; the degree of individual variation in the level of initial earning capacity and human wealth; and the socioeconomic determinants of optimal schooling and of various dimensions of measured ability. Although the theoretical model is not restricted to concepts with measurable counterparts, some additional assumptions must be imposed to render the model empirically tractable. The empirical model is consistent not only with the theoretical model developed in the last section, but with many other theoretical models as well. Perhaps the best view of the theoretical model is that it helps us interpret the empirical results. The latter are, however, interesting in their own right.

Several chronologically successive factors which affect human wealth will be analyzed in a recursive structural model. In the first stage of the recursive system, characteristics of family and social background are considered as determinants of several measured dimensions of ability; these ability indexes were measured just after high school. The family background variables include father's and mother's years of schooling, number of siblings, religion, and number of family moves during youth. In the second stage of the model, background factors and various measured abilities are analyzed as determinants of the length of formal schooling. In the third stage, earnings at each age in the life cycle are related to background, abilities, and years of schooling; these three are determinants of the lifetime earnings profile and thus of the resulting human wealth. By interpreting these empirical patterns, we can verify the theoretical notions and the degree of variability in the theoretical quantities estimated. I begin with a description of the data because they determine some aspects of the empirical model.

The Data

The empirical work is based on the NBER-TH sample. A useful feature of this sample is that it includes earnings data for the same individuals at several points in their lifetime (ages 19 to 54); measures of several specific types of abilities; and detailed socioeconomic background data. Rarely is this much lifetime data available for individuals, especially in combination with the other personal data. The general characteristics of the sample are discussed in detail in several places. The original data are described in Thorndike and Hagen (1959), and recently acquired additional information is described in Taubman and Wales (1974) and Hause (1975).

The results reported here are based on a group of 4,699 men for whom two to five age-earnings points were observed between 1943 and 1970. Most of the men had been born between 1917 and 1925; accordingly, their ages ranged from 19 to 57 over the years of observation (fewer than 1 percent were outside the range from 19 to 55). All had volunteered for air force pilot, navigator, and bombardier programs in 1943. In 1955, Thorndike and Hagen (1959) sampled 17,000 men by mail and included questions on schooling and 1955 earnings. In 1969, NBER did the same for a subset of these 17,000, and included additional questions on initial job earnings, earnings in later years, and schooling. The data include five separate, approximately equidistant points on the age-income profile as well as the year of initial job, last year of full-time schooling, and total years of schooling. The age-income points are approximately initial job, 1955, 1960, 1964, and 1968. The distribution of observations by year is as follows: 3,844 for 1945–1952; 1,846 for 1953–1957; 3,692 for 1958–1962; 1,231 for 1963–1966; and 4,774 for 1967–1970.

Another distinguishing attribute of this data is the wealth of information on measured ability. The air force tests of applicants for pilot and navigator school yielded twenty indexes of various abilities. A single IQ-type aggregate ability index was obtained by a factor analysis of those ability indexes most nearly corresponding to IQ-related abilities. The tests and factor loadings are presented in Table 2. Also, separate aggregate ability indexes were constructed for mathematical ability (MATH), mechanical dexterity (MECH), physical dexterity (PHYS), and general knowledge (GENKN). Reading comprehension originally had only one index. Again, the separate ability indexes were constructed by a factor analysis of the appropriate original scores. The indexes and factor loadings for each are presented in Table 2. The simple correlations among the indexes and of each index with years of schooling are presented in Table 3.

The individuals in the sample differ from the U.S. male population as a whole in several ways: (1) It is a high-ability group; all of the men completed high school or high school equivalency examinations and passed the initial screening for the air force-flight program. (2) Their general health was better than that of the general population in 1969. (3) They were more homogeneous in height and weight because all had to meet military physical standards. (4) They seem to have had a high degree of self-confidence and self-reliance. Some of these factors may, however, be related to the subjects' high ability. In addition to the factors mentioned, the G.I. Bill was available to all those men to help finance their schooling.

The Recursive Model

The model specifies the structural relationship between a set of predetermined variables, including family background and social characteristics, and a set of recursively related endogenous variables, including a vector of ability measures, years of schooling, and annual earnings at several ages in the life cycle. Figure 3 contains a diagram of the recursive model. The recursive nature is partially determined by the nature of the NBER-TH data. For example, the relationship between schooling level and measured abilities is recursive because the

TABLE 2 Principal Component Weights for Aggregating Individual Test Scores

Mathematics (MATH)	
.302	Math A (advanced arithmetic, algebra, trigonometry)
.317	Math B (arithmetic)
.303	Numerical Operations 1 (speed and accuracy of simple arithmetic operations with whole numbers)
.346	Numerical Operations 2 (same as Numerical Operation 1, more complex problems)
Mechanical Dexterity (MECH)	
.187	Mechanical Principles (pictorial presentation of mechanical problems)
.291	Dial and Table Reading (reading instrument dials)
.321	Speed of Identification (matching perceptual forms)
.334	Spatial Orientation 1 (matching aerial photos)
.348	Spatial Orientation 2 (matching detailed aerial photos)
Physical Dexterity (PHYS)	
.253	Discrimination Reaction Time (motor response to visual response)
.302	2-hand Coordination (speed of adaptation to new psychomotor problems)
.336	Complex Coordination (use of hand and foot controls)
.285	Rotary Pursuit (simple motor skills)
.161	Aiming Stress (muscular steadiness and emotional control)
.253	Finger Dexterity
General Knowledge (GENKN)	
.381	Biographical Data-Pilot (index of information associated with success in pilot training)
.502	General Information-Pilot (knowledge of planes and flying techniques)
.251	Biographical Data-Navigator (index of information associated with success in navigator training)
.431	General Information-Navigator (knowledge of topics in astronomy and science)
I.Q.-Type Index (IQ)	
.727	Math A
.726	Math B
.615	Numerical Operations (1 + 2)
.601	Reading Comprehension
.204	Reading Dummy[a]
.758	Dial and Table Reading
.438	Speed of Identification
.485	Spatial Orientation 1
.542	Spatial Orientation 2

[a]Because the Reading Comprehension test had a floor (very low scores were possible) a dummy variable was added such that a 1 was given to each person who did not score at the lowest possible value and a 0 otherwise. A total of 98.4 percent of individuals received a one. All variables were transformed to have zero mean and unit standard deviation before factor analysis.

TABLE 3 Simple Correlations between Abilities and Schooling

Aggregate Ability Index	Years of Schooling	Reading Compre-hension	Manual Dexterity	Mechanical Dexterity	Math and Num. Operations
Reading Compre-hension	.195				
Manual Dexterity	−.025	.250			
Mechanical Dexterity	.107	.349	.472		
Math and Numerical Operations	.241	.389	.193	.383	
General Information	.168	.477	.277	.421	.213

ability test scores were obtained shortly after high school and the schooling variable is number of years beyond high school. The same applies to the relationship between schooling and earnings because the age-earnings observations were made after all formal schooling had been completed. While each type of ability will be considered empirically, the model is formulated here in terms of the single IQ-type ability index. The formal model is as follows:[30]

$$(30)\quad Y_{it} = \sum_{k=0}^{3} \sum_{j=0}^{2} \sum_{l=0}^{2} (\alpha_{kjl}\, Age_{it}^{k}\, Sch_i^{j}\, IQ_i^{l}) + \sum_{q} (\Gamma_q Soc_{qi}) + \mu_{it}$$

$$(31)\quad Sch_i = \beta_{IQ}\, IQ_i + \sum_{q} (\beta_q Soc_{qi}) + \xi_{si}$$

$$(32)\quad IQ_i = \sum_{q} (\Upsilon_q\, Soc_{qi}) + \xi_{IQi}$$

where Y_{it} is annual earnings in real 1970 dollars of the i^{th} individual at observation t; Age_{it} is the age of individual i at observation t; Sch_i is the number of years of schooling of the i^{th} individual; IQ_i is the i^{th} individual's ability index, and Soc_i is the i^{th} individual's vector of q social variables including father's and mother's years of schooling, number of siblings, number of childhood family moves, and religion dummy variables for the Protestant, Catholic, and Jewish religions. (Other religions, no religion, and no response constitute the omitted class.)

Equation 30 may be considered an approximation to the nonlinear equation 23 in Phase II of the life cycle obtained by a MacLaurin's expansion of the exponential functions of t^* (representing years of schooling), and t (representing age). I will not pursue this notion to try to identify the underlying theoretical parameters.[31] This functional form is so general that it could be approximating many other alternative models. The theory provides a formal interpretation

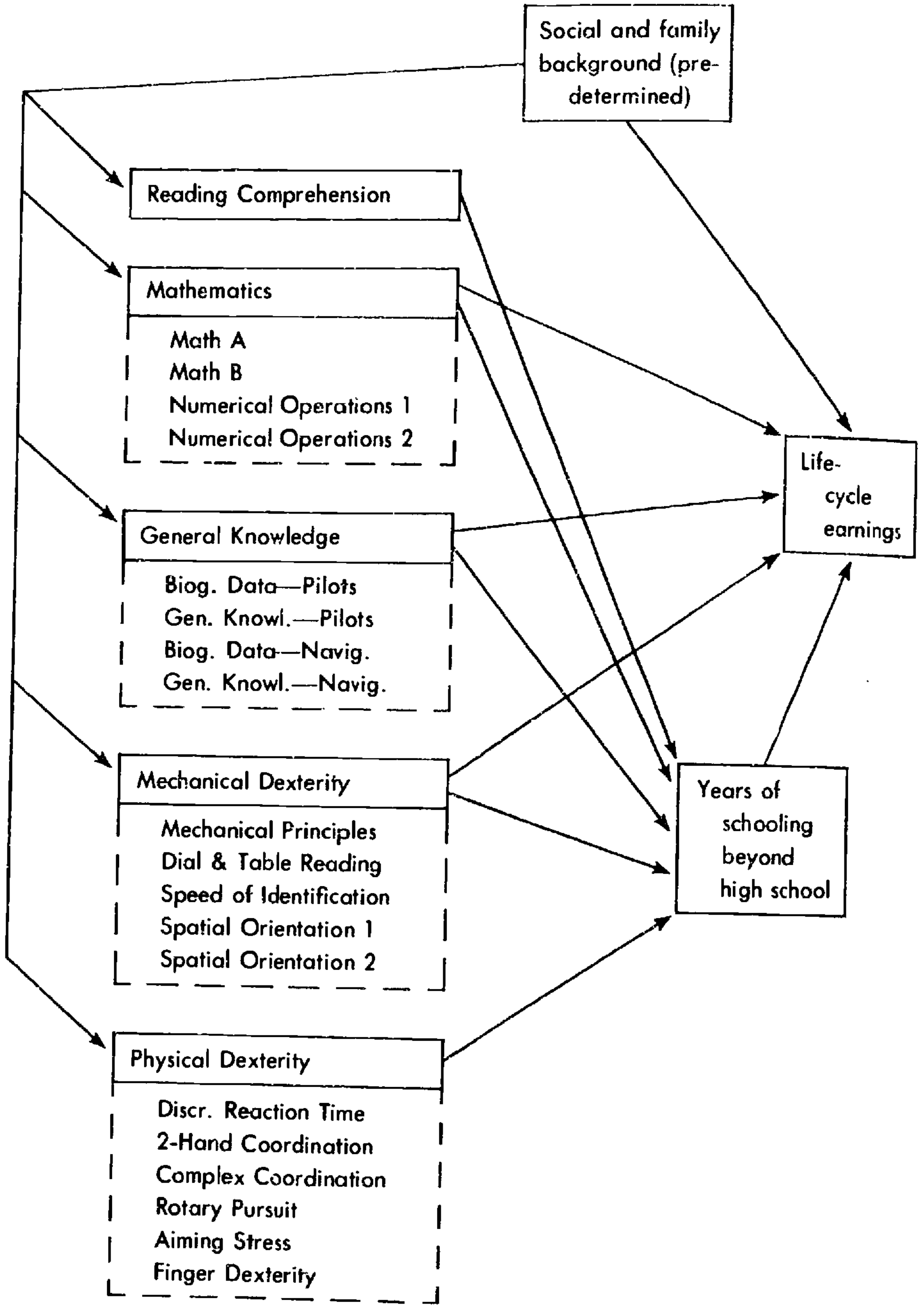


FIGURE 3 Diagrammatic Representation of the Structural Model

of empirical results, and the empirical results are used to verify certain qualitative predications. The degree of polynomial in age, schooling, and ability represented in equation 30 is ascertained empirically as that polynomial surface which "best" fits the data in the sense of minimum variance without excessive order; that is, additional-order polynomials in age, schooling, and ability are introduced until they fail to reduce error variance significantly at the 5 percent level. The best equation is found to be cubic in *Age* and quadratic in *Sch* and in *IQ*. Only age represented a cubic relationship regardless of the order of entering polynomials. The social variables are entered additively arbitrarily.

The recursive nature of the relationships is exploited to justify estimating each equation separately. The schooling and IQ equations are estimated using data on the 4,699 men, and the earnings equation is estimated using the 15,387 pooled age-earnings points, thus combining time series and cross-sectional aspects.

The family background variables are entered linearly into the earnings function. They may be thought of as affecting the individual's stock of human capital at the school-leaving age $[E(t^*)]$ through schooling subsidies and direct resource inputs. They thus affect the level of the earnings function. The probably numerous unmeasured variables that affect the level of earnings are represented by an individual variance component in the error structure of the earnings function. This error structure (μ_{it}) is assumed to be of the form

$$(33)\quad \mu_{it} = \epsilon_i + \eta_{it}$$

where ϵ_i is the i^{th} individual's permanent deviation from the estimated earnings function and η_{it} is the transitory residual. It is assumed that ϵ and η are independent of each other and all measured variables. The repeated observation of each individual over the major part of a lifetime makes possible the analysis of individual differences in lifetime earnings rather than simply an analysis of the lifetime earnings path of the "representative individual" predicted from measured variables alone.

Given this error structure, the more efficient GLS estimates of the parameters α and Γ are obtained and compared to the OLS estimates. Since all individuals are not observed in the same time periods and are indeed not observed the same number of times, it is worthwhile to outline the method of obtaining the GLS estimates. The earnings covariance structure is of the form

$$(34)\quad \operatorname{cov}(\mu_{it}, \mu_{j\tau}) = \begin{cases} \sigma_\epsilon^2 + \sigma_\eta^2 = \sigma_\mu^2 \text{ if } i = j,\ t = \tau \\ \sigma_\epsilon^2 = \rho\sigma_\mu^2 \text{ if } i = j,\ t \neq \tau \\ 0 \text{ if } i \neq j \end{cases}$$

where $\rho = \sigma_\epsilon^2/\sigma_\mu^2$ is the proportion of total earnings variation in any single year which represents permanent differences among individuals. The parameter ρ is also the correlation between any two residuals for the same individual's different years.

The GLS estimates of the parameters α and Γ are effectively weighted averages of estimates which would result from using only within-individual earnings variation or from using only between-individual variation. The GLS estimate weights these two inversely to their error variances. See Nerlove (1971) and Maddala (1971) for a more detailed discussion of these issues. If each individual were observed the same number of times, the weights would be reflected in the parameter

$$(35) \quad \theta = \sigma_\eta^2/(\sigma_\eta^2 + T\sigma_\eta^2)$$

where T is the number of observations on each individual. In the data analyzed here T is between 2 and 5. Since pooled estimates of σ_η^2 and σ_ϵ^2 are obtained by an analysis of within- versus between-individual residual variation, all individuals observed the same number of times will have the same θ, i.e.,

$$(36) \quad \hat{\theta}_i = \hat{\sigma}_\eta^2/(\hat{\sigma}_\eta^2 + T_i\,\sigma_\Sigma^2)$$

The GLS estimates are then obtained by OLS estimation on the transformed data cross-product matrix

$$(37) \quad \left[\sum_{i=1}^{N} \sum_{t=1}^{T_i} X_{it}' X_{it} - \sum_{i=1}^{N} (1 - \hat{\theta}_{T_i})\, T_i \bar{X}_i \bar{X}_i'\right]$$

where X_{it} is the data vector for individual i at observation t including the dependent variable and $\bar{X}$ is a vector of variable means for individual i.

Another objective is to ascertain the degree of dispersion and inequality in human wealth. It is useful and instructive to separate the effect of measured variables and of unmeasured variables, ϵ, on dispersion in human wealth. The expected value of human wealth for any given set of measured variables (schooling, ability, and background) is estimated by summing discounted earnings values predicted from the estimated earnings function of equation 30 over the working life from school-leaving age to the age of full retirement.[32] Retirement is assumed to be at age sixty-five for everyone. All human wealth values are discounted to age sixteen at the same rate for everyone.

These mean human wealth values correspond to the expected value of human wealth for a "representative individual" with the given set of measured variables. Dispersion in these mean values represents dispersion in human wealth caused by variation in the measured variables alone. There is, however, significant variation in the level of earnings, ϵ, and thus in human wealth among individuals alike in their measured characteristics.

It is important to move beyond the notion of a "representative individual" to estimate the total variation in human wealth and to assess the relative importance of schooling, ability, and background as determinants of inequality in human wealth. Predicted earnings and the estimated value of the individual component ϵ are assumed to be orthogonal; hence, the contribution of each to variation in human wealth may be separated, and the sum of the two components equals the total variation.

The total variance in human wealth is estimated in the following manner. First, mean human wealth (MHW) is estimated for each individual in the sample on the basis of his schooling, ability, and background. Next, each individual's human wealth (HW) is estimated by utilizing the individual's own observed earnings history. The mean discounted residual is calculated so that the present value of each individual's transitory earnings component is zero and thus adds nothing to the variance in human wealth. When the discount rate is zero, the resulting mean residual is an unbiased estimate of ϵ_i. An individual's estimated human wealth is then MHW plus the present value of the mean discounted residual, and the variance in human wealth is the sum of the orthogonal variance components.[33] For calculating variances over individuals, each observation is weighted in proportion to the number of age-earnings points observed for that individual.

[III] EMPIRICAL RESULTS

In this section, I explore patterns of lifetime earnings and the resulting human wealth as well as the determinants of schooling levels and abilities that influence these patterns. First, I examine the effects on the age-earnings relationship of schooling, IQ-type measured ability, and social and family background for consistency with the predictions of the theoretical model; and I explore the corresponding contribution of these variables to human wealth at various discount rates. Next, I translate these earnings and human wealth estimates into measures of dispersion for comparisons of inequality in human wealth and inequality in earnings at various stages of the life cycle. The role of individual differences due to unmeasured sources is explicitly analyzed. I then explore the effects of ability and measured social and family background characteristics on years of schooling and the effect of background on ability.

While these results incorporate an aggregate IQ-type ability measure, the detailed effects and determinants of mathematical ability, reading comprehension, mechanical dexterity, physical dexterity, and general knowledge are presented separately.

Life-Cycle Earnings Patterns and Profiles

In this section I present a detailed empirical analysis of individual lifetime earnings patterns and their consistency with patterns predicted by the theoretical

TABLE 4 Earnings Function Parameter Estimates (OLS)

	OLS
Constant	4157.
Sch(*S*)	1935.
Age(*A*)	−785.9
AS	−162.6
A^2	59.4
A^3	−1.09
SA^2	14.3
SA^3	−.23
S^2	−296.1
S^2A	38.0
S^2A^2	−2.9
S^2A^3	.05
Abil(*B*)	−3393.
BS	−2774.
BA	2979.
BAS	−296.2
BA^2	−213.9
BA^3	3.9
BSA^2	21.9
BSA^3	−.47
BS^2	459.7
BS^2A	−33.3
BS^2A^2	2.4
BS^2A^3	−.032
B^2	3533.
B^2S	463.8
B^2A	−2106.
B^2AS	364.9
B^2A^2	149.6
B^2A^3	−2.7
B^2SA^2	−25.4
B^2SA^3	.48
B^2S^2	−139.8
B^2S^2A	−2.9
$B^2S^2A^2$	.17
$B^2S^2A^3$	−.008
FED	84.
MED	101.
NO. SIB	−110.
NO. MOVGS	28.
PROT.	−295.
CATH.	50.
JEW	3852.

NOTE: *Sch* is years beyond ten and *Age* is years beyond sixteen.

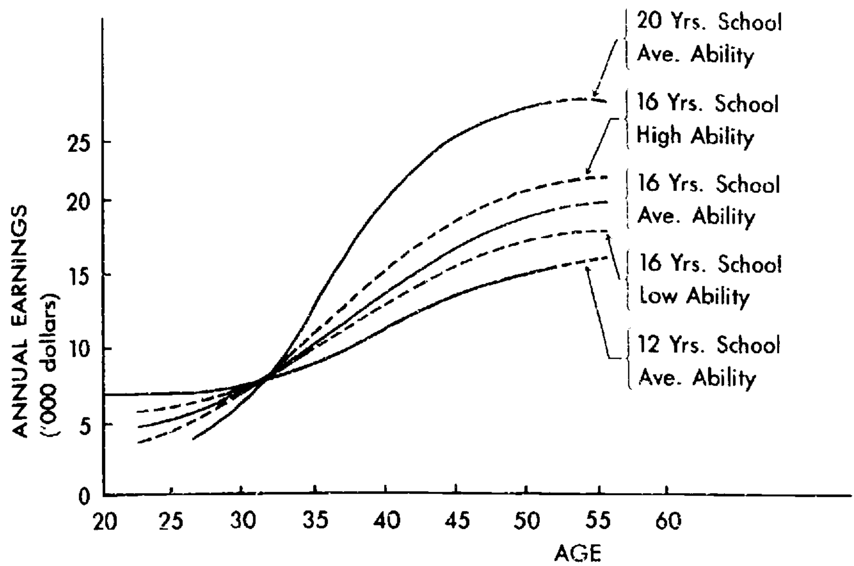


FIGURE 4 Estimated Age-Earnings Profiles for a Protestant with Average Values for His Other Social Variables.

model. The effects of schooling and IQ-type ability on life cycle earnings patterns are represented by their interactions with age (see Table 4). Variation in schooling and ability and their interactions and background account for 30.1 percent of the variation in annual earnings. The estimated standard deviation of μ is \$7,997 with a standard deviation[34] of \$5,224 for η and \$6,054 for ϵ.[35] Hence, 57 percent of residual variation is explained by individual permanent differences.[36] This figure (0.57) may be interpreted as an estimate of the simple correlation between the residuals for any two observations on the same individual. Correspondingly, 70 percent of total variation is explained by measured variables plus the permanent component. As I show later, these variance components play an important role in human wealth variation.

Both OLS and GLS estimates[37] of the parameters α and Γ are presented in Appendix B. Because of the large number of observations, the predicted age-earnings profiles are about the same using either set of estimates.[38] Representative age-earnings profiles based on the OLS earnings function are presented in Figure 4 for a Protestant with average levels of other social variables. The earnings profile is shifted vertically by \$84 for each additional year of father's education, by \$101 for each additional year of mother's education, by −\$110 for each additional sibling, by \$28 for each childhood family move, and by \$345 for Catholic and \$4,147 for Jewish religion (relative to Protestant).

The life-cycle earnings patterns and the differences in those patterns due to schooling and ability levels are clearly evident. Earnings rise over the lifetime and they rise more rapidly, the more educated and more able the individual. For example, between ages forty and forty-five, given mean ability, earnings rise at a rate of $556 per year for a college graduate, at a rate of $366 for a high school graduate, and at $880 for a professional or Ph.D. For a college graduate over the same age range, earnings rise at a rate of $494 per year for an individual one standard deviation below mean ability and $627 for an individual one standard deviation above the mean.

Both the more educated and the more able have lower earnings prior to age thirty, perhaps because their levels of job training investment are higher. That in turn causes future earnings to rise more rapidly and to be higher after age thirty. This empirical relationship illustrates the finding of previous studies, e.g., Griliches and Mason (1974) and many works cited in Jencks (1972), that measured cognitive ability has little effect on earnings at early ages. It is important to note, however, that most studies of the effect of ability on earnings have been for young men under thirty-five years of age. Since ability has its greatest effect late in the life cycle, either using samples of the young or ignoring interaction with age substantially understates the effect of ability.

Another important finding is that ability and schooling have a strong positive interaction with each other,[39] which operates primarily on the age-earnings profile; the higher an individual's ability, the greater the impact of his schooling on the age-earnings relationship, and the higher an individual's schooling, the greater the impact on his ability. These same positive interactions are also quite evident in their effect on human wealth.

These results can be interpreted in the context of the theoretical model. Remember that a change in the initial endowment of human capital, or earning capacity, has the effect of shifting the earnings profile up or down in the same direction as the change in endowment. If we interpret the background variables (all pre-high school except religion) as proxies for some of the effects of early public and family investments in children, then the effect of those variables will represent differences in initial endowment.

The large individual variance component (ϵ) in the earnings function residual is consistent with unmeasured differences in initial earning capacity. It is also consistent with unmeasured differences in investment patterns which are not exactly compensated for in present value. As measured, it includes both effects.

One way of interpreting the term "ability" is by examining differences in the efficiency with which additional human capital can be produced by the individual. The differences may represent individual differences in production inputs that are not under the control of the individual, including genetic endowments as well as production inputs provided by society or family. Differences in post-schooling ability or production efficiency result in earnings profiles which are initially lower for the more able, due to a greater level of investment, but rise more rapidly and surpass the earnings of the less able and remain greater throughout the life cycle. The greater investment by the more able is more than compensated in present value. If measured ability (measured just after high school) represents post-schooling production efficiency, this prediction is clearly verified by the data. The predicted earnings profile changes just as expected, and human wealth increases with increased ability.

Increased schooling, representing increased investment given initial endowment and ability, increases the period of forgone earnings, which is compensated for by greater earnings growth and increased earnings late in the life cycle. We thus have the prediction that some earnings inequality is compensated through differential investment and patterns of returns but that some inequality in human wealth is expected to persist because of differences in endowments, constraints, and abilities. Some of the differences in these are represented by measured variables; some are unmeasured, but are captured in the component for individual residual variance, which is the dominant source of the estimated inequality in human wealth. This finding indicates a need for further research.

Earnings and the Disaggregated Dimensions of Ability

In an exploration of the effects on the lifetime pattern of earnings of the various disaggregated dimensions of ability, the separate effect of reading comprehension, mathematics, mechanical dexterity, physical dexterity and general knowledge were studied rather than the IQ index. The primary finding was that mathematical ability affects the lifetime pattern of earnings in precisely the same way as the aggregate IQ index. None of the other ability indexes significantly affects the lifetime pattern but they do affect the level of earnings. (See Table A-2 for the estimated earnings function including the disaggregated ability variables.) Reading comprehension fails to show any net effect on earnings, given schooling and the other abilities. Mechanical dexterity and general knowledge each affect earnings, generally positively, but with a negative interaction between them. The role of physical dexterity is only to interact positively with general knowledge and negatively with mechanical dexterity, that is,

$$(39)\quad \partial Y/\partial(MECH) = 4{,}089 - 1{,}866(GENKN) - 1{,}826(PHYS)$$

$$\partial Y/\partial(GENKN) = 1{,}815 - 1{,}866(MECH) + 1{,}509(PHYS)$$

The effect of a change of one standard deviation in mechanical dexterity or general information at specific levels of other abilities is given in Table 5. While general knowledge always has a positive effect on earnings, mechanical dexterity has a negative effect at high levels of physical dexterity and general information.

TABLE 5 Changes in Annual Earnings for Given Changes in Ability (real 1970 dollars)

	∂ Y/∂ (MECH) for Given Values of GENKN			∂ Y/∂ (GENKN) for Given Values of MECH		
PHYS	0.75	1.00	1.25	0.75	1.00	1.25
0.75	333	216	99	388	271	154
1.00	219	102	−15	482	365	248
1.25	105	−12	−129	576	459	342

NOTE: Each ability score has a mean of 1.0 and a standard deviation of 0.25.

Earnings, Human Wealth, and the Life Cycle

An understanding of the relationship between variance in human wealth and in annual earnings may be developed by considering some straightforward illustrations. For simplicity, assume away exogenous earnings growth over time and differences in the length of working life and retirement age. Also assume a zero discount rate so that present values are sums. Consider first an example of how the variance in earnings can exceed the variance in human wealth. Assume that all individuals in a given population have the same lifetime earnings profile (earnings rise with age), but they differ in age. Therefore, at any specified age, there is zero variation both in human wealth and in earnings but at a point in time there is positive variation in earnings among individuals in the population. In addition, there is positive covariance between earnings values of adjacent years. Those with high earnings in the first year are older than the rest of the given population and have high earnings in the second year as well. If there are several earnings streams with the same present value but different rates of growth of earnings with age, these conclusions are unaltered except that there will be positive variance in the earnings of individuals of the same age, and earnings early and late in life will be negatively correlated among individuals. Clearly, in this illustration the coefficient of variation and the Gini coefficient of concentration will indicate equality of human wealth and positive inequality of earnings by age group or aggregated over ages.

Secondly, consider how the variance in human wealth can exceed the variance in earnings. Assume, contrary to the first illustration, that earnings do not vary with age (flat age-earnings profiles) but do vary among individuals. Variation in earnings will be the same at all ages and aggregated over ages. The variance in human wealth must exceed the variance in earnings since human wealth is the discounted sum of the constant earnings value. In this particular case the coefficient of variation in earnings at any given age exactly equals the coefficient of variation in human wealth. Inequality in earnings would then be an appropriate index of inequality in human wealth. If the age-earnings profiles are allowed to slope upward (but remain parallel to each other), any cross-sectional earnings distribution aggregated over all ages will have a larger variance than that of the earnings distribution at any particular age. The variance of the aggregate earnings distribution will depend on the age distribution of members of the aggregate as well as the distribution of profiles among members. The inequality in earnings at any age still accurately reflects inequality in human wealth even though the variance in human wealth is larger than the variance in earnings at any age.

Clearly, when the features of these two extreme illustrations are combined, i.e., when individual profiles differ in both mean level and lifetime pattern, either extreme may dominate. The major difference between the two illustrations is the degree to which differences in lifetime earnings profiles among individuals are compensated or uncompensated in present value and the degree of variation in uncompensated differences. The model to be considered empirically incorporates all of these features: various shapes of age-earnings profiles due to measurable variables, differences in human wealth due to unmeasured variables, and stochastic variation in earnings from year to year.

Mean Human Wealth

The expected value of human wealth for a given set of measured variables is estimated by summing discounted earnings values predicted from the estimated earnings function. The human wealth values presented in Table 6 in 1970 dollars are discounted to age sixteen, and full retirement is assumed at age sixty-six.[40] These values result from analysis of the effect of measured variables on the human wealth of a "representative individual."

A striking result is that while schooling has a greater effect on annual earnings, at any age, than does ability, the effect of schooling on mean human wealth is much more sensitive to discounting than is the effect of ability. At a zero discount rate schooling clearly has the dominant effect on lifetime earnings. However, cognitive ability continues to have a positive effect on human wealth at discount rates beyond which the effect of schooling has turned negative. The reason for the difference in sensitivity to discounting is that forgone earnings increase with additional schooling but not with greater ability. An increase in ability for a given schooling level is accompanied by an initial period of slightly lower earnings followed by greater earnings for the remainder of the life cycle, but no change in the age at which earning begins. Additional schooling may be thought of as an investment, while additional ability may be thought of as a greater endowment. The effect of a greater endowment of ability is consistent with greater on-the-job training investment, which is more than compensated.

TABLE 6 Mean Human Wealth, Assuming Full Retirement at Age Sixty-Six for a Protestant with Average Values for Other Social Variables (1970 dollars)

	Years of Schooling								
Ability[a]	12	13	14	15	16	17	18	19	20
Discount rate = zero									
Low	561,734	570,387	587,666	604,956	620,697	634,832	648,042	661,958	679,159
Average	590,258	590,257	604,210	624,534	649,015	677,003	708,208	742,350	780,300
High	620,864	629,259	645,082	666,390	692,553	722,883	756,962	794,537	835,340
Discount rate = 3 percent									
Low	252,447	251,049	252,256	253,773	255,099	256,116	256,899	257,786	259,380
Average	260,149	255,425	255,719	258,868	264,081	270,992	279,337	288,874	299,693
High	270,434	267,564	268,909	273,106	279,615	288,035	298,072	309,510	322,146
Discount rate = 5 percent									
Low	163,305	159,107	156,424	154,280	152,388	150,610	148,886	147,276	145,965
Average	166,144	160,014	156,981	155,992	156,557	158,374	161,195	164,796	169,117
High	170,960	165,399	163,198	163,166	164,790	167,737	171,754	176,643	182,230
Discount rate = 7 percent									
Low	113,200	107,650	103,253	99,566	96,390	93,570	90,985	88,578	86,363
Average	113,848	107,091	102,601	99,773	98,245	97,741	98,027	98,897	100,236
High	115,908	109,217	105,424	103,441	102,776	103,129	104,270	106,020	108,228

[a]The ability index is distributed with a mean of 1.0 and a standard deviation of 0.25. Low = 0.75; average = 1.00; and high = 1.25.

TABLE 7 Contribution of Schooling and Ability Variables to Mean Human Wealth (1970 dollars)

Source	Discount Rate 0%	3%	5%	7%
College vs. high school				
Low (0.75) ability	58,968	2,652	−10,917	−16,810
Average (1.00) ability	58,757	3,932	−9,587	−15,603
High (1.25) ability	71,689	9,181	−6,170	−13,132
Ph.D. or professional vs. college				
Low (0.75) ability	58,462	4,281	−6,423	−10,027
Average (1.00) ability	131,285	35,612	12,560	1,991
High (1.25) ability	142,787	42,531	17,440	5,452
Average to low ability				
High school	28,524	7,702	2,839	648
College	28,318	8,982	4,169	1,855
Ph.D. or professional	101,141	40,313	23,152	13,873
High vs. average ability				
High school	30,600	10,285	4,816	2,060
College	43,538	15,534	8,233	4,531
Ph.D. or professional	55,040	22,453	13,113	7,992

Table 7 clearly illustrates the strong positive interaction between ability and schooling in their effect on mean human wealth. The gain in human wealth from additional schooling increases with ability. The returns to ability are greater at successively higher levels of schooling. Similarly, the returns to schooling increase rather than decrease with more schooling and the return to a higher measured ability index is an increasing function of measured ability.[41] For example, at a discount rate of 3 percent, the difference in mean human wealth between a college and a high school graduate is $2,652 at low ability and more than three times that figure, $9,181, at high ability. The corresponding values for Ph.D. versus college are $4,281 and $42,531 respectively.

While the set of background data used here is quite limited, we can gain some ideas of their relative importance to human wealth from Table 8. Mother's education has a 20 percent greater effect on son's earnings and mean human wealth than does father's education. Consider, for example, that these estimates imply that the mother's attainment of a college degree versus a high school degree is associated with an increase of $17,776 in undiscounted lifetime earnings, compared to $14,784 for the same difference in attainment in father's education. These estimates are roughly 30 percent as large as the effect of the son's own attainment of college over high school for an average-ability son. The effect of parents' education is enhanced by their strong positive correlation with each other. The number of siblings has a negative effect

TABLE 8 **Contribution[a] of Social Variables to Mean Human Wealth (1970 dollars)**

Background Variable	Discount Rate 0%	3%	5%	7%
Father's education				
Each additional year	3,696	1,714	1,107	753
Mother's education				
Each additional year	4,444	2,061	1,331	905
Number of siblings				
Each additional sibling	−4,840	−2,245	−1,449	−986
Number pre-high-school moves				
Each additional move	1,232	571	369	251
Religion				
Jewish vs. Protestant	182,468	84,620	54,637	37,157
Catholic vs. Protestant	15,180	7,040	4,545	3,091

[a]Contributions are for college graduates.

on earnings and mean human wealth while the number of pre-high-school family moves has an insignificant positive effect. Religion, particularly if the person is Jewish, has by far the largest background effect.[42]

The direct effect of these background variables on earnings and mean human wealth appears to be rather small compared to schooling and ability. However, background variables also indirectly affect earnings and human wealth through their effects on schooling and ability, which are not considered here.

Inequality in Earnings versus Inequality in Human Wealth

Human wealth is substantially more equally distributed among members of the sample birth cohort than earnings within narrow life-cycle age ranges (see Table 9). Inequality in earnings at any stage of the life cycle beyond age 30 as measured by either the coefficient of variation or the Gini coefficient is 50 percent larger than inequality in human wealth.[43] This conclusion is not affected by changes in the discount rate.[44]

Since the members of the NBER-TH sample are slightly more homogeneous than all members of the 1917–1925 birth cohort with at least a high school degree, it is useful to compare them with a similar group from the 1960 Census population. The sample cohort group would be 35 to 43 years old in 1960.[45] The corresponding income (from all sources including earnings) inequality among those in the 1960 Census population who were 35 to 44 years old and had at least a high school degree was 0.69 as measured by the coefficient of variation and 0.33 by the Gini coefficient.[46] These differences are not exces-

TABLE 9 **Distribution[a] of Human Wealth and of Earnings (1970 dollars; figures in parentheses are upper-bound values on human wealth inequality)**

	Mean	Standard Deviation	Coefficient of Variation	Gini Coefficient	Skewness
		Distribution of Human Wealth, Assuming Full Retirement at Age 66			
Discount Rate					
0%	$674,146	$289,380	.43	.191	2.69
		($401,685)	(.60)		
3	277,533	115,878	.42	.191	2.94
		(181,305)	(.65)		
5	166,895	69,632	.42	.186	3.18
		(122,331)	(.73)		
7	106,775	45,483	.43	.187	3.38
		(87,603)	(.82)		
		Distribution of Earnings			
Age Group					
30–34	10,284	6,115	.59	.254	6.18
35–39	12,429	7,396	.60	.281	4.41
40–44	15,110	9,037	.60	.285	3.18
45–49	18,795	12,260	.65	.310	3.10

[a]Skewness is measured by the square root of M_x/S_x^3. The coefficient of variation is $S_x/\bar{X}$. Individual observations are weighted by the number of observed age-earnings points. $\bar{X}$ is the mean, S_x is the standard deviation, M_x is $\Sigma(X-\bar{X})^3/N$, and N is the number of observations.

sively large and are in the expected direction, since the NBER-TH group is more homogeneous than the total population.

The difference in inequality between earnings and human wealth is partly due to compensated differences in lifetime earnings profiles. Inequality in human wealth (HW) is largely dominated by the magnitude of variation in the persistent individual differences (ϵ). Variation in ϵ accounts for 40 percent of the total earnings variation, 57 percent of residual earnings variation, and 88 percent of the variation in undiscounted HW.

The importance of variation in ϵ is readily illustrated: schooling, ability, and background account for 10 to 12 percent of the total variation in human wealth, as measured by Var MHW/Var HW; the remainder (88 to 90 percent!) is attributed to variation in ϵ. Note that, within the narrow age groups, the variation in earnings explained by schooling, ability, and background, as measured by Var(MY/Age)/Var(Y/Age), is also 10 to 12 percent.

To illustrate further the importance of variation in ϵ consider the hypothetical alternative extreme values of zero and 100 percent of residual earnings variation due to ϵ. If all the residual variation were purely random, i.e., $\sigma_\epsilon^2 \equiv 0$, even within observations for the same individual, then inequality in human wealth would be solely due to the measured variables:[47] schooling, ability, and background. Under this restriction both the coefficient of variation and Gini coefficient are reduced to one-third their former levels (15 and 7 percent, respectively). At the other extreme, all residual differences persist over a lifetime, i.e., $\sigma_\mu^2 \equiv \sigma_\epsilon^2$. Under this assumption the upper bound on the coefficient of variation, presented in Table 9, is 50 to 100 percent greater than the estimated true values. The upper bound also ranges from about the same level to 50 percent larger than the coefficient of variation for earnings within the narrow age groups.

One may reasonably be interested in inequality within schooling or ability groups. The only subgroups with greater human wealth inequality than the aggregate are those with 13, 14, or 15 years of schooling (those who attended college but did not graduate): their respective coefficients of variation are 0.53, 0.48, and 0.49, as compared to an overall coefficient of 0.43 for undiscounted HW values. The greater inequality in those subgroups is due to greater dispersion rather than to a lower mean relative to other subgroups that are more equally distributed. The greater dispersion is in turn due to greater dispersion in the individual variance component, ϵ, rather than to schooling, ability, and background. Across schooling classes, the coefficient of variation of human wealth declines slightly with increased schooling, and across ability groups it declines slightly with increased ability. Again, this fall in inequality comes about because the rise in dispersion due to ϵ is less than proportionate to the rise in mean human wealth with increased ability or schooling. Inequality in annual earnings within schooling and ability subgroups is at least 50 percent greater than inequality in human wealth within the same subgroups.

Determinants of Years of Schooling

Years-of-schooling is used to measure the length of the period of specialization in investment in human capital. The effects of background variables and of ability on years of schooling are also examined. The OLS regressions of years of schooling on background and ability variables are presented in Table 10. Those background variables which reflect greater access to schooling subsidies and educational inputs are expected to be positively related to years of schooling. Those ability indexes[48] that are important in the production of human capital in the schooling environment will also have a positive effect on years of schooling.

To begin, each parent's schooling level has a significant positive effect on son's education which is dominated by the strong positive interaction of FED and MED. The partial effect of either parent's years of schooling depends on

TABLE 10 Schooling Regressions (OLS)
(dependent variable is years of schooling; figures in parentheses are absolute values of t statistics)

Independent Variables	(1)	(2)	(3)
Constant	13.106	13.17	14.88
	(8.7)	(9.6)	(15.6)
MATH	1.69		
	(11.1)		
RDG	0.720		
	(4.5)		
MECH	0.053		
	(0.3)		
PHYS	−1.070		
	(7.0)		
GENKN	0.699		
	(4.2)		
IQ		1.92	
		(14.0)	
FED	−0.059	−0.054	−0.046
	(1.9)	(1.7)	(1.4)
MED	−0.073	−0.063	−0.051
	(2.6)	(2.2)	(1.8)
FED * MED	0.0118	0.011	0.011
	(4.2)	(4.0)	(3.9)
CATH	−0.107	−0.081	−0.120
	(1.3)	(1.0)	(1.4)
JEW	0.169	0.242	0.305
	(1.1)	(1.6)	(2.0)
OTH RELG	0.169	0.403	0.435
	(3.2)	(3.2)	(3.4)
NO. OF SIB	0.028	0.041	0.044
	(1.6)	(2.2)	(2.4)
NO. MOVES	−0.019	−0.021	−0.011
	(1.0)	(1.0)	(0.6)
MOTH WK FULL (0–5)	0.234	0.188	0.186
	(1.1)	(0.8)	(0.7)
MOTH WK SOME (0–5)	−0.55	−0.092	−0.097
	(0.4)	(0.7)	(0.8)
MOTH WK FULL (6–14)	0.246	0.289	0.240
	(1.3)	(1.5)	(1.2)
MOTH WK SOME (6–14)	0.119	0.59	0.165
	(0.9)	(1.2)	(1.3)
MOTH WK NR (6–14)	−0.297	−0.272	−0.377
	(2.1)	(1.9)	(2.6)

TABLE 10 (concluded)

Independent Variables	(1)	(2)	(3)
Father's occup.			
White collar	0.080	0.104	0.127
	(1.0)	(1.3)	(1.6)
Other	−0.181	−0.186	−0.269
	(1.5)	(1.6)	(2.3)
OWN HOME	0.020	0.014	0.019
	(0.2)	(0.2)	(0.2)
OWN ROOM	0.007	0.008	0.043
	(0.09)	(0.1)	(0.5)
ELEM PRIV	0.292	0.183	0.441
	(0.6)	(0.4)	(0.9)
HS PRIV	0.327	0.332	0.412
	(1.4)	(1.3)	(1.7)
HS VOC	−1.17	−1.29	−1.42
	(11.1)	(12.3)	(13.3)
R^2	.144	.126	.089

[a] The composition of the six ability variables is given in Table 2. The omitted religious class is Protestant. MOTH WK is mother's work status during the indicated age range of the son (in parentheses): 0–5 or 6–14 years. The omitted class did not work at all. OWN HOME is a dummy variable to indicate that the family owned its home. OWN ROOM is a dummy variable to indicate that the person had his own room as a youth. PRIV indicates private school, VOC indicates vocational school, and the omitted type of school attended is parochial school.

the other parent's years of schooling in such a way that they complement each other; i.e.,

$$(40)\quad \partial S/\partial MED = -0.073 + 0.0118\,FED$$

$$\partial S/\partial FED = -0.059 + 0.0118\,MED$$

Therefore,

$$\partial S/\partial MED = \begin{cases} 0.005 \text{ for } FED = 6.5 \\ 0.043 \text{ for } FED = 10.0 \\ 0.081 \text{ for } FED = 13.5 \end{cases}$$

(41)

$$\partial S/\partial FED = \begin{cases} 0.018 \text{ for } MED = 6.5 \\ 0.059 \text{ for } MED = 10.0 \\ 0.100 \text{ for } MED = 13.5 \end{cases}$$

While the effect of parents' education is statistically very significant, it is nevertheless too small to be of any economic importance. At the mean level of the other parent's education, the mean effect of a parent with six years of schooling (sixth grade) versus a college education (sixteen years) is roughly half a year. The small effect of background in general and parents' schooling in particular reflects the unusually high ability of this group of men and the availability of the GI bill, which lowered capital costs for everyone in the sample.

The effect of mother's work status and father's occupation is negligible. Being of Catholic religion has a negative effect of 0.11 year and being Jewish has a positive effect of 0.17 year relative to the Protestant religion. Attendance at a vocational high school has a negative effect of 1.17 years. Interestingly, the number of siblings in the family does not significantly affect the years of the respondent's schooling attainment, even when older and younger siblings are distinguished.

The ability variables are individually more significant than any particular background variable. The disaggregated ability most strongly related to schooling attainment is mathematics, followed by reading comprehension and general knowledge. Mechanical dexterity has no effect, and physical dexterity has a strong negative relation. While these abilities have a significant effect, the magnitudes of the effects are fairly small. The effect on schooling of a change of one standard deviation in ability is 0.48 year for the aggregate IQ index, 0.42 for mathematics, 0.18 for reading comprehension, −0.27 for physical dexterity, 0.17 for general knowledge, and zero for mechanical dexterity.

Determinants of IQ-Type Ability

The regressions relating the various ability indexes to background variables are presented in Table 11.[49] The fairly comprehensive set of background variables explains nearly 6 percent of the variation in the IQ measure. Parents' schooling attainment affects ability positively. The effect of an additional year of mother's education is about 35 percent larger than the effect of an additional year of father's education. Men whose fathers were in white-collar occupations had slightly higher IQs, but those men whose fathers were in other than standard white-collar or blue-collar occupations had significantly lower IQs. If the mother worked full time, there is a small, statistically insignificant, negative impact on IQ. The negative effect is greater, but still not statistically significant, if the son was 6–14 years old when his mother worked than if he was younger (0–5 years old). The small size of this effect is itself notable. Having had his own room as a child significantly increases the IQ score. Attendance at a private instead of a public school has a significant positive effect on IQ; this effect is nearly three times as large for private elementary school attendance as for private high school attendance. Attendance at a vocational high school has a significant negative effect on IQ. On average, Jewish men scored significantly higher in IQ, and Catholic men significantly lower, than Protestants. Additional family moves before high school completion significantly increased IQ.

TABLE 11 Ability Regressions[a] (OLS)
(coefficients are in X 10² terms; figures in parentheses are absolute values of t statistics)

Independent Variable	IQ	MATH	RDG	MECH	PHYS	GENKN
Constant	88.73	95.05	90.86	89.16	101.88	79.08
	(27.1)	(28.7)	(27.6)	(27.2)	(30.9)	(25.1)
FED	0.452	0.261	0.18	0.506	−0.230	0.703
	(1.3)	(0.8)	(0.5)	(1.5)	(0.7)	(2.2)
MED	0.619	0.378	0.489	0.611	−0.331	1.15
	(2.0)	(1.2)	(1.6)	(2.0)	(1.1)	(3.9)
FED * MED	0.0020	−0.0071	0.0099	−0.0019	0.048	0.0048
	(0.07)	(0.2)	(0.3)	(0.06)	(1.6)	(0.2)
NO. MOVES	0.522	0.163	0.737	0.553	0.613	0.931
	(2.5)	(0.8)	(3.6)	(2.7)	(3.0)	(4.7)
NO. OF SIB	0.142	0.580	−0.020	−0.262	−0.586	−0.0145
	(0.73)	(3.0)	(−0.1)	(1.4)	(3.0)	(0.2)
CATH	−2.05	0.711	−4.37	−3.82	−4.80	−6.20
	(2.3)	(0.8)	(4.9)	(4.3)	(5.4)	(7.2)
JEW	3.25	9.68	−2.77	−6.47	−5.14	−8.57
	(2.0)	(5.9)	(1.7)	(4.0)	(3.1)	(5.5)
OTHER RELG	1.63	−0.388	4.76	0.693	−0.114	0.420
	(1.2)	(0.3)	(3.6)	(0.5)	(0.08)	(0.3)
OWN HOME	0.279	−1.40	1.74	1.63	0.584	2.24
	(0.32)	(1.6)	(2.0)	(1.9)	(0.7)	(2.7)
OWN ROOM	1.78	0.892	0.555	2.11	1.01	3.79
	(2.1)	(1.0)	(0.7)	(2.5)	(1.2)	(4.6)
ELEM PRIV	13.38	8.77	2.12	15.19	6.26	6.23
	(2.6)	(1.7)	(0.4)	(2.9)	(1.2)	(1.3)
HS PRIV	4.65	5.07	4.39	−1.33	0.291	−4.11
	(1.8)	(1.9)	(1.7)	(0.5)	(0.1)	(1.7)
HS VOC	−6.64	−9.05	−3.93	0.772	4.85	−2.45
	(5.9)	(8.0)	(3.5)	(0.7)	(4.3)	(2.3)
Father's occup.						
White collar	1.20	0.764	1.86	0.489	−0.991	1.40
	(1.4)	(0.9)	(2.2)	(0.6)	(1.1)	(1.7)
Other	−5.33	−3.71	−6.11	−3.88	−2.85	−4.24
	(4.2)	(2.9)	(4.8)	(3.1)	(2.2)	(3.5)
Mother's work status						
MOTH WK SOME (0–5)	−0.084	−2.12	0.866	2.13	7.27	1.26
	(0.03)	(0.8)	(0.3)	(0.8)	(2.7)	(0.5)
MOTH WK FULL (0–5)	−0.268	−1.55	0.313	1.21	1.84	0.115
	(0.2)	(1.2)	(0.2)	(0.9)	(1.4)	(0.09)
MOTH WK SOME (6–14)	−2.55	−1.33	−1.93	−3.77	−4.68	−2.45
	(1.2)	(0.6)	(0.9)	(1.8)	(2.2)	(1.2)

TABLE 11 (concluded)

Independent Variable	Dependent Variable					
	IQ	MATH	RDG	MECH	PHYS	GENKN
MOTH WK FULL	0.314	1.06	1.65	−1.64	−0.776	1.16
(6–14)	(0.2)	(0.8)	(1.2)	(1.2)	(0.6)	(0.9)
Work status						
nonresponse	−5.497	−4.25	−3.38	−4.01	−3.66	−3.07
(6–14)	(3.7)	(2.8)	(2.3)	(2.7)	(2.4)	(2.1)
R^2	.0569	.0386	.0502	.0457	.0255	.1118

[a]The composition of the six ability variables is given in Table 2. MOTH, OWN HOME, OWN ROOM, PRIV, and VOC are described in Table 10, footnote a.

Determinants of the Specific Types of Ability Background variables. Mother's and father's schooling positively affects each type of ability except physical dexterity. The mother's schooling is more important in size and significance for each variable, with the strongest effect on general knowledge. If the mother worked full time when the respondent was 6–14 years old, each ability type variable is lowered, but if his mother worked full time when he was younger than 6, his mean ability is increased in each case except mathematics. If his father was employed in a white-collar occupation instead of a blue-collar occupation, each type of ability is increased except physical dexterity. If he had his own room, each ability score is increased, with the largest effect on general knowledge, followed by mechanical dexterity. If he attended a private school, the mathematical and reading comprehension scores are increased, but not the others. Additional family moves increase each type of ability.

The overall explanatory power of the background variables with respect to ability scores is poor. The best is for general knowledge, with 11 percent. The worst is for physical dexterity, 2 1/2 percent.

APPENDIX A: THEORETICAL MODEL: SOME SPECIAL CASES

There are a few special cases for which the solutions are different from the one presented here, including (a) the case of no deterioration of human capital, i.e., $\delta = 0$; (b) no purchased inputs, i.e., $\beta_2 = 0$; (c) both no deterioration and a zero discount rate; (d) no production decision, i.e., $\beta_1 + \beta_2 = 0$; and (e) constant returns to scale, i.e., $\beta_1 + \beta_2 = 1$. The solution is simplified when $\beta_1 + \beta_2$ equals a rational number, especially $\beta_1 + \beta_2 = (J-1)/J$ where J is an integer, and in particular, $\beta_1 + \beta_2 = 1/2$. In the latter case some additional implications can be derived.

These are special cases often assumed by other authors, and it is important to note their effects. Haley (1973) analyzes in detail the special cases where there are no purchased inputs ($\beta_2 = 0$) and, in particular, where $\beta_2 = 0$, $\beta_1 = 1/2$ and includes some simulation results.

No Deterioration of Human Capital

This case must be considered separately because setting $\delta = 0$ involves division by zero in many of the equations of the general solution. Clearly, equation 2 becomes $\dot{E} = Q(K, D)$, so its solution in Phase I becomes

$$\text{(A.1)} \quad E = [E_0^{\Delta} + \Delta U t]^{1/\Delta}$$

Hence, $\dot{E} > 0$ and $\ddot{E} > 0$ for every $t < t^*$.

Phase II solutions for I and E may be obtained directly by substituting $\delta = 0$ into equations 15 through 21, and 23. Equation 22 becomes

$$\text{(A.2)} \quad E(t^*) = [E_0^{\Delta} + \Delta U t^*]^{1/\Delta}$$

so that the implicit solution for t^* is contained in

$$\text{(A.3)} \quad E_0^{\Delta} = -\Delta U t^* + (\beta'/r)\beta_1^{1-\Delta} (\beta_1 + \beta_2)(R\beta_2/P\beta_1)^{\beta_2} (1 - e^{r(t^*-N)})$$

Earning capacity and net earnings rise over the entire life cycle, i.e., in Phase II

$$\text{(A.4)} \quad \dot{E} = U^{1/\Delta} [(1 - \Delta)/r]^m (1 - e^{r(t-N)})^m > 0$$

and

$$\text{(A.5)} \quad N\dot{Y} = R\dot{E} - \dot{I} > 0$$

Earning capacity rises at a decreasing rate and is concave, i.e.,

$$\text{(A.6)} \quad \ddot{E} = -rU^{1/\Delta} [(1 - \Delta)/r]^m e^{r(t-N)} (1 - e^{r(t-N)})^m < 0$$

while net earnings has the possibility of an initial convex stage where possibly $\ddot{I} < 0$ is large enough to offset the concave growth in earning capacity, i.e.,

$$\text{(A.7)} \quad N\ddot{Y} = R\ddot{E} - \ddot{I} > 0 \text{ if } \ddot{I} < 0 \text{ and } \ddot{I} > |R\ddot{E}|$$

The more likely case and necessarily so late in the life cycle is that $N\ddot{Y} < 0$. See the discussion of equation 7 for more detail.

Both $\delta = 0$ and $r = 0$

In the case of both no deterioration and zero interest rate, some equations need further alteration. The Phase I solutions are the same as for $\delta = 0$ alone, since r does not enter the solution.

In Phase II, however, the shadow value of human capital accumulation decreases linearly with time; so equation 15 becomes

(A.8) $\lambda_1 = R(t - N)$

The solution for the path of optimal investment, equation 17, becomes

(A.9) $I = R(\beta_1 + \beta_2)/\beta_1 \, [(N - t)\beta''\beta_1 \, (R\beta_2/P\beta_1)^{\beta_2}]^{1/\Delta}$

Since investment is a negative linear function of time to a power greater than 1.0, investment declines at a decreasing rate at all ages, i.e., $\dot{I} < 0$ and $\ddot{I} > 0$. Equation 18 becomes

(A.10) $\dot{E} = U^{1/\Delta} \, (1 - \Delta)^{1/\Delta} \, (N - t)^{(1-\Delta)/} > 0$

so that

(A.11) $E = (E_0^{\Delta} - \Delta U t^{*})^{1/\Delta} + U^{1/\Delta} \, (1 - \Delta)^{1/\Delta} \, [(N - t^{*})^{1/\Delta} - (N - t)^{1/\Delta}]$

for t > t^{*}. Clearly earning capacity rises at a decreasing rate, i.e.,

(A.12) $\ddot{E} = -(1 - \Delta)\dot{E}/\Delta(N - t)$

In this case, the net earnings function is also strictly concave.

No Production Decisions, i.e., $\beta_1 - 0$ and $\beta_2 - 0$

This trivial case is presented only for completeness. When β_1 and β_2 are zero, and thus $\beta_1 + \beta_2 = 0$, Phase I and indeed investment make no sense and earnings (equal to earning capacity) simply decline exponentially at the depreciation rate. If $\beta > 0$, earning capacity is augmented in each period by a constant; therefore, $\dot{E} = \beta - \delta E$ and

(A.13) $E = (\beta/\delta) + [E_0 - (\beta/\delta)] \, e^{-\delta t} = E_0 e^{-\delta t} + (\beta/\delta)(1 - e^{-\delta t})$

The exponential decline in earnings is partially offset by the linear additions to human capital in each period. When $\beta = 0$, earnings simply decline as $E = E_0 e^{-\delta t}$. If depreciation is zero, then earnings are constant or rise linearly if $\beta \neq 0$.

Constant Returns to Scale, i.e., $\beta_1 + \beta_2 - 1$

With constant returns to scale the optimal rule is to specialize in the production of human capital until the last instant of life and then use all of the capital to obtain earnings instantaneously.

During Phase I, equation 10 becomes

(A.14) $\dot{E} = (U - \delta)E$

which has the solution

$$(\text{A.15}) \quad E = E_0 e^{(U-\delta)t}$$

Earning capacity rises exponentially. In Phase II, $\lambda_2 = 0$ and the solution for λ_1 is unchanged, as in equation 15. Substituting (8) and (15) into first-order condition 6b and simplifying yields the condition

$$(\text{A.16}) \quad (1 - e^{(r+\delta)(t-N)})K = 0$$

which is satisfied only at $t = N$.

Alternatively, let $\beta_1 + \beta_2$ approach 1 in equations 16 and 17. Optimal K and I increase monotonically as $\beta_1 + \beta_2$ increases with the ratio β_1/β_2 held constant. For every $\beta_1 + \beta_2 < 1$ there exists some Phase II interval of length $N - t^*$; however, $N - t^*$ shrinks to zero as $\beta_1 + \beta_2$ approaches 1.

Integer Values of 1/Δ Greater than One

The solution to Phase II becomes somewhat simpler for integer values of $1/\Delta$ greater than one. If $1/\Delta = J$ contains integers >1 then $(1 - \Delta)/\Delta = J - 1$ and $(\beta_1 + \beta_2) = (J - 1)/J$. Returns to scale take the values 1/2, 2/3, 3/4, 4/5,... . The simplification occurs because the right-hand side of the differential equation 17 can be represented as a finite sum of J terms rather than the more general infinite binomial series. Equation 18 becomes

$$(\text{A.17}) \quad \dot{E} + \delta E = U \sum_{i=0}^{J-1} (-1)^i \binom{J-1}{i} e^{i(r+\delta)(t-N)}$$

which has a solution as in equation 21:

$$(\text{A.18}) \quad E = E(t^*)\, e^{\delta(t^*-t)} + U_2 \sum_{i=0}^{J-1} \frac{(-1)^i}{\delta + i(r+\delta)} \binom{J-1}{i} e^{i(r+\delta)(t-N)} \left(1 - e^{[\delta + i(r+\delta)](t^*-t)}\right)$$

This is quite a simplification if J is small; e.g., if $J = 4$, then returns to scale $= 3/4$ and there are only four terms in equation A.18, including the initial value. The solution is obviously more amenable to analysis and potential estimation.

Returns to Scale ($\beta_1 + \beta_2$) of One-half

When $\beta_1 + \beta_2 = 1/2$, the solution becomes extremely simple. The human capital path in Phase I, equations 12 and 22, simplify to

$$(\text{A.19}) \quad E = [(U/\delta)(1 - e^{-1/2\delta t}) + \sqrt{E_0}\, e^{-1/2\delta t}]^2$$

where

$$(\text{A.20}) \quad U = \beta\sqrt{2\beta_1}\,(R\beta_2/P\beta_1)^{\beta_2}$$

In Phase II investment, equation 17 becomes

$$\text{(A.21)} \quad I = [R/2(r + \delta)]U_2 (1 - e^{(r+\delta)(t-N)})^2$$

Differential equation 18 simplifies to

$$\text{(A.22)} \quad \dot{E} + \delta E = U_2 (1 - e^{(r+\delta)(t-N)})$$

with the very simple solution

$$\text{(A.23)} \quad E = [(U_1/\delta)(1 - e^{-1/2\delta t^*}) + \sqrt{E_0}\, e^{-1/2\delta t^*}]\, e^{\delta(t^*-t)}$$
$$+ (U_2/\delta)(1 - e^{-\delta t}) + [U_2/(r + 2\delta)]e^{(r+\delta)(t-N)} (e^{(r+\delta)t^*}\, e^{-(r+2\delta)t} - 1)$$

Equation 26, which specifies the implicit solution for t^*, simplifies to

$$\text{(A.24)} \quad \sqrt{E_0} = \beta' \sqrt{2\beta_1}\, (R\beta_2/P\beta_1)^{\beta_2} \left[\frac{1}{\delta} - \frac{(2r + \delta)}{2\delta(r + \delta)}\, e^{1/2\delta t^*}\right.$$
$$\left. \frac{1}{-2(r + \delta)}\, e^{-(r+\delta)N}\, e^{(r+3\delta/2)t^*}\right]$$

NOTES

1. These works have been carefully reviewed by Mincer (1970).
2. Similarly, Paglin (1975) recently suggested including in measured inequality only variation around the aggregate cross-sectional mean earnings-age profile.
3. The model requires a number of restrictive assumptions to be feasible. Individuals are assumed to have perfect knowledge of themselves and the world and face no uncertainty. Furthermore, they receive no income other than from the rental of their human capital, and they have no initial assets. All problems involving leisure time are avoided by assuming that a fixed amount of time in each period is allocated between investment in human capital and labor market earnings. Under these assumptions, consumption and investment decisions are separable, and individuals act in such a way as to maximize their human wealth. Thus, it is not necessary to posit a utility function.
4. This model can be equivalently formulated in terms of the fraction of time or human capital invested in production, S; then $K = S \cdot E$ and S is the decision variable. In another interesting formulation, used by Rosen (1975), Weiss (1975), and Blinder and Weiss (1974), earnings are a direct function of the human capital stock and its rate of change at any given time. Under this formulation $NY(E, \dot{E}) = R[E - (\dot{E} - \delta E^{1/(1-\Delta)})/\beta]$.
5. This is a pedogogically opposite extreme from the constraint proposed by Ben-Porath (1967): $R[E(t) - K(t)] - g^2(t) = 0$. The Ben-Porath form implies that the only constraint on production is that an individual cannot invest more human capital than his total stock, i.e., direct purchases can be financed by borrowing. Since both models yield the same qualitative results except during the specialization period and since the model using equation 3 can be fully solved, the analysis is confined to include equation 3.
6. All the results derived in this section generalized perfectly to the case of any number of purchased inputs and corresponding prices. These are ignored here to simplify the presentation.

7. From this point forward the t subscript will be dropped for notational convenience.
8. The special cases of 0 and 1 are considered separately in Appendix A.
9. Although efficiency may vary more generally over the life cycle, I assume that it differs only between the period of specialization (schooling) and the rest of the life cycle.
10. This result is proved more generally in Ishikawa (1973).
11. $\beta_1 + \beta_2\ (U/\delta) - E^{\Delta} > 0$ is a sufficient condition for the existence of Phase I, as will be proved after the Phase II discussion.
12. Many Phase II results are presented by Ben-Porath (1967) in an alternative format. While a closed form solution was not derived there, most qualitative predictions are valid up to initial conditions at the end of Phase I. An important consideration is the point where phases I and II meet, which will be discussed in detail later.
13. Note that $1/\Delta > 1$ since $\Delta = 1 - \beta_1 - \beta_2$ and thus $\ln(1/\Delta) > 0$. Also consider the point of inflection for the following parameter combinations for the length of the convex investment region from $N - [\ln(1/\Delta)/(r + \delta)]$:

	$\beta_1 + \beta_2$				
$r + \delta$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$
0.20	0.7	1.5	3.5	7.0	10.4
0.15	1.1	2.3	5.3	10.5	15.6
0.10	1.3	2.9	6.9	13.9	20.8
0.05	2.6	5.8	13.8	27.8	41.6
0.01	13.1	29.0	69.3	138.6	207.9

Clearly, for high returns to scale and low interest and depreciation rates, the concave region may not occur. Also since $Q = (R\beta_2/P\beta_1)^{\beta_2} K^{1-\Delta}$, $\dot{Q} < 0$, and $\dot{Q}$ has the sign of $\ddot{I}$ in the same ranges.

14. This approach to the solution is suggested by Haley (1974). Haley also illustrates by simulation that the infinite sum is strongly dominated by the first few terms for reasonable parameter values. Note that

$$\binom{a}{b} = \frac{a!}{b!(a-b)!}$$

15. An interesting special case obtains when Phase I fails to exist, that is, it is optimal for the individual to leave school at the end of the period of compulsory school attendance (age 16 in most states). Alternatively an individual's life cycle may be presumed to begin ($t = 0$) at some age after specialization ends (e.g., after age 40). In the latter case comparative statics can only be considered over parameter ranges which still preclude the existence of Phase I; i.e., the individual is "old enough" to rule out specialization. In this special case $E(t^*)$ is replaced by E_0 in equation 21.

   Note also that equation 21 can be expressed alternatively as

$$E = E(t^*)e^{-\delta(t-t^*)} - U_2 e^{-\delta t} \int e^{\delta t^*} (1 - e^{(r+\delta)(t^*-N)})^m \, dt^*$$

$$+ U_2 e^{-\delta t} \int e^{\delta t} (1 - e^{(r+\delta)(t-N)})^m \, dt$$

16. This point is discussed in detail later. Alternative formulations yielding similar jump points at t^* are developed in Johnson (1974) and Haley (1975).
17. In fact, $RE(t^*)/I(t^*) = (\beta^I/\beta^{II})^{1/\Delta}$. If $\beta^I = \beta^{II}$, then net earnings begin at zero as in the original Ben-Porath (1967) model.

18. Equation 23 may be expressed alternatively as

$$NY = RE(t^*)e^{-\delta(t-t^*)} + RU_2 e^{-\delta t} [\int e^{\delta t}(1 - e^{(r+\delta)(t-N)})^m \, dt$$

$$- \int e^{t^*} (1 - e^{(r+\delta)(t^*-N)})^m \, dt^*] - RU_2' \frac{\beta_1 + \beta_2}{r + \delta} (1 - e^{(r+\delta)(t-N)})^{m+1}$$

19. When an individual is deciding whether to specialize or not, the relevant efficiency parameter is β^I rather than β^{II}, since applicable conditions will be those of Phase I should he decide to specialize.
20. For empirical purposes, a stochastic specification of t^* and NY may be more reasonable, assuming t^* can vary for unspecified reasons, in which case $(\delta E/\delta X)|_{t^*}$ is also interesting. These comparative statics are presented at the end of the section.
21. From (23), the partial effect of E_0 on t^* can be ascertained more formally by implicit differentiation: With E_0^{Δ} moved to the right-hand side of the equation, multiply by $-(\delta/U)$, and call the result Z; that is,

$$\partial t^*/\partial E_0 = -(\partial Z/\partial E_0)/(\partial Z/\partial t^*)$$

where

$$Z = 1 - e^{-\delta\Delta t} (1 - \delta E_0^{\Delta}/U) - [\delta(1 - \Delta)(1 - e^{(r+\delta)(t^*-N)})/(r + \delta)]$$

and U includes $\beta = \beta^I$. The denominator of this term will be the same for all parameters, as follows:

$$\partial Z/\partial t^* = e^{-\delta\Delta t^*} [1 - (\delta E_0^{\Delta}/U)] + \delta(1 - \Delta) e^{(r+\delta)(t^*-N)} > 0$$

$\partial Z/\partial t^*$ is necessarily positive, from equation 22b, when Phase I exists. The sign of partial effects on t^* is then determined by (and is opposite in sign from) partial effects on Z. In this case

$$-\partial Z/\partial E_0 = -(\delta\Delta E_0^{\Delta-1} e^{\delta\Delta t^*}/U) < 0$$

Hence, increasing E_0 has the posited negative effect on the length of the specialization period.
22. The effects of β_1 and β_2 on t^* and the optimal paths are ambiguous otherwise.
23. Special cases of 0, 1/2, 1, and integer values of $1/\Delta$ are presented in detail elsewhere. The cases of $\beta_2 = 0$ (i.e., there are no purchased inputs) and $\beta_1 = 1/2$ are discussed in detail by Haley (1975).
24. More formally, $-(\partial Z/\partial \beta) = \beta^I E_0^{\Delta}\delta/U_2 > 0$.
25. E_0 and β^I must move in the same direction to maintain the same value of t^*, since a rise in E_0 shortens time in specialization while a rise in β^I lengthens it.
26. That is,

$$-\partial Z/\partial r = \delta(1 - \Delta) \{[1 + (N - t^*)(r + \delta)]e^{(r+\delta)(t^*-N)} - 1\}/(r + \delta) < 0$$

27. More formally, $-\partial Z/\partial N = \delta(1 - \Delta) e^{(r+\delta)(t^*-N)} > 0$.
28. That is, $\partial I/\partial P = (-\beta_2 I/P\Delta) < 0$.
29. That is, $\partial I/\partial R = [1 + (\beta_2/\Delta)]I/R > 0$.
30. It should be noted that in this form of the earnings function, any differences among birth cohorts within the narrow 1917–1925 cohort group are ignored. This issue will be explored only briefly in the empirical section. More importantly, since earnings represent repeated observation of the 1917–1925 cohort group, any exogenous real earnings growth over the

period 1943–1970 will be confounded in the age variable. See Weiss and Lillard (1978) for a more detailed discussion of confounding of vintage, experience, and time effects.

31. For a discussion of the identification in similar approximations to nonlinear models, see Fisher (1967).

32. Estimates of human wealth exclude consideration of earnings while in school, for which no data are available, and of the respondents' reduction in earnings during military service. Age-earnings profiles are assumed to be flat beyond the upper end of the sample range, about age 54, since the profiles in Figure 4 appear to peak there.

33. This procedure for calculating individual human wealth is analogous to estimating the earnings function with discounted earnings values as the dependent variable. Alternative estimates of the variance in human wealth obtained by ignoring the earnings function and estimating each individual's human wealth directly from his observed earnings values were very close to those reported here.

34. The standard deviations of mean discounted residuals are \$6,054, \$5,283, \$4,871, and \$4,555, respectively, for discount rates of zero, 3, 5, and 7 percent.

35. To make the corresponding estimates unbiased, all variance estimates presented here are weighted for unequal numbers of observations for each individual and are corrected for the finite number of multiple observations.

Similar results from panel data on a wider range of birth cohorts observed over a shorter period are reported by Lillard and Weiss (1977) for a sample of Ph.D. scientists observed over the decade 1960–1970 and by Lillard and Willis (1977) for a national sample of men from the Michigan Income Dynamics Panel, who were observed over the period 1967–1973.

36. One source of variation in ϵ is cohort differences within the 1917–1925 cohort group due to differences in, say, schooling quality or exogenous wage growth. This source is clearly evident when the mean values of ϵ across cohorts are compared: \$1.020 for 1925; \$800 for 1924; \$18 for 1923; −\$62 for 1922; −\$875 for 1921; −\$745 for 1920; −\$347 for 1919; −\$1,329 for 1918; and −\$1,924 for 1917. The variances, however, do not vary systematically among cohorts.

37. For $T_i = 5$; $\hat{\theta} = 0.13$ and $1 - \hat{\theta} = 0.87$; for $T_i = 4$, the figures are 0.16 and 0.84; for $T_i = 3$, 0.20 and 0.80; and for $T_i = 2$, 0.27 and 0.73.

38. While the parameters themselves may appear to be quite different, once the nonlinearities and interactions are accounted for, the predicted profiles are quite close.

39. This positive interaction is enhanced by a positive simple correlation between schooling and ability in the data of 0.245.

40. Varying the retirement age between fifty-four and seventy made no differences in the inequality conclusions.

41. This result may be partially due to the composition of the sample studied which includes only highly able and well-educated men.

42. Part of the effect of this variable may be due to the city size of the respondent's residence; much of the Jewish population resides in the New York metropolitan area which has substantially higher wages than most other parts of the United States.

43. For a detailed discussion of alternative inequality indexes, see Atkinson (1970). As measured by the coefficient of variation, inequality among individuals in single-year cohorts rather than the 1917–1925 cohort group is 0.39 for 1925; 0.39 for 1924; 0.40 for 1923; 0.44 for 1922; 0.48 for 1921; 0.47 for 1920; 0.53 for 1919; 0.43 for 1918; and 0.49 for 1917. The major source of these differences is mean human wealth rather than its standard deviation. The greater mean human wealth for the younger cohorts is due to differences in mean human wealth caused by schooling, ability, and background differences and by differences in mean δ (as indicated in footnote 36).

44. Several retirement ages were considered, including mean retirement age (based on labor

force participation rates) in each schooling group. It made virtually no difference in the inequality conclusions reached here.

45. These inequality estimates may be compared to the more usual cross-sectional inequality figures. Since earnings are roughly uniformly distributed over ages within the sample (except ages over 57), a simple aggregate of the 15,387 earnings points over all ages crudely approximates the distribution of earnings of a cross section but with only a narrow cohort observed. If there are no cohort or exogenous wage growth effects it is precisely analogous to a cross-sectional earnings distribution. Inequality in this aggregate is 0.75 as measured by the coefficient of variation and 0.353 by the Gini coefficient.

46. Calculated from Final Report PC(2)-5A, Census of Population: 1960, U.S. Bureau of the Census, 1963.

47. For positive discount rates the corresponding assumption must be that all residual variation is exactly compensated in present value.

48. In the context of the model, ability can have several commonly used meanings. A common use is "the ability to produce earnings." This is ambiguous since it may be interpreted to mean either net earnings or earning capacity. This use could imply the current stock of human capital, if all of it were allocated to work, or the capacity to produce a future stream of earnings, if all of current capacity were invested in producing more human capital. These two interpretations are related but distinct. Clearly, net earnings are directly affected by current investment. One person may have more earning capacity than another at the same age but lower net earnings because of a larger investment in human capital. If earnings capacity is meant then it should be measured at some common age to reflect a common position in the life cycle. A convenient age is the school-leaving age, t^*. Another common conception of ability is "the rate at which an individual accumulates earning capacity." The actual rate of accumulation of human capital is represented in the model by $\dot{E}(a)$. Another interpretation of ability intimately related to these and suggested by the model is the efficiency with which the individual produces new human capital, represented by the production parameter, β. That is, the index of ability is the relative efficiency with which an individual can use a given set of inputs to produce new human capital.

49. It should be made clear that the background variables are related to a weighted combination of test scores representing an identifiable ability type. Regression results must be interpreted with caution since the underlying test scores represent ordinal rankings rather than a cardinal measurement.

REFERENCES

Becker, G. 1964. *Human Capital.* New York: NBER.

Ben-Porath, Y. 1967. "The Production of Human Capital and the Life Cycle of Earnings." *Journal of Political Economy,* August.

Blinder, A. S., and Y. Weiss. 1976. "Human Capital and Labor Supply: A Synthesis." *Journal of Political Economy,* June.

Fisher, F. 1976. *The Identification Problem in Econometrics.* Huntington, N.Y.: Krieger.

Griliches, Z., and W. M. Mason. 1972. "Education, Income, and Ability." *Journal of Political Economy,* May/June.

Haley, W. J. 1973. "Human Capital: The Choice Between Investment and Income." *American Economic Review,* December.

______. 1975. "Estimation of the Earnings Profile from Optimal Human Capital Accumulation." Michigan State University. Workshop Paper 7402.

Hause, J. 1975. "Ability and Schooling as Determinants of Lifetime Earnings, or If You're So Smart, Why Aren't You Rich?" In *Education, Income, and Human Behavior*. Other Conference 9. New York: NBER.

Houthakker, H. S. 1959. "Education and Income." *Review of Economics and Statistics*, February.

Ishikawa, T. 1973. "A Simple Jevonian Model of Educational Investment Revisited." Harvard Institute of Economic Research. Discussion Paper 289. April.

Jencks, C. 1972. *Inequality*. New York: Harper & Row.

Johnson, T. 1970. "Returns from Investment in Human Capital."*American Economic Review*, September.

———. 1974. "Time in School: The Case of the Prudent Patron." North Carolina State University. November.

Lillard, L. A. 1973. "An Explicit Solution to the Human Capital Life Cycle of Earnings Model and Its Implications to Earnings Distributions." Ph.D. diss. North Carolina State University.

Lillard, L. A., and Y. Weiss. "Error Components in Longitudinal Earnings Data: American Scientists 1960–70." *Econometrica*. forthcoming.

Lillard, L. A., and R. Willis. "Dynamic Aspects of Earnings Mobility." *Econometrica*, forthcoming.

Maddala, G. S. 1971. "The Use of Variance Components Models in Pooling Cross Section and Time Series Data." *Econometrica*, March.

Mincer, J. 1970. "The Distribution of Labor Incomes: A Survey with Special Reference to the Human Capital Approach." *Journal of Economic Literature* 8, No. 1: 1–26.

———. 1974. *Schooling, Experience, and Earnings*. New York: NBER.

Mincer, J., and S. Polachek. 1974. "Family Investments in Human Capital: Earnings of Women." *Journal of Political Economy*, March, Part II.

Nerlove, M. 1971a. "Further Evidence on the Estimation of Dynamic Relations from a Time Series of Cross Sections." *Econometrica*, March.

———. 1971b. "A Note on Error Components Models." *Econometrica*, March.

Paglin, M. 1975. "The Measurement and Trend of Inequality. A Basic Revision." *American Economic Review*, September.

Rosen, S. 1973. "Income-generating Functions and Capital Accumulation." Harvard Institute of Economic Research, Discussion Paper 306, June.

———. "Toward a Theory of Life Earning." *Journal of Political Economy*, forthcoming.

Taubman, P., and T. Wales. 1974. *Higher Education and Earnings: College as an Investment and Screening Device*. New York: NBER.

Thorndike, R. L., and E. Hagen. 1959. *Ten Thousand Careers*. New York: Wiley.

Wallace, T. D., and L. A. Ihnen. 1975. "Full Time Schooling Life Cycle Models Human Capital Accumulation." *Journal of Political Economy*, February.

Wallace, T. D., and A. Hussain. 1969. "The Use of Error Components Models in Combining Cross Section with Time Series Data." *Econometrica*, January.

Weiss, Y., and L. A. Lillard. "Vintage, Experience, and Time Effects in the Growth of Earnings: American Scientists 1960–70." *Journal of Political Economy*, forthcoming.

Wilkinson, B. W. 1966. "Present Values of Lifetime Earnings for Different Occupations." *Journal of Political Economy*, December.

APPENDIX B: EMPIRICAL RESULTS

TABLE B-1 OLS and GLS Estimates of Earnings Function Parameters

Variable	OLS	GLS
Constant	5432.54	6476.80
Age (*A*)	140.60	−337.97
Sch (*S*)	−768.66	−1492.40
AS	79.24	277.36
A^2	−14.40	5.54
A^3	.275	.075
SA^2	11.42	3.39
SA^3	−.232	−.142
S^2	109.48	223.76
S^2A	−23.35	−52.84
S^2A^2	−.353	1.23
S^2A^3	.010	−.015
Abil (*B*)	2827.34	2030.03
BS	232.29	233.32
BA	−235.68	232.90
BSA	−284.55	−428.86
BS^2	−145.52	−133.80
BA^2	16.97	−3.21
BA^3	−.220	−.099
BSA^2	.687	4.67
BS^2A	41.86	60.26
BS^2A^2	−.0018	−1.07
BS^2A^3	−.0005	.018
B^2	−2019.02	−1439.59
B^2S^2	62.22	26.60
B^2A	54.98	−177.64
B^2A^2	1.60	8.84
B^2SA	110.48	147.48
B^2SA^3	−.012	−.044
B^2S^2A	−16.86	−17.04
FED.	62.22	122.64
MED.	136.52	142.88
NO. OF SIB.	−148.12	−144.90
NO. MOVES	38.81	37.28
PROT.	−402.69	−596.98
CATH.	67.34	−372.70
JEW.	5206.86	5665.88

NOTE: *Sch* is years beyond ten and *Age* is years beyond sixteen. In these equations highly collinear polynomial terms have been deleted. The estimated weighting factors, $\hat{\theta}$, are presented in footnote 37.

TABLE B-2 Earnings Function Incorporating Five Ability Measures (OLS)

Variable	Coefficient
Constant	4116.07
Age (A)	−672.88
Sch (S)	−569.27
AS	259.60
A^2	19.45
A^3	−.116
SA^2	2.38
SA^3	−.099
S^2	37.93
S^2A	−29.22
S^2A^2	.277
S^2A^3	−.0041
$MATH$ (M)	−2405.7
MS	264.01
MA	1263.4
MSA	−569.23
MS^2	−23.54
MA^2	−30.57
MA^3	.121
MSA^2	6.13
MS^2A	50.81
MS^2A^2	−.473
MS^2A^3	.0093
M^2	581.71
M^2S^2	2.31
M^2A	−581.36
M^2A^2	14.26
M^2SA	230.46
M^2SA^3	−.062
M^2S^2A	−20.22
$MECH$	4089.0
$GENKN$	1814.9
$MECH \cdot GENKN$	−1865.9
$MECH \cdot PHYS$	−1862.2
$GENKN \cdot PHYS$	1508.9
R^2	.277