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The Consumer Expenditure Function

ABSTRACT: A consumer expenditure function which integrates pure consumption and household investment in durable goods is formulated and estimated. A considerable increase in ability to explain consumer expenditures—relative to multiequation models—results from reduced reliance on the official classification of commodities as durable or nondurable. Further empirical investigation provides strong evidence that (1) private-sector income is significantly better than disposable personal income for explaining consumer expenditures, (2) the M_1 definition of money is similarly superior to both the M_2 and M_3 definitions, and (3) the weight of current income in permanent income is about 10 percent per year. A data appendix is included.

[I] INTRODUCTION AND SUMMARY

The functional relationship of aggregate consumer expenditures to income and other variables is one of the central elements of macroeconomic dynamics. Theoretical work, however, has been almost entirely devoted to models of pure consumption of service flows. But most cyclical variation in consumer expenditures would appear to arise in the adjustments of the stocks of consumer durable and semidurable goods and not in fluctuations in the growth of pure consumption. So macroeconomists should be concerned with a consumer ex-

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penditure function that integrates the asset adjustment function and the pure consumption function.

A few economists—most notably Franco Modigliani in the MPS model—have concerned themselves with the distribution between consumer expenditures and consumption. Their approach has been to estimate separate equations for pure consumption and for consumers' investment in durable goods. Consumption data are estimated as consumer expenditures less expenditures on durables plus an imputed rental value of the stock of durable goods. Expenditures on durables are in some models broken down further—such as for automobiles and for other durable goods. Such a multiequation approach depends critically upon the completeness of the empirical definition of consumer expenditures for durable goods. To the extent that goods which are behaviorally durable are in fact classified as nondurable, the model will be misspecified and omit a portion of the cyclical variation in the consumer expenditures. In my restatement of the permanent income theory (1974), it was shown that on the order of half of the behaviorally defined durable goods are classified in the official data as nondurable goods and services.[1] So the standard approach indeed suffers from specification biases.

The most obvious approach would have been to correct the definition of durable goods so that a multiequation approach can be directly applied. This was impossible because of both the lack of the required finely disaggregated data and the generality of durability in a behavioral sense. To take a simple example related to the concept of human capital, surely a vacation is a durable good yielding benefits for many years in the form both of memories and of slide shows inflicted on relatives. A more promising approach followed in this paper is to formulate a model in which the role of specification bias is minimized. As it happens, an integrated consumer expenditure function not only serves this role but also refocuses attention on the basic macroeconomic concept.

The integrated consumer expenditure function is derived in section II by inverting the standard theoretical definition of pure consumption so that consumer expenditures are defined in terms of pure consumption, household net investment in durable goods, and the yield on the stock of durable goods held at the beginning of the period. This definition is converted into a consumer expenditure function by substitutions based upon the permanent income theory of pure consumption and a generalized stock adjustment model of household durables investment. Consumer expenditures are determined primarily by permanent income, transitory income, the real money stock, and the stock of consumers' durable goods, with the long-term interest rate and relative price of durables playing minor roles because of their effect on stock demands. The model provides expected signs for most of these variables and explicates the relationships among their coefficients.

In section III, the model is applied to postwar U.S. data with remarkably favorable results. The estimated coefficients do not differ significantly from expectations and are consistent with the secular relation of consumption to saving. The most surprising finding is that the marginal propensity to spend (excess) real money balances is somewhat larger than the marginal propensity to spend current income for a one-year period. The theoretical model is shown to hold up well when disaggregated by use of estimated pure consumption and household durables investment. Most importantly, the explanatory power of the integrated model is considerably better than one based on separate consumption and household durables investment equations.

In section IV, the consumer expenditure function is used to investigate three outstanding empirical questions unrelated to the definition of durables: (1) Which concept better explains consumer expenditures: personal income or private-sector income? (2) Which of the money definitions—M_1, M_2, or M_3—is best at explaining consumer expenditures? (3) What is the weight (β) of current income in the formation of permanent income? These questions were studied simultaneously by maximum likelihood estimation for each combination of income and money definitions for both quarterly and annual data. The data provided the following answers: Private-sector income and M_1 (currency plus demand deposits) do significantly better than alternative definitions. The likelihood function is rather flat for values of β between zero and 20 percent per year but falls sharply for higher values of β; hence, the β weight of 10 percent per year previously estimated for a pure consumption model is retained.

Concluding remarks and suggestions for future research are contained in section V. The data appendix makes available to other researchers a considerable investment in constructing private-sector income, permanent income, and the stock of household durable goods from the national income accounts as well as monthly M_3 data based on the Federal Reserve definition for 1947-1958.

[II] THE THEORETICAL MODEL

This section contains an elaboration of the integrated model of consumer expenditures presented in Darby (1975). First a general framework is derived suitable for integrating all three-equation models of pure consumption, c_t^n; household investment in durable goods, Δd_t; and the (end-of-period) stock of consumers' durable goods, d_t. A specific but empirically quite general model is then substituted into this framework to obtain the basic equation used in the empirical investigations.

The real stock, d_t, of consumers' goods ("the durables stock") at the end of period t is computed by applying a depreciation rate of δ per period:

(1) $d_t = (1 - 0.5\,\delta)c_t^d + (1 - \delta)d_{t-1}$

where the coefficient of durable goods expenditures, c_t^d, adjusts for intraperiod depreciation on gross investment.[2] It follows directly that the net investment in durables, Δd_t, is

(2) $\Delta d_t = (1 - 0.5\,\delta)c_t^d - \delta d_{t-1}$

The usual definition of pure consumption, c_t^f, is total consumer expenditures, c_t^x, less the net investment in durables plus an imputed yield at the rate r per period on the average durables stock for the period:

(3) $c_t^f = c_t^x - \Delta d_t + 0.5r(d_t + d_{t-1})$

$= c_t^x - (1 - 0.5r)\Delta d_t + rd_{t-1}$

Solving for c_t^x shows that consumer expenditures equal pure consumption plus net durables investment (adjusted for intraperiod yield[3]) less the yield on the beginning durables stock:

(4) $c_t^x = c_t^f + (1 - 0.5r)\Delta d_t - rd_{t-1}$

Equation 4 is converted from an identity to a theory by substituting behavioral functions into the right-hand side. Since the real value, d_{t-1}, of the durables stock at the beginning of period t is predetermined by past changes in that stock,[4] functions must be specified only for pure consumption, c_t^f, and household investment in durable goods, Δd_t.

For aggregate time series data, the permanent income hypothesis is an appealing explanation of pure consumption:

(5) $c_t^f = ky_{Pt}$

Pure consumption is assumed to be a constant fraction, k, of permanent income, y_{Pt}. A nonzero constant on the right-hand side might be present without affecting the form of the equation ultimately estimated below. The permanent income concept appears to provide a relatively accurate method for estimating aggregate wealth (inclusive of human capital) as compared to direct estimates nomally used in life-cycle models.[5] This specification also allows further empirical study of the reformulated permanent income theory presented in Darby (1974). Some other approach might in fact produce superior empirical results, but that is an open issue for future research.

The change in the stock of durable goods is of the nature of a portfolio adjustment problem. Households will increase their holdings of durable goods in response to the increase in total assets from normal saving in order to make up part of any remaining discrepancy between the desired and beginning stocks, in response to unexpected saving due to windfalls (transitory income, y_{Tt}), and as a temporary response to disproportionately large money balances:

(6) $\Delta d_t = (\Delta d_t)^e + \lambda_1[d_t^* - (\Delta d_t)^e - d_{t-1}] + \lambda_2 y_{Tt} + \lambda_3(m_t - m_t^*)$

Standard models of stock adjustment in the form $\lambda(d_t^* - d_{t-1})$ are strictly applicable only to a no-growth world since they otherwise imply that no one ever learns to plan ahead. Given the definition of planned investment, $(\Delta d_t)^e$, below, the difference between the models is only one of regression coefficient interpretation. Wachtel (1972) has a similar model of consumer portfolio balances inclusive of durable goods. The model captures the main elements that are generally supposed in the literature to affect changes in the stock of durable goods.[6]

The model is completed by specifying the long-run durables stock demand, d_t^*; the planned change in durable goods, $(\Delta d_t)^e$; and real money demand, m_t^*. Durables stock demand is assumed to be a linear function of permanent income; the relative price of durable goods P_{Dt}/P_{NDt}; and the long-term interest rate, i_t:[7]

$$(7) \quad d_t^* = \alpha_0 + \alpha_1 y_{Pt} + \alpha_2 \frac{P_{Dt}}{P_{NDt}} + \alpha_3 i_t$$

The planned change in durable goods through normal saving is approximately proportional to permanent income:

$$(8) \quad (\Delta d_t)^e = \eta y_{Pt}$$

The demand for real money balances is assumed to be a linear function of permanent income, transitory income,[8] and the long-term interest rate:

$$(9) \quad m_t^* = \gamma_0 + \gamma_1 y_{Pt} + \gamma_2 y_{Tt} + \gamma_3 i_t$$

Substitution into equation 6 yields the consumers' durable goods investment function

$$(10) \quad \Delta d_t = (\lambda_1 \alpha_0 - \lambda_3 \gamma_0) + [(1 - \lambda_1)\eta + \lambda_1 \alpha_1 - \lambda_3 \gamma_1] y_{Pt}$$
$$+ (\lambda_2 - \lambda_3 \gamma_2) y_{Tt} + \lambda_3 m_t - \lambda_1 d_{t-1} + \lambda_1 \alpha_2 \frac{P_{Dt}}{P_{NDt}} + (\lambda_1 \alpha_3 - \lambda_3 \gamma_3) i_t$$

The coefficient of real money balances is unambiguously positive and the coefficients of the lagged real durable goods stock and the relative price of durable goods are unambiguously negative. The signs of the other coefficients are ambiguous.

Finally equations 5 and 10 are substituted into equation 4 to obtain the consumer expenditure function:

$$(11) \quad c_t^e = \beta_0 + \beta_1 y_{Pt} + \beta_2 y_{Tt} + \beta_3 m_t + \beta_4 d_{t-1} + \beta_5 \frac{P_{Dt}}{P_{NDt}} + \beta_6 i_t$$

where

$$\beta_0 = (1 - 0.5r)(\lambda_1 \alpha_0 - \lambda_3 \gamma_0)$$
$$\beta_1 = k + (1 - 0.5r)[(1 - \lambda_1)\eta + \lambda_1 \alpha_1 - \lambda_3 \gamma_1]$$

$$\beta_2 = (1 - 0.5r)(\lambda_2 - \lambda_3\gamma_2)$$
$$\beta_3 = (1 - 0.5r)\lambda_3 > 0$$
$$\beta_4 = -r - (1 - 0.5r)\lambda_1 < 0$$
$$\beta_5 = (1 - 0.5r)\lambda_1\alpha_2 < 0$$
$$\beta_6 = (1 - 0.5r)(\lambda_1\alpha_3 - \lambda_3\gamma_3)$$

Although unambiguous signs are assigned only to β_3, β_4, and β_5, it would be surprising if the direct positive effects of permanent and transitory income were completely offset by their indirect effects operating through the demand for money. Variations in the magnitudes of $\lambda_3\gamma_0$, $\lambda_3\gamma_1$, $\lambda_3\gamma_2$, and $\lambda_3\gamma_3$ will cause some variation in the estimates, below, of β_0, β_1, β_2, and β_6 for alternative money definitions.[9]

In sum, equation 11 serves as a reasonably straightforward method of incorporating standard notions about factors influencing pure consumption and household investment in consumers' durable goods into a consumer expenditure function. Alternative routes could be used to derive the same equation with somewhat different interpretations placed on the coefficients, but the current approach seems the most attractive one to me.

Equation 11 provides an alternative to the use of separate regression equations for consumption and consumers' investment in durables, that is, to separate estimation of equations 5 and 10. The great advantage of the integrated equation 11 is due to the difficulty of trying to classify goods and services as either durable or nondurable. If equations 5 and 10 are estimated separately, the half of behaviorally durable goods classified as nondurable goods and services is not allowed to respond to such "durable" variables as transitory income and real money balances. This misclassification problem does not arise in the combined consumer expenditure function approach.

Some data problems and biases remain. Some classification is necessary because empirical use of (11) still requires estimates of the real stock and relative price of durable goods. But the *stock* of officially designated durable goods is likely to be a very good proxy for the stock of all behaviorally defined durable goods since both respond to identical determinants except for possibly different relative prices and depreciation rates. It is certainly not clear whether those durable goods included in the official definition (such as automobiles and radios) depreciate at a much slower or faster rate than those excluded (such as suits and once-in-a-lifetime vacations), but the bias from such a difference would appear to be trivial. The only substantial problem arises in the relative price of durables, where there is no reason to suppose price movements of officially defined durables to be a good proxy for excluded durables; hence, this coefficient will be biased toward zero. Thus, the importance of specification bias and of bias due to errors in the variables is indeed substantially reduced.

[III] ESTIMATION OF THE MODEL

Basic estimates of the model and a comparison with the multiequation approach are presented in this section. Discussion of some important empirical issues concerning the definitions of income and money and the computation of permanent income is taken up in section IV.

Data Definitions[10]

A major empirical finding of this paper is that the way durable goods, income, and money are defined makes a real difference in the explanatory power of the functions used. Hence it is necessary to devote particular attention to the precise definitions of data sources used. Some important series have been constructed and are made available in the data appendix for use by others. Four basic series are available directly:

c_t^* = personal consumption expenditures in constant (1958) dollars (quarterly data at seasonally adjusted quarterly rates—SAQR);

c_t^d = personal consumption expenditures for durable goods in constant (1958) dollars (quarterly data at SAQR);

m_t = money supply, M_1 (average of monthly data), deflated by the implicit price deflator for personal consumption expenditures;

i_t = yield on long-term U.S. government bonds (average of monthly data).

The stock of durable goods at the end of quarter t is computed according to equation 1 for $\delta = 0.05$ as follows:[11]

$$(12) \quad d_t = 0.975\, c_t^d + 0.95\, d_{t-1}$$

Annual regressions use end-of-year (fourth-quarter) data extracted from the quarterly estimates.

Two alternative current income measures are compared in section IV, one corresponding to the accrual of purchasing power and the other to cash receipts. Each is adjusted for an imputed 10 percent per year real yield, r, on the beginning durables stock.[12] The basic accrual concept of income is private-sector income, y_t^{PS} (see Darby 1976, chap. 2), which is the amount (implicit in the national income accounts) available to the private sector (ultimately consumers) for consumption or addition to wealth.[13] The cash receipts concept is based on disposable personal income, y_t^{DP}. Both series are deflated by the implicit price deflator for personal consumption expenditures (1958 = 1.000), and quarterly observations are at SAQR. Thus, on the accrual definition current income is

$$(13) \quad y_t = y_t^{PS} + r d_{t-1}$$

where $r = 0.10$ for annual data and 0.025 for quarterly data. Where the cash receipts definition is used, y_t^{DP} replaces y_t^{PS} in (13).

Permanent income is computed in the usual way as

$$(14) \quad y_{Pt} = \beta y_t + (1 - \beta)(1 + g)y_{Pt-1}$$

The implied geometrically declining weights were shown in Darby (1974) to be implied by a perpetual inventory model of total (human and nonhuman) wealth, where β is the real yield on wealth and g is the trend growth rate of income.[14] The value of β is estimated by search over the interval $0 \leqslant \beta \leqslant 1$ for the value which minimizes the sum of squared residuals in the consumer expenditure regression.

Transitory income is computed as the difference between the estimates of current and permanent income:

$$(15) \quad y_{Tt} = y_t - y_{Pt}$$

The relative price of durable to nondurable goods and services is computed by dividing the implicit price deflator for personal consumption expenditures on durable goods by the corresponding deflator for nondurable goods and services. The latter unpublished deflator is derived as the ratio of expenditures on nondurable goods and services in current dollars to the expenditures in constant (1958) dollars.[15]

For purposes of comparison with the multiequation approach for explaining consumer expenditures, estimates of household investment in durable goods, Δd_t, and pure consumption, $c_t^{\prime}$, are based on the Commerce Department definitions of durable goods:

$$(16) \quad \Delta d_t = d_t - d_{t-1}$$

$$(17) \quad c_t^{\prime} = c_t^{x} - (1 - 0.5r)\Delta d_t + r d_{t-1}$$

where the imputed yield on durable goods, r, is the same as that used in estimating current income.

Estimates of the Consumer Expenditure Function

The consumer expenditure function (11) was estimated by ordinary least squares in both quarterly and annual versions for the entire period 1947–1973 for which complete data were available. For reasons to be discussed in section IV, the basic estimates are based on the accrual (private-sector) income definition and the narrow (M_1) money definition.

The use of OLS (ordinary least squares) regressions raises the standard questions of possible simultaneous equation bias. Leaving aside small-sample objections to alternative simultaneous equation estimators, I would argue that there is little problem here anyway. It appears to me that the concurrent effects of the consumer expenditure function disturbance on the other right-hand-side variables must be close to negligible. I base this judgment on two considerations: (1) The disturbances in equation 11 appear very small indeed relative to the exogenous shifts in the other variables. (2) Reduced form estimates of the effects of government expenditures on income suggest at most weak multiplier effects in the first quarter and the first year. Presumably, small quarter-to-quarter disturbances in consumer expenditures would be met out of inventories to at least as great an extent as in the case of government expenditures. This judgment must ultimately be tested by embedding the consumer expenditure function in a macroeconometric model.

The annual estimate is[16]

$$(18) \quad c_t^* = \underset{(-2.57)}{-148.9} + \underset{(16.69)}{1.08}\, y_{Pt} + \underset{(6.87)}{0.406}\, y_{Tt} + \underset{(4.96)}{0.681}\, m_t - \underset{(-5.29)}{0.376}\, d_{t-1} + \underset{(0.80)}{29.0}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.11)}{1.49}\, i_t$$

$$\hat{\beta} = 0.15\,[0, 0.23];\ SEE = 1.98;\ R^2\,(\text{adj.}) = 0.9996;\ D.W. = 2.39$$

The corresponding quarterly regression is

$$(19) \quad c_t^* = \underset{(-3.21)}{-28.52} + \underset{(27.16)}{0.90}\, y_{Pt} + \underset{(12.67)}{0.455}\, y_{Tt} + \underset{(7.59)}{0.189}\, m_t - \underset{(-4.39)}{0.042}\, d_{t-1} + \underset{(0.53)}{2.95}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.66)}{0.37}\, i_t$$

$$\hat{\beta} = 0.01\,[0, 0.06];\ SEE = 0.744,\ R^2(\text{adj.}) = 0.9992;\ D.W. = 1.08$$

The two estimates correspond very closely when it is recalled that because of the stock-flow relationships c_t^*, y_{Pt}, and y_{Tt} are measured at quarterly rates in the quarterly regression.[17] The low quarterly Durbin-Watson statistic suggests autocorrelation of the residuals, but that is not present in the annual regression. This autocorrelation may be due either to correlated data errors such as from the seasonal adjustment or else to an omitted variable such as lagged transitory income, which is not important at the annual level. Since autocorrelation suggests overly optimistic standard errors, the discussion below will emphasize the more reliable annual regression.

Because of the important trend element, the adjusted R^2 is a meaningless measure of explanatory power.[18] More useful is the ratio of the standard error of estimate to the mean value of the dependent variable. This value is 0.58 percent for the annual regression and 0.86 percent for the quarterly one. If the consumer expenditure functions were converted to private saving functions by use of the identities (see Darby 1975, eq. 12), the standard errors would be

5.0 percent of mean private saving for annual data and 7.5 percent for quarterly data. Further, the annual standard error of estimate is only 34.0 percent of the standard error for the naive model of footnote 18 and 50.5 percent of the standard error for a Keynesian consumption function.

In the annual regression, the coefficient of y_{Pt} exceeds 1.0 because the effect operating through the stock demand for durables is large relative to the offset due to the demand for money. The long-run effect of permanent income would include induced effects on the durables and money stocks. Of special interest are the implied long-run values for the ratio, k, of pure consumption to total accrued income and the ratio, σ, of private saving to private-sector income (exclusive of the imputed yield on the durables stock). These values are estimated at 0.90 and 0.08 respectively on the basis of regression 18.[19] In view of the nonlinear transformations and auxiliary information used in their computations, these rather standard values are better regarded as rough checks on the consistency of the regression than as good estimates of k and σ.

The short-run marginal propensity to consume is given by

$$(20)\quad \frac{dc_t^*}{dy_t} = \frac{dc_t^*}{dy_{Pt}}\frac{dy_{Pt}}{dy_t} + \frac{dc_t^*}{dy_{Tt}}\frac{dy_{Tt}}{dy_t}$$

$$= (1.08)(0.15) + (0.406)(0.85) \simeq 0.51$$

For the quarterly regression, the corresponding value is 0.46. The lower quarterly value reflects the smaller impact on permanent income of a one-quarter change in current income as compared to a one-year change. The estimates of this β weight bracket the value of 0.1 per year (0.025 per quarter) which was estimated in Darby (1974) on the basis of pure consumption. They will be analyzed further in section IV.

The coefficient of real money balances is quite significant in both the economic and statistical senses. Its value is much too high for a wealth effect. This would appear to support the substitution hypothesis of the real balance effect. This may be interpreted in two equivalent ways: (1) Bonds and durables are substitutes in the household portfolio, and the demands for both are affected by an excess supply of money. (2) Money supply, given its demand, is a good proxy for the unobservable real yields on substitutes for durable goods. Another possibly complementary liquidity hypothesis would stress the critical role of cash balances in providing down payments for the purchase of consumers' durable goods because of the illiquidity of other forms of assets.

To see whether the interest rate would pick up most of the explanatory power of the real money supply—as suggested by the Keynesian approach—regression 18 was re-estimated with the coefficient of m_t constrained to zero. Not only did the standard error of estimate increase by \$0.9 billion (45 percent), but the coefficient of the interest rate was virtually unchanged. The coefficient of transitory income increased to 0.57, however.

The negative coefficient on the real durables stock is significantly larger than the yield on the stock. This indicates that both the direct substitution of the yield on durables for nondurable goods and services and the indirect adjustment of the durables stock affect consumer expenditures. Abnormally high durables sales during a boom imply a period of abnormally low durables sales later, while low sales during a recession imply high sales later.

The coefficient of the relative price of durable goods is insignificant and of the wrong sign. Although not surprising—given that about half of behaviorally defined durable goods are represented in the denominator—this result is disappointing. An unsuccessful attempt was made to estimate durables stock and price series inclusive of clothing and shoes. Although the relative price coefficient became negative, insurmountable difficulties in estimating the initial stock and depreciation resulted in a slight deterioration in the standard error. Even were a definitionally "pure" estimate available, there would be two other factors making for an insignificant, or even perversely signed, coefficient for the relative price of durables: (1) The behavior of the relative price of durables is dominated by a downward trend over the postwar period. This is probably due to relatively rapid quality improvements in durables which mask the real price changes and bias the coefficient toward zero. (2) If, contrary to the usual macroeconomic assumption, the supply curve of durable goods is not infinitely elastic at a given price, the relative price coefficient would reflect the interaction of demand and supply effects and be of indeterminant sign.

The nominal interest rate coefficient is slightly positive. This indicates that the positive effect (from decreasing the demand for money) slightly outweighs the negative one (from decreasing the demand for durable stock). Since no attempt was made to adjust for expected inflation, the nominal interest rate would not be expected to have much effect on the durables stock demand. It is perhaps surprising then, if money demand is significantly interest-elastic. that the interest rate coefficient is so low.

The early part of the period, say from 1947 through 1953, appeared suspect for four possible reasons: (1) the constraint on durables goods purchases during World War II, (2) possible inaccuracies in the starting benchmarks for permanent income and the durables stock, (3) the effect on the demand for money prior to 1951-1953 of pegged interest rates on government bonds, and (4) the Korean War and associated price controls. The equations were reestimated for 1954-1973, but there was no hint of a structural change or even a significant change in any of the coefficients.[20] So the entire period is retained for the statistical analysis.

Disaggregation into Consumption and Durables Investment Equations

To illustrate the value of the integrated consumer expenditure function approach, the underlying consumption function (5) and consumers' durables in-

TABLE 1 Disaggregation of the Consumer Expenditure Function Using Estimated Pure Consumption and Household Durables Investment

Regr. No.	Dep. Var.	Frequency of Observation	Regression Coefficients for: Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	β	SEE	R^2 (adj.)	D.W.
1	c_t^f	Annual		0.880 (418.2)						.15	4.53	.9982	0.28
2	c_t^f	Annual	−44.4 (−1.08)	0.976 (21.4)	0.219 (5.22)	0.208 (2.14)	−0.190 (−3.78)	−2.95 (−0.12)	3.51 (3.69)	.15	1.40	.9998	2.06
3	Δd_t	Annual	−110.0 (−2.08)	0.106 (1.80)	0.198 (3.66)	0.498 (3.98)	−0.090 (−1.39)	33.7 (1.02)	−2.12 (−1.73)	.15	1.81	.9358	2.01
4	c_t^f	Quarterly		0.876 (619.3)						.01	1.53	.9967	0.08
5	c_t^f	Quarterly	−7.65 (−1.36)	0.794 (38.0)	0.265 (11.7)	0.078 (4.96)	0.008 (1.32)	−1.24 (−0.35)	0.857 (6.01)	.01	0.47	.9997	1.05
6	Δd_t	Quarterly	−21.1 (−3.01)	0.104 (4.00)	0.193 (6.78)	0.112 (5.70)	−0.025 (−3.32)	4.24 (0.96)	−0.488 (−2.74)	.01	0.59	8951	0.95

NOTE: Figures in parentheses are t values. β estimates are from equations 18 and 19.

vestment function (10) were also estimated separately. For this purpose, the official definition of durable goods was used to construct estimates (as explained in "Data Definitions," above) of pure consumption, c_t', and household durables investment, Δd_t.

Table 1 contains the regression results. Equation 5 is estimated by regressions 1 and 4 for annual and quarterly data respectively. The previous indirect calculation of k as 0.90 corresponds well to the direct estimate of 0.88. Since it was argued that a pure consumption estimate based on the official durables definition would in fact include considerable household investment in misclassified durables, regressions 2 and 5 apply the consumer expenditure function to estimated "pure consumption." Regressions 3 and 6 apply the household durables investment function (10) to estimated net investment in (officially classified) durable goods.

In comparing regressions 2 and 3, it is clear that the estimated net investment contains about half of total net investment in a behavioral sense.[21] The only significant problem—not present in the quarterly regressions—is that the coefficient on the lagged durables stock is larger in regression 2 than in 3. This apparently offsets a slightly high estimated β weight of current income in permanent income, while quarterly regressions 5 and 6 display the opposite bias, owing to a low β weight.[22] The signs of the coefficients of the relative price of durables are just the reverse of what would be expected, but not much can be made of the statistically insignificant results for that variable.

In sum, the disaggregated version of the model is very much what would be guessed from its derivation and the estimates of the integrated consumer expenditure function. The only significant divergence between the annual and quarterly results—autocorrelation aside—is apparently due to the use of a slightly too high value of β in the annual regressions and a slightly too low value in the quarterly regressions.

The disaggregation done in Table 1 takes advantage of the estimated β weight of current income in permanent income from the integrated consumer expenditure function. A standard multiequation model would make separate estimates of equations 5 and 10 and combine them by use of identity 4 if a prediction of total consumer expenditures were required. The β estimate of the durables investment function will be unbiased but imprecise because of the low coefficient of permanent income. Since the estimates of pure consumption include elements of durables investment, the demonstration of the upward bias of β (from Darby 1974) applies directly. Nevertheless, the biased permanent income estimates will provide more accurate predictions of c_t^f than regressions 1 and 4. In practice an even more favorable estimate of c_t^f based on the Koyck transformation would likely be used instead of equation 5:

$$(21) \quad c_t^f = a_1 + a_2 y_t + a_3 c_{t-1}^f$$

The square roots of the mean squared error, 1947–1973, for the annual and quarterly consumer expenditure functions (regression equations 18 and 19) are 1.704 and 0.720. The corresponding figures for the maximum likelihood estimates of equations 5 and 10 combined by equation 4 are 3.458 and 1.150. For the maximum likelihood estimates of equations 21 (Koyck) and 10 combined by equation 4, the figures are 2.635 and 0.731. The integrated consumer expenditure function does much better than either disaggregated approach for the annual data. But for quarterly data, the method utilizing the Koyck transformation does nearly as well. The quarterly national income accounts data appear to spread receipts and expenditures over adjacent quarters, however; so the Koyck transformation in this case displays a spurious accuracy.

The consumer expenditure function has been successfully estimated in this section with no significant departures from expected signs or magnitudes of coefficients. The estimated coefficients are internally consistent. The disaggregated estimates are consistent with the original hypothesis that all coefficients other than permanent income enter because of household investment in durable goods but that nearly half of durable goods in a behavioral sense are included in the official data on nondurable goods and services. As a result, disaggregate estimates of consumer expenditures derived from separate models of pure consumption and household durables investment compare poorly with the estimates of the integrated consumer expenditure function.

[IV] ANALYSIS OF THREE EMPIRICAL ISSUES

The consumer expenditure function is used in this section to investigate further three empirical issues: (1) the definition of current income that best explains consumer expenditures, (2) the definition of money that best explains consumer expenditures, and (3) the value of β, the weight of current income in the determination of permanent income.

The two income definitions compared are the accrual and cash receipts concepts.[23] These two definitions reflect two basic and alternative conceptions of consumer behavior. The accrual concept is consistent with a view of the consumer as a rational decision maker constrained by total wealth. The cash receipts concept is sensible if consumers spend nearly all the money they receive. Until recently, use of the latter concept (disposable personal income) was the standard practice. A number of studies in the last decade have moved toward the accrual concept by adding undistributed corporate profits (as an estimate of accrued capital gains).

There are many other income definitions which could be considered. For example Barro (1974) and Kochin (1974) have recently argued that government bonds may not be viewed by the private sector as net wealth. In that case an accrual definition of income would be essentially net national product less government expenditures for goods and services plus the increase in high-powered (base) money.[24] Feldstein (1974) on the other hand argues for inclusion of an estimate of increases in "social security wealth." Another issue concerns the transfer of purchasing power to the government through inflation. This would suggest subtracting the rate of inflation times high-powered money and government bonds (if government bonds are included in net wealth). In view of the high estimation costs of dealing with many alternative income definitions simultaneously with the other two main empirical issues, it was decided to compare only the basic accrual and cash receipts definitions, leaving for further research comparison of finer differences conditional on a particular money definition and β weight.

In section III I used the M_1 (currency plus demand deposits) definition of money. In this section, I compare M_1 with two other money definitions that have received considerable attention by monetary economists: M_2 (M_1 plus time deposits at commercial banks exclusive of large negotiable certificates of deposit) and M_3 (M_2 plus savings and loan and mutual savings bank deposits).

The M_2 data used are an average of the monthly data deflated by the implicit price deflator for personal consumption expenditures. Unfortunately, Federal Reserve data for M_3 are available only from January 1959 on while the Friedman and Schwartz (1970) data contain no series using the official M_3 definition. Monthly estimates of M_3 for 1947 through 1958 were made on the basis of the Friedman and Schwartz data on savings and loan and mutual savings bank deposits.[25] The M_3 data used in this series are averages of that monthly data deflated by the implicit price deflator for personal consumption expenditures.

In Darby (1974), removal of the specification bias resulted in an estimated β weight of 0.1 per year in terms of an essentially pure consumption model. In this section I examine whether that estimate holds up in the consumer expenditure function under alternative definitions of income and money. Were β not estimated for each combination it could bias the choice of the best combination of income and money.

These three empirical issues are examined simultaneously, using the regressions reported in tables 2 (annual data) and 3 (quarterly data). The message of these tables is very clear: The accrual income concept and the M_1 money concept do much better in explanatory power (as judged by the sum of squared residuals or standard error of estimate) than the alternatives. Further, the β weight of 0.1 per year previously estimated on the basis of pure consumption continues to hold up in the consumer expenditure function.

Consider first the definition of income: For each money definition and for both annual and quarterly data, the accrual definition of income does better than the cash receipts definition.[26] *SSR* for the best cash receipts definition regression exceeds that of the corresponding accrual definition by 41.8 percent for the annual data and 10.6 percent for the quarterly data.[27] Given the success

TABLE 2 Consumer Expenditure Functions Estimated for Alternative Income and Money Definitions, Annual Data, 1947–1973

Regr. No.	Income Concept	Money Concept	Regression Coefficients for: Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	$\hat{\beta}$	*SEE*	R^2 (adj.)	*D.W.*	*SSR*
7	Accrual[a]	M_1	−148.9	1.08	0.406	0.681	−0.376	29.0	1.49	.15	1.98	.9996	2.39	78.4
			(−2.57)	(16.69)	(6.87)	(4.96)	(−5.29)	(0.80)	(1.11)					
8	Cash	M_1	−90.0	1.09	0.630	0.461	−0.402	15.3	−0.159	.10	2.36	.9995	2.25	111.2
			(−1.35)	(15.71)	(7.46)	(2.62)	(−5.38)	(0.36)	(−0.10)					
9	Accrual	M_2	−14.7	0.92	0.410	0.183	−0.247	−14.1	2.08	.15	2.33	.9995	2.29	108.4
			(−0.28)	(14.04)	(5.57)	(3.51)	(−3.49)	(−0.36)	(1.32)					
10	Cash	M_2	−8.20	1.03	0.628	0.116	−0.363	−10.9	−0.08	.125	2.48	.9994	2.36	123.4
			(−0.15)	(13.46)	(6.46)	(1.96)	(−4.82)	(−0.26)	(−0.05)					
11	Accrual	M_3	6.75	0.83	0.337	0.146	−0.203	−16.6	2.65	.225	2.28	.9995	2.26	104.2
			(0.13)	(9.14)	(4.18)	(3.35)	(−2.58)	(−0.43)	(1.59)					
12	Cash	M_3	2.41	0.95	0.581	0.106	−0.331	−10.4	0.91	.175	2.40	.9995	2.32	115.5
			(0.05)	(8.86)	(5.89)	(2.15)	(−3.77)	(−0.25)	(0.52)					

NOTE: Figures in parentheses are t values. *SSR* indicates the sum of squared residuals.
[a]Regression 7 was discussed as equation 18 in section III.

TABLE 3 Consumer Expenditure Functions Estimated for Alternative Income and Money Definitions, Quarterly Data, 1947–1973

Regr. No.	Income Concept	Money Concept	Regression Coefficients for: Constant	y_{Pt}	y_{Tt}	m_t	d_{t-1}	P_{Dt}/P_{NDt}	i_t	$\hat{\beta}$	SEE	R^2 (adj.)	D.W.	SSR
13	Accrual	M_1	−28.5	0.90	0.455	0.189	−0.042	2.95	0.37	.01	.744	.9992	1.08	55.9
			(−3.21)	(27.16)	(12.67)	(7.59)	(−4.39)	(0.53)	(1.66)					
14	Cash	M_1	−18.3	0.96	0.597	0.138	−0.061	1.30	−0.14	.00	.782	.9991	0.94	61.8
			(−2.02)	(25.76)	(11.38)	(5.08)	(−5.84)	(0.22)	(−0.61)					
15	Accrual	M_2	−0.29	0.82	0.444	0.054	−0.035	−4.37	0.50	.02	.797	.9991	0.95	64.2
			(−0.04)	(23.06)	(10.73)	(6.10)	(−3.52)	(−0.76)	(2.06)					
16	Cash	M_2	1.50	0.92	0.615	0.039	−0.060	−3.86	−0.03	.01	.810	.9990	0.90	66.3
			(0.20)	(21.51)	(10.92)	(4.20)	(−5.57)	(−0.66)	(−0.11)					
17	Accrual	M_3	3.12	0.79	0.381	0.042	−0.044	−4.61	0.60	.06	.784	.9991	0.94	62.1
			(0.42)	(14.45)	(8.42)	(6.00)	(−3.63)	(−0.82)	(2.31)					
18	Cash	M_3	6.01	0.79	0.571	0.036	−0.040	−3.84	0.30	.01	.785	.9991	0.93	62.2
			(0.83)	(14.25)	(9.98)	(5.04)	(−3.54)	(−0.68)	(1.17)					

NOTE: Figures in parentheses are t values. SSR indicates the sum of squared residuals.
[a]Regression 13 was discussed as equation 19 in section III.

of the model, which posits rational consumers faced with a wealth constraint, it would have been disconcerting to discover that the accrual definition of income did not do considerably better than the cash receipts definition.

As to the empirical definition of money, the results are similar. For either definition of income, the M_1 definition does better than either M_2 or M_3. Compared with the best M_1 estimate, *SSR* for the best alternative, M_3, is 32.9 percent higher for annual data and 11.1 percent higher for quarterly data. The coefficients of real money balances would be expected to decline in moving from M_1 to M_2 to M_3 (because of the increasing absolute magnitudes). However, the standard errors decline less rapidly (hence, the t values fall), suggesting that M_2 and M_3 are properly interpreted as proxies for M_1. In addition, M_3 does a bit better than M_2 suggesting that consumers find bank and nonbank time deposits much better substitutes for each other than they find all kinds of time deposits for M_1.

As already discussed in section III, the estimates of β bracket, but in no case significantly differ from, the previous estimate of 0.1 per year or 0.025 per quarter. As discussed in footnote 22 below, the high correlation of the durables stock and permanent income for low β weights make precise estimation impossible. However, it is clear from the behavior of the likelihood function that the actual β weight must lie in the neighborhood of 0.1 per year (0.025 per quarter). This is illustrated in figures 1 and 2, which are graphs of the sum of squared residuals as a function of the β weights for annual and quarterly data, respectively. The critical value of *SSR* for a two-tailed likelihood ratio test at the 90 percent significance level is indicated in each figure by SSR^{crit}. Between zero and 0.2 for annual data and, equivalently, between zero and 0.05 for quarterly data, the *SSR* is rather flat; hence, the minimizing β weights of 0.15 and 0.01, respectively, are little better than any other value within that range. From 0.2 to 0.6 per year (0.05 to 0.2 per quarter), *SSR* rises very rapidly to a much higher plateau. So the estimation of β is imprecise within the range from 0 to 0.2 per year, but any value much above that range, including Friedman's original (1957) biased estimate of 0.35 per year, can be easily rejected (see Darby 1974, especially pp. 233–234).

Regressions 7 and 13 (presented earlier as equations 18 and 19) have inconsistent β weights. Since the average of the two β weights is 9.5 percent per year and there is no reason to reject the previous figure of 10 percent per year based on a pure consumption model, consistent estimates of the consumer expenditure function were made based on a β weight of 0.10 per year and 0.025 per quarter. The annual estimate is[28]

$$(22)\quad c_t^* = \underset{(-2.52)}{-147.5} + \underset{(17.42)}{1.005}\, y_{Pt} + \underset{(7.98)}{0.446}\, y_{Tt} + \underset{(5.07)}{0.729}\, m_t$$

$$- \underset{(-4.60)}{0.289}\, d_{t-1} + \underset{(0.83)}{30.3}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.47)}{1.96}\, i_t$$

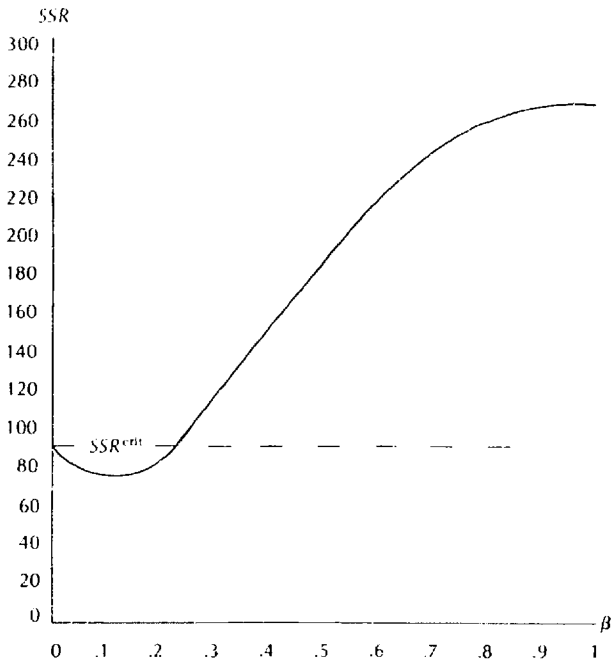

FIGURE 1 Sum of Squared Residuals as a Function of the β Weight: Annual Consumer Expenditure Functions

$\hat{\beta} = 0.1$; $SEE = 2.00$; $R^2(\text{adj.}) = 0.9996$; $D.W. = 2.39$

The corresponding quarterly estimate is

$$(23) \quad c_t^x = \underset{(-3.37)}{-30.47} + \underset{(25.88)}{0.971}\, y_{Pt} + \underset{(12.85)}{0.460}\, y_{Tt} + \underset{(7.51)}{0.187}\, m_t$$

$$- \underset{(-6.01)}{0.065}\, d_{t-1} + \underset{(0.49)}{2.76}\, \frac{P_{Dt}}{P_{NDt}} + \underset{(1.43)}{0.33}\, i_t$$

$\hat{\beta} = 0.025$; $SEE = 0.746$; $R^2(\text{adj.}) = 0.9992$; $D.W. = 1.07$

Strong evidence has been presented in this section for the following empirical propositions: (1) The accrual (private-sector income) definition of income explains consumer expenditures better than the cash receipts (disposable personal income) definition. (2) The narrow, M_1, definition of money is an important determinant of consumer expenditures and significantly better in explanatory power than either broad definition, M_2 or M_3. (3) The weight, β, of current income in permanent income lies in the range from zero to 20 percent per year.

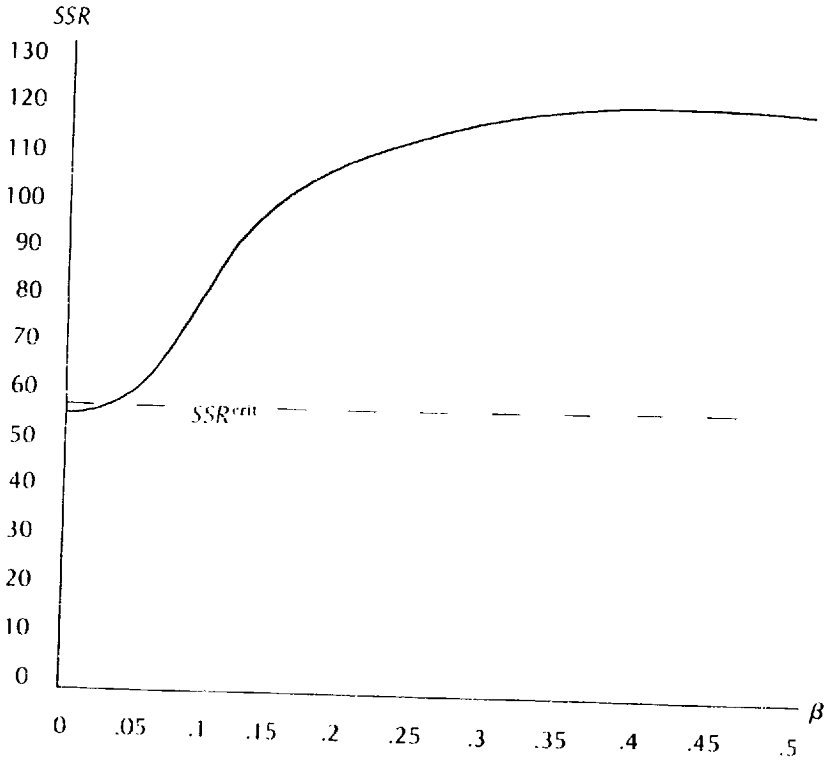

FIGURE 2 **Sum of Squared Residuals as a Function of the β Weight: Quarterly Consumer Expenditure Functions**

[V] CONCLUDING REMARKS

The central theme of this paper is the empirical value of an integrated consumer expenditure function in explaining consumer expenditures. The theoretical value of the function is that it concentrates directly on the variable of prime interest to macroeconomists. But the alternative treatment of household investment in durable goods as a component of an enlarged definition of total investment also has theoretical appeal. The basic attraction is therefore the empirical one. The integrated approach is much less subject to biases introduced by the essentially arbitrary classification of commodities between durable and nondurable goods and services.

An empirical question can be answered only by examination of the data. An unusually clear answer was provided by the research reported here: The consumer expenditure function explains the data well and significantly better than the multiequation, pure consumption-household investment approach. The reason for this superior performance is that the official data on durable goods expenditures include only about half of total durables expenditures as defined behaviorally.

The data also provided strong evidence that (1) an accrual (private-sector) definition of income better explains consumer expenditures than a cash receipts (disposable) personal income definition; (2) the narrow, M_1, definition similarly does better than either M_2 or M_3; and (3) the β weight of current income in the formation of permanent income lies somewhere in the range from zero to about 20 percent per annum. While there is no a priori presumption about the best money definition, the results based on the income definition and the β weight reinforce the basic conception underlying the model—that consumers are rational decision makers constrained by total (human and nonhuman) wealth as estimated by permanent income. The rationality of consumers would certainly be questionable if they responded to cash receipts rather than accrued income. A β weight of about 10 percent per annum, which is the estimated real yield on total wealth, is preferred to the higher weights estimated in many previous studies.[29]

The empirical advantages of an integrated consumer expenditure function seem clear. Future research might be directed at substituting a life-cycle model for the permanent income explanation of pure consumption to compare their explanatory powers. Other areas for possible improvement would be either the generalized stock adjustment hypothesis (6) or the underlying stock demand functions (7) and (9). A somewhat different line of research would utilize the consumer expenditure function to examine finer definitions of accrued income adjusted for increases in government debt, in social security wealth, or the inflationary tax on base money and possibly government debt.

DATA APPENDIX

Several data series of general applicability were estimated in the course of this project. In order to make them available for future research by others in this and other areas, the most important are reproduced here with instructions for updating as revised data become available.

Table A-1 contains annual data for nominal and real private-sector income, the current and permanent (real) income on the accrued definition, the real durables stock, and the nominal M_1 money supply. Table A-2 contains quarterly data for the same series. Table A-3 contains the monthly nominal M_1 data through 1959, when they tie in with the Federal Reserve Board's published data.

TABLE A-1 Annual Data Series, 1946-1973

	Y_t^{PS} (1)	y_t^{PS} (2)	y_t^{PSDY} (3)	y_{Pt} (4)	d_t (5)	M_{3t} (6)
1946	163.22	-	-	223.77	72.43	-
1947	176.05	225.96	233.20	232.43	81.33	172.71
1948	200.45	243.36	251.49	242.35	90.01	176.74
1949	199.35	244.00	253.00	251.78	99.09	178.10
1950	215.17	259.67	269.58	262.24	112.21	183.88
1951	235.42	265.77	276.99	272.76	119.75	191.82
1952	246.85	272.81	284.79	283.37	125.42	204.35
1953	258.70	282.12	294.66	294.27	134.05	215.89
1954	263.97	285.46	298.87	304.88	141.22	229.28
1955	285.02	307.21	321.33	317.04	154.10	240.53
1956	300.37	317.00	332.41	329.51	162.61	250.40
1957	314.90	322.48	338.74	341.79	169.93	261.83
1958	322.82	322.81	339.80	353.38	172.69	278.57
1959	345.65	341.29	358.56	366.08	180.20	295.62
1960	355.80	345.78	363.80	378.48	187.32	305.57
1961	369.60	355.63	374.36	391.12	192.38	326.65
1962	392.97	374.70	393.94	404.89	201.22	350.86
1963	411.05	387.31	407.43	419.11	212.54	380.07
1964	447.52	416.86	438.12	435.46	226.49	410.07
1965	486.17	446.71	469.36	453.87	244.78	444.83
1966	526.20	471.67	496.15	473.75	264.24	476.19
1967	556.55	486.46	512.88	494.00	281.17	511.70
1968	596.87	503.97	532.09	514.84	302.65	554.49
1969	633.10	512.47	542.74	535.39	323.88	588.93
1970	683.57	528.42	560.81	556.39	339.49	613.94
1971	744.15	553.84	587.78	578.72	360.34	692.55
1972	804.55	581.95	617.99	602.60	388.58	778.92
1973	905.25	620.52	659.37	629.06	419.04	862.13

NOTE: Columns 1 through 5 are based on final data through 1971. Federal Reserve estimates of M_3 are subject to change back to 1959.

Col. 1: Private sector income in billions of current dollars. To update or extend: Y_t^{PS} = net national product less the following: government purchases of goods and services, government surplus (NIA [national income accounts] basis), statistical discrepancy (NIA), federal government transfer payments to foreigners (net), personal transfer payments to foreigners.

Col. 2: Private sector income in billions of 1958 dollars. To update or extend: $y_t^{PS} = Y_t^{PS}$ deflated by the implicit price deflator for personal consumption expenditures.

Col. 3: Private sector income in billions of constant (1958) dollars adjusted for the imputed yield on the stock of consumers' durable goods—the accrual concept of income. To update or extend: $y_t^{PSDY} = y_t^{PS}$ $+ 0.1d_{t-1}$.

Col. 4: Permanent income based on y_t^{PSDY} and a β weight of 0.1 (billions of 1958 dollars). To update or extend: $y_{Pt} = 0.1y_t^{PSDY} + 0.9344862y_{Pt-1}$.

Col. 5: Stock of consumers' durable goods at the end of the year (billions of 1958 dollars). To update or extend: Fourth-quarter data from Table A-2.

Col. 6: Money stock, M_3, in billions of current dollars. To update or extend: Average of monthly Federal Reserve data beginning 1959.

TABLE A-2 Quarterly Data Series, 1964:4–1973:4

	Y_t^{PS} (1)	y_t^{PS} (2)	y_t^{PSDY} (3)	y_{P_t} (4)	d_t (5)	M_{1t} (6)
1946:4	169.9	–	–	227.17	72.43	–
1947:1	170.4	223.62	230.87	229.39	74.56	169.19
1947:2	172.0	223.96	231.41	231.58	76.76	171.88
1947:3	179.3	228.99	236.67	233.88	78.89	174.06
1947:4	182.5	227.27	235.16	236.09	81.33	175.72
1948:1	190.5	234.90	243.03	238.47	83.63	176.74
1948:2	199.5	243.29	251.66	241.03	85.83	176.27
1948:3	205.1	246.22	254.80	243.63	88.02	176.96
1948:4	206.7	249.04	257.84	246.26	90.01	176.99
1949:1	201.1	244.35	253.35	248.74	91.77	177.05
1949:2	198.9	243.15	252.33	251.15	94.01	178.04
1949:3	199.8	245.76	255.16	253.60	96.48	178.34
1949:4	197.6	242.75	252.40	255.94	99.09	178.97
1950:1	209.5	257.69	267.60	258.63	101.86	180.75
1950:2	209.7	256.67	266.86	261.25	104.59	183.50
1950:3	217.3	260.24	270.70	263.93	109.11	184.87
1950:4	224.2	264.08	274.99	266.67	112.21	186.42
1951:1	226.0	257.11	268.33	269.20	115.28	187.97
1951:2	234.6	265.99	277.51	271.93	117.00	190.00
1951:3	240.0	271.19	282.89	274.74	118.43	192.80
1951:4	241.1	268.78	280.63	277.46	119.75	196.50
1952:1	241.9	268.78	280.75	280.13	121.08	199.80
1952:2	242.6	269.26	281.36	282.78	122.51	202.66
1952:3	248.0	273.73	285.98	285.51	123.40	205.85
1952:4	254.9	279.50	291.84	288.33	125.42	209.08
1953:1	258.0	282.58	295.13	291.20	127.75	211.72
1953:2	260.1	284.26	297.04	294.06	129.97	214.98
1953:3	259.6	282.17	295.17	296.84	132.05	217.26
1953:4	257.1	279.46	292.66	299.51	134.05	219.60
1954:1	260.6	281.42	294.83	302.19	135.61	229.13
1954:2	261.3	282.18	295.74	304.86	137.34	225.62
1954:3	263.6	285.28	299.01	307.56	139.08	229.51
1954:4	270.4	292.96	306.86	310.42	141.22	232.87
1955:1	276.6	298.70	312.83	313.38	144.08	236.60
1955:2	283.7	306.37	320.78	316.49	147.45	239.40
1955:3	287.4	309.36	324.11	319.64	151.00	241.87
1955:4	292.4	314.41	329.51	322.88	154.10	244.27
1956:1	293.9	313.99	329.40	326.06	156.46	246.53
1956:2	297.8	315.80	331.45	329.24	158.63	249.10
1956:3	302.3	317.21	333.07	332.41	160.50	251.50
1956:4	307.5	320.98	337.03	335.63	162.61	254.47
1957:1	311.1	321.72	337.98	338.83	164.84	257.63

TABLE A-2 (continued)

	Y_t^{PS} (1)	y_t^{PS} (2)	y_t^{PSDY} (3)	y_{Pt} (4)	d_t (5)	M_{3t} (6)
1957:2	314.2	322.92	339.40	342.01	166.72	260.60
1957:3	318.0	324.16	340.83	345.18	168.37	263.43
1957:4	316.3	321.12	337.95	348.22	169.93	265.67
1958:1	314.4	315.66	332.65	351.09	170.72	269.60
1958:2	317.6	317.60	334.67	353.96	171.20	276.47
1958:3	325.1	324.77	341.89	356.97	171.83	281.80
1958:4	334.2	333.20	350.38	360.14	172.69	286.43
1959:1	339.8	337.77	355.04	363.38	174.35	290.90
1959:2	348.1	344.99	362.43	366.75	176.43	294.73
1959:3	345.3	339.86	357.50	369.95	178.60	297.87
1959:4	349.4	342.55	360.41	373.17	180.20	299.00
1960:1	354.1	346.14	364.16	376.43	182.25	299.80
1960:2	356.9	347.52	365.74	379.68	184.26	302.07
1960:3	357.2	346.80	365.22	382.87	186.01	307.50
1960:4	355.0	342.66	361.27	385.91	187.32	312.93
1961:1	357.7	344.60	363.34	388.95	188.11	318.27
1961:2	366.0	352.94	371.75	392.15	189.24	323.97
1961:3	372.6	358.27	377.19	395.44	190.62	329.43
1961:4	382.1	366.70	385.76	398.90	192.38	334.93
1962:1	386.9	370.24	389.48	402.39	194.48	341.47
1962:2	391.6	374.02	393.47	405.93	196.48	348.33
1962:3	394.4	375.62	395.27	409.46	198.77	353.27
1962:4	399.0	378.92	398.79	413.02	201.22	360.37
1963:1	402.6	381.25	401.37	416.59	203.88	368.63
1963:2	406.7	383.68	404.07	420.17	206.60	376.50
1963:3	414.1	389.92	410.58	423.85	209.53	383.70
1963:4	420.8	394.38	415.33	427.60	212.54	391.43
1964:1	433.5	405.52	426.77	431.58	215.95	397.73
1964:2	445.7	415.38	436.97	435.75	219.61	404.83
1964:3	453.2	421.97	443.93	440.02	223.35	414.27
1964:4	457.7	424.58	446.92	444.31	226.49	423.47
1965:1	469.0	433.46	456.11	448.76	231.03	432.20
1965:2	477.0	438.42	461.52	453.27	235.15	439.50
1965:3	493.6	452.84	476.36	458.08	239.75	448.37
1965:4	505.1	462.12	486.10	463.07	244.78	459.27
1966:1	513.5	465.97	490.45	468.08	250.26	468.13
1966:2	520.0	467.21	492.23	473.06	254.71	474.77
1966:3	529.6	473.28	498.75	478.12	259.60	478.63
1966:4	541.7	480.23	506.19	483.29	264.24	483.23
1967:1	544.5	480.58	507.01	488.40	268.12	492.40
1967:2	550.8	484.01	510.82	493.53	272.75	505.17
1967:3	560.7	488.41	515.69	498.69	276.98	519.33
1967:4	570.2	492.83	520.52	503.90	281.17	529.90
1968:1	579.6	496.23	524.35	509.12	286.27	538.60

TABLE A-2 (concluded)

	Y_t^{PS} (1)	y_t^{PS} (2)	y_t^{PSDY} (3)	y_{Pt} (4)	d_t (5)	M_{3t} (6)
1968:2	595.7	504.83	533.46	514.49	291.45	548.13
1968:3	602.3	506.99	536.13	519.84	297.21	558.90
1968:4	609.9	507.83	537.55	525.14	302.65	572.33
1969:1	613.8	506.44	536.70	530.34	308.46	582.90
1969:2	626.5	510.59	541.44	535.57	314.00	589.30
1969:3	642.7	517.06	548.45	540.90	319.02	590.73
1969:4	649.4	515.81	547.71	546.12	323.88	592.80
1970:1	660.6	518.12	550.51	551.34	328.28	594.87
1970:2	681.3	529.78	562.61	556.77	332.78	604.30
1970:3	695.2	536.01	569.28	562.29	337.01	619.60
1970:4	697.2	529.79	563.49	567.58	339.49	637.00
1971:1	722.5	544.46	578.41	573.15	344.38	659.47
1971:2	741.4	552.87	587.31	578.86	349.19	685.43
1971:3	748.9	554.74	589.66	584.54	354.60	703.80
1971:4	763.8	563.27	598.73	590.36	360.34	721.50
1972:1	775.2	566.67	602.70	596.19	366.65	744.03
1972:2	791.4	574.73	611.39	602.14	373.42	765.97
1972:3	809.1	583.35	620.69	608.23	380.78	790.67
1972:4	842.5	603.08	641.16	614.74	388.58	815.00
1973:1	873.3	617.61	656.47	621.53	397.72	835.87
1973:2	893.3	619.06	658.83	628.27	406.03	854.50
1973:3	915.1	622.52	663.12	635.01	413.59	870.47
1973:4	939.3	622.88	664.24	641.68	419.04	887.70

NOTE: Columns 1 through 4 are at seasonally adjusted annual rates; divide by 4 to obtain the seasonally adjusted quarterly rates used in the text. Columns 1 through 5 are based on final data through 1971:4. Federal Reserve estimates are subject to change back to 1959.

Cols. 1–3: As defined in the corresponding notes to Table A-1. To extend or update, see Table A-1.

Col. 4: Permanent income based on y_t^{PSDY} and a β weight of 0.025 per quarter (billions of 1958 dollars). To update or extend: $y_{Pt} = 0.025 y_t^{PSYD} + 0.9843473 y_{PT-b}$.

Col. 5: Stock of consumers' durable goods at the end of the quarter (billions of 1958 dollars). To update or extend: $d_t = 0.24375 c_t^d + 0.95 d_{t-1}$, where c_t^d is consumption expenditures for durable goods in constant (1958) dollars at seasonally adjusted annual rates.

Col. 6: See corresponding note in Table A-1.

TABLE A-3: Estimates of M_3 Money Stock, Monthly, 1947–1959 (seasonally adjusted monthly averages in billions of dollars)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1947	168.4	169.0	170.2	171.1	171.8	172.8	173.2	174.0	175.0	175.2	176.0	176.0
1948	176.7	176.9	176.6	176.3	176.1	176.4	176.9	177.1	177.0	177.1	176.9	176.9
1949	176.7	177.2	177.2	177.7	178.3	178.2	178.3	178.4	178.3	178.6	178.9	179.4
1950	179.9	180.7	181.6	182.7	183.7	184.1	184.6	184.9	185.1	185.8	186.5	186.9
1951	187.5	187.9	188.5	189.5	190.0	190.5	191.8	192.5	194.1	194.8	196.9	197.8
1952	198.9	199.7	200.7	201.7	202.6	203.6	204.6	205.8	207.1	207.9	209.2	210.2
1953	210.7	211.4	213.0	214.1	215.3	215.5	216.5	217.4	217.9	218.7	219.7	220.4
1954	231.5	232.4	223.5	224.1	225.9	226.8	228.4	229.6	230.6	231.7	233.0	233.9
1955	235.3	237.1	237.4	238.4	239.6	240.2	241.1	241.7	242.8	243.7	244.1	245.0
1956	245.8	246.5	247.3	248.4	248.9	250.0	250.6	251.3	252.6	253.4	254.5	255.5
1957	256.6	257.6	258.7	259.6	260.7	261.5	262.6	263.5	264.2	264.9	265.7	266.4
1958	266.9	269.8	272.1	274.4	276.3	278.7	280.0	282.0	283.4	284.8	286.6	287.9
1959	290.0	290.7	292.0	293.4	294.8	296.0	297.5	297.8	298.3	298.5	299.1	299.4

NOTE: $M_3 = M_2$ money stock plus deposits at mutual savings banks and savings and loan associations (Federal Reserve Board definition). Data for 1947 through 1958 are estimated by adding to M_2 the Friedman and Schwartz (1970) data on mutual savings bank and savings and loan association deposits. Monthly savings and loan data were interpolated between annual (1947–1949) and quarterly (1950–1954) benchmarks by the use of mutual savings bank deposits. Data for 1959 are Federal Reserve figures.

NOTES

1. A rough definition of behavioral durability is responsiveness to transitory income. Consumer expenditures for durable goods and for clothing and shoes are about equally responsive to changes in transitory and permanent income, but all other expenditures are only about one-quarter as responsive to changes in transitory compared with permanent income. Thus the official Commerce Department definition does appear to capture goods significantly more durable than those classified as nondurable goods and services (with the exception of clothing and shoes). Given the relative magnitudes, however, the remaining durable elements in "nondurable goods and services" are nevertheless quite significant.
2. In my earlier work (1972, 1975), this adjustment was neglected because of the small difference from unity for quarterly data (with $\delta = 0.05$, $1 - 0.5\delta = 0.975$).
3. This adjustment, too, is small for quarterly data. Using $r = 2.5$ percent per quarter or 10 percent per year, $1 - 0.5r = 0.9875$ for quarterly data or 0.95 for annual data.
4. I am assuming a constant real yield, r, here. In fact, r would vary over time—particularly when the stock of durables is not at its long-run optimum. The observed willingness of consumers to use durables to absorb transitory income shocks suggests that the actual real yield variations are negligible.
5. Darby (1974) demonstrates the interpretation of permanent income as a perpetual inventory of wealth.
6. Other possible influences have been omitted either here or below in completing the specification because of the difficulty in obtaining good data and the paucity of true degrees of freedom. The empirical results that follow do not seem to have suffered much.
7. A real interest rate is correct here. If π is the expected inflation rate and τ the marginal tax rate on interest (see Darby 1976, pp. 74–75), then the a_3 term should be $a_3\{i - [\pi/(1 - \tau)]$. Carrying through to equation 10, this implies that a term $-[\lambda_1\alpha_3/(1 - \tau)]\pi$ is omitted from the specification. Since π and i are positively correlated and $-\lambda_1\alpha_3/(1 - \tau)$ is positive, this imparts a positive bias to the estimated interest rate coefficient. The task of including an estimated inflation rate is left for future research.
8. The coefficient of permanent income will capture the effect of wealth and of secular trends in institutions, payments technology, and so forth. The coefficient of transitory income reflects both effects of windfalls on portfolio adjustment and of cyclical variations in transactions (see Darby 1972).
9. Unfortunately, even given the value of r, the only structural parameters that can be recovered from the regression coefficients are λ_1, λ_3, and α_2. The other ten parameters cannot be separately identified.
10. Basic data series were all drawn from the NBER data bank
11. This amounts to the usual ten-year-life, double-declining-balance method. The initial value for December 31, 1946, was computed from Raymond Goldsmith's (1962) data as 72.43 billion 1958 dollars. See Darby (1972, pp. 931–932) for details. This calculation requires that c_t^d be measured at *quarterly* rates in order to integrate flows into stocks.
12. A theoretically more attractive definition would be to base the imputation on the average durables stock for the period $d_{t-1} + 0.5\,\Delta d_t$. This was not done because it may impart spurious correlation, particularly in the disaggregated estimation of the Δd_t equation.
13. That is, private-sector income equals disposable personal income + undistributed corporate profits + wage accruals less disbursements + corporate inventory valuation adjustment less other personal outlays. For computational purposes, an equivalent definition is net national product less taxes net of transfers (i.e., government purchases of goods and services + NIA [national income accounts] surplus) less government and private transfers to foreigners less statistical discrepancy.

14. This growth rate is implicit in saving plans. The required growth rates and initial values y_{P0} (0 = 1946 for annual data and 1946:4 for quarterly data) were estimated by a log-linear trend (see Darby 1974 for details) as:

Income Concept	$\hat{g}$	$\hat{y}_{P0}$
Accrual (annual data)	0.03832	223.773
Accrual (quarterly data)	0.00959	56.7934
Cash receipts (annual data)	0.04003	233.973
Cash receipts (quarterly data)	0.01001	54.3573

15. This series is the most problematical. It was pointed out in the introduction that roughly half of behaviorally durable goods are included among nondurable goods and services. Thus their prices will be included in the denominator instead of the numerator. Also, the durable goods price deflator is generally believed biased relative to the nondurables deflator because of more rapid quality improvements in the former.

16. The t values are given in parentheses. The square brackets indicate the confidence interval for a probability of more than 90 percent computed on the basis of the asymptotic distribution of the logarithm of the likelihood function. For the annual regression, $\hat{\beta} = 0, 0.025, 0.050, \ldots, 0.975, 1.000$ was searched for the value that maximized the likelihood function. For the quarterly regression, $\hat{\beta} = 0, 0.01, 0.02, \ldots, 0.99, 1.00$ was used.

17. See the analysis of the coefficients below equation 11. The annual coefficients should be approximately four times the quarterly ones except for those of y_{Pt} and y_{Tt}. Only the amount by which β_1 exceeds k is multiplied by 4 for y_{Pt}. The coefficient β_2 of y_{Tt} should be essentially unchanged, as the lower quarterly value of λ_3 is offset by a higher value of γ_2; hence, the only expected change is due to the slightly higher quarterly value of the $(1 - 0.5r)$ adjustment factor.

18. For example, a naive first order autoregression shows a very high R^2 for the annual data:

$$c_t^s = \underset{(-1.14)}{-4.72} + \underset{(88.47)}{1.054}\, c_{t-1}^s$$

where $SEE = 5.82$; R^2(adj.) $= 0.9968$; and $D.W. = 2.13$.

19. The value of k is estimated on the basis of $k = \hat{\beta}_1 - (1 - 0.5r)[(1 - \lambda_1)\eta + \lambda_1\alpha_1 - \lambda_3\gamma_1]$. The imputed yield $r = 0.1$. From regression 18, $\hat{\beta}_1 = 1.08$; $\hat{\lambda}_1 = -(\hat{\beta}_4 + r)/(1 - 0.5r) \approx 0.29$; $\hat{\lambda}_3 = \hat{\beta}_3/(1 - 0.5r) \approx 0.72$. The values of $\hat{\alpha}_1$ and $\hat{\gamma}_1$ are estimated by dividing the total sample-period change in the durables stock and the real money stock, respectively, by the total change in permanent income. So $\hat{\alpha}_1 \approx 0.79$ and $\hat{\gamma}_1 \approx 0.087$. The estimated value of $\hat{\eta}$ is computed as $\eta = (\Delta d_t)^e/y_{Pt} = \alpha_1(\Delta y_{Pt}/y_{Pt})^e \approx \hat{\alpha}_1\,[\hat{g}/(1 + \hat{g})] \approx 0.029$, where $\hat{g} = 0.03832$ from footnote 12 above. Substitution yields $k \approx 1.08 - 0.95\,(0.021 + 0.229 - 0.063) \approx 0.90$. The estimate of α is computed by noting that in the long-run $y_t^{ps} = y_{Pt} - rd_{t-1}$ and

$$(1 - \sigma) = \frac{c_t^s}{y_t^{ps}} = \frac{y_{Pt}}{y_t^{p} - rd_{t-1}}\,\frac{c_t^s}{y_{Pt}}$$

Substituting equation 4 for c_t^s yields

$$1 - \sigma = \frac{1}{1 - (rd_{t-1}/y_{Pt})}\left[k + (1 - 0.5r)\frac{\Delta d_t}{y_{Pt}} - r\,\frac{d_{t-1}}{y_{Pt}}\right]$$

Taking $d_{t-1}/y_{Pt} \approx \hat{\alpha}_1 - \hat{\eta} \approx 0.76$,

$$1 - \hat{\sigma} = (1/0.924)[0.90 + (0.95)(0.029) - (0.10)(0.76)] = 0.92$$

So $\hat{\sigma} = 0.08$.

20. The estimates of the β weight of current income in permanent income were a bit closer to the value of 0.1 per year estimated in Darby (1974). The only other noticeable—though statistically insignificant—changes were generally higher (in absolute value) estimates for the coefficients of money and the durables stock. All these changes are consistent with the hypothesized shift, but the standard errors of estimate actually deteriorated slightly in the truncated period.
21. The coefficient of transitory income, y_{Tt}, is higher in regression 3 than in 2, reflecting the larger offset in (3) due to its higher money coefficient. Note that, rounding error aside, regressions 2 plus 3 less 0.1 d_{t-1} equal regression equation 18. Similarly, regressions 5 plus 6 less 0.025 d_{t-1} equal regression equation 19.
22. I am indebted to Thomas Mayer for the observation that for low β weights, permanent income and the durables stock are closely related because of the high correlation between transitory income and fluctuations in household durables investment. A high estimate of β applies too low a weight to past transitory income and can be offset by a more negative coefficient on the durables stock. This correlation is the probable explanation for the relatively flat likelihood function at the low end of the β range, as discussed in section IV. In regressions based on a β weight of 0.1 per year (but not reproduced here), the coefficient of d_{t-1} is −0.092 for the c_t^d dependent variable and −0.102 for the Δd_t dependent variable. Consumer expenditure functions for $\beta = 0.1$ per year and 0.025 per quarter are presented in section IV.
23. As explained in the first part of section III, the accrual concept is private-sector income adjusted for the yield on the durables stock, while the cash receipts concept is disposable personal income with the same adjustment. The conclusions as to the relative merits of the two concepts are not affected by omission of the durables yield adjustment.
24. To be precise, transfers to foreigners and the statistical discrepancy should also be subtracted.
25. The monthly M_3 data for January 1947 through December 1959 are reported in the Data Appendix. Monthly savings and loan deposits were interpolated between annual (1947–1949) and quarterly (1950–1954) benchmarks by the use of mutual savings bank deposits.
26. The quarterly sums of squared residuals are biased downward by the autocorrelation indicated by the low Durbin-Watson statistics. Note that this statistic is 1.08 for the accrual definition and 0.94 for the cash receipts definition (regressions 13 and 14 respectively). Only regressions 17 and 18 for the quarterly M_3 comparison are close to a dead heat. That presumably reflects some peculiarity in the data which also accounts for the unusually high β estimate of 0.06 per quarter in regression 17.
27. In comparing definitions such as these, the hypotheses are not strictly nested and no generally acceptable significance test exists. Consider the following, however. If the difference between the accrual and the cash receipts definitions were allowed to enter with a weight, μ, (to be estimated) between 0 and 1, the cash receipts definition would be nested (with the restriction $\mu = 1$) in the more general hypothesis that income is the sum of the accrual concept plus μ times the difference. For this model, SSR could not be greater than SSR for $\mu = 0$ (the accrual income definition). If we suppose that this upper limit on the unconstrained sum of squares is the actual value—which is favorable to accepting the cash receipts definition—the likelihood ratio test could be used. The critical value at the 5 percent significance level for the excess sum of squares would then be 15.3 percent for annual data

and 3.6 percent for quarterly data. So even on this apparently generous test, the cash receipts definition is significantly worse than the accrual definition. The same argument and critical values would apply to the money definitions discussed below.

28. The t values are given in parentheses. The greater than 90 percent confidence interval for β is [0, 0.23] for annual data and [0, 0.06] for quarterly data.

29. The analysis of this specification bias is contained in Darby (1974). A 10 percent per year β weight was estimated there on the basis of a pure consumption model.

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