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10 Index Hedge Performance: Insurer Market Penetration and Basis Risk

John A. Major

Index-based financial instruments bring transparency and efficiency to both sides of risk transfer, to investor and hedger alike. Unfortunately, to the extent that an index is anonymous and commoditized, it cannot correlate perfectly with a specific portfolio. Thus, hedging with index-based financial instruments brings with it basis risk. The result is "significant practical and philosophical barriers" to the financing of property/casualty catastrophe risks by means of catastrophe derivatives (Foppert 1993). This study explores the basis risk between catastrophe futures and portfolios of insured homeowners' building risks subject to the hurricane peril.[1]

A concrete example of the influence of market penetration on basis risk can be seen in figures 10.1–10.3. Figure 10.1 is a map of the Miami, Florida, vicin-

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1. Section 10.4 below comments on the all-lines, all-coverages, all-perils context.

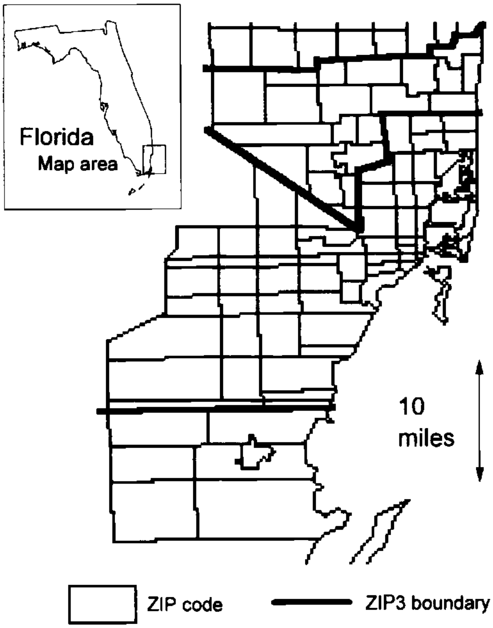

Fig. 10.1 Miami, Florida, vicinity zip codes

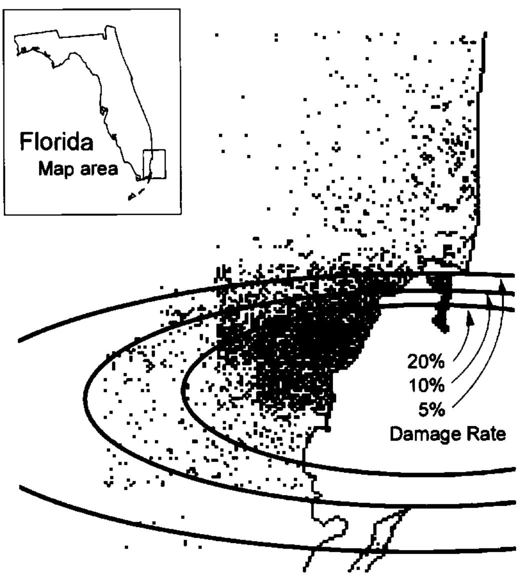

Fig. 10.2 Effect of Hurricane Andrew on the homeowner's-insurance industry
Note: Each dot represents $1 million of losses to the insurance industry.

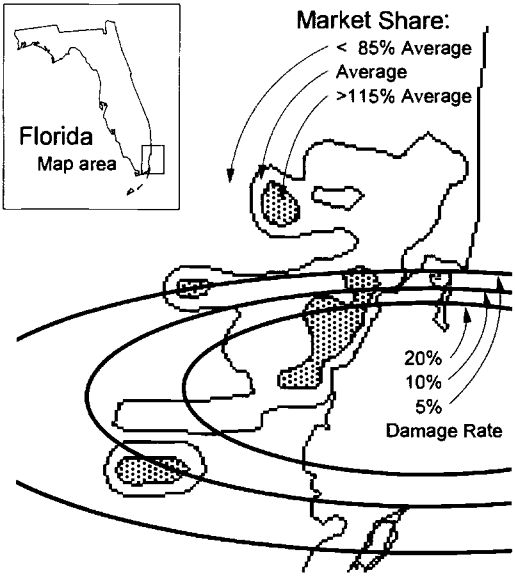

Fig. 10.3 Market penetration vs. damage rate

ity, from just south of Ft. Lauderdale to just north of Key Largo. The polygons depict zip codes, with the bolder lines marking the boundaries of zip sectional centers.[2] Figures 10.2 and 10.3 register to the same scale and placement as figure 10.1.

Figure 10.2 shows the effect of Hurricane Andrew on the homeowner's-insurance industry in Florida. Three levels of contours represent the damage rate (losses divided by exposed value) caused by the hurricane. Near the center of the contours, the damage rate approached 50 percent. Each dot represents $1 million of losses to the industry. The densest portion of losses is not centered in the damage-rate contours because of the population-density gradient. The highest concentration of exposed value, the center of the Hialeah–Miami–Miami Beach metropolitan area, is to the north of the 5 percent contour line. The population density decreases steadily as one moves south. Had the hurricane made landfall ten miles north or south of where it did, there could have been a factor-of-two change in the industry outcome.

Figure 10.3 shows the market penetration of a particular homeowner's insurer at the time of the hurricane. Imagine that this company had held a commoditized cat contract instead of reinsurance, a contract that would reimburse it a share of the industry losses from a natural disaster equal to its share of

2. A zip (zone improvement plan) code is a unit of geography defined by the U.S. Postal Service and designated by a five-digit number. On average, residential zip codes contain about two thousand households. Sectional centers are administrative units composed of all zips with the same first three digits.

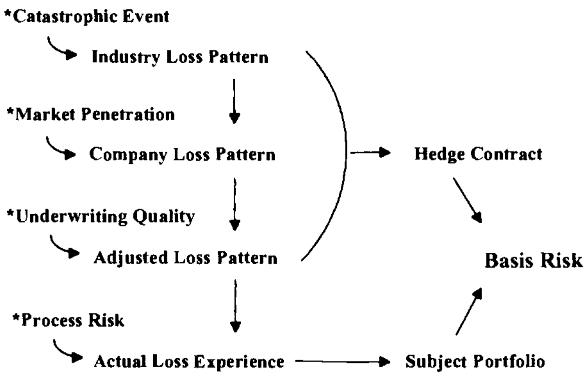

Fig. 10.4 Influences on hedge performance

exposures in southeastern Florida. Basis risk emerges in the mismatch between expectations and outcomes: its share of exposures is not uniform across the area. This company had the misfortune to have its peak market penetration almost coincide with the peak damage rates. Its actual damages were about 25 percent higher than its expected industry share, so the hedge would have underperformed, leaving the company with 20 percent of its losses not covered. On the other hand, if the hurricane had come in just ten miles south of where it did, the hedge would have netted the company a profit.

The influences on both the losses experienced by a portfolio of insured risks and the recoveries available from an index-based hedge, as illustrated in figure 10.4, can be classified as follows:

The catastrophic event itself. This is the industrywide pattern of losses arrayed in space.

The market penetration of the subject portfolio. Multiplying this pattern by industry losses produces an estimate of the subject company's losses.

Underwriting quality. This includes nonspatial characteristics of the subject portfolio, such as deductibles, policy forms, risk-selection standards, and claim-settlement practices. These characteristics contribute to loss variation, even after the event and market penetration have been taken into account. To the extent that they are understood, a hedge can be adjusted for them.

Process risk. This is the ultimately unpredictable component of loss variation.

Basis risk is the random variation of the difference between the hedge-contract payout and the actual loss experience of the subject portfolio. Two types of basis risk are considered. *Conditional basis risk* refers to the variation due to the influence of factors other than the events. It addresses such questions as, How would the hedge perform if an event like Andrew were to occur? Given an implicit class of events—"like Andrew"—the relation between the portfolio's loss and the contractual recovery has a random character. For some events in the class, the insurer is "lucky" in that, say, its penetration is low in the most heavily damaged areas. This variation is termed *conditional basis risk.* Considering all sources of variation, that is, allowing varying events to influence outcomes as well, yields the more familiar definition of basis risk. Here, the term *unconditional basis risk* is used to emphasize the distinction.

To operationalize the notion of conditional basis risk, this study uses an equivalence principle: the subject portfolio is a random draw from a class of portfolios sharing the same *market characteristics.* Conditional basis risk can then be treated as the sampling behavior of hedge outcomes in a specific event when portfolios are drawn from the market-characteristics equivalence class. The motivation for this definition is that, ex ante, the hedger posing the question about conditional performance has an event firmly in mind but does not know where it will strike the portfolio. Intuitively, the idea is to "shift the event" to different parts of the subject portfolio yet maintain all industry-specific attributes of the event. The equivalence principle does this by exchanging the portfolio for an equivalent version. Section 10.1 defines the market-characteristics equivalence class by showing how to characterize a portfolio in a small number of parameters, the *market-characteristics vector.* It also develops a model of the distribution of this vector as well as a model of process risk.

This study models catastrophe indices built up from the insurance industry's catastrophe losses from an event, along with corresponding exposures, by zip code. A hedging instrument is some linear combination of the zip-by-zip losses reported by the index. For a futures contract based on a single-valued statewide index (e.g., the Property Claim Services [PCS] cat index), that linear combination consists of a single constant applied to all zip codes in a state.

Section 10.2 shows the use of these models in a Monte Carlo simulation (Metropolis and Ulam 1949; Rubinstein 1981) of catastrophe-index hedging. It explores the influence of insurer market penetration on basis risk by contrasting the performance of statewide and zip-based contracts. Conditional and unconditional measures of basis risk, correlation coefficients, and optimal hedge ratios are presented.

This study assumes no underwriting-quality influence. The boundary between underwriting quality and process risk is indistinct; it is determined by the extent to which risk characteristics can be taken into account. If an insurer kept and made use of detailed records concerning the construction and materials of each house in its portfolio, these factors would be considered part of underwriting quality. If not, they would be considered part of process risk. To the extent that a hedger understands its own practices and portfolio characteristics vis-à-vis the industry and their implications for loss experience, it can adjust its use of index contracts accordingly.[3] In this sense, the study assumes perfect self-knowledge; hedge-performance estimates reported here should therefore be considered upper bounds.

Section 10.3 discusses related work. Section 10.4 makes concluding remarks. The appendix discusses the modeling of insurer market penetration in detail.

10.1 A Model of the Underlying

10.1.1 Introduction

Consider a state consisting of a set of zip codes symbolized by z. The companies doing business in the state are symbolized by c. Let $R_{z,c}$ denote the risk count (number of insured homes) that company c has in zip code z. Let R_z denote the sum across all companies, that is, the total market, in zip code z. Let R_c and R be the respective sums over all the zips in the state. $M_c = R_c/R$, the risk-count share of company c, is a key parameter throughout this study.[4] Let $v_{z,c,i}$ be the insured value of the ith risk and $\bar{v}_{z,c}$, $\bar{v}_z$, and $\bar{v}$ the average values for the respective groups.

The objective of sections 10.1.2–10.1.4 below is to characterize the pattern of penetration and values for company c in a small number of parameters, the *market-characteristics vector.* In section 10.1.5 below, a second level of modeling considers the distribution of the market-characteristics vector itself.

A catastrophic event is represented as a schedule of damage rate (expected loss–to–value ratio) by zip code. Section 10.1.6 below factors damage rate into its frequency and severity components. Loss can then be modeled as a compound Poisson process (Beard, Pentikäinen, and Pesonen 1984).

10.1.2 The Market Characteristics of One Company

The simplest model of risk count is to assume that $R_{z,c}$ follows a Poisson distribution.[5] Since zip codes vary in population, it makes sense to represent the Poisson mean as $\lambda_{z,c} = \pi_c \cdot R_z$. However, constant penetration π_c is implausible because realized penetration varies much more than Poisson. This leads to a two-stage hierarchical model $\lambda_{z,c} = \pi_{z,c} \cdot R_z$, where $\pi_{z,c}$ is itself a random variable. A logical choice would be lognormal, yielding $\lambda_{z,c} = \exp(\mu_c + \xi_{z,c}) \cdot R_z$, where μ_c is constant, and $\xi_{z,c} \sim \text{Normal}(0, \sigma_c^2)$. Even this is inadequate be-

3. Underwriting quality also influences the reported index values. If less than the full industry is incorporated into an index, there is sampling error. Careful index construction and the central limit theorem can assure that this error remains small relative to the hedger's variation.

4. While *market share* is usually defined as the share of premiums received by the company, it will be convenient to deal with shares of risk count and shares of total insured value.

5. Since market share rarely exceeds 20 percent, the Poisson process can be used rather than the more technically correct but cumbersome binomial process.

cause penetration is spatially autocorrelated.[6] A case study illustrating spatial autocorrelation is presented in the appendix.

There are two fundamental approaches to the analysis of spatial structure.[7] Metric methods (Whittle 1954) deal with distance relations between observations. Occurrence methods (Grieg-Smith 1964; Kershaw 1964) count observations in quadrats (random samples of areas). This study treats zip codes as quadrats at one level of scale and zip sectional centers (three-digit zip codes) as quadrats at a larger scale. This leads to a model of the form

$$\lambda_{z,c} = \exp(\mu_c + \xi_{z,c} + \zeta_{z3,c}) \cdot R_z,$$

$$R_{z,c} \sim \text{Poisson}(\lambda_{z,c}),$$

with μ_c constant, $\xi_{z,c} \sim \text{Normal}(0, \sigma_c^2)$, $z3$ symbolizing three-digit zips, and $\zeta_{z3,c} \sim \text{Normal}(0, \tau_c^2)$.

The selection of risks within a zip is not homogeneous either. Another two-level hierarchical component of the model accounts for differential selection of homes by value:

$$\bar{v}_{z,c} = \exp(\varepsilon_{z,c}) - \bar{v}^{(1-\beta_c)} \cdot \bar{v}_z^{\beta_c},$$

$$v_{z,c,i} = \bar{v}_{z,c} \cdot \exp(\phi_{z,c,i}),$$

where $\varepsilon_{z,c} \sim \text{Normal}(\alpha_c, \upsilon_c^2)$, and $\phi_{z,c,i} \sim \text{Normal}(-\frac{1}{2} \cdot \omega_c^2, \omega_c^2)$.[8]

The pattern of risk count and values for a particular company in a particular state is thus abstracted as being the result of a random process characterized by a *market-characteristics* vector of seven parameters, $\vartheta = <\mu, \sigma, \tau, \alpha, \beta, \upsilon\ \omega>$.

10.1.3 Market-Characteristics Data

As part of its risk-management consulting, Guy Carpenter has acquired portfolio descriptions of many property-casualty insurers. Ten companies were chosen at random from the database. Property-exposure data, dating from 1988 to 1995, were extracted for nine states to obtain a total of twenty-four company-state combinations. States were chosen by importance sampling (Kahn 1950), where the weighting was proportional to the PCS historical catastrophe loss totals, with some concession to the presence of data exhibited by the companies. A Latin Square protocol (Cochran and Cox 1957) was fol-

6. Zip codes near one another have more similar penetration rates than widely separated zip codes.

7. Typically, the first step in analyzing spatial autocorrelation is to consider exogenous characteristics of the geographic areas, themselves spatially autocorrelated, as potential regressors. For example, one might test whether penetration is a function of median housing value. Company-independent factors are unavailable for this purpose because the study adopts the stance that 100 percent of the market is penetrated by 100 percent of the companies.

8. The choice of $-\frac{1}{2} \cdot \omega_c^2$ for the mean of $\phi_{z,c,i}$ enforces $E[v_{z,c,i}] = \bar{v}_{z,c}$.

Table 10.1 **Penetration Study Data**

	Zip Codes		Company									
State	All	> 50 OOHU	A	B	C	D	E	F	G	H	I	J
FL	941	907	√	√	√							
MA	564	545	√			√	√					
MD	484	449		√				√	√			
MN	946	895									√	√
MT	375	296								√	√	
NC	859	819			√	√				√		
NJ	626	610				√		√	√			
NY	1,890	1,750		√	√							
TX	1,968	1,804	√				√			√		

Note: OOHU = owner-occupied housing units.

Table 10.2 **Distribution of Risk-Count Share $M_c = R_c/R$**

Size Class	Risk-Count Share (%)	Number of Company-States
1	.2–.5	5
2	.5–1.0	3
3	1.0–2.0	7
4	2.0–5.0	5
5	5.0–10.0	4
Total		24

lowed so that every state could be paired with at least two companies and every company with at least two states.[9] Zip codes where the 1990 Census of Population and Housing reported at least fifty owner-occupied housing units were kept.[10] This typically retained 99.9 percent of the housing units in the state. Tables 10.1 and 10.2 summarize.

10.1.4 Fitting the Market Characteristics

Maximum-likelihood estimation of hierarchical models is notoriously difficult, involving high-dimensional integration of analytically intractable integrands (Hill 1965; Tiao and Tan 1965). Modern methods to solve these problems include integral approximations (Tierney and Kadane 1986), the EM algorithm (Dempster, Laird, and Rubin 1977), and Gibbs sampling (Gelfand and Smith 1990). Computational efficiency is important in this study because of the number of observations (sometimes over fifteen hundred zip codes) and the need to make estimates for each of the twenty-four cases.[11]

9. There was a failure to meet the criterion; one company had usable data in only one state.
10. Zip-code tabulations of census data were obtained from Claritas, Inc.
11. In addition, ninety-six estimation cycles were executed in bootstrap replications.

Table 10.3 **Occurrence of Zero Risk Count among Zips**

Size Class	Risk-Count Share (%)	Frequency of $R_{z,c} = 0$ (%)
1	.2–.5	42.4
2	.5–1.0	44.0
3	1.0–2.0	12.6
4	2.0–5.0	8.0
5	5.0–10.0	6.6

Could one substitute observed risk count $R_{z,c}$ for the Poisson parameter $\lambda_{z,c}$ and estimate μ_c, σ_c, and τ_c by performing analysis of variance (ANOVA) (Fisher 1942) directly on $\ln(R_{z,c}/R_z)$? This has some justification; variation among the $R_{z,c}$ is much greater than Poisson. However, it fails because $R_{z,c}$ is very often zero. Table 10.3 summarizes, pooling cases in each size class.

Therefore, a more sophisticated approach to estimating the $\lambda_{z,c}$ is used: an empirical Bayes technique (Robbins 1951, 1955) treats the "true" unobservable penetration rates $\pi_{z,c}$ as parameters that underlie the observed Poisson risk counts. These parameters, as random variables in their own right, follow a prior distribution driven by hyperparameters estimated from the "crude" observed penetration rates. The estimated prior allows a Bayes posterior point estimate of the realized-penetration-rate parameters $\pi_{z,c}$. This procedure is detailed in the appendix.

The next step in the analysis is to conduct a random-effects ANOVA with the following model:

$$\ln(\pi_{z,c}) = \ln(\lambda_{z,c}/R_z) = \mu_c + \xi_{z,c} + \zeta_{z3,c},$$

where μ_c is constant, $\xi_{z,c} \sim \text{Normal}(0, \sigma_c^2)$, and $\zeta_{z3,c} \sim \text{Normal}(0, \tau_c^2)$.

Henderson's Method I (Henderson 1953) is used to estimate μ_c and the variance components σ_c^2 and τ_c^2. While not the most sophisticated approach, it has the advantage of being very fast computationally because it relies on sums-of-squares equations and uses no iteration.

The final step is to develop the distribution of insured property values. The model[12]

$$\ln(\bar{v}_{z,c}/\bar{v}) = \alpha_c + \beta_c \cdot \ln(\bar{v}_z/\bar{v}) + \varepsilon_{z,c},$$

where $\varepsilon_{z,c} \sim \text{Normal}(0, \upsilon_c^2)$, is fitted by weighted least squares, with weights proportional to $R_{z,c}$.

Unfortunately, the data consist of aggregates by zip code and so are insufficient to fit

$$\ln(v_{z,c,i}) = \ln(\bar{v}_{z,c}) + \phi_{z,c,i},$$

12. Median housing values in the census data were used in preference to means.

where $\phi_{z,c,i} \sim \text{Normal}(-\frac{1}{2} \cdot \omega_c^2, \omega_c^2)$. To estimate ω_c, detailed risk-by-risk data are needed. Fortunately, such detail was available from another study of one company in one state, and the estimated value there was $\omega_c = 0.51$.

By the above process, the twenty-four cases were abstracted to points ϑ_c in a seven-dimensional parameter space. Since there is only the single fixed value for ω, it is not useful to include it in the parameter space. It is, however, useful to augment the parameterization by the new variable $\kappa_c = \ln(M_c) = \ln(R_c/R)$, representing the observed risk-count share of the company. Given such a point $\vartheta_c = \langle\kappa, \mu, \sigma, \tau, \alpha, \beta, \upsilon\rangle$, simulation can construct a pattern of risk count and aggregate value with the specified spatial characteristics according to the hierarchical model

$$
\begin{aligned}
\xi_z &\sim \text{Normal}(0, \sigma^2), \\
\zeta_{z3} &\sim \text{Normal}(0, \tau^2), \\
\lambda_z &= \exp(\mu + \xi_z + \zeta_{z3}) \cdot R_z, \\
R_{z,c} &\sim \text{Poisson}(\lambda_z), \\
\varepsilon_z &\sim \text{Normal}(0, \upsilon^2), \\
\bar{v}_{z,c} &= \exp(\alpha + \varepsilon_z) \cdot \bar{v}^{(1-\beta)} \cdot \bar{v}_z^{\beta}, \\
\omega &= 0.51, \\
\phi_{z,i} &\sim \text{Normal}\left(-\frac{1}{2} \cdot \omega^2, \omega^2\right), \\
v_{z,c,i} &= \bar{v}_{z,c} \cdot \exp(\phi_{z,i}).
\end{aligned}
$$

10.1.5 The Distribution of Market Characteristics

The next step is to investigate the multivariate distribution of ϑ. Table 10.4, above the diagonal, shows the correlations of the ϑ parameters. (The lower-left triangle is discussed below.) For $N = 24$ cases, the two-tail 5 percent critical value is $|\rho| > 0.4043$; six pairs show significant correlations. That κ and μ show a high correlation is not surprising: the expected value of M according to the model is $\exp[\mu + \frac{1}{2} \cdot (\sigma^2 + \tau^2)]$.

Other correlations suggest that companies of different overall penetration behave differently in the details of their penetration. For example, negative correlation of σ and τ with κ and μ suggests that higher penetrations are more uniformly achieved. Figure 10.5 shows this relation in more detail. There was no apparent distinction between agency companies and direct writers.

Following this lead, σ, τ, and α were regressed against κ. For σ and α, simple least squares was used. Because the standard errors of the τ estimates were relatively high, weighted least squares was used with weights proportional to the inverse of the standard errors for τ values fitted in section 10.1.4

Table 10.4 Correlation Coefficients between Penetration Model Parameters

	μ	σ	τ	α	β	υ	
κ	.94	−.32	−.59	−.38	−.24	.13	κ
		−.51	−.72	−.50	−.19	.12	μ
			.51	.31	−.14	−.09	σ
τ^*		−.40		.34	.09	.15	τ
α^*		.21	−.15		.21	.01	α
β		−.23	.00	.13		−.36	β
υ		−.06	−.24	.06			
		σ^*	τ^*	α^*			

Note: See text for key.

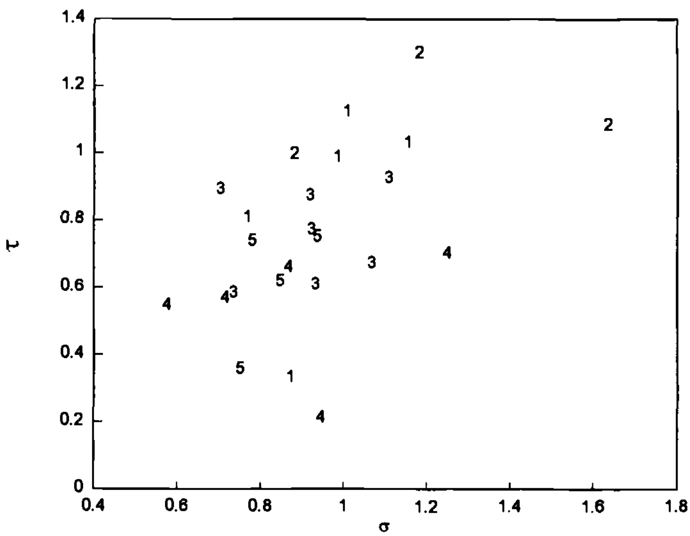

Fig. 10.5 Sigma vs. tau by risk-count share class

above. Table 10.4, below the diagonal, shows correlations where the (starred) κ-modeled parameters have been replaced by their residuals. No remaining correlation is significant.

The conditional distribution of ϑ on κ is therefore represented as

$$\mu = \kappa - \frac{1}{2} \cdot (\sigma^2 + \tau^2),$$

$$\sigma \sim \text{Normal}(m = 0.6674 - 0.0649 \cdot \kappa,\ s = 0.2146),$$

$$\tau \sim \text{Normal}(m = 0.2783 - 0.0913 \cdot \kappa,\ s = 0.2327),$$

$$\alpha \sim \text{Normal}(m = -0.2569 - 0.1387 \cdot \kappa,\ s = 0.3851),$$

$$\beta \sim \text{Normal}(m = 0.5497,\ s = 0.1514),$$
$$\upsilon \sim \text{Normal}(m = 0.4303,\ s = 0.0852).$$

10.1.6 Process Risk

The total insured value that company c has exposed to catastrophe risk in zip code z is given by $v_{z,c} = \sum_i v_{z,c,i} = R_{z,c} \cdot \bar{v}_{z,c}$. Given a particular catastrophic event in the state, let the random variable representing the loss incurred by the ith risk be $L_{z,c,i}$. The damage rate (loss-to-value ratio) is $d_{z,c,i} = L_{z,c,i}/v_{z,c,i}$. Let $d_{z,c}$, d_z, and d_c be damage rates for the aggregates. In particular, note that d_z represents the aggregate industry damage rate in zip code z.

Assume that $d_{z,c,i}$ are conditionally independent and identically distributed given z and c.[13] Let $f_{z,c}$ be the probability that the ith risk incurs a loss $L_{z,c,i} > 0$. This is the *damage-rate frequency.* Let $m_{z,c}$ and $s^2_{z,c}$ be the mean and variance of $d_{z,c,i}|(L_{z,c,i} > 0)$, the *damage-rate severity.* Because $L_{z,c,i} = d_{z,c,i} \cdot v_{z,c,i}$ and d and v are assumed independent, treat $L_{z,c}$ as a compound Poisson process with rate $l_{z,c} = R_{z,c} \cdot f_{z,c}$, expected severity $E[L_{z,c,i}\ |L_{z,c,i} > 0] = m_{z,c} \cdot v_{z,c}$, and variance of severity $\text{Var}[L_{z,c,i}\ |L_{z,c,i} > 0] = \bar{v}^2_{z,c} \cdot s^2_{z,c} + (m^2_{z,c} + s^2_{z,c}) \cdot \text{Var}[v_{z,c,i}]$.

Event descriptions specify values for d_z, but the equation $E[d_{z,c}] = f_{z,c} \cdot m_{z,c}$ needs to be factored, and $s^2_{z,c}$ needs to be estimated. First assume that process risk in zip code z operates on all companies homogeneously: drop the subscript c, and reduce the problems to factoring $E[d_{z,c}] = d_z = f_z \cdot m_z$ and estimating s^2_z. This assumes away company-specific differences in the expected damage rate in a given zip code, the "underwriting-quality effects" mentioned above.

Friedman (1984) offers a solution to the factoring and estimating problems. To separate frequency from severity, refer to his table 2. His columns 2, 3, and 4 correspond to this study's f_z, m_z, and d_z, respectively.[14] He shows an almost perfect loglinear relation between f_z and d_z, namely, $f = 2.155 \cdot (d)^{0.6132}$. Adopt that, cap f_z at one, and derive $m_z = d_z/f_z$.

For s^2_z, refer to Friedman's figure 14. This graph shows selected percentage points of the distribution of severity as a function of wind speed. Beta distributions with coefficients of variation of 1.0 fit this reasonably well; therefore, $s^2_z = m^2_z$ can be assumed.

In summary, given a zip code z with company exposure specified by $R_{z,c}$ and $\bar{v}_{z,c}$ and an event specified by a damage rate d_z, model the losses, $L_{z,c}$, as a compound Poisson process with Poisson rate $l_{z,c}$ and severity parameters $n_{z,c}$ (mean) and $t^2_{z,c}$ (variance), where

$$a = 2.155,\quad b = 0.6132,$$
$$f_z = \min[1,\ a \cdot (d_z)^b],$$
$$m_z = d_z / f_z,$$

13. Studies at Guy Carpenter have found only a mild dependence of d on v.
14. Friedman uses counties, not zip codes, as the unit of geography.

$$\ell_{z,c} = R_{z,c} f_z,$$

$$n_{z,c} = m_z \bar{v}_{z,c},$$

$$t^2_{z,c} = m^2_z \cdot (\bar{v}^2_{z,c} + 2 \cdot \mathrm{Var}[v_{z,c,i}]).$$

In particular, this implies

$$E[L_{z,c}] = R_{z,c} \cdot d_z \cdot \bar{v}_{z,c},$$

$$\mathrm{Var}[L_{z,c}] = 2 \cdot R_{z,c} \cdot d^2_z / f_z \cdot \bar{v}^2_{z,c} \cdot \exp(\omega^2).$$

10.2 Simulating Hedge Performance

10.2.1 Introduction

This section uses the model of the underlying to analyze an insurer hedging its portfolio. Section 10.2.2 below describes three nested cycles of the simulation. The innermost cycle, starting with a specific event and company, simulates conditional basis risk. The intermediate cycle, starting with a specific company, iterates events through the innermost cycle to simulate unconditional basis risk. The outermost cycle samples market characteristics across a spectrum of companies. How events and companies are sampled is discussed in section 10.2.3 below.

There are several statistics of hedge performance available. The correlation coefficient $\rho(L, G)$ between the risk being hedged L and the hedge instrument G is most often quoted. The optimal hedge ratio, that is, the value of α minimizing the variance of outcomes $\mathrm{Var}[L - \alpha \cdot G]$, is also frequently used.[15] The minimum variance is a true *measure* of performance.

Standard deviation, the square root of variance, is more easily intuited than variance because it is expressed in the same units as the hedge. This study therefore represents basis risk in terms of the *volatility* of the hedge, defined here as $\varsigma(\alpha, G) = (\mathrm{Var}[L_c - \alpha \cdot G])^{1/2}/E[L]$. In particular, $\varsigma(0, G)$, the "unhedged volatility," is the coefficient of variation of L; $\varsigma(\alpha_{opt}, G)$, the "attained volatility," is the minimum standard deviation of the hedge, expressed relative to $E[L]$. Hedge-performance results are presented in sections 10.2.4–10.2.6 below.

10.2.2 Theory

Consider a group of reporting companies $g = 1, 2, \ldots, N$, and let $U_z = \Sigma_g v_{z,g}$ and $K_z = \Sigma_g L_{z,g}$ represent the group's total exposed values and losses incurred from the event in zip code z, respectively.

Consider two loss indices tailored to company c. First is the statewide version:

15. These are related by $\alpha_{opt} = \rho(L, G) \cdot (\mathrm{Var}[L]/\mathrm{Var}[G])^{1/2}$.

$$H_c = (\Sigma_z K_z) \cdot (\Sigma_z v_{z,c})/(\Sigma_z U_z).$$

This corresponds to a situation where only the statewide aggregate losses and values are reported. The company's hedge can only be a simple proportion of the reported losses.

Second is the ZIP-based index:

$$I_c = \Sigma_z I_{z,c} = \Sigma_z (K_z \cdot v_{z,c}/U_z).$$

This corresponds to a situation where individual zip-code losses and values are reported. The hedge reflects the company's specific pattern of zip-by-zip penetration.

Assume that the index reporting group consists of the entire industry, $U_z = v_z$. This permits the following simplifications: $d = (\Sigma_z d_z \cdot v_z)/(\Sigma_z v_z)$, $H_c = d \cdot \Sigma_z v_{z,c}$, and $I_c = \Sigma_z (d_z \cdot v_{z,c})$, where v_z is the aggregate industry value insured in zip code z.

Because of the way it was defined (see sec. 10.1.6 above), $E[L_{z,c}] = I_{z,c}$; therefore, a zip-based hedge, as modeled, is able to reflect the loss experience of the subject company, except for the effects of process risk. A hedge based on the statewide index, on the other hand, is also subject to market-penetration heterogeneity effects.

The objective is, then, to evaluate $\mathrm{Var}[L_c - \alpha \cdot H_c]$ and $\mathrm{Var}[L_c - \alpha \cdot I_c]$ for arbitrary hedge ratios α.[16] The accumulation of sufficient statistics proceeds via Monte Carlo simulation.

The inner loop of the simulation takes as input the specification of a company, ϑ_c, and the specification of an event $\eta = \{d_z\}$ as it affects a state $\{\bar{v}, \{< R_z, \bar{v}_z >\}\}$.[17] Fifty realizations of a pattern of market penetration $\{< R^{(s)}_{z,c}, \bar{v}^{(s)}_{z,c} >\}_{s=1,\ldots,50}$ corresponding to ϑ_c are drawn by simple random sampling according to the models of section 10.1.4 above. Hedge results $H^{(s)}_c$ and $I^{(s)}_c$ are computed according to the preceding discussion. Process risk in each zip code, $\mathrm{Var}[L^{(s)}_{z,c}|\eta]$, is computed according to section 10.1.6 above and totaled across zips for $\mathrm{Var}[L^{(s)}_c|\eta]$. Then the (event-conditional) expectations and variance-covariance matrix of L, H, and I, are tabulated. Since process risk is uncorrelated with anything else, the only moment affected by process risk is $\mathrm{Var}[L_c \mid \eta] = \mathrm{Var}_s[E[L^{(s)}_c|\eta]] + E_s[\mathrm{Var}[L^{(s)}_c|\eta]] = \mathrm{Var}_s[I^{(s)}_c|\eta] + E_s[\mathrm{Var}[L^{(s)}_c|\eta]]$. Optimal hedge ratios, correlation coefficients, and conditional basis-risk measures are then computed. Note that the optimal hedge ratio for the zip-based index, conditional on event, is one.[18]

The intermediate loop of the simulation takes as input the specification of a

16. While it may be possible to improve the zip-based hedge by allowing α to vary between zip codes, this refinement will not be considered here.

17. For events that cross state lines, the state components are simulated individually, and the appropriate moments are summed together.

18. This is because the covariance between I and L is equal to the variance of I: $\mathrm{Covar}[L_c, I_c |\eta] = \mathrm{Covar}_s[E[L^{(s)}_c|\eta], I^{(s)}_c |\eta] + E_s[\mathrm{Covar}[L^{(s)}_c, I^{(s)}_c|\eta]] = \mathrm{Covar}_s[I^{(s)}_c, I^{(s)}_c |\eta] + 0 = \mathrm{Var}_s[I^{(s)}_c |\eta]$.

company, ϑ_c, and produces as output the unconditional moments, hedge ratios, correlations, and basis-risk measures. This requires sampling events in a framework of possible hedge contracts.

Two common types of catastrophe reinsurance contracts are *annual aggregate* and *per occurrence* contracts. The former type responds to the total catastrophe losses experienced by an insured portfolio during a year. The latter, more common, type responds to the losses caused by a single event. Per occurrence contracts also usually have *reinstatement* provisions that allow the contract to be renewed (for a price) after a claim has been submitted.

In this study, index hedging emulates the per occurrence type of contract. An event is therefore defined as one of the following: (1) a hurricane with a minimum specified damage[19] to the industry or (2) an entire year without such a hurricane.

Conditional hedge statistics are combined across events, taking event probability into account, to obtain unconditional hedge statistics.[20] For example, the calculation of covariance between L and H proceeds as

$$\begin{aligned}\mathrm{Cov}[L,\ H] = {} & \Sigma_\eta P_\eta \cdot \mathrm{Cov}[L,\ H|\eta] + \Sigma_\eta P_\eta \\ & \cdot E[L|\eta] \cdot E[H|\eta] - (\Sigma_\eta P_\eta \cdot E[L|\eta]) \cdot (\Sigma_\eta P_\eta \cdot E[H|\eta]),\end{aligned}$$

where P_η represents the sampling probability of event η. Note that the optimal *unconditional* hedge ratio for the zip-based index is also one.[21]

Analysis across the spectrum of insurers occurs in the outer layer of simulation, where a random sample of market-characteristics vectors, ϑ_c, is drawn according to section 10.1.5 above.

10.2.3 Sampling Issues

Historical damages[22] from catastrophes since 1949 were obtained from PCS and adjusted for socioeconomic growth by Guy Carpenter actuarial staff (Mahon 1995) to restate them as contemporaneous events. Hurricanes were isolated from other types of events. The threshold hurricane selected was Hurricane Allen, which struck Texas on 4 August 1980 and inflicted \$58 million (1980 dollars) in insured damages. Since 1949, there were thirty-seven hurri-

19. The effect of a threshold level of damage is to approximate smaller events by zero.

20. It might be argued that the proper way to compute unconditional results is to compute variances conditional on particular realizations of market penetration $< R^{(s)}_{z,c}, \bar{v}^{(s)}_{z,c} >$ and then to take expectation with respect to all the realizations in a market-characteristics class ϑ_c. This approach makes it difficult to assess conditional hedge performance and raises issues of possible "overfitting" of hedge ratios to the limited number of events in the simulation. Numerical studies found that this alternative approach produced state-based hedge-volatility estimates approximately 10–12 percent lower than the estimates shown later in this paper.

21. Again, this is because the covariance between I and L is equal to the variance of I: $\mathrm{Cov}[L, I] = \Sigma_\eta P_\eta \cdot \mathrm{Cov}[L, I|\eta] + \Sigma_\eta P_\eta \cdot E[L|\eta] \cdot E[I|\eta] - (\Sigma_\eta P_\eta \cdot E[L|\eta]) \cdot (\Sigma_\eta P_\eta \cdot E[I|\eta]) = \Sigma_\eta P_\eta \cdot \mathrm{Var}_s[I^{(s)}|\eta] + \Sigma_\eta P_\eta \cdot E_s[I^{(s)}|\eta]^2 - (\Sigma_\eta P_\eta \cdot E_s[I^{(s)}|\eta])^2 = E_\eta[\mathrm{Var}[I|\eta]] + \mathrm{Var}_\eta[E[I|\eta]] = \mathrm{Var}[I]$.

22. Historical damages included all lines and coverages.

Table 10.5 Simulated Hurricanes

Hurricane	Date	States Used
Connie	11 August 1955	Virginia, North Carolina, Delaware, Maryland
Debra	24 July 1959	Texas
Donna	9 September 1960	Florida, Massachusetts, New York, Rhode Island
Carla	9 September 1961	Texas
Betsy	7 September 1965	Florida, Louisiana
Alicia	16 August 1983	Texas
Elena	30 August 1985	Florida, Alabama, Mississippi
Andrew	24 August 1992	Florida
Erin	1 August 1995	Florida

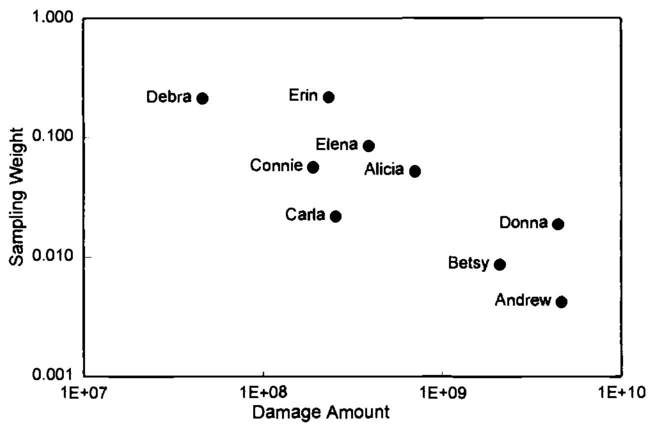

Fig. 10.6 Simulated hurricanes

canes causing at least as much (contemporary-equivalent) damage and eighteen years without such a hurricane.

Nine hurricanes were chosen as an importance sample of the thirty-seven candidates, where the importance weight was proportional to the adjusted damage estimate. They are listed in table 10.5. Their patterns,[23] that is, homeowner's-building-coverage damage rates $\{d_z\}$, were obtained from USWIND.[24] Figure 10.6 shows the hurricane simulated industry damage versus sampling weight P_η. The nonevent, a year with no hurricane, is not shown. It has a sampling weight of $18/55 = 0.327$.

For the outer simulation loop, six values of risk-count share (M or, equivalently, κ) were selected. For each value, Latin Hypercube stratified sampling (McKay, Conover, and Beckman 1979) was used to draw a sample of twenty

23. In cases where a hurricane caused damage to several states, not all states were used for detailed simulation. The worst-hit states were selected so as to account for at least 90 percent of all the simulated damages.

24. USWIND is a hurricane catastrophe model developed by EQECAT Inc.

Table 10.6 Simulated Market-Share Classes (%)

Risk-Count Share	Average TIV Share	Minimum TIV Share	Maximum TIV Share
.20	.43	.18	.85
.50	.95	.40	1.76
1.00	1.72	.78	3.21
2.00	3.20	1.24	7.52
5.00	6.88	2.56	13.80
10.00	12.51	5.85	23.30

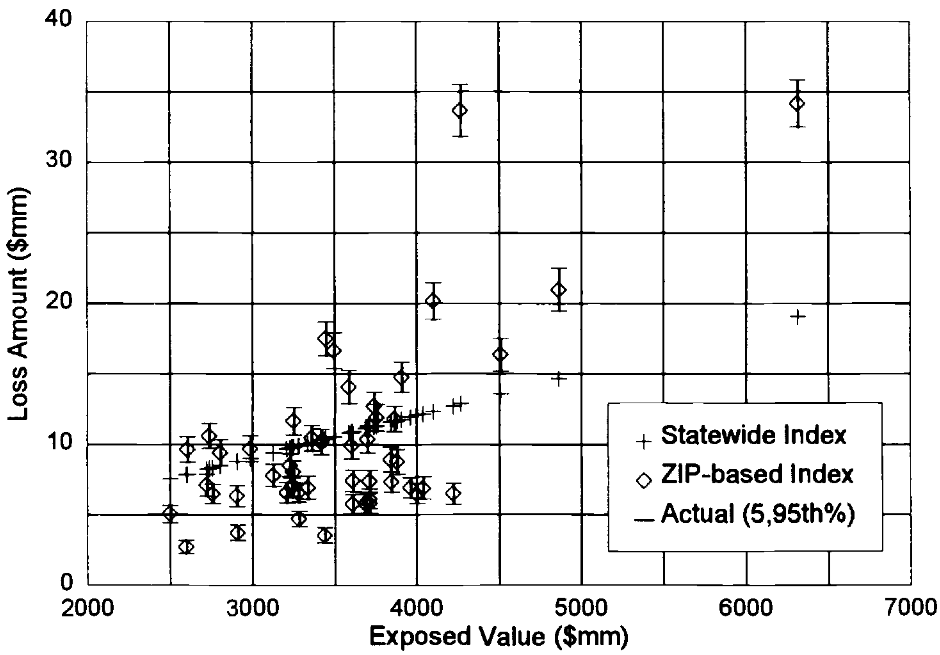

Fig. 10.7 Simulated loss and index outcomes, 1 percent company in Hurricane Alicia

values from the distribution of ϑlκ. The values are presented in table 10.6. Note that the share of total insured value (TIV) varies within risk-count share classes. This is visible in figure 10.12 below.

10.2.4 Example

Figure 10.7 shows the simulation of a 1 percent risk-count share company in Hurricane Alicia. The simulated outcomes occur in fifty sets of three vertically collinear symbols. The horizontal axis represents the total (statewide) exposed value. Vertically, the crosses represent the value of the statewide index; it is

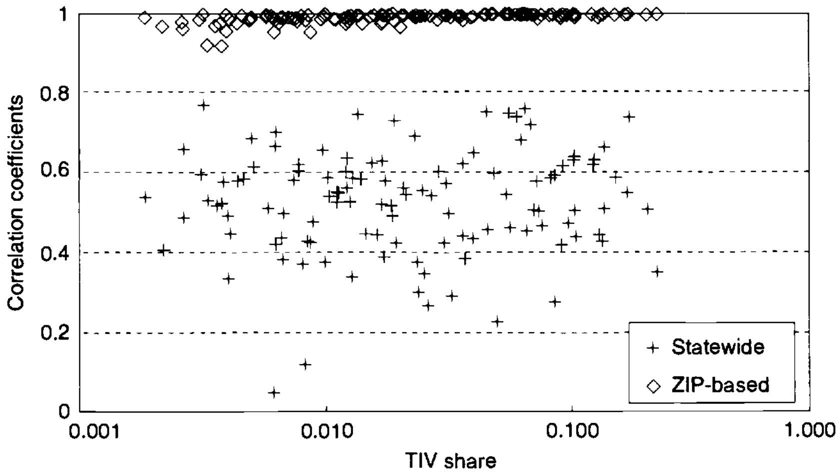

Fig. 10.8 Correlation coefficients between loss experience and indices, conditional on Hurricane Alicia

proportional to exposed value, at the 0.3 percent statewide damage rate. The diamonds represent the value of the zip-based index. The bars span the fifth to the ninety-fifth percentile values for the actual loss.[25]

The correlation between the statewide index and the losses is 0.661; between the zip-based index and the losses it is 0.996. The optimal hedge ratio for the statewide index is 2.17. These outcomes correspond to an unhedged volatility of 0.627, attained volatility of 0.471 for the statewide hedge, and attained volatility of 0.056 for the zip-based hedge.

10.2.5 Conditional Basis-Risk Results

Figure 10.8 shows the correlation coefficients, conditional on Hurricane Alicia, between loss experience and both the statewide index H and the zip-based index I for all simulated companies. Figure 10.9 shows the associated optimal hedge ratios for the statewide hedge.[26] Figures 10.10, 10.11, and 10.12 show basis risk as attained volatilities. Figure 10.12 repeats the information in figure 10.11 with the vertical axis rendered on a logarithmic scale. Table 10.7 summarizes for the six risk-count share classes. Conditionally, the statewide hedge reduces the volatility of results by a modest amount, whereas the zip-based hedge achieves a dramatic reduction.

Figure 10.13 shows attained volatility, relative to unhedged volatility, for all events. The vertical bars represent the mean, plus or minus one standard devia-

25. Recall that actual losses are not simulated in detail; rather, the expected value of losses and process risk variance are simulated. A normal distribution is assumed in locating the percentage points on this graph.

26. Recall that the optimal hedge ratio for the zip-based hedge, conditional on event, is always one.

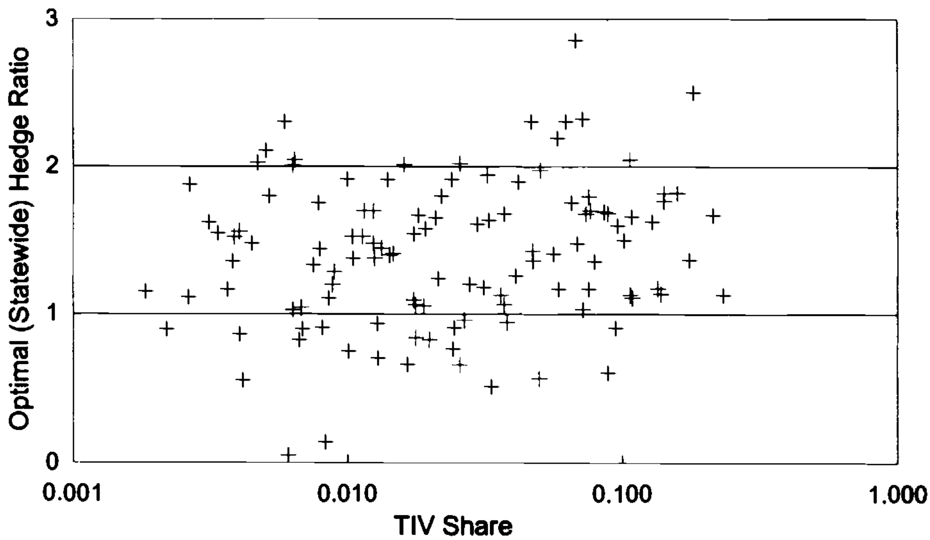

Fig. 10.9 Optimal conditional hedge ratios, statewide index, conditional on Hurricane Alicia

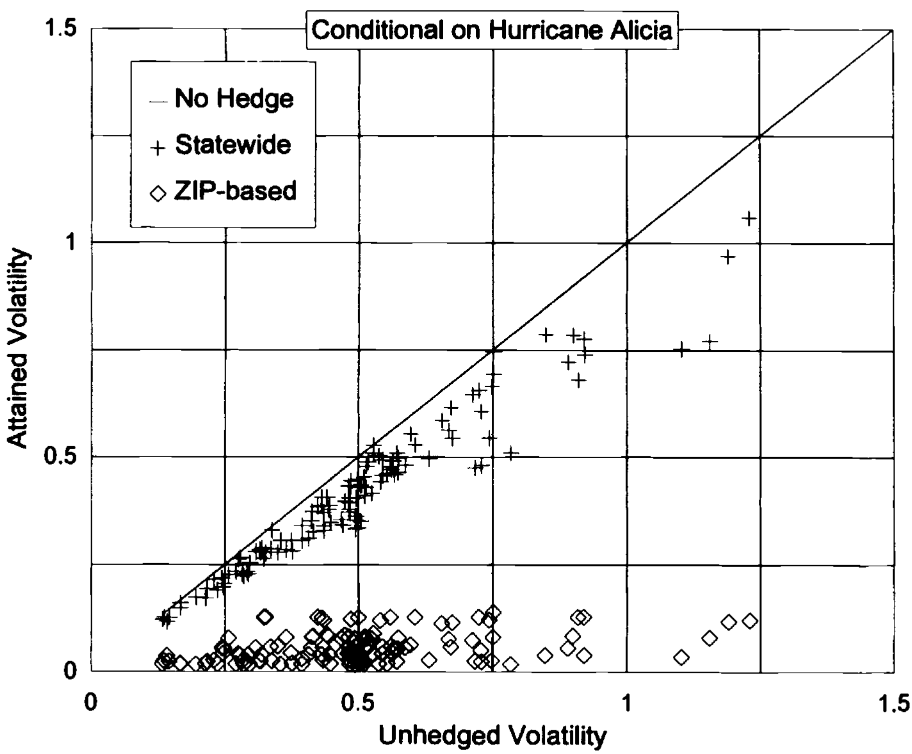

Fig. 10.10 Conditional volatility comparison

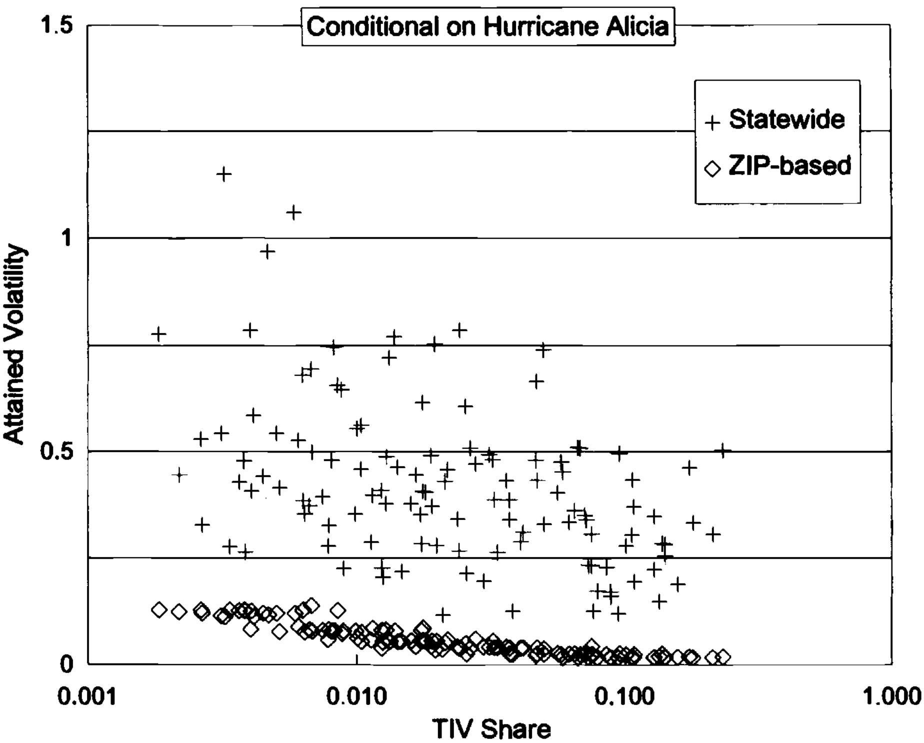

Fig. 10.11 Attained conditional volatility by TIV share

tion, of this ratio across all simulated companies. Hurricane Alicia is fairly typical of the simulated events. Only in Hurricane Donna does the statewide hedge achieve results consistently better than a 25 percent reduction in conditional volatility. The zip-based hedge typically achieves over a 70 percent reduction in volatility. Figure 10.14 shows the mean unhedged volatilities themselves. Again, Alicia is typical, and Donna exhibits an unusually low value.

10.2.6 Unconditional Basis-Risk Results

This section addresses unconditional hedge performance across all events, including the nonevent (a year without a severe hurricane).

Figure 10.15 shows the correlation coefficients between loss experience and both the statewide index H and the zip-based index I for all simulated companies. Figure 10.16 shows the associated optimal hedge ratios for the statewide hedge.[27] Figures 10.17 and 10.18 show basis risk as attained volatilities. The two figures differ in that, in figure 10.18, the vertical axis is logarithmic. Table

27. Recall that the optimal unconditional hedge ratio for the zip-based hedge is always one.

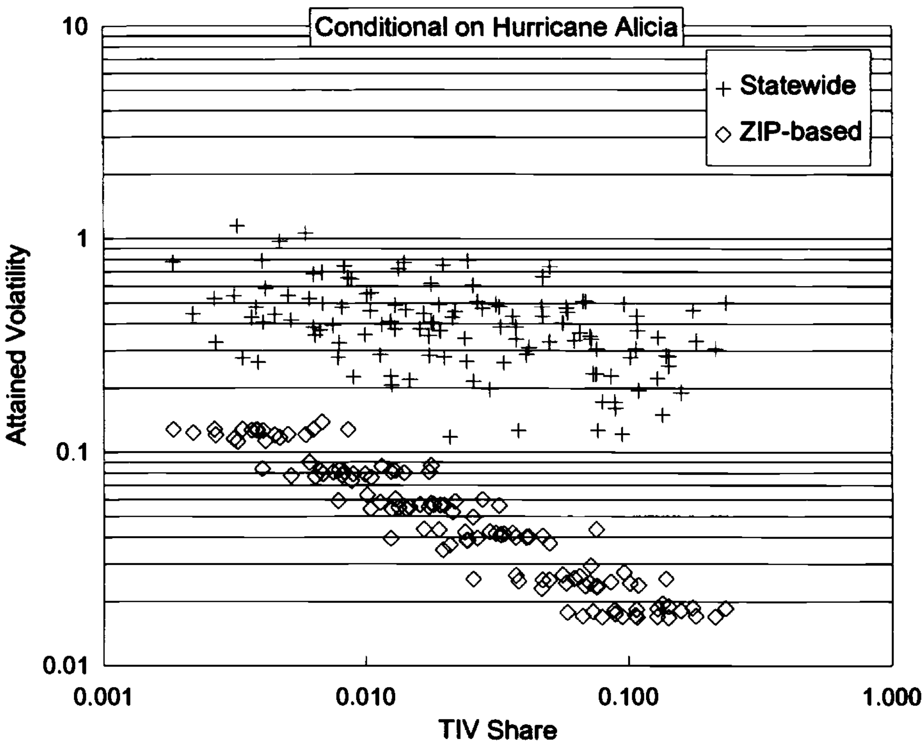

Fig. 10.12 Attained conditional volatility by TIV Share, logarithmic scale

Table 10.7 Average Performance Statistics, Conditional on Hurricane Alicia

Risk-Count Share (%)	Optimal Hedge Ratio Statewide	Correlation Coefficient		Attained Volatility		
		Statewide	Zip Based	No Hedge	Statewide	Zip Based
.2	1.57	.530	.973	.711	.582	.124
.5	1.25	.500	.986	.555	.466	.081
1.0	1.44	.522	.991	.503	.426	.056
2.0	1.40	.496	.993	.483	.409	.040
5.0	1.53	.543	.996	.424	.339	.025
10.0	1.58	.545	.998	.371	.299	.018

10.8 summarizes for the six risk-count share classes. Figure 10.19 shows attained volatilities expressed as a fraction of unhedged volatility. Table 10.9 summarizes.

On an unconditional basis, both hedges achieve meaningful reductions in volatility. However, whereas the statewide index typically reduces volatility

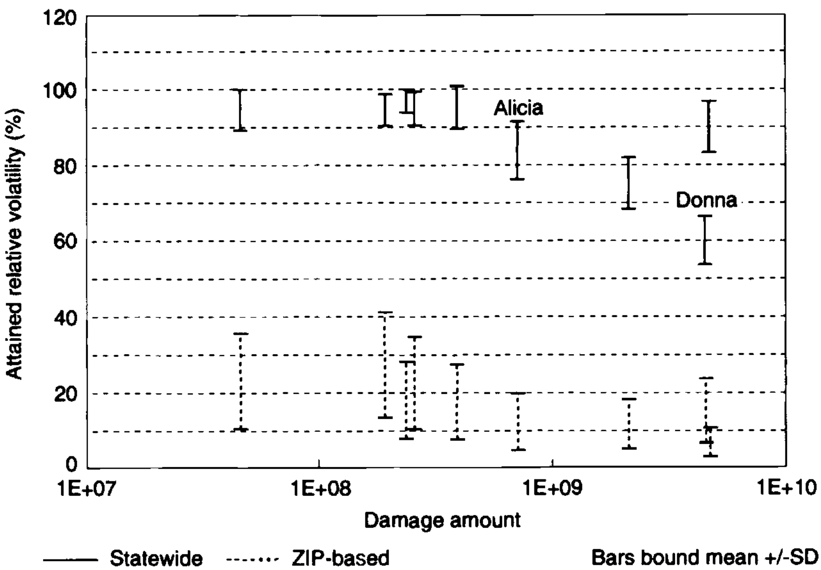

Fig. 10.13 Attained relative conditional volatility by event

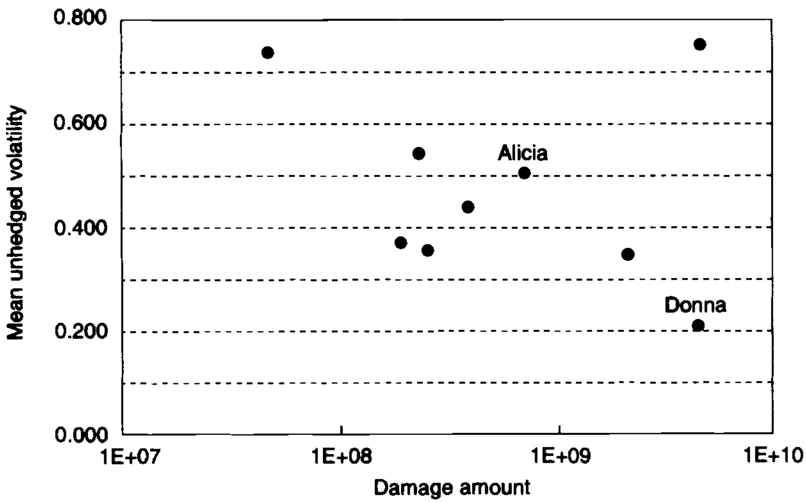

Fig. 10.14 Unhedged conditional volatility by event

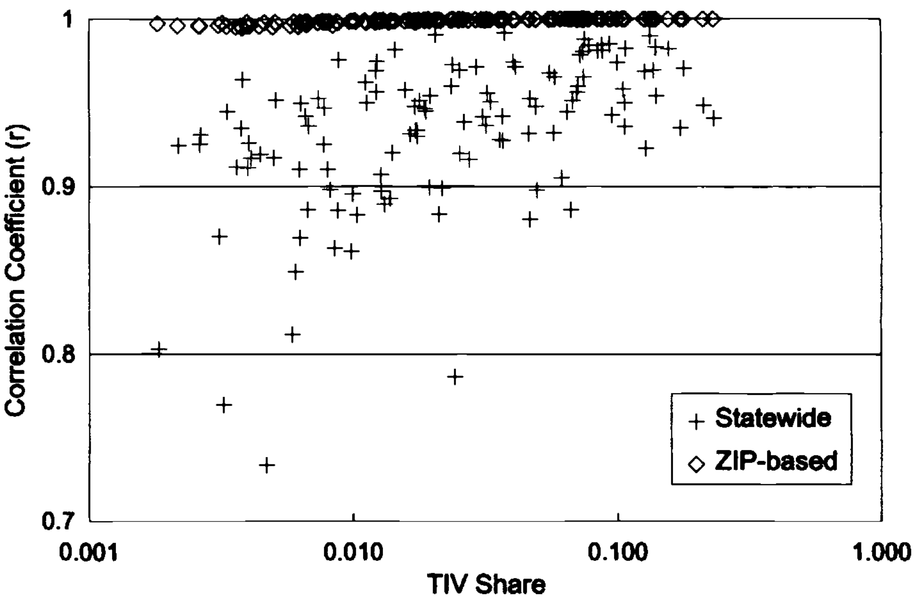

Fig. 10.15 Unconditional correlation coefficients between loss experience and indices

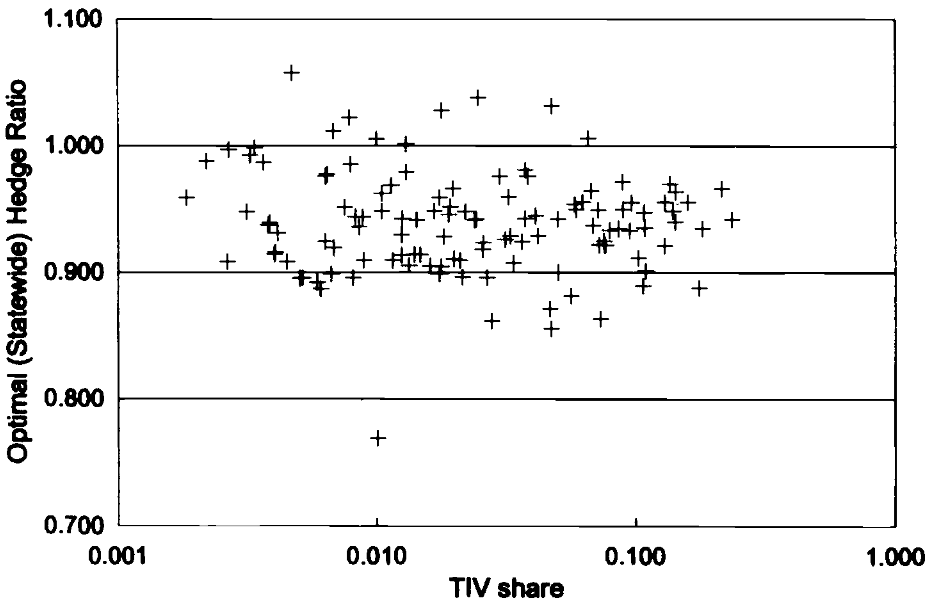

Fig. 10.16 Optimal unconditional hedge ratios, statewide index

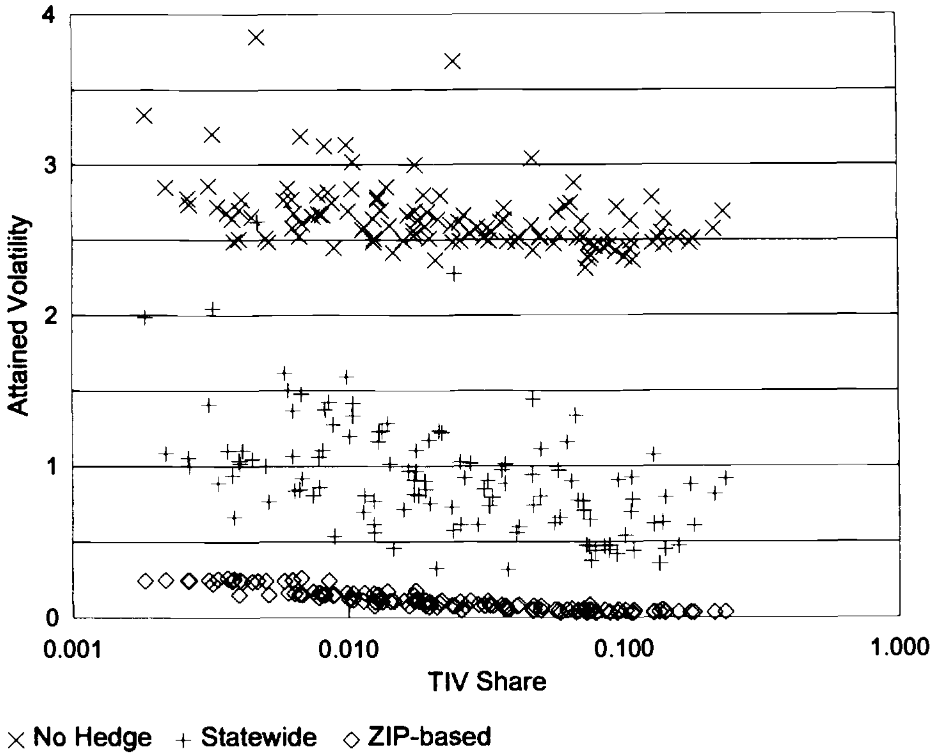

Fig. 10.17 Attained unconditional volatility by TIV share

50–75 percent, the zip-based hedge reduces it 90–99 percent. The reason for this is the conditional volatility, which is nearly unaffected by the statewide hedge but is reduced 70 percent or more by the zip-based hedge.

Differences in hedge performance among the simulated companies appear in both conditional and unconditional analyses. The attained volatility of the zip-based hedge shows clear "banding" by risk-count share, with no evidence of trending with TIV or even much variation within risk-count share class. The statewide hedge attained volatilities show a generally decreasing trend with TIV share and considerable variation, which can be interpreted as sensitivity to the details of the market-characteristics parameters. The unhedged volatility shows a less pronounced downward trend with increasing TIV share and perhaps a bit less sensitivity to market characteristics.

10.3 Related Work

The theory of hedging with financial futures and other derivatives is well established in the finance literature. More recent work in applying this theory

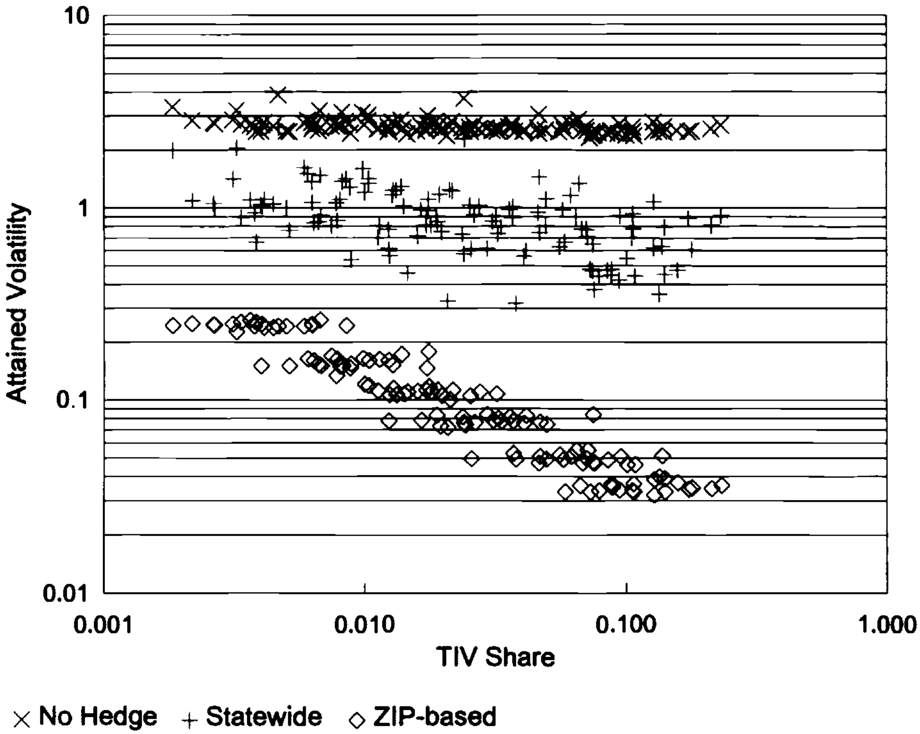

Fig. 10.18 Attained unconditional volatility by TIV share, logarithmic scale

Table 10.8 **Average Performance Statistics, across All Events, for Hedges Based on a Hurricane Futures Index**

Risk-Count Share (%)	Optimal Hedge Ratio Statewide	Correlation Coefficient		Attained Volatility		
		Statewide	Zip Based	No Hedge	Statewide	Zip Based
.2	.955	.886	.996	2.842	1.295	.245
.5	.938	.921	.998	2.722	1.045	.158
1.0	.927	.929	.999	2.626	.955	.112
2.0	.934	.942	1.000	2.612	.850	.078
5.0	.941	.954	1.000	2.568	.744	.050
10.0	.939	.960	1.000	2.540	.682	.035

to the case of property-casualty insurance exposures and catastrophe instruments includes Bühlmann (1996) and Meyers (1996). In both of these, the parameter of interest is the correlation coefficient between the experience of the subject portfolio and that of the catastrophe instrument being purchased.

For both the subject portfolio and the hedging instrument, Bühlmann as-

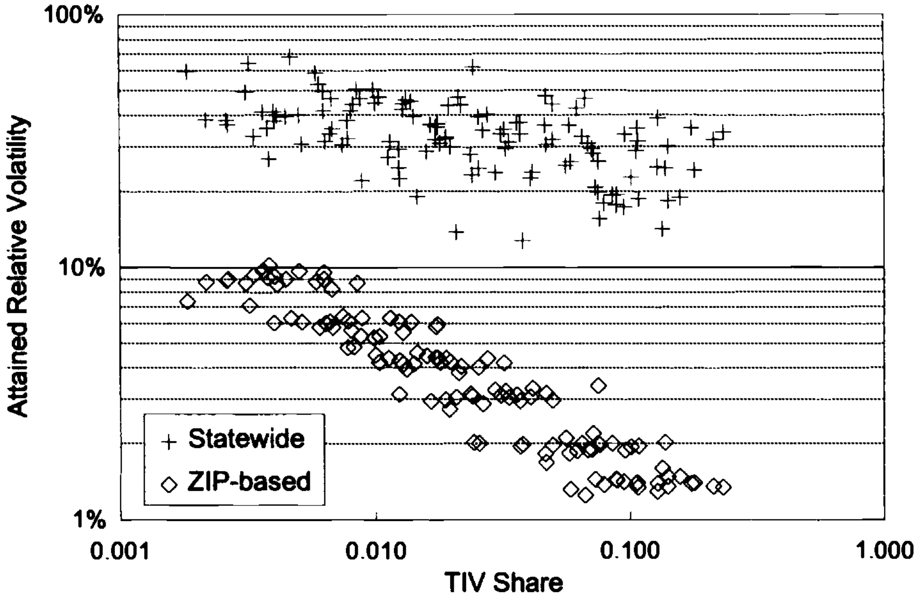

Fig. 10.19 Unconditional attained relative volatility

Table 10.9 Mean Reduction in Volatility for Hedges Based on a Hurricane Futures Index (%)

Risk-Count Share	Mean Attained Volatility/ Unhedged Volatility		Risk-Count Share	Mean Attained Volatility/ Unhedged Volatility	
	Statewide	Zip Based		Statewide	Zip Based
.2	44.6	8.7	2.0	31.7	3.0
.5	38.0	5.8	5.0	28.5	1.9
1.0	36.1	4.3	10.0	26.5	1.4

sumes a mixture of catastrophe and noncatastrophe risk. Meyers models a futures contract based on the (catastrophe-only) PCS index but, like Bühlmann, considers a mixed-subject portfolio. Generally, catastrophe experience is assumed independent of noncatastrophe experience, and all noncatastrophe experience is assumed mutually independent. Correlation coefficients between mixed portfolios can therefore be easily derived from those of the corresponding pure catastrophe components. In both papers, the authors use artificial examples to motivate the discussion.

There have been numerous empirical assessments of hedge performance. D'Arcy and France (1992) correlate underwriting profits with national PCS losses. Hoyt and Williams (1995) and Harrington, Mann, and Niehaus (1995) analyze loss ratios hedged against national or broad regional industry indices. Weber and Belonsky (1996) correlate company losses with national and regional PCS indices. Correlations between large company experience and regional indices in hurricane-prone areas were found to be typically in the 0.6–0.7 range. More recently, Harrington and Niehaus (1997) compare hedge performance of regional indices to tailored state-level indices. They find a state-based PCS hedge to attain correlations typically in the 0.75–0.8 range.

The use of empirical Bayes methods in the actuarial literature is not new. Lamm-Tennant, Starks, and Stokes (1992) use an empirical Bayes technique to assess the profitability of insurers. Major and Riedinger (1992, 1995) use empirical Bayes estimates in a computerized search for insurance-claim fraud. The implicit connections with Bayesian credibility theory (Herzog 1990; Klugman 1992) point to a large body of literature.

The use of simulation in the analysis of catastrophe risk is certainly not new. Friedman's pioneering work (Friedman 1969, 1972, 1975, 1979, 1984; Friedman and Mangano 1983) is the template from which subsequent analysts have started. Recently, Insurance Services Office (1996) made use of simulation to assess the volatility of catastrophe losses relative to surplus. In particular, they found coefficients of variation of 2.8 for earthquakes in the industry as a whole and 1.8–7.4 for all perils combined in selected insurer groups.

Analysis of the occurrence of hurricanes spans a considerable segment of the literature, including Friedman's work, Ho et al. (1987), Ho, Schwerdt, and Goodyear (1975), Schwerdt, Ho, and Watkins (1979), Georgiou, Davenport, and Vickery (1983), Twisdale, Vickery, and Hardy (1994), and Major and Mangano (1995).

10.4 Concluding Remarks

Within the context and limitations of the study, hedging with a statewide catastrophe index was shown to be afflicted by substantial basis risk caused by the variation in market penetration of insured portfolios. This analysis was limited in a number of dimensions, however. In particular, it explored only one type of peril (hurricane) and one type of insured risk (residential buildings). Drawing inferences to the general case of all lines requires projecting the behavior of the components of basis risk to other structures and coverages.

Drawing inferences to the general case of all perils requires dealing with other damage-generating mechanisms. Among 213 PCS-recorded events with contemporary severity to the industry at least as great as Hurricane Allen, only thirty-seven were hurricanes. Only three were earthquakes. While representing only 19 percent of the events, these two types accounted for roughly half the loss dollars. Nonhurricane wind events (tornadoes, thunderstorms, winter storms, tropical storms, etc.) accounted for most of the remaining events and

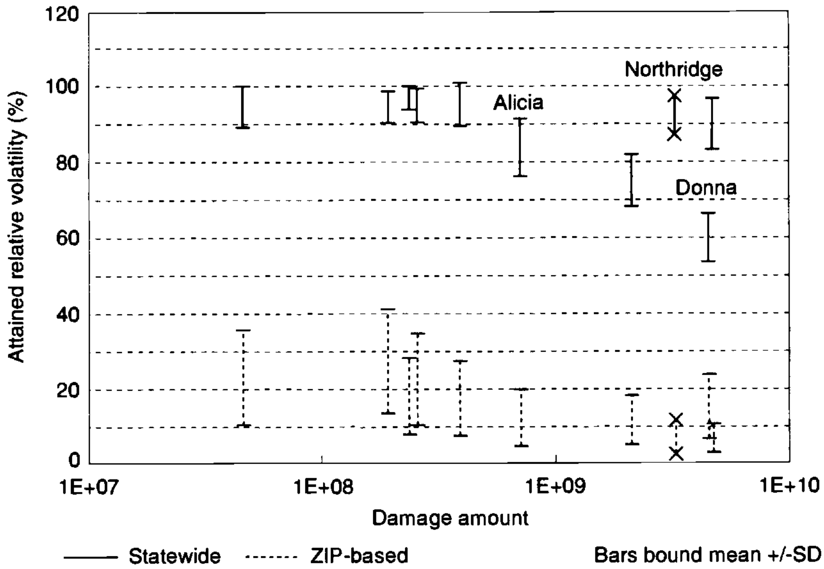

Fig. 10.20 Attained relative volatility by event, including Northridge

loss dollars.[28] The largest 14 of these 213 events, at least fifteen times as severe as Hurricane Allen,[29] accounted for half the losses. Of those, ten were hurricanes (65 percent of the losses), and one was an earthquake (12 percent of the losses).

The largest earthquake was that in Northridge on 17 January 1994, causing $12.5 billion in insured damages. Figures 10.20 and 10.21 reproduce the information in figures 10.13 and 10.14 above, with simulated Northridge results superimposed.[30]

Northridge does not appear strikingly different from the simulated hurricanes.[31] If Northridge is typical of large earthquakes in this regard, then the conclusions of the study would not change significantly by extending the scope of the perils because the dominant events are hurricanes and earthquakes. On the other hand, if earthquakes are subject to larger variation of market penetration and process risk, then hedge performance would be worse than reported here. However, the overall conclusion, that market-penetration variation causes substantial basis risk, seems irrefutable.

28. But most of the *total* damage to portfolios was located away from hurricane- and earthquake-prone areas.

29. This cutoff is roughly the median of the nine events simulated here.

30. A uniform earthquake-insurance coverage factor of approximately 20 percent was assumed, whereas 100 percent of homes were assumed covered for hurricane losses.

31. Of course, the simulation is based on the underlying model fitted to homeowner's penetration, not earthquake-insurance penetration. Nonetheless, the spatial scale of damages turns out to be typical of hurricanes.

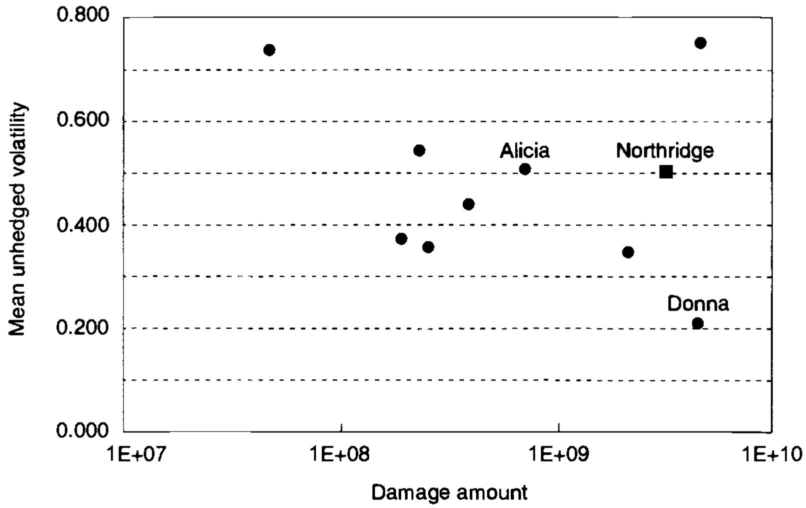

Fig. 10.21 Unhedged conditional volatility by event, including Northridge

Underwriting quality is also not dealt with in this study. No hedgers will have perfect knowledge of the effects of their portfolios' physical and financial characteristics or their own underwriting risk-selection standards and claim-settlement practices. Unhedged variation may be substantially understated in this study. The performance results shown here must be regarded as best case.

Appendix
An Empirical Bayes Approach to Imputing Penetration Rates

Methodology

Searle, Casella, and McCulloch (1992) define empirical Bayes estimation as "using a marginal distribution to estimate parameters in a hierarchical model, and substituting these estimates for their corresponding parameters in a formal Bayes estimator." This section describes such an approach to estimating $\pi_{z,c}$, the underlying (unobservable) risk-count penetration rate in zip code z for company c.

The estimate is based on the posterior modal value for $\psi_{z,c} = \ln(\pi_{z,c})$, given the observed risk count $R_{z,c}$ and total industry risk count R_z, that $R_{z,c}$ is Poisson distributed with expectation $\lambda_{z,c} = \exp(\psi_{z,c}) \cdot R_z$, and a normal prior distribution for $\psi_{z,c}$.

To obtain the prior, values of $\ln(R_{z,c}/R_z)$ are first examined across all zips where $R_{z,c} > 0$. This subset is treated as a left-censored sample from a normal distribution. The rate of censoring, along with the twenty-fifth and seventy-fifth percentiles of the sample, allows the estimation of the mean and standard deviation of the full, uncensored distribution.[32] The same procedure is applied to subsamples grouped by zip sectional center (zip3). The prior for $\psi_{z,c}$ is then taken to be a normal with standard deviation equal to that estimated from the statewide sample and mean equal to a weighted average of the statewide and zip3 estimates. The statewide weight is the observed rate of censoring in the zip3.

Given the prior mean μ and standard deviation σ from the previous calculation, the posterior modal $\psi_{z,c}$ is the solution to the equation

$$\partial[L(\psi,\ R_{z,c}|R_z)]/\partial\psi = 0,$$

where the joint likelihood of the data and the parameter is proportional to

$$L(\psi,\ R_{z,c}|R_z) = \exp\left\{-\frac{1}{2}\cdot[(\psi - \mu)/\sigma]^2\right\} \cdot \exp[-\exp(\psi)\cdot R_z]\cdot[\exp(\psi)\cdot R_z]^{R_{z,c}}.$$

Since $\lambda = \exp(\psi)\cdot R_z$, solving for the zero derivative reduces to solving

$$\ln(\lambda) + \sigma^2\cdot\lambda = \mu + \sigma^2\cdot R_{z,c} + \ln(R_z).$$

As $R_{z,c}$ gets large, the estimated value for $\lambda_{z,c}$ converges to $R_{z,c}$. When $R_{z,c}$ is zero, the estimated penetration rate $\lambda_{z,c}/R_z$ decreases as R_z increases.

A Market-Penetration Case Study

Consider a case from the data presented in section 10.1.3 above. The subject company has a risk-count share of 1–2 percent in the state of Maryland, and 12.9 percent of the zips have a zero risk count. The observed penetration rates, $R_{z,c}/R_z$ are shown in figure 10A.1. Each symbol in the charts represents one zip code. The points were randomly perturbed horizontally for better visibility.

The presence of spatial autocorrelation can be seen by means of analysis of variance where zip sectional center (zip3) is taken as the treatment effect in table 10A.1.

The F-statistic has $p = 2.2\cdot 10^{-14}$, which is highly significant. However, the significance test is predicated on normally distributed errors, which is clearly not the case.

The next step is to impute the underlying penetration rates, filtering out, as it were, the variation arising from Poisson sampling. Figures 10A.2 and 10A.3

32. Using the interquartile range improves robustness of the estimates. Also, the entire sample cannot be used because the censoring has an element of randomness to it.

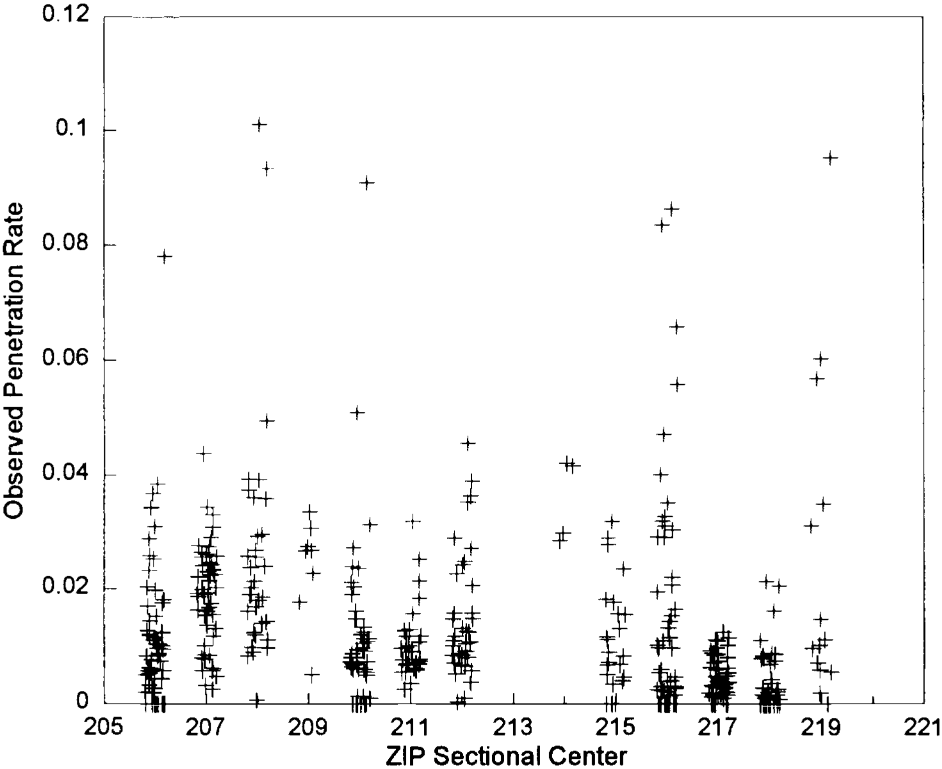

Fig. 10A.1 Example of observed penetration rates by zip sectional center

Table 10A.1 Analysis of Variance of Penetration Rate

Source of Variation	df	SS	MS	*F*
Zip3	12	.0192	.00160	8.42
Error	435	.0828	.00019	
Total	447	.1020		

Note: SS = sum of squares; MS = mean square.

show the results. Each symbol represents one zip code. Figure 10A.3 zooms in on those zips with an estimated penetration rate less than half a percent. No zip code is (ever) estimated to have a zero penetration rate.

Figure 10A.4 shows the resulting estimated penetration rates ($\pi_{z,c}$) by three-digit zip. The vertical axis is logarithmic. This figure makes spatial autocorrelation visually apparent. For example, penetration rates in zips 208xx (Bethesda)

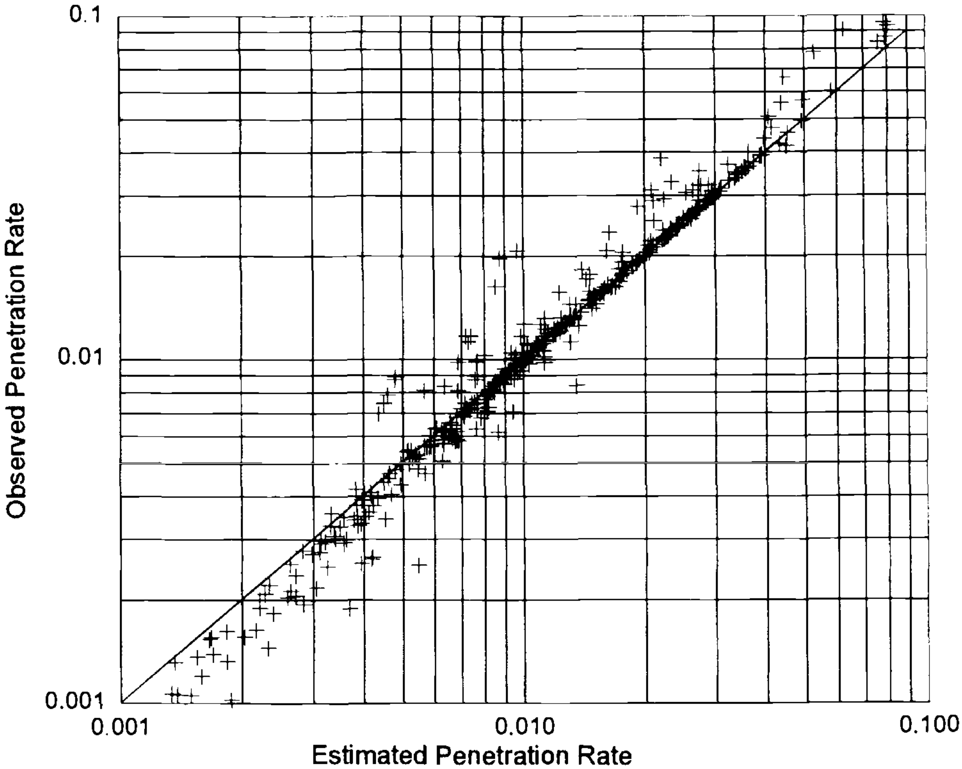

Fig. 10A.2 Estimated vs. observed penetration rates

tend to be above 1 percent, while those in zips 217xx (Frederick) are almost all below 1 percent.

Analysis of variance of the logs of $\pi_{z,c}$ results are given in table 10A.2.

This has $p = 7.7 \cdot 10^{-38}$. Figure 10A.4 suggests that normality assumptions are reasonable in the logarithmic domain. Therefore, zip3 has an effect; therefore, spatial autocorrelation is present. All twenty-four cases had zip3 effects significant at the 1 percent level or better.

For the risk-count penetration parameters of ϑ, the variance components are computed by

$$\text{zip (error) effect: } \sigma^2 = \text{SS(ERROR)/df(ERROR)} = 0.544,$$

$$\text{zip3 effect: } \tau^2 = [\text{SS(ZIP3)} - \text{df(ZIP3)} \cdot \sigma^2]/[N - (\Sigma N_i^2)/N] = 0.337,$$

which correspond to $\sigma_c = 0.74$ (.03) and $\tau_c = 0.58$ (.24), where standard errors (Searle 1956) are expressed as coefficients of variation.

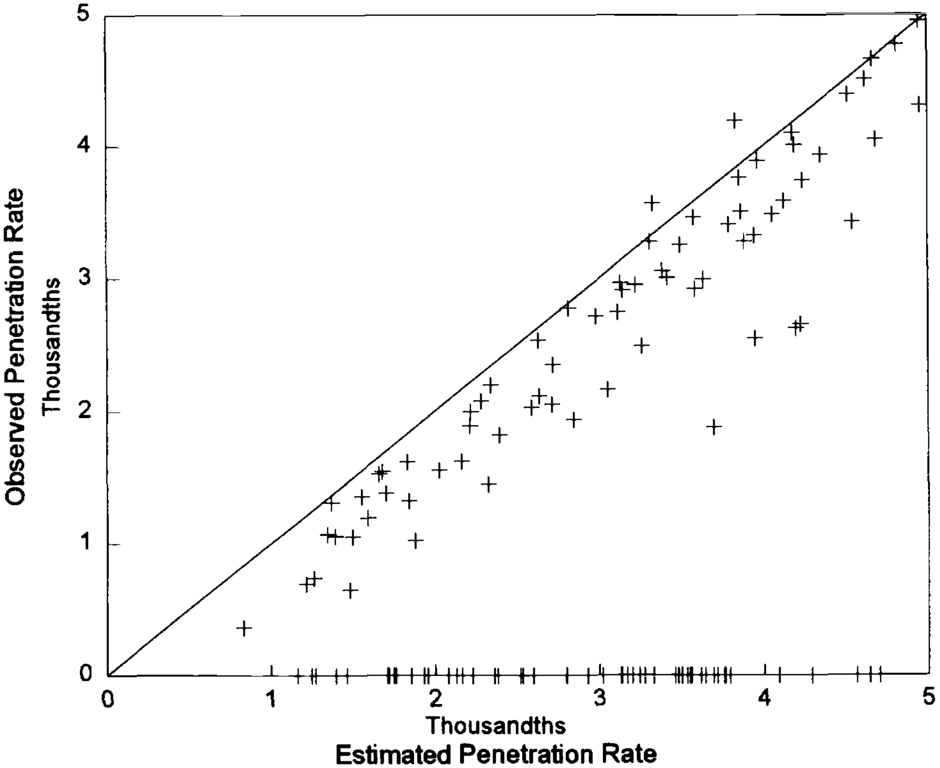

Fig. 10A.3 Estimated vs. observed penetration rates, close up

Table 10A.2 Analysis of Variance of Log Imputed Penetration Rate

Source of Variation	df	SS	MS	*F*
Zip3	12	142.93	11.9112	21.9
Error	435	236.56	.5443	
Total	447	379.49		

Note: SS = sum of squares; MS = mean square.

The Question of Bias

To assess the estimation procedure, twelve of the twenty-four cases were replicated eight times each by parametric bootstrap (Efron and Tibshirani 1993) and reestimated. The median bias among the cases was −2.75 percent for the zip effect and −8.0 percent for the zip3 effect. While the former was statistically significant, it was judged small enough to ignore. The latter was not statistically significant.

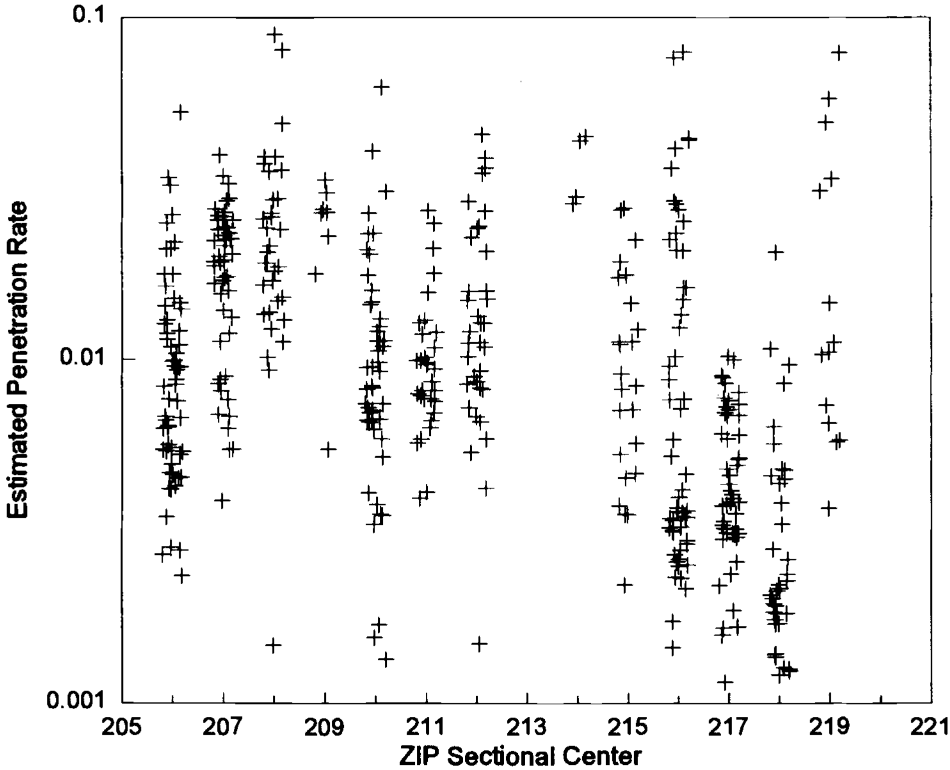

Fig. 10A.4 Example of estimated penetration rates by zip sectional center

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Comment André F. Perold

Basis risk has been a central issue in many of the papers and discussions at this conference. For example, we have heard claims that basis risk is a main source of insurer profitability but that it is also the risk that limits capacity; that broad-market hedging instruments are too coarse to effectively reduce basis risk and that finer hedges such as ones based on zip-code-level indices are needed; that large insurers can manage basis risk through broader distribution but that small insurers will require intermediation of basis risk; that reinsurers will earn spreads by buying and packaging idiosyncratic risks and hedge themselves by selling standardized, broad-market risks; and that the creation of reinsurance contracts, written on standardized zip-code-level indices, is necessary for the development of a deep and liquid market for contracts written on marketwide indices.

John Major's paper seeks to inform this discussion by empirically estimating basis risk. The paper estimates insurer-specific deviations from statewide indices and then examines the hedging effectiveness of contracts based on zip-code-level indices. Being the first study of its kind, the paper makes an important contribution to our understanding of basis risk.

In Major's model, firms are homogeneous with respect to underwriting quality, and their exposures within a zip code are near homogeneous in that they exhibit only small deviations from the index ("process risk"). Except for process risk, a firm's exposure within a zip code is thus proportional to its penetration of that zip code, and its statewide exposure is determined by its vector of zip-code-level penetrations. For example, if firm A writes insurance only in zip code 1, and if firm B writes insurance only in zip code 2, firm A's exposure will differ from firm B's to the extent that the losses in zip codes 1 and 2 are unrelated. A statewide hedge based on the sum of losses in zip codes 1 and 2 might therefore be quite ineffective. On the other hand, hedging instruments based on zip-code-level indices will allow firms A and B to hedge all but process risk.

The paper provides conditional as well as unconditional estimates of basis risk. *Conditional* refers to losses in a specific hurricane, while *unconditional* refers to losses simulated over multiple hurricanes. The difficult part is to model the variation in loss exposures for a given event. That is, different hurricane paths may result in the same total statewide damage, and the distribution of damage across these paths must be estimated. The paper does so by assuming that this distribution is the same as the distribution of market penetration across firms. In the example given above, this corresponds to assuming that a hurricane would hit zip code 1 or zip code 2 but not both.

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To clarify, let v_z be the industry coverage of zip code z, let d_z be the industrywide loss, and let p_z be the penetration of a given firm. Ignoring process risk, the firm's losses are given by $\Sigma_z v_z p_z d_z$, while the industry's losses are given by $\Sigma_z v_z d_z$. The firm's basis—if using a hedging instrument based on the statewide index—is given by

$$B = \Sigma_z v_z p_z d_z - h \Sigma_z v_z d_z (\Sigma_z v_z p_z / \Sigma_z v_z),$$

where h is the hedge ratio, and the term in parentheses is the ratio of the firm's statewide exposure to the industry's statewide exposure. The firm's unconditional basis risk is given by the variation in B as a function of the vector $\{d_z\}$. The firm's conditional basis risk is determined by the variation in B as a function of $\{d_z\}$, holding fixed industry statewide losses $\Sigma_z v_z d_z$.

The key to understanding the paper's results is to note that basis risk is a symmetrical function of the vector of losses $\{d_z\}$ and the vector of market penetrations $\{p_{cz}\}$; that is, interchanging $\{d_z\}$ and $\{p_z\}$ leaves B unaffected. Thus, the variation in basis across firms (i.e., across realizations of market penetration) will be the same as the variation in basis across hurricane paths if the market-penetration vector $\{p_z\}$ is sampled from the same distribution from which the loss vector $\{d_z\}$ is drawn.

For reasons presumably of data availability, the paper simulates variation in basis risk by sampling from the distribution of market penetrations across firms. While this is a clever idea, my intuition is that the approach *understates* true basis risk. I suspect that firms are relatively more homogeneous with respect to market penetration than are the hurricane paths that result in a given amount of statewide damage.

The basic results of the paper are that unhedged losses have an unconditional coefficient of variation in the range 2.5–2.8. With the use of statewide hedging instruments, unconditional coefficients of variation are in the range 0.7–1.3; and, with the use of zip-code-level hedging instruments, the range is only 0.04–0.2. That these last numbers are not zero reflects the fact that the only risk remaining—process risk—is modeled as being very small.

In Major's simulations, the correlation between individual firm exposures and the statewide index is very high, ranging from 89 to 96 percent, while the correlation between individual firm exposures and the zip-code-level indices is extremely high, in excess of 99.5 percent. The extremely high zip-code-level correlation simply reflects the low level of process risk, while the high correlation with the statewide index might be the result of relatively low variation among firm market penetrations, as discussed above.

A Simple Model of Insurance Origination and Hedging

The results of the paper raise the important question of how the correlation between the insurer's portfolio and the hedging instrument might affect the optimal hedge ratio. The paper presents minimum-variance hedge ratios, which are independent of the cost of hedging. Quite possibly, however, instruments based on zip-code-level indices might be considerably less liquid than instruments based on statewide-level indices. For example, adverse selection likely will become more significant for finer hedges—especially if these hedges are used by firms with significant market penetration—resulting in these instruments having greater bid-offer spreads.[1] Thus, even though zip-code-level instruments might be better hedges because of higher correlations, the demand for these instruments might be dampened by higher transaction costs. In what follows, I attempt to model the trade-off between higher hedging costs and higher correlations in a simple way.

Consider a one-period model in which the firm originates q units of coverage, with expected profits $P(q)$ before hedging costs and deadweight capital costs. The firm hedges a fraction h (or hq units) of the risks that it has insured with an instrument that has correlation R with these risks. The hedging instrument is denominated so that its per unit variance, σ^2, is the same as the per unit variance of the risk being hedged. Absent hedging costs, the minimum-variance hedge ratio is optimal and is equal to R.

The per unit cost of hedging is S, which includes normal transaction costs such as the bid-ask spread as well as any "abnormal premium" in the pricing of the instrument. The abnormal premium is the instrument's expected return in excess of the risk-free rate, before transaction costs. There should be no abnormal premium in an efficient capital market if catastrophe risk is uncorrelated with priced factors. However, as discussed in Froot and O'Connell (chap. 5 in this volume), the pricing of catastrophe hedging instruments may presently contain significant abnormal premiums. The cost S is assumed here to be invariant to the quantity hedged, h.

The firm's demand for hedging stems from the deadweight cost of risk capital that the firm bears because of agency and information costs (see Merton and Perold 1993; Merton 1993; and Froot and Stein 1998). Applying the model developed in Perold (1998), these deadweight costs are proportional to the total risk of the firm. The functional form is

$$\text{Deadweight cost of risk capital} = k\sigma_F,$$

where k is a constant, and σ_F is the standard deviation of the firm's end-of-period cash flows. σ_F is given by

$$\sigma_F^2 = \sigma^2 q^2\{1 + h^2 - 2Rh\}.$$

The value of the firm[2] is $P(q) - hqS - k\sigma_F$, which can be expressed as

$$\text{Firm value} = P(q) - q\{hS + k\sigma(1 + h^2 - 2Rh)^{1/2}\}.$$

1. For a discussion of the analogous problem in the stock market, see Gammill and Perold (1989).

2. Here, *firm value* refers to expected excess profits after deadweight costs of hedging and of risk capital. It represents the premium over *book value*, or invested capital, of the firm.

Maximizing over h yields the optimal hedge ratio

$$h^* = R - S\{(1 - R^2)/(k^2\sigma^2 - S^2)\}^{1/2}.$$

At the optimal hedge ratio, the value of the firm is

$$\text{Firm value} = P(q) - q\{SR + [(1 - R^2)(k^2\sigma^2 - S^2)]^{1/2}\}.$$

These results relate the optimal hedge ratio and firm value to the correlation and cost of the hedging instrument as follows. First, the firm hedges less as the cost of hedging rises; that is, h^* is decreasing in S. In addition, no hedging occurs if the cost of hedging is large relative to the firm's deadweight cost of risk capital, that is, if $S > k\sigma R$. Full hedging occurs when $R = 1$ even if $S > 0$, provided that $S < k\sigma R$. Firm value is increasing in R and decreasing in S.

This model can now be tied to the empirical findings of the paper. If the results are correct that the statewide index and zip-code-level index correlations differ by only 4–11 percent, then it may easily be that the benefits of the finer zip-code-based hedges are offset by higher costs of hedging. A numerical example illustrates the trade-off.

Let $P(q) = \$100$, $q = 100$, and $k\sigma = \$1.00$. With these values, if $R = 0$ so that the firm does no hedging, its deadweight cost of risk capital evaluates to $qk\sigma = \$100$, with the result that the value of the firm is zero. At the other extreme, if $S = 0$ and $R = 1$, all risk can be fully and costlessly hedged, and the firm therefore bears no deadweight cost of risk capital. The value of the firm then is \$100.

Tables 10C.1 and 10C.2 calculate the optimal hedge ratio and firm value, respectively, for various values of S and R. The tables show that the hedging costs need to be significant if these costs are to negate the benefits of higher correlations. For example, suppose that a *costless* broad-market hedging instrument is available and that its correlation with the firm's risks is $R = 0.7$.

Table 10C.1 **Optimal Hedge Ratio**

	Cost of Hedging (S)				
R	0	.1	.3	.5	.7
1.0	1.00	1.00	1.00	1.00	1.00
.9	.90	.86	.76	.65	.47
.8	.80	.74	.61	.45	.21
.7	.70	.63	.48	.29	.00
.6	.60	.52	.35	.14	.00
.5	.50	.41	.23	.00	.00
.4	.40	.31	.11	.00	.00
.3	.30	.20	.00	.00	.00
.2	.20	.10	.00	.00	.00
.1	.10	.00	.00	.00	.00

Table 10C.2 **Firm Value with Optimal Hedging**

	Cost of Hedging (S) ($)				
R	0	.1	.3	.5	.7
1.0	100.0	90.0	70.0	50.0	30.0
.9	56.4	47.6	31.4	17.3	5.9
.8	40.0	32.3	18.8	8.0	1.2
.7	28.6	21.9	10.9	3.2	.0
.6	20.0	14.4	5.7	.7	.0
.5	13.4	8.8	2.4	.0	.0
.4	8.3	4.8	.6	.0	.0
.3	4.6	2.1	.0	.0	.0
.2	2.0	.5	.0	.0	.0
.1	.5	.0	.0	.0	.0

The optimal hedge ratio for this instrument is 0.7, and the value of the firm is \$28.60. If a finer instrument is available with correlation $R = 0.9$ and the cost of hedging is $S = 0.3$, the optimal hedge ratio is 0.76. The firm hedges $qh = 76$ units and incurs a hedging-related cost of $qhS = \$22.80$. Moreover, hedging reduces the firm's deadweight cost of risk capital from \$100 to \$45.80, and the value of the firm is therefore \$100 − \$45.80 − \$22.80 = \$31.40. Thus, improving the correlation from 0.7 to 0.9 has economic significance even if the cost of hedging is large. At an even larger cost of hedging, say, $S = 0.5$, the hedge ratio is still high, at 0.65, but the value of the firm is much lower, at \$17.30.

These results are obviously most sensitive to the magnitude of the deadweight capital costs—which drive the demand for hedging in the first place. For example, if $k\sigma$ is small (e.g., $k\sigma = 0.1$), then the cost of hedging must only exceed 0.1 for no hedging to occur.

Conclusion

John Major's paper estimates the potentially significant increase in correlation, and consequent reduction in basis risk, that might be achieved with the use of hedging instruments based on zip-code-level indices. The paper does so by devising a simulation technique based principally on variation in market penetration across firms and the assumption that this variation is a good proxy for the distribution of damage across hurricane paths. The implications of the results for the demand for hedging with zip-code-level instruments are not obvious, however. There almost certainly will be greater costs associated with the use of these instruments. Depending on the reasons that firms hedge in the first place, these greater costs may offset the superior covariance properties of zip-code-level instruments.

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