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Chapter Title: Stochastic Problems in the Simulation of Labor Supply

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2 Stochastic Problems in the Simulation of Labor Supply

Jerry A. Hausman

Suppose that we have successfully estimated a structural model of labor supply. Given the large amount of public interest in the question of income tax reform, an important use of the estimated model would be to assess the possible effects of proposed reforms on the labor supply, tax revenues, and individual welfare. These evaluations are sometimes performed using local elasticity estimates. However, such a simplified analysis may not be very accurate for the rather large changes contemplated in many tax reform proposals. Another problem with simplified elasticity calculations is that they often ignore the considerable heterogeneity of the population response. A better approach would seem to be to use the estimated structural model to predict the effect of the tax changes. Thus we would need to derive analytically the statistical expectation of the population response under the proposed changes; or if analytical derivation proves to be mathematically intractable, a Monte Carlo approach would provide the results.

But an important potential problem arises when such simulations are conducted. This problem arises because of the nonlinear, and often nonconvex, budget sets which are a consequence of progressive income taxation as well as other tax and transfer policies. In a nonlinear econometric model with nonlinearities of this type, it is not necessarily the case that the sources of stochastic variation have an additive zero expectation term within a simulation exercise. Nor is it the case that such effects are necessarily small, since R^2 values in labor supply models are typically not that high; i.e. much unexplained residual variation remains after the

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model has been specified and estimated. Thus for a particular individual we might well expect that careful treatment of the stochastic specification in calculating the appropriate expectation would be quite important. Yet for the population at large, or, equivalently, a very large sample, the importance of the stochastic components is unclear. In the sample, if the variation of the exogenous variable is sufficiently large and the fit of the equation sufficiently good, the effect of the stochastic component may be small. Perhaps a more promising approach is to realize that extremely accurate computation for each individual may not be needed, because a law-of-large-numbers type of result may hold for the entire sample. That is, rather crude computational techniques may be used for each individual, but the sample mean values can still be quite accurate. Significant computational savings occur because say only one Monte Carlo draw is done for each individual. While the variance of the predicted response of that given individual may be large, in the complete sample the large variance may not be important, because of a large-number type of averaging. This sort of technical question is the major focus of this paper.

The plan of the paper is as follows. In section 2.1, we outline the problem of labor supply with nonlinear budget constraints. We also specify and use estimates of a linear supply model. This section and the estimates within it follow from Hausman (1981*b*). In section 2.2, the stochastic problems which arise in simulation of nonlinear budget set models are studied. Both analytical and Monte Carlo approaches are considered. Comparative statistics for computer times are given to indicate potential savings from the use of simple computational techniques. Then, in section 2.3, we consider tax reform proposals. The type of tax reform proposal considered is a reduction of tax rates by 10% to 30%. Here not only do we consider labor supply effects and welfare effects, but also we look at tax revenue considerations. It is important to emphasize at the outset that all analysis takes place within a partial equilibrium framework. Thus general equilibrium effects which might be quite important, especially in long-run response, are not treated.

2.1 The Econometrics of Labor Supply with Taxes

The essential feature which distinguishes econometric models of labor supply with taxes from traditional demand models is the nonconstancy of the net, after-tax wage. Except for the case of a proportional tax system, the net wage depends on hours worked because of the operation of the tax system. Also, the marginal net wage depends on the specific budget segment that the individual's indifference curve is tangent to. Thus econometric techniques need to be devised which can treat the nonlinearity of the budget set. An econometric model needs to take the exogenous nonlinear budget set and explain the individual choice of desired hours.

We first describe such a model for convex and nonconvex budget sets. As expected, the convex case is simpler to deal with. We then consider other issues of model specification such as variation in tastes and fixed costs to working.

Econometric estimation is quite straightforward in the case of a convex budget set. Convex budget sets arise from the operation of a progressive tax system. Let us first analyze the simplest case, that of a progressive tax on labor income so that the marginal tax rate is nondecreasing. In figure 2.1 three marginal tax rates are considered, t_1, t_2, t_3, which lead to three after-tax net wages, w_1, w_2, w_3, where $w_i = w(1 - t_i)$. y_1 denotes nonlabor income. H_1 and H_2 correspond to kink point hours which occur at the intersection of two tax brackets. But an important addition to the diagram are the "virtual" incomes y_2 and y_3, which follow from extension of a given budget segment to the vertical axis. They are denoted as virtual income because if the individual faced the linear budget set $B_2 = (w_2, y_2)$, he would still choose hours of work h^* as in figure 2.1. An important property of such convex budget sets in the presence of strictly quasi-concave preferences is that only one tangency (at most) will exist between the individual indifference curves and the budget set. Hausman (1979) uses this result to demonstrate that only a specification of the labor supply

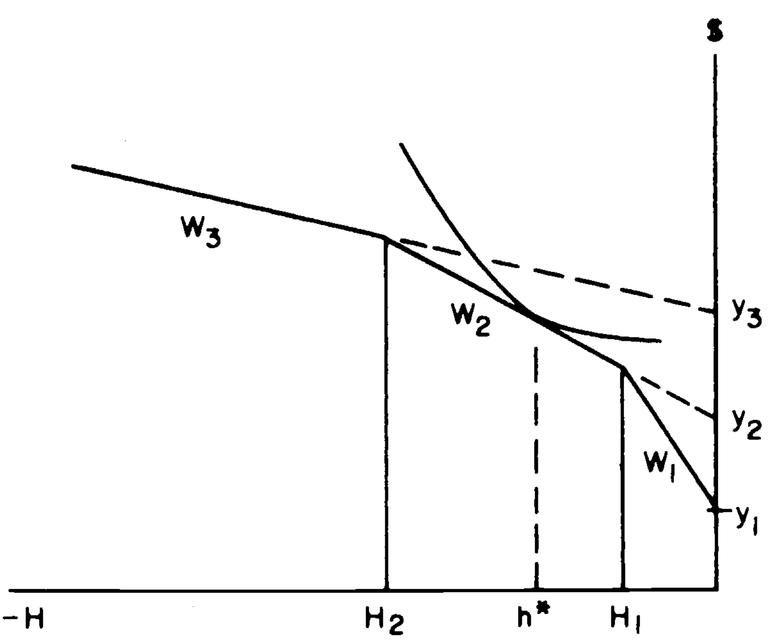

Fig. 2.1

function is necessary for estimation. The form of the underlying utility function is not necessary.

Since a unique tangency or a corner solution at zero hours will determine desired hours of work, we need only determine where the tangency occurs. To do so we begin with a slight generalization of the usual type of labor supply specification:

$$h = \tilde{g}(w,y,z,\beta) + \epsilon = h^* + \epsilon \ , \tag{1.1}$$

where w is a vector of net wages, y is a vector of virtual incomes, z are individual socioeconomic variables, β is the unknown vector of coefficients assumed fixed over the population, and ϵ is a stochastic term which represents the divergence between desired hours h^* and actual hours. The typical specification that has been used in $\tilde{g}(\ )$ is linear or log linear and scalar w and y corresponding to the market wage and nonlabor income. The stochastic term is assumed to have classical properties so that no quantity constraints on hours worked exists. However, $0 \leq h \leq \bar{H}$, where H is a physical maximum to hours worked. We also assume that when the β are estimated the Slutsky conditions are satisfied so that $\tilde{g}(\ )$ arises from concave preferences.

The problem to be solved is to find h^* when the individual is faced with the convex budget set B for $i = 1, \ldots, m$. To find h^* we take the specification of desired hours on a given budget segment B_i:

$$h_i^* = g(w_i,y_i,z,\beta) \ . \tag{1.2}$$

Calculate h_1^*; if $0 \leq h_1^* \leq H_1$, where the H_i are kink point hours in figure 2.1, then h_1^* is feasible and represents the unique tangency of the indifference curves and the budget set. If $h_1^* \leq 0$, then zero hours is the desired amount of work. However, if h^* exceeds H_1, it is not feasible, so we move on to try the next budget segment. If $H_1 \leq h_2^* \leq H_2$, we again would have the unique optimum. If we have bracketed the kink point so that $h_1^* > H_1$ and $h_2^* < H_1$, then $h^* = H_1$ so that desired hours fall at the kink point. Otherwise we go on and calculate h_3^*. By trying out all the segments we will either find a tangency of find that $h_i^* > H_i^*$ for all i, in which case $h^* = \bar{H}$. Then a nonlinear least squares procedure or Tobit procedure to take account of a minimum at zero should be used to compute the unknown β parameters. The statistical procedure would basically minimize the sum of $\Sigma_{j=1}^{N} (h_j - h_j^*)^2$, where j represents individuals in the sample. Perhaps a better technique would be to use Tobit, which enforces the constraint that $h_j \geq 0$.

The case of the nonconvex budget is more complicated because equation (1.2) can lead to more than one feasible tangency, which leads to many potential values of h_i^*. Nonconvex budget sets arise from the presence of government transfer programs. The four most important programs of this type are low-income tax credit, Aid for Dependent Children

(AFDC), social security benefits, and a negative income tax (NIT) program. In figure 2.2, in which we indicate a common type of nonconvex budget set, we have two tangencies of the indifference curves with the budget set.

How can we decide which of these feasible h_i^* is the global optimum? Burtless and Hausman (1978) initially demonstrated the technique of working backward from the labor supply specification of equation (1.2) to the underlying preferences, which can be represented by a utility function. The basic idea was to make use of Roy's identity, which generated the labor supply function from the indirect utility function $v(w_i,y_i)$:

$$\text{(1.3)} \qquad \frac{\partial v(w_i,y_i)}{\partial w_i} \Bigg/ \frac{\partial v(w_i,y_i)}{\partial y_i} = h_i^* = g(w_i,y_i,z,\beta)$$

along a given budget segment. So long as the Slutsky condition holds, $v(w_i,y_i)$ can be recovered by solving the differential equation (1.3). In fact, $v(\quad)$ often has a quite simple closed form for commonly used labor supply specifications.

For the linear supply specification $h_i^* = \alpha w_i + \beta y_i + z\gamma$ which is used in this paper, Hausman (1980) solved for the indirect utility function:

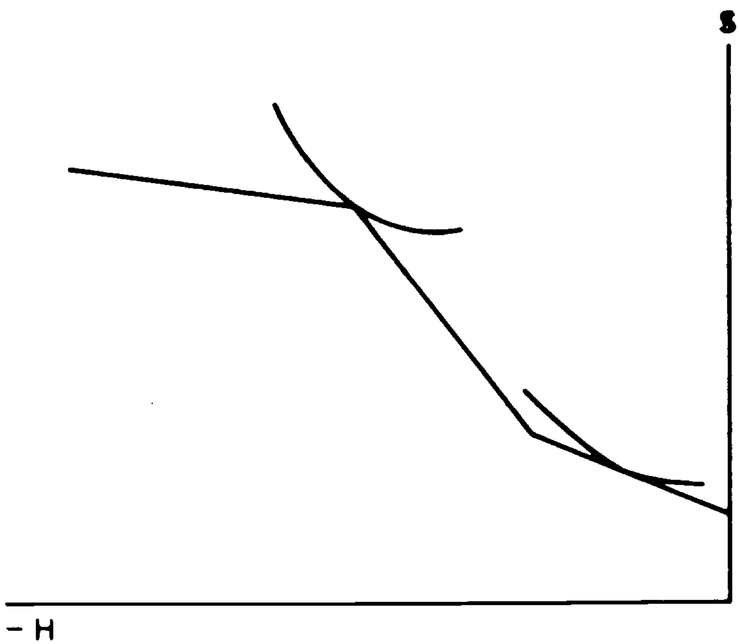

Fig. 2.2

$$v(w_i,y_i) = e^{\beta w_i}\left(y + \frac{\alpha}{\beta}\, w_i - \frac{\alpha}{\beta^2} + \frac{z\gamma}{\beta}\right). \tag{1.4}$$

Given the indirect utility function, all of the feasible tangencies can be compared, and the tangency with highest utility is chosen as the preferred hours of work h^*. Then, as with the convex budget set case, we can use either nonlinear least squares or a Tobit procedure to estimate the unknown coefficients. While using a specific parameterization of the utility function is upsetting to some people, it should be realized that setting down a labor supply function as in equation (1.2) is equivalent to setting down a utility function under the assumption of utility maximization. To the extent that the labor supply specification yields a robust approximation to the data, the associated utility function will also provide a good approximation to the underlying preferences. The utility function allows us to make the global comparisons to determine the preferred hours of labor supply. The convex case needs only local comparisons, but the nonconvex case requires global comparisons because of the possibility of multiple tangencies of indifference curves with the budget set.

We next introduce the possibility of variation in tastes. In the labor supply specification of equation (1.1), all individuals are assumed to have identical values of β so that variation of observationally equivalent individuals must arise solely from ϵ. However, empirical studies seem to do an inadequate job of explaining observed hours of work under the assumption of the representative individual. Burtless and Hausman (1978) allowed for variation in preferences by permitting β to be randomly distributed in the population. Their results indicated that variation in β seemed more important than variation in α. They also found that variation in β represented approximately eight times as much of the unexplained variance as did variation in ϵ. An even more satisfactory procedure would be to allow all the taste coefficients to vary in the population. At present the requirement of evaluating multiple integrals over nonrectangular regions for the more general specification has led to the use of the simple case of variation of one or two taste coefficients. Further research is needed to determine whether this more complex specification would be an important improvement over current models.

Another consideration which can have an important effect on the budget set for women's labor force participation is fixed costs of working. Transportation costs, the presence of young children, and search costs of finding a job can lead to a fixed cost element in the labor supply decision. The basic effect of fixed costs is to introduce a nonconvexity in the budget set at the origin. Thus, even if the original budget is convex as in figure 2.1, the presence of fixed costs leads to a minimum number of hours H_0, which depends on the wage below which an individual will not choose to work. In figure 2.3 nonlabor income is y_1, with the original convex budget set denoted by the dotted line. However, the presence of fixed costs

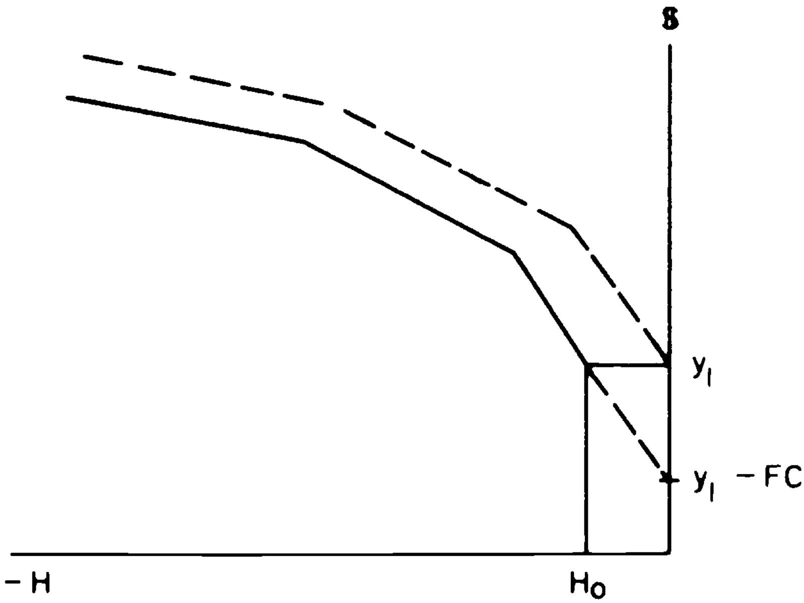

Fig. 2.3

lowers the effective budget set to the point y_1 − FC. The individual would not choose to work fewer than H_0 hours because she would be better off at zero hours. This nonconvexity invalidates the simple reservation wage theory of labor force participation since hours also need to be accounted for. Hausman (1980) in a labor force participation study of welfare mothers found average fixed costs to be on the order of $100 per month. The importance of fixed costs could explain the often noted empirical fact that very few individuals are observed working fewer than ten or fifteen hours per week.

We estimated a model of labor supply (Hausman 1981*b*) which takes full account of the effect of taxation for two groups in the population. The labor supply of husbands and wives is considered for 1975 for a sample from the Michigan Income Dynamics Data. Budget sets were constructed using both federal and state tax regulations (cf. Hausman 1981*b*). It is important to note that we did not have access to actual tax return data. Instead, we imputed deductions beyond the standard deduction using population averages. At present no data source has both all the necessary labor supply data and actual income tax return information.[1] At the

1. Sample selection criteria and budget set assumptions are discussed in Hausman (1981*b*). We note that farmers, the self-employed, and severely disabled individuals are excluded from the sample. Potential problems of tax evasion and tax avoidance should be decreased by our sample selection procedures. Also, for families with incomes which place

current stage of model development only a single person can be considered so that the husband was treated as the primary worker in a family, with a wife as the secondary worker. A model which allows for joint family labor supply decisions seems the obvious next goal of our research. For both husbands and wives we consider each of two cases: a convex budget set where the effects of FICA, the earned income credit, and the standard deduction are averaged to produce a convex budget set and a complete nonconvex budget set where the effect of each program is to introduce a nonconvexity.

Along each segment the basic labor supply model used is linear in wages and virtual income:

$$h_i^* = \alpha w_i + \beta y_i + z\gamma \ , \tag{1.5}$$

where h_i^* is desired hours, w_i is the net wage on segment i, y_i is virtual income for segment i, and z are socioeconomic variables. For fixed α, β, and γ, desired hours h_i^* may not be feasible since h_i^* may be greater or less than the hours at the end points of the budget segment H_{i-1} and H_i. If desired hours are feasible, then we have a tangency of the indifference curve and the budget segment. In the case of a convex budget set this tangency is unique, and we then use our stochastic specification for the deviation of actual hours from desired hours for person j as

$$h_j = h_{ij}^* + \eta_j \ . \tag{1.6}$$

Since observed hours $h_j \geq 0$, the stochastic term η_j is assumed to be independent and truncated normal across individuals in the population. Thus we have a Tobit specification for the hours worked variable. However, if $h_j^* = 0$, we assume that the individuals do not choose to work and so set $h_j = 0$ also. Since the final model has two sources of stochastic variation, the interpretation of η_j differs from standard models. Here we picture the individual faced by a choice among a set of jobs that differ in normal (long-run) hours worked. He chooses that job closest to his h_j^*. But observed h_j may differ because of unexpected layoffs, short time, overtime, or poor health. As an empirical matter we find the standard deviation of η_j to be reasonably small, which indicates that individuals are successful in matching jobs to their desired hours of work.[2]

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them above the range of the standard deduction, we used data from the *Statistics of Income* which should capture a large proportion of tax avoidance procedures. But data problems will nevertheless remain. It certainly seems preferable, however, to account for taxes rather than to ignore them as is the typical tradition in the labor supply literature, e.g. Smith (1980), in which only one of seven papers recognizes the existence of income taxation.

2. I disagree with my discussant's (Heckman's) remarks about his evidence on the piling up of labor supply at kink points for two reasons. First, the presence of η_i reduces to zero the probability that anyone is *observed* at a kink point. We should still observe a dispersion of individuals over the budget set. Second, since the kink points differ for each individual, I do not see how a casual look at the data can give us more evidence. Last, he is incorrect in his claim that the econometric procedures depend critically on exact knowledge of the location of the kink points.

If the budget set is nonconvex, h_j^* is not necessarily unique, because multiple tangencies can occur between the indifference curves and the budget set. Then h_{ij}^* is chosen as the tangency leading to maximum utility, which is determined by use of the corresponding indirect utility function from equation (1.4). We again use the stochastic specification of equation (1.6) to express the deviation of actual hours from desired hours of work. It is interesting to note that, although certain interior kink points in figure 2.2 in the nonconvex case cannot correspond to desired hours, we might still observe them as actual hours of work due to the stochastic term η_j in the model.

The second source of stochastic variation in the model arises from a distribution of tastes in the population. In line with our previous research we specify β to be a truncated normal random variable which falls in the interval $(-\infty, 0)$. An upper limit of zero is specified since we assume that leisure is a normal good. Thus, as β ranges over the permissible interval, there is a certain probability that any amount of hours corresponds to desired hours. As an empirical matter β turns out to be the major source of stochastic variation in the model, which confirms our previous findings reported in Burtless and Hausman (1978).[3]

The estimated results for husbands are presented along with asymptotic standard errors in table 2.1. The coefficients are generally estimated quite precisely, especially the wage and nonlabor income coefficients. The socioeconomic variables have coefficients of reasonable magnitude except the house equity, which perhaps reflects factors in the mortgage credit market and the special tax treatment of houses. We first note that the uncompensated wage coefficient is essentially zero. Not only is the estimate close to zero, but the estimated standard error is quite small. In the extreme case of two standard deviations from the estimate for the nonconvex case, a change in the net wage of \$1.00 along a budget segment leads to an expected increase in annual hours worked of 32.5, which is less than 2% of the sample mean. The expected change in hours

3. This specification of different tastes for leisure is perhaps the most controversial part of the model since it represents the most marked departure from usual labor supply models where coefficients are assumed identical across individuals. There, all population heterogeneity arises through the additive disturbance term η_j, e.g. the labor supply models contained in Smith (1980). A further discussion is contained in Hausman (1981*b*). To test for robustness of the specification, I tried different functional forms for the probability distribution in Hausman (1980). Also, Burtless and Hausman (1978) and Hausman (1981*b*) used instrumental variable techniques which do not depend on normality assumptions. Nor do they depend on the normal good assumption for leisure. The results were quite similar to the full maximum likelihood model estimates. I disagree with my discussant's remarks on the robustness of the procedure. My investigations lead me to believe that the procedures I use are considerably *more robust* than the reservation wage model of labor supply with its unsupported proportionality assumption. For instance, in his latest estimates, which ignore the existence of taxes (Heckman 1980, p. 229), the estimate of the uncompensated labor supply elasticity for wives changes from 2.1 to 4.8, with only a minor change in econometric specification. Both estimates are quite high, with the latter estimate absurdly so. My estimates are considerably more robust to econometric specification, as the labor supply elasticities for the three different budget sets of table 2.2 indicates.

Table 2.1 **Husbands' Annual Hours of Work ($\times$ 1,000)**

Variable	Convex	Nonconvex
μ_β – nonlabor income (1,000s)	2.037	1.061
	(.0729)	(.245)
σ_β	.6242	.4541
	(.0234)	(.0570)
Wage	.0002	.0113
	(.0090)	(.0106)
Constant	2.4195	2.366
	(.0589)	(.153)
Children < 6	–.0039	.0113
	(.0255)	(.0635)
Family size	.0341	.0657
	(.0170)	(.0310)
(Age – 45,0)	–.0011	–.0055
	(.0108)	(.0235)
House equity	.0026	.0036
	(.0009)	(.0008)
Bad health	–.1387	–.0520
	(.1436)	(.564)
σ_η	.2794	.2862
	(.0178)	(.0540)
Mean β	–.166	–.153
Standard deviation of β	.156	.141
Median of β	–.120	–.113

Note: Asymptotic standard errors are presented in parentheses below each estimated coefficient.

is only 11.3, while in the convex case the expected change in annual hours is 0.2. The finding of an extremely small uncompensated wage effect is in accord with the previous empirical findings. Thus the direct effect of income taxation that reduces the net wage has almost no effect on hours worked among husbands.

However, our results do differ from previous studies in indicating a significant income effect. Remember that we allow a distribution of preferences in the population. The estimated probability density for the nonconvex case is shown in figure 2.4. The distribution has substantial skewness since it is the extreme left tail of the truncated normal distribution with the standard deviation approximately equal to the mean in magnitude. My previous work has also found this general form even when different probability densities are used, e.g. Hausman (1980), where a Weibull density is used. The underlying parameters of the preference distribution are estimated quite precisely so that the finding is not likely to be an accidental occurence.

Next we present the empirical results for a sample of married women. Our sample consists of the wives of the males used in the previous section.

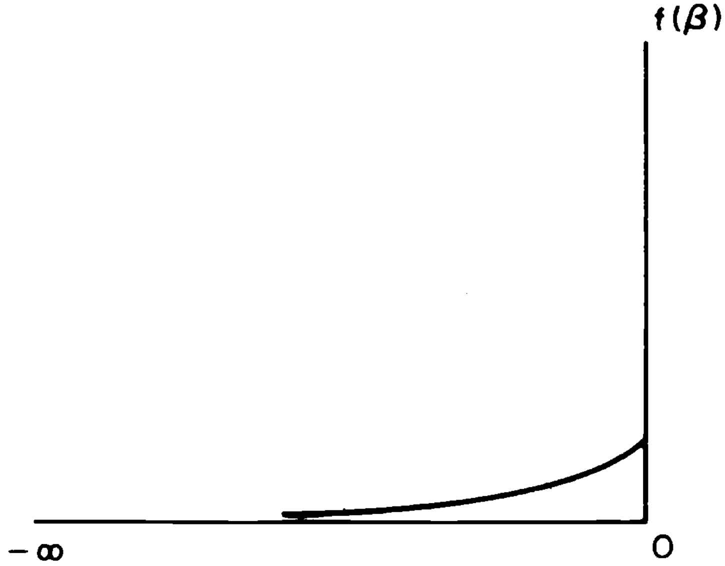

Fig. 2.4

Previous research has indicated that married women's labor supply decisions are sensitive to the net wage so that we would expect to find that taxes create both an important uncompensated wage effect and an income effect, as they do for husbands. As previously stated, we treat wives' labor supply decisions conditional on husbands' earned income. Thus wives are considered to be secondary workers, which may not be a proper assumption. Since in our sample labor force participation of husbands is near 100% while that of wives is near 50%, perhaps treating wives' earnings conditional on husbands' earnings is not a particularly bad assumption. However, the crucial question is whether husbands' earnings should enter the wives' labor supply decision as exogenous nonlabor income. It is probable that some jointness in decision making takes place when the husband adjusts his hours of work to his wife's earnings. A family labor supply model would be able to treat these problems better, but here we only provide estimates for the conditional model.

We turn now to the estimates of the labor supply equations which are presented in table 2.2. We present estimates for a convexified budget set, for the complete nonconvex budget set, and for a nonconvex budget set with fixed costs included. First, note that we find substantial uncompensated wage and income elasticities. For the average woman who is working full time we find the uncompensated wage elasticity to be 0.995 for the

Table 2.2 **Wives' Annual Hours ($\times$ 1,000)**

Variables	Convex	Nonconvex	Nonconvex with Fixed Costs
μ_β – income (1,000s)	2.0958	1.7519	2.0216
	(.1389)	(.1475)	(.1186)
σ_β	.5390	.4836	.5262
	(.0460)	(.0490)	(.0711)
α – wage	.4951	.5058	.4608
	(.2310)	(.0932)	(.1062)
Intercept	.5790	.3501	.6234
	(.9517)	(.4907)	(.5766)
Family size	.2387	.2202	.2144
	(.1270)	(.0773)	(.1259)
Children < 6	−.1695	−.1123	.1472
	(.3426)	(.2239)	(.1576)
College education	−.7851	−.7205	−.6903
	(.4216)	(.2390)	(.4389)
Age (35–45)	.2328	.0733	.0824
	(.1102)	(.0349)	(.0436)
Age (45+)	−.1066	−.1043	−.1989
	(.0644)	(.0539)	(.0660)
Health	−.4771	−.3139	−.3581
	(.7274)	(.4753)	(.4647)
Equity	−.0221	−.0150	−.0210
	(.0172)	(.0039)	(.0113)
Fixed costs: intercept	...	...	1.2125
			(.3570)
Fixed costs: kids < 6	...	...	.1720
			(.9541)
Fixed costs: family size	...	...	−.2118
			(1.6106)
σ_η	.3086	.2907	.2801
	(.2388)	(.2099)	(.2386)
Mean of β	−.125	−.118	−.123
Standard deviation of β	.112	.109	.113
Median of β	−.089	−.085	−.088

nonconvex results, and a similar magnitude for the convex results, 0.978, is found. When fixed costs are added, the uncompensated wage elasticity falls to 0.9065. Thus all three estimates indicate that the effect of the income tax in decreasing the net, after-tax wage is important in determining wives' labor supply. Since wives' net wage is lowered substantially by the presence of the "marriage tax," the tax effect may be much greater than if wives' earnings were not added to husbands' earnings for tax purposes. On the other hand, we also find an important effect of nonlabor income (and actual income). The elasticity at the means is approximately − 0.2. This effect causes more wives to go to work, because their husbands' earnings are reduced by taxes. The two effects have opposite signs so that a simulation is needed to evaluate the net effect of the marriage tax.

In this section we have presented our specification of the labor supply in the presence of nonlinear convex budget sets and nonconvex budget sets. The stochastic specification has been emphasized since it will play an important role in the simulation results. We now consider how the results can be used to simulate the effects of tax reform. We emphasize computational considerations so that the simulations can be conducted at low or moderate costs of computer time.

2.2 Tax Change Evaluation

In this section we develop formulas for expected hours of work, expected tax revenue, and expected deadweight loss given our model of labor supply and the estimates of the previous section. The main question that we attempt to answer is how much attention must be paid to the stochastic components of the specification to obtain accurate estimates. We consider both analytical and Monte Carlo approaches to the problem. We want to find accurate and low-cost computational techniques which permit the use of simulation methodology. At the same time keep in mind the typically large samples which are involved in a simulation. These large samples make computational techniques an important consideration. But the large samples may also allow possible simplifications in computational techniques because of the averaging process used in the calculation of simulation results.

For a given person j the desired hours of work on budget segment i is specified to be

$$h_{ij} = \alpha w_{ij} + \beta y_{ij} + Z_j\gamma + \eta_j = h^*_{ij} + \eta_j \ , \tag{2.1}$$

where w_{ij} is the net, after-tax wage on segment i and y_{ij} is virtual income for segment i, i.e. the intercept of segment i extended back to the vertical axis in figure 2.1. The vector z_j represents socioeconomic characteristics of individual j. Now if w_{ij} and y_{ij} were determined exogenously and α, β, and γ were fixed coefficients, then we could use the standard linear expectation rules to derive $Eh_{ij} = \alpha w_{ij} + \beta y_{ij} + Z_j\gamma$. Of course, we specify β to be distributed randomly in the population in the intervals $(-\infty,0)$. But the extension to stochastic β does not create much difficulty because again, given exogenous w_{ij} and y_{ij},

$$\begin{aligned} Eh_{ij} &= \alpha w_{ij} + y_{ij}E\beta + Z_j\gamma \\ &= \alpha w_{ij} + \left(\mu_\beta - \sigma_\beta\left[\frac{\phi(\mu_\beta/\sigma_\beta)}{1-\Phi(\mu_\beta/\sigma_\beta)}\right]\right)y_{ij} + Z_j\gamma \\ &= \alpha w_{ij} + \bar{\beta} y_{ij} + Z_j\gamma \ , \end{aligned} \tag{2.2}$$

where μ_β and σ_β are the underlying parameters of the nontruncated distribution for β, respectively, while ϕ and Φ are the standard normal density function and distribution function, respectively. The problem to be faced, then, is that w_{ij} and y_{ij} are determined by the budget segment B_{ij}, which depends on two stochastic components, η_j and $v_j = \beta_j - \bar{\beta}$. Thus we have the problem that the variables on the right-hand side are not determined exogenously. Nor do we have a simple formula for their expectation as we would in the linear simultaneous equations case. Thus, not unexpectedly, we need to consider the complete budget set when calculating the conditional expectation of hours worked, tax revenue paid, or deadweight loss and account for the "endogeneity" of w_{ij} and y_{ij}. It turns out to be the nonlinearity of the budget set together with the distribution of preferences specification which cause the significant costs of labor supply simulations. As we indicate below, the solution for that part of the β distribution which corresponds to a given budget segment is a nontrivial calculation.

We first consider the analytical conditional expectation for hours worked. The expectation is

$$Eh_j = \sum_{i=1}^{m} \left[\int_{\beta_{i-1,j}}^{\beta_{ij}} \int_{q_{ij}(\beta)}^{\infty} (h_{ij}^*(\beta) + n_j) f(\eta_j) f(\beta) d\eta_j d\beta \right] + \sum_{i=1}^{m-1} \left[\int_{r_{i,j}}^{\infty} (h_{ij}^* + \eta_j)(F(\beta_{ij}) - F(\beta_{i-1,j})) f(\eta_j) d\eta_j \right]. \tag{2.3}$$

As we discussed in the last section for $\beta < \beta_j^*$, the minimum β which causes desired hours to be positive ($h_{ij}^* > 0$), we assume that actual hours $h_j = 0$. Thus equation (2.3) calculates the expectation of actual hours h_j over the range for which desired hours h_{ij}^* are positive. The first sum in the equation corresponds to the case where desired hours fall along a budget segment $i = 1, m$. The range of β values which causes this to happen is denoted $(\beta_{i-1,j}, \beta_{ij})$. Note that in the nonconvex case some segments may have the intergral end points equal, which means that desired hours will not fall anywhere on the segment. It is basically this calculation which leads to the greatest expense in simulation since calculation of the univariate and bivariate integrals is not that costly. The nonconvex budget set of figure 2.2 indicates the possibility of an indifference curve that is tangent to two budget segments simultaneously. Thus in the nonconvex case there are portions of the budget segment which cannot correspond to desired hours. For this possibility to happen, the indirect utility function of equation (1.4) is equal for a given β for two sets of values of w_i and y_i. Calculation of these β values for each nonconvexity in the budget set requires the iterative solution of a nonlinear equation. Given the further facts that the points of mutual tangency are unknown and that complete budget segments may be skipped over, the computation of the $(\beta_{i-1,j}, \beta_{ij})$ for all budget segments $i = 1, m$ is a rather complicated task. Thus we look for possible simplifications in simulation to reduce both the computer costs and the required programming time.

The outer integral in equation (2.3) determines desired hours $h^*_{ij}(\beta)$. But actual hours $h_{ij} = h^*_{ij}(\beta) + \eta_j$. The inner integral accounts for this second source of stochastic variation. Note that for large negative values of η_j we have $h_{ij} < 0$. Thus $q_{ij}(\beta) = h^*_{ij}(\beta)$, the minimum value of η which keeps actual hours positive. The second sum in equation (2.3) corresponds to desired hours falling at one of the $m - 1$ curves, or kink points, of the budget set. The lower limit to the integral, r_{ij}, again determines the range for positive h_{ij}.

Evaluation of the integrals in equation (2.3) is not especially difficult, even given the bivariate integrals. Conditioning formulas can be used, and known partial fraction expansions for univariate integrals lead to quick evaluation. These simplifications follow basically from the linear specifications of h_{ij} in equation (2.1). Unfortunately, because of the nonlinearity of the expenditure function, the computation of integrals becomes considerably more complicated for calculating deadweight loss. The expenditure function which corresponds to the indirect utility function of equation (1.4) is

$$e(w_{ij}, U_j) = \exp(-\beta w_{ij})\ U_j - \frac{\alpha}{\beta}\ w_i + \frac{\alpha}{\beta^2} - \frac{Z_j\gamma}{\beta}\ . \tag{2.4}$$

The nonlinearity arises from β appearing in both the exponential and the denominators of the coefficients. For a given β, deadweight loss is measured by calculating either the compensated or equivalent variation via the expenditure function of equation (2.4) and then subtracting off compensated taxes paid, using the definition of Diamond and McFadden (1974).[4] Hausman (1981*a*) has demonstrated the necessity of doing the correct Hicksian measure of consumer surplus because use of the incorrect Marshallian measure can lead to very large errors in calculation of the deadweight loss. For calculation of deadweight loss, equation (2.3) is altered to account for the deadweight loss for β values which correspond to zero hours of work. Otherwise, the general formula remains the same, with the main difference that the nonlinear calculation required for deadweight loss makes computation considerably more slow than in the case of hours worked, which is a linear function of β. Conditioning formulas for the integrals are no longer applicable, and quadrature methods to evaluate the univariate and bivariate integrals are now required.

4. Other definitions are discussed in Auerbach and Rosen (1980).

To evaluate computation techniques we tried four approaches listed in order of decreasing computational burden on a sample of men in 1975 from the Panel Study of Income Dynamics (PSID) data base:

1. Analytical evaluation, via the computer, of the integrals in equation (2.3). For the nonlinear deadweight loss calculation we took the $\tilde{\beta}_{ij}$ corresponding to the mean β on the interval $(\beta_{i-1,j}, \beta_{ij})$ so that complete quadrature methods were not necessary to evaluate the integrals.

2. The distribution of β was still integrated over, but one Monte Carlo draw from a normal distribution was done for η_j.

3. The distribution of β was integrated over, and η_j was set to zero.[5]

4. β was taken at its mean value $\bar{\beta} = \mu_\beta - \sigma_\beta \phi(\mu_\beta/\sigma_\beta)/[1 - \Phi(\mu_\beta/\sigma_\beta)]$. Corresponding to $\bar{\beta}$ we find $h^*_{ij}(\beta)$, and η_j is set to zero. This technique also removes any need for integration for taxes paid or calculation of deadweight loss.

Note that the second approach leads to unbiased (or consistent) estimates of the expectation. Such estimates will have more variance than the actual expectations of the first approach because of the variance created by the Monte Carlo draws. However, we consider a sample of 200 men to see whether the appropriate law of large numbers works fast enough for this consideration not to be important. Potential bias is created by approach (3) since the expectation of η_j is positive and decreases along each segment as β_j increases. Last, approach (4) creates additional bias because it runs afoul of the rule that the expectation of a nonlinear function is not equal, in general, to the function of the expectation. Potential problems arise here for both hours of work and deadweight loss because of the nonlinearity of the budget set.

In table 2.3 we consider the four techniques on the first five men in our simulation file to see what happens at the individual level. The column labeled "hours" gives actual hours, while the next four columns calculate the expectation of hours corresponding to methods (1)–(4). The next two sets of columns correspond to the expectation of taxes paid and the expectation of deadweight loss using the equivalent variation measure.

Since method (1) leads to the correct evaluation of the expectation, it provides the standard of comparison for methods (2)–(4). For labor supply we see that method (2) leads to considerable variance, as expected. Method (3), which sets $\eta = 0$, gives identical results to method (1). Method (4), which takes the mean $\bar{\beta}$, leads to some bias, although only a small amount. For expected taxes paid, methods (3) and (4) again have a bias which is somewhat larger in this case. Last, deadweight loss seems most sensitive to the technique used. Techniques (3) and (4) are off

5. The women's sample might be better than the men's sample for testing this option because the sensitivity around zero hours for a man is probably quite small. Thus biases are not apt to be important for men. However, subsequent simulations have indicated that, while the bias is slightly larger for women, it is still probably small enough to be ignored.

Table 2.3 Individual Evaluation Methods (× 1,000)

Individual	Actual Hours	Expected Hours				Expected Taxes Paid				Expected Deadweight Loss			
		$H(1)$	$H(2)$	$H(3)$	$H(4)$	$T(1)$	$T(2)$	$T(3)$	$T(4)$	DWL(1)	DWL(2)	DWL(3)	DWL(4)
1	2.708	2.393	2.136	2.393	2.389	1.105	.883	1.104	1.100	.162	.395	.151	.156
2	1.928	2.097	2.679	2.097	2.101	5.208	7.661	5.165	5.058	2.971	.993	2.784	2.848
3	1.994	1.900	2.114	1.900	1.887	2.560	2.934	2.543	2.455	.449	.401	.401	.496
4	2.310	2.233	2.559	2.223	2.245	3.082	3.857	3.061	3.063	1.081	.337	1.022	1.015
5	2.121	2.201	1.455	2.201	2.142	1.713	.814	1.704	1.602	.314	1.974	.287	.382
Means	2.212	2.163	2.189	2.163	2.153	2.734	3.230	2.715	2.656	.995	.820	.929	.997

by about 7% in these calculations. Thus our tentative conclusion for individual calculations is that for labor supply and taxes paid method (3) is probably an appropriate technique to use.[6] However, for deadweight loss, full analytical evaluation of the integrals seems necessary for accurate calculation of the expectation.

We now turn to the major use of simulation for tax changes. We simulate over a file of approximately 225 men from the PSID file to see what happens to accuracy for mean changes. This file was found large enough to capture the limiting behavior of the different evaluation methods. Note that a substantial amount of computer processing time is involved here.[7] Taking the amount of time to do method (1) as unity, we find that method (2) takes 0.560 while method (3) takes 0.500 and method (4) takes 0.360 as long. Where many simulations are done over tax files that have thousands of entries, these time considerations can become quite important. Given the nonlinearity of the problem, the simulations can take up large amounts of computer time.

Simulation results are given in table 2.4. We now find that method (2) gives almost identical results to method (1) for hours and taxes. This result is as expected since the Monte Carlo method should give accurate computations once the law of large numbers has had time to take effect. Method (3) is fine for hours, but it is note quite as good for taxes. Moreover, it offers only a very slight savings over method (2). Method (4) probably can be rated as unsatisfactory given the size of tax changes that we are usually interested in evaluating. For deadweight loss calculations, method (2) is off by about 4%. The other two methods are off by double that amount. Here we might conclude that larger samples are probably needed for method (2) to be sufficiently accurate. Methods (3) and (4) might be rejected as too inaccurate to evaluate proposed tax changes. Thus we may conclude this section with the finding that methods (2) and (3) are both appropriate for use in the evaluation of tax change on labor supply and taxes. For computation of deadweight loss, where nonlinearities become important, only method (2) is approximately accurate. However, for samples of the size we are considering, method (1), which involves calculation of all the integrals involved in the expectation, provides the only truly accurate method. The appropriate next step in this line of research is to develop formulas for the (asymptotic) standard errors which correspond to the results in table 2.4. Given the nonlinearities inherent in the calculation of hours, taxes, and deadweight loss, asymptotic expansions would be used to account for the uncertainty in the parameter estimates. But the accuracy of these techniques might be

6. Method (4) may also be satisfactory for a first approximation.

7. While relative computer costs are difficult to compare, a simulation on the full sample of 1,000 families on the Massachusetts Institute of Technology IBM 370 computer costs around $60.

Table 2.4 Full Sample Evaluation Methods: Mean Response (× 1,000)

Method	Hours	Taxes	DWL
1	2.169	2.268	.652
2	2.165	2.267	.676
3	2.169	2.251	.608
4	2.138	2.153	.709

questionable here. Evaluation of the accuracy of the expansions would require a full-scale Monte Carlo study in itself. Yet such information might be very helpful, especially if the standard errors for the calculations in table 2.4 turn out to be sizable. We need to remember that "parameter uncertainty" does not average out by a large-numbers type of result in simulations because of perfect correlation across sample draws in the use of parameter values. This area seems to be an important aspect of future research in the field. We now turn to evaluation of some proposed tax changes. Method (3) is used for expected hours and expected taxes, while we use method (1) to evaluate expected changes in deadweight loss.

2.3 Simulation Results

In this section we consider the effect of two different types of tax systems. The first type of tax is the current federal tax on labor income including both the income and payroll tax. We compare it to a no-tax situation. To measure the change in labor supply we calculate the change in expected hours of work using equation (2.3). The appropriate choice for the change in individual welfare is not quite as clear. We use the equivalent variation calculated from the expenditure function of equation (2.4). Choice of the equivalent variation as the measure of deadweight loss, or the excess burden of taxation, seems appropriate since we later consider changes from the current system to an altered tax system. Since in the altered tax system individual welfare may be higher, we want to know the cost (in utility) of staying with the current system. But two possible objections to our measure is that we aggregate across individuals, giving each individual the same weight in the implicit social welfare function, and that different individuals are allowed different coefficients in their expenditure functions. The problems created for analysis of vertical equity considerations by these choices are discussed in Atkinson and Stiglitz (1976). The latter problem may not be especially serious since parameter differences arise from a distribution of preferences which is common to the entire population.

The other type of tax system that we consider involves a cut in tax rates of a given percentage. We consider the expected change in labor supply, the expected change in tax revenue, and the expected change in dead-

Table 2.5 **Mean Tax Results for Husbands**

Market Wage	DWL	DWL/Tax Revenue	DWL/Net Income	Change in Labor Supply
$3.15	$ 66	9.4%	.8%	−4.5%
$4.72	$ 204	14.4%	2.0%	−6.5%
$5.87	$ 387	19.0%	3.1%	−8.5%
$7.06	$ 633	23.7%	4.5%	−10.1%
$10.01	$1,749	39.5%	9.9%	−12.8%

weight loss from the current system. Much recent attention has focused on the revenue effects of a change in the income tax rates. It is important to note that our analysis is wholly *partial equilibrium* in nature. We look only at changes in expected labor supply. Thus potentially important factors such as changes in market wages and changes in inputs of other factors of production are not considered. A more complex general equilibrium model is needed to answer these questions. Also, since tax revenues will be decreasing, the problem of compensation arises. The problem of potential versus actual compensation was the basis of the Kaldor-Hicks-Scitovsky-Samuelson-Little debate of the 1940s. Without the choice of an explicit social welfare function we cannot resolve this problem. But we assume no posttax redistribution of income among individuals, since such actual (rather than potential) compensation is unlikely to take place.

In table 2.5 we look at the effect of the current tax system for five categories of husbands defined by their market wage.[8] Overall, we find that the tax system decreases the labor supply by 8.5% and that the mean deadweight loss as a proportion of tax revenue raised is 28.7%. We note important differences among the five categories. First, we see that deadweight loss rises rapidly with the market wage as expected. In terms of the welfare cost of the tax we see that the ratio of deadweight loss to tax revenue raised starts at 9.4% and rises to 39.5% by the time we reach the highest wage category. We see that the cost of raising revenue via the income and payroll taxes is not negligible. In terms of a distributional measure we see that the ratio of deadweight loss to net income also rises rapidly. In fact, this measure indicates that individuals in the highest wage category bear a cost about ten times the lowest category while individuals in the second highest category bear a cost five times as high. Without a specific social welfare measure, we cannot decide whether the

8. When we refer to the current tax system, we are actually using the 1975 data, which the model was estimated with. However, except for the rise in social security contributions, the taxation of labor income has not changed significantly since 1975. Of course, individuals on average have moved into higher marginal tax brackets because of the lack of indexation of the income tax.

current tax system has too much, too little, or about the right amount of progressiveness. But the measures of table 2.5 seem important in thinking about the problem. Last, note that the change in labor supply from the no-tax situation again rises with the wage category. The high marginal tax brackets have a significantly greater effect on labor supply than do the low tax brackets.

We now do a similar set of calculations for our sample of wives. While we found both significant deadweight loss and an important effect on labor supply for husbands compared to the no-tax situation, the situation is more complicated for wives. First, about half of all wives do not work. In the absence of an income tax, the net wage would rise, causing some of them to decide to work and others to increase their labor supply. But at the same time their husbands' after-tax earnings would also rise, which has the opposite effect on labor force participation. Thus both effects must be accounted for in considering the effects of the income tax. Overall for wives, (in table 2.6), we find the ratio of deadweight loss to tax revenue to be 18.4%. But it should be remembered that this ratio understates the effect on labor force participants alone. For labor supply, we find that taxes serve to increase the labor supply in the lowest wage category but decrease the labor supply as the wage rises. Overall, they decrease the labor supply by 18.2%. Thus, again for wives we see that the current income tax system both has an important labor supply effect and imposes a significant cost in welfare terms for raising tax revenue.

We now turn to a consideration of tax proposals of the Kemp-Roth type. We will consider two levels of tax cuts, 10% and 30%. The question which has been focused on most is what effect these tax cuts would have on tax revenues. Our results are partial equilibrium so that general equilibrium effects are not accounted for. The main effect here arises from the change in labor supply. But increased hours also move some individuals into higher tax brackets. Both effects need to be accounted for. In table 2.7 we present two Kemp-Roth simulation results. For the 10% tax deduction the mean hours of labor supply for husbands rise 22.5 hours, or 1.1%. Tax revenues fall by 7.4%. Even given the fact that our

Table 2.6 **Mean Tax Results for Wives**

Market Wage	DWL	DWL/Tax Revenue	DWL/Net Income	Change in Labor Supply
$2.11	$ 23	4.6%	.3%	+31.2%
$2.50	$ 119	15.3%	1.3%	−14.2%
$3.03	$ 142	15.9%	1.5%	−20.3%
$3.63	$ 184	16.5%	1.7%	−23.8%
$5.79	$1,283	35.7%	8.6%	−22.9%

Table 2.7 **Kemp-Roth Tax Cut Proposals for Husbands**

	10% Tax Cut			30% Tax Cut		
Market Wage	DWL/Tax Revenue	DWL/Net Income	Change in Labor Supply	DWL/Tax Revenue	DWL/Net Income	Change in Labor Supply
$3.15	8.5%	.7%	+.4%	6.8%	.4%	+1.3%
$4.72	13.3%	1.7%	+.5%	10.9%	1.1%	+1.6%
$5.87	17.4%	2.6%	+.9%	14.5%	1.8%	+2.7%
$7.06	21.8%	3.8%	+1.1%	17.9%	2.5%	+3.1%
$10.01	36.1%	8.2%	+1.4%	29.5%	5.3%	+4.6%

model is partial equilibrium, rudimentary calculations demonstrate that general equilibrium effects are very unlikely to be large enough to cause tax revenues from decreasing significantly in the short run, as our results show. In terms of the welfare cost of the tax we see that the deadweight loss falls significantly. The ratio of mean deadweight loss to tax revenue falls from 22.1% under the current system to 19.0% under the 10% tax cut plan. For the 30% tax cut labor supply increases by 2.7% while tax revenue falls by 22.6%. Again, we see that deadweight loss decreases significantly and the ratio of deadweight loss to tax revenues raised decreases to 15.4%. In terms of distributional changes the top quintile has the greatest increase in utility as a ratio to net income. Thus, as expected, decreasing taxes by a constant percentage reduces deadweight loss but does so in a manner most beneficial to those individuals who face the highest tax rates. Kemp-Roth type tax cuts have large effects both in terms of decreasing deadweight loss and in decreasing government revenue. Without knowledge of marginal government expenditure, it is difficult to evaluate the trade-off. But we cannot recommend Kemp-Roth cuts on welfare grounds alone, given the substantial fall in government revenue.

For wives we do not present detailed quintile results because the overall pattern is similar to the results for husbands. The mean results are given in table 2.8. Overall, we see that the labor supply response to a tax cut is greater for wives than for husbands. We expect this since the wage

Table 2.8 **Overall Kemp-Roth Tax Cut for Wives**

Tax Cut	Change in Tax Revenue	Change in DWL	Change in Supply (hours)
10%	−3.8%	−10.6%	+50.2
30%	−16.2%	−17.4%	+117.0

elasticity is about twice the income elasticity, so we should have a net increase in labor supply. Furthermore, the difference in the elasticities is about four times that of husbands, and we do observe a significantly larger response. For the 10% tax cut case, labor supply increases by 4.1% and tax revenues fall by 3.8%. For the 30% tax cut case, labor supply increases by 9.4% and tax revenues fall by 16.2%.

Our overall evaluation of the Kemp-Roth tax proposals is that while tax revenues will decrease by significantly less than the tax cut, overall government revenue from the income and payroll tax will decline. An argument might be made that general equilibrium results may be large enough to reverse this conclusion, but I doubt that it is a valid argument, especially in the short run. Thus, unless a strong argument can be made for reducing government expenditures with little welfare loss from the recipients, the Kemp-Roth tax cut proposals cannot be supported on the basis of our results alone. They certainly do not have the "free lunch" properties claimed by some of their supporters.

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Comment James J. Heckman

In his paper Hausman applies econometric methods developed in the literature on sample selection bias and censored samples to estimate labor supply behavior in response to various forms of tax policies. "Kinked" convex and nonconvex budget constraints receive the lion's share of the attention in the analysis. Various methods of simulating the estimates are then proposed and implemented.

I have little to say about Hausman's simulation procedures. To discuss simulation of the estimates before discussing the quality of the estimates puts the cart before the horse. I have reservations about the input used in the simulation procedures and feel that it is premature to use the estimates offered by Hausman as a serious guide to assessing the impact of alternative tax policies on labor supply.

My principal reservations about his estimates focus on the specification of the budget set confronting individuals that is used in the empirical work and on the econometric specification of the labor supply equations. Before turning to these issues, however, it is useful to place the current work in context.

Kosters's (1967) pioneering work on labor supply was based on the following key assumptions: (*a*) taxes are proportional, (*b*) a worker is free to choose any hours of work at "his" wage, (*c*) the income of one spouse is predetermined in the labor supply of the other (for married workers). Boskin (1973) and Hall (1973) relaxed (*a*) and (*c*) while retaining (*b*). The standard tax deduction formula is used to compute effective marginal tax rates. Tax schedule "kinks" are ignored as a first approximation, and the linearization device employed in the Hausman paper for a kinked constraint (section 2.1) was used to parameterize the effective after-tax wage confronting the worker. Both Boskin and Hall replace the income of the spouse (where appropriate) with the theoretically more correct wage. The main empirical findings reported in these and other papers done at the same time suggested a backward-bending male labor supply function (for hours worked for most groups) and strong positive wage effects for most female groups (both hours worked and participation). A persistent empirical problem in this literature is the often statistically insignificant and sometimes positive effect of measures of "exogenous income" on labor supply. The range of male labor supply estimates is 0.19 to 0.07 for the uncompensated substitution effect, but this range is by no means universally accepted. (For a survey see Heckman, Killingsworth, and MaCurdy 1981.)

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Rosen (1976) relaxed (*b*) using a hedonic model that arises from explicit consideration of employer interests in employee hours of work. His empirical findings on the wage-hours locus have been confirmed in later work. Rosen also utilizes a variant of the Boskin-Hall procedure to compute effective taxes facing individuals.

Hausman follows in the tradition of Boskin and Hall but focuses on several important problems overlooked in their work. First, Boskin and Hall both ignore the simultaneity problem that arises in using computed marginal and average tax rates. The point here is simple but empirically important. Given a nonproportional tax, the computed tax rate depends on the error in the labor supply equation. Putting tax-adjusted wage rates and virtual income levels on the right-hand side of a labor supply equation (as do Boskin and Hall) creates a standard simultaneous bias problem. Hausman's procedure attempts to avoid this sort of bias.[1] Second, Boskin and Hall both ignore the kinks at various levels of adjusted gross income in the official tax tables. In a progressive system there should be bunching at the kinks. Individuals at these kinks are at a corner equilibrium in their labor supply so that the textbook theory of labor supply must be modified, albeit in a straightforward way. Because of these kinks, the standard instrumental variable solution to the first problem does not work. Besides addressing these issues, Hausman also explicitly allows for individual heterogeneity in preferences following up on the work of Hall (1975) and Heckman (1974). He demonstrates, as had the papers cited, that there is considerable dispersion in preferences for work in the population. Finally, Hausman follows Cogan (1980, 1981) in introducing fixed costs into the analysis of labor supply.

There are few original ideas contained in this paper. However, the synthesis of the work of others is interesting. The main contribution of the paper is the development of computational algorithms. Hausman takes the textbook one-period labor supply model and imposes it onto his data in order to secure estimates and generate policy simulations. In doing so he ignores a considerable body of accumulated empirical evidence that casts doubt on the validity of the textbook model. Hausman's procedures critically depend on access to data that he does not have and that economists are unlikely to have in the near future.

This paper is a microeconometric counterpart of the standard macroeconometric exercise that was conducted in the 1960s when the consensus view was that the remaining research agenda in that field was a matter of "fine tuning." Hausman adopts the view that was assumed then, that

1. Rosen also discusses this point, but his solution—evaluating tax rates at a standard hours of work position—trades a simultaneous equation bias for an induced measurement error bias that is likely to be very sizable. This general problem in the Boskin-Hall procedure has been noted by many analysts. See the survey in Heckman, Killingsworth, and MaCurdy (1981) for a discussion of this point.

there is agreement on the validity of the simple theory, that the basic empirical facts are well known, and that all that is required to produce policy forecasts is a simply computed algorithm. Would that it were so.

My comments on this paper are directed toward the general research strategy and the specific procedures used to achieve the estimates reported in Hausman's paper. In particular, I discuss the following topics: (1) The specification of the choice set facing individuals. (2) The arbitrary, and sometimes very controversial, functional forms and distributional assumptions that are imposed onto the data in order to secure estimates. (3) The economic interpretation of consumer surplus measures and deadweight losses in the presence of heterogeneity in consumer preferences.

The Choice Set

There is no question that kinks appear in adjusted gross income tax schedules if they are properly measured. Kinked nonconvex constraints characterize many social programs and social experiments.

I am surprised to find so much discussion of the econometrics of kinks in the Hausman paper without empirical demonstration of the importance of the problem. Specifically, I refer to the bunching that one expects to find in the presence of convex preferences and kinks. There should be some piling up of labor supply at kink points. I know of no such evidence, and in looking casually at the CES data for 1972 I find no evidence of such bunching. Of course one reason for finding no bunching is that it is very difficult to compute the correct kink points for a consumer unless we known itemizations and deductions. Hausman's econometric procedures rely critically on the assumption that kink points are known to the econometrician, a point I elaborate below. I question, in practice, whether they are in fact known. Another "reason" for the absence of evidence on the importance of kinks is the ad hoc assumption built into Hausman's model that workers are forced off their preferred labor supply curve by exogenous shocks that are independent of the preferences and resources of workers.

I am also surprised to find so little attention devoted in this paper and in the literature in general to the problem of tax avoidance and labor supply. The problem strikes me as more important than the problem of kinks. Rational economic behavior suggests that individuals will devote resources to avoid taxes and that they will take advantage of tax subsidies on goods such as housing. Tax rates computed from standard tables (as Hausman computes his tax rates) will overstate the true tax rate paid. Dollar taxes paid will understate the true cost of the tax by the direct avoidance costs (even abstracting from labor supply adjustments). Dollar taxes estimated from a tax schedule will overestimate true taxes paid (including avoidance costs). In the appendix I sketch a very simple model to demonstrate these points.

There I demonstrate that for a plausible tax avoidance function, estimates, such as Hausman's, that ignore tax avoidance behavior understate true income and overstate estimated income effects, leading to an overestimate of welfare losses. The true kink point confronting consumers varies in a manner that depends on consumer preferences for goods that are subsidized by the tax system (e.g. housing) and resources available to the consumer. This means that Hausman's econometric procedures, which require the econometrician to compute individual taxes, are inappropriate in the absence of detailed information about consumption behavior.

There is the additional complication that, because some goods are subsidized by the tax system, the simple two-dimensional labor supply analysis utilized by Hausman is inappropriate. It is appropriate to analyze labor supply and a composite good "consumption" only if the tax system does not subsidize the consumption of specific items such as housing. But it does. Numerical estimates of the bias from this source are not yet possible. The Hausman model is misspecified because it omits such relative price effects induced by the tax system. This point helps to explain the apparently (perverse) positive effect of home equity on labor supply reported in table 2.1.

In the appendix, I present a model for incorporating tax avoidance effects into the analysis of labor supply and offer some rough estimates of the empirical importance of the phenomenon. If my numbers hold up in further investigation, the bias from neglecting tax avoidance is considerable.

I next turn to a point to which I have alluded several times: *that Hausman's econometric procedures require that the budget set confronting the consumer be known to the econometrician.*[2] Because of considerable unobserved variability in deductions and exemptions that is not accounted for in Hausman's tax computation algorithm, and because for some groups of workers (primarily females) wages must be estimated, the true position of the budget set is not known to the econometrician. Hausman only allows two types of variability in the model (in the income coefficient β and in the discrepancy between actual and desired hours of work). Measurement error in specifying the budget set introduces a third type of variability that cannot be represented as either of the first two types. Thus *his maximum likelihood procedure*, which requires a full accounting for all sources of variability in order to deliver consistent estimates of the structural parameters of the model, *does not produce*

2. Hausman claims that this is not so in his footnote 2, but there is no demonstration of this claim there or in any of his papers because the claim is false.

consistent estimates. This is so because the after-tax wages used in his formula (1.4) are measured with error as are the segments on the labor supply axis that purport to correspond to the after-tax wages. Using imputed after-tax wages from his procedure, one computes the wrong probability that an individual desires to be on a given branch of a budget set if the budget set is not correctly specified. As a consequence, Hausman's estimators are inconsistent.

These problems are not insurmountable, but they are not addressed, much less solved, in any of Hausman's papers. An exact analysis of the magnitude of the bias that results from this source is difficult. Roughly speaking, in a group of otherwise identical individuals those with more taste for work have greater incentive to avoid taxes. Under Hausman's imputation scheme, such people are allocated to a higher tax bracket than they actually face. Substitution effects are overstated, leading to an overestimate of computed welfare losses. Neglecting dispersion in wage rates by assigning average wages to individuals tends to lead to a downward bias (in absolute value) in estimated substitution effects. This second effect would be most pronounced for women (for whom wages are more likely to be imputed), perhaps accounting in part for Hausman's relatively low estimated substitution elasticity for women. (See Heckman, Killingsworth, and MaCurdy 1981 for a survey of recent results on the labor supply of women.)

For more detailed discussion of this topic, see Heckman and MaCurdy (1981, pp. 88–95, especially pp. 92–93). The essential point made there is that errors in variables problems in general nonlinear models, such as Hausman's, require more careful analysis than has been accorded to them in the literature. In light of this point, I cannot help but speculate that previous empirical procedures such as those of Boskin and Hall that incorporate less (false) information into the estimation procedure may be more robust than procedures such as Hausman's which assume information that does not exist and which produce inconsistent estimators if the information is false.

The Imposition of Functional Forms and Prior "Information"

In light of the long-standing empirical controversy surrounding the sign of the income effect (β), I feel that it is inappropriate to impose negativity onto the estimates as Hausman does. This point is particularly important in the estimates of the male labor supply equation. Hausman's estimated substitution effect (α) is essentially zero. By imposing a negative income effect onto the data by his econometric procedure, Hausman guarantees that his procedure will produce a larger compensated substitution effect—and hence a larger welfare loss—than other studies have. At a minimum, I think that unrestricted estimates should be reported and a test of Hausman's assumptions that the mean of β and the largest population value of β are negative should be performed. This point is especially relevant in light of previous published results by Burtless and Hausman (1978, p. 1124), who report that when β is permitted to become positive, the estimated mean value of β is not statistically significantly different from zero.

Hausman justifies the imposition of a nonpositive value of β by appealing to the argument that leisure is a normal good. Much previous research indicates that when β is not restricted, it frequently is estimated to be positive (as in fact is the coefficient on house equity in Hausman's table 2.1—which coefficient can be interpreted as estimating β multiplied by the rate of return on housing stock). There are two reasons for this (Heckman 1971; Greenberg and Kosters 1973; Smith 1980): (*a*) the endogeneity of assets in a life-cycle model of labor supply, and (*b*) the correlation between preferences for work and savings. These papers indicate that the standard one-period model of labor supply as used by Hausman must be modified to produce useful results. Hausman chooses to ignore all of this research and decides the matter by fiat. For this reason his estimates, which ignore life-cycle phenomena and the endogeneity of assets, are not to be taken as serious guides to policy.

A similar remark applies to the use of functional forms to secure estimates of fixed costs and other unobservables. The new game in labor supply, pioneered by Cogan (1980, 1981), is to interpret departures of estimated labor supply functions from a simple functional form as evidence for the presence of fixed costs and the dispersion of preferences. In the appendix, I indicate how this game can be played to produce estimates of tax avoidance parameters.

There is much accumulating evidence (Heckman and Singer 1982; Goldberger 1981; Duncan 1981) that parameter estimates of nonlinear models of the sort estimated by Hausman are very sensitive to the choice of functional form of the model and the distributions of unobservables. In light of this recent work, I am very uneasy that so much mileage is obtained from imposing arbitrary, and intrinsically untestable, nonlinearities and distributional assumptions onto the data in order to secure labor supply estimates. I am not as negative as Hausman on the more modest empirical procedures used by previous analysts who make less grandiose claims about the validity of their models and use less "information" in securing labor supply estimates. Given our current state of knowledge about labor supply, there should be less "fine tuning" and more insight if the sort of bold claims made in the Hausman paper are to be taken seriously. Much more evidence on the robustness of the estimates reported in this paper to departures from the assumptions of the model is required.

The Implications of Heterogeneity on the Calculation of Welfare Losses

In light of Hausman's finding (and that of previous analysts Hall 1975 and Heckman 1974) that there is considerable heterogeneity in consumer marginal rates of substitution between goods and leisure, the interpretation to be placed on the reported estimates of welfare loss is not clear. Hausman notes this point in his paper, but does not discuss it in any detail. His estimates *overstate* the true amount of compensation that the society must be paid to be just as well off as before a tax change because the society can redistribute income among heterogeneous consumers, and Hausman's estimates assume that no redistribution occurs.

Miscellaneous Comments and a Summary

I agree with Hausman's conclusion that the Kemp-Roth tax cut proposals will not produce a free lunch. A reading of the estimates derived in the pre-Hausman literature on labor supply supports this conclusion. Hausman's numerical results offer another shred of evidence that supports the view held by most economists that the Laffer curve has no empirical foundation in the labor supply literature.

I am less convinced by the estimates of welfare losses due to income taxes that are presented in his paper. For reasons already advanced, Hausman's procedures tend to produce inflated estimates of welfare losses.

Appendix *A Simple, Econometrically Tractable Model of Tax Avoidance and Labor Supply*

Let $U(X,L)$ be the preference function of the consumer. X is goods consumption, and L is leisure. $0 \leq L \leq 1$. Let the tax function facing the consumer be proportional. The after-tax fraction of income received is $\theta(A)$, where A is dollar avoidance costs, $\theta'(A) \geq 0$, $\theta''(A) \leq 0$, $\lim_{A \to \infty} \theta(A) \leq 1$. For simplicity, avoidance is assumed to be nondeductible and nonutility bearing.[3] The wage is W, and the consumer can freely choose his hours of work. R is his unearned income.

3. The specification of the tax avoidance function warrants some discussion. A more general analysis would write taxes paid after avoidance $\tilde{T}$ as a function of taxes paid with no avoidance T and avoidance expenditure A: $\tilde{T} = F(T, A)$ with $F_A < 0$ and $F(T,0) = T$. The consumer's problem is to maximize tax saving less avoidance cost $(T - \tilde{T}) - A$. A sufficient condition for an interior solution is $F_{AA} > 0$ and $\partial F / \partial A < -1$ for $A = 0$ and all $T > 0$.

Specializing F so that $\tilde{T} = T\varphi(A)$ with $\varphi(0) = 1$, $\varphi' < 0$, $\varphi'' > 0$, for a strictly proportional income tax t, the θ function adopted in the text is $\theta(A) = 1 - t\,\varphi(A)$.

For a kinked tax schedule expressed in terms of total income Y,

$$
\begin{aligned}
Y &= R + W(1 - L) , \\
T &= t_1 Y \quad Y \leq Y_1 , \\
T &= t_1 Y_1 + t_2(Y - Y_1) \quad Y > Y_1 ,
\end{aligned}
$$

we define the fraction of income retained after taxes as

The consumer's problem is

$$\max U(X,L) \text{ s.t. } \theta(A)(R + W(1-L)) - PX - A = 0 .$$

There is a prior maximization problem: First choose A. The first-order condition is

$$\theta'(A)(R + W(1 - L)) = 1 .$$

For $R > 0$, $\theta''(A) < 0$ is sufficient. As income increases, avoidance expenditures increase. The standard tax table reveals $\theta(0)$. The computed marginal tax rate, as a function of income, is $\theta(A)$, which is always less than $\theta(0)$. $1 - \theta(A)$ is the effective marginal tax rate.

The effect of tax avoidance is to make the true tax function more concave. A progressive tax table may appear as regressive after tax avoidance occurs. Consider a proportional tax. Figure C2.A.1 displays the table after-tax income after avoidance for the case $\theta''' < 0$. Figure C2.A.2 demonstrates the apparent budget set confronting the consumer, the tax table schedule, and the true constraint (inclusive of tax avoidance). In investigating the labor force participation decision using a constraint computed from the standard table, we understate true income and overstate estimated income effects. Using the apparent schedule (ignoring A) underestimates income effects. Substitution effects are overestimated. The effect on computed welfare losses (ignoring A) of estimates computed from the standard tables is to overstate the true welfare loss for two reasons: (*a*) the estimated compensated substitution effect is overestimated and (*b*) the true tax change of any computed tax change is smaller. Including A, this effect is partially but never completely offset. Below, I present preliminary estimates that suggest that these effects may be empirically quite strong.

Analogous results hold for the case of a kinked progressive standard tax table. The true constraint has a kink, and in the neighborhood of each kink point the slope of the true schedule to the right of the kink exceeds the slope of the true schedule to the left of the kink (see footnote 3). For sufficiently high after-tax income, the true marginal tax rate to the left of the kink may be *less* than the true marginal tax rate to the right of the

---

$$\theta(A,Y) = 1 - t_1\varphi(A), \quad Y \leq Y_1 ,$$

$$\theta(A,Y) = 1 - \left[t_2 + (t_1 - t_2)\frac{Y_1}{Y}\right]\phi(A), \quad Y > Y_1 .$$

For optimal values of A, θ is a continuous function of Y but is not a continuously differentiable function of Y. In the neighborhood of Y_1, the derivative of income after taxes and avoidance cost to the left of Y_1 exceeds the derivative of income after taxes and avoidance cost to the right of Y_1. To see this, note that optimal A is a continuous function of Y and to the right of Y, using the envelope theorem, income after taxes and avoidance cost $E(= Y - \tilde{T} - A)$ has the derivative $\partial E/\partial Y = 1 - t_2\phi(A)$, $Y > Y_1$, while the derivative of E with respect to Y to the left of Y_1 is $\partial E/\partial Y = 1 - t_1\phi(A)$.

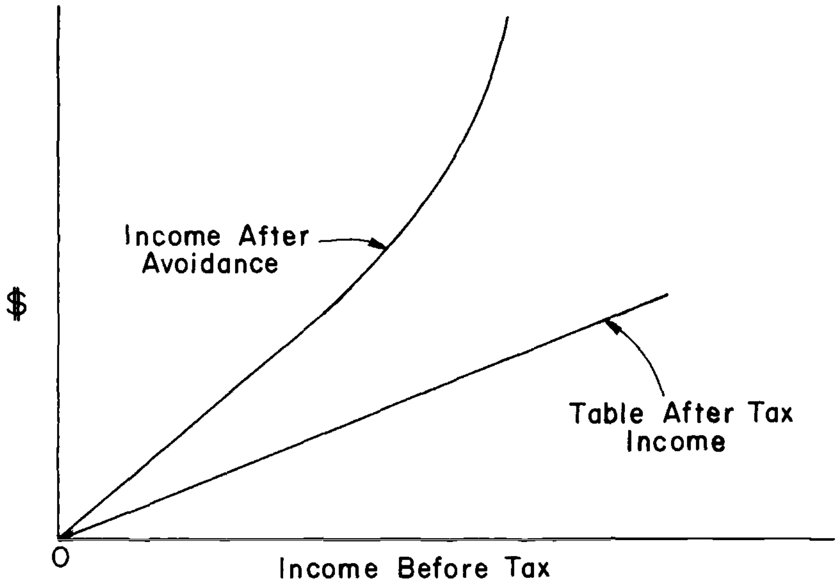

Fig. C2.A.1 Income after avoidance and table after-tax income as functions of income before tax.

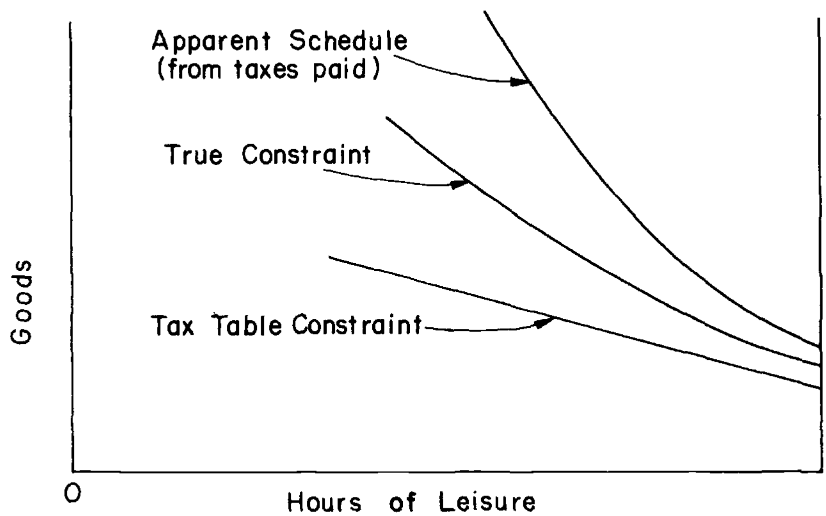

Fig. C2.A.2 Apparent constraint, true constraint, and tax table constraint for $R > 0$.

kink. This case is illustrated for a one-kink standard tax table in figure C2.A.3. There is a further complication that may be empirically quite important and which may cause the kink to disappear from the data. If tax avoidance costs can be written off in part and we cannot observe A, we may not be able to locate the abscissa of the kink in the figure. The same is true if avoidance is broadened to consider a variety of purchases and deductions not directly observed that affect adjusted gross income. The true kink point may vary in a population of consumers with the same wage and unearned income.

If A has a utility-bearing aspect (e.g. the subsidy on owned homes versus rented homes), a slightly more complicated analysis is required. The price of A depends on income. In a multigood world, the Hicks composite commodity theorem no longer applies so that the simple composite good used to derive indifference curves (or labor supply functions) and to specify the constraint set no longer holds. The after-tax price of A belongs in the labor supply function, and the computation of welfare loss requires a multidimensional analysis.

Is any of this empirically important? Since A cannot be observed, it may be argued that the preceding analysis is of little empirical relevance. This argument is incorrect. If the sort of strong functional form assumptions used by Hausman and Cogan to estimate unobservable fixed costs are adopted, it is also possible to estimate A. For the sake of brevity we only consider an apparent proportional tax case.

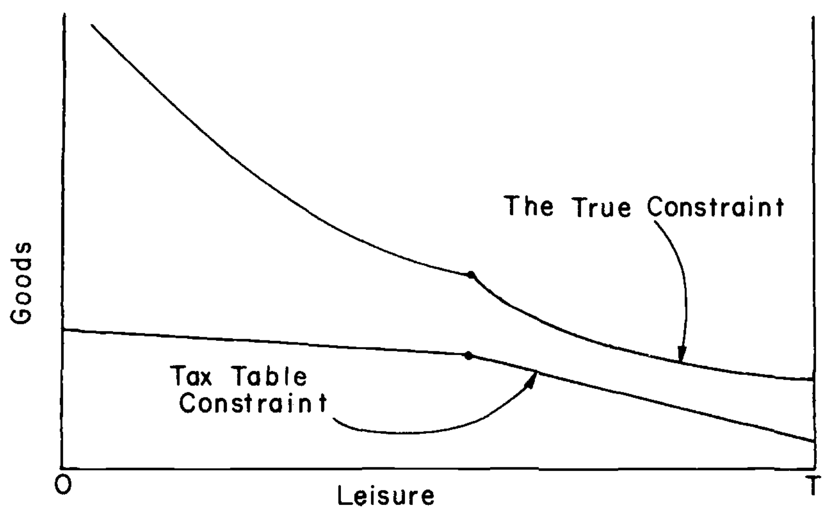

Fig. C2.A.3 True constraint and tax table constraint for the case of a one-kink progressive tax and $R > 0$.

Let $\theta(A) = k - b/(A + 1)$, so the apparent tax rate is $k - b$ (for $A = 0$). Then income after taxes and tax avoidance is (letting $h = 1 - L$)

$$[Wh + R][k] - b^{1/2}[Wh + R]^{1/2} + 1 \ .$$

Adopt a linear labor supply specification as a maintained assumption. Then, linearizing the true budget constraint in the fashion suggested by Hall and Boskin, the true marginal wage is

$$W\left[k - \frac{1}{2}\, b^{1/2}\,[Wh + R]^{-1/2}\right]$$

and virtual income is

$$1 + Rk - b^{1/2}\,[Wh + R]^{1/2} + Whb^{1/2}\,\frac{1}{2}\,[Wh + R]^{-1/2} \ ,$$

so the following labor supply equation may be fitted:

$$\begin{aligned} h = \alpha W&\left[k - \frac{1}{2}\, b^{1/2}\,[Wh + R]^{-1/2}\right] \\ &+ \beta\left[Rk - b^{1/2}\,[Wh + R]^{1/2} + Whb^{1/2}\right. \\ &\left. \times \frac{1}{2}\,[Wh + R]^{-1/2} + 1\right] + Z\gamma + \epsilon \ . \end{aligned}$$

Since we know $k - b$, we can estimate α, b, k, and β. Thus we can compute welfare losses. Because of the nonlinearity of the reduced form h function, we may use polynomials in Z as instruments. Modification of the analysis to a kinked convex case is a trivial extension.

If we have access to reported taxes, a simpler procedure is available so that it is not necessary to resort to arbitrary functional form restrictions on labor supply functions to estimate tax avoidance. Reported taxes are

$$[Wh + R]\left[1 - k + \frac{b}{A + 1}\right] .$$

Since $A + 1 = b^{1/2}[Wh + R]^{1/2}$ from the first-order condition for optimal tax avoidance, observed income after taxes is

$$[Wh + R]\left[1 - k + \frac{b^{1/2}}{[Wh + R]^{1/2}}\right].$$

Since we know $k - b = \phi$, substitute in the tax function to reach

$$[Wh + R][1 - \phi - b] + b^{1/2}\,[Wh + R]^{1/2} \ .$$

Regressing reported taxes on income generates estimates of $b^{1/2}$. This procedure can be used for the kinked constraint case.

How important is this problem? Without a serious analysis of the data on reported taxes versus taxes computed from standard tables, it is impossible to say. A very rough estimate obtained from the 1972 CES data in which taxes paid are reported suggests that b is roughly 10. This implies that a person earning $16,000 in adjusted gross income in 1972 spent about $400 on tax avoidance. This implies that the true marginal tax rate for this person is 25.5% rather than the quoted 28% and that true taxes paid (including avoidance) come to about $500 less than what is estimated from the standard form. If these results hold up in a thorough study, they suggest that the effect of ignoring tax avoidance may be empirically quite important.

It is interesting to note that the effect of ignoring tax avoidance behavior is to overstate estimated welfare losses. Moreover, a more comprehensive view of the tax system suggests that more effort might profitably be devoted to specifying the correct choice set for consumers and that econometric methods, such as those advocated by Hausman, that require exact information on the constraint set will generate biased estimates of tax response.

It is important to point out that different conclusions would be produced by other functional forms for the after-tax fraction of income function $\theta(A)$. Much further empirical and theoretical work is required before we can be sure that tax avoidance behavior is of any empirical importance.

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