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# 5 The Bequest Process and the Causes of Inequality in the Distribution of Wealth

Michael C. Wolfson

## Introduction

The objective of this paper is to assess, in a reasonably realistic manner, the quantitative importance of various patterns of intergenerational wealth transmission on the overall level of wealth inequality. The basic ingredients of the analysis are: a set of models of the main processes involved in the evolution of the size distribution of wealth; detailed microdata drawn from two Statistics Canada surveys for the distribution of wealth and the pattern of saving; and a specially developed computer simulation program.

The analysis starts with the distribution of wealth as observed in Canada in May, 1970, and projects it to the year 2000 under a range of alternative assumptions.

The model developed for the analysis involves a new methodological approach. For example, it is not based on the Orcutt, et al. (1976) style of microsimulation which operates at the level of individuals and families. Instead, the population density function representing the wealth distribution for each age/family size group is the basic building block or object of analysis. As a result, the model consists of a set of component processes that are defined, in mathematical terms, directly as operations on distributions. In this way, restrictive assumptions such as omission of age specific distributional detail (e.g., Atkinson 1971) and reliance on specific inequality measures or fixed function forms (e.g., Oulton 1976; Blinder 1973) can be avoided. At the same time, the

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simulation model is considerably smaller and less costly to run than that developed by Orcutt, et al. (1976). The model constitutes a development of the "continuum" approach to models of size distributions rather than the "fixed identity" approach as described by Vaughan (1975) and Wolfson (1977).

The plan of the paper is as follows: First, the general structure of the model will be described. Then the component processes specifically associated in the intergenerational transmission of wealth will be developed. Finally, the results of a set of computer simulations focusing on the bequest process will be presented and discussed. The interested reader is referred to Wolfson (1977) for a detailed discussion of the model, data, and other simulation results.

## 5.1 General Structure of the Model

The time series model of the evolution of the distribution of wealth starts with an "observation" of this distribution. This observation is drawn from the 1970 Statistics Canada Survey of Consumer Finance. The population was divided into two family size groups—one-adult families (including single parent families) and two-or-more adult families (all of which are assumed to be exactly two-adult nuclear families)—and twelve five-year age groups, the youngest being 20–24 and the eldest 85–89. For each of these twenty-four age/family size groups, a wealth density function was tabulated using thirty given wealth intervals. The twenty-four wealth densities then constituted the starting point for the model.

The time series model projects this disaggregated wealth distribution five years at a time. Figure 5.1 displays the general structure of the model where $W_{at}$ represents the wealth densities for age group $a$ $(1 \leq a \leq A)$ at time $t$. (A subscript for family size has been omitted for notational convenience.) It should be noted that the ordering of these processes is somewhat arbitrary; for example, marriage could precede divorce. It is assumed that reordering would not affect our results significantly.

The first step in the evolution of the set of wealth densities is the saving process. This process is quite complex. Actual saving rates and distributions of saving by age, family size, and wealth category were obtained by special analysis and tabulation of the microdata from the 1970 Statistics Canada Survey of Family Expenditures. These observed saving patterns as well as assumed rates of return are combined with the wealth densities in a complex of operations that include scaling and convolution.

In the growth step, the two main concerns are to keep the wealth densities in constant dollars and to account for population growth. It is assumed that population is growing at 10 percent, nominal saving at 25

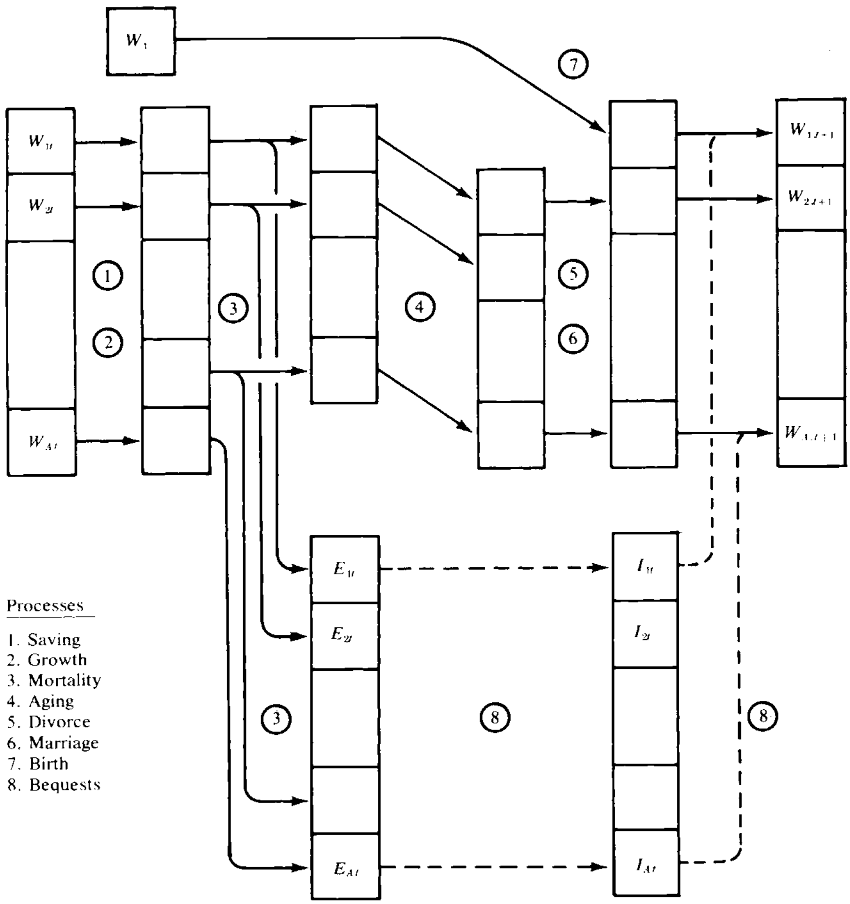


**Fig. 5.1** General Structure of the Time Series Model. Variables: $W_{at}$ = distribution of wealth; $E_{at}$ = distribution of estates; $I_{at}$ = distribution of inheritance.

percent, and prices at 12.5 percent, all per five years. The pattern of saving (i.e., the "shapes" of the saving density functions by age, family size, and wealth category) is assumed fixed as observed in 1970. The growth process therefore involves a set of vertical and horizontal scaling operations on the wealth and saving densities.

The mortality step is based on observed age and sex specific mortality rates. Note that when only one spouse in a two-adult family dies, it becomes a one-adult family; if both spouses die it becomes an estate. (The annual mortality rates have been appropriately transformed into five-year rates.) The mortality rate for the 85–89 age group is assumed to be 100 percent. Mortality has also been assumed to be independent of wealth and marital status (admittedly an incorrect assumption, e.g., see Shorrocks 1975). The wealth densities of survivors and estates are then obtained by a combination of scaling and vertical addition operations.

The aging process involves simply bumping each age group's wealth density down to the next slot in the full age-wealth joint density.

Divorce and marriage are based on observed rates and the assumption that for divorce the wealth is divided equally between the two parting spouses. In the case of marriage, two polar assumptions have been considered: random mating and perfectly assortative mating (rich marry rich, poor marry poor, and so on). The latter assumption will be used in the simulations presented below. Its importance is examined briefly at the end of the paper.

Finally, the birth step involves giving an initial wealth distribution to subsequent "newborn" cohorts. It is assumed that the wealth distribution observed in 1970 for those under 25 consisted entirely of "prenatal" saving and can therefore be used for $W_{1t}$ in subsequent years.

The final step in the process of generating the set of $W_{a,t+1}$ ($1 \leq a \leq A$) from the set of $W_{at}$ is to augment the wealth densities using the distributions of inheritances. The details of the process are developed in the next section. It should be noted that the model assumes no gifts *inter vivos*. They could be incorporated into the model, but it has been assumed that as a first step, the analysis would be more transparent if all intergenerational transfers arose as the result of bequests.

## 5.2 The Bequest Process

The basic determinant of the bequest process in practice is the way in which people draw up their wills. In a will, the decedent specifies (among other things) who the beneficiaries of the estate are to be, and how much each will receive. Our intent here is to describe this process in such a way that it is easy to pose hypothetical questions of the following form: What would the distribution of wealth be like if will writing behavior differed from the usual pattern with regard to some aspect such as x?

There are two basic difficulties that must be overcome, however. The first is a general lack of data (see, e.g., Shoup 1966; Jantscher 1967; Cheng, Grant, Ploeger, no date). For example, it is not well known how the average number of heirs varies with the size of the estate. In cases like this, our strategy will be to define a set of polar cases. These cases are ones that would be expected, intuitively, to constitute bounds on the kinds of behavior most likely to be observed.

This procedure of constructing bounding assumptions is like that used for the pattern of marriage above where, in the absence of reasonable data, we identified assortative and random mating as *a priori* bounding polar cases.

A second difficulty is that where polar assumptions regarding bequests have already been discussed in the theoretical literature, they are not stated in a form suitable for our methodological approach. This problem stems from the distinction between the "fixed identity" and "continuum" approaches. For example, we just referred to the division of an estate among the heirs. The conventional polar assumptions are primogeniture and equal division. But these cases are typically defined for an average or representative estate—the fixed identity approach. Our problem will be to define corresponding polar cases directly in terms of distributions of estates—the continuum approach.

The starting point for the bequest process in the context of figure 5.1 is the set of estate distributions $\{E_{at}(x)\}$, where $a$ indicates the age of the decedents. The end point of the process is the set of wealth distributions $\{W_{a,t+1}(x)\}$ that have been augmented by inheritance. The basic assumption to begin with is that between these two endpoints, the bequest process can be divided into three broad steps which are independent of each other. These steps will then form the framework within which the polar assumptions will be constructed. These three steps are: the transformation of estates into bequests (how each estate is divided); the transformation of bequests into inheritances (how the ages of decedents and inheritors are related); and the association of inheritances and inheritors (how within age groups the size of the inheritance tends to be related to the wealth level of the inheritor).

Clearly, any detailed description of how these steps operate in reality is necessarily very complex. The requirement for our model then is to construct a concise and relatively simple set of assumptions. These assumptions should be formulated in accordance with three main objectives: they should span the full range of behavior likely to be observed within each step; they should be easily translated into simple and efficient computer algorithms; and they should be parameterized in such a way that a relatively small number of "points" span or fully explore all possible combinations of polar cases.

We turn now to a discussion of each of the three main steps of the bequest process and the particular assumptions that will be employed.

### 5.2.1 Estates to Bequests

Recall that it has already been assumed that if only one spouse in a family dies, all wealth passes to the surviving spouse. Thus estates arise only when individuals, or both spouses in a family, die. There are two basic assumptions that will be made to start. First, we will make no distinction between estates coming from deceased individuals and those coming from "deceased" families. Thus, a preliminary step of the bequest process is actually to combine the estates (i.e., vertically add the two distributions) for the two family size groups within each age interval. The second assumption concerns legal practice by which it is impossible to inherit debt. The "estates" of individuals or families dying in debt are therefore ignored. However, the plight of these (negative) estates' creditors is also ignored (less than 0.04 percent of aggregate net worth in 1970–74).

The polar possibilities with regard to disequalizing or equalizing tendencies, in the case of a single estate (the fixed identity approach), are that it is either left intact and passed to a single heir (age and sex are ignored), or divided equally among some larger number of heirs, $h$. These two cases will be called primogeniture and equal division, respectively. Perhaps a more realistic situation is what might be called "modified primogeniture." In this case, some proportion $p$ of the estate goes to one heir while the remaining part $(1\text{-}p)$ is divided equally among the remaining $h\text{-}1$ heirs. Note that $p = 1/h$ corresponds to equal division, and $p = 1$ to primogeniture. It would also seem realistic to expect some relationship between the size of the estate and the way it is divided. For example, if only the very wealthy were concerned about keeping their estates intact, one might expect more primogeniture at the upper end of the wealth spectrum than at the lower end. (A more formal way to describe this last example is "differential division," the assumption that the size range of estates is partitioned into a set of wealth intervals, and within each interval a different pattern of division occurs: primogeniture in one, equal division among $h$ heirs in another.)

The discussion so far has been in terms of single or representative estates. But our concern is to formulate these assumptions directly in terms of distributions. To this end, let $B_a(x)$ be the density function of bequests coming from the estates of decedents age $a$. We then seek ways of relating $B_a(x)$ to $E_a(x)$. (The time subscript has been dropped for notational convenience only.)

Let us consider five such relationships, based on the discussion above of the possibilities with regard to a single estate:

a. Primogeniture: $B_a(x) = E_a(x)$

b. Equal Division: $B_a(x) = h^2 E_a(hx)$

c. Modified Primogeniture, i.e., a bequest of size $x$ may have come either from an estate of size $x/p$, or from an estate of size $(h-1)x/(1-p)$: $B_a(x) = E_a(x/p) + (h-1)^2 E_a[(h-1)\ x/(1-p)]$

d. Class Primogeniture, i.e., below some wealth level $w$ there is equal division while above it there is primogeniture:

$$\text{Let } E_a^1(x) = \begin{cases} E_a(x) & \text{for } x < w \\ 0 & \text{otherwise} \end{cases}$$

$$E_a^2(x) = \begin{cases} E_a(x) & \text{for } x \geq w \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Then } B_a(x) = h^2 E_a^1(hx) + E_a^2(x)$$

e. Differential Division, i.e., the wealth spectrum is divided into $k$ intervals; within each interval there is equal division among $h_i$ heirs; $h_i = 1$ implies primogeniture in the $i$th interval:

$$\text{Let } E_a^i(x) = \begin{cases} E_a(x) & \text{for } c_i < x \leq c_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Then } B_a(x) = \sum_{i=1}^{k} h_i^2 E_a^i(h_i w)$$

From these descriptions it is clear that $a$ and $b$ are special cases of $c$, indeed polar cases of $c$ with regard to their equalizing or disequalizing tendencies. Also, $a$, $b$, and $d$ are special cases of $e$. Because of its relative flexibility $e$, differential division, has been chosen as the general parametric form for the model of this part of the bequest process. The parameters are then:

$k$ = the number of distinct wealth classes from the point of view of bequest behavior (this number has nothing to do with the wealth classes used by the cross-sectional saving function);
$c_i$ = the lower limit (in dollars) of the $i$th wealth class (assumed fixed in real terms); and
$h_i$ = the number of heirs in the $i$th wealth class.

In the simulation runs described below, only two wealth classes ($k = 2$) will be used with a wealth cutoff of $c_2 = \$40{,}000$ in 1970 and $h_i$ taking values in the set $\{1, 2, 4, 6\}$. Specific assumptions for this step will be denoted "heirs $= h_1, h_2$." Since the number of heirs in each wealth class is the parameter of greatest interest, $c_2$ is not included in this shorthand notation. Given this value of $c_2$, $i = 1$ can be interpreted as a reference to poor or middle class families, and when $i = 2$ the reference is to the rich.

The situation with heirs = 2,1 ($h_1 = 2$ and $h_2 = 1$) is illustrated in figures 5.2 and 5.3, in terms of the population density function and Lorenz curve, respectively. It seems clear from the comparison between figures 5.3a and 5.3b that the distribution of bequests is more unequal after this differential division than after "universal primogeniture" where $h_1 = h_2 = 1$. However this conclusion is complicated by the fact that the number of bequests is different in the two cases. If $n_1$ is the number of "poor" estates and $n_2$ is the number of rich estates, then differential division in this particular case results in $n_1$ more more bequests than universal primogeniture. As an alternative, if the Lorenz curves for bequests for heirs = 2,1 and heirs = 1,1 with a suitable number ($n_1$ in this case) of zero inheritances included are compared, then it is clear that differential division (heirs = 2,1) results in a *more* equal distribution of bequests ("higher" Lorenz curve) than universal primogeniture. This point is illustrated in figure 5.3c.

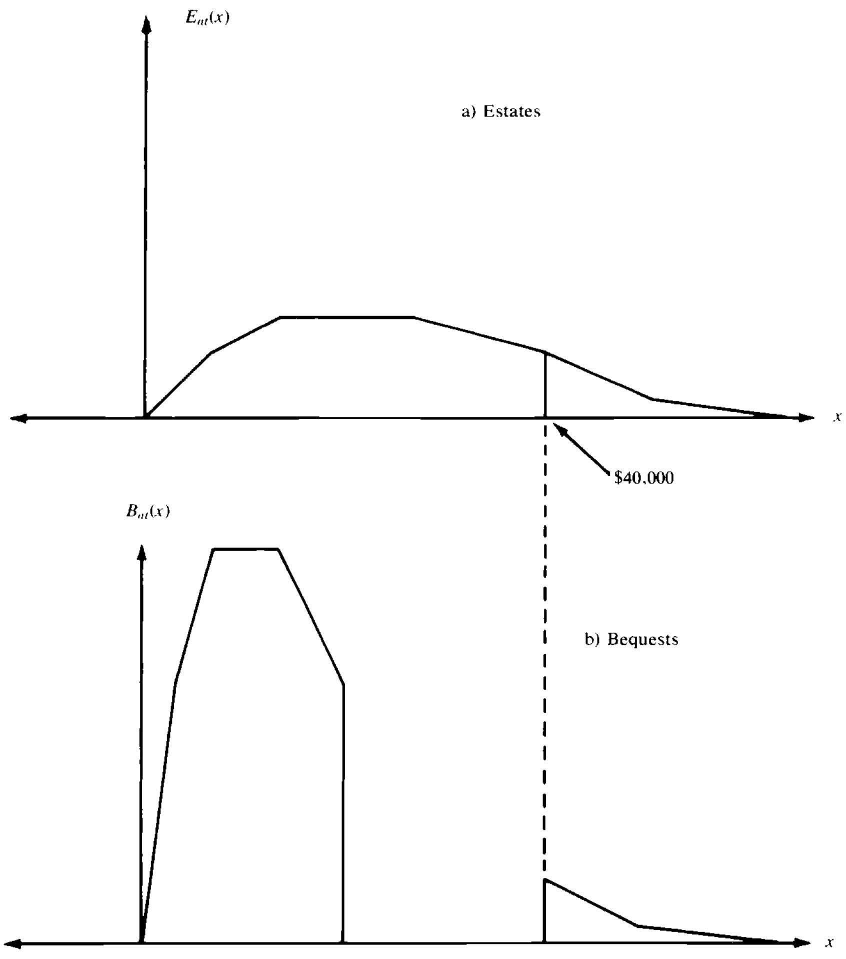


**Fig. 5.2** Population Densities for Estates and Bequests Given Differential Division. Hypothetical case: heirs = 2,1.

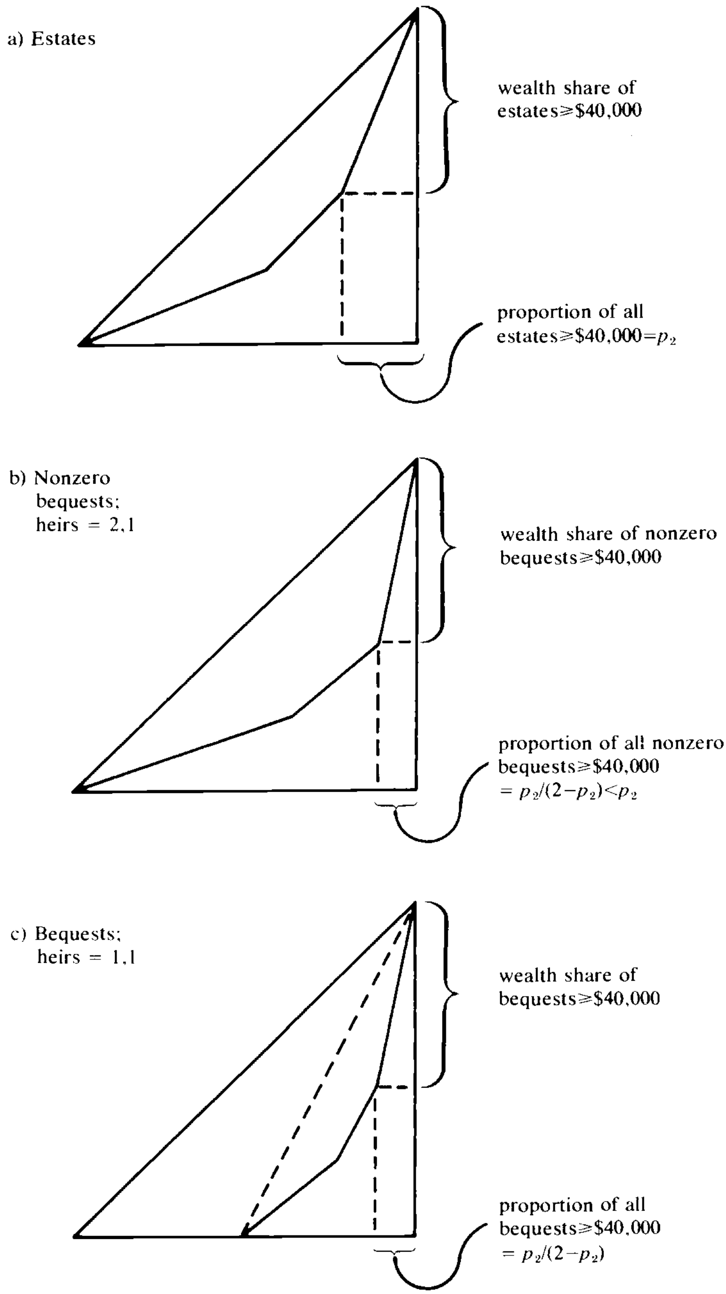


**Fig. 5.3** Hypothetical Lorenz Curves for Differential Division

In fact, a more general proposition is possible. If the two alternative assumptions heirs $= g_1,g_2$ and heirs $= h_1,h_2$ are being compared, where $g_1 \leq h_1$ and $g_2 \leq h_2$ and strict inequality holds at least once, then heirs $= h_1,h_2$ will result in a more equal distribution of bequests provided that the distribution of bequests from heirs $= g_1,g_2$ is "padded out" with enough zero bequests so that the total number of bequests is the same for the two distributions. (This proposition is proved in Wolfson 1977.) However, it is not possible to say in general which distribution of bequests is more equal, for example, when comparing heirs $= 1,4$ and heirs $= 4,1$.

5.2.2 Bequests to Inheritances

In our model, the basic distinction between bequests and inheritances lies in the significance of the age subscript. The "age" of a bequest refers to the age of the decedent, while the "age" of an inheritance refers to the age of the inheritor. The problem in this step of the bequest process is to define a general parametric relationship between the two sets of distributions. One major issue here concerns "generation skipping." It appears that many of the wealthy, to reduce total tax liability over several generations, leave substantial portions of their estates (in the form of trusts) to their grandchildren (Shoup 1966, p. 41). Lampman (1962, p. 239) suggests that this behavior has an equalizing effect. It also appears that many estates are divided among heirs of two or three different generations.

Given this range of bequest behavior, it seems important to be able to examine the effects of age differences between decedents and inheritors. This will be done in a highly simplified way: all the heirs of decedents aged $a$ will be assumed to be the same age, and this age will be $d$ years less than the age of the decedent. And if decedents are age $a \leq d$ (i.e., their heirs would be in age group $a - d < 20$), then their heirs will be assumed to be in the first age group (20–24). Thus, the possibility that heirs of the same estate, or generally of decedents of the same age, may be of different ages has been ignored. And by implication, the possibility that heirs of certain ages may be more likely to inherit larger portions of the estate than heirs of other ages (e.g., children versus siblings of the decedent) will also be ignored, since all the heirs of any given estate will be in the same age group. But as a starting point, this assumption still allows an interesting range of "polar" cases to be examined, defined in terms of the single parameter $d$. More formally, the assumption will be:

$$I_1(x) = \sum_{a=1}^{d+1} B_a(x)$$

$$I_a(x) = B_{a+d}(x) \text{ for } 2 \leq a \leq A - d,$$

$$I_a(x) = 0 \qquad \text{for } A - d < a \leq A,$$

where $I_a(x)$ is the density function of inheritances destined for inheritors aged $a$ and $0 \leq d < A$. In general, this assumption will be denoted "age-diff $= d$." The values that will be examined are 0, 25, 45, and 65. This assumption is illustrated in figure 5.4. Note that if one set of distributions of bequests by age group is more equal than another (in the general sense of their all having higher Lorenz curves) then the corresponding set of distributions of inheritances by age group will also be more equal (see Wolfson 1977).

### 5.2.3 Inheritances to Inheritors

There are actually three parts to this step in the bequest process: allocating inheritances between the two family size categories; choosing the subset of each age/family size category who will inherit; and associating inheritances by size with the wealth levels of inheritors. Given

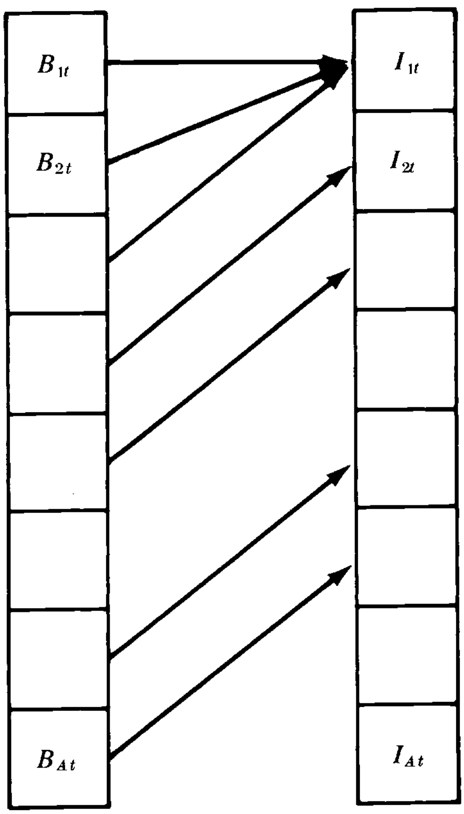


**Fig. 5.4** Bequests to Inheritances

the general objectives of comprehensiveness and simplicity, the following assumptions will be made:

a. Inheritances are divided between the two family size groups in proportion to the number of family units in each. Thus, if $p_{au}$ is the number of unattached individuals aged $a$ and $p_{af}$ is the number of families aged $a$, then $(p_{au}/(p_{au} + p_{af}))\ I_a(x)$ is the distribution of inheritances destined for unattached individuals aged $a$.

b. Two main methods for choosing inheritors will be distinguished. In both it is assumed that no inheritor receives more than one inheritance.[1]

Let $p_a$ = number of family units aged $a$ (ignoring family size for notational convenience);

$q_a$ = number of inheritances destined for inheritors aged $a$, assumed less than $p_a$;

$J_a(x)$ = wealth distribution of inheritors aged $a$;

$W_a(x)$ = wealth distribution of all family units aged $a$; and

$w_a$ = wealth level above which there are $q_a$ family units aged $a$.

These definitions imply the following relationships:

$$p_a = \int W_a(x)\mathrm{d}x$$

$$q_a = \int I_a(x)\mathrm{d}x = \int J_a(x)\mathrm{d}x$$

$$w_a = \min\left\{ w\colon \int_w^\infty W_a(x)dx \leq q_a \right\}$$

Two polar assumptions regarding the choice of inheritors can now be easily defined, one highly egalitarian in its implications and the other implying disequalizing tendencies. Formally, the assumptions require a relationship between $J_a(x)$ and $W_a(x)$. The two assumptions are:

*Random* (equalizing)—the probability of inheriting is independent of wealth level. Thus,

$$J_a(x) = (q_a/p_a)\ W_a(x)$$

*Select-R* (disequalizing)—only the richest within each age group become inheritors. Thus,

$$J_a(x) = \begin{cases} W_a(x) & \text{for } x \geq w_a \\ 0 & \text{otherwise} \end{cases}$$

Figure 5.5 illustrates these two alternatives. Note that there is a third possibility that would actually have stronger equalizing tendencies than random choice of inheritors—namely if the poorest $q_a$ were always chosen to inherit. However this possibility seems as unlikely as the assumption of "perfectly perverse assortative mating" (rich systematically marrying poor) and it will not be considered further.

c. The final part of this step of the bequest process is the manner in which the distributions of inheritances $I_a(x)$ and inheritors $J_a(x)$ are combined (ignoring family size for notational convenience). We shall again define two polar cases:

*Random* (equalizing)—the probability of inheriting any particular amount of wealth, given that one inherits, is independent of current wealth.

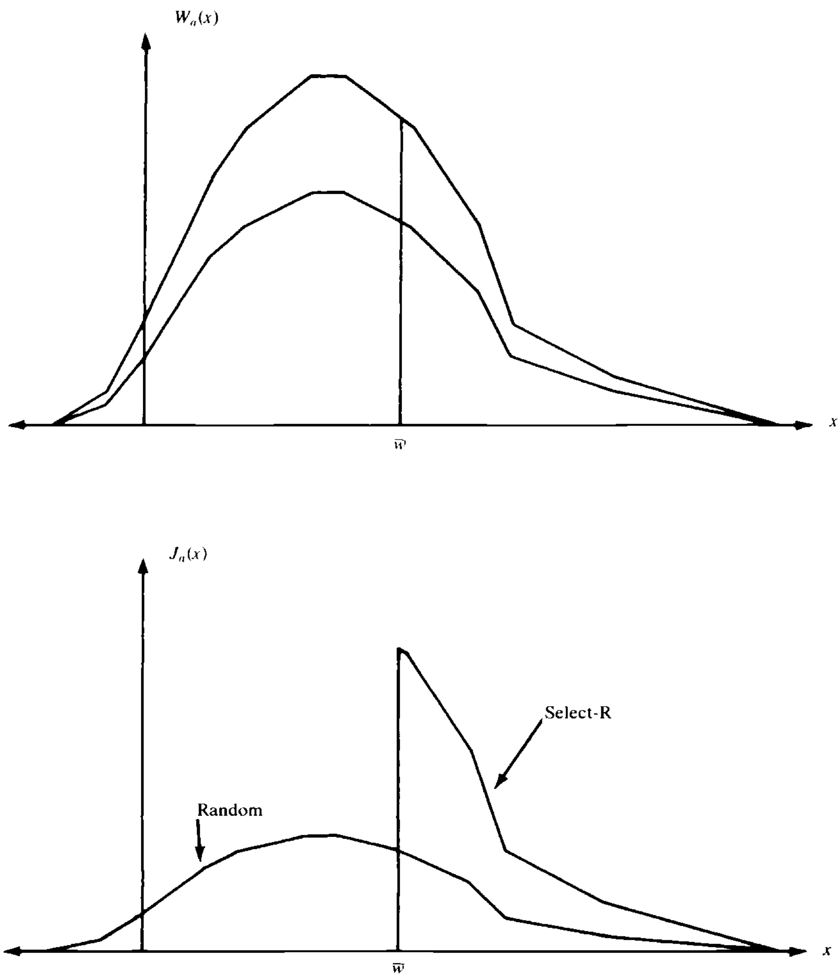


**Fig. 5.5** Hypothetical Example of Polar Cases for the Selection of Inheritors

*Assortative* (disequalizing)—the richest inheritor receives the largest inheritance, the second richest inheritor receives the second largest inheritance, and so on.

The random case corresponds to the mathematical operation of convoluting $I_a(x)$ and $J_a(x)$, and it can therefore use the algorithm already developed for the saving process and random mating. However, the assortative case here does not correspond to the case of assortative mating in the demographic model. There, it was possible to make use of the fact that the wealth distributions of prospective husbands and wives were identical.[2] But the shapes of the distributions of inheritances and inheritors will not be the same in general. Fortunately, there is a simple mathematical operation corresponding to this process of assortative combination: the cumulative density function of perfectly assortatively combined inheritances and inheritors is the horizontal sum of the cumulative density of inheritances and the cumulative density of inheritors.

Given a total of one assumption in part (a), and two assumptions each in parts (b) and (c), there are a total of four possible combinaations of assumptions in this third step of the bequest process. However, to reduce the combinatorial problems of having many possible assumptions, we shall focus on two (compound) polar assumptions regarding the receipt of inheritances by inheritors:

*Equal*—The most equalizing case for combining inheritors and inheritances is first to choose inheritors randomly from the wealth distribution of potential inheritors, and then to match inheritances with inheritors in a random manner.

*Unequal*—The most disequalizing polar case for combining inheritors and inheritances is first to select only the richest potential inheritors, and then to match the largest inheritances with the richest inheritors assortatively.

For convenience, these two assumptions for combining inheritors with inheritances will be denoted "comb = equal" and "comb = uneq," respectively.

It is clear that for any particular distribution of inheritances, comb = equal will result in a more equal "post inheritance" distribution of wealth than comb = uneq. But consider a second question. Suppose it is known that one distribution of inheritances is more equal (in the sense of a higher Lorenz curve) than another. Will the corresponding postinheritance distribution of wealth also be more equal? The answer is yes for both comb = equal and comb = uneq (see Wolfson 1977).

We have now completed the description of the model of the bequest process. There are three main steps. First, estates are divided into bequests. A general structure allowing differing numbers of heirs by wealth class is used. Second, bequests are transformed into inheritances by considering the differences in age between decedents and inheritors. Third, and finally, inheritors are selected and their wealth is augmented by inheritances in either an equalizing or a disequalizing manner.

Formally, any particular assumption for the bequest process can be summarized in terms of the following parameters:

$k =$ number of wealth classes for bequest behavior of decedents;

$c_i =$ lower limit (in 1970 dollars) of $i$th wealth class;

$h_i =$ number of heirs in $i$th wealth class;

$d =$ age difference between decedents and inheritors; and

$\begin{Bmatrix} \text{Equal} \\ \text{Unequal} \end{Bmatrix} =$ polar methods for matching inheritances and inheritors

However, we shall always assume $k = 2$ and usually assume that $c_2 =$ \$40,000 (in 1970). (More wealth classes could have been simulated, but it was not felt that any further interesting results would emerge.) The shorthand notation for the three assumptions will then be:

$$\text{heirs} = h_1,h_2, \text{ for } h_i \in \{1,2,4,6\};$$

$$\text{age-diff} = d, \text{ for } d \in \{0,25,45,65\}; \text{ and}$$

$$\text{comb} = \text{equal or comb} = \text{uneq.}$$

## 5.3 Computer Simulation Results

Before launching into a discussion of the simulation results, it is first necessary to explain how these results will be summarized. Typically, a single simulation of the time series model will cover a period of thirty years. It therefore generates a sequence of six age/family size/wealth joint densities, in addition to the initial joint density for 1970. Furthermore, an anlysis of a particular parameter may involve as many as five or six such simulated wealth sequences at a time. There is, as a result, a nontrivial problem of "data reduction"—selecting the key indicators of the results of any simulated sequence of wealth distributions.

Our approach to the problem is the following. First, the primary concern will be with the aggregate wealth distribution, i.e., the distribution for all age/family size groups combined. Second, for any wealth distribution the focus will be on six summary statistics: the mean level of net worth, three summary measures of inequality, and two inequality indicators. The three inequality measures are the well-known Gini coefficient and squared coefficient of variation (CV), and a specially designed measure we have called the exponential measure (EXP). The two inequality indicators are the wealth shares of the top 1 percent and next 4 percent of the population.[3] Finally, the main interest will be in the wealth distribution at the end of the sequence, in the year 2000. This approach is clearly a dramatic simplification. From a total of 168 (2 family size groups, 14 age groups, 6 years) wealth densities, six scalar magnitudes will be distilled. However, when it is necessary to the discussion of various simulation results we shall refer to the more disaggregated data.

The model of the bequest process has three basic parameters: heirs, age-diff, and comb. For each of these parameters, a range of values was specified. Both the number of parameters and the number of values each would take was kept small so that the total number of combinations did not grow too large. The main reason for concern over the number of possible combinations is the expectation that there could well be significant interaction among the parameters. This expectation affects the experimental strategy. If there were no significant interactions among the main parameters, it would be possible to proceed by first defining a "base run" of the simulation model. Then variations around it, one parameter at a time, could be explored. But in the case of the bequest process, this approach is unacceptable. We must be able to check whether or not there is significant interaction. Having a relatively small set of parameter combinations makes this task easier.

Given the three main parameters and their range of values, it is possible to display the set of combinations as nodes on a three-dimensional grid—or two two-dimensional grids, one each for comb = equal and comb = uneq. The two 2-D grids are displayed in figure 5.6 below.
The nodes marked by heavy dots indicate the combinations of parameters which have been simulated. For example, (heirs = 1,1; comb = uneq; age-diff = 45) has been simulated, while (heirs = 4,1; comb = uneq; age-diff = 25) has not. This diagram, therefore, displays the program of experiments with the bequest model. This set of simulations will be analyzed in two stages.

The first parameter that will be examined is "heirs," with the set of "experimental values" corresponding to the two horizontal rows of dots for age-diff = 45 in figure 5.6. The results of these simulations are displayed in table 5.1, collected into three groups.

A preliminary observation concerns the level of inequality in the year 2000 compared with 1970. Even with universal primogeniture (heirs = 1,1) and unequal combination (comb = uneq), when age-diff = 45 inequality in the upper tail of the distribution is reduced by the year 2000 (CV = 5.0 versus CV = 5.8, and top 1% = 17.0 versus top 1% =20.3, comparing run 1 and the 1970 values), though inequality increases in the lower and middle ranges of the wealth spectrum (EXP

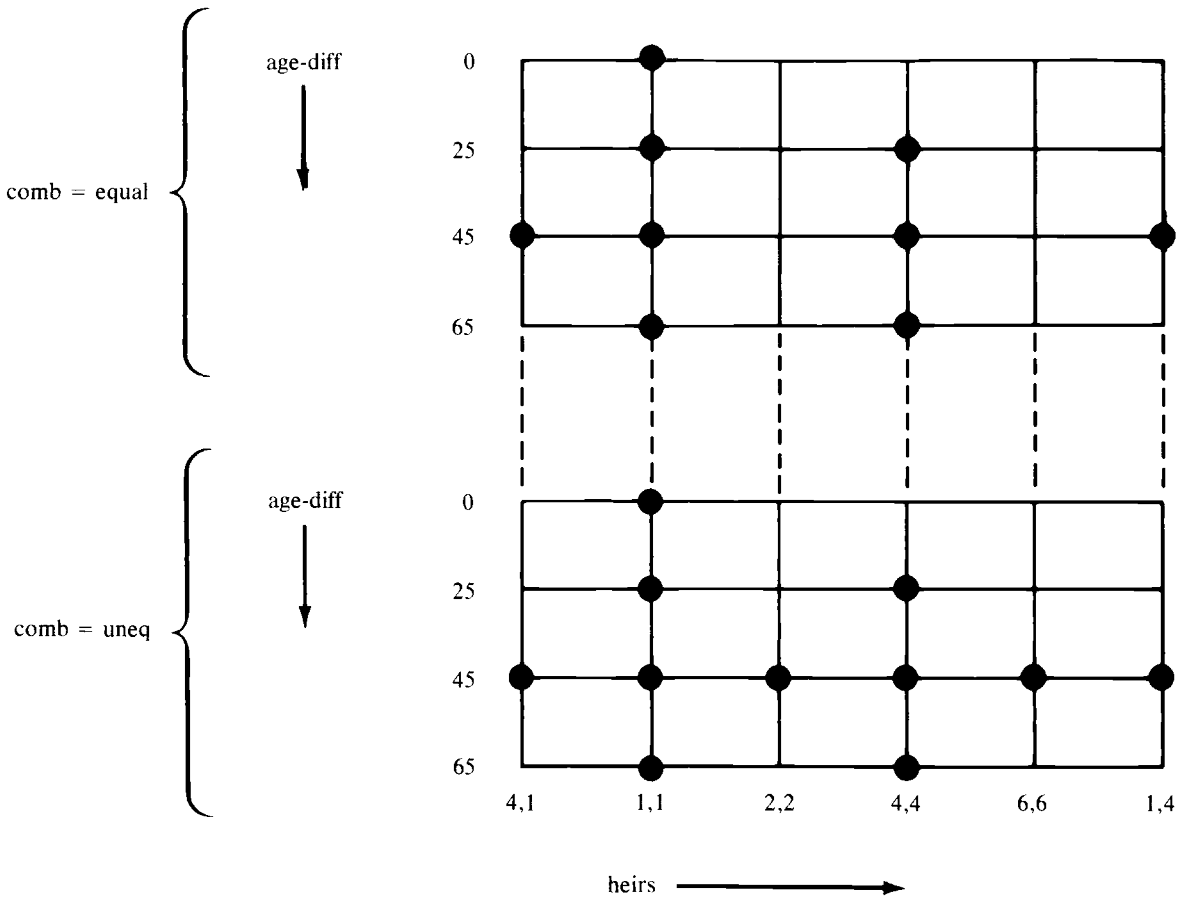


**Fig. 5.6** Space of Parameters for the Bequest Model

**Table 5.1** **Time Series Results, Bequests Part 1**

| Run | Heirs | Age-Diff | Comb | Mean Wealth | Inequality Gini | CV | Exp | Wealth Shares Top 1% | Next 4% |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1, 1 | 45 | uneq | 18.2 | .81 | 5.0 | .70 | 17.0 | 25.1 |
| 2 | 2, 2 | 45 | uneq | 18.4 | .79 | 4.0 | .68 | 14.2 | 24.2 |
| 3 | 4, 4 | 45 | uneq | 18.5 | .76 | 3.4 | .66 | 12.6 | 22.6 |
| 4 | 6, 6 | 45 | uneq | 18.5 | .74 | 3.1 | .65 | 12.0 | 21.6 |
| 5 | 4, 1[a] | 45 | uneq | 18.1 | .79 | 4.8 | .68 | 17.1 | 23.4 |
| 6 | 4, 1 | 45 | uneq | 18.2 | .78 | 4.6 | .68 | 16.7 | 22.5 |
| 1 | 1, 1 | 45 | uneq | 18.2 | .81 | 5.0 | .70 | 17.0 | 25.1 |
| 3 | 4, 4 | 45 | uneq | 18.5 | .76 | 3.4 | .66 | 12.6 | 22.6 |
| 7 | 1, 4 | 45 | uneq | 18.4 | .79 | 3.7 | .68 | 13.1 | 24.6 |
| 8 | 4, 1 | 45 | equal | 18.3 | .72 | 3.2 | .64 | 12.5 | 21.4 |
| 9 | 1, 1 | 45 | equal | 18.4 | .75 | 3.3 | .65 | 12.4 | 21.8 |
| 10 | 4, 4 | 45 | equal | 18.3 | .68 | 2.6 | .61 | 10.9 | 19.3 |
| 11 | 1, 4 | 45 | equal | 18.5 | .71 | 2.7 | .63 | 10.9 | 19.8 |
| | 1970 | | | 15.4 | .75 | 5.8 | .65 | 20.3 | 20.9 |

*Note*: [a]Cutoff at \$25,000; all others at \$40,000.

= .70 versus EXP = .65, and Gini = .81 versus Gini = .75). Intuitively, it appears that the dispersion in saving along with the other components of the model is sufficient to generate increasing inequality over time in the lower and middle wealth ranges. But with these bequest model parameters, the share of the top 1 percent falls. (These inequality results do not hold when age-diff = 25, however, as will be seen in the next set of simulation results.) It is also the case that with the given growth rate assumptions and no taxation, average "real" wealth grows slowly over the thirty-year period. (The geometric average growth rate is 0.56% per year.)

The first main observation is that as one would expect, increasing the number of heirs reduces inequality quite substantially at all points in the wealth spectrum. This result holds for both equal and unequal combination (runs 1 to 4 and runs 9 and 10). The only apparent interaction between the comb and heirs parameters is in the upper tail of the distribution: runs 1 and 3 do not show a consistently larger or smaller change in inequality values, either absolutely or relatively, than do runs 9 and 10 except for the CV and share of the top 1 percent. This result (that more heirs implies less inequality) is as expected, since division among a larger number of heirs implies that more family units have their wealth augmented by smaller amounts.

The second main observation is that differential division (heirs = 4, 1) does not always lead to unambiguously (i.e., in terms of Lorenz curves) lower inequality than universal primogeniture. It appears to be the case for run 6 versus run 1. This result is what one would expect, given the earlier theoretical analysis. But in run 5 versus run 1, the share of the top 1 percent is higher. The explanation must be that despite the fact that (initially, in 1970–74) the postinheritance distributions of wealth are more equal in run 5 than in run 1 (i.e., the upper tail of the distribution is not so elongated), more family units are moved above the cutoff dividing middle and rich for the saving process. Thus, the disequalizing tendencies of the saving and yield differences outweigh, in this case, the effects of a more equal postinheritance distribution of wealth. A similar result (the share of the top 1% increases) occurs in run 8 versus run 9 with comb = equal.

If we turn to the interactions between the heirs and comb parameters, we find that in the case of comb = equal, all three summary measures agree on the ranking heirs = 1,1 > heirs = 4,1 > heirs = 1,4 > heirs = 4,4 (runs 8 to 11). The same ranking holds when comb = uneq for the CV; but the EXP is equal for heirs = 1,4 and heirs = 4,1 (runs 6 and 7); and the Gini reverses their order. However, a more important interaction between the heirs and comb parameters would seem to be revealed by the CV and share of the top 1 percent. When comb = uneq, having fewer heirs (e.g., heirs = 1,1 versus heirs = 4,4) has a much more pronounced effect in the upper tail of the distribution than when comb = equal (runs 1 and 3 versus runs 9 and 10). These results correspond to the intuition that the disequalizing effects of primogeniture (relative to equal division) are highlighted and concentrated in the upper tail of the distribution when comb = uneq, but muted and spread throughout the distribution when comb = equal.

We turn now to focus on the effects of age-diff, the age difference between decedents and inheritors. As figure 5.6 indicates, there are four sets of runs that can be assembled to explore the age-diff parameter for alternative heirs and comb assumptions. The results of these runs are set out in table 5.2 (runs 2, 6, 9, and 12 have already appeared in the previous table as runs 1, 3, 9, and 10, respectively).

As a preliminary observation, note that both run 3 and run 4 have all indicators showing greater inequality than the 1970 values. But the main observation to be drawn from these simulation results is that almost without exception, lower values of age-diff are associated with higher levels of inequality. (The only exception is the share of the top 1% in runs 12 and 13.) In other words, a general shift to more generation skipping would decrease the level of inequality over the next thirty years, in agreement with Lampman's (1962, p. 239) suggestion.

A range of factors must be combined to explain this phenomenon. The first fact to be kept in mind is that almost 80 percent of the (nonnegative) estates in the model are from decedents aged 70–89. The average size of these estates is about $16,000 in 1970–74. Their average level of inequality is relatively low (Gini = .56, CV = 1.8, EXP = .53,

**Table 5.2 Time Series Results, Bequests Part 2**

| Run | Heirs | Age-Diff | Comb | Mean Wealth | Inequality | | | Wealth Shares | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Gini | CV | Exp | Top 1% | Next 4% |
| 1 | 1, 1 | 65 | uneq | 18.2 | .76 | 3.5 | .66 | 13.2 | 22.2 |
| 2 | 1, 1 | 45 | uneq | 18.2 | .81 | 5.0 | .70 | 17.0 | 25.1 |
| 3 | 1, 1 | 25 | uneq | 17.8 | .83 | 6.0 | .71 | 21.4 | 25.1 |
| 4 | 1, 1 | 0 | uneq | 17.5 | .83 | 8.8 | .71 | 24.7 | 21.8 |
| 5 | 4, 4 | 65 | uneq | 18.1 | .68 | 2.7 | .61 | 11.4 | 19.7 |
| 6 | 4, 4 | 45 | uneq | 18.5 | .76 | 3.4 | .66 | 12.6 | 22.6 |
| 7 | 4, 4 | 25 | uneq | 18.0 | .81 | 4.5 | .70 | 15.6 | 24.8 |
| 8 | 1, 1 | 65 | equal | 18.2 | .73 | 3.2 | .64 | 12.4 | 21.1 |
| 9 | 1, 1 | 45 | equal | 18.4 | .75 | 3.3 | .65 | 12.4 | 21.8 |
| 10 | 1, 1 | 25 | equal | 18.3 | .77 | 3.5 | .67 | 12.7 | 22.7 |
| 11 | 1, 1 | 0 | equal | 17.5 | .80 | 4.2 | .69 | 14.5 | 24.8 |
| 12 | 4, 4 | 65 | equal | 18.1 | .66 | 2.5 | .59 | 11.0 | 18.9 |
| 13 | 4, 4 | 45 | equal | 18.3 | .68 | 2.6 | .61 | 10.9 | 19.3 |
| 14 | 4, 4 | 25 | equal | 18.4 | .72 | 2.8 | .64 | 11.2 | 20.4 |
| | 1970 | | | 15.4 | .75 | 5.8 | .65 | 20.3 | 20.9 |

top 1% = 10, next 4% = 18) compared with both overall inequality and levels of inequality within most age groups. When age-diff is high, most of these bequests are concentrated in the younger age groups rather than being spread among more and older age groups (recall figure 5.4). Since these young age groups tend to have below average wealth, the main effect of the inheritances is to raise their average level of wealth. By bringing it closer to the overall mean, the between-age-group component of aggregate inequality is reduced. Had these bequests been spread among older age groups, more wealth would have gone to age groups already owning closer to average or above average wealth. Thus, lower values of age-diff tend to distribute inheritances in such a way that "between group" inequality is reduced less or even increased. This point is illustrated by the figures given in table 5.3.

A second point is that as age-diff decreases (e.g., from 65 to 45), bequests are spread among a wider range of age groups. In the case of unequal combination (of inheritors and inheritances) this means, for example, that an eightieth-percentile family unit in the 20–24 age group may no longer be an inheritor while a ninety-seventh-percentile family unit in the 35–39 age group (who is typically wealthier) may become an inheritor—clearly a disequalizing change.[4] And as table 5.2 shows, with primogeniture (heirs = 1,1) and comb = uneq, the decrease in age-diff has a much more pronounced effect on the upper tail of the distribution, indicated by the CV and share of the top 1 percent, than in the other cases (runs 1 to 4 versus runs 5 to 14).

Equal rather than unequal combination has the anticipated equalizing effect on the aggregate wealth distribution, for all combinations of heirs and age-diff assumptions. Similarly, four-way equal division rather than primogeniture continues to show equalizing effects for all combinations of age-diff and comb assumptions.

However, the relative impact of these two pairs of specific alternatives depends on age-diff. If the "distance" covered by each inequality measure in moving from unequal combination and primogeniture to equal combination and four-way division is examined, more of this distance is covered by the move to four-way division when age-diff = 65. But when age-diff is 45 or 25, more of the distance is covered by moving from unequal to equal combination. This point is illustrated in figure 5.7. If $A$, $B$, $C$, and $D$ are the differences in the values of a particular inequality measure for the given pairs of runs, it is clear that $A + D = B + C$. What the figure shows *is* that $A > B$ and thus $D < C$ when age-diff = 65 for the Gini, CV, EXP, and share of the top 1 percent and 5 percent. But when age-diff = 45 or 25, $B > A$.

Intuitively, the explanation is that the number of heirs has a greater impact on overall inequality than the manner of combination when all these heirs are concentrated in the 20–24 age group. However, as the heirs are spread among a wider range of age groups, the move to comb = equal has a greater effect in reducing overall inequality than the move

**Table 5.3** **Numbers and Sizes of Estates Compared with 1970 Age-Wealth Distribution**

| | Family Size 1 | | Family Size 2+ | | Estates | |
|---|---|---|---|---|---|---|
| Age Group | Number | Mean ($000s) | Number | Mean ($000s) | Number | Mean ($000s) |
| 20–24 | .0741 | .6 | .0775 | 2.2 | .0002 | 2.3 |
| 25–29 | .0333 | 1.2 | .0869 | 6.7 | .0001 | 3.8 |
| 30–34 | .0248 | 8.7 | .0812 | 13.0 | .0001 | 12.6 |
| 35–39 | .0217 | 10.0 | .0738 | 16.9 | .0001 | 14.9 |
| 40–44 | .0210 | 11.7 | .0658 | 21.7 | .0002 | 15.1 |
| 45–49 | .0206 | 10.4 | .0582 | 33.4 | .0004 | 15.1 |
| 50–54 | .0216 | 16.6 | .0502 | 27.0 | .0006 | 22.1 |
| 55–59 | .0218 | 13.2 | .0426 | 37.8 | .0010 | 19.2 |
| 60–64 | .0227 | 15.2 | .0347 | 25.1 | .0016 | 22.7 |
| 65–69 | .0237 | 13.8 | .0266 | 31.7 | .0025 | 17.4 |
| 70–74 | .0248 | 13.3 | .0183 | 26.3 | .0040 | 15.8 |
| 75–79 | .0237 | 14.4 | .0110 | 23.4 | .0056 | 16.3 |
| 80–84 | .0198 | 14.4 | .0051 | 23.4 | .0068 | 16.1 |
| 85–89 | .0131 | 14.4 | .0016 | 23.4 | .0114 | 16.2 |
| All | .3666 | 9.4 | .6334 | 18.9 | .0346 | 16.6 |

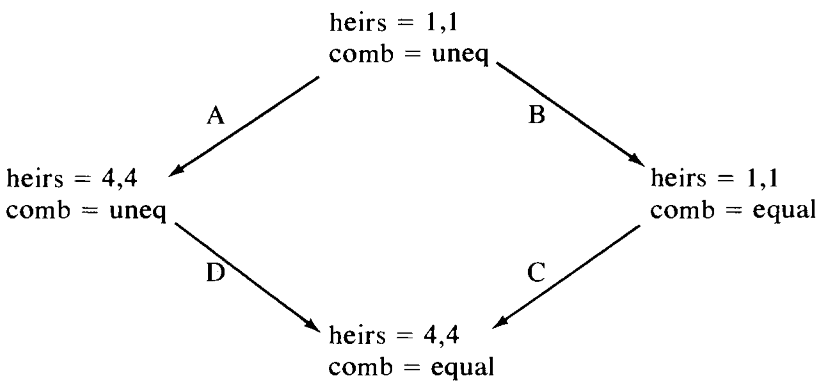


**Fig. 5.7** Movement of Inequality Measures

from heirs = 1,1 to heirs = 4,4. Since the 20–24 age group has average wealth substantially below the overall average, the implication is that (obviously) the main factor in accounting for the level of inequality is the wealth position of inheritors in the overall distribution. An inheritor in the upper tail of the distribution for the 20–24 age group (comb = uneq, age-diff = 65) may well own less wealth than the average family unit in the 60–64 age group (comb = equal, age-diff = 25).

As a final point, the average level of wealth in the year 2000 is influenced by two main factors: the extent to which inheritances go to age/wealth groups with higher accumulation rates (saving rate × yield) and the extent to which inheritances cause inheritors to move up to the next wealth class. For example, if the effect of a lower value of age-diff is to redirect an inheritance from a middle class aged 20–24 family unit well below the wealth cutoff to an aged 30–34 family unit just below this cutoff, average wealth should increase because the inheritance will be subject to a higher accumulation rate with the latter family unit.

As a matter of further interest the pattern of marriage will be considered. Three pairs of simulation runs will be examined, corresponding to three sets of assumptions regarding the bequest process. The three sets of assumptions are: A) there are no capital transfers at all (i.e., 100%

estate tax); B) there are no wealth taxes and all heirs are in the 20–24 age group (age-diff = 65); and C) there are no wealth taxes and the age difference between decedents and inheritors is always twenty-five years (age-diff = 25).

In addition, except for the first pair of runs, it is assumed that all estates have exactly one heir (heirs = 1,1) and that inheritors and inheritances are combined in a "disequalizing" manner (comb = uneq). Otherwise, the simulation runs use identical inputs. The results for the aggregate wealth distributions in the year 2000 are displayed in table 5.4.

**Table 5.4** **Time Series Results, Pattern of Marriage**

| Pattern of Marriage | Bequest Assumption | Mean Wealth | Inequality | | | Wealth Share | |
|---|---|---|---|---|---|---|---|
| | | | Gini | CV | EXP | Top 1% | Next 4% |
| Assort. | A | 14.3 | .80 | 3.9 | .70 | 13.8 | 22.5 |
| random | A | 14.3 | .79 | 3.8 | .68 | 13.4 | 22.3 |
| Assort. | B | 18.2 | .76 | 3.5 | .66 | 13.2 | 22.2 |
| random | B | 18.1 | .74 | 3.2 | .65 | 12.5 | 21.6 |
| Assort. | C | 17.8 | .83 | 6.6 | .71 | 21.4 | 25.1 |
| random | C | 17.8 | .82 | 6.4 | .70 | 20.8 | 25.1 |

The pattern of marriage, unlike the pattern of bequests, appears to be of small quantitative significance, in contrast to Blinder's (1973, p. 624) conclusion. Its quantitative effect is strongest in the second pair of runs. Intuitively, this observation is quite plausible: concentrating all inheritance in the youngest age group gives the greatest chance, of the three runs, for the pattern of marriage to have an effect. The reason, of course, is that the youngest age group has the most marrying yet to do.

## 5.4 Summary and Conclusions

In this paper, the general structure of the time series model developed in Wolfson (1977) has been briefly described and the basic structure of the bequest submodel has been specified. The task of the bequest part of the time series model is to transform the distribution of estates generated by mortality into a set of wealth distributions of survivors where inheritors have been found and their wealth has been augmented by the amounts of their inheritances. This process has been divided into three main steps: transforming estates into bequests, transforming bequests into inheritances, and matching inheritances with inheritors. Correspondingly, three main parameters of the bequest process have been defined: the heirs parameter describes the way in which estates are divided into bequests, using either primogeniture or equal division for either of two wealth classes; the age-diff parameter gives the age difference in years between decedents and their heirs, as an indication of the extent of generation skipping; and the comb parameter determines whether inheritors are to be selected and matched (combined) with their inheritances in an equalizing or disequalizing manner.

The basic conclusions are:

a. Equal division among a large number of heirs reduces inequality. This effect is most pronounced in the upper tail of the distribution when there is unequal combination. When there is equal combination, the strength of this effect is relatively unaffected by the extent of generation skipping (age-diff). When there is unequal combination, the effect is stronger in the upper tail of the distribution (CV and share of the top 1%) but weaker elsewhere (Gini and EXP) when there is less generation skipping (age-diff is lower).

b. Differential division generally results in less inequality than universal primogeniture, and more inequality than equal division. Exceptions can arise because of differential yields and saving behavior by wealth class.

c. More extensive generation skipping reduces inequality. This effect is strongest when there is unequal combination. Given unequal combination, the strongest effect in the upper tail occurs with universal primogeniture. Given equal combination, the strength of this effect is relatively unaffected by the pattern of division.

d. Unequal rather than equal combination results in greater inequality. For both primogeniture and equal division, this effect is much stronger when there is no generation skipping. For various amounts of generation skipping, this effect is not greatly affected by the pattern of division (heirs = 1,1 versus heirs = 4,4).

e. The levels of inequality generated for the year 2000 fall on both sides of the levels observed in 1970.

f. With no capital transfer taxes, real average wealth increases from 1970 to 2000 by about 0.6 percent per year.

g. The pattern of marriage appears relatively unimportant.

## Notes

1. Note that this assumption implies a constraint on the acceptable values of the parameters $\{h_i\}$ and $d$ defined above—so that the number of inheritances never exceeds the number of potential inheritors. These parameter values will always be chosen so that this constraint is not binding.

2. In that case, the "assortatively married" distribution was simply the husbands' (or wives') distribution horizontally scaled by a factor of two (and scaled vertically by one-half and again by one-half).

3. The Gini is most sensitive to inequality near the mode; the CV is relatively more sensitive in the upper tail of the distribution; while the EXP is relatively

more sensitive to inequality at the lower end of the wealth spectrum. An inequality measure always obeys the Pigou-Dalton condition of transfer, while an inequality indicator satisfies only the weaker condition of never violating the Pigou-Dalton condition. These five statistics have been chosen to give the most complete picture of the aspects of the "shape" of a wealth density in which we are most interested. For a more complete discussion, see Wolfson (1977, chap. 3), and Love and Wolfson (1976).

4. More precisely, when heirs = 1,1, age-diff = 65 implies that 22.8 percent of the 20–24 age group inherit. But age-diff = 45 implies that 7.1 percent of the 20–24 age group inherit and 11.9 percent of the 35–39 group inherit. These figures can be computed from table 5.3.

# Comment Martin David

Michael Wolfson should be complimented for a major breakthrough in understanding the process of intergenerational wealth transmission and its impact on inequality of wealth. His approach is commendable for working with distributions, for keeping the number of parameters in his simulation to a minimum, and for exploring the sensitivity of results with a number of extreme cases.

Before embarking on a critique of the specific simulation that Wolfson has developed, it is useful to categorize the kinds of information that can be obtained from simulation and to answer the question, What do we wish to know about the process of transfer of wealth between generations? Five areas of research appear to be relevant:

(1) the concentration and deconcentration of wealth; (2) the share of wealth held by the very rich that represents taxable capacity and the share of wealth held by the poor that represents a resource which should be considered in transfer payment programs; (3) the transmission of wealth through human capital investments; (4) the transmission of entrepreneurial activity through family enterprises; and (5) the maintenance of economic power in a kinship grouping through purposive creation of family dynasties.

Wolfson's paper tells us about the first two areas for a sample of Canadian families. The technique that he develops appears useful for investigation of at least some aspects of the other three categories.

There appears to be particularly little concern over these five areas, in this conference. While intellectual curiosity may be satisfied by a view into the affairs of the wealthy, support for serious study of the wealth distribution requires that we indicate clearly how knowledge of wealth

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can increase the target efficiency of government redistribution programs, and that we relate the consequences of changes in the progression of taxation on the wealthy to the level of investment in human and physical capital.

I would like to organize my remarks into two classes: those that can be handled within the limited framework describing the bequest process that has already been outlined; and those that require the addition of one or more new routines in the simulation process, but which would appear to add greatly to the realism of the results.

## Alterations of the Model

### *Savings Rates*

A major flaw in the work presented is that we have no data on the sensitivity of the findings to the savings rates assumed for the age/wealth/family size groups. Table C5.1 reproduces the savings rates on which Wolfson bases the accumulation that occurs in the simulation. These rates were extracted from the Statistics Canada FAMEX expenditure study for 1969. Several aspects of the table are troublesome. The savings are derived as a residual from income and expenditure reports in the survey and are therefore subject to the response errors that are well known (and carefully discussed by Ferber [1966] and Modigliani and Ando [1960]). The author also had available separate estimates of net change in assets and liabilities, and it would have been desirable to incorporate those estimates into alternative simulations.

**Table C5.1** **Average Propensity to Save Out of Disposable Income (%)**

| Age Category | Family Size = 1 | | | | Family Size ≥ 2 | | | | All FS |
|---|---|---|---|---|---|---|---|---|---|
| | Poor | Middle | Rich | All $W$* | Poor | Middle | Rich | All $W$* | All $W$* |
| < 25 | | | | 0.0 | 1.8 | 8.5 | | 2.7 | 1.9 |
| 25–29 | | | | 0.9 | 5.5 | 9.5 | | 6.8 | 6.2 |
| 30–34 | | | | 6.3 | 4.4 | 6.7 | 17.5 | 6.2 | 6.2 |
| 35–39 | | | | 10.2 | 1.5 | 5.4 | | 4.8 | 5.1 |
| 40–44 | | | | 5.3 | 2.5 | 6.6 | 14.0 | 6.3 | 6.3 |
| 45–49 | | | | 8.0 | 4.2 | 10.2 | 12.2 | 8.7 | 8.7 |
| 50–54 | 5.6 | 13.8 | | 8.6 | 7.1 | 9.9 | 12.6 | 9.5 | 9.4 |
| 55–59 | 7.0 | 12.2 | | 9.4 | 6.2 | 11.5 | 11.2 | 10.1 | 10.0 |
| 60–64 | .1 | − .3 | | − .1 | 8.2 | 9.8 | 9.9 | 9.4 | 8.2 |
| 65–69 | 3.5 | −3.5 | | −1.1 | 4.7 | 1.3 | 8.3 | 4.0 | 3.0 |
| 70–74 | 2.5 | 2.5 | | 2.5 | 1.2 | 1.6 | 10.0 | 4.0 | 3.6 |
| ≥ 75 | −2.3 | 1.1 | | −1.6 | 4.3 | −1.0 | 11.9 | 3.3 | 1.7 |
| All ages | 3.0 | .4 | 16.0 | 3.5 | 4.3 | 7.8 | 12.1 | 7.0 | 6.6 |

*Source*: Wolfson (1977, p. 128).
*$W$ defined by investment income classes.

A second problem with the table is that from age 60 to age 70 there is a peculiar trough of low savings or dissavings that disappears in most groups at age 75. I would guess that this is in part a phenomenon of aggregation. As Shorrocks (1976) has pointed out, the mortality risks tend to be less for persons with higher incomes. Thus cross-sectional age differences in the table show differential selection (within each wealth class) of those with higher earnings and higher savings rates. The difference between savings rates of those aged 65–69 and those aged 70–74 is thus more likely to reflect differences in individuals and their income level than a shift in behavior.

Both of the foregoing problems might be attacked by using an explicit model of the lifetime accumulation pattern to generate a savings function that is smoothed across age groups and wealth groups. The same model could then more explicitly deal with a richer family size classification. A second advantage of the use of a model on group mean average savings data is that it would overcome one of the difficulties that Wolfson faced in deriving savings rates: the expenditure survey did not contain data on net worth. The table actually classifies families by amount of income from investment rather than by net wealth. With an identical matrix of cells defined on the net worth survey, it would be possible to validate a wealth effect from an aggregated model.

One strength of the savings rate should be noted. The savings rate is net of any gifts, so that *inter vivos* transfers to children are properly excluded from the amounts accumulated into the estates of decedents by the simulation.

*Number of Heirs*

A demographer would gasp at the manner in which Wolfson selects the number of heirs. We have a choice of 1, 2, 4, or 6 uniformly across the population. No rationale is offered for the choice of these numbers, except in the case of primogeniture. It seems apparent that some effort should be made to tie the number of heirs to the distribution of eligible persons. Failing that, some effort should be made to relate the number of heirs and their age to some likely expectations in the population. Fortunately, Menchik (1976) offers some evidence on the distribution of heirs by category of relationship to the head (see table C5.2). An average of 7.6 bequests are made in each estate (over $40,000) included in the sample. Roughly one-third are bequests to spouses, children, and grandchildren. Another six percent go to brothers or sisters. Thus data on completed family size are useful for distributing about four-tenths of the total number of bequests. U.S. data from Blau and Duncan (1967) indicate that there would thus be approximately 2.45 children per completed family and therefore eligible to receive bequests from each of the two natural parents.

**Table C5.2 Proportion of Beneficiaries by Relationship to Decedent**

| Beneficiary's Relationship to Decedent | Ratio of Number of Beneficiaries to Number of Estates (× 100) |
|---|---|
| Spouse | 43.1 |
| Child | 126.0 |
| Grandchild | 78.4 |
| Total | 247.5 |
| Brother | 20.0 |
| Sister | 27.7 |
| Niece or nephew | 135.5 |
| Total all beneficiaries | 763.5 |

*Source*: Menchik (1976, p. 144).

Perhaps more interesting, and a useful factor to consider in simulating the inequality of wealth distribution, is that expected completed family size is inversely related to socioeconomic status (see tables C5.3 and C5.4). While Blau and Duncan point out that the differential fertility associated with education has diminished in more recent cohorts, the differentials shown for farm and nonfarm residents are large and could be significant factors in changing the inequality of wealth distribution. Since Wolfson's data include information on occupation, education, and place of residence, it would be easy to vary the number of heirs according to such variables and assess changes in the inequality of the results.

*Primogeniture*

Wolfson offers simulations in which bequests are concentrated on a single heir as one polar extreme. I find that possibility very unlikely, and would like to see some evidence that primogeniture is still a factor in the bequest process. (Menchik (1976) finds little evidence of primogeniture in his Connecticut probate sample.) The principal motivation for

**Table C5.3 Children Ever Born According to Husband's Father's Occupation**

| Husband's Father's Occupation | Children Ever Born per Wife |
|---|---|
| All couples | 2.45 |
| Higher white-collar | 1.98 |
| Lower white-collar | 1.99 |
| Higher manual | 2.39 |
| Lower manual | 2.33 |
| Farm | 2.84 |
| N.A. | 2.45 |

*Source*: Blau and Duncan (1967, p. 366).

**Table C5.4** **Children Ever Born per Wife by Educational Attainment of Wife and Farm Residence and Background of Couple**

| Years of School Completed by Wife | Total | Nonfarm Residence: Nonfarm Background | Nonfarm Residence: Farm Background | Farm Residence |
|---|---|---|---|---|
| Total | 2.45 | 2.21 | 2.58 | 3.34 |
| *Elementary* | | | | |
| 0 to 4 | 3.96 | 2.30 | 4.24 | 5.15 |
| 5 to 7 | 3.07 | 2.39 | 3.39 | 3.85 |
| 8 | 2.71 | 2.43 | 2.77 | 3.53 |
| *High school* | | | | |
| 1 to 3 | 2.47 | 2.38 | 2.46 | 3.26 |
| 4 | 2.11 | 2.09 | 2.02 | 2.70 |
| *College* | | | | |
| 1 to 3 | 2.14 | 1.99 | 2.24 | 2.62 |
| 4 or more | 1.98 | 1.98 | 1.91 | 2.18 |

*Source*: Blau and Duncan (1967, p. 382).

primogeniture is the indivisibility of assets involved in some closely held family enterprise, a farm or a business. Thus it might be of use to separate the share of wealth that is in such enterprises and allocate it to a single heir, while dividing the remaining estate among several beneficiaries. Table C5.5 gives some indication of the importance of a primo-

**Table C5.5** **Mean Net Worth and Equity in Business within 1969 Income Class (all households, Canada, 1969)**

| 1969 Income Group (lower bound of interval in $000s) | Average Net Worth: Excluding Business Equity | Average Net Worth: Including Business Equity | Percent with Equity in Business | Median Equity in Business (holders only) | Ratio: Bus. Equity to Total Net Worth |
|---|---|---|---|---|---|
| $-\infty$ | 4.0 | 7.4 | 13.5 | 19.9 | 45.9 |
| +1 | 6.9 | 8.9 | 7.2 | 12.6 | 22.5 |
| 2 | 8.4 | 11.1 | 15.6 | 15.5 | 24.3 |
| 3 | 10.2 | 12.8 | 17.2 | 13.7 | 20.3 |
| 4 | 10.2 | 13.6 | 16.6 | 11.1 | 25.0 |
| 5 | 9.6 | 11.3 | 11.2 | 10.8 | 15.0 |
| 6 | 10.8 | 12.8 | 13.5 | 8.4 | 15.6 |
| 7 | 12.7 | 14.8 | 11.1 | 10.6 | 14.2 |
| 10 | 18.1 | 20.8 | 13.2 | 13.9 | 13.0 |
| 15 | 33.9 | 39.4 | 21.6 | 16.6 | 14.0 |
| 25 | 94.8 | 205.2 | 53.4 | 60.0 | 53.8 |
| Total | 14.4 | 18.4 | 14.0 | 13.3 | 21.7 |

*Source*: Statistics Canada (1973b, tables 73, 93, 97).

geniture relating only to business assets. For Canada as a whole about one-fifth of net wealth is in business equity (proprietorships, partnerships, or closely held corporations). The percent is particularly high in the top income group and the lowest income group, suggesting that the proposed modification of the rule could lead to substantial differences from the bounding simulations that Wolfson shows in table 5.1.

One last suggested alteration of Wolfson's model is that wealth be divided between the surviving spouse and children at the time of death of the first marriage partner. As far as I can see, this would be possible within the framework that Wolfson has already derived. Table C5.6 delineates the nature of Wolfson's assumption that assets pass exclusively to the surviving spouse. The options marked w can be simulated within Wolfson's assumptions, depending upon the age difference assumed between decedent and beneficiary. The asterisks indicate additional possibilities for splitting the estate, possibilities which may do nothing to lessen the inequality of wealth among family dynasties, but which may go a great length to lessening the degree of inequality in the distribution of wealth among households (see Menchik 1976, chap. 4). These additional possibilities are important in several ways. Table C5.7 addresses the question of how much present value is left to children when the spouse is provided for by a generation skipping trust through which the spouse has a lifetime interest while the children have a remainder interest. The table shows, given the distribution of age at death, that the trust mechanism passes a healthy percent of the decedent's wealth to the children. Considering the infrequency with which the principal of trusts is invaded, the value of the conditional wealth represented by the present value of the remainder interest ought to be counted part of the wealth of the children rather than wealth of the surviving spouse. This would generally produce an equalizing change in the simulation outcomes.

Table C5.8, taken from Menchik's sample of Connecticut probate records, indicates the proportion of the estate going to spouse, children, and grandchildren by wealth class of the estate. (The values passed to

**Table C5.6 Alternative Beneficiaries for the Estate**

| Case | Decedent Ever Married? | Surviving Spouse? | Children Ever Born? | Eligible Heirs: Spouse | Children | Grandchildren | Others |
|---|---|---|---|---|---|---|---|
| A | No | | | | | | w |
| B | Yes | Yes | No | w | | | * |
| C | | | Yes | w | * | *(possibly) | * |
| D | | No | No | | | | w |
| E | | | Yes | | w | w | w |

children and grandchildren are present values of remainder interests, such as those contained in table C5.7 when the bequest is in trust.) Confirmation of this pattern was reported by Jantscher (1967), who shows that trusts involving spouse-children, and children-grandchildren as income and remaindermen account for a large share of the total wealth passing

**Table C5.7** **Present Value of a Remainder Interest in an Estate Left to the Children, with a Life Interest to the Surviving Spouse**

| | Age at Death of Spouse | | | |
|---|---|---|---|---|
| | 25 | 45 | 65 | 75 |
| *Widower* | | | | |
| Life expectancy | 45.6 | 27.4 | 13.0 | 8.1 |
| Present value of the principal interest discounted at | | | | |
| 5% | .108 | .267 | .530 | .674 |
| 7% | .046 | .157 | .415 | .578 |
| *Widow* | | | | |
| Life expectancy | 51.8 | 32.9 | 16.3 | 9.6 |
| Present value of the principal interest discounted at | | | | |
| 5% | .079 | .201 | .452 | .626 |
| 7% | .030 | .108 | .332 | .522 |

**Table C5.8** **Mean Bequest and Share of Bequest to Beneficiaries within Wealth Class (Connecticut probate sample)**

| | Wealth Class | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Mean bequest ($000) | 48.8 | 77.8 | 138.9 | 278.5 | 615.0 | 1,941.6 |
| *Share of bequest given to* | | | | | | |
| Spouse | .23 | .24 | .20 | .23 | .17 | .15 |
| Children | .42 | .34 | .35 | .27 | .31 | .30 |
| Grandchildren | .03 | .04 | .04 | .05 | .07 | .06 |
| Total | .68 | .62 | .59 | .55 | .55 | .51 |
| Brothers | .05 | .06 | .04 | .03 | .01 | .02 |
| Sisters | .07 | .05 | .04 | .05 | .02 | .04 |
| Nieces and nephews | .09 | .11 | .12 | .13 | .12 | .09 |

*Source*: Menchik (1976, pp. 148, 149).

into trusts at death (see tables C5.9 and C5.10). Both Menchik's and Jantscher's studies show that the number of heirs increases with increasing estate size, and that there is an increased tendency to generation skipping (which Menchik demonstrates to be tax induced). While these findings, peculiar to U.S. institutions in transfer taxation, may not be directly applicable to the Canadian tax environment, they suggest that several extensions of the model are highly desirable: variable numbers of heirs should be generated by a distribution of completed family size; estates should be divided among persons from several generations; the number of generations involved in a single transfer should be made conditional on the size of the estate. Each extension appears to be a desir-

**Table C5.9** **Bequests in Spouse-Children and Children-Grandchildren Trusts and Value of Such Bequests, as a Percentage of Total Bequests, All Decedents, 1957 and 1959, by Size of Estate**

| | Estate Size | | |
|---|---|---|---|
| Trust Type | Small | Medium | Large |
| *Spouse-children* | | | |
| All decedents bequeathing property | 5.2 | 10.4 | 9.0 |
| Trust-creating decedents bequeathing property | 34.6 | 26.8 | 16.1 |
| Total value of bequests in trust | 35.0 | 24.7 | 11.2 |
| *Children-grandchildren* | | | |
| All decedents bequeathing property | 2.0 | 6.2 | 13.4 |
| Trust-creating decedents bequeathing property | 13.1 | 16.0 | 24.2 |
| Total value of bequests in trust | 13.4 | 16.1 | 25.8 |

*Source*: Jantscher (1967, p. 68).

**Table C5.10** **Bequests in Spouse-Children and Children-Grandchildren Trusts and Value of Such Bequests, as a Percentage of Total Bequests, Husbands, 1957 and 1959, by Size of Estate**

| | Estate Size | | |
|---|---|---|---|
| Trust Type | Small | Medium | Large |
| *Spouse-children* | | | |
| All husbands bequeathing property | 9.8 | 19.8 | 17.4 |
| Trust-creating husbands bequeathing property | 53.8 | 44.2 | 29.4 |
| Total value of bequests in trust | 52.9 | 41.8 | 22.0 |
| *Children-grandchildren* | | | |
| All husbands bequeathing property | 0.5 | 3.4 | 11.0 |
| Trust-creating husbands bequeathing property | 2.6 | 7.7 | 18.5 |
| Total value of bequests in trust | 1.3 | 6.4 | 16.9 |

*Source*: Jantscher (1967, p. 71).

able and more realistic specification of the bequest process than what can be captured in the bounding simulations involving award of bequests to a single age difference in relation to the decedent and a uniform number of heirs within each of two wealth classes.

Finally, no simulation is complete without an accounting of the transmission of human capital. Inclusion of an algorithm for intergenerational transmission of education would be extremely valuable, as it is the joint distribution of human and nonhuman capital that is of the greatest policy significance.

## Further Comment Michael C. Wolfson

Professor David, in his comment, has indicated a number of useful points and directions for further work. Let me first reply to some of his specific criticisms. He suggests that a major flaw in my paper is the absence of a sensitivity analysis with respect to the saving rates used in the simulations. In fact, in my thesis on which this paper is based (Wolfson 1977), a fairly extensive sensitivity analysis was performed. The results showed, for example, that assuming a uniform 10 percent saving rate for all age/wealth/family size groups made almost no difference to the simulation results.

Considerations of space did not permit any explanation in the paper of the saving process actually used in the model. It is the case, however, that the saving rates displayed in Professor David's table C5.1 comprise only one part of the saving function. The general saving function used in the model is given by the following equation (all items disaggregated by the two family size categories; time subscript omitted for convenience).

$$W_{a+s}(x) = \sum_{i=1}^{k} \int_{V_i}^{V^{i+1}} W_a(z)\, S^E_{ai}[x - (1 + s_{ai}r_{ai})z]dz$$

where $a$ = age group

$i$ = wealth class

$V_i$ = lower limit of $i$th wealth class

$W_a(x)$ = wealth density before saving

$W_{a+s}(x)$ = wealth density after saving

$s_{ai}$ = saving rate out of income

$r_{ai}$ = yield on wealth after tax

$S^E_{ai}(y)$ = probability distribution for saving out of earnings

The saving rates in Professor David's table C5.1 refer only to the $s_{ai}$, though his comments about the definition of saving apply equally to the derivation of the $s_{ai}$ and the $S^{E}_{ai}(y)$.

A second concern raised by Professor David is the definition of saving. The difference between the definition actually used and net change in assets and liabilities less net capital receipts (gifts received less gifts given), an alternative suggested by Professor David, is indicated by the following sum: payments for insurance, annuities, and private and registered pension plans + net accumulation of motor vehicles — one-half of pension and private annuity income. These items were included in our definition of saving first to eliminate the difference in the definition of wealth between the SCF(1973) and the FAMEX (1973) in the case of motor vehicles, and second to capture pension saving and dis-saving.

With regard to the patterns shown by the saving rates in Professor David's table C5.1, it is not clear that Professor David's concerns are entirely appropriate. His point about mortality selection is obviously relevant. However, it is not necessarily a correct interpretation of the table to infer a peculiar trough in savings in the 60 to 70 age range. For all wealth groups combined, there is a fairly clear pattern of declining but always positive saving rates from age 55 on. There is greater variability within the columns associated with specific family size/wealth categories. However, this could be the result of movement of family units from family size group 2 to size group 1 as they age as a result of mortality, or from movement from higher to lower wealth classes, both of which actually occurred (see Wolfson 1977, p. 124).

In my paper, I used quite arbitrary choices for the number of heirs in the simulations, as Professor David has pointed out. The Menchik data he cites are certainly interesting in this regard, though I was unaware of them when the simulations were being run. In any case, the range of simulations actually run gives results that are clear enough, and the range chosen is not unreasonable given the figures presented in Professor David's table C5.2.

Finally, Professor David has indicated a number of directions in which the model could be extended. Of course, in any exercise like that of my thesis, a number of choices must be made regarding the areas where more or less detailed effort should be applied—everything cannot be done at once. It is hoped that some of the extensions to the model that he has suggested can be incorporated sometime in the future.

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