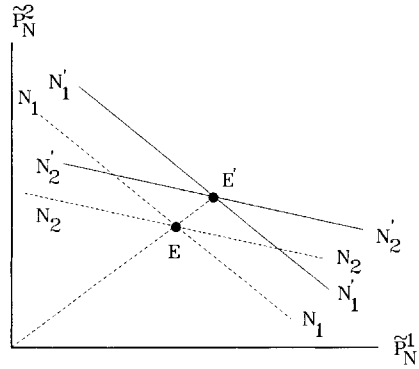

Fig. 7.2

diagram with consumer prices of N on the axes, the effect of a VAT is to shift both N_1N_1 and N_2N_2 out, to $N_1'N_1'$ and $N_2'N_2'$, respectively; the new equilibrium is at E, with the price of the nontraded good increased by a fraction τ in both periods.

To see why this must be true, we first note by inspection of (12) that, if consumer prices of all goods rise exactly in proportion to the VAT, there is no effect on production incentives. So, if all prices rise so as to offset the VAT, there will be no change in the allocation of resources or production.

Second, we argue that under the hypothesized solution there will be no effect on demand. The simplest way to see this is to notice that the welfare function (3) implies a set of compensated demand functions,

$$C_I^t = H_I^t(p, W), \quad I = X, M, N, \quad t = 1, 2, \tag{13}$$

where p is the vector of present-value consumer prices. The functions $H(\cdot)$ are homogeneous of degree zero in p; so, if all consumer prices rise in the same proportion while welfare is unchanged, then demand will be unchanged. But, if nothing changes, nothing changes, including welfare; so, when all prices rise by τ, the market for nontraded goods continues to clear in each period.

An idealized VAT, then, has no allocative effects. In particular, it is neither procompetitive nor anticompetitive; whatever your definition of competitiveness, it has no effect at all.

Many general equilibrium results, such as the equivalence of a VAT without an export rebate to an import tariff, to which we will refer in the next section, depend on the assumption that nominal price levels do not matter. Thus, their practical relevance depends either on price flexibility or on an appropriate exchange rate adjustment. The assertion that a VAT is neutral with regard to competitiveness does not, however, require even this

much defense. Because consumer prices rise precisely in proportion to the tax, the net prices to producers are unchanged. The marginal revenue product of factors of production must also be unchanged. So (to step slightly outside the model), even if factor prices and/or producer prices are sticky and the exchange rate is fixed, a VAT will still have no competitive effect.

Perhaps the surprising point is that this absence of a competitive effect occurs despite the rebate of VAT on exports, which is widely regarded as a kind of export subsidy. In fact, as we show in the next section, in the absence of an export rebate a VAT would distort allocation, definitely reduce export production, and probably shift resources on net away from traded goods sectors.

7.3 The Role of Border Tax Adjustments

The controversy over VATs is largely generated by the impression that the border tax adjustments—the fact that imports are subject to the tax while exports have the tax rebated—constitute a policy favoring a country's traded goods sectors. It is therefore interesting to ask how a VAT would function without these adjustments.

Perhaps the simplest case would be a system with no border adjustments at all—that is, no VAT collected on imports, no rebate on exports. This would in effect shift the tax from a "destination" basis to an "origin" basis. The effects of such a system may be derived immediately by the following observations. First, the prices to consumers of exports and imports will clearly remain unchanged; thus, the price to producers net of taxation must fall in proportion to the VAT rate. Clearly, if the price of the nontraded good also remains the same to consumers, that is, if the price net of taxes to firms falls by the size of the tax, then producers will have no incentive to change their output mix. At the same time, if no relative prices change, then at unchanged utility consumers will also leave their choices unchanged. But, if nothing happens, nothing happens; so the VAT without border tax adjustments is neutral in the same way as a VAT with these adjustments.[2]

The difference in this case is, of course, that the *nominal* marginal product of factors of production in foreign currency falls. Thus, in the case without border tax adjustments, there must be either price flexibility or (more plausibly) a currency depreciation in order for the neutrality of the VAT to hold. This in turn helps explain why in practice VATs do in fact include border adjustments.

It is also true that, given the general preference among authorities for a subtraction method of administration, it would be awkward to exempt imports from the tax. Firms would be given an imputation of taxes paid on imports, as opposed to showing proof of actual payment on domestic inputs; this would raise the odd prospect of firms preferring to use imports because

of the lower administrative costs. Partly for this reason, it seems likely that a country pressured into avoiding any border adjustments would end up without an export rebate but would still tax imports. In this case, the VAT would have a distortionary effect on the allocation of resources. Perhaps surprisingly, this effect is essentially protectionist—a VAT without an export rebate is equivalent to an import tariff.

The difference between a VAT with and without an export rebate may be seen in the export pricing condition. Without the rebate, arbitrage will ensure that the consumer price of exportables equals the world price because the producer pays the tax whether the good is exported or sold domestically. Thus,

$$\tilde{P}_X^t = P_X^*, \quad t = 1, 2. \tag{14}$$

Comparing this with (11), we see that the rebate-less VAT leads to a lower export price. This is not surprising since we have in effect added an export tax to the idealized VAT described before.

The internal price of exports relative to imports is of course lower in this case—or, to reverse the point, the relative price of imports is higher. It is a general proposition, the so-called Lerner symmetry theorem, that an export tax and an import tax are equivalent in their general equilibrium effects. So an ideal VAT without an export rebate is like a protectionist policy.

We should note, however, that the equivalence between import and export taxes is one of those propositions that depends either on nominal prices not mattering or on an appropriate exchange rate adjustment. Note that the effect of a VAT without a rebate is to lower the price to producers of the exported good, when measured in foreign currency; a tariff would of course raise the price of the imported good instead. Thus, these are only equivalent, given either an exchange rate adjustment or sufficient price flexibility.

We see, then, that a VAT without an export adjustment would in effect be a protectionist measure. Will it increase or decrease "competitiveness" as measured by the size of the traded goods sector? The answer is ambiguous, but a presumption may be offered that the size of the traded goods sector as a whole will decrease. To see this, it is helpful to collapse the model into a single period, ignoring the intertemporal aspect (which is in any case unimportant for this question). Equilibrium in the one-period version of the model may be analyzed using a diagram suggested by Dornbusch (1974) and shown in figure 7.3. On the axes are the consumer prices of X and M, relative to the consumer price of N. The curve NN represents a locus of points for which the market for nontraded goods clears: it is downward sloping because a rise in either traded good's relative price will shift demand onto and resources out of the nontraded sector. The ray OT has a slope equal to the consumer price of imports relative to exports, which is determined by world prices and the tax system. Equilibrium occurs where this ray crosses NN, at E.

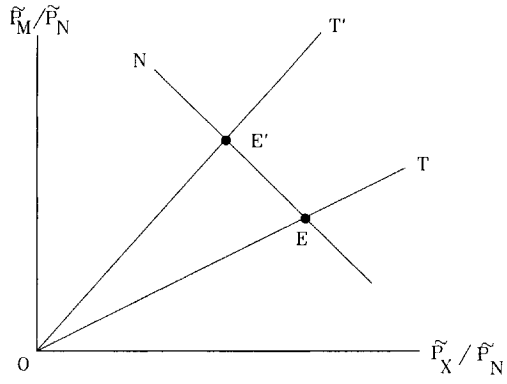

Fig. 7.3

Now suppose that the rebate on exports were to be removed from a VAT. Then the ratio of import to export prices would rise by the fraction τ, corresponding to a counterclockwise rotation of OT to OT'. Equilibrium would shift from E to E'.

Clearly, the resulting rise in P_M/P_N would tend to shift resources out of the nontraded sector, while the fall in P_X/P_N would tend to shift resources into N. The overall effect on the size of N is therefore ambiguous. However, we may offer a presumption that the net effect on N is positive and therefore that the net effect on traded goods sectors as a whole is negative.

The reason for this presumption is the probable relative importance of demand and supply adjustment in the exporting and import-competing sectors. A tariff reduces exports and imports by an equal amount. The reduction comes about through a combined reduction in demand and increase in supply for the importable and on the export side through a combination of increased demand and reduced supply. Initially, however, demand exceeds supply for the importable, while supply exceeds demand for the exportable. Thus, more of the exportable side will tend to come from supply and less from demand than on the import side—that is, we would expect exportable production at world prices to fall more than import-competing production rises. Thus, the size of the tradable sector as a whole will typically fall.

A specific example may make the point. Consider an economy that produces but does not itself consume its export good and that consumes but does not produce its import good—an extreme form of the general proposition that countries must have excess supply for exportables and excess demand for importables. When such an economy imposes a tariff or export tax, the export sector necessarily shrinks, and, since there is no import-competing production, the nontraded sector expands. Thus, in this

extreme case, the effect of a tax on trade, such as a VAT without an export rebate, will unambiguously be to shrink the size of the traded goods sector. Adding some import-competing production and some domestic demand for exportables will remove the certainty of this outcome, but it will still be a presumption.

We see, then, that the widespread belief that the use of export rebates in a value-added tax system is questionable and perhaps an unfair protectionist device is very nearly the opposite of the truth. In fact, the export rebate is necessary if the VAT is not to have a protectionist effect, reducing the volume of trade and probably reducing the size of the tradable sector.

7.4 The Idealized VAT as a Substitute for an Income Tax

The best case for arguing that a VAT enhances competitiveness is not what it does but what it doesn't do: a VAT, unlike an income tax, does not place a tax on saving. Thus, to the extent that a VAT substitutes for an income tax, it will tend to reduce the current propensity to consume. As many economists have pointed out (see, in particular, Frenkel and Razin 1988), to the extent that a value-added tax that substitutes for an income tax reduces current consumption, it will in turn will tend to lead to a trade surplus in the short run. A trade surplus, other things equal, tends to increase the size of the traded goods sector.

In order to demonstrate this point, we introduce an income tax into our basic model.

We already know that an idealized VAT does not distort the economy, relative to a no-tax equilibrium. Thus, in making the comparison of a VAT and an income tax, it is sufficient to consider the effects of an income tax. So we now examine the effects of imposing on our economy an income tax at a proportional rate π. Proceeds of this tax, like those of the VAT considered earlier, are assumed to be redistributed in a nondistorting fashion.

It is important to specify how profit income is calculated for tax purposes. The most natural assumption here is that both earnings on foreign investments and earnings on capital are treated as part of second-period income, with profits calculated as the difference between sales and factor costs plus depreciation on capital—but, since the economy only lasts two periods, the whole capital stock is depreciated. There is a potential issue over whether depreciation should be calculated at historical or replacement cost, but our assumption of constant prices on world markets allows us to ignore the issue here.

Income in the first period, then, is the value of production less taxes, plus whatever transfer the government makes:

$$I^1 = (1 - \pi)[P_X^1 Q_X^1 + P_M^1 Q_M^1 + P_N^1 Q_N^1] + L^1, \tag{15}$$

where L_1 is the rebate from the government.

Income in the second period is

$$(16)\quad I^2 = (1 - \pi)[P_X^2 Q_X^2 + P_M^2 Q_M^2 + P_N^2 Q_N^2] + (1 - \pi)r^*[P_X^1 Q_X^1 - P_X^1 C_X^1 + P_M^1 Q_M^1 - P_M^1 C_M^1 + P_N^1 Q_N^1 - P_N^1 C_N^1] + L^2.$$

Here, the first term represents factor income, that is, gross domestic product. The second term represents capital consumption allowances. The third term represents the income from net foreign investment. Finally, the fourth term represents the rebate from the government.

Now consider an individual's budget constraint. In the first period, the individual accumulates wealth equal to the difference between income and consumption expenditures:

$$(17)\quad W = I^1 - [P_X^1 C_X^1 + P_M^1 C_M^1 + P_N^1 C_N^1].$$

In the second period, the value of consumption equals income plus wealth:

$$(18)\quad P_X^1 C_X^1 + P_M^1 C_M^1 + P_N^1 C_N^1 = W(1 + r^*) + I^2.$$

From inspection of (16)–(18), it is now immediately apparent that the presence of the tax distorts the incentives of a consumer. An individual who takes the government rebates as given faces a rate of return of $r^*(1 - \pi)$ rather than r^* on deferred consumption. For a small income tax, which will have a second-order effect on welfare, the result must be a substitution effect that induces consumers to consume more in the first period and less in the second.

To analyze the trade consequences of this disincentive to save, we turn once again to the diagrammatic analysis of nontraded goods prices. In figure 7.4, the curves N_1N_1 and N_2N_2 represent market clearing for the nontraded goods market in the first and second period, respectively. Imposing an income tax shifts consumption from the second period to the first. Thus, other things being equal, demand for first period N rises, shifting N_1N_1 up to $N_1'N_1'$; other things being equal, demand for second period N falls, shifting N_2N_2 down to $N_2'N_2'$. Thus, the result is to shift the equilibrium from E to E', raising P_N in the first period and lowering it in the second. The initial effect of an income tax is, therefore, to draw resources out of the traded goods sectors and into the nontraded sector, thereby reducing exports and the production of import substitutes.

In passing, it may be worth noting that, in an economy such as this, which, although small in world goods and financial markets, does produce a nontraded good, it is not the case that changes in the saving rate affect only the balance of payments, without affecting domestic real interest rates. It is true that the real rate of interest in terms of traded goods remains fixed at r^* by assumption. A real interest rate defined in terms of a basket of either domestic production or domestic consumption will, however, change whenever P_N^1/P_N^2 changes. In particular, the rise in P_N^1/P_N^2 that results from an

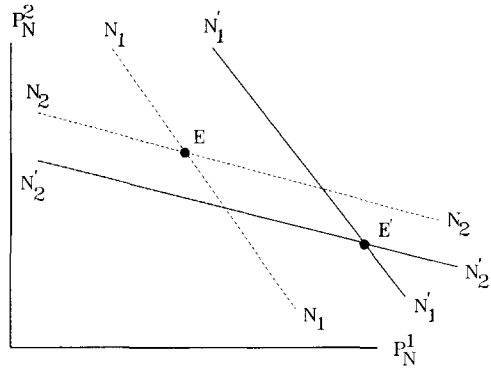

Fig. 7.4

income tax will imply deflation of domestic prices relative to world prices from period 1 to period 2, and will thus be measured as a rise in the real domestic interest rate. In this sense, the income tax produces domestic crowding out as well as a shift toward trade deficit.

We have now seen that an income tax, in contrast to a VAT, does reduce the size of the traded goods sector. It is now straightforward to analyze the effect of introducing a VAT that substitutes for an income tax. The VAT has no competitive effect; the reduction in the income tax expands trade. Thus, the overall effect is to shift resources into tradables.

It is important, however, to note that this is true only in the first period. In the second period, P_N falls, and the traded goods sector is presumably smaller. The point is that the short-term increase in net exports leads to an accumulation of overseas assets that eventually finances an excess of imports over exports.

7.5 Effects of a Selective VAT

We have so far considered only an idealized VAT that succeeds in taxing all consumption at the same rate. In practice, value-added taxation does not fall equally on all activities. In part, this is because of practical difficulties: nonmarketed production, ranging from do-it-yourself repairs to the services of owner-occupied housing and consumer durables, cannot be taxed. Also, social considerations, rightly or wrongly, frequently lead to exemptions for medical care, education, and various other activities that are deemed inappropriate for taxation. As a matter of practice, many other services are frequently exempted from VATs. Among OECD countries with value-added taxes, the VAT typically applies to only about two-thirds of total consumption and often has lower rates for some products than for others.

For the purposes of this paper, the important point is that the de facto and de jure exemptions from a VAT are likely to fall primarily on nontraded rather than traded goods and services. This is necessarily true of nonmarketed production and for one reason or another is also true of most of the marketed areas that are likely to be exempted or subject to reduced taxation.

The effect of a selective VAT is, therefore, to increase nontradable consumption and production at the expense of tradable. Imports and exports are both reduced by the imposition of the typical VAT.

To see this more formally, we return to our basic model. It will simplify matters at no cost if we take advantage of the assumption of an unchanged relative price of exports and imports to aggregate X and M into a composite traded good T. We represent the differential taxation of nontraded and traded goods in extreme form by supposing that, while domestic consumption of T is subject to a value-added tax at a rate τ, consumption of N is nontaxed.

Firms in the economy will maximize the present value of production after taxes,

$$V = (1 + \tau)^{-1}\tilde{P}_T^1(Q_T^1 - K_T) + \tilde{P}_N^1(Q_N^1 - K_N) + (1 + r^*)^{-1}[(1 + \tau)^{-1}\tilde{P}_T^2 Q_T^2 + \tilde{P}_N^2 Q_N^2]. \tag{19}$$

Clearly, the presence of the tax acts as a disincentive to produce traded goods.

To think about the equilibrium that results, it is helpful once again to start by collapsing the model into a single period. In figure 7.5, the curve QQ represents the economy's production possibility frontier between N and T. In a one-period model, trade must be balanced, implying equality of supply and demand for T as well as N; thus, consumption must lie on this production possibility frontier. The optimum consumption is shown as E, where the

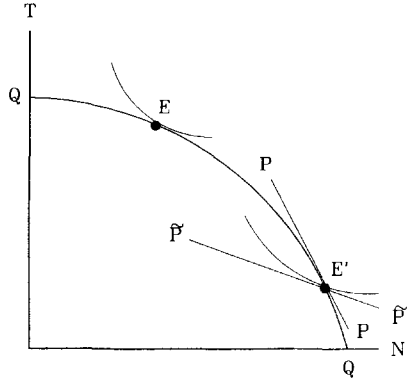

Fig. 7.5

PPF is tangent to the highest possible indifference curve. With a selected VAT on traded goods, however, consumption is distorted; the equilibrium is at a point like E', where $\tilde{P}\tilde{P}$ represents consumer prices and *PP* the marginal rate of transformation in production. As shown, the relative price of tradable faced by consumers is higher than that faced by firms, and the result is a smaller traded goods sector.

In the two-period model, the basic effect is the same. Figure 7.6 shows initial equilibrium loci at N_1N_1 and N_2N_2, respectively. The effect of the VAT, other things being equal, is to raise the demand for the nontraded good in each period. Thus, both schedules shift out. While it is possible that the net effect could be to lower P_N in one period, ordinarily both prices will rise. Meanwhile, the net price of T to producers will remain unchanged since producers must remain indifferent between producing for the domestic and the world market. Thus, the rise in the price of N will induce a shift of resources out of the traded goods sector.

A selective VAT that falls most heavily on traded goods, then, will tend to hurt the traded goods sectors of an economy—the reverse of the common belief. In addition, there is the effect noted in the last section: to the extent that a VAT substitutes for an income tax, while it will in the short run encourage saving and therefore net exports, in the long run the resulting accumulation of net foreign assets will have the opposite effect on net exports.

7.6 Conclusions

There is a widespread belief that value-added taxation, because it is levied on imports and rebated on exports, acts as a combination of protection and export subsidy, giving the traded goods sectors of countries with VATs an advantage over the corresponding sectors of countries that rely on income taxation. In this paper, we have used a simple model to show that this view is

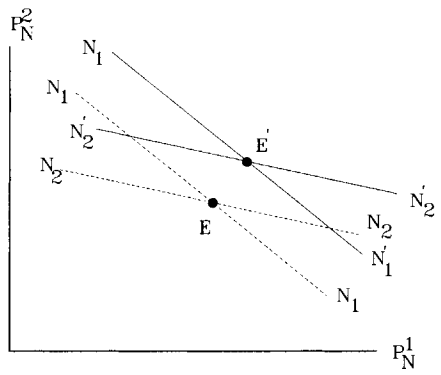

Fig. 7.6

almost completely wrong. A VAT is not a protectionist measure; indeed, the allegedly procompetitive device of export rebates is necessary if the VAT is not to act as an export tax, which in turn is actually a protectionist measure that would reduce both imports and exports. To the extent that a VAT does improve competitiveness, it does so in the short run by offering less bias against saving than an income tax, which, other things being equal, tends to improve the trade balance—but which is far from the common belief about why VATs are helpful in international competition. Moreover, in the longer term, the resulting accumulation of foreign investment would lead to an increase of imports in excess of exports. In practice, moreover, a VAT would almost surely fall more heavily on traded rather than nontraded goods, which would constitute a bias *against* both exports and imports.

Notes

1. An early treatment is Shibata (1967). For a modern and especially neat statement of the point, see Grossman (1980); for a brief statement, see Dixit (1985).

2. Hamilton and Whalley (1986) have pointed out that, given the nonuniformity of tax rates across goods in practice, there is a difference between destination and origin systems. To take an extreme example, imagine a country that places a VAT on importables but not exportables. In a VAT with border tax adjustments, such a system is in effect a consumption tax on the importable, with no tax on domestic producers; without the border adjustments,it becomes a production tax, with no tax on consumers. We abstract from this issue in this paper; Hamilton and Whalley demonstrate that it is relatively unimportant quantitatively.

References

Dixit, A. 1985. Tax policy in open economies. In *Handbook of public economics*, ed. A. Auerbach and M. Feldstein. Amsterdam: North-Holland.

Dornbusch, R. 1974. Tariffs and nontraded goods. *Journal of International Economics* 4:177–85.

Frenkel, J., and A. Razin. 1988. *Fiscal policies and the world economy.* Cambridge, Mass.: MIT Press.

Grossman, G. 1980. Border tax adjustment: Do they distort trade? *Journal of International Economics* 10:117–28.

Hamilton, B., and J. Whalley. 1986. Border tax adjustments in US trade. *Journal of International Economics* 20:377–83.

McLure, C. 1987. *The value-added tax: Key to deficit reduction?* Washington, D.C.: American Enterprise Institute.

Shibata, H. 1967. The theory of economic unions: A Comparative analysis of customs unions, free trade areas, and tax unions. In *Fiscal harmonization in common markets*, ed. C. S. Shoup. New York: Columbia University Press.

Comment

Avinash Dixit

A long line of literature on the value-added tax (VAT) has exposed the fallacy of the common view that, because a VAT is levied on imports and rebated on exports, it constitutes a tariff–cum–export subsidy: an unfair advantage if other nations practice it and something desirable if we do it. McLure (1987, 56) says of the common view, "Although this patently absurd argument is heard less frequently now than in earlier episodes of the continuing debate of the pros and cons of the VAT, it is encountered often enough that it deserves brief discussion." Feldstein and Krugman begin with a lengthy discussion; one would have hoped that a briefer one would have sufficed. Then some of their new contributions could have been discussed in greater depth.

I particularly liked two points that are a very substantial advance over previous work. The first is the treatment of short-run and disequilibrium situations. They examine the consequences of stickiness of nominal prices and exchange rates in a much clearer manner than the catalogs that one finds in the literature. The second is their analysis of a selective VAT. Previous work sets up the benchmark of a uniform VAT and makes some informal remarks about what would happen in the absence of uniformity. Feldstein and Krugman offer a more complete model.

Their focus is on the consumption versus income tax distinction and on production shifts among the export, import, and nontradable sectors. Hence, a two-period, three-sector model. I need hardly say that it is deployed with great skill and elegance; one expects that from these authors. Let me concentrate on what the model leaves out.

First a minor point. In this model, the idealized VAT has no allocative effects at all. This is because labor supply is fixed exogenously. All of net present value of production becomes the rent income of some unspecified fixed factors, and the ideal uniform VAT acts as a tax on pure rent. In a more general model, it would have some distorting effects.

Second, while the nature of capital as a produced input is properly taken into account in the two-period setting, no other produced inputs are recognized. In fact, the treatment of intermediate inputs is a vital aspect of a VAT and deserves more attention. This becomes especially important when tax rates are not uniform across goods and in particular when some sectors are exempt. In the rest of my discussion, I shall extend the Feldstein-Krugman model to handle this issue.

In the usual invoice or credit method of administering VAT, there is a distinction between exemption and zero rating, and the two have different effects. Suppose the production of nontraded goods uses traded goods as

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intermediate inputs. Under zero rating, the producer can claim a refund of the tax paid at the earlier stage. With plain exemption, sometimes called exemption without credit, such a producer is off the VAT register, not liable to pay tax but unable to claim a refund of the tax paid at earlier stages. Both systems are used in practice. In the United Kingdom, for example, food is zero rated, but insurance and finance are exempt. In most systems, exports are zero rated; nonmarketed commodities are by their nature exempt.

The Krugman-Feldstein analysis extends easily to intermediate inputs when nontraded goods are zero rated. But exemption brings new problems. Since the producers of an exempt good pay the tax-inclusive price for their purchases of inputs of taxed goods, a production distortion is introduced. In their figure 7.5, the new production point E' moves into the interior of the feasible set.

If an exempt good is further used in the production of other taxed goods, it breaks the chain of tax credits. Thus, an element of value added is taxed twice, compounding the production distortion. There is also the suspicion that an exempt activity sandwiched between two taxed activities will be at an actual disadvantage (see McLure 1987, 73). This seems to suggest that the tax can lower the outputs of both kinds of goods—traded and nontraded. This is the possibility that I proceed to examine.

The economy produces two kinds of goods, traded (labeled t) and nontraded (labled n). Labor is the only mobile primary factor. Each good is produced using labor, another primary factor that is specific to the sector, and intermediate input of the other good.

The traded good is subject to VAT at rate τ; the nontraded good is exempt (not zero rated). Fix the world price of the traded good at one; then the domestic producer price is one and the domestic consumer price is $(1 + \tau)$. To focus on the production effects, assume a constant domestic marginal rate of substitution in consumption, and normalize it at unity. Then, for the nontraded good, the domestic price (consumer as well as producer) is $(1 + \tau)$. Let w denote the wage rate.

Assume that the cost function for the traded good is

$$Q_t^{1+\mu_t}\,\phi^t(w,\ 1 + \tau),$$

where ϕ^t is the usual increasing, concave, linearly homogeneous cost function, and $\mu_t > 0$ because of the presence of the fixed factor. (This assumes a production function that is Cobb-Douglas in the fixed factor and a labor-nontraded composite. This is a special form, but one that yields results in instructive parametric form.) Similarly, suppose the cost function for the nontraded good is

$$Q_n^{1+\mu_n}\,\phi^n(w,\ 1 + \tau).$$

Note that the tax-inclusive price must be paid for traded good inputs.

In each sector, price equals marginal cost:

$$1 = (1 + \mu_t)\,Q_t^{\mu_t}\,\phi^t(w,\ 1 + \tau), \tag{1}$$

and

$$1 + \tau = (1 + \mu_n)\, Q_n^{\mu_n}\, \phi^n(w, 1 + \tau). \tag{2}$$

Finally, suppose the supply of labor is exogenously fixed at L. Then the labor market equilibrium condition is

$$Q_t^{1+\mu_t}\, \phi_w^t(w, 1 + \tau) + Q_n^{1+\mu_n}\, \phi_w^n(w, 1 + \tau) = L. \tag{3}$$

Equations (1)–(3) determine Q_t, Q_n, and w.

Note that the tax parameter τ affects the equilibrium in three roles. One is by raising the producer price of the nontraded good—the left-hand side of (2). This is the role studied by Krugman and Feldstein. The second is by raising the cost of nontraded inputs for the traded good sector—the right-hand side of (1). The third is the cost of traded inputs for the nontraded good sector—the right-hand side of (2)—which arises because the traded sector is exempt rather than zero rated. It is conceptually useful to separate these roles. Therefore, I shall label the $(1 + \tau)$ occurring in the three places differently when carrying out the comparative statics. Let α, β, γ be the labels for the three roles mentioned above, in that order. Then the equilibrium conditions are

$$1 = (1 + \mu_t)\, Q_t^{\mu_t}\, \phi^t(w, \beta), \tag{1'}$$

$$\alpha = (1 + \mu_n)\, Q_n^{\mu_n}\, \phi^n(w, \gamma), \tag{2'}$$

and

$$Q_t^{1+\mu_t}\, \phi_w^t(w, \beta) + Q_n^{1+\mu_n}\, \phi_w^n(w, \gamma) = L. \tag{3'}$$

Total logarithmic differentiation gives

$$0 = \mu_t\, \hat{Q}_t + \theta_t\, \hat{w} + (1 - \theta_t)\hat{\beta}, \tag{4}$$

$$\hat{\alpha} = \mu_n\, \hat{Q}_n + \theta_n\, \hat{w} + (1 - \theta_n)\hat{\gamma}, \tag{5}$$

and

$$\lambda_t[(1 + \mu_t)\hat{Q}_t - (1 - \theta_t)\sigma_t(\hat{w} - \hat{\beta})] + \lambda_n[(1 + \mu_n)\hat{Q}_n - (1 - \theta_n)\sigma_n(\hat{w} - \hat{\gamma})] = 0. \tag{6}$$

For $i = n, t$, the θ_i are the distributive shares of labor in the labor-intermediate composite, the σ_i are the elasticities of substitution between labor and the intermediate input, and the λ_i are the proportions of labor employed in the sectors.

Substitute for the $\hat{Q}_i$ from (4) and (5) into (6), and simplify. Let

$$\nu_i = (1 + \mu_i)/\mu_i > 1,$$

and

$$\Delta = \lambda_t[\theta_t\nu_t + (1 - \theta_t)\sigma_t] + \lambda_n[\theta_n\nu_n + (1 - \theta_n)\sigma_n].$$

Then

$$\hat{w} = \frac{1}{\Delta}[\lambda_n\nu_n\hat{\alpha} - \lambda_t(1 - \theta_t)(\nu_t - \sigma_t)\hat{\beta} - \lambda_n(1 - \theta_n)(\nu_n - \sigma_n)\hat{\gamma}]. \tag{7}$$

Using this in (4) and (5) gives the solutions for $\hat{Q}_t$ and $\hat{Q}_n$.

The expression (7) clarifies the different roles played by the tax. Most important, the effect of each input-cost-increasing role on the demand for labor is governed by a balance between the diseconomies of scale and the elasticity of substitution parameters. A large ν_i leads to a large reduction in the scale of production and therefore a reduction in labor demand; a large σ_i means a more rapid switch to labor-intensive techniques.

I shall omit further elaboration of these different effects and merely state what happens when we recognize that in fact

$$\hat{\alpha} = \hat{\beta} = \hat{\gamma} = \widehat{1+\tau}.$$

We find

$$\hat{w}/\widehat{1+\tau} = 1 - \lambda_t\nu_t/\Delta.$$

Then (4) gives

$$\mu_t\hat{Q}_t/\widehat{1+\tau} = -\theta_t[1 - \lambda_t\nu_t/\Delta] - (1 - \theta_t) \tag{8}$$

$$= \theta_t\lambda_t\nu_t/\Delta - 1 < 0.$$

Similarly, from (5) we have

$$\mu_n\hat{Q}_n/\widehat{1+\tau} = 1 - \theta_n[1 - \lambda_t\nu_t/\Delta] - (1 - \theta_n) \tag{9}$$

$$= \theta_n\lambda_t\nu_t/\Delta > 0.$$

Thus, the suspicion that the sector producing an exempt good that both uses and is used in the production of a taxed good might actually be harmed by the tax on the other sector is not borne out. In the limiting case where either of the σ_i goes to infinity, however, the right-hand side of (8) goes to negative one and that of (9) to zero; then the gross output of the traded good is reduced without any increase in that of the nontraded good.

Reference

McLure, Charles E., Jr. 1987. *The value added tax: Key to deficit reduction?* Washington, D.C.: American Enterprise Institute.