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Cyclical Wages in a Search-and-Bargaining Model with Large Firms

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When employment rises over the business cycle in the United States, real wages tend to rise somewhat as well. However, the size of these increases appears to be too modest to be consistent with a variety of models. This paper shows it is possible to rationalize this modest elasticity of wages with respect to employment in a model with flexible wages. The model follows Mortensen and Pissarides (1994) in supposing that workers and firms must incur costs to find one another and that wages are set by bilateral bargaining. Rather than considering perfectly competitive firms that each employ one worker, the model considers large, imperfectly competitive firms.

The existence of imperfectly competitive firms allows one to consider changes in market power as a source of business fluctuations, and it turns out that this helps to make real wages less procyclical. The reason is straightforward. Unlike what happens when technology fluctuates, firms that raise their output for nontechnological reasons have a lower (rather than a higher) marginal product of labor. This means that, when bargaining, workers must moderate their wage demands. The result is that wages rise less when employment rises.

While this chapter models changes in market power as the result of changes in the elasticity of demand facing the typical firm, changes in market power can also be equated with situations where firms adapt to changes in their demand by changing their quantities instead of changing their prices. The result, as described here, is that the ratio of price to marginal cost is affected, and this is the broad definition of a change in market power that motivates the analysis. As discussed by Rotemberg and Woodford (1991), seeing output fluctuations as due to fluctuations in market power is closely related to the view that these fluctuations are due to demand rather than being due to supply.

The basic logic—that reductions in market power leads to increased employment, which is associated with a weaker bargaining position for workers, is present in Rotemberg (1998) as well. That model, however, is static so that real wages actually decline when output increases. In the present chapter, as in the Mortensen-Pissarides framework, there are two additional forces that tend to cause wages to rise with employment. The first is that jobs become easier to find in booms, so workers are less desperate for a job than they are in recessions. This strengthens the bargaining position of workers and leads them to obtain higher wages. This force is quantitatively important, and motivates Hall and Milgrom (2005) to study bargaining solutions for wages that are less sensitive to worker's alternative options. In this chapter, by contrast, the Nash bargaining approach used in Mortensen and Pissarides (1994) is utilized, so this force remains important.

The second force is that, in standard specifications of the costs of hiring workers, the increased labor market tightness in booms raises the cost of recruiting workers. When bargaining, workers realize that their employer would have to replace them if the workers were to depart. An increase in recruiting costs, thus, strengthens the bargaining position of workers, and leads to higher real wages. The quantitative importance of this force depends on the size of the economies of scale in the posting of vacancies.

In the Mortensen and Pissarides model each firm hires just one worker and can post at most one vacancy. In Pissarides' (2000, ch. 3) extension of the model to large firms, however, each firm can post multiple vacancies. Pissarides (2000) assumes that each of these vacancies has the same cost, but it is easy to imagine that the technology of posting jobs is subject to economies of scale. For example, an advertisement for many employees might not cost much more than an advertisement for fewer. One of the contributions of this chapter is to demonstrate that these economies of scale have profound implications. The reason is they imply that the marginal recruitment cost in booms, when firms hire many workers, may not be significantly larger than in recessions, when firms hire fewer of them. This obviously reduces the extent to which real wages are procyclical.

The literature on the extent to which real wages are procyclical is voluminous. As shown in the Abraham and Haltiwanger (1995) survey, the results depend on how real wages are measured, as well as on the sample period. Using aggregate data, the specification they report that leads to the most procyclical real wages has an elasticity of the real wage with respect to employment of under 0.3. Using individual data, estimates tend to be higher, and a finding of a unit elasticity is not uncommon. It is arguable, however, that neither the elasticity of aggregate wages nor the elasticity of individual wages with respect to employment corresponds to the wage elasticity implied by the model. The reason is that both individuals and firms differ in their characteristics. While the following model abstracts from these differences, an important property of bargaining models is that differences in firm characteristics tend to translate into wage differences for identical workers (a phenomenon often referred to as *rent sharing*). This means that it should be straightforward to generalize the model, so that different firms pay different wages.

The average change in wages experienced by people who change jobs would then depend on the kinds of industries that expand and contract. A measure of wage change that seems more robustly related to the model that follows is, therefore, the average wage change experienced by people who stay in their jobs. Bils (1985) shows that these wages are much less procyclical than those of job changers. His estimates suggest a 1 percent increase in the unemployment rate reduces the wages of people who stay in their jobs by between 0.4 and 0.6 percent. Bils and McLaughlin (2001) show that people who stay in the same industry see their wage rise by about 0.2 percent, when aggregate employment rises by 1 percent. This suggests that it would be desirable to have models that are consistent with an elasticity of the wage with respect to employment in the range 0.2 to 0.5. Given the uncertainty involved, it would also be attractive if small variations in the parameters could generate both higher and lower elasticities.

Supposing employment fluctuations are due to changes in market power also helps to rationalize the relatively weak tendency of labor productivity to be procyclical. Shimer (2005a) and Mortensen and Nagypál (2007) show that rather substantial changes in technological opportunities are needed, if one is to suppose that such changes account for the bulk of cyclical movements in the U.S. labor markets. The reason is that, while improvements in technology raise labor demand, they raise real wages as well, so firms have only a moderate incentive to hire additional workers. Large increases in technological opportunities are then needed to rationalize even moderate increases in employment. This implies that labor productivity rises substantially as well.

By contrast, reductions in market power that lead firms to hire additional workers do not necessarily lead to large increases in labor productivity. Indeed, one might imagine that the existence of diminishing returns to labor implies that labor productivity would actually have to fall when employment rises. However, as emphasized by Hall (1988), the increasing returns (that tend to be synonymous with market power) can imply that productivity rises when the labor input is increased. In this chapter, the level of increasing returns in production is chosen to match the extent to which labor productivity tends to rise with employment.

As can be seen in Shimer (2005a) and Yashiv (2006), the literature on matching models in macroeconomics is extensive. This chapter is most directly related to Shimer (2005a), whose main conclusion is that observed labor productivity movements are not large enough for matching models to rationalize movements in labor market variables. Another reason Shimer (2005a) provides an ideal point of reference for discussing the strengths and weaknesses of the approach presented here, is that both models predict a strong negative correlation between vacancies and unemployment. In the detrended data he presents, this correlation is –0.95, suggesting that a stable Beveridge curve is a highly desirable feature of a model that purports to explain labor markets.

This chapter shares some common ground with Yashiv (2006), who also lets firms post multiple vacancies and allows shocks other than technology shocks to affect hiring. Yashiv (2006) considers the effects of changes in interest rates (which are modeled as affecting the discount rate) and of changes in separations. He finds, however, that the combination of these shocks generates a labor share (and thus a real wage) that is much more procyclical than in the data. His models implied elasticity of the labor share with respect to employment exceeds three, when this elasticity is actually slightly negative in U.S. data.

In highlighting disturbances to market power in a search and matching framework, this chapter is related to Chéron and Langot (2000), Trigari (2004), Krause and Lubik (2007), and Walsh (2005). These papers consider firms with sticky prices whose ratio of price to marginal cost varies with monetary policy. Chéron and Langot (2000) show that the combination of sticky prices and the search and bargaining framework can simultaneously generate stable Phillips and Beveridge curves. At the same time, their model does generate real wages that are very strongly related to employment, compared to their raw data results.

Trigari (2004) and Walsh (2005) do not focus on the extent to which real wages are procyclical. Rather, they show that replacing competitive labor markets with a search and bargaining framework enhances the ability of sticky price models to explain the response of output, employment, and inflation to monetary disturbances. On the other hand, Krause and Lubik (2007) stress the unrealistic implications of their model concerning both the procyclical movements in real wages and the joint behavior of vacancies and unemployment. One difference between their specification and the one considered here is that they presume that the productivity of each job is independent of how many other employees the firm has hired. This means that increases in employment do not reduce the marginal product of labor of existing jobs, and thus do not exert downward pressure on wages.

Rigid prices may well provide the most empirically plausible reason for cyclical fluctuations in market power. Nonetheless, this paper takes a more direct route, and considers fluctuations in market power that are due to fluctuations in the elasticity of demand facing the typical firm. Such fluctuations are of interest in their own right, and a valuable recent analysis rationalizing them is provided in Ravn, Schmitt-Grobe, and Uribe (2006). It is interesting to learn whether changes in the elasticity of demand, with empirically plausible characteristics, can explain aggregate fluctuations.

The chapter proceeds as follows. The next section lays out the dynamic equations of the model. Section 2.2 considers steady-states. Section 2.3 focuses on how steady-states change as either market power or technology changes. Looking at differences between steady-states both provides clearer intuition (because the steady-state equations are particularly simple), and gives meaningful elasticities (because the economy converges to its steady-state relatively quickly). This latter point is established numerically in section 2.4, which analyzes the dynamic behavior of the model economy near its steady-state, and section 2.5 concludes.

2.1 Model

Worker preferences and the matching of workers to firms are based on a discrete-time version of Mortensen and Pissarides (1994). A constant number of individuals $\overline{H}$ would like to work at the current wage w_t, but only H_t of them actually work. The rest (u_t) are unemployed so that:

$$u_t = \overline{H} - H_t. \tag{1}$$

Those who are unemployed at t have a probability of having a job at t + 1 equal to f_t, so this job finding probability varies over time. Meanwhile, those who have a job at t have a probability s of being unemployed at t + 1, where this separation probability is kept constant on the grounds that Shimer (2005b) and Hall (2006) have argued, that this is a good approximation to employment dynamics. This approximation simplifies the analysis considerably, and it seems worthwhile to know whether an economy that experiences only fluctuations in the finding rate can replicate some of the cyclical features of actual economies. In this approximation, the dynamics of unemployment are given by:

$$u_{t+1} = s(\overline{H} - u_t) + (1 - f_t)u_t. \tag{2}$$

As in Pissarides (2000) and Shimer (2005a), the finding rate f_t is assumed to depend on the ratio of vacancies posted by firms v_t to unemployment u_t. For small fluctuations, this function can be approximated by a power function so that:

$$f_t = \left(\frac{v_t}{u_t}\right)^{\eta}, \tag{3}$$

where η is a positive parameter.

Each consumer at t has overall lifetime utility given by:

$$E_t \sum_{j=0}^{\infty} \beta^j (C^i_{t+j} + \tilde{\lambda}\delta^i_{t+j}), \tag{4}$$

where C^i_τ is the consumption of individual i at τ, $\tilde{\lambda}$ is a parameter, and δ^i_τ is an indicator that equals one if the individual is unemployed at τ and zero otherwise. Letting w_t denote the wage at t in terms of time t goods, and supposing that individuals have access to a financial asset that has a real return r, individual i's asset holdings at the beginning of $t + 1$ are:

$$A^i_{t+1} = (1 + r)[A^i_t - C^i_t + w_t - T_t + \delta^i_t(\hat{\lambda} - w_t)],$$

where T_t represents lump sum taxes, and $\hat{\lambda}$ represents unemployment insurance payments at t. For individuals to not all prefer to consume zero at certain dates, $\beta(1 + r)$ must be equal to one.

The linearity of the utility function (4) implies a constant real rate r, as in Shimer (2005a). Since Andolfatto (1996), several researchers have considered search and bargaining models that let the real rate vary because consumers' utility functions have the constant relative risk aversion (CRRA) form. With differences in people's employment histories, the CRRA becomes more manageable if one presumes that perfect insurance against being unemployed is available, so that ex ante identical individuals all have the same consumption ex post. However, this insurance implies that people prefer being unemployed to working, which is somewhat in opposition with supposing that individuals search for work and threaten their employers with departure in order to increase their wages.[1]

If one postulates CRRA preferences with the property that period utility is separable in consumption and leisure, the level of consumption affects the amount of additional income that makes people indifferent between working and not working. When consumption rises, the marginal utility of income falls and reservation wages rise, which leads to more procyclical wages. It may be possible to weaken this effect by removing the perfect insurance assumption. This effect can also be eliminated by following den Haan, Ramey, and Watson (2000) and supposing that, as in function (4), period utility depends on a linear combination of consumption and leisure. Reservation wages are then constant. Thus, two benefits of function (4) are the constancy of reservation wages and the tractability of the model, even in the absence of perfect insurance.

Given these preferences, let U_t^u denote the value to a worker of being unemployed at the beginning of t, while $(U_t^e + w_t)$ denotes the value of being employed. Letting $\lambda = \tilde{\lambda} + \hat{\lambda}$ be the flow benefit of being unemployed at t, U_t^u and U_t^e satisfy:

$$U_t^u = \lambda + E_t\beta[f_t(U_{t+1}^e + w_{t+1}) + (1 - f_t)U_{t+1}^u], \text{ and}$$

$$U_t^e = E_t\beta[(1 - s)(U_{t+1}^e + w_{t+1}) + sU_{t+1}^u],$$

where the operator E_t takes expectations, conditional on information available at the beginning of period t. Taking the difference between the second and the first of these equations, and letting $\Delta_t \equiv U_t^e - U_t^u$:

$$\Delta_t = E_t\beta(1 - s - f_t)(\Delta_{t+1} + w_{t+1}) - \lambda. \tag{5}$$

The behavior of firms depends on the cost of recruiting. Because this chapter departs somewhat from standard assumptions concerning this cost, it is discussed in some detail before turning to other determinants of firm profitability. In the Pissarides (2000) analysis of large firms, vacancies are supposed to have a constant cost c, and all vacancies are equally likely to be filled. If total vacancies are v_t, and the total number of people hired at t is $u_t f_t$ (as in the previous analysis) the probability that any one vacancy is filled is $u_t f_t / v_t$. For a large firm that can post many vacancies, the expected cost of recruiting a worker is then $cv_t / u_t f_t$.

In the case of large firms, however, it need not be the case that the cost of posting v^i vacancies is linear in v^i. Indeed, whether this cost is interpreted as the cost of advertising openings in an information source, or as the cost of deciding how tasks need to be split up among workers to obtain the outcomes that the firm seeks, it is easy to imagine that this cost is subject to economies of scale.[2] For this reason, it is worth considering a more general recruiting cost, where the cost to an individual firm of posting v^i vacancies is given by $R(v^i)$, where:

$R(v^i) = c(v^i)^{\varepsilon_c}.$

The case of $\varepsilon_c = 1$ then corresponds to the Pissarides (2000) analysis of large firms.[3]

Vacancies that are posted at t allow the firm to increase its employment at $t + 1$ beyond $(1 - s)H_t^i$, where H_t^i is its employment at t and where a fraction s of these employees depart. Following the analysis of Pissarides (2000), a firm that posts v_t^i vacancies can expect to hire $v_t^i u_t f_t / v_t$ additional workers.[4] The analysis is simplified by assuming that firms have access to a technology where the posting of these vacancies ensures that exactly $v_t^i u_t f_t / v_t$ new employees are hired.[5] It follows that:

$$H_{t+1}^i - (1 - s)H_t^i = \frac{v_t^i u_t f_t}{v_t}. \tag{6}$$

Total hiring costs for this firm are thus $c\{v_t[H_{t+1}^i - (1 - s)H_t^i]/u_t f_t\}^{\varepsilon_c}$ so that the marginal cost of hiring an additional worker for $t + 1$ is:

$$\phi_t^i \equiv \frac{dR}{dH_{t+1}^i} = c\varepsilon_c \left(\frac{v_t}{u_t f_t}\right)^{\varepsilon_c} [H_{t+1}^i - (1 - s)H_t^i]^{\varepsilon_c - 1}. \tag{7}$$

At a symmetric equilibrium, each firm's total hiring equals $u_t f_t / N$, where N is the number of firms. Marginal hiring costs at such a symmetric equilibrium thus equal:

$$\phi_t = \frac{c\varepsilon_c N^{1-\varepsilon_c} v_t^{\varepsilon_c}}{u_t f_t}. \tag{8}$$

Since the elasticity of ϕ_t with respect to v_t equals ε_c, it decreases when this parameter decreases. By contrast, the elasticity of ϕ with respect to $(u_t f_t)$ remains –1, regardless of ε_c. A 1 percent increase in uf always lowers by 1 percent the increase in vacancies that is needed to attract an additional worker, so it reduces the cost of these extra vacancies by about 1 percent. A 1 percent increase in v, by contrast, has two effects. While it raises the amount by which vacancies must increase to attract an additional worker, it also raises the number of vacancies the firm must post to attract its standard share of the uf workers available for hire. With $\varepsilon_c < 1$, this increase in the baseline level of vacancies reduces the percent that costs rise when vacancies are increased to hire an additional worker.

The upshot of this discussion is that one cannot generally determine the size of changes in ϕ from changes in v and u, even if one has fitted a matching function to recover empirically the way that f responds to vacancies and unemployment. What remains possible is to use equation

(2) to compute the unemployment rates induced by changes in f_t. Knowing these unemployment rates, one can utilize knowledge of f to obtain the necessary changes in vacancy rates. Increases in f tend to raise uf because the percentage decline in u is not as large as the percentage increase in f. So, f can only increase if v rises as well. The increase in uf exerts a negative influence on ϕ, while the influence of the increase in v is positive. With sufficiently low values of ε_c, however, this latter effect is small, so the overall effect on ϕ is ambiguous. In the extreme case where ε_c is close to zero, an increase in total hiring actually leads to a decline in the cost of obtaining an additional worker.

The timing of moves by firms and workers is the following. At the end of period t, firms are assumed to learn the productivity and market power conditions for $t + 1$. They then choose their price for that period, and the vacancies v_t^i they post at t. To simplify the analysis, the cost of posting these vacancies is paid at $t + 1$, when the recruitment effort of the firm bears fruit. Each worker then bargains individually with the firm. Because this bargaining is efficient and λ is less than the marginal product of labor, all workers stay at the firm with which they are matched. The typical firm finds itself with H_t^i workers. Assuming its capital is fixed at K^i, its output Y_t^i in period t is:

$$Y_t^i = z_t[F(K^i, H_t^i) - \Phi], \tag{9}$$

where the function F is homogeneous of degree one in both arguments, z_t is an economy-wide indicator of productivity, and Φ is a fixed cost. This fixed cost can be set to zero when there is perfect competition among firms, but needs to be positive to ensure that profits are zero if firms have market power.

This market power, in turn, is the result of imperfect substitutability of the goods produced by different firms. Thus, aggregate output y_t and consumption are aggregators of the output and consumption of individual goods. Supposing the average price charged by all other firms at t is p_t, a firm that charges P_t^i finds itself with a demand of:

$$D_t^i = \frac{y_t}{N}\left(\frac{P_t^i}{p_t}\right)^{-\varepsilon_{dt}}, \tag{10}$$

where ε_{dt} is the elasticity of demand, which is allowed to vary over time. The reason the relative price term is multiplied by y/N is that each firm sells an equal share of total output y if they all charge the same price.

Firm i's real flow of profits at t, π_t^i is given by:

$$\pi_t^i = (P_t^i/p_t)\min(Y_t^i, D_t^i) - w_t^i H_t^i - r_t K^i - R(v_{t-1}^i), \tag{11}$$

where w_t^i is the firms real wage. Goods cannot be stored, so the firm finds it in its interest to set $Y_t^i = D_t^i$ along the equilibrium path. Off the equilibrium path, worker departures do lower Y_t^i relative to D_t^i. Moreover, a firm that loses a worker at t and does not change its number of vacancies at the end of t can expect to end up with $(1 - s)$ fewer workers at $t + 1$. It follows that a firm that loses a worker must anticipate it may have to increase its hiring at the end of t, to make up for this loss. The departure of a worker thus leads to an expected flow of losses equal to:

$$(P_t^i/p_t)z_t F_H(K^i, H_t^i) + E_t \beta(1 - s)\phi_t^i - w_t^i, \tag{12}$$

where the first term represents the marginal product of labor (Pissarides 2000), while the second term represents the additional expected recruiting costs. Note that the recruiting costs ϕ_t^i correspond to vacancies posted in period t. These recruitment costs are discounted because they are paid at $t + 1$, and they are multiplied by $(1 - s)$ to take account of the possibility that these recruitment costs would have been incurred anyway with probability s.[6]

As in Pissarides (2000), the wage is set through a generalization of Nash bargaining, and maximizes a weighted geometric average of the gains of the two parties. Workers are assumed to bargain individually and simultaneously. One can think of each worker as bargaining with a separate representative of the firm. Thus, each worker and the representative that he bargains with assume (at the time of bargaining) that the firm will reach a set of agreements with the other workers that leads these to remain employed. They also assume that the price has been set so that all the worker's output is sold, if the worker stays with the firm.

The perceived gain to the representative of the firm of keeping a worker is then given in equation (12), while the gain to the worker from employment is $\Delta_t + w_t$. The bargaining solution thus maximizes:

$$(\Delta_t + w_t^i)^\alpha[(P_t^i/p_t)z_t F_H(K^i, H_t^i) + E_t \beta(1 - s)\phi_t^i - w_t^i]^{1-\alpha},$$

where α represents the bargaining strength of the workers. The solution of this maximization problem is:

$$w_t^i + \Delta_t = \alpha[(P_t^i/p_t)z_t F_H(K^i, H_t^i) + E_t \beta(1 - s)\phi_t^i + \Delta_t]. \tag{13}$$

At a symmetric equilibrium, all firms charge the same price and the marginal product of labor $z_t F_H(K^i, H_t^i)$ is equal to a common value ρ_t, so this equation becomes:

$$w_t = -\Delta_t + \alpha[\rho_t + E_t \beta(1 - s)\phi_t + \Delta_t]. \tag{14}$$

When firm i decides, at the end of $t-1$, on vacancies as well as prices and capital for t, it maximizes:

$$U_t^i = E_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^i.$$

Given its choice of capital and labor, the optimal price is the one that ensures that the firm's output Y_t^i is equal to the firm's demand D_t^i.[7] Using this price, as well as the wage given by equation (13), the firm's flow of profits at t is given by:

$$\pi_t^i = \left(\frac{y_t}{N}\right)^{1/\varepsilon_{dt}} [z_t F(K^i, H_t^i) - z_t \Phi]^{1-1/\varepsilon_{dt}} - r_t K^i$$

$$- R\left\{\frac{v_{t-1}[H_t^i - (1-s)H_{t-1}^i]}{u_{t-1} f_{t-1}}\right\}$$

$$- H_t^i \left\{\alpha\left[\left(\frac{p_t^i}{p_t}\right) z_t F_H + E_t \beta(1-s)\phi_t^i\right] + (\alpha - 1)\Delta_t\right\}.$$

Since the path of Δ involves payoffs of workers outside the firm, the firm treats this path as exogenous. The firm's first-order condition with respect to H_t^i is thus:

$$\frac{dU_t^i}{dH_t^i} = 0 = \left(1 - \frac{1}{\varepsilon_{dt}}\right)\left\{\frac{y_t/N}{z_t[F(K^i, H_t^i) - \Phi]}\right\}^{1/\varepsilon_{dt}} z_t F_H - w_t^i - \phi_{t-1}^i$$

$$+ E_t \beta(1-s)\phi_t^i$$

$$+ \alpha H_t^i \left(\left[\frac{y_t/N}{z_t F(K^i, H_t^i) - z_t \Phi}\right]^{1/\varepsilon_{dt}} \left\{\frac{(z_t F_H)^2}{\varepsilon_{dt}[z_t F(K^i, H_t^i) - z_t \Phi]} - z_t F_{HH}\right\}\right.$$

$$\left. - E_t \beta(1-s)\frac{d\phi_t^i}{dH_t^i}\right).$$

A positive value of $d\phi_t^i/dH_t^i$ discourages hiring, because it implies that an increase in hiring raises wages at the bargaining stage. Using equation (7),

$$\frac{d\phi_t^i}{dH_t^i} = (1-s)(1-\varepsilon_c)\frac{\phi_t^i}{H_{t+1}^i - (1-s)H_t^i},$$

which is indeed positive when $\varepsilon_c < 1$. The reason is that extra hiring at t implies lower expected hiring at $t+1$, which raises marginal recruiting costs if $\varepsilon_c < 1$.

Combining these equations, and noting that $z_t[F(K^i, H_t^i) - \Phi]$ equals y/N at a symmetric equilibrium, the first-order condition (with respect to H_t^i) at such an equilibrium becomes:

$$\rho_t\left(1 - \frac{1 - \alpha\mu_t x_H}{\varepsilon_{dt}} + \frac{\alpha x_K}{e}\right) = w_t + \phi_{t-1} \qquad (15)$$
$$- E_t\beta(1 - s)\phi_t\left[1 - \frac{\alpha(1 - s)(1 - \varepsilon_c)H_t}{u_t f_t}\right],$$

where μ_t is the ratio of zF to output $(zF - \Phi)$, $x_H \equiv HF_H/F$, and $x_K \equiv KF_K/F$ is equal to $(1 - x_H)$ due to the homogeneity of F. This homogeneity also implies that $HF_{HH} = -KF_{HK} = -x_K F_H/e$, where e is the elasticity of substitution between capital and labor.

Because the firm's hiring costs are concave if $\varepsilon_c < 1$, it is important to check that the firm satisfies its second-order condition for an optimum, at least along a path where all firms satisfy (15). Using equation (14) in the above expression for dU_t^i/dH_t^i, this derivative becomes:

$$\frac{dU_t^i}{dH_t^i} = \left\{\frac{y_t/N}{z_t[F(K^i, H_t^i) - \Phi]}\right\}^{1/\varepsilon_{dt}}\left(1 - \alpha - \frac{1 - \alpha\mu_t s_{Ht}}{\varepsilon_{dt}} + \alpha\frac{s_{Kt}}{e}\right)z_t F_H$$
$$+ (1 - \alpha)\Delta_t - \phi_{t-1}^i + E_t\beta(1 - s)\left[(1 - \alpha)\phi_t^i - \alpha H_t^i\frac{d\phi_t^i}{dH_t^i}\right].$$

Assuming a constant elasticity of substitution e between capital and labor, the second derivative of firm welfare, therefore, equals:

$$\frac{d^2U_t^i}{dH_t^{i2}} = -\left[\frac{d\phi_{t-1}^i}{dH_t^i} - E_t\beta(1 - \alpha)(1 - s)\frac{d\phi_t^i}{dH_t^i}\right] \qquad (16)$$
$$- E_t\alpha\beta(1 - s)\left(\frac{d\phi_t^i}{dH_t^i} + H_t^i\frac{d^2\phi_t^i}{dH_t^{i2}}\right)$$
$$- \left\{\frac{y_t/N}{z_t[F(K^i, H_t^i) - \Phi]}\right\}^{1/\varepsilon_{dt}}\left(1 - \alpha - \frac{1 - \alpha\mu_t s_{Ht}}{\varepsilon_{dt}} + \alpha\frac{s_{Kt}}{e}\right)\left(\frac{s_{Kt}}{e} + \frac{\mu_t s_{Ht}}{\varepsilon_{dt}}\right)\frac{z_t F_H}{H_t^i}$$
$$+ \left\{\frac{y_t/N}{z_t[F(K^i, H_t^i) - \Phi]}\right\}^{1/\varepsilon_{dt}}\alpha s_{Ht}\left[\frac{\mu_t}{\varepsilon_{dt}}\left(1 - \mu_t s_{Ht} - \frac{s_{Kt}}{e}\right) + \frac{s_{Kt}}{e}\left(\frac{1}{e} - 1\right)\right]\frac{z_t F_H}{H_t^i}.$$

For an e not too far from one, the last two terms in this equation are negative, as is required for the second-order condition to hold.[8] The concavity of R makes the first term in brackets positive, which could potentially lead to violations of this condition. However, equation (7) also implies that $d^2\phi_t^i/dH_t^{i2}$ is positive when $\varepsilon_c < 1$, so that the first term in parentheses is positive as well; and this contributes to satisfying the second-order condition. Because this term is quantitatively important in the calculations reported in the following, a brief discussion seems worthwhile.

When $\varepsilon_c < 1$, firms are somewhat discouraged from hiring workers at t, because doing so raises future hiring costs and thereby raises wages. The last terms in equation (16) show this effect becomes even more important as firms increase their hiring. The reason is that recruiting costs rise faster with recruitment for low levels of hiring. As the firm increases its hiring and needs ever less future hiring, it then makes the derivative of recruiting costs with respect to hiring larger. This discourages hiring, and leads the optimum to correspond to the point that satisfies the first-order condition. It is worth comparing the first-order condition (15) to the corresponding condition in the more standard model (Pissarides 2000), where firms have one worker at most, and where ϕ_t is simply the expected cost of recruiting a worker at t for $t + 1$. Free entry then implies that the expected profit from spending these resources on recruiting equals zero, or that:

$$\phi_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j (1 - s)^j (\rho_{t+j} - w_{t+j}).$$

This implies:

$$\rho_t - w_t - \phi_{t-1} + E_t\beta(1 - s)\phi_t = 0. \tag{17}$$

Comparing conditions (15) and (17), it is apparent that the coefficients of w_t and ϕ_{t-1} are the same. However, the coefficients of the marginal product of labor and of $E_t\beta\phi_t$ are different. The differences in these coefficients give insights into the changes introduced by my large-firm assumptions. First, supposing the firm's demand curve is less than infinitely elastic lowers the attractiveness of hiring workers. This represents the standard monopolistic distortion. This tendency to hire fewer than the efficient number of employees is tempered somewhat by Nash bargaining, because workers absorb in lower wages a fraction of the reduction in price that is induced by expanding output.

Interestingly, two differences between conditions (15) and (17) remain, even if one assumes that firms are perfectly competitive. The first is that, with $x_K > 0$ and $e < \infty$, a firm lowers its marginal product of labor by hiring additional workers (where this reduction in the marginal product of labor is larger when the elasticity of substitution e is smaller and when the share of capital is larger). This provides an inducement to overhire.[9]

The second difference is that, with $\varepsilon_c < 1$, marginal recruiting costs fall when the firm recruits more workers. A firm that increases its employment at t tends to increase its wage as a result of needing to recruit less heavily at $t + 1$. The firm, therefore, faces a reduced incentive to hire workers at t. It should be apparent that the sum total of these effects on the path of employment (for given levels of ϕ_{t+j}) depends on the parameters that one chooses. It is important to stress, however, that these changes (relative to the standard model) need not by themselves have any important effect on the extent to which real wages are procyclical. This issue is discussed further in the following.

If the marginal product of labor ρ_t is exogenous, a symmetric equilibrium is a path for u_t, v_t, f_t, ϕ_t, Δ_t, and w_t that satisfies conditions (2), (3), (5), (8), (14), and (15). If $\rho_t = z_t F_H(K, H_t)$, one must include this equation, as well as equation (1), among the equilibrium conditions, and must solve for the path of ρ_t, H_t, u_t, v_t, f_t, ϕ_t, Δ_t, and w_t.

This model is analyzed in several steps. First, in the next section, its overall steady steady-state is computed.

2.2 *Steady-State*

The steady-state implication of condition (5) is:

$$\Delta[1 - \beta(1 - s - f)] = \beta(1 - s - f)w - \lambda,$$

where the unsubscripted values of Δ, w, and f represent their steady-state values. The steady-state implication of condition (14) is:

$$\Delta(1 - \alpha) = -w + \alpha(\rho + \beta(1 - s)\phi).$$

Together, these equations imply:

$$[1 - \alpha\beta(1 - s - f)]w = (1 - \alpha)\lambda + \alpha[1 - \beta(1 - s - f)][\rho + \beta(1 - s)\phi]. \tag{18}$$

This equation can be interpreted as giving the *bargaining wage*. This wage is a linear combination of the value of leisure, the marginal product of labor, and the cost of replacing the worker by recruiting a new one.

Since condition (2) implies that $uf = (\overline{H} - u)s$ in a steady-state, it follows from the definition of unemployment in equation (1) that total hiring uf in a steady-state equals total separations sH. Using this in condition (15) gives the steady-state relationship:

$$\left(1 - \frac{1 - \alpha\mu x_H}{\varepsilon_d} + \frac{\alpha x_K}{e}\right)\rho = w + m_\phi \phi, \tag{19}$$

where $m_\phi \equiv 1 - \beta(1-s)[1-\alpha(1-s)(1-\varepsilon_c)/s]$. This equation can be interpreted as a hiring equation, where the firm equates the benefit of hiring an additional worker (which is related, though not necessarily identical, to the marginal product of labor) to its marginal cost (which includes a wage and a hiring cost component).

For a given replacement rate λ/w, the bargaining and hiring equations (18) and (19) are linear in w/ρ and ϕ/ρ. Thus, one can readily solve for these two ratios as a function of the parameters $m, s, f, \alpha, e, x_H, \mu, \varepsilon_c$, ε_d, and λ/w. Once one has the ratios w/ρ and ϕ/ρ, one can use equation (16) to check whether the second-order condition holds for the representative firm at this steady-state. It does so for all the parameters considered in the following.

The calibrated parameters used in the analysis are given in table 2.1.

Table 2.1
Calibrated Parameters

	Baseline (BB)	Shimer (2005a) Alternative (AA)
β: Discount rate	0.996	0.996
s: Steady-state separation rate	0.034	0.034
f: Steady-state finding rate	0.45	0.45
η: Elasticity of finding rate with respect to v/u	0.28	0.28
α: Worker's weight in bargaining	0.72	0.72
λ/w: Value of leisure relative to work	0.9	0.4
x_H: Importance of labor in production $\left(= \frac{HF_H}{F}\right)$	0.66	1.00
e: Elasticity of substitution of H for $K \left(= \frac{FF_{HK}}{F_H F_K}\right)$	1.00	0.33
ε_d: Elasticity of demand	2.00	10,000
μ: Index of returns to scale in production $\left(= \frac{F}{F - \Phi}\right)$	1.545	1.00
ε_c: Elasticity of recruiting costs	0.2	1.00

Under the assumption that a period lasts one month, the first five of these (β, s, f, η and α) are taken from Shimer (2005a) and do not vary across specifications. In the specification that is labeled alternative, the other calibrated parameters are also close to the values in Shimer (2005a).[10]

In the baseline specification, by contrast, some of the parameters are different. The replacement rate λ/w has been the subject of some discussion. Shimer (2005a) sets λ/ρ (which is very similar to λ/w for his parameters) equal to 0.4 on the basis that, on average, unemployment insurance in the United States typically pays workers somewhat less than four-tenths of their regular wage. Hagedorn and Manovskii (2005) have emphasized that λ/w should be higher than the fraction of wages covered by unemployment insurance, because people also give up their utility from leisure when they work, and this utility flow should be included in λ.

For any value of λ/w below one, workers prefer working to not working, so they are involuntarily unemployed whenever they do not have a job. Setting this ratio very close to one, on the other hand, would be inconsistent with the observation that reported well-being falls substantially when workers become unemployed (Di Tella, MacCulloch, and Oswald 2003). This leads Mortensen and Nagypál (2007) to criticize Hagedorn and Manovskii (2005) for using parameters such that $\lambda/w = 0.983$, which implies that workers gain only 1.7 percent of flow utility by going from unemployment to employment. Keeping in mind this criticism, while also taking into account the fact that low values of λ/w tend to make real wages too procyclical, my baseline simulations are computed under the assumption that $\lambda/w = 0.9$.

To allow for diminishing returns to labor, the baseline parameters involves $x_H = HF_H/F < 1$. Because of its use in other studies, and because insight is gained by keeping this parameter constant across several different specifications, the baseline value of this parameter is set to two-thirds even though the current model is not one where this parameter necessarily equals the share of income paid to labor. The equilibrium labor share equals $x_H\mu w/\rho$ so that it is also affected by μ and w/ρ. This means that the specifications considered in the following do not all have the same steady-state labor share.

For the most part, the elasticity of substitution between capital and labor e is set to one. However, the relevant elasticity for wage setting is the short run substitutability (after capital has been installed), and this is arguably much smaller. For this reason, the case where $e = 1/3$ is also considered.

An equally important production function parameter that needs to be calibrated is μ, the steady-state value of μ_t. In a symmetric equilibrium with fixed capital, equation (9) implies:

$$Y_t = z_t[F(K, H_t) - \Phi]. \tag{20}$$

In a steady-state with constant z, the parameter $\mu = zF/Y$ is related to the ratio of fixed costs over output Φ/Y by the relationship $\mu - 1 = \Phi/Y$. If z and K are constant, and the log deviations of H_t around the steady-state are relatively small, the percentage deviations of output and employment from their steady-state values satisfy:

$$\tilde{Y}_t = \mu x_H \tilde{H}_t, \tag{21}$$

where a tilde represents a log deviation from a steady state.

This equation implies that, if z fails to vary cyclically, x_H is known, the cyclical value of employment is correctly measured, and the cyclical value of Y is subject to measurement error, one can estimate μ from a regression of $\tilde{Y}_t$ on $\tilde{H}_t$. Using Bureau of Economic Analysis (BEA) data of output, hours, and employment from the business sector, regressions were run by detrending the three variables, using the method outlined in Rotemberg (2003). Using quarterly data from 1950:1 to 2002:1, the coefficient of employment in the regression of output on employment was 1.11. If $x_H = 2/3$, this coefficient is consistent with equation (21) when $\mu = 1.7$. This parameter value has the advantage of allowing the model without technology shocks to account for these cyclical productivity movements.[11] The baseline value for μ is somewhat smaller ($\mu = 1.545$) to ensure that the labor share is below one, in the main specifications in the following section.[12]

Given these increasing returns, one can expect firms to have market power. This market power is important for the analysis because it allows markups to fluctuate. For illustration, this market power is captured by letting the steady-state elasticity of demand ε_d equal two. While it could be feared that this would lead to implausibly large markups of price over marginal cost. It is shown in the following that this is not the case.

When there are short-run technology shocks, the coefficient in the regression of output on employment need not recover μ because technology shocks affect both output and employment. Moreover, imperfect competition is not needed in this case for output to fluctuate. For this reason, several results concerning the effect of technology disturbances are also presented for the case where where firms have constant returns to scale in production, and face perfectly competitive output markets.

The last parameter that needs to be calibrated is ε_c. As discussed previously, it is standard in search models to suppose that $\varepsilon_c = 1$. Kramarz and Michaud (2003) provide some evidence on this parameter. They use French firm-level data on hiring and on certain hiring expenditures, namely expenditures on job advertising and search-firm fees. They run regressions of the change in these expenditures on the change in hiring between 1992 and 1996, and include a quadratic term in their regressions.

This term allows them to reject the hypothesis that hiring costs are linear in hiring. However, their estimated degree of returns to scale is small. Starting at the mean of their sample, a firm whose hiring was 1 percent larger experienced about a 0.97 percent increase in its hiring costs. The true value of ε_c could be lower, however, if the degree of economies of scale were larger in the component of hiring costs that involves the firm's own employees or output. Also, the cross-sectional variability of changes in hiring costs across firms might be driven by cross-sectional differences in the extent to which firms open new plants. New plants may have a different effect on hiring costs than do changes in the number of employees who are associated with a fixed capital stock, and it is the latter who are most relevant for the model. For purposes of illustration, results are reported for $\varepsilon_c = 0.2$, as well as for the conventional case where $\varepsilon_c = 1$. The results are monotone in the values of this parameter, so these examples ought to be informative about the general effect of this parameter.

Table 2.2 reports the steady-state values of w/ρ and ϕ/ρ for several combinations of parameter values. These ratios of wages relative to the marginal product of labor, and marginal recruiting costs relative to the marginal product of labor can be used to compute the ratio of price to marginal cost. If the marginal costs of labor were simply taken to be the wage, w/ρ would be the inverse of the markup of price over marginal cost. Given the existence of recruiting costs, and their influence on wages, the inverse of the markup equals $(w + m_\phi \phi)/\rho$, and this is reported in the last column of this table.

As can be seen in this table, there are substantial differences in wages and marginal recruiting costs relative to the marginal product of labor across these specifications. To gain intuition for these differences, it is worth starting with specification (6), which is close to the one used in Shimer (2005a). In this specification, price is equal to marginal cost, as in models where firms take prices as given in both goods and labor markets. Changing from this specification to (7), where capital is used in

Table 2.2
Steady-State Values

Spec	Parameters	Wage over marginal product of labor w/ρ	Marginal recruitment cost over marginal product ϕ/ρ	Marginal cost over price $(w + m_{\phi}\phi)/\rho$
(1)	Baseline (BB)	0.94	0.01	1.11
(2)	BB with $\varepsilon_c = 1$	1.10	0.19	1.11
(3)	BB with $\lambda/w = 0.4$	0.69	0.03	1.11
(4)	BB with $e = 1/3$	0.96	0.04	1.59
(5)	BB with $\varepsilon_c = 1$ and $\lambda/w = 0.4$	1.08	0.63	1.11
(6)	Shimer (2005a) alternative (AA)	0.98	0.47	1.00
(7)	AA with $x_H = 2/3$ and $e = 1$	1.21	0.82	1.24
(8)	AA with $\lambda/w = 0.9$	0.997	0.08	1.00
(9)	AA with $\lambda/w = 0.9$ and $\varepsilon_c = 0.2$	0.93	0.004	1.00

Note: ε_d is the elasticity of demand facing the typical firm, z is an indicator of technology, ε_c is the elasticity of vacancy posting costs with respect to vacancies, λ/w is the steady-state ratio of the workers opportunity cost of working to the wage, e is the elasticity of substitution of capital for labor, and x_H is a measure of the importance of the labor input.

production with $x_K = 1/3$ and $e = 1$, so that the production function takes the Cobb-Douglas form with plausible capital costs leads to a substantial increase in both w/ρ and ϕ/ρ. The reason is that the dependence of the marginal product of labor on the amount of labor hired now leads to overhiring. This has the effect of lowering the marginal product of labor both relative to the real wage and relative to recruiting costs. More formally, an increase in x_K/e raises the left-hand side of equation (19) so that ρ must fall relative to a linear combination of w and ϕ. Since condition (18) requires the wage to be an unchanged linear combination of ρ and ϕ, the wage must rise relative to ρ while falling relative to ϕ (so that ϕ/ρ rises more than w/ρ). The overall effect of this is to lead price to be substantially lower than marginal cost. Given that the labor share is less than one, this does not need to be inconsistent with zero profits (as long as the required payments to capital are sufficiently low).

Going from specification (7) to specification (5) involves increasing market power, which lowers the left-hand side of equation (19) and reduces overhiring. While this does indeed lower w/ρ and ϕ/ρ, and the inverse of the markup of price over marginal cost, the size of this effect is relatively modest. Raising λ/w from 0.4 to 0.9 (when going from specification [5] to [2], or from specification [3] to [1]) raises w/ρ for the simple reason that workers have access to a superior alternative. This increase in wages, relative to the marginal product of labor implied by condition (18), requires that ϕ/ρ fall to satisfy equation (19). In other words, the increase in the wage reduces the attractiveness of obtaining a worker, so the marginal recruiting cost must fall relative to the marginal product of labor.

Lowering ε_c when going from specification (5) to specification (3) (or from specification [2] to [1]) reduces both w/ρ and ϕ/ρ. The reason is that the lower value of ε_c makes marginal hiring less attractive by lowering the right-hand side of equation (19). This tends to raise the marginal product of labor relative to the wage. The result is that the baseline specification has almost the same w/ρ as the alternative specification (6). The higher value of ε_c, and the lower value of λ/w in the latter, tend to raise w/ρ, but this is offset by the effect of the higher value of x_K/e in the former. The main difference between the baseline and the alternative is that ϕ/ρ is much larger in the latter. The reason is that increases in ε_c have a particularly large positive effect on recruiting costs.

Specifications (8) and (9) are designed to analyze the effects of setting $\varepsilon_c = 0.2$ in a standard Mortensen-Pissarides model in which output is a linear function of the labor input, and employment fluctuations are due exclusively to fluctuations in technology. These specifications, therefore, feature firms that sell their output in a perfectly competitive market. Specification (8) departs from specification (6) only in raising λ/w, so that it is a variant of the analysis in Hagedorn and Manovskii (2005). Specification (9) then lowers ε_c, and sheds light on the effects of this parameter.

The purpose of specification (4), instead, is to analyze the effects of lowering the elasticity of substitution of labor for capital relative to the baseline specification. This change also raises the left-hand side of equations (19), so it leads to more hiring, higher w/ρ, and higher ϕ/ρ. The result is that this specification has considerably lower markups of price over marginal cost, and its implied labor share is only slightly below one.[13]

Now turn to the analysis of employment fluctuations. These are modeled as either being due to changes in technological opportunities (as in most analyses based on Mortensen and Pissarides 1994), or as being due to changes in the elasticity of demand ε_d. Changes in this demand elasticity provide a particularly simple modeling device for non technological changes in labor demand, but the analysis should also be relevant for other settings where labor demand changes without changes in z. It should, in particular, be adaptable to models of sticky prices where the demand for labor changes when firms with rigid prices face a change in the demand for their products. Following the lead of Mortensen and Pissarides (1994), it is assumed that λ stays constant in the face of the fluctuations in z and ε_d.

Two different approaches are considered for calculating the model's implications regarding the effects of changes in z and ε_d on employment and wages. Both these methods rely on approximations near a steady-state, so they both apply only when fluctuations in the driving variables are relatively small. In one method, it is supposed that the variables in the model always obey the steady-state relationships (18) and (19), and the approximations of these relations around a particular point are considered. Since it is important that variables not depart too much from this point, it is convenient to suppose that this approximation is taken around the point that describes the equilibrium when the exogenous variables take on their mean values. Because of the supposition that these steady-state relations always hold, this is labeled the *stochastic steady-state* method for computing the behavior of the model.

An obvious alternative is to rely on the dynamic equilibrium relations, and approximate these around a steady-state. This is the second method used here. It might seem that this second method is superior since the dynamic equations do not imply that the economy always obeys steady-state equations (18) and (19). However, the first method has some benefits, and is discussed first.

3.3 Stochastic Steady-States

Using U.S. data on unemployment duration, Shimer (2005a) and Hall (2006) infer f_t from the likelihood that people who have been unemployed for less than one month (in a particular survey month) remain unemployed in subsequent surveys. Shimer's (2005a) resulting estimate of f_t averages 0.45, so that nearly half of the unemployed find jobs within a month. Since the coefficient of lagged unemployment in equation (2) equals $(1 - s - f_t)$, such a high finding rate implies that unemployment converges quickly toward the steady-state implied by f_t. This stochastic steady-state is given by:

$$u_t = \frac{s}{f_t + s}\overline{H}, \tag{22}$$

so the implied unemployment rate equals $s/(f_t + s)$. Figure 2.1 shows the actual U.S. unemployment rate and this implied unemployment rate.

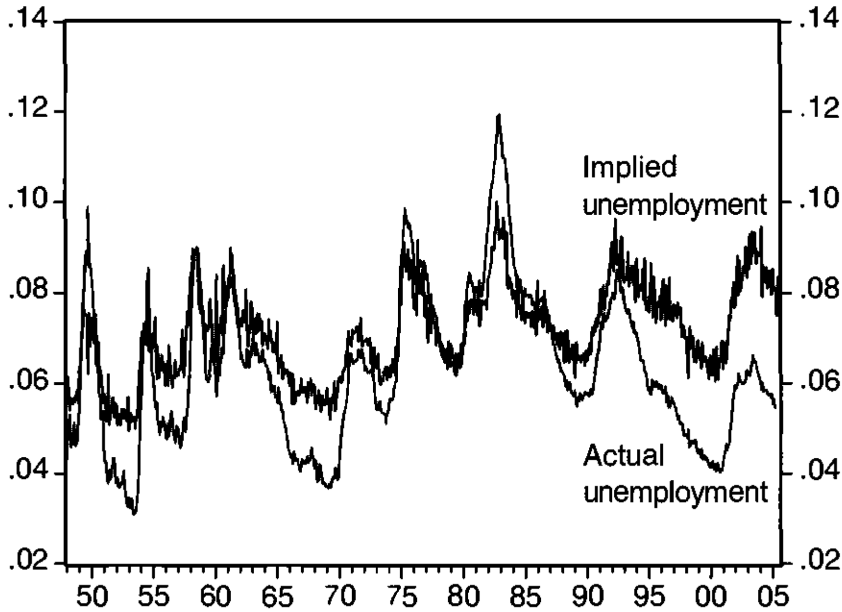

Figure 2.1
Actual and Implied Unemployment Rates in the U.S.

The implied rate is computed using Shimer's (2005a) method for obtaining f_t and setting s equal to the average separation rate in this sample, where this separation rate is also computed using his method. The actual and implied unemployment rates have similar cyclical movements, though the implied rate is somewhat less variable than the actual rate.[14] While the fit is far from perfect, figure 2.1 suggests it would be worthwhile to know whether a model that can generate these implied movements in the unemployment rate is also consistent with weak procyclical movements in the real wage.

Because this is a model where convergence to the stochastic steady-state appears to be rapid, the stochastic steady-state may be a good approximation to the dynamic equilibrium of the model. Indeed, the following results indicate that calculations based on these stochastic steady-states closely approximate the simulations of the nonlinear version of the model reported in Shimer (2005a). In addition, the next section shows that the elasticity of wages (with respect to employment) is similar in this stochastic steady-state to the corresponding elasticity in a dynamic equilibrium model. Aside from these similarities, the main virtue of analyzing stochastic steady-states is their simplicity. All that is required to understand the behavior of the variables in the model are the two equations (18) and (19), so it is easy to gain intuition for the results. In particular, it becomes easy to understand what features of U.S. data lead a low ε^c to be necessary for the procyclical movements in real wages to be mild.

Using a tilde to denote logarithmic deviations around a mean outcome and unsubscripted variables to denote the mean outcome, equation (22) implies:

$$\tilde{u}_t = \frac{-f}{f+s}\tilde{f}_t. \tag{23}$$

Combined with equation (1), equation (22) also provides a simple connection between employment and $\tilde{f}$. This is represented by:

$$\tilde{H}_t = \frac{s}{f+s}\tilde{f}_t = \tilde{u}_t + \tilde{f}_t, \tag{24}$$

where the second equality follows from equation (23).

The linearization of equation (3) is:

$$\tilde{f}_t = \eta(\tilde{v}_t - \tilde{u}_t). \tag{25}$$

Combined with equation (23), this implies that vacancies must satisfy:

$$\tilde{v}_t = \left(1 - \frac{1 + s/f}{\eta}\right)\tilde{u}_t, \tag{26}$$

which gives a downward sloping Beveridge curve for $\eta \leq 1 + s/f$.

The variables $\tilde{u}_t$, $\tilde{v}_t$, and $\tilde{f}_t$ are perfectly correlated in this stochastic steady-state. This implies that the ratio of the standard deviation of $\tilde{v}$, to the standard deviation of $\tilde{u}$ can be read from equation (26) to equal $(1 + s/f)/\eta - 1$. Similarly, the ratio of the standard deviation of $\tilde{f}_t$ to that of $\tilde{u}_t$ can be obtained from equation (24) and equals $1 + s/f$. These ratios of standard deviation can then be compared to those in U.S. data as well as those from the nonlinear simulations in Shimer (2005a).

These comparisons are provided in table 2.3, which also shows the relative variabilities of v/u and labor productivity. The first row of this table shows ratios of standard deviations reported in Shimer (2005a, p. 28) using U.S. quarterly data from 1951 to 2003.[15] In these data, u, v, and f are nearly perfectly correlated so that the ratios of the standard deviations are essentially equal to the bivariate regression coefficients between the corresponding variables.

Table 2.3
Ratios of Standard Deviations Relative to Standard Deviation of Unemployment

	v	v/u	f	Labor productivity
Shimer (2005a) U.S. data	1.06	2.01	0.62	0.11
Implication of stochastic steady-state	2.84	3.84	1.08	
Shimer (2005a) simulations of productivity shocks	3.00	3.89	1.11	

The second row of table 2.3 reports the relationships among labor market variables implied by the stochastic steady-state model (the analysis of the model's implications for labor productivity are taken up later in this chapter). Since the implied connections between u, v, and f depend only on the parameters s, f, and η, they are identical across all the specifications considered. The final row reports ratios based on the nonlinear simulations of the effects of productivity shocks reported in Shimer (2005a, 39). The last two rows are quite similar, demonstrating that the stochastic steady-state version of the model provides a good approximation for computing these statistics. The correspondence with U.S. data is not nearly as good. Indeed, several of these model-generated moments become more similar to the observations if η is lowered to 0.5. The ratio of the standard deviation of v to that of u, in particular, falls to 1.15, which is quite close to the observed value of 1.06. On the other hand, the value of $\eta = 0.28$ has the advantage that it comes close to matching the empirical ratio of the standard deviation of v/u, to the standard deviation of f. This equals about 3.2 in the data, while it equals 3.5 in this calibration. With $\eta = 0.5$, this ratio falls to 2, and this considerably understates this particular relative variability.

The tendency of $\tilde{v}$ to change dramatically whenever there is a small change in aggregate employment is visible figure 2.2, which plots the logarithms of both help wanted advertisement and employment. The mean was subtracted from both series, and this allows them to be displayed with the same scale. To gain a sense of the differences in variability that are involved, employment dropped by 2 percent from its peak in 1979:11 to its trough in 1982:12, while the index of help wanted advertisement dropped from a value of 100 in 1979:11 to a value of fifty-one in 1982:12. Using detrended monthly data, the regression coefficient of the logarithm of help wanted advertisements on total nonfarm employment is around eight.[16]

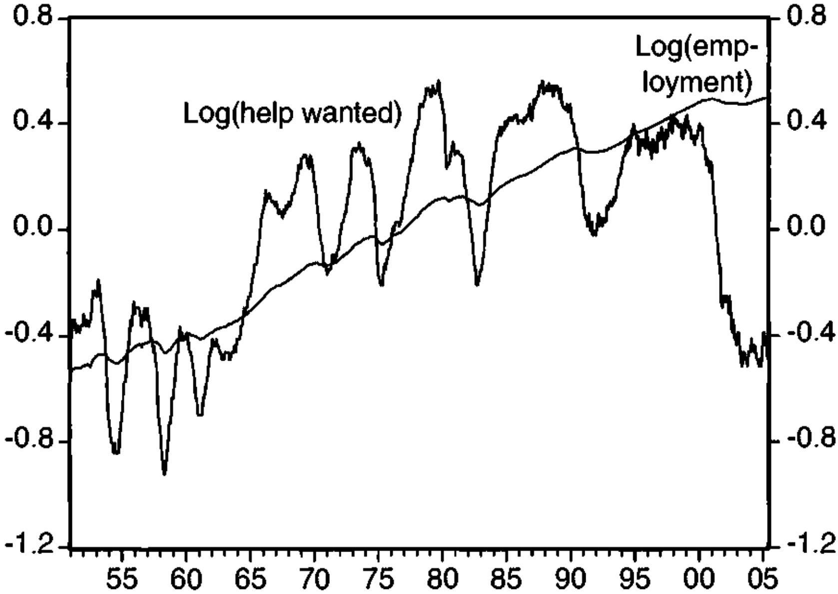

Figure 2.2
Logarithms of Help Wanted Advertisement and Employment

These movements affect the model's predictions regarding wages. This can be seen by linearizing equation (8), which gives:

$$\tilde{\phi}_t = \varepsilon_c \tilde{v}_t - \tilde{u}_t - \tilde{f}_t = \varepsilon_c \tilde{v}_t - \tilde{H}_t, \tag{27}$$

where the second equality follows from equation (24). The rise in $\tilde{v}$ implied by equation (26) leads this equation to imply that if $\varepsilon_c = 1$, $\tilde{\phi}$ rises by about 7 percent when employment rises by 1 percent. Equation (18) then requires the wage to increase sharply. With a lower value of ε_c this effect is muted, and it is possible for $\tilde{\phi}$ not to rise at all.

Using equation (25) to substitute for $\tilde{v}_t$ in equation (27), and using equation (23) to substitute for $\tilde{u}_t$ in the resulting equation, $\tilde{\phi}_t$ becomes a function of $\tilde{f}_t$ only:

$$\tilde{\phi}_t = \left[\varepsilon_c\left(\frac{1}{\eta} - \frac{f}{f+s}\right) - \frac{s}{f+s}\right]\tilde{f}_t. \tag{28}$$

The level of ε_c, such that the marginal hiring cost is unaffected by the finding rate, makes the expression in square brackets zero and satisfies:

$$\varepsilon_c = \frac{\eta s}{(1-\eta)f + s},$$

which equals 0.0266 for the calibrated values of the other parameters. For higher values of ε_c, $\tilde{\phi}$ is increasing in $\tilde{f}$.

The log-linearization of the bargaining equation (18) yields:

$$[1 - \alpha\beta(1 - s - f)]\frac{w}{\rho}\tilde{w}_t = \alpha\beta f\left[1 + \beta(1 - s)\frac{\phi}{\rho} - \frac{w}{\rho}\right]\tilde{f}_t \qquad (29)$$

$$+ \alpha[1 - \beta(1 - s - f)]\left[\tilde{\rho}_t + \beta(1 - s)\frac{\phi}{\rho}\tilde{\phi}_t\right].$$

Using the definition $\rho_t = z_t F_H$, the deviation in the marginal product of labor $\tilde{\rho}$ is given by:

$$\tilde{\rho}_t = \tilde{z}_t - \frac{x_K}{e}\tilde{H}_t. \qquad (30)$$

By using equation (30) to substitute for $\tilde{\rho}_t$ in equation (29), using equation (28) to substitute for $\tilde{\phi}_t$ in equation (29), and then using equation (24) to replace $\tilde{f}_t$ by $\tilde{H}_t$, one obtains an equation relating $\tilde{w}_t$ to $\tilde{H}_t$ and $\tilde{z}_t$. If there are no changes in technological opportunities so that $\tilde{z}_t = 0$, this equation gives the elasticity of the wage with respect to employment.

Lest this argument seem too mechanical, it is worth understanding the economic logic that allows one to compute this key elasticity using just the bargaining steady-state relation. Suppose a change in the elasticity of demand leads to an increase in the job-finding rate. Steady-state considerations allow one to pin down how much this increases employment, and (thus) the extent to which the marginal product of labor falls if z is unchanged. The wage also depends on how much the marginal recruiting cost is affected, and this depends not only on the finding rate and on unemployment (which is determined by the finding rate) but also on the level of vacancies. However, if one knows how the finding rate depends on vacancies and unemployment, one also knows the level of vacancies that is consistent with the given combination of the unemployment and finding rate. This vacancy rate can then be used along with the unemployment and finding rate to compute the marginal hiring cost ϕ. In bargaining, the wage depends only on the marginal product of labor, the finding rate, and hiring costs. Since all three of these determinants of wages can be derived from the finding rate (or the level of employment), one can compute how the wage is related to employment from this equation alone. Interestingly, this calculation does not depend on the original impulse that leads firms to hire labor, as long as this impulse affects ρ only through its effect on employment.

Table 2.4
Stochastic steady-state: Elasticities with respect to employment and relative variabilities

		Variable ε_d		Variable z		
Spec.	Parameters	Elasticity of w	Elasticity of ε_d	Elasticity of w	Elasticity of z	$\sigma_{\text{lab.prod.}}$ over σ_u
(1)	Baseline (BB)	0.43	10.65	2.47	3.44	0.26
(2)	BB with $\varepsilon_c = 1$	3.96	27.95	8.51	9.03	0.68
(3)	BB with $\lambda/w = .4$	3.37	30.25	11.20	9.77	0.74
(4)	BB with $e = 1/3$	0.15	28.57	3.01	4.95	0.38
(5)	BB with $\varepsilon_c = 1$ and $\lambda/w = .4$	14.76	96.36	30.68	31.14	2.36
(6)	Alternative (AA)	—	—	29.92	30.04	2.27
(7)	AA with $x_H = 2/3$ and $e = 1$	—	—	31.25	31.72	2.37
(8)	AA with $\lambda/w = .9$	—	—	4.99	5.08	0.38
(9)	AA with $\lambda/w = .9$ and $\varepsilon_c = .2$	—	—	1.96	2.28	0.17

Note: ε_d is the elasticity of demand facing the typical firm, z is an indicator of technology, ε_c is the elasticity of vacancy posting costs with respect to vacancies, λ/w is the steady-state ratio of the workers opportunity cost of working to the wage, e is the elasticity of substitution of capital for labor, and x_H is a measure of the importance of the labor input.

Table 2.4's first results column displays these elasticities for selected parameter values. For the baseline case, this elasticity is around 0.4, which is close to the microeconomic evidence of the wages of people who keep their jobs. The table also shows that changes in the parameters towards those employed in Shimer (2005a), increases this elasticity to the point that it becomes too large (relative to the empirical evidence).

Raising ε_c so that it equals one implies that recruiting costs rise substantially with employment, so the elasticity of the wage increases to 4.5. With $\varepsilon_c = 0.2$, by contrast, these recruiting costs rise less. If ε_c is lowered to the value of 0.0266, where marginal recruiting costs are independent of employment, the wage still rises even though the marginal product of labor falls. The reason is that an increase in employment is also associated with a higher finding rate for jobs, and this improves the bargaining position of workers.

This explains why λ/w has such a strong effect on this elasticity. When $\lambda/w = 0.4$, workers vastly prefer employment to unemployment. Workers are therefore, in a very weak bargaining position when the finding rate is low, so they accept low real wages. An increase in the finding rate has a big effect on the bargaining position of the workers (since they now have less to lose from not forming a bond with a particular employer), so the bargained wage rises substantially. By contrast, when $\lambda/w = 0.9$

the bargaining position of workers is not so different between the boom and the bust, so real wages are less procyclical.

Specification (4) is interesting because it shows that the real wage becomes nearly acyclical when the changes in labor demand are due to ε_d, and the elasticity of substitution between capital and labor is lowered to one-third. The reason this elasticity has such a large effect is that it governs the extent to which the marginal product of labor falls when employment rises. With a lower value for e, the marginal product of labor falls more, and this keeps the rise in the real wage small. When labor demand is driven by technology shocks, lowering the elasticity of substitution of capital for labor does not have this effect, because firms are not led to hire workers who reduce the marginal product of labor.

To compute the changes in ε_d that give rise to changes in employment when z is constant, or to compute the effect of changes in z, one must also use equation (19). When e is not equal to one the labor and capital shares (HF_H/F and KF_K/F) depend on the level of employment, and it is worth recording this dependence before approximating equation (19) as a whole. In particular:

$$d\left(\frac{KF_K}{F}\right) = \frac{KF_K}{F}\left(\frac{HF_{HK}}{F_K} - \frac{HF_H}{F}\right)\frac{dH}{H} = x_K x_H\left(\frac{1}{e} - 1\right)\frac{dH}{H}.$$

Since $KF_K + HF_H = F$, the derivative of HF_H/F has the same magnitude and the opposite sign. As a result, the log-linear approximation of equation (19), around the mean outcome, is:

$$\frac{w}{\rho}\tilde{w}_t + \frac{m_\phi \phi}{\rho}\tilde{\phi}_t = \left(1 - \frac{1 - \alpha\mu x_H}{\varepsilon_d} + \frac{\alpha x_K}{e}\right)\tilde{\rho}_t \tag{31}$$

$$+ \alpha x_K x_H\left(\frac{1}{e} - \frac{\mu}{\varepsilon_d}\right)\left(\frac{1}{e} - 1\right)\tilde{H}_t + \frac{1 - \alpha\mu x_H}{\varepsilon_d}\tilde{\varepsilon}_{dt}.$$

In the case of constant z, equation (31) can be used to compute the extent to which the elasticity of demand must change for any given change in employment. To carry out this computation, one uses equations (30), (27), and (25) to substitute for $\tilde{\rho}_t$, $\tilde{\phi}_t$, and $\tilde{f}_t$ (respectively), as well as equation (29) to substitute for the wage $\tilde{w}_t$. The resulting response of $\tilde{\varepsilon}_d$ to changes in employment is displayed in the second results column of table 2.4.

The first thing to note about these percentage increases in the elasticity of demand that are needed to increase employment by 1 percent, is that they are large. They are, in particular, much larger than the changes in demand elasticity that are needed to vary labor demand by the same amount if firms have access to a competitive labor market. Recall that such firms would set price equal to $\varepsilon_d/(\varepsilon_d - 1)$ times marginal cost, which is in turn equal to the wage divided by the marginal product of labor. Thus, for such a firm:

$$\frac{\varepsilon_d}{\varepsilon_d - 1} w = zF_H.$$

The linearization of this equation near a particular outcome yields:

$$\tilde{\varepsilon}_{dt} = (\varepsilon_d - 1)\left(\tilde{w}_t + \frac{x_K}{e}\tilde{H}_t - \tilde{z}_t\right). \tag{32}$$

This implies that, with the baseline values of ε_d and s_k/e, a one percent increase in employment that is accompanied by a 0.4 percent increase in the wage would require less than a 0.75 percent increase in the elasticity of demand. By contrast, in the baseline case, a 1 percent increase in employment (together with the implied 0.4 percent increase in the wage) requires more than a 17 percent increase in the elasticity of demand. One reason for this large difference is that bargaining implies that workers' wages fall when the firm's price falls as it increases output. This makes the firm less sensitive to its elasticity of demand.

Table 2.4 also shows that the size of the increase in the elasticity of demand that is needed to raise employment by one percent is larger when $\varepsilon_c = 1$, or when $\lambda/w = 0.4$. As discussed previously, both of these modifications imply that wages rise more with employment. The increases in labor demand that are needed to rationalize a given increase in employment are larger, so ε_d must rise by more. In addition, when $\varepsilon_c = 1$, ϕ rises with the level of employment. These higher recruiting costs act as an additional brake on hiring so that ε_d must rise even more.

When $\varepsilon_c = 1$, the elasticity of demand must rise by 47 percent to increase employment by 1 percent, and this seems excessive. However, it is important to remember that the elasticity of demand is mainly a modeling device to capture the effect of changes in product market distortions that could be due to other causes. Still, these changes appear more plausible when the model implies only that the elasticity of demand must rise by 6 percent for each percent increase in employment.

As the previous analysis suggests, it is possible to reduce the required increase in the elasticity of demand by lowering α. However, this reduction in the bargaining power of workers is not a panacea; it tends to increase the elasticity of the wage with respect to employment when employment fluctuations are due to changes in the elasticity of demand. This effect of α may seem surprising, because the bargaining equation (18) shows that the wage becomes a constant equal to the reservation wage λ when α equals zero. It might, thus, be suspected that the wage becomes less variable as α drops.

This intuition would be correct if ρ and ϕ were held constant. However, holding these variables constant implies that a reduction in α also lowers the wage relative to the reservation value λ. As previously shown, reductions in λ/w do lower the elasticity of the wage with respect to employment. It is also possible to lower α while keeping all the other parameters in Table 1 (including λ/w) constant. A reduction in α then leads to a decline in the steady state values of w/ρ and ϕ/ρ. The resulting increase in ρ, relative to λ, ensures that λ/w stays constant even though the bargaining power of workers has fallen.

With this adjustment in w/ρ and ϕ/ρ, a lower α starting from the baseline parameters implies that wages are less affected by the marginal product of labor and more affected by the the net benefit from being unemployed ($-\Delta$). This net benefit rises in booms because unemployed workers expect to find jobs sooner. When employment variations are due to changes in ε_d, the decline in the marginal product of labor has a smaller effect on the real wage, so real wages are more procyclical.[17]

The log-linearized equations (29) and (31) can also be used to study the effect of technology shocks, or, as Shimer (2005a) has framed the question, the size of the technological changes that are needed to rationalize employment movements. To see this, follow Shimer (2005a) and suppose that the elasticity of demand is constant. Then, after substituting for $\tilde{\phi}$, $\tilde{\rho}$, and $\tilde{f}$, these two equations have three unknowns $\tilde{H}$, $\tilde{w}$, and $\tilde{z}$. They can, therefore, be solved for $\tilde{z}$ and $\tilde{w}$ as a function of the log deviation of employment.

The resulting elasticities of the wage and z, with respect to employment, are displayed in the third and fourth columns of table 2.4. One implication of the model discussed previously, is that real wages are more procyclical when employment is driven by changes in z than when it is driven by changes in z than when it is driven by changes in ε_d, and the table shows that this difference is quantitatively important. The elasticity of the wage with respect to employment is always at least twice as large in the former case.

Using Shimer's parameters in specification (6), z must rise by 30 percent to induce a 1 percent increase in employment. This is another way of phrasing Shimer's (2005a) central conclusion that productivity does not fluctuate sufficiently to justify the observed fluctuations in labor market variables.[18]

Shimer's (2005a) focus is on the variability of the average product of labor, and on the relationship between this variability and that of labor market variables like unemployment, vacancies, and the ratio v/u. Differentiating the production function (20), the deviation of labor productivity from its steady-state value is:

$$\tilde{Y}_t - \tilde{H}_t = \tilde{z}_t + (\mu x_H - 1)\tilde{H}_t, \tag{33}$$

which reduces to $\tilde{z}_t$ in the Mortensen-Pissarides case where $x_H = 1$. In specifications (1)–(6), $\mu x_H = 1.03$. The term in parentheses, which also equals the elasticity of labor productivity with respect to employment is in the case of constant z, thus equals 0.03. This term is not significant relative to the variations in $\tilde{z}$ reported in the fourth column of table 2.4. To obtain the model's prediction concerning the relative standard deviation of labor productivity and unemployment, it suffices to divide the elasticity of labor productivity with respect to employment by f/s. The reason is that the combination of equations (23) and (24) implies that f/s equals the ratio of standard deviation of unemployment to that of employment. The result is displayed in the last column of table 2.4.

That an increase in λ/ρ (like the one that causes the difference between specification [5] and specification [2], or from [6] to [8]) helps to reduce the required variations in productivity was shown already by Hagedorn and Manovskii (2005). As suggested earlier, a higher value of λ/ρ makes recessions less costly for unemployed workers, so their wages do not fall as much. This reduces the extent to which labor costs rise in booms, so productivity need not increase as much to rationalize a given increase in employment.

Reducing the value of ε_c (as when going from specification [5] to specification [3], or from [2] to [1], or from [8] to [9]) also reduces the elasticity of z with respect to employment considerably. When ϕ increases with employment (because $\varepsilon_c > 0.02666$), there are two effects that require higher increases in z. First, the higher value of ϕ leads workers to obtain higher wages because it is more costly to replace them. Second, the higher value of ϕ acts directly as a reason to keep hiring low. Both of these must be offset by larger increases in productivity for the firm to increase its employment in the first place.

Reducing the elasticity of substitution between capital and labor also reduces the extent to which productivity must rise, though it turns out that this effect is quantitatively significant only when ε_c is low.[19] The source of this effect is the following: KF_k/F rises with employment when $e < 1$. As we saw earlier, the extent to which firms wish to overhire rises with this share. Increases in employment thus reduce the profitability of additional hiring less than would otherwise be the case. This implies that productivity need not rise as much to induce the firm to carry out this extra hiring.

This raises the question of whether the parameters in equation (1) solve the central puzzle raised by Shimer (2005a). This is that the observed standard deviation of labor productivity shown in table 2.3 is too low, relative to the observed standard deviations for the labor market variables to be consistent with the standard Mortensen-Pissarides model. Comparing the last column of table 2.4 with the observed standard deviation of productivity, with respect to employment, shows that the model with $\varepsilon_c = 0.2$ goes a considerable way toward resolving this particular puzzle. Interestingly, the match is somewhat closer if one keeps $x_H = 1$, rather than introducing capital in the way shown here.

The model's predicted variability of productivity would appear even less excessive if one compared it to the variabilities of v, f, and v/u. This is because, as can be seen in table 2.3, this calibration tends to overpredict the fluctuations in these variables. It therefore predicts relatively low values of the standard deviation of productivity relative to v, f, and v/u.

Before closing this section, it is worth pointing out that, when subject only to shocks of ε_d, the calibrations in table 2.4 predict relative movements in productivity that are much smaller in size than the observed ones. This follows from the fact that, as discussed previously, the elasticity of labor productivity with respect to employment equals 0.03 when z is constant. This implies that the predicted standard deviation of labor productivity, over that of unemployment, equals $0.03 * s/f$, which is negligible relative to the observed value of 0.11. Interestingly, μ was calibrated so that it came close to explaining a different feature of the relationship between productivity and employment.

Nonetheless, the model subject to changes in ε_d might benefit from modifications that increased the magnitude of its predicted productivity movements. At the moment, the model predicts that real wages should be more procyclical than productivity so that the labor share is procyclical. In aggregate data, it is well known that the labor share is somewhat countercyclical. This need not constitute decisive evidence against the model, however, because the model applies to individual firms, and aggregate data are affected by changes in composition. Still, if it were found that the labor share is countercyclical at the typical individual firm, this would be inconsistent with the model.

3.4 Approximate Equilibria near a Steady-State

In this section, dynamic simulations of the full model around a steady-state are considered. The model consists of equations (1), (2), (3), (5), (7), (14), and (15) and an equation specifying how ρ_t depends on H_t. In the case where $e = 1$, this equation takes the Cobb-Douglas form $\rho_t = z_t\tilde{\rho}H^{x_H}$, although it takes the CES form when $e = 1/3$. This gives seven equations in H_t, u_t, v_t, f_t, ϕ_t, Δ_t, w_t, ε_{dt}, and z_t. These equations have just one state variable, namely, the lagged value of u. They can be solved for the effects of technology by treating z_t as exogenous and fixing ε_{dt}, or for the effects of variable market power by fixing z_t and treating ε_{dt} as exogenous. Equivalently, the stochastic processes for either z_t or ε_{dt} that are needed to rationalize a set of plausible stochastic processes for the log of H_t, h_t are considered.

The stochastic processes for h_t are based on the behavior of detrended employment in the business sector. Using data from 1950:1 to 2002:1, a regression of (quarterly) detrended employment on its own lag yields a coefficient of 0.941, while a regression on two lags gives a coefficient of 1.55 on the first lag and −0.64 on the second. This AR(2) specification fits better, in that the second coefficient is highly statistically significant, and that the Durbin-Watson statistic rises from 0.78 to 1.99 when two lags are included instead of one. Still, it is standard in analyzing Mortensen-Pissarides models to study AR(1) processes, and for this reason two specifications are considered that differ in the order of the autocorrelation that describes h_t.

It is still supposed that a period lasts one month (so that the steady-state finding rate remains 0.45, for example) and the two specifications are:

$$h_t = .98h_{t-1} + v_t^1, \tag{34}$$

$$h_t = 1.76h_{t-1} - .78h_{t-2} + v_t^2. \tag{35}$$

The first of these is simply the monthly analogue of the AR(1) model estimated with quarterly data, so its coefficient is the cubic root of the estimated coefficient discussed previously. The second is more loosely based on the quarterly AR(2) specification. The two models do have in common that the peak response of employment to a shock, in quarter t, occurs in quarter $t + 2$.[20]

This model is simulated using DYNARE, which uses a method of approximating the behavior of the model near a steady-state that is close to Collard and Juillard (2001). Because these calculations involve a second-order approximation, the variance of the shocks v^1 and v^2 affect the results. These variances are chosen so the standard deviation of h is approximately 0.02, the standard deviation of cyclical log employment in the U.S. business sector.

One simple way of presenting the resulting simulations is to consider regressions of wages, z, and ε_d on employment with simulated data. These can readily be computed from the impulse-response functions, and the results of these theoretical regressions are presented in table 2.5.

The elasticities of the wage and z look quite similar to those of table 2.4, though the required responses of ε_d are even larger. One obvious question that arises at this point is why the numerical implications of this fully dynamic model are so similar to those of its steady-state counterpart. The reason is that, with a high value of f_t, neither the future nor the past exert a strong influence on the models current predictions. It has already been noted that the coefficient of lagged unemployment in equation (2) is $(1 - s - f_t)$, which is small when f is high. Moreover, $(1 - s - f_t)$ is also the coefficient of future Δ in equation (5). Thus, a high value of f also implies that Δ is mostly affected by developments in the very near future.

The remaining dynamic equilibrium condition is (15), and this too is

Table 2.5
Elasticities with Respect to Employment near Steady-State

		Variable ε_d		Variable z	
Spec.	Parameters	Elasticity of w	Elasticity of ε_d	Elasticity of w	Elasticity of z
(1)	Baseline (BB) AR(2)	.41	41.3	2.54	3.68
(2)	Baseline (BB) AR(1)	.41	50.8	2.72	3.88
(3)	BB with $\varepsilon_c = 1$, AR(2)	5.37	40.9	11.5	10.7
(4)	BB with $\varepsilon_c = 1$, AR(1)	5.22	41.4	11.7	11.0
(5)	BB with $e = 1/3$, AR(2)	.13	125.1	3.05	5.20
(6)	BB with $e = 1/3$, AR(1)	.13	129.9	3.32	5.48

Note: ε_d is the elasticity of demand facing the typical firm, z is an indicator of technology, ε_c is the elasticity of vacancy posting costs with respect to vacancies, and e is the elasticity of substitution of capital for labor.

consistent with employment and wages being near their steady-state as long as there is not much difference between current and future hiring costs. Since equation (8) implies that hiring costs depend only on contemporaneous variables, slow-moving changes in employment like those implied by conditions (34) and (35) are consistent with having the other variables in the model near the steady-state values that correspond to the current level of employment. This also explains why the statistics in table 2.4 are not affected very strongly by whether one seeks to rationalize AR(1) or AR(2) stochastic processes for employment. Table 2.4 also shows that, as before, lowering the elasticity of substitution between capital and labor reduces the extent to which real wages are procyclical when employment fluctuations are due to changes in ε_d. While the calculations are not reported, the elasticities reported in Table 4 also seem robust to plausible changes in the standard deviations of the shocks affecting employment in conditions (34) and (35).

One advantage of computing these approximations near a steady-state, is that they allow one to look at impulse responses. One can then see the pattern of movements in either z or ε_d that are needed to justify the stochastic processes for h. The changes in z and ε_d, together with the responses of log employment and the log real wage, are depicted for the more interesting AR(2) case in figures 2.3 and 2.4.

Like the response of employment itself, the required responses of z and ε_d in the AR(2) case are hump shaped. However, while employment rises immediately (and then keeps rising for some time), both z and ε_d are required to fall somewhat on impact. Only later do z and ε_d rise, with their peak increases actually coming somewhat after the peak changes in employment. These results emerge because the baseline parameter values imply that the marginal cost of adding employees rises disproportionately as employment reaches its peak (when future employment declines). The relevant combination of current and expected future adjustment costs does not rise as rapidly when increases in employment are followed by further increases. As a result, the prospect of a future increase in labor demand (because of future increases in either z or ε_d) leads firms to increase their hiring immediately. The actual initial increase in employment is not quite as large, so the model requires that there be an opposing force that discourages initial employment.

The initial fall in ε_d that is required, is equal to only about a quarter of the eventual peak rise in ε_d. By contrast, the initial fall in z_t is nearly half as large in absolute value as the ultimate increase in this productivity indicator. The underlying reason for this larger response is that bargain-

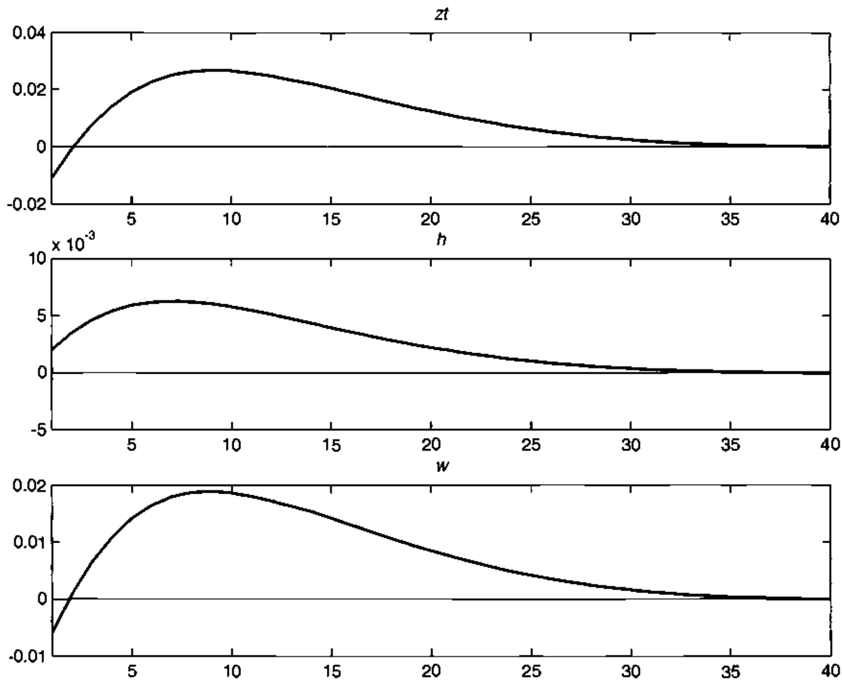

Figure 2.3
Impulse Responses to Technology Shocks, Baseline Parameters—AR(2) Employment

ing between workers and firms leads wages to fall when z falls. Small reductions in z, which are accompanied by reductions in w in equilibrium, are therefore not sufficient to discourage hiring by the requisite amount. To track the actual increase in initial employment, z (and the real wage) must fall significantly.

3.5 Conclusions

This chapter has shown that, in the context of matching models, variations in market power have some advantages relative to variations in technology shocks for explaining the relatively weak procyclical movements in productivity and real wages. While variations in market power emerge as an attractive source of aggregate fluctuations in employment, the particular source of these variations considered here does not. In particular, the variations in the elasticity of demand that are needed to explain employment fluctuations are too large. While this paper has not

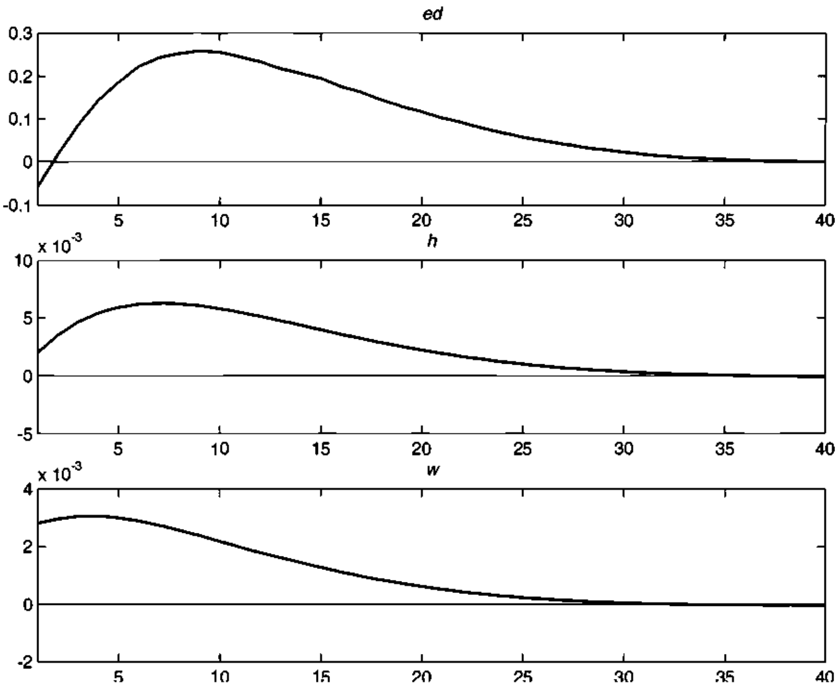

Figure 2.4
Impulse Response to Demand Elasticity Changes, Baseline Parameters—AR(2) Employment

considered sticky prices explicitly, the findings suggest that it may be easier to rationalize the needed market-power fluctuations in such a setting. With constant prices, a firm that sees its demand fall by 1 percent should lower output by close to 1 percent, and such a change would not seem dramatic to the agents involved. By contrast, the model suggests that a very large reduction in the elasticity of demand is necessary to induce the firm to reduce its output by 1 percent.

This chapter has focused on matching the regression coefficients implied by the model (which are simply the correlation multiplied by the appropriate ratio of standard deviations), to those that one finds in actual data. The more usual approach (see, for example, Shimer 2005a) is to try to match ratios of standard deviations in the model and in the data. Models with a single shock tend to imply correlations near one, so the model-generated regression coefficients are close to the ratio of standard deviations. In the data, however, many correlations—particularly those involving real wages and productivity—are smaller than one, and so the approach followed here is not identical to one that focuses on ratios of standard deviations.

In particular, matching the regression coefficient of wages on employment in a single shock model leads to a real wage that is less variable than observed aggregate wages. Not surprisingly, obtaining a model that matches a single labor market statistic still leaves one far from having a complete model of labor market dynamics. A more complete model would incorporate multiple shocks. The regression coefficient of wages on employment would then equal the weighted average of the regression coefficients from models that have only one of the included shocks, with the weights being related to the extent to which the individual shocks contribute to fluctuations in employment.

It is, therefore, possible (in principle) to have a small overall regression coefficient of wages on employment that results from some shocks that lead to large positive responses of wages to employment, and other shocks that lead to large falls in wages when employment rises. Two studies focusing on the responses of real wages to exogenous monetary and fiscal disturbances both find small procyclical wages, however.[21] This suggests that a mechanism that induces small procyclical real-wage movements such as the one presented here, may well play a role also in a more complete model with multiple shocks.

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Notes

1. Merz (1995) adopts the fiction of a family whose members all insure each other against unemployment. This fiction does not avoid the problem if individual family members are somewhat selfish. In this case, moral hazard would lead their search effort to be inefficiently low.

2. Vacancies are often measured by the conference board help-wanted index. Interestingly, having this index increase by 1 percent may well increase the costs of the firms placing these adds by less than 1 percent. Abraham and Wachter (1987, 209) report that this index is obtained by counting the total monthly number of job advertisements placed in major newspapers. This total number rises when advertisers repeat their advertisements

a larger number of times. Placing an advertisement n times does not generally cost n times the amount it costs to place an advertisement once. For example, the *Boston Globe*'s May 2005 rates indicate the cost of placing an advertisement for four additional days within a week is zero, once the advertisement runs for Sunday and two additional weekdays (the Sunday rate per agate line is twenty-five dollars, the daily rates once an ad appears on Sunday is five dollars, and the weekly rate is thirty-five dollars). See http://bostonworks.boston.com/mediakit/ratecards/. This example indicates that even the marginal cost of placing an additional advertisement may fall when more advertisements are placed.

3. Yashiv (2006) also considers costs that do not rise linearly with vacancies, though he only studies the case of diminishing returns to scale.

4. This requires that firms be sufficiently small so that the effect of their vacancies on the ratio uf/v can be neglected. In the case where the number of firms N is large, one obtains essentially identical formulas by neglecting the effect of changes in a firms vacancies on total hiring, and supposing instead that the total number of workers hired $(u_t f_t)$ distributes itself evenly over the total number of vacancies posted by firms.

5. This requires that there be a slight negative correlation between the probability of success of the different vacancies that are posted by a particular firm. One advantage of this specification is that it ensures that all the symmetric firms in the model remain the same size, since they all post the same number of vacancies in equilibrium.

6. The probability of leaving the firm (s) is assumed to be identical with the workers probability of becoming unemployed, as in Shimer (2005a). This is only a simplification because many workers move from one job to another, so the probability that a worker separates from a firm is higher than the probability that this worker separates from active employment. One attraction of incorporating both separation rates explicitly is that, even if the two rates are assumed to be constant, more workers would transition from job to job in booms. This would mean that vacancies are more productive in booms, and could thereby further reduce the calibrated procyclicality of ϕ_t.

7. This requires that the firm have nothing to gain in the bargaining stage by having the capacity to produce more output than is demanded at the price that it has set. No such benefit exists if, as assumed previously, wages are set under the supposition that all workers are needed to produce the quantity demanded. This still raises the question of why the firm does not recruit some workers just so they can be ready to carry out the job of any worker who leaves. This possibility can be neglected if one assumes that vacancies attract workers only if they involve a specific task that is not already carried out by another worker. See Rotemberg (1998) for further discussion.

8. The second-order condition is only a necessary condition for the first-order condition to be associated with a profit maximum; it does not guarantee that there do not exist other employment paths with even larger values of U^i. One might be particularly concerned that the firm would prefer to hire only occasionally, and keep its hiring equal to zero at other times, as suggested by Kramarz and Michaud (2003). While this is both a realistic possibility, and one that might be optimal if the model were treated as valid globally, it is neglected to maintain the simple representative-firm framework. One way to rule out this behavior (even if it were implied by the equations spelled out in the text) is to suppose that these equations are valid only locally, and that, for example, each period the firm loses some employees whose replacement is essential to keep production positive.

9. See Stole and Zwiebel (1996) for this effect in the context of a (somewhat) different bargaining model. Chapter 3 of Pissarides (2000) avoids this effect by supposing that the firm takes the wage as given, when it decides how many employees to hire. This exogeneity is

not entirely consistent with Nash bargaining, however. Cahuc and Wasmer (2001) show that the Pissarides (2000) solution is correct if firms are free to adjust their capital *after* they have finished bargaining with their workers, and *before* production takes place. In the more plausible case where capital is purchased earlier, so that its level is fixed for the period between bargaining and production, this overhiring force also affects firms choice of capital, since a firm can just as easily lower the marginal product of labor by under employing capital as by over employing labor. Merz (1995) and Andolfatto (1996) first solve optimal planning problems where these issues do not arise. These papers also show that the social planning solutions can be decentralized as competitive equilibria. However, the decentralization results of Merz (1995) and Andolfatto (1996) are derived under the assumption that wages are independent of the amount of labor and capital that firms employ.

10. The two minor differences are that ε_{it} is set to a large finite value rather than to infinity, and that λ/w is set to 0.4 rather than having λ/ρ set to this value. Also note that e is irrelevant when $x_H = 1$, as in Shimer (2005a).

11. The coefficient of total hours in a regression of output on total hours was only 1.07 and implies a μ of only about 1.6. This may be a more accurate measure of returns to scale, because hours per worker are well known to be procyclical. Unfortunately, fluctuations in hours per worker are beyond the scope of the model.

12. The relationships between wages and employment discussed subsequently are quite similar if μ is set to 1.7 instead.

13. Except for specifications (2) and (5), whose role is to illustrate the effects of changes in parameters, all the specifications in the table have labor shares below one.

14. The actual and implied series overlap considerably more if the implied series is given by $s_t/(s_t + f_t)$, so that it includes variations in the separation.

15. Note that since the relationship between these variables is linear and deterministic according to the stochastic steady-state model, it does not matter whether the data it is compared with are quarterly or monthly.

16. This was done using monthly data from 1951:1 to 2005:5. Because the data are monthly, I modified the parameters of Rotemberg (2003) so that the objective function involves the covariance between the cycle at t and the cycle sixty-four months hence, while the constraint is that the cycle at t be uncorrelated with the difference between the current trend and the average of the trends at $t + 20$ and $t - 20$.

17. When, instead, employment variations are due to changes in z, the fact that a lower α reduces the sensitivity of wages to the marginal product of labor makes wages less procyclical. This effect is modest, however, because ρ/w falls together with α. Take, for example, the parameters in specification (8), and consider lowering α to the relatively small value of 0.1. This lowers the elasticity of the wage, with respect to employment, from the value of 2.85 (in table 2.4) to 1.97.

18. Shimer (2005a) shows that net productivity must rise by about 1 percent for each 1 percent increase in the ratio of vacancies to unemployment. The reason this implies productivity must rise by about 30 percent for each percent increase in employment, can be seen as follows. A 1 percent increase in net productivity corresponds to about a 0.6 percent increase in productivity itself, given a λ/ρ ratio of 0.4. At the same time, $\eta = 0.28$ implies that a 1 percent increase in v/u raises the job finding rate by 0.28 percent. A 1 percent increase in the job finding rate, therefore, requires a 1/0.28 percent increase in v/u, and a 0.6/0.28

(approximately 2) percent increase in productivity. Equation (24) implies that the finding rate must rise by (about) 15 percent for each 1 percent increase in employment, so a 1 percent increase in employment does indeed require a 30 percent increase in productivity.

19. Indeed, using the alternative parameters AA, with $x_H = 2/3$ and $e = 1/3$ actually raises the elasticity of z (with respect to employment) to thirty-two.

20. In the monthly model, a shock that raises the average level of employment by 1 percent in the initial quarter raises it by 1.82 percent two quarters after this shock first has an impact. In the estimated quarterly model, this figure equals 1.74, which is somewhat lower. On the other hand, the estimated quarterly models response after three quarters equals 1.57, which is somewhat higher than the 1.61 percent response implied by the monthly model. Thus, while the responses are similar in both cases, they are not identical.

21. See Christiano, Eichenbaum, and Evans (2005) for responses to monetary policy, and Rotemberg and Woodford (1992) for responses to shocks to military purchases.

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Comment

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Rotemberg's paper is a very interesting and thoughtful work that will certainly generate a large amount of follow-up research. It develops a dynamic general equilibrium (DGE) model with search and matching frictions and period-by-period Nash bargaining that explains the modest elasticity of wages to employment observed in U.S. data when changes in market power are assumed to be driving economic fluctuations. There are two possible ways to see this paper's main contribution. One can see it as providing evidence that variations in market power have some advantage relative to technology shocks as a source of fluctuations in employment. A different line of interpretation is that this paper offers a solution to Shimer's (2005) critique of the baseline Mortensen and Pissarides (1994) model. While the paper is mainly presented along the first line of interpretation, most of this discussion follows the second. Thus, after briefly reviewing Shimer's critique and the subsequent debate, I go over the solution offered by Rotemberg and clarify which new mechanisms are at work. I then discuss a few open questions and conclude.

Shimer (2005) shows that the conventional Mortensen - Pissarides (MP) model cannot explain the magnitude of the cyclical fluctuations in labor market activity when business cycles are assumed to be driven by technology shocks of a plausible magnitude.[1] The main reason for this failure, he argues, is period-by-period Nash bargaining which makes wages too flexible. Under reasonable parameter values, wages absorb almost all changes in productivity, leaving little incentives for firms to increase their hiring activity.

A rapidly growing literature has emerged to address Shimer's critique. Hall (2005) and Shimer (2005) show that with the introduction of ad hoc wage rigidity, in the form of either a constant wage or a partially smoothed wage rule, the baseline MP model is able to explain the observed volatility in U.S. unemployment and vacancies. Along these lines, Gertler and Trigari (2006) modify the MP framework to allow for staggered multiperiod wage contracting, and show that a reasonable calibration of the model can account well for the cyclical behavior of wages and labor market activity. Further, a number of authors have explored axiomatic foundations for wage rigidity, building directly on assumptions about the information structure of the economy.[2] Finally, others have considered flexible wage alternatives. Hagedorn and Manovskii (2006) propose an alternative parameterization of two key parameters of the model: the workers' bargaining power, and the relative flow value from unemployment. Mortensen and Nagypal (2006) investigate the role of job destructions shocks, job-to-job flows, and hiring costs. See Hall (2006) for an extended survey.

Rotemberg's solution to the volatility puzzle belongs to the area of flexible wage explanations. Before reviewing it, it is useful to gain some more understanding of what factors account for the procyclicality of Nash bargained wages in the baseline MP model. Using Rotemberg's notation yields:

$$w_t = \alpha[\rho_t + \beta(1 - s)\phi_t] + (1 - \alpha)[\lambda + (f_t - 1 + s)\beta\Delta_{t+1}],$$

where the real wage is a convex combination of what a worker contributes to the match and what he loses by accepting a job, and where the weights depend on the worker's relative bargaining power, α. The worker's contribution is the marginal product of labor, ρ_t, plus savings on future hiring costs, $\beta(1 - s)\phi_t$, where ϕ_t denotes the marginal cost of hiring a new worker at time $t + 1$, s denotes the separation rate, and β is the discount factor. The foregone benefit from unemployment, in turn, is the flow value of unemployment, λ, plus the expected discounted gain of moving from unemployment in this period to employment in the next period, $(f_t - 1 + s)\beta\Delta_{t+1}$ (with f_t denoting the job finding rate, and Δ_{t+1} the future surplus from employment).[3]

Accordingly, there are three main factors that cause the wage to move procyclically in response to an increase in labor productivity. First, the marginal product of labor ρ_t is procyclical. Second, when firms increase vacancies after a productivity shock, the probability of finding a job f_t raises, leading in turn to an increase in the worker's outside option. Third, under the conventional assumption of a fixed cost per vacancy, the marginal hiring cost ϕ_t increases linearly with the number of vacancies posted, putting further upward pressure on wages during booms.

Rotemberg augments the conventional MP framework with three new features. First, he drops the assumption of one worker per firm, and instead allows firms to hire a continuum of workers. At the same time, he assumes diminishing returns to labor. Second, he assumes imperfectly competitive firms with variable market power. Third, he drops the conventional assumption of a fixed cost per vacancy opened, and instead assumes that firms face concave costs of posting vacancies. Why do these new features help to explain the weak procyclicality of wages? There are two mechanisms at work: economies of scale in posting vacancies, and market power shocks with diminishing returns to labor. Both will be described below.

First, Rotemberg assumes that the hiring cost function takes the form $c(v_t)^{\varepsilon_c}$, where v_t denotes vacancies and c and ε_c are two positive parameters. This leads to an expression for marginal hiring cost, ϕ_t, which is proportional to $(v_t)^{\varepsilon_c}$. Note that this functional form nests the conventional assumption of linear costs in vacancy posting when $\varepsilon_c = 1$. This paper, in contrast, assumes $\varepsilon_c < 1$. That is, the technology of posting vacancies is subject to economies of scale. Thus, because of concavity, when vacancies and employment raise in booms, recruiting costs ϕ_t do not increase as much as in the linear cost case. This makes the worker's effective bargaining power less procyclical, and reduces the extent to which real wages increase in expansions. Second, because in Rotemberg's model, firms are imperfectly competitive, it is possible to consider economic expansions that are generated by reductions in market power. The additional assumption of diminishing returns to labor implies the model has different implications depending on whether business cycles are driven by technology or by market power shocks. Increases in productivity cause both employment and the marginal product of labor to go up, putting upward pressure on wages. In contrast, reductions in market power induce an increase in employment, but produce a reduction in the marginal product of labor. This puts downward pressure on wages. Thus, having market power shocks driving fluctuations in employment rather than technology shocks, everything else equal, implies wages will be less procyclical.

Rotemberg's paper differs from the existing literature when it comes to evaluating the model against the data. Shimer (2005), and the authors who address his critique, focus on the ability of the model to match the observed correlations and volatilities of a set of selected labor market variables (unemployment, vacancies, labor market tightness, labor share, etc.) when productivity shocks are assumed to drive the cycle. Rotemberg, instead, proceeds as follows. First, he feeds into the model an estimated stochastic process for employment and solves for the exogenous driving force process. In particular, he assumes that either changes in market power (generated by changes in the elasticity of demand facing the typical firm), or changes in productivity are the sole driving force of the economy. Then, he compares the model ability to match one selected statistics in particular—the elasticity of wages to employment—under the two assumptions. The main result is that, given the change in the driving forces (demand elasticity or productivity) needed to rationalize the process for employment, the elasticity of wages to employment is always lower when business cycles are driven by market power shocks. In addition, under the baseline parameterization with market power shocks, the elasticity in the model is very close to Rotemberg's preferred estimate in the data. This is exactly the sense in which the paper provides evidence in favor of market power shocks as a source of fluctuations.

One interesting question, which is already partially answered in the paper, is whether the MP model augmented with the additional features proposed by Rotemberg can solve the Shimer's puzzle. To answer this question, I next explore how well a version of this papers economy (with no capital) is able to account for the volatility in the U.S. labor market in face of technology shocks, and compare it to the empirical performance of the baseline MP model.[4] I use Rotemberg's baseline calibration displayed in table 2.6 of the chapter. In addition, the baseline MP model employed for comparison, essentially the one in Shimer (2005), is simply obtained by changing the parameterization to have linear hiring costs, a lower relative flow value from unemployment, perfect competition, and constant returns to scale. Details are shown in table 2C.1.

Table 2C.2 displays the results. The first three rows report the standard deviation for the five key labor market variables (wages, employment, unemployment, job finding rate, and vacancies) in the U.S. economy for the period 1964–2004, in the baseline MP model and in Rotemberg's model. The standard deviations are normalized relative to output. The table shows that Rotemberg's model does better than the baseline model for all variables considered. Associated with a lower relative volatility of wages (0.56 versus 0.95), there is a significantly larger

Table 2C.1
Parameterization

Rotemberg's model	$\varepsilon_c = 0.2$	$\lambda/w = 0.9$	$\varepsilon_d = 2$	$s_h = 2/3$
Baseline MP model	$\varepsilon_c = 1$	$\lambda/w = 0.4$	$\varepsilon_d \to \infty$	$s_h = 1$

Table 2C.2
Standard Deviations (Relative to Output)

	w	H	u	f	v
US Data, 1964–2004[a]	0.52	0.60	5.15	3.44	6.30
Baseline MP model	0.95	0.03	0.39	0.44	1.22
Rotemberg's model	0.56	0.20	2.68	3.07	8.72
Baseline MP + $\varepsilon_c = 0.2$	1.01	0.08	1.09	1.27	3.65
Baseline MP + $\lambda/w = 0.9$	0.77	0.14	1.91	2.15	5.97
Baseline MP + $\varepsilon_d = 2.0$	0.94	0.03	0.38	0.43	1.19
Baseline MP + $s_h = 0.66$	0.95	0.03	0.38	0.42	1.18

[a]Most of the U.S. data is from the BLS. Output is production in the non farm business sector. The wage w is average hourly earnings of production workers in the private sector, deflated by the CPI. Employment H is all employees in the non farm sector. Unemployment u is civilian unemployment 16 years old and over. Vacancies v are based on the help wanted advertising index from the conference board. Finally, the data are HP-filtered with a conventional smoothing weight.

relative volatility of all indicators of labor market activity. Moreover the volatility of wages comes very close to matching the data (0.56 in the model versus 0.52 in the data). Note, however, that the model captures only one third of employment volatility (0.20 versus 0.60 in the data) and half of unemployment volatility (2.68 versus 5.15 in the data).

To illustrate what features are important, the baseline MP model is augmented with the additional features added one at a time. The last four rows of table 2C.2 show the results. The two features that help the model solve Shimer's critique are concave hiring costs ($\varepsilon_c = 0.2$), and a large relative flow value from unemployment ($\lambda/w = 0.9$). Imperfect competition ($\varepsilon_d = 2$), and diminishing returns to labor ($s_h = 2/3$) do not affect the results in any significant way when productivity shocks drive the cycle. There are two important observations.

First, the table shows concave hiring costs are quantitatively less important than the large relative flow value from unemployment of 90 percent. The latter feature essentially makes labor supply highly elastic. Thus, similar to Hagedorn and Manovskii (2006), Rotemberg's solution to Shimer's puzzle relies largely on a value of the flow opportunity cost of employment that is quite far from what conventional analyses suggest may be reasonable, typically 40 percent of the wage.

Second, while Rotemberg's analysis sheds light on the importance of alternative assumptions about hiring costs, a number of open questions remain about their exact behavior. Hiring costs refer to costs incurred at all stages of recruiting. They include the costs of advertising and screening that pertain to all vacancies v_t, and the costs of training and disrupting production that only pertain to actual hires or matches m_t. Recent work by Yashiv (2006) assumes that both types of costs are present. Specifically, in the context of a DGE framework with matching frictions, Yashiv estimates a hiring cost function of the following form:

$$\frac{c}{\varepsilon_c}\left[\frac{\phi v_t}{H_t} + \frac{(1-\phi)m_t}{H_t}\right]^{\varepsilon_c} H_t,$$

where the parameter ϕ measures the relative importance of advertising versus training costs and the parameter ε_c the curvature of the function. Although the parameter ε_c applies jointly to both types of costs, differently from Rotemberg where it only applies to advertising costs, Yashiv estimates convex hiring costs ($\varepsilon_c > 1$) rather than concave. In addition, he also estimates that a larger weight appears on costs relating to actual hires ($\phi < 0.5$), which are ignored in Rotemberg's paper. Note, finally, that Rotemberg's analysis relies on a quite large degree of concavity in vacancy posting costs as he assumes $\varepsilon_c = 0.2$. In general, further research seems to be needed to fully understand the behavior of these costs.

One final remark concerns the model's evaluation strategy and results. As acknowledged in the paper, changes in the elasticity of demand needed to rationalize the employment process are quite large relative to what seems to be plausible. In a sense, this shifts Shimer's puzzle in a different direction, toward the need for implausible fluctuations in the elasticity of demand rather than in productivity. Rotemberg, however, convincingly argues that this problem would be significantly tempered in the case where market power fluctuations are due to nominal price rigidities rather than to changes in the elasticity of demand.[5] It may then be worthwhile for future research to modify the model in this direction, and evaluate quantitatively the role of sticky prices. Furthermore, it would seem important for future work to study whether Rotemberg's model with plausible changes in both market power and productivity, and possibly other sources of fluctuations, is capable of explaining the overall cyclical behavior of U.S. labor markets.

Notes

1. Costain and Reiter (2003) develop a similar argument in an independently written but less well-known paper.

2. Examples include Menzio (2005), Kennan (2006), and Shimer and Wright (2004).

3. One minor difference in notation from Rotemberg is that here Δ_t is defined to include the wage received at time t if employed.

4. In this exercise, I assume an AR(1) monthly process for technology with autoregressive parameter of $0.95^{1/3}$.

5. See Krause and Lubik (2005), Trigari (2004, 2006), and Walsh (2005) for examples of DGE models with sticky prices and search frictions.

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