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# The Consumption-Tightness Puzzle

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## 1.1 Introduction

This paper analyzes the properties of the Mortensen and Pissarides (1994) and Pissarides (2000) style labor market matching model, extended with a labor market participation choice embedded in a stochastic growth model. The bulk of modern business cycle theories assume instantaneous and costless matching of employers and workers (Christiano and Eichenbaum 1992; Hansen 1985; and Prescott 1986). Labor market matching models, as an alternative, realistically assume it takes time and resources to match firms wishing to fill job vacancies with workers looking for jobs. This labor market matching process introduces frictional unemployment and it places the labor market in a central role in the transmission of shocks over time and across agents. Therefore, it is not surprising that this framework, which has proven extremely successful as a tool for understanding the long-run determinants of unemployment (Ljungqvist and Sargent 2005), is receiving growing interest in the business cycle literature (Andolfatto 1996; Cheron and Langot 2004, den Haan, Ramey, and Watson 2000; Gertler and Trigari 2005; Hall 2005; Merz 1995; and Shimer 2005).

In the Mortensen-Pissarides setup, the labor market matching process is modeled on the basis of a matching function that relates the number of new job matches to the number of search-active unmatched agents and to the number of job vacancies posted by firms. When deciding on the number of job vacancies to post, firms consider the cost of setting aside resources to open a job vacancy relative to the expected benefits that a successful job match produces. Thus, on the part of firms, matching models allow for variations in the extensive search margin.

On the part of workers, most applications of matching and search the-

ories in the business cycle literature assume the labor market participation rate is constant. Therefore, variations in the extensive search margin occur only through changes in the net hiring rate (the difference between the number of new job matches and the termination of existing job-worker relationships). This assumption might seem natural, given that the labor market participation rate does not vary much over the business cycle. We argue that this latter argument is misleading on several grounds. First, consistent with the theory that we propose, U.S. labor market participation rates display procyclical movements. Second, it is important to understand whether the relatively low volatility of the participation rate is consistent with economic theory. Third, in order to ask whether theory can account for the empirically observed moments of unemployment, the measurements of unemployment (in the data and in theory) need to be consistent, and this requires the introduction of a participation choice. Finally—and this is a main contribution of this paper—we show that the matching model extended with a participation choice provides a series of strong predictions for indicators that are central in labor market matching models, and for variables that are at the heart of business cycle research.

We assume that in order to participate in labor market activities, agents need to give up leisure that enables them to search for a job. In return, consistent with Flinn and Heckman (1983), search active agents face a (potentially) more favorable labor market outcome than nonparticipants. In particular, we assume the matching probability of the former group of agents is higher than the matching probability of the latter. Therefore, the participation choice is based on the trade-off between giving up leisure to be search active, versus the expected benefits of being search active.

We are not the first to introduce participation choice into models of labor market search and labor market matching. Burdett et al. (1984) analyze and estimate a three-state labor market search model with participation choice (Bowlus 1997). Andolfatto and Gomme (1996) study a search model with participation choice, in order to analyze the effects of labor market policies. Following Pissarides (2000), a number of papers have analyzed matching models with participation choice. Garibaldi and Wasmer (2005), Haefke and Reiter (2006), Pries and Rogerson (2004), and Yip (2003) all analyze dynamic search models with participation choice, in which shocks to the value of nonparticipation (relative to participation) generates flows in and out of the labor market. Each of these papers examine models without savings and assume incomplete

markets. The current paper (instead) introduces production and savings. As discussed by Hall (2006), savings and self-insurance are key when accounting for the search incentives of the unemployed. We assume complete markets, since this gives rise to a much simpler framework than more complicated incomplete markets settings. Furthermore, it appears that the complete markets setting closely emulates the main properties of the perhaps more realistic incomplete markets self-insurance model (Hall 2006). Moreover, our analysis allows for risk aversion, and we show that this is a key parameter. Similar complete markets settings have been analyzed by Veracierto (2003) and by Ravn (2005). Veracierto (2003) introduces a labor market participation choice into a Lucas-Alvarez type (island) search model, with production and savings assuming complete markets. Ravn (2005) estimates a more complicated version of the model, which is analyzed in the current paper. The main innovation of the current paper, relative to Veracierto (2003) and Ravn (2005), is that we are able to derive a simple and robust relationship between labor market tightness and consumption, which appears to have been overlooked in previous research.

The model we study introduces a symmetry between firms' and workers' search activities, since both sides of the labor market vary their search efforts at the extensive margin. This symmetry is shown to have important consequences, and (surprisingly) is of considerable analytical convenience. When allowing for variations in the labor market participation rate, the first-order condition for households' search intensity along the extensive margin resembles the more familiar vacancy posting condition that derives from the firms' problem. In particular, variations in households' search intensity along the extensive margin equalize the marginal costs of the search (the utility value of a loss of leisure) with the expected marginal benefit of labor market search, which is the product of the probability that the labor market search produces a match and the marginal benefit of being employed.

When this insight is combined with the assumption that wages are determined according to a (postmatch) Nash bargain, it implies a linear relationship between labor market tightness and the marginal utility of consumption. We refer to this result as the *consumption-tightness puzzle*. This allows us to fully characterize the cyclical variations in labor market tightness on the basis of the cyclical variations in consumption. Therefore, a great advantage of our analysis is that we derive a simple, testable relationship that does not depend upon the source of shocks to the economy, nor on the persistence of these shocks.

We frame this relationship as a puzzle for the following reasons. First, it implies a very low volatility of the  $vu$ -ratio (or extreme volatility of consumption), since the standard deviation of the logarithm of the  $vu$ -ratio should equal the standard deviation of the logarithm of consumption times the curvature of the marginal utility of consumption. The latter is the coefficient of relative risk aversion (or the inverse of the intertemporal elasticity of substitution [IES]), and standard estimates of this parameter are small, and values above five are usually claimed to be implausible. In contrast, in U.S. quarterly data, the standard deviation of the  $vu$ -ratio is around twenty times higher than the standard deviation of consumption in the business cycle frequencies. Thus, theory can account for a maximum of 25 percent of the observed volatility of the  $vu$ -ratio. Expressed differently, the model implies procyclical unemployment, since vacancies are not only slightly more procyclical than realistic measures of the marginal utility of consumption, but also display much higher volatility. Alternatively, this latter insight can be formulated in terms of the slope of the Beveridge curve, which is positive in the model but negative in the data.

The rationale for why the matching model, with an extensive search margin, implies a positive correlation between unemployment and vacancies is straightforward. Consider a situation in which firms decide to post more vacancies. This increases households' payoff from labor market participation since, for a given unemployment rate, the probability increases that a job search will result in a job match. Therefore, there will be a positive correlation between vacancies and labor market participation. Moreover, since higher unemployment increases the returns from posting job vacancies, firms react by increasing job vacancies. This mechanism introduces a positive correlation between unemployment and vacancies, unless the variations in labor market tightness are related to large (inversely signed) variations in the marginal utility of consumption, and we argue that the latter is empirically implausible. This positive correlation between unemployment and vacancies also explains why the volatility of labor market tightness is low.

We show that these insights are robust, and study four extensions of the model. First, we introduce an intensive search margin. We assume that agents can vary their search effort, but that a higher search effort is costly. We show that this extension leaves the consumption-tightness puzzle unaltered for plausible parametrizations of the search effort costs. Next, we introduce home production. In this setup, the participation choice is a trade-off between forgoing the benefits of labor market

search, and giving up the resources generated by home production. This implies a modification to the relationship between consumption (of market goods) and tightness, but we argue it might possibly worsen the consumption-tightness puzzle. The reason is that the implied relationship no longer depends on the intertemporal elasticity of substitution of consumption.

The final two extensions alter the assumptions of the matching framework. We first allow for passive search, that is, for the possibility that non-participants might be matched with a job vacancy despite not actively searching for a job. This setup is (potentially) consistent with the fact that there are substantial flows from out of the labor force into employment. This extension does address the consumption-tightness puzzle, because there is less incentive to become search active when nonparticipation also allows agents to find jobs. Nevertheless, for realistic assumptions regarding the size of flows into employment from unemployment and non-participation, the consumption-tightness puzzle is approximately unchanged. In the second setting, we assume the matching technology is duration dependent. In particular, we assume unemployed workers might be faced with either an efficient or an inefficient matching technology, where the latter is associated with a smaller matching probability than the former. This setup gives rise to a relationship between consumption and an altered version of tightness, defined as the ratio of vacancies to the measure of search-active workers faced with the inefficient matching function (who are, on average, long-term unemployed). In this case, we leave it open whether duration dependence is important for the consumption-tightness puzzle, because it is hard to match the implied measure of unemployment with official unemployment statistics.

A key aspect of the labor market matching model with an extensive search margin is that the participation rate should be procyclical (positively correlated with consumption). Such procyclical movements in the participation rate can actually be observed in U.S. data. In particular, the secular rise in the participation rate that has occurred in the United States over the last sixty years slowed down in each of the recessions dated by the NBER Business Cycle Dating Committee. Furthermore, we show that a positive correlation exists between consumption and the participation rate at the business cycle frequencies. However, participation rates lag around a year after consumption, and the elasticity of the participation rate to consumption is very low. We argue that future research needs to look into the reasons why labor market participation, although procyclical, varies little over the business cycle.

The remainder of this chapter is structured as follows. Section 1.2 presents the basic model and derives the main result on the relationship between consumption and labor market tightness. Section 1.3 extends the basic setup to include, in turn, an intensive search margin, homework, passive search, and duration-dependent matching functions. Section 1.4 discusses the implications for variations in participation rates, and section 1.5 concludes and summarizes.

## 1.2 The Model

We study a stochastic optimal growth model combined with a labor market matching modeling of the labor market similar to that of Andolfatto (1996) and Merz (1995). We introduce a participation choice modeled as a trade-off between forgoing the opportunity of finding a job and the cost of giving up leisure in order to engage in labor market search activities. We show that introducing the extensive search margin (the participation choice) has fundamental implications.

### 1.2.1 Preferences and Technology

There is a measure one of a household. A household consists of a continuum of agents, and it is assumed that households pool the idiosyncratic labor market risk of their members. At any point in time a measure ( $n_t$ ) of the household members are employed and earn labor income, a measure ( $u_t$ ) are nonemployed but actively searching, and a measure ( $1 - n_t - u_t$ ) are out of the labor force. Unemployment is measured by the second group of agents. Thus, consistent with the measurement of U.S. unemployment, we define unemployed agents as characterized by (1) not matched with an employer, but (2) actively searching for a job.

Employed household members supply  $l_t$  hours of work, and, as in the labor-hoarding model of Burnside and Eichenbaum (1996), there is a fixed leisure cost of  $s \geq 0$  of engaging in labor market activities. Non employed actively searching household members also face the fixed cost ( $s$ ) of participating in labor market activities. Nonparticipants, instead, enjoy all their time endowment as leisure.

The period utility function of a household member is given as:

$$u(c_{it}, e_{it}) = G(c_{it}) + H(e_{it}), \quad (1)$$

where  $c_{it}$  denotes consumption, and  $e_{it}$  denotes leisure (given labor market status  $i = n, u, l$ ). We denote by  $i = n$  that the household member is

employed, by  $i = u$  that the household member is unemployed, and by  $i = l$  that the household member is not participating. The time endowment is normalized to one unit. It follows that  $e_{nt} = 1 - l_t - s$ ,  $e_{ut} = 1 - s$ , and  $e_{lt} = 1$ .

The flow utility of a representative household is then given as:

$$u(c_t, e_t) = G(c_t) + n_t H(1 - l_t - s) + u_t H(1 - s) + (1 - n_t - u_t) H(1).$$

Here we have used the risk-sharing principle that—due to separability of preferences—implies each household member consumes the same amount of goods regardless of his or her labor market status.

The subutility functions  $G$  and  $H$  are assumed to be increasing and strictly concave. We restrict  $G(c_t)$  to be of the form  $G(c_t) = c_t^{1-\eta}/(1-\eta)$  for  $\eta > 0$  and  $\eta \neq 1$ , or  $G(c_t) = \ln c_t$ . The parameter  $1/\eta$  is the intertemporal elasticity of substitution (IES). Utility is assumed to be additively separable over time:

$$W_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_{t+j}, e_{t+j}),$$

where  $E_t$  denotes the expectations operator conditional on information available at date  $t$ , and  $\beta < 1$  is the subjective discount factor.

Firms with vacancies and unemployed workers meet randomly in an anonymous matching market. Matches are formed according to the following matching function:

$$m_t = M(v_t, u_t), \tag{2}$$

where  $m_t$  is the measure of new matches between a measure of  $u_t$  unemployed workers and  $v_t$  vacant jobs in period  $t$ . The function  $M$  is assumed to be increasing and concave in each of its arguments, and to be homogeneous of degree one in vacancies and unemployment jointly. Given the constant returns assumption, we can express the matching function as:

$$m_t = u_t \varphi(\theta_t),$$

where  $\theta_t = v_t/u_t$  is the ratio of vacancies to unemployment and  $\varphi(\theta_t) = M(\theta_t, 1)$ . Thus, the probability that an actively searching worker finds a job vacancy,  $\gamma_t^h = m_t/u_t = \varphi(\theta_t)$ , is an increasing function of  $\theta_t$ , while the probability that a job vacancy is matched with an unemployed worker,  $\gamma_t^f = m_t/v_t = \varphi(\theta_t)/\theta_t$ , is a decreasing function of  $\theta_t$ . It follows that  $\gamma_t^h/\gamma_t^f = \theta_t$ . Hence, it is clear that the  $vu$ -ratio (labor market tightness), is a key variable, since it determines the matching market prospects of firms and workers.



This matching technology assumes that nonparticipants do not receive any job offers.<sup>1</sup> We later examine the consequences of allowing for passive search as well, that is, assuming job offers might arrive for nonparticipants as well.

Each period, firms and employed households face an exogenously given probability that their match is terminated. This probability is given by  $\sigma_t \in (0, 1)$ . Thus, the transition equation for employment is given as:

$$n_{t+1} = (1 - \sigma_t)n_t + u_t\varphi(\theta_t). \quad (3)$$

We assume that the job-separation rate follows an autoregressive process:

$$\ln \sigma_{t+1} = (1 - \rho_\sigma)\ln \sigma + \rho_\sigma \ln \sigma_t + \varepsilon_{t+1}^\sigma, \quad (4)$$

where  $\rho_\sigma \in (-1, 1)$ ,  $\sigma > 0$  denotes the unconditional mean of  $\sigma_t$ ,  $\varepsilon_{t+1}^\sigma$  is assumed to be normally and independently distributed over time with mean 0 and variance  $v_\sigma$ .

Output is produced using inputs of labor (the product of employment and hours worked per employee),  $n_t l_t$ , capital,  $k_t$ , and is subject to stochastic productivity shocks,  $z_t$ . We assume that firms take capital rental rates,  $(r_t)$  and the price of output (the numeraire) for given. As in Andolfatto (1996), we assume firms have a number of different jobs that may either be filled, posted in the vacancy market, or dormant. If firms decide to post a vacancy they must pay a resource cost ( $\kappa > 0$ ) per vacancy per period. In equilibrium, firms determine the optimal number of vacancies by maximizing their profits and taking into account the costs and benefits of vacancy postings. The firms are owned by the households, and their profits are paid out to the households as dividends.

The production function is specified by:

$$y_t = f(k_t, n_t l_t, z_t), \quad (5)$$

which we assume satisfies the Inada conditions, is increasing and strictly concave in  $k_t$  and in  $n_t l_t$ , and homogeneous of degree 1 in  $(k_t, n_t l_t)$ . The process for productivity shocks is assumed to be stationary, but possibly persistent:

$$\ln z_{t+1} = (1 - \rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z, \quad (6)$$

where  $\rho_z \in (-1, 1)$ ,  $\bar{z} > 0$  denotes the unconditional mean of  $z$ , and  $\varepsilon_{t+1}^z$  is assumed to be normally and independently distributed over time with mean 0 and variance  $v_z$ .

The capital stock evolves over time according to the standard neo-classical specification:

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (7)$$

where  $\delta \in (0, 1)$  denotes the depreciation rate, and  $i_t$  is gross investment.

The resource constraint of the economy is then given by:

$$y_t \geq c_t + i_t + \kappa v_t. \quad (8)$$

We assume wages are determined according to a standard Nash bargaining over the joint match surplus of a worker-job pair. We let  $\vartheta$  denote the bargaining weight of the workers. We do not impose the Hosios (1990) condition, since our results will hold regardless of this efficiency consideration.

We will now derive the implications of this model on the basis of the competitive search equilibrium. Given the recursive structure of the model, we remove time indices and use the notation  $x'$  to denote the next-period value of the variable  $x$ .

### 1.2.2 The Households' Problem

The maximization problem of the representative household can be formulated on the basis of the following Bellman equation:

$$J(k, n) = \max_{(c, k', u, n')} \{c^{1-\eta}/(1-\eta) + nH(1-l-s) + uH(1-s) + (1-n-u)H(1) + \beta EJ(k', n')\}, \quad (9)$$

$$c + k' \leq (1 - \delta + r)k + wnl + \pi \quad (10)$$

$$n' = (1 - \sigma)n + \gamma^h u. \quad (11)$$

The representative households' value function is denoted by  $J(k, n)$ , which depends on its holdings of capital, and the share of the household members that are employed.<sup>2</sup> We use the notation  $Ex'$  to denote the expectation of  $x'$  conditional on all available current information (including the transition laws for the exogenous shocks and the aggregate state variables). Equation (10) is the budget constraint, which states that total spending on consumption ( $c$ ) and capital for the next period ( $k'$ ) cannot exceed the sum of the value of its remaining capital stock ( $k - \delta k$ ), rental income from capital ( $rk$ ), labor income ( $wnl$ ), and the dividends received from its ownership of the firms ( $\pi$ ).

Equation (11) is the households' employment transition function. It

relates the share of household members who are employed next period ( $n'$ ), to this period's employment ( $n$ ) corrected for net new employment. The latter is given by the number of new worker job matches ( $\gamma^h u$ ) minus the separations of currently employed household members from their jobs ( $\sigma n$ ). Importantly, individual households take the matching probability ( $\gamma^h$ ) for given.

The first-order conditions for  $c$ ,  $k'$ ,  $u$ , and  $n'$ , in that order, are given by:

$$c^{-\eta} = \lambda_c \quad (12)$$

$$\lambda_c = \beta E J_k(k', n'), \quad (13)$$

$$H(1) - H(1 - s) = \gamma^h \lambda_n, \quad (14)$$

$$\lambda_n = \beta E J_n(k', n'), \quad (15)$$

and the envelope conditions are:

$$J_k(k, n) = \lambda_c(1 - \delta + r), \quad (16)$$

$$J_n(k, n) = \lambda_c w l + (1 - \sigma) \lambda_n + H(1 - l - s) - H(1). \quad (17)$$

Combining the first-order conditions for  $u$  and  $n'$  implies that:

$$\gamma^h \beta E J_n(k', n') = H(1) - H(1 - s). \quad (18)$$

Equation (18) is key. The right-hand side of this expression is the utility loss associated with a marginal change in the share of household members that are search active, rather than nonparticipating. This utility loss comes from the active search agents need to spend time on search activities that nonparticipants instead enjoy as leisure. The left-hand side of the expression is the expectation of the change in the value of employment produced by a marginal change in the number of search-active household members. This is given by the probability that an active search agent is matched with a vacancy ( $\gamma^h$ ), times the expected marginal value of employment next period ( $E J_n$ ), discounted at the rate of  $\beta$ .

Combining condition (18) with condition (17) gives us:

$$\begin{aligned} & \frac{H(1) - H(1 - s)}{\gamma^h} \\ &= \beta E \left\{ w l' c'^{-\eta} + (1 - \sigma') \frac{H(1) - H(1 - s)}{\gamma^{h'}} - [H(1) - H(1 - l' - s)] \right\}. \quad (19) \end{aligned}$$

This is similar to the more familiar vacancy creation condition (which we derive in the following). It sets the cost of labor market search equal to the expected benefits. The latter consists of the sum of the (utility value of the) marginal increase in labor income, and the future search costs savings minus the utility value of the loss of leisure (associated with working rather than enjoying the entire time as leisure).

### 1.2.3 The Firms' Problem

Bellman's equation for the firms' problem is given as:

$$Q(n) = \max_{k,v,n'} \left[ F(k, nl) - wnl - \kappa v - rk + \beta E \frac{u'_c}{u_c} Q(n') \right], \quad (20)$$

$$n' = (1 - \sigma)n + \gamma^f v, \quad (21)$$

where  $Q(n)$  is the value of a firm with  $n$ -filled jobs. The objective function consists of the current profit flow ( $\pi = F(k, nl) - wnl - \kappa v - rk$ ), plus the discounted expected future value. The maximization takes place subject to the job transition function, which links the future number of filled jobs to the current stock of filled jobs plus net hiring where the latter is the difference between new hires ( $\gamma^f v$ ), and exogenous terminations of current jobs ( $\sigma n$ ).

The first-order conditions for this problem can be formulated as:

$$F_k = r \quad (22)$$

$$\frac{\kappa}{\gamma^f} = \beta E \frac{u'_c}{u_c} [Q_n(n')], \quad (23)$$

and from the envelope condition it follows that:

$$Q_n(n) = F_n - wl + (1 - \sigma)\beta E \frac{u'_c}{u_c} [Q_n(n')]. \quad (24)$$

Combining this with equation (23) gives us:

$$\frac{\kappa}{\gamma^f} = \beta E \frac{u'_c}{u_c} \left[ F_{n'} - w'l' + (1 - \sigma') \frac{\kappa}{\gamma^{f'}} \right]. \quad (25)$$

Condition (22) equalizes the rental rate of capital with the marginal product of capital. Equation (23) is the condition for the optimal number of vacancy postings. The latter sets the vacancy posting cost ( $\kappa$ ) equal to

the expected discounted value of posting a vacancy, which is given by the probability that a vacancy results in a new hire ( $\gamma^f$ ) times the marginal value of filling a vacancy [ $Q_n(n')$ ] discounted by  $\beta u'_c/u_c$ . The value of filling a vacancy, in turn, is the sum of the marginal profit (the difference between the marginal product of a hire and the marginal wage cost) plus the expected future vacancy posting cost savings. Combining these expressions gives us the condition for vacancy postings given in equation (25).

#### 1.2.4 Wages

Wages are determined by ex post (after matching) Nash bargaining. This implies that employers and workers share the joint match surplus according to their bargaining power. Let  $\vartheta \in (0, 1)$  denote the firms' bargaining power and let  $S_n$  denote the joint match surplus. The match surplus is given as:

$$S_n = Q_n(n) + \frac{1}{c^{-\eta}} J_n(k, n),$$

and the surplus is divided so that:

$$\vartheta J_n(k, n) = c^{-\eta}(1 - \vartheta)Q_n(n), \quad (26)$$

where  $Q_n$  and  $J_n$  were derived above. Evaluating condition (26) for the next period, and taking expectations given today's information set, we have:

$$\vartheta \beta E c^\eta J_n(k', n') = (1 - \vartheta) \beta E \frac{c'^{-\eta}}{c^{-\eta}} Q_n(n').$$

This condition is simplified by using the first-order conditions from the households' and the firms' problems. In particular, we have:

$$(1 - \vartheta) \beta E \frac{c'^{-\eta}}{c^{-\eta}} Q_n = (1 - \vartheta) \frac{\kappa}{\gamma^f}$$

$$\vartheta \beta E c^\eta J_n = \vartheta \frac{H(1) - H(1 - s)}{\gamma^h} c^\eta.$$

Therefore the Nash bargaining outcome implies:

$$(1 - \vartheta) \frac{\kappa}{\gamma^f} = \vartheta \frac{H(1) - H(1 - s)}{\gamma^h} c^\eta, \quad (27)$$

which is the key relationship that we discuss below.

Using these, we can derive the equilibrium wage bill per employee as:

$$wl = (1 - \vartheta)F_n + \vartheta c^n [H(1) - H(1 - l - s)],$$

which determines the wage as a weighted average of the marginal product of employment, and the utility-weighted leisure cost of working rather than enjoying the time as leisure.

### 1.2.5 The Consumption-Tightness Puzzle

We can now derive the key result, which is summarized by the following proposition:

**PROPOSITION 1.** *In the competitive search equilibrium, independent of the source of shocks to the economy, the  $vu$ -ratio is related to consumption throughout the following condition:*

$$\theta = \frac{\vartheta}{1 - \vartheta} \omega c^n, \quad (28)$$

where  $\omega$  is a constant given by  $[H(1) - H(1 - s)]/\kappa$ .

**PROOF.** The result follows simply from rearranging condition (27) using  $\gamma^h/\gamma^f = (m/u)/(m/v) = \theta$ .

This equation summarizes (in a simple way) the central implications for variations in unemployment and vacancies in the labor market matching model, with an endogenous participation choice. As we will show, the relationship implies: (a) low volatility of the  $vu$ -ratio, (b) a strong tendency for procyclical movements in unemployment, and (c) a positively sloped Beveridge curve. Before we show these results, it is worth mentioning that the relationship between labor market tightness and consumption, derived above, does not depend on stochastic processes for job-separation shocks and technology shocks, and it also does not depend on the absence of capital adjustment costs (or on the production technology).

Table 1.1 reports some selected moments of U.S. aggregate output and labor market variables at the business cycle frequencies. We present moments of quarterly data for the sample period 1964–2004. In order to isolate movements in relevant variables at the business-cycle frequencies, data were detrended with either the Hodrick and Prescott (1997) filter or the Baxter and King (1999) approximate band-pass filter.<sup>3</sup> We ex-

**Table 1.1**  
U.S. Business Cycle Statistics, 1964–2004

Variable	Standard deviation %	Correlations with	
		Output	Consumption
<i>HP filtered Data</i>			
Output	1.56	1	—
Consumption	1.23	0.87	—
Total hours	1.75	0.91	—
Unemployment	10.81	−0.87	−0.71
Vacancies	13.18	0.91	0.81
<i>vu</i> -ratio	23.66	0.90	0.77
Ratio of vacancies to unempl. > 15 weeks	34.71	0.85	0.68
Participation rate	0.31	0.45	0.27
<i>BK filtered Data</i>			
Output	1.51	1	—
Consumption	1.22	0.89	—
Total Hours	1.77	0.88	—
Unemployment	10.97	−0.89	−0.73
Vacancies	13.15	0.93	0.81
<i>vu</i> -ratio	23.85	0.92	0.78
Ratio of vacancies to unempl. > 15 weeks	35.52	0.87	0.70
Participation rate	0.28	0.48	0.29

amine the properties of aggregate output, aggregate consumption, aggregate hours worked, aggregate unemployment, and vacancies, all as ratios of the U.S. civilian noninstitutional population. The table also reports the moments of the *vu*-ratio. Consumption is measured as U.S. private sector consumption of non durables and services. Hours worked are aggregate hours worked in the non farm part of the economy. Unemployment is the total number of unemployed persons as reported by the Bureau of Labor Statistics. Vacancies are measured on the basis of an index of help wanted advertisements.<sup>4</sup> The table reports the percentage standard deviations of these variables, and some selected cross-correlations.

In the United States, unemployment is strongly countercyclical and very volatile. At the business-cycle frequencies, the standard deviation

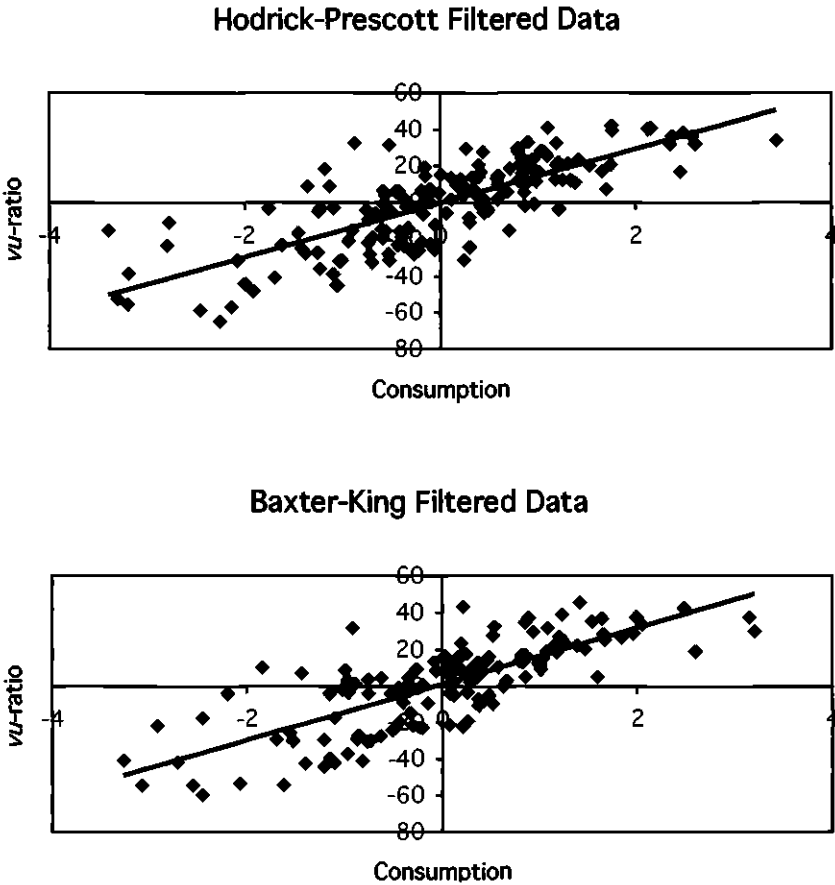
of unemployment is close to 11 percent per quarter, or more than seven times higher than that of output (nine times that of consumption). The contemporaneous correlation between unemployment and output is close to  $-0.90$ . Vacancies are even more volatile than unemployment (its standard deviation is above 13 percent per quarter), and display very procyclical behavior at the business cycle frequencies (the correlation between vacancies and output is above 90 percent). The strong negative contemporaneous correlation between unemployment and vacancies, which forms part of the classic Beveridge curve relationship, then implies high volatility of the  $vu$ -ratio (its standard deviation is sixteen times that of output, or around twenty times that of consumption) and a contemporaneous positive correlation with output in excess of 0.90.

Consistent with condition (28), the  $vu$ -ratio and consumption are positively correlated (the cross-correlation is approximately 80 percent in the U.S. data). Figure 1.1 illustrates consumption, plotted against the  $vu$ -ratio for the two detrending methods. The figure clearly visualizes the positive correlation between them. The  $R^2$  measure of fit is (as high as) 60 percent and 62 percent for Hodrick-Prescott (HP) filtered data and Baxter-King (BK) filtered data, respectively.

However, for realistic degrees of the intertemporal elasticity of substitution, theory can account for only a small fraction of the observed volatility of the  $vu$ -ratio. Notice that condition (28) implies that regressing the (logarithm of the)  $vu$ -ratio on (the logarithm of) consumption should give an estimate of the inverse of the IES, or alternatively, that the standard deviation of the  $vu$ -ratio implied by the model is equal to the inverse of the IES, times the standard deviation of consumption. The slopes of the regression lines in figure 1.1 imply estimates of the inverse of the IES equal to 14.8 and 15.3, respectively, for the two panels of figure 1.1. The ratio of the standard deviations instead implies values of  $\eta$  of 19.2 and 19.5 for HP filtered and BK filtered data, respectively. These estimates are far above the values of  $\eta$  normally considered realistic. Estimates by Eichenbaum, Hansen, and Singleton (1988), Friend and Blume (1975), Neely, Roy, and Whiteman (2001), and many others, indicate realistic estimates of  $\eta$  are in the range of 0.5–3 (see Mehra and Prescott 2003, for an extensive discussion). Said differently, for standard values of the IES, using the observed volatility of consumption, the model can account for only a small fraction (less than 25 percent) of the volatility of the  $vu$ -ratio.<sup>5</sup>

Another way of expressing these insights is in terms of the covariance





**Figure 1.1**  
The Consumption–VU-ratio Relationship

implications. In particular, for realistic second moments of vacancies and consumption, the labor market matching model implies procyclical unemployment. To see this, note that condition (28) can be expressed as:

$$u = \frac{v}{c^\eta} \frac{1 - \alpha}{\alpha \omega}.$$

Taking logarithms gives us:

$$\text{cor}(\hat{u}, \hat{c}) = \frac{\text{cov}(\hat{c}, \hat{v})}{[\text{var}(\hat{u})\text{var}(\hat{c})]^{1/2}} - \eta \left( \frac{\text{var}(\hat{c})}{\text{var}(\hat{u})} \right)^{1/2},$$

where a variable with a hat ( $\hat{\cdot}$ ) indicates the logarithm of the variable.



**Figure 1.2**

**Actual versus Implied Unemployment**

*Note:* This figure illustrates the actual U.S. unemployment level, with the unemployment level implied by theorem 1. The diamonds (squares) illustrate the relationship when assuming  $\eta = 1$  ( $\eta = 10$ ). The linear regression lines show that there is a negative relationship between the actual and predicted unemployment levels.

Suppose the model would be able to reproduce the empirical estimates of the moments of the data that enter on the right hand side of this expression. In this case, using the estimates in Table 1, the cross-correlation between unemployment and consumption would equal approximately  $0.99 - \eta/10$ .<sup>6</sup> Taking a value of  $\eta$  in the upper end of the empirically plausible estimates,  $\eta = 3$ , implies that  $\text{cor}(\hat{u}, \hat{c}) = 0.69$ . In U.S. data, this correlation is  $-0.70$  (see table 1.1). Therefore, even if the model could reproduce the correlation between vacancies and consumption and the variances of consumption, vacancies, and unemployment, it would require very large and unrealistic values of  $\eta$  to account for the countercyclical movements in unemployment observed in the data.

In order to visualize the extent to which the actual and implied unemployment rates differ, figure 1.2 plots the actual unemployment rate against the unemployment rate implied by the above relationship for  $\eta = 1$  (a realistic value) and for  $\eta = 10$  (an unrealistically high value) on the basis of HP-filtered U.S. data. In both cases, there is a strong negative association between the actual and implied unemployment rate.

The intuition for the tendency for procyclical unemployment is straightforward. In this model, while employment is predetermined, unemployment is not a state variable, since households can adjust the number of agents that are actively searching through variations in the

participation rate. An increase in vacancies increases the expected payoff from labor market search, since the probability of being matched with a vacancy rises. Therefore, the participation rate increases, which leads to a tendency for procyclical unemployment. This effect is moderated only by the extent to which the underlying shock lowers the marginal utility of consumption (which lowers the payoff from search activities). The latter effect, however, is only quantitatively important when the curvature of the utility function is very large, and we argue that plausible estimates of the IES imply moderate curvature. Therefore, fluctuations in vacancies tend to induce equally signed fluctuations in unemployment through variations in the participation rate. In other words, the Beveridge curve is—counterfactually—positively sloped when we allow for a participation choice.

In sum, once one allows agents to choose whether to actively search or not, the labor market matching model gives rise to a consumption-tightness puzzle in the sense that unrealistically high degrees of risk aversion (low degrees of intertemporal elasticity of substitution) are required to account for (a) the volatility of the  $vu$ -ratio, and (b) the countercyclical movements in unemployment (and a negatively sloped Beveridge curve).

### 1.3 Extensions

We now examine a number of extensions of the basic model, in order to gauge the robustness of the consumption-tightness puzzle highlighted in the previous section. As we will show, the qualitative features of the results are robust.

#### 1.3.1 Variable Search Effort

The first extension introduces variable search intensity into the previous model. We assume that, for given levels of unemployment and vacancies, when more resources are spent on a job search, more matches will be produced between unemployed workers and firms with vacant jobs. As in Merz (1995), higher search effort is assumed to give rise to a resource cost.<sup>7</sup> Allowing for variable search effort may, therefore, moderate the previous results, since the tendency for households to devote more resources to search activities when vacancies rise can also be achieved through variations in the intensive search margin.

With variable search effort, the matching technology is given as:

$$m_t = M(v_t, h_t u_t),$$

where  $h_t$  denotes search effort. We assume that the matching technology displays constant returns to  $v_t, h_t u_t$ , jointly. Thus, the probability that an actively searching worker finds a job vacancy,  $m_t/u_t = h_t \phi(\theta_t/h_t)$ , is an increasing function of  $\theta_t$  and  $h_t$ .

We assume that higher search effort (along the intensive margin) gives rise to a resource cost,  $d(h_t)$  per search-active household member. The economy's resource constraint now reads:

$$y_t \geq c_t + i_t + \kappa v_t + u_t d(h_t),$$

where  $d$  is an increasing and convex function.

The first-order necessary conditions for the households' problem (see Appendix A for details) imply that optimal search effort ( $h$ ) is determined such that:

$$c^{-\eta} \frac{\partial d(h)}{\partial h} = \beta \gamma^h E J_n(k', n'), \quad (29)$$

where  $\gamma^h = m/(uh)$ .

This condition states that, in the optimum, marginal search costs equal the probability that a new match is formed times the marginal value of a match. Combining this equation with the households first-order condition for the choice of  $u$  implies that:

$$H(1) - H(1 - s) = c^{-\eta} [\psi(h) - 1] d(h), \quad (30)$$

where  $\psi$  is the elasticity of  $d(h)$ ,  $\psi(h) = [\partial d(h)/\partial h][h/d(h)]$ . Thus, if the elasticity of the search effort costs is constant,  $c^{-\eta}$  and  $d(h)$  will be perfectly negatively correlated. In other words, under these conditions search effort will be positively correlated with consumption, regardless of the source of shocks to the economy.<sup>8</sup>

After some algebra, we can show that when we allow for variable search efforts at both the intensive and the extensive margins, the following condition must hold:

$$\theta = \frac{\alpha}{1 - \alpha} \omega c^\eta \frac{\psi(h)}{\psi(h) - 1}. \quad (31)$$

This expression differs from the one derived under the assumption of constant search effort only by the term  $\psi(h)/[\psi(h) - 1]$  that appears on the left-hand side. When  $\psi(h)$  is constant, the model with variable search effort delivers exactly the same predictions regarding the volatility of

the  $vu$ -ratio and the cyclical features of unemployment as the model with constant search effort.

### 1.3.2 Homework

Next we consider an extension of the basic model with homework (see, e.g., Benhabib, Rogerson, and Wright 1991).<sup>9</sup> This extension modifies the trade-off between labor market search and nonparticipation, since agents now have the opportunity of spending part of their time endowment on home production.

Agents consume two types of goods: market goods ( $c_m$ ) and goods produced in the home sector ( $c_h$ ). Both goods are produced using inputs of capital and labor, and are subject to productivity shocks. Goods produced in the home sector are used for consumption only.

Per capita hours supplied to the home sector are given as:

$$\mu = n\mu_n + u\mu_u + (1 - n - u)\mu_l,$$

where  $\mu_n$  denotes hours worked at home of an employed worker,  $\mu_u$  hours worked at home of an unemployed household member, and  $\mu_l$  hours worked at the home of a nonparticipant. The home production resource constraint is given as:

$$c_h \leq g[(1 - x)k, \mu, z^h], \quad (32)$$

where  $c_h$  is the consumption of home-goods, and  $x$  denotes the fraction of the aggregate capital stock that is used for production in the market sector. The variable  $z^h$  represents temporary productivity shocks to the home production technology, which we assume are generated by a first-order autoregressive process with innovations that are possibly correlated with the innovations to  $z$ . We assume that  $g$  is increasing and concave in  $(1 - x)k$  and in  $\mu$ , and that it is homogeneous of degree one in  $[(1 - x)k, \mu]$  jointly.

The period utility function is given as:

$$u(c, e_i) = c^{1-\eta}/(1-\eta) + H(e_i),$$

where  $e_n = 1 - s - l - \mu_n$ ,  $e_u = 1 - s - \mu_u$ , and  $e_l = 1 - \mu_l$ ; and  $c$  is an aggregate of the consumption of the two goods:

$$c = C(c_m, c_h).$$

We assume that  $C$  is increasing, concave, and homogeneous of degree one. Finally, the resource constraint for the market sector now reads:

$$c_m + k' + \kappa v \leq f(xk, nl, z) + (1 - \delta)k. \quad (33)$$

The households' problem can now be expressed as choosing sequences of consumption, capital stocks, hours worked in the home sector, the share of active search agents, and the division of capital between sectors to solve:

$$J(k, n) = \max_{(c, n', k', u, \mu_i, x)} [c^{1-\eta}/(1-\eta) + nH_w(1-l-s-\mu_n) + uH_u(1-s-\mu_u) + (1-n-u)H_n(1-\mu_i) + \beta EJ(k', n')], \quad (34)$$

subject to the constraints:

$$c_m + k' \leq (1 - \delta + rx)k + wnl + \pi,$$

$$n' = (1 - \sigma)n + \gamma^h u,$$

$$c_h \leq g[(1-x)k, \mu, z^h].$$

The first-order conditions are described in detail in Appendix B. A key implication follows from the first-order condition for hours devoted to homework, which is given as:

$$\frac{\partial H(x_i)}{\partial \mu_i} = \frac{\partial C / \partial c_h}{\partial C / \partial c_m} \frac{\partial g}{\partial \mu} \beta EJ_k(k', n').$$

Notice that the right-hand side of this expression does not depend on the labor market status. Therefore, under the condition that  $H$  is strictly concave, it follows that:

$$\mu_i = \mu_u + s = \mu_n + l + s. \quad (35)$$

In other words, leisure does not depend on labor market status. Thus, agents that are nonparticipants compensate for their lack of hours devoted to market activities by working  $s$  more hours at home than agents that are search active, and  $s + l$  hours more at home than agents that are employed. This reflects risk sharing: the marginal disutility of work is equalized across agents (that differ by their labor market status).

We follow Gomme, Kydland, and Rupert (2001) and assume that the consumption aggregator ( $C$ ) is given by a Cobb-Douglas function:

$$c = c_m^\xi c_h^{1-\xi}, \quad \xi \in (0; 1), \quad (36)$$

and that the home-good production function is given by a Cobb-Douglas production function:

$$g[(1-x)k, \mu, z^h] = z^h[(1-x)k]^\tau \mu^{1-\tau}, \tau \in (0; 1). \quad (37)$$

We can now derive the following result:

**PROPOSITION 2.** *In the homework economy with a Cobb-Douglas consumption aggregator and Cobb-Douglas home production function, the  $vu$ -ratio and consumption of market goods are related as:*

$$\theta = \frac{c_m}{\mu} \frac{\vartheta}{1-\vartheta} \omega_h, \quad (38)$$

where  $\omega_h = [(1-\xi)/\xi](1-\tau)(s/\kappa)$ .

**PROOF.** Using  $\mu_l = \mu_u + s = \mu_n + l + s$ , the first-order conditions for the optimal choices of  $u$ ,  $\mu_l$ ,  $c_m$ , and  $c_h$ , and the outcome of the wage bargaining implies that:

$$\frac{\partial C}{\partial c_h} (\mu_l - \mu_u) \frac{\partial g}{\partial \mu} = \frac{1-\vartheta}{\vartheta} \kappa \frac{\partial C}{\partial c_m} \theta,$$

where equation (35) implies that  $\mu_l - \mu_u = s$ . Using function (37) we have:

$$\frac{\partial g[(1-x)k, \mu, z^h]}{\partial \mu} = \frac{(1-\tau_h)c_h}{\mu},$$

and from function (36) it follows that:

$$\frac{\partial c}{\partial c_h} \bigg/ \frac{\partial c}{\partial c_m} = \frac{1-\xi}{\xi(c_m/c_h)}.$$

Inserting these gives equation (38).

This relationship differs from condition (28) in two ways. First, it no longer involves the risk aversion parameter  $\eta$ . Second, the relationship also involves the number of hours supplied to the home sector ( $\mu$ ). The latter aspect might, potentially, help address the consumption-tightness puzzle. In particular, a negative covariance between consumption of market goods and hours supplied to the home sector induces volatility in the  $vu$ -ratio. Most models with home production do imply strongly countercyclical movements in hours supplied in the home sector (Gomme, Kydland and Rupert 2001).

Quantitatively, however, even a substantial negative covariance between the consumption of market goods and homework hours is un-

likely to help explain the gap between the observed volatility of the  $vu$ -ratio and that implied by the growth model with labor market frictions and a participation choice. The model, therefore, still has a strong tendency for procyclical movements in unemployment and for a positive contemporaneous correlation between unemployment and vacancies. To see this, consider the following calculation. The standard deviation of HP-filtered (per capita) hours worked in the market sector is around 1.75 percent per quarter in the United States (see table 1.1). The volatility of hours worked in the home sector is unlikely to be higher than this. Thus, even if consumption of market goods and hours worked in the home sector were perfectly negatively correlated, the implied standard deviation of the  $vu$ -ratio would be no higher than 2.59 percent (around ten times lower than the standard deviation of the  $vu$ -ratio in the U.S. data).<sup>10</sup> For the same reasons, the model with homework implies a positive correlation between unemployment and vacancies and procyclical unemployment.

Therefore we conclude that the introduction of homework does not impact the consumption-labor market tightness puzzle. On the contrary, it may even worsen.

### 1.3.3 *Passive Search*

The matching technology analyzed so far assumes that nonparticipants do not receive any job offers. However, in U.S. data there are substantial flows from out of the labor force directly into employment; see Davis, Faberman and Haltiwanger (2006) for a recent review. For this reason, Andolfatto and Gomme (1996), Pries and Rogerson (2004), and Yip (2003) assume that wage offers might be received by both unemployed workers and by nonparticipants.<sup>11</sup>

We now extend the model of section 1.2, by allowing for passive search. We assume that the aggregate matching function is given as:

$$m_t = M^u(v_t, u_t) + M^l(v_t, 1 - n_t - u_t).$$

We will assume that  $m_t^u/u_t \geq m_t^l/(1 - n_t - u_t)$ , in order to be consistent with the observation that the matching frequency of unemployed workers is much higher than the matching frequency of nonparticipants. This assumption also compares well with Flinn and Heckman's (1983) finding that unemployment helps facilitate job searches relative to nonparticipation.



The households' problem is given as:

$$J(k, n) = \max_{(c, k', u, n')} \{c^{1-\eta}/(1-\eta) + nH(1-l-s) + uH(1-s) + (1-n-u)H(1) + \beta EJ(k', n')\}, \quad (39)$$

$$c + k' \leq (1 - \delta + r)k + wnl + \pi, \quad (40)$$

$$n' = (1 - \sigma)n + \gamma_1^h u + \gamma_2^h (1 - n - u), \quad (41)$$

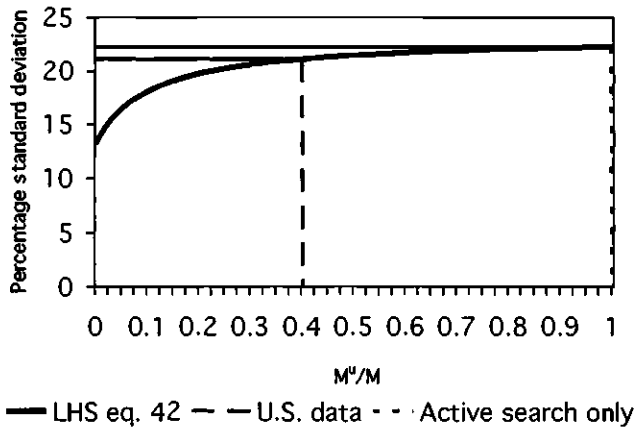
where equation (41) takes into account that nonparticipants as well as unemployed household members might become matched. In this equation we define  $\gamma_1^h = m^u/u$  and  $\gamma_2^h = m^l/(1-n-u)$ .

The firms' problem is unchanged (apart from the change in the probability that a vacancy is filled). Going through the same steps as in section 1.2 gives us the following condition:

$$\theta \frac{u + (1-n)\frac{m^u}{m}}{1-n-u} = \frac{\vartheta}{1-\vartheta} \omega c^\alpha, \quad (42)$$

which is identical to condition (28), apart from the ratio  $[u + (1-n)(m^u/m)]/(1-n-u)$  that appears on the left-hand side. Note that this ratio is equal to 1 when  $m^u/m = 1$  as we assumed in section 1.2. When  $m^u/m$  approaches 0 instead, this ratio becomes equal to  $u/(1-n-u)$ . In this case, the left-hand side of condition (42) becomes equal to the ratio of vacancies to nonparticipation.

According to Fallick and Fleischman (2004), the mean flow from non-employment into employment is (approximately) equal to 4.6 million in the United States, and the mean flow from unemployment accounts for around 1.8 million of these new job findings. Thus,  $m^u/m$  is equal to approximately 40 percent. Assuming that this ratio is constant, we can then compute the left-hand side of condition (42). Figure 1.3 illustrates the implied standard deviation of (HP filtered values of) the left-hand side of condition (42) for alternative values of  $m^u/m$ . When  $m^u/m$  approaches zero the implied percentage standard deviation of the left-hand side is 13.2, which is 40 percent lower than the standard deviation of tightness itself. However, when we set  $m^u/m$  equal to its mean U.S. value, the implied standard deviation of the left-hand side of condition (42) is 21.2, which is only marginally lower than the volatility of tightness. Therefore, while allowing for passive search helps address the consumption-tightness puzzle, quantitatively this feature does not appear to matter



**Figure 1.3**  
 Passive Search Model

*Note:* The full drawn line illustrates the percentage standard deviation of HP-filtered values of the left-hand side of equation (42) for alternative values of the share of total matches due to passive search ( $M^u/M$ ). The mean value of  $M^u/M$  in U.S. data is around 40 percent. The model of section 2 of the paper assumes that  $M^u/M$  is equal to 1.

significantly because the left-hand side of condition (42) is insensitive to  $m^u/m$  unless this ratio becomes very small.

Therefore, we conclude that allowing for passive search is not essential for the results.

### 1.3.4 Duration-Dependent Matching Functions

The previous section analyzed a setting in which active search agents and nonparticipants face heterogenous matching functions. An alternative modeling is that unmatched agents differ in their labor market prospects, even when actively searching. In particular, recently unemployed agents may face more efficient matching functions than longer-term unemployed workers or out of the labor force. We now analyze such a setting.

We assume there are two types of unemployed workers who differ in their prospects of being matched with vacancies, short-term unemployed, and long-term unemployed. Long-term unemployed workers face a less efficient matching technology than the short-term unemployed, and this group of agents may choose to become nonparticipants.

As in section 1.2, we assume that only active search agents receive job offers.

The labor market flow dynamics are as follows. Every period, a fraction ( $\sigma$ ) of the currently employed worker job matches are terminated, and a measure ( $M$ ) of new matches is formed. Workers who experience a termination of their matches enter into short-term unemployment.<sup>12</sup> A short-term unemployed household member may either remain short-term unemployed, become matched with a vacancy, or experience a transition to long-term unemployment. We assume that the latter event occurs with probability  $\mu \in (0; 1)$ . New matches are formed between vacant jobs and search active unmatched agents, but the number of matches depends on both labor market tightness and on the structure of unemployment.

Formally, we assume that the aggregate number of matches is given as:

$$M(v, u_1, u_2) = m_1(v, u_1) + m_2(v, u_2),$$

$$m_1(v, u) > m_2(v, u) \text{ for } \forall v, u > 0,$$

where  $u_1$  denotes the measure of short-term unemployed workers, and  $u_2$  the measure of long-term unemployed.

The employment transition equation is now given as:

$$n' = (1 - \sigma)n + m_1 + m_2, \quad (43)$$

and the transition equation for short-term unemployment is given as:

$$u_1' = (1 - \phi)u_1 + \sigma n - m_1, \quad (44)$$

where  $\phi$  is the probability that a currently short-term unemployed worker becomes long-term unemployed.

Bellman's equation for the households' problem is:

$$J(k, n, u_1) = \max_{(c, k', u)} \{c^{1-\eta}/(1-\eta) + nH(1-l-s) + (u_1 + u_2)H(1-s) + (1-n-u_1-u_2)H(1) + \beta EJ(k', n', u_1)\}, \quad (45)$$

where we note that short-term unemployment is now a state variable. The Bellman equation is maximized subject to the constraints:

$$c + k' \leq (1 - \delta + r)k + wnl + \pi, \quad (46)$$

$$n' = (1 - \sigma)n + \gamma_1^h u_1 + \gamma_2^h u_2, \quad (47)$$

$$u_1' = (1 - \phi)u_1 + \sigma n - \gamma_1^h u_1. \quad (48)$$

The variable  $\gamma_1^h$  denotes the probability that a short-term unemployed actively search household member is matched with a vacancy, and  $\gamma_2^h$  is the equivalent probability for a long-term unemployed worker.

In this model, the participation choice is relevant for long-term unemployed household members. Under mild conditions of  $\gamma_1^h$  relative to  $\gamma_2^h$ , household members are better off searching as long as they are faced with the more efficient matching technology.

The first-order condition for the optimal choice of  $u_2$  is given by:

$$\gamma_2^h \beta E J_n(k', n') = H(1) - H(1 - s). \quad (49)$$

This condition is equivalent to condition (18), derived in section 1.2, apart from the definition of the matching market prospect. The marginal value of employment, however, now takes into account the multiple matching functions. It is given as:

$$J_n(k, n, u_1) = c^{-\eta} w l - [H(1) - H(1 - l - s)] + (1 - \sigma) \beta E J_n(k', n', u_1) \\ + \sigma \beta E J_{u_1}(k', n', u_1).$$

This determines the marginal value of a job as a sum of the utility value of the labor income, the expected marginal value of being employed the next period times the probability that the match survives (discounted one period), the expected marginal value of short-term unemployment times the probability that the match is terminated, and less the utility value of the loss of leisure of working rather than enjoying the time endowment as leisure. The marginal value of short-term unemployment, in turn, is given as:

$$J_{u_1}(k, n, u_1) = \gamma_1^h \beta E J_n(k', n', u_1) + [(1 - \phi) - \gamma_1^h] \beta E J_{u_1}(k', n', u_1) \\ - [H(1) - H(1 - s)].$$

A short-term unemployed worker finds a job match with probability  $\gamma_1^h$ , which gives him or her the value  $\beta E J_n(k', n', u_1)$ . With probability  $[(1 - \phi) - \gamma_1^h]$ , a currently short-term unemployed worker is still unemployed next period, giving her or him a value of  $\beta E J_{u_1}(k', n', u_1)$ . Finally, actively searching rather than nonparticipating gives rise to a utility loss  $[H(1) - H(1 - s)]$ , due to the search effort that must be exerted.

The firms' problem is now:

$$Q(n) = \max_{k,v} \{F(k, nl) - wnl - \kappa v - rk + \beta E \frac{u'_c}{u_c} Q(n')\}, \quad (50)$$

subject to:

$$n' = (1 - \sigma)n + (\gamma_1' + \gamma_2')v. \quad (51)$$

Note that we assume firms cannot target any of the two matching markets individually. The first-order condition for the choice of capital is identical to the model in section 1.2. The vacancy posting condition, however, is now:

$$\kappa = (\gamma_1' + \gamma_2')\beta E \frac{u_c'}{u_c} (Q_n), \quad (52)$$

where

$$Q_n = F_n - wl + (1 - \sigma)\beta E \frac{u_c'}{u_c} (Q_n).$$

It is important to notice that the relevant first-order condition for households' search efforts at the extensive margin involves the probability that long-term unemployed household members find a job match, while firms' first-order condition for vacancy postings involve the probability of meeting any unmatched search active worker. If possible, firms would prefer target vacancies in the matching market that yield the highest possible probability of a match with an unemployed worker. This possibility is, however, ruled out by assumption, and this creates the wedge between the relevant matching market first-order conditions.

Wages are (again) determined by an ex post Nash bargain. Following the same steps as in the previous models gives us, in equilibrium:

$$\theta \frac{u}{u_2} = \frac{\vartheta}{1 - \vartheta} c^n \frac{H(1) - H(1-s)}{\kappa}, \quad (53)$$

where  $\theta = v/(u_1 + u_2)$ .

This relationship is similar to the one derived in the previous subsection, in equation (42), since it implies a modification to the appropriate measure of tightness. In the current setting, the consumption-tightness puzzle involves the ratio of vacancies to long-term unemployment rather than the standard definition of tightness that enters equation (28).

If  $u_2$  is literally interpreted as long-term unemployment, this model leads to an even bigger consumption-tightness puzzle than the model we analyzed in section 1.2. The reason is that longer-term unemployment is even more volatile than overall unemployment. In table 1.1 we report, for example, the moments of vacancies to unemployment above fifteen weeks of duration. The standard deviation of this ratio is around

35 percent per quarter, which is 59 percent higher than the standard deviation of tightness itself.

However, this calculation might be misleading, for two reasons. First,  $u_2$  does not directly measure long-term unemployment, although the unemployment duration of agents faced with the inefficient matching technology will (on average) be longer than the mean duration of agents faced with the more efficient matching technology. Second, the measurement of the duration of unemployment (applied by the Bureau of Labor Statistics) defines the duration of unemployment as the length of "in-progress spell of joblessness." The duration of unemployment of an active search agent faced with the inefficient matching technology, who was previously out of the labor force, will (therefore) be measured by the duration of the current job search (rather than the length of time since the last job match).

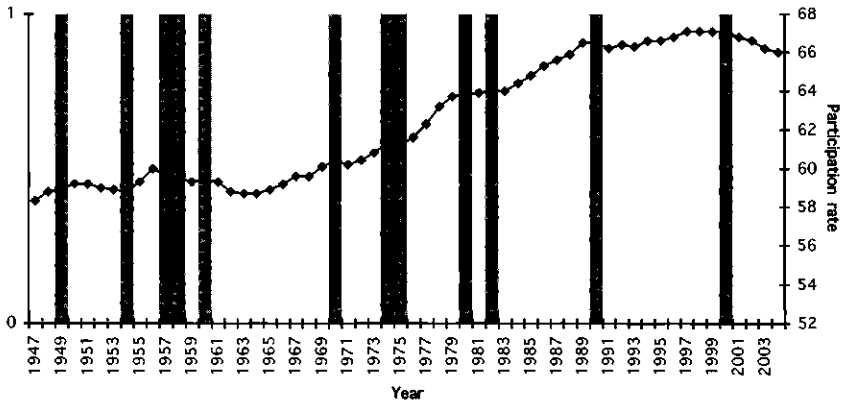
For these reasons, the measurement of  $u_2$  on the basis of long-term unemployment might be misleading. In essence,  $u_2$  denotes the measure of agents who, despite being faced with a potentially inefficient matching technology, still find it worthwhile to be actively searching. It is not clear how to match this measure up with the data, and we (therefore) leave it open whether duration dependence of the matching market prospects is important for accounting for the consumption-tightness puzzle.

#### 1.4 Discussion

The previous analysis has illustrated the robust relationship between the marginal utility of consumption and labor market tightness that we derived in section 1.2. We now discuss some wider aspects of the result, and its implications.

The low volatility of labor market tightness and procyclical movements in unemployment derive from the variations in labor market participation. In the set up that we study, households optimally choose to increase labor market participation in response to increases in labor market tightness. It is this mechanism that implies low volatility of labor market tightness in equilibrium.<sup>13</sup>

Hence, it is clear that variations in the participation rate are key, and the introduction of an extensive search margin leads to a strong tendency for procyclical variations in labor market participation. Figure 1.4 illustrates the U.S. labor market participation rate from 1947 onward. The figure clearly illustrates the secular increase in the U.S. participation rate. It rose from around 58 percent in the late 1940s to approximately 67



**Figure 1.4**  
U.S. Participation Rate

*Note:* The graph illustrates the civilian noninstitutional labor force, as a share of the civilian noninstitutional population of age 16 and above. The shaded areas are recessions as defined by the NBER dating committee.

percent by the 2000s, an increase that is dominated by the increase in the employment rate (from 56 percent to 64 percent).

Figure 1.4 also illustrates (shaded areas) the recessions of the U.S. economy, according to the NBER business-cycle dating committee. The figure indicates that the secular increase in the participation rate predominantly took place during periods of high activity. In particular, the secular increase in the participation rate either slowed down or was reversed during each of the recessions. Thus, consistent with the theory, there appears to be some cyclical features of the movement in the participation rate.

To examine this further, the last rows of the two panels of table 1.1 report the moments of HP filtered and BK filtered participation rates. The participation rate is procyclical, but displays low volatility at the business cycle frequencies, regardless of the detrending method. In particular, relative to trend, the standard deviation of the participation rate is around one fourth of the standard deviation of consumption at the business cycle frequencies, and the cross-correlation between these variables is just below 30 percent.

Figure 1.5 illustrates (in the top panel) the HP filtered U.S. data for consumption and the participation rate. This clearly illustrates that the participation rate is much smoother than consumption at the business cycle frequencies. The figure also hints that there might be a phase shift

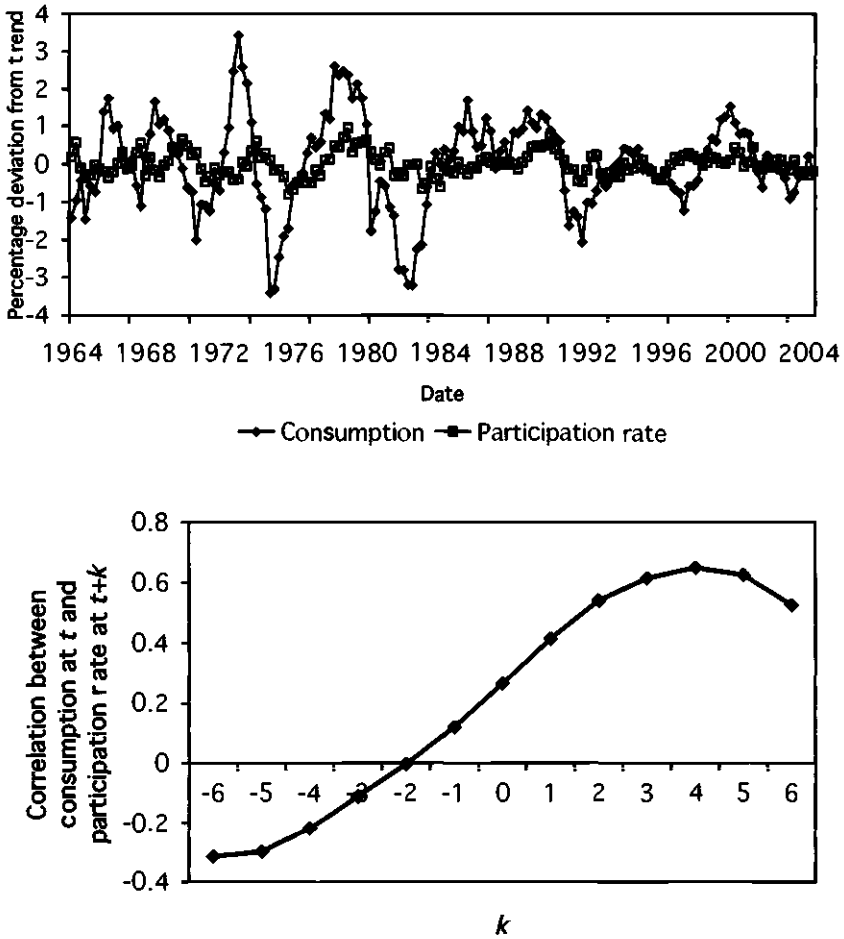
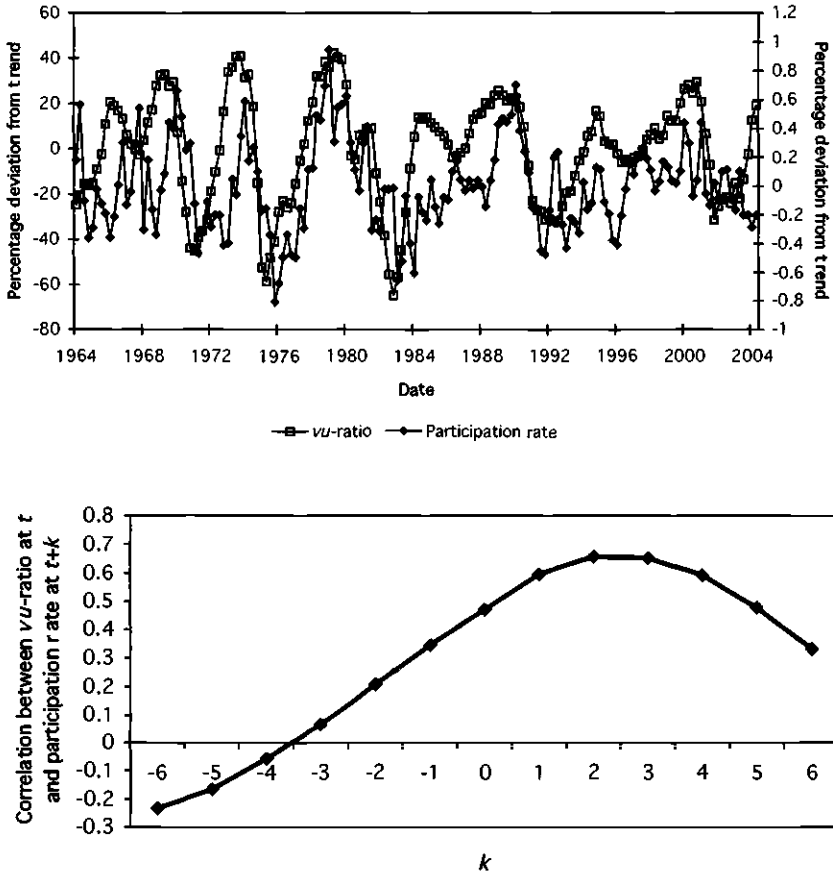


Figure 1.5  
Consumption and Participation Rate

between consumption and participation rates. In particular, with the exception of the late 1970s, the participation rate appears to lag behind the fluctuations in consumption. The lower panel illustrates the cross-correlation function between consumption and leads and lags of the participation rate. The results indicate that the participation rate lags about four quarters later than consumption. Moreover, with a four-quarter lag, the cross-correlation is as high as 65 percent.

Nevertheless, despite this high correlation, the elasticity of the participation rate compared to consumption is still estimated low. Using the





**Figure 1.6**  
Labor Market Tightness and Participation Rate

*Note:* The top panel illustrates percentage deviations from an HP-trend of the *vu*-ratio (left scale), and of labor market participation (right scale).

estimates from table 1.1, the elasticity of the participation rate (with respect to consumption) is around 17 percent (with a four-quarter lag). Thus, even large cyclical fluctuations in consumption are associated with small variations in participation rates.

Similarly, figure 1.6 illustrates the relationship between the *vu*-ratio and labor market participation rates. The top panel shows the deviations from (Hodrick-Prescott) trends of the *vu*-ratio and of the participation rate. Given the large difference in their volatility, the *vu*-ratio

is plotted against the left axis and the participation rate against the right axis. Consistent with the model we have analyzed, these two variables are clearly positively related. The lower panel shows the cross-correlation function in leads and lags. As above, there is a substantial positive correlation, and it occurs with a lag (but slightly shorter than above). With a two-quarter lag (of the participation rate), the cross-correlation is close to 70 percent.

This suggests that perhaps the findings of this paper are related to costs of entering and exiting the labor force. Such costs might explain why the participation rate moves so little (in response to variations in the benefits of job search), and why the participation rate appears to lag behind output and consumption over the business cycle. Such costs also appear realistic for some parts of the agents who compose the out of the labor force group. Young people who are still in school, or workers who have to move in order to search for a job (for example) might find it costly to change their labor market status. On the other hand, using a limited information approach, Ravn (2005) estimates such costs to be very large in order to account for the labor market movements over the business cycle, which casts doubt on this aspect being the sole explanation for the findings of this paper.

An alternative assumption, adopted by Garibaldi and Wasmer (2005) and Haefke and Reiter (2006), is that agents differ in their evaluation of leisure (or in their productivity in homework). This implies that the non-participants will be heterogenous with respect to how close their assessment of nonparticipation is to their assessment of labor market search. In particular, those agents that value leisure highly may find it optimal to remain out of the labor force, even for large increases in the value of searching. Through this mechanism, heterogeneity in the valuation of leisure (or in home productivity) can limit the tendency for procyclical movements in labor force participation that have been derived in this paper. However, this setup appears to be in contradiction to the large observed flows of agents from nonparticipation into employment (and into unemployment), which we discussed in section 1.3.3, unless there are large idiosyncratic shocks to preferences or home productivity. We find such idiosyncratic preferences shocks hard to interpret.<sup>14</sup>

Therefore, we find it more promising to explore (in more detail) the type of settings we studied in sections 1.3.3 and 1.3.4. Here, duration dependence and passive search jointly imply that: (a) there is less incentive to join the labor force in response to increases in the value of search, and

(b) some of the nonparticipants might find it unprofitable to actively search for a job (not because they value leisure highly, but because they face little prospect of finding a job through active search).

## 1.5 Summary and Conclusions

An important line of research in business cycle theory has studied the effects of matching frictions in the labor market. This is an important development in business cycle theory, since fluctuations in labor are key for understanding the business cycle (Kydland 1995). The matching frictions assumed in the Mortensen and Pissarides (1994) setup, places the labor market in a central role of the propagation of shocks over time and across agents.

This chapter has analyzed the effects of introducing a labor market participation choice, and, surprisingly, has shown that the introduction of a labor market participation choice is of considerable analytical convenience, since it allows us to derive a very simple testable relationship between labor market tightness and consumption. Moreover, this relationship is robust in the various extensions of the baseline model that we proposed. The advantage of this result is that it involves only observable variables, which gives rise to a relationship that does not depend on the properties of the stochastic processes of exogenous variables.

A standard perception from such labor market matching models is that unemployment, consistent with the data, behaves countercyclically as job matches increase in good times when firms increase their investment in job hiring activities. This paper has shown, however, that once one introduces an endogenous labor market participation choice, there is a strong tendency for procyclical behavior of unemployment. The reason is that labor market nonparticipants have an incentive to enter the labor market, that is, begin an active search, when labor market prospects improve. These procyclical movements in labor market participation rates imply low volatility of the ratio of vacancies to unemployment, and a positive slope of the Beveridge curve. Evidently, in U.S. data, although participation rates do move procyclically, the elasticity of the participation rate is very low.

Understanding why this is the case is an important issue for further research. There are various avenues open to address this. One possibility is to introduce costs of entering and exiting the labor force. Another possibility is to introduce some of the aspects that we examine in section 1.3. It is possible when these features are joined, and possibly combined

with other extensions (such as habit persistence, nonseparable preferences, incomplete markets), that this will yield a solution to the consumption-tightness puzzle. We will examine this in future research.

## Acknowledgments

Helpful comments from Kai Christoffel, Wouter Den Haan, Bob Hall, Julio Rotemberg, seminar participants at ISOM 2006, ESSIM 2006, Boston College, Duke University, the ECB, Universidad Carlos III de Madrid, and at University of Paris I are gratefully acknowledged.

## Notes

1. Bowlus (1997) makes the same assumption in a search model as Haefke and Reiter (2006) also do.
2. We simplify the notation slightly for presentational purposes. The state variables of the households also include the aggregate capital stock, aggregate employment, and the stochastic variables  $z$  and  $\sigma$ .
3. As is standard in the business cycle literature, we use a value of 1,600 for the smoothing parameter in the HP filter. For the BK filter we use a moving average length of twelve quarters, and the cut-off frequencies are chosen as six quarters and thirty-two quarters (respectively).
4. Table A1 reports the definitions and sources of the data.
5. The model can easily be extended to include productivity growth. In this case condition (28) is still valid but relates the  $vu$ -ratio to consumption, relative to the level of productivity. Therefore, one may wonder whether the calculations should not relate the level of the  $vu$ -ratio to detrended consumption. Following this strategy, however, implies even higher and more unrealistic estimates of  $\eta$ . Using Baxter and King's (1999) filtered consumption, for example, the slope of the regression lines implies a value of  $\eta$  of twenty, and the ratio of standard deviations of the  $vu$ -ratio and consumption gives a value of forty for  $\eta$ .
6. To get this expression, express  $\text{cor}(\hat{u}, \hat{c})$  as  $[\text{var}(\hat{v})/\text{var}(\hat{u})]^{1/2} \text{cor}[(\hat{c}, \hat{v})] - \eta[\text{var}(\hat{c})/\text{var}(\hat{u})]^{1/2}$ . Inserting the estimates, on the basis of the HP-filtered data in table 1.1 implies the formula in the text.
7. Search effort, therefore, has the interpretation of costs of, for example, filling in job applications, travelling to job interviews, and so on. Alternatively, one can assume search efforts give rise to leisure costs (Andolfatto 1996). However, the latter modeling implies that search effort is constant in the optimum (see note 8) and is less interesting for our purposes.
8. Merz (1995) also finds procyclical search intensity in a standard labor market matching model without the participation choice. Her result is derived from the impulse responses in a numerically solved version of the model. For the Andolfatto (1996) specification of leisure costs of search effort, we would assume  $\hat{d}(h) = 0$  and that  $x_u = (1 - h - s)$ . This implies, however, that optimal search effort is constant, since the first-order condition for  $h$  can be expressed as:  $-\partial H(1 - h - s)/\partial h = [H(1) - H(1 - h - s)]/h$ , which involves only  $h$  and constants. Therefore the optimal  $h$  is constant.

9. Garibaldi and Wasmer (2005) also introduce homework into a matching framework with a participation choice. Cooley and Quadrini (1999) include homework in a matching framework with limited-asset market participation.
10. To get this number, assuming a Cobb–Douglas matching technology, it follows from equation (38) that the standard deviation of the logarithm of the  $vu$ -ratio is given as  $[\sigma_c^2 + \sigma_r^2 - 2\text{cov}(c, r)]^{1/2}$ . Assuming that  $\text{cov}(c, r) = -\sigma_c\sigma_r$ , and using the values for  $\sigma_c$  and  $\sigma_r$  from table 1.1 for the HP filter data gives the number in the text.
11. In principle, time aggregation might account for the recorded flows from out of the labor force to employment, even if nonparticipants have to become active searchers to find a job match.
12. Strictly speaking, the use of the terms *short-term unemployment* and *long-term unemployment* is misleading, since the transition from the former to the latter group occurs independently of the duration of unemployment. However, on average, the former group will have experienced shorter unemployment spells than the latter.
13. Notice, however, that the response of unemployment to vacancies may lead to high volatility of vacancies itself.
14. A similar mechanism can be introduced into the setup studied in this chapter by allowing for shocks to the marginal rate of substitution between leisure and consumption. Denoting such a taste shock for  $\zeta_t$ , the equivalent of condition (28) becomes  $\theta = [\partial/(1 - \partial)]\omega\zeta^c$ . A large variance of the taste shock may, therefore, break the link between consumption and tightness.

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## Appendix A

### *Derivation of the Results for the Model with Variable Search Effort*

In this model, the households problem is given as:

$$J(k, n) = \max_{(c, k, u, n, h)} [c^{1-\eta}/(1-\eta) + nH(1-l-s) + uH(1-s) + (1-n-u)H(1) + \beta EJ(k', n')],$$

subject to:

$$c + k' \leq (1 - \delta + r)k + wnl + \pi - ud(h),$$

$$n' = (1 - \sigma)n + \gamma^h u h.$$

We let  $\lambda_c$  denote the multiplier on the first constraint, and  $\lambda_n$  the multiplier on the second constraint. The first-order conditions are given by:

$$c : c_n = \lambda_c$$

$$k' : \lambda_c = \beta E J_{k'}(k', n')$$

$$h : \lambda_c \frac{\partial d(h)}{\partial h} = \lambda_n \gamma^h$$

$$k : J_k(k, n) = \lambda_c (1 - \delta + r)$$

$$u : H(1) - H(1 - s) = \gamma^h \lambda_n h - \gamma d(h) \lambda_c$$

$$n' : \lambda_n = \beta E J_{n'}(k', n')$$

$$n : J_n(k, n) = \lambda_c w l + (1 - \sigma) \lambda_n + H(1 - l - s) - H(1).$$

Combining the first-order conditions for  $h$  and  $n'$  gives us:

$$c^{-\eta} \frac{\partial d(h)}{\partial h} = \beta \gamma^h E J_{n'}(k', n'),$$

which is equation (29) in the text.

Next, combining the conditions for  $u$ ,  $c$ , and  $h$  implies:

$$H(1) - H(1 - s) = \gamma^h \lambda_n h + d(h) c^{-\eta} \Rightarrow$$

$$H(1) - H(1 - s) = c^{-\eta} \gamma d(h) [\psi(h) - 1],$$

which is equation (30) in the text. The firms' problem is unchanged relative to the basic model. The Nash wage bargaining, therefore, implies:

$$\vartheta J_n = c^{-\eta} (1 - \vartheta) Q_n,$$

where:

$$J_n(k, n) = \lambda_c w l + H(1 - l - s) - H(1) + (1 - \sigma) \beta E J_{n'}(k', n'),$$

$$Q_n(n) = F_n - w l + (1 - \sigma) \beta E \frac{u'_c}{u_c} [Q_{n'}(n')],$$

and from the envelope conditions, we have:

$$\beta E \frac{u'_c}{u_c} [Q_{n'}(n')] = \frac{\kappa}{\gamma^f}.$$

$$\beta E J_{n'}(k', n') = \lambda_n.$$



From the first-order condition for  $h$  we have, we can express the latter as:

$$\beta EJ_n(k', n') = \frac{H(1) - H(1 - s) + \gamma d(h)c^{-\eta}}{\gamma^h h}.$$

Therefore, it follows that:

$$\vartheta \frac{H(1) - H(1 - s) + \gamma d(h)c^{-\eta}}{\gamma^h h} = (1 - \vartheta)c^{-\eta} \frac{\kappa}{\gamma^f},$$

which can be rearranged to give us:

$$\theta = \frac{\vartheta}{1 - \vartheta} c^{\eta \kappa} [H(1) - H(1 - s)] \left[ \frac{\psi(h)}{\psi(h) - 1} \right],$$

which corresponds to equation (31).

## Appendix B

### Homework

The firms' problem is again unchanged so we concentrate on the households' problem. It can be formulated as:

$$J(k, n) = \max_{(c, n', k', u, r, x)} [c^{1-\eta}/1 - \eta) + nH(1 - l - s - \mu_w) + uH(1 - s - \mu_u) \\ + (1 - n - u)H(1 - \mu_n) + \beta EJ(k', n')].$$

subject to:

$$c_m + k' \leq (1 - \delta + rx)k + wn l + \pi,$$

$$n' = (1 - \sigma)n + \gamma^h u,$$

$$c_h = g[(1 - x)k, n\mu_n + u\mu_u + (1 - n - u)\mu_l, z^h].$$

We denote the multipliers on these restrictions (in that order) by  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ . The first-order conditions (and envelope conditions) are given as:

$$c_m : c^{-\eta} \frac{\partial C}{\partial c_m} = \lambda_1$$

$$c_h : c^{-\eta} \frac{\partial C}{\partial c_h} = \lambda_3$$

$$k' : \lambda_1 = \beta EJ_{k'}(k', n')$$

$$u : \lambda_2 \gamma^h + \lambda_3 \frac{\partial g}{\partial \mu} (\mu_u - \mu_l) = H(1 - \mu_l) - H(1 - s - \mu_u)$$

$$\mu_n : nH'(1 - l - s - \mu_n) = \lambda_3 \frac{\partial g}{\partial \mu} n$$

$$\mu_u : uH'(1 - s - \mu_u) = \lambda_3 \frac{\partial g}{\partial \mu} u$$

$$\mu_l : (1 - n - u)H'(1 - \mu_l) = \lambda_3 \frac{\partial g}{\partial \mu} (1 - n - u)$$

$$x : \lambda_1 rk = \lambda_3 \frac{\partial g}{\partial k_n} k$$

$$n' : \lambda_2 = \beta EJ_{n'}(k', n')$$

$$n : J_n = H(1 - l - s - \mu_n) - H(1 - \mu_l) + \lambda_1 wl + \lambda_2 (1 - \sigma) + \lambda_3 \frac{\partial g}{\partial \mu} (\mu_n - \mu_u)$$

$$k : J_k = \lambda_1 (1 - \delta + rx) + \lambda_3 \frac{\partial g}{\partial k} x.$$

The first-order conditions for  $\mu_n$ ,  $\mu_u$  and  $\mu_l$  immediately imply that:

$$\mu_l = \mu_u + s = \mu_n + l + s,$$

since:

$$H'(1 - l - s - \mu_n) = H'(1 - s - \mu_u) = H'(1 - \mu_l).$$

Turn now to the wage bargaining. We have:

$$\vartheta J_n(k, n) = \lambda_1 (1 - \vartheta) Q_n(n),$$

where:

$$J_n(k, n) = \lambda_1 wl + \lambda_3 \frac{\partial g}{\partial \mu} (\mu_n - \mu_u) + (1 - \sigma) \beta EJ_{n'}(k', n'),$$

$$Q_n(n) = F_n - wl + (1 - \sigma) \beta E \frac{u'_c}{u'_c} [Q_{n'}(n')],$$

and the envelope conditions imply:

$$\beta E \frac{u'_c}{u_c} [Q_{n'}(n')] = \frac{\kappa}{\gamma^{f'}}$$

$$\beta E J_{n'}(k', n') = \frac{\lambda_3 \frac{\partial g}{\partial \mu} (\mu_l - \mu_u)}{\gamma^h}.$$

Therefore, we get:

$$\vartheta \beta E \frac{1}{\lambda_1} J_{n'}(k', n') = (1 - \vartheta) \beta E \frac{\lambda'_1}{\lambda_1} Q_{n'}(n') \Rightarrow$$

$$\vartheta \frac{1}{\lambda_1} \frac{\lambda_3 \frac{\partial g}{\partial \mu} (\mu_l - \mu_u)}{\gamma^h} = (1 - \vartheta) \frac{\kappa}{\gamma^{f'}} \Rightarrow$$

$$\frac{\partial C}{\partial c_h} (\mu_l - \mu_u) \frac{\partial g}{\partial r} = \frac{1 - \vartheta}{\vartheta} \kappa \frac{\partial C}{\partial c_m} \theta.$$

We now use the Cobb-Douglas assumptions, and the result from above, that  $r_l - r_u = s$ , which allows us to express this condition as:

$$(1 - \xi) \frac{C}{c_h} s (1 - \tau) \frac{c_h}{\mu} = \frac{1 - \vartheta}{\vartheta} \kappa \frac{C}{c_m} \theta \xi,$$

which can be re arranged to give us equation (38):

$$\theta = \frac{c_m}{\mu} \frac{\vartheta}{1 - \vartheta} \frac{1 - \xi}{\xi} (1 - \tau) \frac{s}{\kappa}$$

**Table 1A.1**

Definitions and Sources of Data (sample period: 1964 Q.1–2004 Q.1)

Name	Definition	Source
Civilian non-institutional pop.	Civilian noninstitutional population 16 years of age and above	Economagic, Fed. of St. Louis
Output	Gross Domestic Product in chained year 2000 prices divided by civilian noninstitutional population	Economagic, Fed. of St. Louis
Consumption	Personal Consumption Expenditure in chained year 2000, prices divided by civilian noninstitutional population	Economagic, Fed. of St. Louis
Total hours	Nonfarm hours divided by civilian noninstitutional population	DRI database
Unemployment	Total unemployment divided by civilian noninstitutional population	Economagic, Fed. of St. Louis
Vacancies	Index of help wanted advertising in newspapers divided by civilian noninstitutional population	Economagic, Fed. of St. Louis
<i>vu</i> -ratio	Vacancies divided by unemployment	—
Unemployment > 15 weeks	Civilians unemployed 15 weeks and above divided by civilian noninstitutional population	Economagic, Fed. of St. Louis
Participation rate	Sum of civilian employment and unemployment divided by civilian noninstitutional population	Economagic, Fed. of St. Louis

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## *Comment*

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The previous chapter introduces an endogenous labor market participation choice into a Mortensen and Pissarides (1994) matching model, embedded into a stochastic growth model. It provides a very clear and instructive analysis of an important margin in the labor market. While several papers stress the importance of endogenous labor market participation for long-run economic performance, the business cycle applications of endogenous participation have been less successful. This paper contributes to bridging the gap between labor market participation in the medium and long run, and understanding the business cycle frequency fluctuations of participation. In the chapter, it is shown that the introduction of the participation margin, into the labor market model, allows for an analytical expression of labor market tightness (vacancies divided by unemployed) as a linear function of (the marginal utility of) consumption. The consumption-tightness puzzle is formulated by showing that, under plausible parameterizations, the model can only account for a small fraction of the observed volatility of labor market tightness. The introduction of the participation margin has a direct effect on the market tightness condition, and (therefore) on the general dynamics of the model. However, these modifications have some counterfactual implications. The unemployment rate is procyclical, and the model fails to reproduce a (negatively sloped) Beveridge curve. The author acknowledges these shortcomings, and provides some modifications of the basic model. Though these modifications are not sufficient to overcome the dynamic inconsistencies, they offer some further insight into the dynamics of labor market flows.

I will divide my comments into three parts. First, I will give a brief summary on the main contribution of the paper. I will then highlight the dynamic inconsistencies of the model. Third, I will discuss modifications of the model and their implications on the main inconsistencies.

## Summary

The introduction of the endogenous labor participation decision has several implications on the dynamic properties of the model. As in the standard Mortensen-Pissarides matching model the firms post vacancies until the cost ( $\kappa$ ), associated with the posting of an additional vacancy equals the expected discounted revenue from a filled vacancy (weighted by the matching probability).

$$\frac{\kappa}{\gamma^f} = \beta E \frac{u'_c}{u_c} [Q_n(n')], \quad (1)$$

where  $Q(n)$  gives the value of a firm with  $n$  filled jobs, and  $\gamma^f$  gives the probability a vacancy of a firm is filled.

The introduction of endogenous participation allows us to write the labor market participation decision in an equivalent form. Due to the endogenous participation choice, the household includes the unemployment rate as a control variable in the optimization problem. The associated first-order conditions for unemployment and next period employment are:

$$H(1) - H(1 - s) = \gamma^h \lambda_n, \quad (2)$$

and

$$\lambda_n = \beta E J_n(k', n'). \quad (3)$$

where  $J(n, k)$  is the value function of the household (contingent on the capital holdings) and the number of employed household members, and  $\lambda_n$  denotes the Lagrange multiplier of the evolution of employment.

Combining first-order conditions (2) and (3), we arrive at the following:

$$\frac{H(1) - H(1 - s)}{\gamma^h} = \beta E J_n(k', n'). \quad (4)$$

This condition states that nonparticipating household members will enter the labor market until the utility loss from labor market participation equals the expected additional value from labor market participation. This expected value is given by the marginal increase in the value function, due to an additional employed household member weighted by the probability that a searching worker is matched to a firm.

It is important to note the symmetry of conditions (1) and (4). In both expressions, the derivative of the respective value function is directly re-

lated to model variables. To relate the first-order conditions of households and firms, we can write the standard surplus sharing rule (in expectations) resulting from the wage bargaining as follows:

$$\vartheta \beta E c^n J_n(k', n') = (1 - \vartheta) \beta E \frac{c'^{-\eta}}{c^{-\eta}} Q_n(n'). \quad (5)$$

Plugging conditions (1) and (4), into equation (5) we get an equation for market tightness:

$$\theta = \frac{\gamma^h}{\gamma^f} = \frac{\vartheta}{1 - \vartheta} \frac{H(1) - H(1 - s)}{\kappa} c^n. \quad (6)$$

Labor market tightness is written as a linear function of the marginal utility of consumption. Under plausible parameterizations the model can only explain a small fraction of the observed fluctuations in  $\theta$ .

### Dynamic Implications

To get a better understanding of the labor market dynamic, it is instructive to analyze the flows into and out of unemployment. The first flow into unemployment occurs due to the exogenous separation of existing work relations at rate  $\sigma$ . Vacancy posting, and labor market participation decisions imply two further flows which are closely related. If the profits of the firm  $\pi^f = F(k, nl) - wnl - \kappa v - rk$  increase, firms will react by posting additional vacancies according to condition (1). In addition to the usual employment effects of vacancy posting, the increased number of vacancies in the market is also inducing nonparticipating workers to enter the labor market. Using equation (4) we can decompose the participation decision into two effects. First, the increase in profits implies a higher joint surplus of employment relations, increased wages, and an increase in the value function of the households. Second, the increased vacancy posting implies a higher probability that a searching worker will be matched to a firm. Under endogenous participation, these effects drive up labor market participation. The participation decisions have an immediate effect on unemployment, while the creation of new matches is costly and time consuming such that unemployment increases after a positive shock to the profits of firms. These effects imply a positive correlation of consumption and unemployment. The very same effect is also causal for the positive correlation between vacancies and unemployment, implying a positive slope in the Beveridge curve. It is important to note that this effect is generated by the endogenous labor market

participation decision but does not exist in the standard Mortensen-Pissarides model.

Furthermore, the endogenous participation choice is also affecting the vacancy posting intensity and could contribute to the vacancy fluctuation discussion initiated by Shimer (2005). Under constant participation, additional posted vacancies increase market tightness and decrease the probability that a vacancy is matched. This congestion externality limits the fluctuations in vacancies. Under endogenous participation the congestion externality is much less severe. Additional vacancies trigger the market entry of nonparticipating households, such that market tightness reacts to a lesser extent. The decreased congestion externality (due to the participation margin) could provide a channel to explain observed vacancy and employment fluctuations in the Mortensen-Pissarides model.

In order to relate these channels to the data, Ravn relies on the market tightness condition (6). While the theoretical model implies a largely proportionate volatility of consumption and labor market tightness, the data shows that labor market tightness is around twenty times as volatile as consumption. As discussed above, the reason for this effect is the strong and procyclical response of participation and unemployment. As a corollary of this relationship, the downward sloping Beveridge curve relation in the data cannot be reproduced by the model. In the data vacancies and unemployment are negatively correlated, indicating that in periods of increased economic activity firms post more vacancies and unemployment goes down.

We find that the main reason for these dynamic inconsistencies can be found in the strong reaction of participation and its implications on unemployment.

## Extensions

Section 3 in the chapter offers several extensions to the basic model, in order to overcome the dynamic inconsistencies.

Section 1.3.3 introduces passive search into the model. This implies that nonparticipating household members face a certain probability to become employed, even though they do not enter the formal search market. This approach seems to be promising from an empirical point of view. As cited in the chapter, Fallick and Fleischman (2004) find that only 40 percent of the flow into employment is actually coming from unemployment. Considering this evidence calls for a modeling of signifi-



cant flows, from inactivity directly into employment. To model these flows a second matching function is introduced. The number of matches in the second matching function is determined by the number of posted vacancies, and by the number of nonparticipating households ( $1 - n - u$ ). This specification has some implications in the Mortensen-Pissarides modeling approach. A lot of research focuses on the failure of the model to produce an appropriate degree of vacancy and employment fluctuation. Under passive search and endogenous participation it is possible to explain further employment fluctuations without having to rely solely on the degree of vacancy fluctuations.

In the chapter it is assumed that the ratio of matches ( $m_{u,t}/m_t$ ) is constant. Under this assumption, the dynamics of the model are not strongly affected by the introduction of passive search. Equation (7) gives the market tightness condition for the model with passive search:<sup>1</sup>

$$\theta_t \frac{(1 - n_t) \frac{m_{u,t}}{m_t} - u_t}{1 - n_t - u_t} = \frac{\vartheta}{1 - \vartheta} \frac{H(1) - H(1 - s)}{\kappa} c_t^n. \quad (7)$$

Only if the ratio of matches from unemployment to total matches is small will the dynamics of the model be significantly affected by the fraction on the left side of equation (7). To check the validity of this result, it could be useful to relax the assumption of a constant ratio of matches from unemployment to total matches. Using the matching functions  $m_{u,t} = \varphi_u \nu_t^\psi u_t^{1-\psi}$ , and  $m_{i,t} = \varphi_i \nu_t^\psi (1 - n_t - u_t)^{1-\psi}$  with  $\varphi_i \leq \varphi_u$ , we can write the ratio of matches in equation (7) as:

$$\frac{m_{u,t}}{m_{u,t} + m_{i,t}} = \left[ 1 + \frac{\varphi_i}{\varphi_u} \left( \frac{1 - n_t - u_t}{u_t} \right)^{1-\psi} \right]^{-1}. \quad (8)$$

Plugging this expression into equation (7) shows that the labor market tightness condition is no longer exclusively related to consumption, but is also determined by the dynamics of the ratio of unemployment to that of the nonparticipating labor force.

This extension introduces the important flow from inactivity directly into employment. After a positive shock to firm profits, vacancy posting increases. This increases the matching probability for both the participating and the nonparticipating household members. The higher efficiency of the matching function, for the participating household members, gives incentives to enter the labor market. The flow from nonparticipation into unemployment implies that the unemployment rate will remain procyclical.

Further extensions aim to curb the response of participation. An extension proposed in the paper assumes that the efficiency of the matching functions depends on being short-term or long-term unemployed, where the movements in and out of participation are restricted to the second group. An alternative method to dampen the participation decision is proposed by Haefke and Reiter (2006), who build on Garibaldi and Wasmer (2005). They construct a home production model, where they model a cross-sectional density of productivity in the home sector. The main implication of this modification is that changes in the labor market conditions will affect only the household members close to the participation margin. They claim that, once the cross-sectional density is calibrated properly, the labor market participation reaction is attenuated, and the counterfactual implications of the model disappear.

## Conclusion

The chapter introduces an endogenous labor market participation choice into a standard Mortensen-Pissarides business cycle matching model. In view of the important role of labor market participation, for medium- and long-run supply side dynamics as well as for business cycle fluctuations, it is important to model the associated labor market flow in a business cycle model. The analytical tractability allows for an instructive and intuitive analysis of endogenous participation. The counterfactual dynamic properties of this class of model are, however, persisting. In the data, labor market participation displays mildly procyclical fluctuations and is lagging the cycle by several quarters. In the model the reaction of labor market participation is immediate, and too strong.

To consolidate the model dynamics with the data, it is necessary to curb the model response of labor market participation. The chapter proposes several ways to achieve this aim, including passive search and heterogenous search functions. An important and interesting extension of the model could include further modifications along the mentioned developments to enhance the model properties. Furthermore, it could be interesting to model home production under heterogenous productivity levels, to allow for a flexible modeling of the participation margin.

## Notes

1. In the following exposition I am reintroducing the time indices to distinguish between variables and parameters.

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## Comment

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This chapter provides a valuable addition to the search and matching literature by considering a model where people choose whether to be out of the labor force, or unemployed. While not the first model of endogenous labor participation, its features are somewhat different from those considered previously. Interestingly, the model proves to have spectacularly counterfactual predictions: the ratio of unemployment to vacancies is predicted to fluctuate very little, and unemployment is predicted to be larger in booms than in recessions. This shows that the model's mechanism to induce changes in endogenous participation is unattractive.

My discussion concentrates on two points. This first is that the simplest way to avoid the counterfactual implications of the model is to allow for variations in the marginal cost of job search, that is the cost of the job search for the last person that shifts from being out of the labor force to being unemployed. Put differently, the model's difficulties seem due (at least in part) to the supposition that the cost of being unemployed, rather than being out of the labor force, is constant and the same for everyone. What this implies is that small changes in the benefit from working lead everyone to prefer to join the labor force—so that the variations in labor force participation are much larger than those observed in reality. This implication can be avoided by following Haefke and Reiter (2006), so that people differ in the extent to which they value searching for a job (through unemployment) as opposed to being out of the labor force. At any given moment there are then people who are indifferent between the unemployed (U), and the out of the labor force (N) state. However, small increases in the benefits from being in the employed (E) state do not, in this case, lead to a massive desire to switch from N to U.

The second point is that the behavior of the six flows between the three states E, N, and U, as reported in Blanchard and Diamond (1990),

seems at odds with the model in important respects. First, these flows reflect substantial heterogeneity that is not present in the model. In any given month, for example, one finds both people giving up on searching (that is moving from U to N), and people joining the labor force (that is going from N to U). More importantly, it is not clear that the model captures the underlying flows that stand behind the observed reductions in labor force participation that one observes in recessions.

Consider an abbreviated version of Ravn's basic model. The household utility at  $t$  depends on two stocks, namely the capital stock  $k_t$  and employment  $n_t$ . If household member  $i$  is unemployed, his disutility from unemployment equals  $z_t^u$ . The disutility of working, instead, is equal to  $z_t^w$ . The utility of the household is then given by:

$$J(k_t, n_t) = E_t \sum_{\tau=t}^{\infty} \beta^\tau \left[ \frac{c_{t+\tau}^{1-\eta}}{1-\eta} - z_t^w n_t - z_t^u u_t \right],$$

where  $\beta$ ,  $\eta$ , and  $z^w$  are parameters,  $E_t$  takes expectations based on information at  $t$ , and  $c$  is each member's consumption.

As is standard, the matching technology makes the probability that an unemployed worker finds a job ( $\gamma_t^h$ ) be equal to  $\phi(v_t/u_t)$ , where  $v_t$  is the vacancy rate. With a separation rate of  $\sigma_t$ , the dynamics for employment satisfy:

$$n_{t+1} = (1 - \sigma_t)n_t + \gamma_t^h u_t. \quad (1)$$

The novelty of letting participation be endogenous, is that the household realizes it can increase expected employment by  $\gamma_t^h$  by letting one more member be unemployed. For the household to be indifferent, with respect to asking member  $i$  to join the unemployment pool, it must be that:

$$\beta E_t \frac{dJ(k_{t+1}, n_{t+1})}{dn_{t+1}} = \frac{z_t^u}{\gamma_t^h}.$$

This states that the marginal benefit of employment to the household is equal to its expected marginal cost, which is that more members will have to be unemployed.

Now consider matters from the perspective of firms. Their present discounted value of profits also depends on employment, and can be written as  $Q(n_t)$ . Firms have the option of posting vacancies, and each of these has a cost  $\kappa$ . Posting an additional vacancy at  $t$  leads to a probability  $\gamma_t^f$  of having an additional employee at  $t + 1$ , where equation (1) implies that  $\gamma_t^f v_t = \gamma_t^h u_t$ . We expect that (in equilibrium) firms should be

indifferent between posting and not posting an additional vacancy, and this implies:

$$E_t \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\eta} \frac{dQ(n_{t+1})}{dn_{t+1}} = \frac{\kappa}{\gamma_t^f} \tag{3}$$

where  $\beta(c_{t+1}/c_t)^{-\eta}$  is the rate at which firms discount the future.

Nash bargaining (between the firm and the household) implies that the benefits to the firm of hiring an additional employee must be proportional to the benefits to a household of having an additional worker employed. Supposing that  $\kappa$  is denominated in consumption units, the change in  $Q$  from extra employment at  $t + 1$  is  $c_{t+1}^{-\eta} dQ/dn_{t+1}$ , in units of utility, so that Nash bargaining requires:

$$(1 - \xi) E_t c_{t+1}^{-\eta} \frac{dQ(n_{t+1})}{dn_{t+1}} = \xi E_t \frac{dJ(k_{t+1}, n_{t+1})}{dn_{t+1}},$$

where  $\xi$  is a constant. Combining this with conditions (2) and (3) yields:

$$\xi \frac{z_t^u}{\gamma_t^h} = (1 - \xi) \frac{\kappa}{\gamma_t^f} c_t^{-\eta},$$

or, since  $\gamma_t^f/\gamma_t^h = u_t/v_t$ ,

$$\frac{u_t}{v_t} z_t^u = \tilde{\xi} c_t^{-\eta},$$

where  $\tilde{\xi}$  is a constant.

Equation (4) captures the implication that, under Ravn's assumption that  $z_t^u$  is constant, the  $v/u$  ratio cannot move more than consumption in the case of the log utility. The intuition for this result is straightforward. Nash bargaining implies that the benefits to the firm of an additional employee must be proportional to the benefits of employment of an additional member of the household. This, in turn, implies that the cost of employment to a firm (via increasing vacancies) must be proportional to the cost of employment to households (via increasing the number of members looking for work). Because the cost of vacancies is denominated in consumption units, and the cost of unemployment is denoted in units of utility, the ratio of costs of employment to the household (relative to the costs of employment to firms) is allowed to depend on the marginal utility of consumption. The only other determinant of the ratio of the cost of employment for a household, relative to the cost of employment for a firm, is the ratio of  $v$  to  $u$ , with a high  $v/u$  implying that obtaining an additional employee is relatively more costly for a house-

hold. Thus, the ratio  $u/v$  cannot vary more than the marginal utility of consumption.

The easiest way out of this straightjacket is to suppose that  $z^u$  varies over time. The  $v/u$  ratio can then vary in proportion to  $1/z^u$ . If the cost of converting a household member into a searcher is high, the unemployment rate can be low relative to the vacancy rate. The intuition for this is straightforward as well. Even if workers find it easy to find jobs because the unemployment rate is low, households will not send additional members to search if their cost of search is high.

Along the lines of Haefke and Reiter (2006), one can interpret this variation in  $z^u$  as being related to heterogeneity of household members. When the unemployment rate is high, the marginal household member that is being sent to look for work has a low search cost. In the Haefke and Reiter (2006) model, this member has low productivity in household production. Not surprisingly, that model can explain why unemployment would rise in recessions (when the surplus of low search cost workers in the household leads many of them to search), where the Ravn model implies that unemployment should rise in booms.

In a sense, the Ravn model represents too much of a departure from the Mortensen-Pissarides (MP) model. One can interpret the MP model as one with endogenous labor force participation, with  $z^u$  being so low that every member of the household searches when they are not employed (so that the optimum is not interior). Haefke and Reiter (2006) allow some household members to have a low  $z^u$ , though the distribution of these costs ensures that some members are indifferent to searching and being out of the labor force. Ravn is going much further because he supposes that no member strictly prefers to search since they all have a high enough  $z^u$  to be indifferent between searching and being out of the labor force.

Now turn to the broader issues concerning the connection between this model of endogenous labor force participation, and the observations regarding flows in Blanchard and Diamond (1990). An attractive aspect of the Ravn model is that it is consistent with procyclical labor force participation. The defect, of course, is that this happens because fewer people are unemployed in recessions. Blanchard and Diamond (1990, 121) report on what happens to all six flows between E, N, and U as aggregate activity slows down. As might be expected from the fact that the separation rate moves little in Shimer (2005), there is only a small increase in the E to U flow (and this is offset somewhat by a reduction in the E to N flow). Similarly, the procyclical finding rate means

that recessions are associated with reductions in the flows from U to E, and from N to E. Unlike the present model, the reduction in the flow from N to E appears to account for the reduction in the participation rate. The reason is that the flow from N to U actually increases, while the flow from U to N actually falls. In other words, unlike what the model predicts, recessions are not periods where people who would otherwise be unemployed move out of the labor force (or become discouraged).

The question of what accounts for the increased attraction of the U state in recessions, both for people who are initially unemployed (and fail to move to N) and for people who are initially out of the labor force, is a fascinating one. One possible hypothesis is that these movements are related to the absence of insurance markets, and to the existence of credit constraints. As Chetty (2006) has demonstrated, lack of liquidity plays a big role in the behavior of the unemployed by making them more likely to find a job. What might account for the behavior of job flows is that losing a job in a recession is more likely to land people in financial difficulty because the job finding rate is lower. The unemployed are, thus, less willing to pause their job search by joining the N state. Similarly, spouses of people who lose their jobs in recessions might go from the N to the U state, in order to try to avoid the financial problems that the family would encounter if both the original employee and the spouse stayed without employment for long.

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