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3 Corporate Pension Policy and the Value of PBGC Insurance

Alan J. Marcus

Title IV of the Employee Retirement Income Security Act of 1974 established the Pension Benefit Guarantee Corporation to insure the benefits of participants of defined benefit pension plans. The PBGC now insures the pension benefits of more than 28 million employees in single-employer plans and provides less extensive coverage to participants in multi-employer plans. Firms initially were charged a premium of $1.00 per year per employee for this coverage. This premium structure was meant to be temporary, until the data required to establish actuarially balanced plans became available. In 1980, the PBGC raised the premiums to $2.60 per employee per year. In 1982 the PBGC requested a further increase in the premium rate to $6.00, and warned that even this increase might be insufficient to cover prospective PBGC liabilities if several currently precarious large firms fail to regain financial stability (*Wall Street Journal* 1982). This latest request has led to renewed interest in PBGC pricing policy and the assessment of PBGC liabilities. Although the Multiemployer Pension Plan Amendment Act of 1980 directed the PBGC to study the possibility of a graduated premium rate schedule based on risk, such recommendations have yet to be made, and the current proposals for rate changes are still independent of risk.

One approach to valuing PBGC liabilities is provided by the options pricing framework. The formal correspondence between put options

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and term insurance policies has long been noted, and the option pricing methodology has been used to value insurance plans in other contexts (Mayers and Smith 1977; Merton 1977; Sosin 1980; Marcus and Shaked 1984). In fact, several authors (Sharpe 1976; Treynor 1977; da Motta 1979; Langetieg et al. 1982) already have used option pricing methodology to study the valuation of PBGC insurance. The provisions of ERISA allow firms to transfer their pension liabilities to the PBGC in return for pension fund assets plus 30% of the market value of the firm's net worth. Thus, viewing PBGC insurance as a put option, the pension liabilities play the role of the exercise price while the fund assets plus 30% of net worth play the role of the underlying asset or stock price.[1]

However, while the analogy between put options and the option to terminate a pension plan appears straightforward, the correspondence between the two is not at all clear with respect to the effective time to maturity of the pension put. Taken literally, ERISA rules seem to imply that a firm may terminate an underfunded plan, transfer its net liability to the PBGC, and reestablish a new insured plan. Under this reading of the law, firms would immediately terminate any plan that became underfunded by more than 30% of net worth. The option would have instantaneous maturity and be indefinitely renewable.

In practice, however, virtually all terminations of underfunded pension plans occur as a by-product of corporate bankruptcy. The lack of voluntary terminations suggests that there may be hidden costs to termination. Bulow (1982) suggests that voluntary termination might lead to unfavorable government treatment in other matters.[2] Other observers (e.g., Munnell 1982) cite damaged labor relations as an implicit cost of termination. This seems less convincing, however, since the firm may replace the terminated plan with another plan of equal value, in which case both employees and employers can gain at the expense of the PBGC. More explicit costs of termination might arise from legal entanglements. In one widely cited case, the PBGC brought suit to block the voluntary termination and reorganization of the underfunded pension plan of AlloyTek. The two sides ultimately settled out of court in 1981, with the PBGC assuming the underfunded plan and AlloyTek agreeing not to establish a new defined benefit plan. Instead, the firm was allowed to establish a defined contribution plan for its employees by buying Individual Retirement Accounts (IRAs) for them (Munell 1982).

Most authors have chosen to avoid the ambiguity regarding termination provisions. Treynor (1977) analyzes pension finance using a one-period model, in which the fund automatically terminates at the end of the period. Sharpe (1976) also uses a one-period model, which effectively transforms the termination put into a European option. In a similar vein, da Motta (1979) assumes an arbitrary finite maturity date. His model allows firms to drop out of the PBGC insurance program at interim moments when pension funding payments come due, but the firm cannot exercise the PBGC put until an exogenously given maturity date (p. 93). Harrison and Sharpe (1982) also study a multiperiod model in which the PBGC insurance is exercised only at the end of the last period. Bulow (1981, 1983), Bulow and Scholes (1982), and Bulow et al. (1982) generally pass over the issue of termination date per se, and focus instead on contingent liabilities at termination, whenever that may be. Finally, Langetieg et al. (1982) consider PBGC insurance in a general multiperiod contingent claims framework, but examine only the qualitative properties of the insurance, and do not derive a valuation formula for the insurance.

While these models offer several important insights, the issue of the implicit termination date remains problematic. It is clear that any estimates of the value of PBGC liabilities will be sensitive to the conditions that set off a plan termination. The sensitivity of the qualitative conclusions of these models to the imposition of an exogenous termination date remains an open question.

This paper presents two models of the pension insurance program that also use the contingent claims methodology but that do not impose an exogenous maturity date on PBGC insurance. The value of PBGC insurance is derived for two scenarios. In the first, the possibility of corporate bankruptcy is ruled out, and the pension plan is terminated only when that action is value maximizing for the firm. This scenario is motivated by the opportunity for profitable termination which ERISA seems to offer firms. The point of departure for this model is the AlloyTek case, the resolution of which indicates that a firm can terminate an underfunded pension plan with minimum explicit cost once, but only once.[3] A one-time-only termination provision makes the pension put formally identical to an infinite maturity American option, which expires only upon exercise. The cost of termination is the opportunity cost of not being able to terminate in the future for possible greater benefits. The termination decision becomes an optimal timing problem in which the option is exercised only if it is sufficiently in the money. Such a model potentially can explain the existence of underfunded plans which have not yet terminated without resorting to unspecified implicit costs of termination. Given the ability of a firm to replace the terminated defined benefit plan with a defined contribution plan, it is not clear that those costs would be significant for most firms.

The first model yields an upper-bound estimate of the value of PBGC insurance because the plan is terminated only when that action is optimal for the firm. In contrast, the second model should provide a lower bound on the value of the PBGC insurance. In this model, a pension plan terminates only at the occurrence of corporate bankruptcy. The motivation for this approach is twofold: First, it is consistent with the empirical fact that virtually no underfunded-but-solvent firms exercise the pension put. Second, it is consistent with proposals for pension insurance reform that would disallow termination of underfunded plans by solvent firms. The value derived for this scenario should represent a lower bound on the true value of the insurance, since it rules out the possibility for firms to choose a value-maximizing termination rule. The true value of PBGC insurance should lie between the valuation bounds generated by these two models.

The models employed in this paper allow for an analysis and valuation of pension insurance in a model in which plan termination is determined endogenously. The models also offer a framework for studying corporate pension funding and investment policy. The implications of these models confirm and extend those of Bulow (1981) and Harrison and Sharpe (1982), who analyzed pension funding strategies for plans with a given maturity date.

The next section presents a model of pension insurance. The valuation of PBGC liabilities is derived for each scenario, risk-rated pension insurance premium structures are considered, and optimal corporate financial policy is examined. It is shown that a fund can be significantly underfunded before a firm would find termination to be a profitable strategy. It also is shown that even under a bankruptcy-only termination rule, PBGC liabilities can be extremely large and quite sensitive to the pension funding policy of the firm.

Section 3.2 presents empirical estimates of the value of PBGC insurance for a sample of Fortune 100 firms. The results of this section indicate that the pension put has significant value for several firms, and that the true value of PBGC liabilities can differ substantially from the common measure of such liabilities, which is accrued benefits less the sum of fund assets plus 30% of firm net worth. Section 3.3 concludes.

3.1 A Model of Pension Insurance

3.1.1 Valuation of PBGC Pension Liabilities: Voluntary Termination

For simplicity, I will assume that all accrued benefits are vested and fully insured by the PBGC. In fact, guaranteed benefits typically account for between 90% and 95% of vested benefits, while approximately 80% of accrued benefits are vested (Amoroso 1983). This simplification is necessary to derive analytic solutions below; it should not affect the qualitative properties of the solution.

Following Bulow, let A denote the value of accrued benefits, F denote the value of assets in the pension fund, and $.3E$ denote the firm liability beyond assets in the pension fund (i.e., 30% of net worth). F and E are measured as market values, while A is the present value of accrued benefits calculated by discounting at the riskless nominal interest rate. The benefits represent an obligation which will be paid with certainty, either by the firm or by the PBGC.

At a termination, if the plan is sufficiently funded ($F + .3E \geq A$), the firm gains F and transfers assets of value A to the PBGC. Otherwise, the firm is liable only up to the amount $F + .3E$. The net proceeds to the firm at termination therefore equal[4]

$$F - \min(A, F + .3E) \tag{1}$$

or equivalently,

$$F - A + \max[A - (F + .3E), 0]. \tag{2}$$

Expression (2) highlights the nature of the firm's put option. Its net pension liability is $F - A$; however, at the termination date it can transfer its liability of A to the PBGC in return for only $F + .3E$.

There is no explicit maturity date associated with the insurance plan. In this sense, it is isomorphic to an American put option with infinite maturity and exercise price A. Just as the put can be exercised only once, the firm can voluntarily terminate just one defined benefit plan. Thereafter, it may offer its employees only defined contribution plans. These plans are akin to mutual funds in that they neither require nor receive PBGC insurance. Part of the firm's problem will be to choose a rule for voluntary termination that, in conjunction with its other policies, maximizes firm value.

To solve for the value of the pension insurance it first is necessary to specify the dynamics for accrued liabilities and the assets backing the plan. These will differ from conventional specifications because of the effects of firm contributions to the pension fund and the effects of new retirees and deaths on the dynamics for A.

For convenience, use S to denote the sum $F + .3E$. I will assume that S follows the diffusion process

$$dS = (C_S + \alpha_S)S\,dt + \sigma_S S\,dz_S \tag{3}$$

where α_S is a standard drift term attributable to the normal rate of return on the pension fund assets, F, and the firm equity, E, and where C_S is the rate (as a fraction of S) of firm contributions into the pension fund *net* of payments to retirees.[5] Solutions are presented below in which C_S is a function of the funding status of the plan; it need not be constant. If firm funding for accruing benefits exceeds payouts from the pension fund for current retirees, C_S will be positive. In a steady state with no uncertainty, a constant interest rate, and a constant number of retirees, the present value of accrued benefits would be constant over time. A firm administering a fully funded plan could withdraw interest earnings from the plan to help it pay benefits to current retirees and still maintain full funding. In this case, new contributions into the plan would fall short of payouts to retirees by the amount of the interest earnings; C_S would be negative. In fact, if 30% of the firm's equity were not included in the assets backing the fund, C_S would equal the negative of the interest rate. Firm contributions would fall short of current payouts by interest earnings on fund assets, which as a fraction of assets would simply be the interest rate.

The dynamics for A are more complicated. As a base case, consider a situation in which none of the firm's employees have yet retired and in which no further pension benefits will accrue. If the interest rate, r, is constant, then the present value of accrued benefits, A, which is the exercise price of the pension put, will increase at the constant proportional rate r. The growth in the exercise price derives from the definition of A as a present value, and differs from the more conventional situation in which the exercise price is specified as a dollar amount.

If long-term interest rates are stochastic, then so will be the present value of accrued benefits. Denote by α_A the expected rate of return on a bond with a payoff stream identical to that of accrued benefits. This will also be the expected growth rate in the present value of already accrued benefits. If interest rates were nonstochastic, then α_A would equal r.

Demographics also affect the evolution of A. Accrued benefits increase when current workers increase their length of employment and decrease when plan participants die or have benefits paid to them. In a steady state with no uncertainty, and a constant level of accrued benefits, newly accruing benefits plus the increase in the present value of already accrued benefits would exactly offset the decrease in total accrued benefits due to retiree deaths. Denoting the *net* growth rate in accrued benefits attributable to demographic factors as C_A, the total growth rate in A would be $C_A + r$. In the nonstochastic steady state, C_A would equal $-r$, and A would remain constant. The evolution of A can then be summarized by the process

$$dA = (C_A + \alpha_A)A dt + \sigma_A A dz_A. \tag{4}$$

The stochastic component of (4) is due to uncertainty regarding long-term interest rates and the future pattern of additional net accruals. I will denote the correlation coefficient between dz_A and dz_S as ρ.

Following the analysis in Merton (1973), and letting $P(A,S)$ denote the value of the pension put, one can show that P must satisfy the partial differential equation

$$(5)\quad \frac{1}{2}P_{AA}A^2\sigma_A^2 + \frac{1}{2}P_{SS}S^2\sigma_S^2 + P_{AS}AS\sigma_A\sigma_S\rho - rP + (r + C_A)AP_A + (r + C_S)SP_S = 0,$$

where subscripts on P denote partial derivatives and r denotes the rate of return on instantaneously riskless bonds. Equation (5) lacks a term involving calendar time because the put is of infinite maturity (Merton 1973). The terms C_A and C_S have effects analogous to those of (negative) proportional dividends in the standard option pricing model.

The boundary conditions for P are:

a) At a point of exercise of the put (i.e., termination of the plan), $P = A - S$.
b) The limit of P as S approaches infinity is zero.
c) The limit of P as A approaches zero is zero.
d) The rule for voluntary termination is chosen to maximize the value of the pension-insurance put option.[6]

For general specifications for C_A and C_S, (5) must be solved numerically. (See sec. 3.1.1.3.) In the special case that C_A and C_S are constant, (5) has an analytic solution that can be shown to have the general form (McDonald and Siegel 1982):

$$(6)\qquad P(A,S) = (1 - K)A(S/A)^{\epsilon}K^{-\epsilon},$$

where K is the ratio of S/A at which the option is exercised. Equation (6) will satisfy p.d.e. (5) for

$$\epsilon = -\left[\left(\frac{C_S - C_A}{\sigma^2} - \frac{1}{2}\right)^2 - 2\frac{C_A}{\sigma^2}\right]^{1/2} + \left(\frac{1}{2} - \frac{C_S - C_A}{\sigma^2}\right)$$

$$\sigma^2 = \sigma_A^2 + \sigma_S^2 - 2\rho\sigma_A\sigma_S.$$

These conditions are derived by solving the quadratic equation that is generated by substituting (6) into (5). Choosing K to maximize the value of the option results in the condition

$$(7)\qquad K^* = \frac{\epsilon}{\epsilon - 1}.$$

Equation (6) gives the value of the PBGC insurance plan (under the simplifying assumptions of no bankruptcy and constant C_S and C_A). Given estimates of the parameters in (6) and (7) one could assess the value of the insurance to the shareholders of the firm. These values could serve as the basis for a risk-rated premium structure. Two such structures are discussed below in section 3.1.4.

Equation (7) gives the condition for voluntary termination of the pension plan. Second-order conditions require that $\epsilon < 1$. One must further restrict ϵ to be negative since a feasible K^* must be positive (because A and S are always positive). Thus, $\epsilon < 0$, which implies 0

$< K^* < 1$ so that the put will be exercised only for $S < A$, that is, if fund assets plus 30% of net worth fall below accrued benefits. Parameters that result in nonnegative values for ϵ would imply that the option would never be exercised.[7]

Equations (6) and (7) generalize the formula for the perpetual American put option presented in Merton (1973). In the special case that A is nonstochastic, that $C_S = 0$ and $C_A = -r$ (which offsets the growth in A due to the time value of money and thereby causes the dollar value of the "exercise price," A, to be constant), ϵ equals $-2r/\sigma^2$ and (6) reduces to Merton's equation (52).

Comparative Statics for the Closed Form Solution

Although the closed form solution places an unrealistic restriction on the firm's pension-funding policy, it offers the opportunity to examine analytically some properties of the valuation equation. More realistic specifications of funding policy are considered in later sections. It is possible to show analytically for the special case presented in equation (6) that the value of the termination option increases with C_A and decreases with C_S. Conversely, the ratio of S/A at which it is optimal to terminate falls with C_A and increases with C_S. The intuition for these results is straightforward: when the gap between the growth rates of accrued benefits and the assets backing those benefits ($S = F + .3E$) increases, the expected profits from a future exercise of the put option increase and the value of waiting to exercise correspondingly increases. These results are illustrated in table 3.1, in which optimal ratios for pension termination, $K^* = (S/A)^*$, and the values of the pension put, $P(A,S)$, are presented for various combinations of C_A and C_S and for a variance rate of .05.[8] Recall that the certainty equivalent drifts in A and S are $r + C_A$ and $r + C_S$, respectively. Therefore the parameters presented in table 3.1 correspond to combinations of sustained growth rates in the value of the assets and liabilities of the fund ranging from $-.08$ to $+.06$.

The values of PBGC obligations presented in the second panel of table 3.1 are calculated assuming that $A = S = 1.0$. Therefore, these entries may be interpreted as the value of the pension insurance as a fraction of accrued benefits when the pension put is exactly at the money, that is, when the total assets backing the pension fund obligations equal the present value of those obligations. Remember, however, that this condition does not correspond to full funding of the pension fund, since S includes the contingent liability of the firm of $.3E$. Of course, equation (6) could be used to generate actuarially fair values of the insurance for any initial values of A and S.

The table demonstrates that the value of the termination put can be substantial. As a base case, the zero drift configuration of C_A and C_S

Table 3.1 **Termination Ratios and Option Values** ($\sigma^2 = .05$, $S_0/A_0 = 1$)

	Optimal Exercise Ratio, $K = (S/A)^*$							
$r + C_A$:	−.08	−.06	−.04	−.02	0	.02	.04	.06
$r + C_S$								
−.08	.69	.64	.58	.52	.44	.36	.28	.19
−.06	.72	.68	.62	.55	.48	.40	.31	.21
−.04	.75	.71	.66	.59	.52	.43	.34	.23
−.02	.78	.74	.69	.64	.56	.48	.38	.26
0	.80	.77	.73	.68	.61	.52	.42	.30
.02	.82	.79	.76	.72	.66	.58	.47	.37
.04	.83	.82	.79	.75	.70	.63	.53	.39
.06	.85	.84	.81	.78	.74	.68	.59	.46

	Put Value							
$r + C_A$:	−.08	−.06	−.04	−.02	0	.02	.04	.06
$r + C_S$								
−.08	.136	.162	.196	.238	.290	.356	.440	.549
−.06	.120	.144	.174	.214	.264	.328	.412	.523
−.04	.106	.126	.153	.189	.236	.298	.381	.494
−.02	.093	.110	.134	.165	.208	.266	.347	.461
0	.082	.097	.116	.143	.180	.233	.310	.423
.02	.073	.085	.101	.123	.154	.200	.270	.379
.04	.065	.075	.088	.106	.131	.169	.230	.330
.06	.058	.066	.077	.091	.111	.142	.191	.277

gives a pension put value of 18% of the value of accrued liabilities. Therefore even fully funded plans (where funding includes the firm's contingent liability of .3*E*) can pose significant risk to the PBGC. When $r + C_S$ is negative (i.e., when pension assets are being depleted because of payments to retirees) or when $r + C_A$ is positive, pension insurance values increase dramatically.

It is interesting to note that when $C_A = C_S = 0$, $\epsilon = 0$, and the pension put will never be terminated. In this case, the "exercise price," *A*, is growing at an expected rate equal to its cost of capital; therefore, in contrast to the standard put option, waiting to exercise does not impose a time-value-of-money cost.

The table also can be used to examine the effects of equal changes in C_S and C_A. Reading down the diagonals from top left to bottom right demonstrates that the optimal voluntary termination ratio decreases for larger (algebraic) values of these growth rates. The value of the pension put correspondingly increases. These results derive from the effect of scale on the termination decision. If a pension fund is increasing in size (large positive C_A, C_S), then the dollar gain from a termination for any given *ratio* of *S/A* is larger. If the fund is growing, it pays to wait to terminate, and the ratio *S/A* must be smaller to induce early termination. Thus, one should expect termination decisions to be more frequent in declining industries in which pension funds are shrinking. These results can be verified analytically: Equal (algebraic) increases in C_A and C_S always increase the value of $P(A,S)$ and lower the termination ratio, K^*.

Corporate Pension Funding Policy

Bulow (1981) and Harrison and Sharpe (1982) examine pension funding policy in a model with taxes and with an exogenous termination date. They conclude that a firm should fund its plan either to the maximum or the minimum level permitted. This razor's edge characteristic is also a property of the voluntary termination model.

To confirm this point, compute the first and second derivatives of $P(A,S)$ with respect to pension funding, S:

$$\begin{aligned} P_S &= \epsilon(1 - K)S^{\epsilon-1}A^{1-\epsilon}K^{-\epsilon} \\ &= -LK/(S/A)]^{1-\epsilon} \end{aligned} \tag{8}$$

$$P_{SS} = \epsilon(\epsilon - 1)(1 - K)A^{1-\epsilon}S^{\epsilon-2}K^{-\epsilon} > 0, \tag{9}$$

where the final form of equation (8) is obtained by substituting for ϵ from (7). From (8), for any nonterminated plan (i.e., $K < S/A$), we have that $0 > P_S > -1$, so that each dollar contributed reduces the insurance value by less than \$1.00, and by (9), each successive dollar contributed reduces the insurance value by progressively smaller amounts. In contrast, the marginal tax shield arising from contributions to the pension fund is independent of the level of current funding (Black 1980; Tepper 1981). Therefore, the firm will always be forced to a corner solution: At any interior point, if \$1.00 of extra funding results in an incremental tax shield that exceeds the marginal decrease in the value of pension insurance, then so must the next dollar contribution, and so on. Conversely, if marginally decreased funding is optimal in the interior, then so must be further decreases until some statutory limit is reached. See figure 3.1.

Discretionary Funding: Voluntary Termination

The analytic solution studied in sections 3.1.1.1 and 3.1.1.2 imposes passive behavior on the firm in that pension funding always equals a fixed fraction of current assets, S. In fact, one would expect firms to adjust funding as financial circumstances change. Figure 3.2 presents numerical solutions for PBGC insurance values for three behavioral assumptions.[9] Suppose that firm funding behavior can be described by the following specification:

$$C_S = c_0 + c_1[\ln(A/S)]. \tag{10}$$

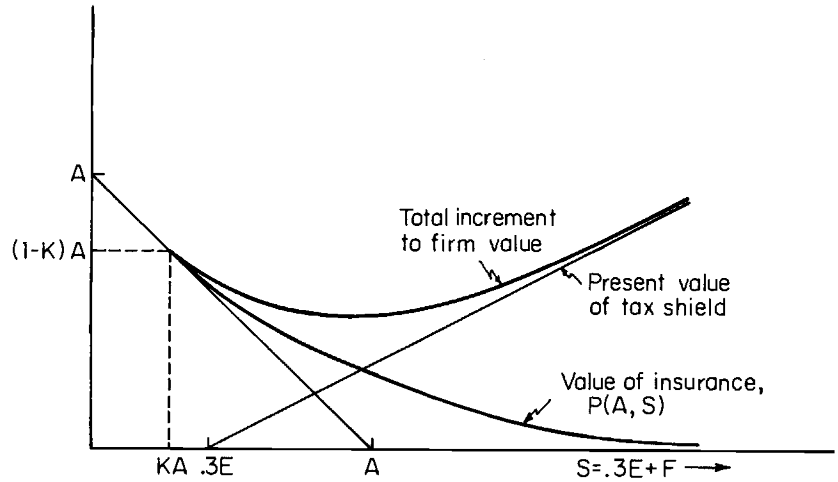

Fig. 3.1 Optimal funding decision

NOTE: The pension plan is terminated when $S/A \leq K$, or at $S = KA$. At termination, the obligation of the PBGC equals $A - S = (1 - K)A$. Before termination, the insurance is worth $P(A,S)$. The tangency at $S = KA$ is the termination point.

The present value of tax savings from pension funding increases with funding, or, holding E fixed, with S. The present value of tax savings is proportional to the level of funding. The total increment to firm value is maximized at either the minimum or maximum permitted funding levels.

For $c_1 = 0$, funding is independent of the current status of the pension plan; this is the passive rule. For $c_1 < 0$, the firm follows an exploitative strategy: if the plan becomes underfunded ($A > S$), then C_S falls, contributions to the plan are reduced, and the value of the pension insurance is increased. Conversely, if the plan is overfunded, then PGBC insurance is less valuable, and funding increases to exploit the tax benefit of further contributions. This specification thus induces the value-maximizing, extreme funding behavior discussed in section 3.1.1.2. Finally, for $c_1 > 0$, the firm follows what might be called socially responsible behavior. Its contribution rate increases when the plan is underfunded and falls when overfunded.

Figure 3.2 presents numerical solutions to equation (5) using parameters $c_0 = -r$ and $c_1 = -.1$, 0, and .1. The figure demonstrates that the value of pension insurance (as a fraction of accrued liabilities) is most variable for the exploitative strategy. For underfunded plans, the value of the insurance is greatest for $c_1 = -.1$, and lowest for $c_1 = .1$. (The values of the pension insurance for $c_1 = .1$ and $c_1 = 0$ are equal for $S/A < .5$, since the pension plan would be terminated at that

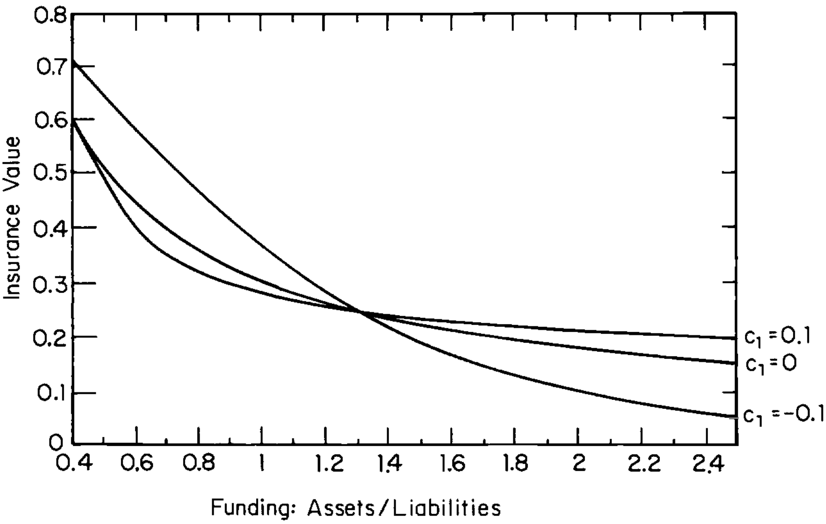

Fig. 3.2 Pension-insurance value as a fraction of plan liabilities (Voluntary-termination scenario)

point for either value of c_1.) Conversely, for overfunded plans, the insurance value is lowest for $c_1 = -.1$.

3.1.2 PBGC Liabilities with Termination Only at Bankruptcy

If the pension plan terminates only when the firm bankrupts, the special put option conveyed by the current pension insurance system is lost. Instead, at bankruptcy, the PBGC simply assumes the pension fund.

The value of the PBGC liability will depend in general upon the exact conditions that set off a bankruptcy. I will assume that bankruptcy is declared when the value of the firm, V, falls below the present value of the debt obligations of the firm, where that value is computed under the assumption that the obligations will be fully met. (This notion of debt, rather than market value, is the appropriate one because limited liability assures that the market value of debt can never exceed V.) Although this definition of bankruptcy is at odds with the technical definition that a firm fails to meet a coupon or principal payment, it still seems a useful way to model bankruptcy for the present purpose. Firms in practice have several overlapping debt issues outstanding with associated sinking fund covenants that would make the modeling of bankruptcy in a legal context exceedingly complex and firm specific. Economic insolvency offers a more straightforward approach.

Denote by D the present value of debt obligations computed by discounting at the riskless-in-terms-of-default interest rate and let $v = V/D$. Then insolvency occurs at the first occurrence of $v \leq 1$. At that moment, the PBGC inherits a net liability of $A - F$, where F denotes the value of the funds in the pension plan. The PBGC's claim to 30% of firm net worth is irrelevant in this instance, since at bankruptcy, when $V \leq D$, equity has no value.

To derive the value of the PBGC insurance, we proceed as before. The dynamics for debt, pension funds, and firm value are taken to be the diffusion processes

$$\begin{aligned} dD &= \alpha_D D dt + \sigma_D D dz_D \\ dF &= (\alpha_F + C_F) F dt + \sigma_F F dz_F \\ dV &= \alpha_V V dt + \sigma_V V dz_V, \end{aligned}$$

where C_F denotes the rate of contributions to the pension fund as a fraction of F. In a nonstochastic steady state with a constant interest rate, C_F would equal $-r$. All fund earnings would be withdrawn to help pay benefits to current retirees so that total fund assets would remain unchanged over time. The covariances between the instantaneous rates of return on the variables will be denoted by σ_{DF}, σ_{DV}, and so on.

Letting $P(v,F,A)$ be the value of the PBGC liabilities, one can show that P must satisfy the p.d.e.

(11)

$$\frac{1}{2}(P_{vv}\,\sigma_v^2\,v^2 + P_{FF}\,\sigma_F^2\,F^2 + P_{AA}\sigma_A^2\,A^2) + P_{vF}\sigma_{vF}vF + P_{vA}\sigma_{vA}vA + P_{FA}\sigma_{FA}FA + P_v rv + P_F(C_F + r)F + P_A\,(C_A + r)A - rP = 0$$

subject to the boundary conditions

a) $P = A - F$ when $v = 1$

b) the limit of P as v approaches infinity is zero

c) the limit of P as A and F approach zero is zero.

These boundary conditions embody the assumption that if a firm with an overfunded plan goes bankrupt, then the PBGC simply inherits the plan together with its surplus. Given this rule, the present value of the PBGC's net liability can be negative. This assumption is likely to be irrelevant in practice, however, since it is highly improbable that a firm with discretionary funding would ever reach bankruptcy with an overfunded pension plan.

For the special case in which C_F and C_A are constant, the solution to this equation is

(12) $$P = Av^{-\phi} - Fv^{-\theta},$$

where

$$\theta = \frac{K}{M} + \left[\left(\frac{K}{M}\right)^2 - \frac{2C_F}{M}\right]^{\frac{1}{2}}$$

$$\phi = \frac{L}{M} + \left[\left(\frac{L}{M}\right)^2 - \frac{2C_A}{M}\right]^{\frac{1}{2}}$$

$$K = -\frac{1}{2}\sigma_V^2 + \frac{1}{2}\sigma_D^2 - \sigma_{DF} + \sigma_{VF}$$

$$\mathrm{L} = -\frac{1}{2}\sigma_V^2 + \frac{1}{2}\sigma_D^2 - \sigma_{DA} + \sigma_{VA}$$

$$M = \sigma_V^2 + \sigma_D^2 - 2\sigma_{DV} = \sigma_v^2,$$

and where the solution is valid for parameters which result in positive values for θ and ϕ.[10]

Optimal corporate pension funding policy in the bankruptcy-only model resembles that in the voluntary termination model. The partial derivative of $P(v,F,A)$ with respect to the funding level, F, is simply $-v^{-\theta}$, which is independent of F. Thus, we again obtain a razor's edge property: If v is sufficiently large, then the tax benefits of additional funding will dominate the transfer of wealth to the PBGC and the firm will fund to the statutory limit. Otherwise, minimal funding will be value maximizing.

Discretionary Funding: Bankruptcy-Only Termination

Bodie et al. (1986) have found some tendency for pension funding policy to vary positively with firm profitability and negatively with the firm's tax-paying status. These results are consistent with the trade-off between the tax and pension-insurance considerations investigated in this paper. In order to explore the implications of discretionary funding policy in the bankruptcy-only termination model, consider the following specification for funding behavior:

(13) $$C_F = c_0 - c_1(D/V).$$

For $c_1 > 0$, funding declines with the firm's debt ratio (and associated probability of bankruptcy) to a minimum possible level of $c_0 - c_1$. Although debt ratios are not perfect measures of firm financial status, especially in interindustry comparisons, this specification does capture the stylized notion that as a firm approaches bankruptcy, its pension funding will decrease and in fact can become negative. Negative contributions are, strictly speaking, disallowed by ERISA. However, de facto negative funding is realized when the pension plan purchases equity or debt of the firm.

Figure 3.3 displays numerical solutions for the value of PBGC insurance as a function of the debt ratio, D/V, for a fully funded plan for three values of c_1.[11] We set c_0 at a level such that at $D/V = 0$, the ratio of plan assets to liabilities would increase at a rate of 2% per year. As D/V increases, the funding rate falls, and eventually the ratio of assets to liabilities will decrease over time.

For extreme values of the debt ratio, the present value of PBGC liabilities equals zero. Because the plan is fully funded, the PBGC faces no liability even if the firm bankrupts (i.e., $D/V = 1$). At the other extreme, as the debt ratio approaches zero, PBGC liabilities fall to zero because the probability of bankruptcy vanishes. For middle-range values of the debt ratio, however, PBGC liabilities can be quite large. If the firm reaches a debt ratio of .6, for example, there is a significant chance of bankruptcy, and until bankruptcy is reached, the firm will

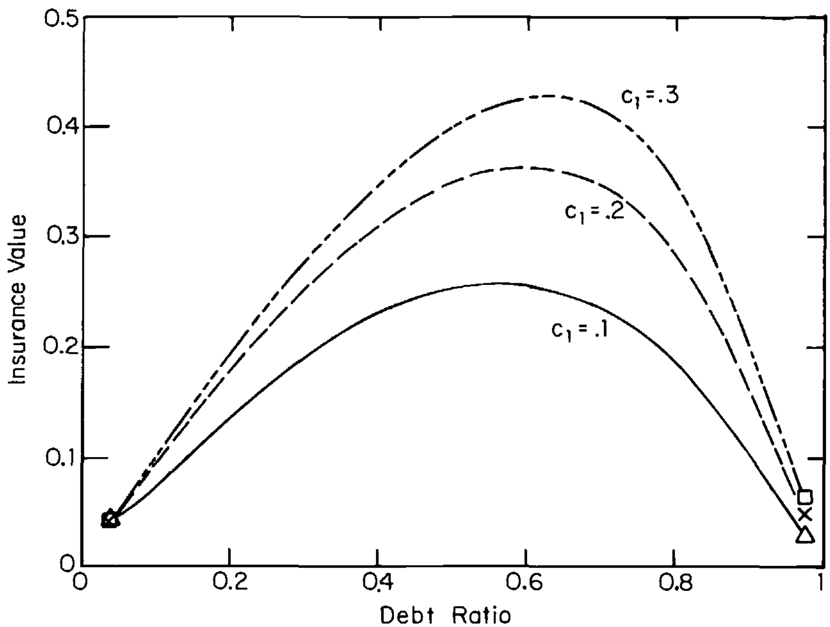

Fig. 3.3 Pension-insurance value as a fraction of plan liabilities (Fully funded plan, bankruptcy-only termination)

continue to drain the pension plan. In contrast, if the firm is fully funded when $D/V = .9$, bankruptcy might be imminent, but there is less time for the firm to extract funds from the plan. The PBGC's liability is correspondingly small.

Figure 3.3 shows that the PBGC's liability can be a significant fraction of vested benefits, even for fully funded plans. Using the conservative assumption that $c_1 = .1$, the value of pension insurance still can rise to more than 20% of benefits. Further, the disparity in insurance values between the $c_1 = .1$ and $c_1 = .3$ curves shows that the value of pension insurance can be quite sensitive to firm funding policy, even without the option for firms to voluntarily terminate plans. These results imply that the common practice of estimating PBGC liabilities as $\max(0, A - S)$ can be quite misleading. They also indicate that spinoff/terminations, which allow firms to recapture the surplus assets from a pension fund and leave the PBGC guaranteeing a fund with no cushion against adverse investment experience, can result in significant PBGC liabilities.

3.1.3 The General Case

A general treatment of PBGC insurance would allow for termination either at the first occurrence of a voluntary termination point or at the first occurrence of corporate bankruptcy. As a general rule, there is no closed-form solution for the value of PBGC pension insurance in this mixed case, even with passive funding policies. The difficulty arises from the effects of debt on the variance rate of the firm's equity. Geske (1979) has shown that the variance rate evolves stochastically in this situation. Because the assets backing pension benefits, S, include 30% of firm net worth, σ^2 in equation (6) could no longer be taken as a fixed parameter, and the solution for the value of the pension insurance consequently would need to be modified. This effect, together with the fact that termination can result from either of two conditions, appears to make a numerical solution technique necessary. Even the numerical approach presents difficulties, however, since the problem would involve four state variables: A, S, F, and v.

Notwithstanding these complications, the above solutions still can be of use in valuing PBGC liabilities. The voluntary termination model should provide an upper bound on the value of pension insurance, since the termination rule is chosen to maximize the value of the insurance. In contrast, the termination-only-at-bankruptcy model provides a lower bound on the value of the insurance.

In practice, underfunded plans are associated with financially troubled firms. The models provide some clues to why troubled firms should tend to maintain underfunded plans. One possibility is that such firms have low marginal tax rates due to loss carry-forward provisions, and therefore derive less tax benefit from pension funding. Another explanation is that underfunding the pension plan represents a source of financing cheaper than that available in outside credit markets. This advantage will be greatest for firms with the highest borrowing rates. Finally, if bankruptcy causes the firm to forfeit the pension assets to the PBGC, overfunding of the plan would create a potential bankruptcy cost to which troubled firms would be more sensitive. This effect was made explicit in section 3.1.2, in which it was shown that firms with large values of D/V will find that minimal funding is value maximizing.

3.1.4 Risk-Rated Premiums

The valuation equations derived in sections 3.1.1 and 3.1.2 provide the present value of PBGC liabilities under different scenarios. They do not, however, provide explicit means to calculate fair annual premium *rates* for pension insurance. Because fund termination dates are stochastic, the premium annuity that has an ex ante present value equal to the present value of PBGC obligations cannot be easily calculated. One approach that might provide a reasonable approximation to the fair premium rate would be first to calculate the expected value of the time to termination, and then to calculate the annuity appropriate to the present value of PBGC obligations using a horizon equal to the expected time until termination and an interest rate equal to that paid on the firm's outstanding debt.

A different approach would require ex post settling up. At the start of each period, the present value of PBGC obligations would be calculated. At period-end, that value would be recalculated, and the firm would pay (or be paid) the change in the value of PBGC liabilities. The advantage of this scheme is that it eliminates most of the moral hazard problems involved in prespecified rate structures. Any increase in risk would induce increased premiums. The firm would always pay a fair price for its pension-put option (or for its limited liability in the bankruptcy model) and would thus lose the ability and the incentive to underfund at the expense of the PBGC.

3.2 Empirical Estimates

Estimates of the value of PBGC insurance will be presented for both the voluntary and bankruptcy-only termination models. For each model, three scenarios are considered, corresponding to different plan-funding strategies. Values in the voluntary termination model are calculated for $c_0 = -r$ and for $c_1 = -.1$ (exploitative strategy), 0 (passive strategy), and .1 (socially responsible strategy). (See eq. [10].) Values in the bankruptcy-only termination model are calculated for coefficients on the debt ratio equal to .1, .2, and .3, and for $c_0 = -r + .02$. (See eq. [13].) The (real) interest rate in all of the calculations is set at .04.

3.2.1 Data

Pensions and Investment Age (July 11, 1983) reports pension fund statistics derived from the 1982 annual reports of the Fortune 100 companies. The survey includes pension fund assets, vested benefits, and the assumed interest rate used to derive the present value of vested benefits.

The survey expresses pension fund assets as market values. The market value of vested benefits can be approximated by multiplying the reported value of benefits by the ratio of the plan's assumed interest rate to the actual long-term market interest rate for 1982. This adjustment assumes that pension benefit payout streams have time paths similar to perpetuities. The average rate on 30-year United States government obligations in 1982 was 12.76%. The market value of equity is easily derived from stock market data at year-end 1982, and total firm value can be approximated as equity plus book value of long-term debt.

The remaining inputs required to estimate the value of PBGC insurance are the variance and covariance rates on underlying securities. Table 3.2 presents the values assigned to these variables. These values are meant to be reasonable guesses only. The low variance rates on A and D and high correlation between the two reflect their similar natures as nominal liabilities. The variance rates on firm value and pension

Table 3.2 Assumptions Used to Compute Value of Insurance

		Variance Rate (annual)
Fund liabilities	A	.01
Fund assets	F	.04
Assets + .3 equity	S	.04
Firm debt	D	.01
Firm value	V	.04

Correlation Matrix

	A	F	S	D	V
A					
F	n				
S	.1	n			
D	.8	.1	n		
V	.1	.5	n	.2	

NOTE: n = correlation coefficient between these variables was not necessary for calculations.

fund assets compare to a historical value for the S&P 500 of approximately .05 annually. The variance rate for V is derived by unlevering the S&P 500 variance using a debt/value ratio of 1/3 and then by doubling that variance to account for the lack of diversification of a single stock relative to the index. The variance rate on fund assets is set slightly below that on the S&P 500: The fund is probably less well diversified than the index, but this effect is offset by debt held in the fund.

3.2.2 Results

Tables 3.3a and 3.3b present estimates of the value of PBGC insurance for 87 of the Fortune 100 firms. Thirteen observations were lost because of missing data. Table 3.3a presents results in which the present values of benefits are calculated using the 12.76% yield on 30-year T-bonds that prevailed during 1982, while table 3.3b uses a 10% interest rate. Columns 1 and 2 of the tables are the present value of vested benefits for each plan, and the level of overfunding of each plan, respectively. Columns 3–8 are the ratios of the value of PBGC insurance to vested benefits for the voluntary termination scenario and the bankruptcy-only scenario under the three assumptions for firm funding behavior. These ratios can be interpreted as the fraction of pension benefits that are financed (in present value terms) by the PBGC. The ratios thus give a measure of the PBGC subsidy per dollar of pension benefits.

The results in tables 3.3a and 3.3b are consolidated in tables 3.4 and 3.5. Table 3.4 presents summary statistics for the voluntary termination model. The table reveals that PBGC liabilities can be extremely sensitive to firm funding policy. At current funding levels, total liabilities for the exploitative strategy ($c_1 = -.1$) are less than one-third their value for the socially responsible strategy ($c_1 = .1$). This result reflects the overfunded status of most plans in 1982. At 1982 funding levels, the exploitative strategy entailed the largest contribution rate into the pension fund (in order to maximize tax benefits), and thus resulted in the smallest insurance values.

Although table 3.4 indicates that most firms derive little value from pension insurance, a small number of "problem firms" derive considerable value from the pension insurance. These tend to be the larger firms: the weighted averages of the insurance values are substantially greater than the means.

As expected, PBGC liabilities are extremely sensitive to the interest rate used in calculating vested benefits. Table 3.4 shows that the total insurance values for the 87 firms in the voluntary termination scenario are 1.4–2.0 times as large for a 10% interest rate as they are for the actual 1982 rate of 12.76%. The total value of PBGC liabilities for the 87 firms is extremely large, ranging from \$6.7 billion to \$20.6 billion

Table 3.3a **PBGC Insurance Values** (Nominal interest rate = 12.76%)

			Insurance Value as a Fraction of Vested Benefits					
			Voluntary Termination			Bankruptcy-Only		
	Vested	Over-	Sensitivity to Funding			Sensitivity to Debt		
Company	Benefits	Funding	−.10	0	.10	.10	.20	.30
ALLIED	551.	259.	0.0687	0.1537	0.1898	0.1417	0.2717	0.3352
ALCOA	1053.	322.	0.0672	0.1501	0.1855	0.1763	0.2917	0.3481
AMER HESS	37.	70.	0.0	0.0292	0.1355	−.2307	0.0504	0.1958
AM BRANDS	239.	97.	0.0029	0.0824	0.1558	0.0991	0.1537	0.1734
AM CAN	655.	247.	0.1511	0.2032	0.2217	0.1352	0.2812	0.3622
AN-BUSCH	149.	165.	0.0001	0.0441	0.1273	0.0597	0.1698	0.2128
ARMCO	842.	328.	0.1218	0.1873	0.2118	0.1586	0.2815	0.3415
ASHLAND OIL	135.	205.	0.0027	0.0765	0.1446	−.2301	0.0375	0.1860
ARCO	878.	635.	0.0005	0.0569	0.1275	0.1047	0.2224	0.2732
BETH STEEL	2472.	−148.	0.3384	0.2878	0.2703	0.2699	0.3751	0.4387
BOEING	1140.	1261.	0.0171	0.1093	0.1651	0.0357	0.0738	0.0866
BORDEN	150.	92.	0.0028	0.0779	0.1473	0.1068	0.1910	0.2238
BURROUGHS	348.	223.	0.0181	0.1161	0.1754	0.1163	0.2283	0.2766
CATERPILLAR	1260.	733.	0.0508	0.1478	0.1931	0.1182	0.2580	0.3263
CHRYSLER	2277.	−329.	0.3380	0.2875	0.2700	0.3055	0.4013	0.4591
COASTAL	37.	71.	0.0006	0.0680	0.1523	−.9242	−.6312	−.4335
COCA-COLA	139.	96.	0.0	0.0253	0.1172	0.0423	0.0588	0.0644
COLG-PALMOL	211.	274.	0.0029	0.0827	0.1564	0.0351	0.1037	0.1274
CONS FOODS	61.	80.	0.0001	0.0463	0.1339	0.0398	0.1340	0.1681
CONTL GROUP	614.	304.	0.0684	0.1528	0.1887	0.1246	0.2700	0.3452
CONTROL DATA	120.	157.	0.0006	0.0621	0.1392	−.5791	−.3450	−.1871
CPC INTL	136.	17.	0.0006	0.0604	0.1352	0.0637	0.0841	0.0909
DEERE	569.	545.	0.0173	0.1110	0.1676	0.0721	0.2060	0.2637
DIGITAL EQ	22.	151.	0.0	0.0	0.0228	0.0420	0.0438	0.0443
DOW CHEM	655.	513.	0.0026	0.0737	0.1392	0.0754	0.2332	0.3103
DRESSER	326.	291.	0.0191	0.1225	0.1850	0.0735	0.1580	0.1894
DU PONT	3586.	4057.	0.0165	0.1054	0.1592	0.0024	0.1908	0.2829
EAST KODAK	1276.	1466.	0.0005	0.0596	0.1336	0.0440	0.0445	0.0446
EXXON	1939.	2306.	0.0006	0.0671	0.1504	0.0431	0.1159	0.1413
FIRESTONE	745.	256.	0.1199	0.1843	0.2084	0.1444	0.2240	0.2564
FORD	4420.	2800.	0.0969	0.1759	0.2073	0.1175	0.2384	0.2925
GEN DYNAMICS	569.	726.	0.0185	0.1187	0.1793	0.0440	0.0445	0.0446
GEN ELEC	4208.	4474.	0.0082	0.0962	0.1604	0.0412	0.0469	0.0488
GEN FOODS	397.	535.	0.0088	0.1041	0.1735	0.0350	0.1652	0.2171
GEN MILLS	221.	102.	0.0031	0.0866	0.1637	0.0681	0.1063	0.1193
GEN MOTORS	13195.	1237.	0.1808	0.2180	0.2307	0.1297	0.1762	0.1933
GEORGIA PAC	97.	122.	0.0	0.0157	0.0726	0.0043	0.1859	0.2705
GETTY OIL	232.	255.	0.0001	0.0378	0.1093	0.0599	0.1699	0.2128
GOODYEAR	983.	590.	0.0501	0.1459	0.1905	0.1189	0.2204	0.2629
WR GRACE	109.	240.	0.0001	0.0436	0.1259	−.2231	0.0600	0.1983
GREYHOUND	656.	326.	0.0913	0.1658	0.1953	0.1359	0.2381	0.2823
GULF OIL	1067.	856.	0.0188	0.1204	0.1818	0.0941	0.2083	0.2561
GULF&WEST	245.	132.	0.0176	0.1128	0.1704	0.0173	0.1899	0.2941
HEWLETT-PACK	230.	270.	0.0	0.0211	0.0976	0.0440	0.0445	0.0446

Table 3.3a (continued)

			Insurance Value as a Fraction of Vested Benefits					
			Voluntary Termination			Bankruptcy-Only		
	Vested	Over-	Sensitivity to Funding			Sensitivity to Debt		
Company	Benefits	Funding	−.10	0	.10	.10	.20	.30
IC INDUS	174.	103.	0.0501	0.1456	0.1902	−.1592	0.0031	0.1126
IBM	2909.	5481.	0.0001	0.0465	0.1345	0.0354	0.0463	0.0499
INTL PAPER	401.	560.	0.0027	0.0754	0.1426	0.0203	0.1723	0.2359
ITT	1039.	625.	0.0330	0.1350	0.1883	0.1140	0.2556	0.3248
J&JOHNSON	146.	218.	0.0	0.0	0.0058	0.0439	0.0444	0.0446
KERR-MCGEE	41.	85.	0.0	0.0211	0.0977	−.1381	0.1076	0.2220
LITTON INDUS	290.	289.	0.0026	0.0741	0.1401	0.0399	0.0795	0.0928
LOCKHEED	1228.	1296.	0.0498	0.1448	0.1892	0.0394	0.2052	0.2824
LTV	1333.	115.	0.2572	0.2480	0.2443	0.1437	0.2543	0.3288
MCDERMOTT	311.	270.	0.0307	0.1253	0.1748	0.0810	0.2192	0.2810
MCDONNELL DO	949.	1052.	0.0309	0.1265	0.1764	0.0432	0.0451	0.0457
3M	330.	403.	0.0	0.0154	0.0713	0.0440	0.0445	0.0446
MOBIL	1315.	1643.	0.0029	0.0830	0.1568	0.0319	0.1856	0.2519
MONSANTO	803.	894.	0.0188	0.1205	0.1820	0.0590	0.1695	0.2126
MOTOROLA	43.	146.	0.0	0.0	0.0075	−.0421	0.0538	0.0861
NABISCO	261.	77.	0.0026	0.0743	0.1404	0.1198	0.1776	0.1991
PEPSICO	111.	172.	0.0	0.0173	0.0803	0.0223	0.1362	0.1786
PHILIP MORRI	195.	296.	0.0	0.0212	0.0983	−.0202	0.1661	0.2493
PHILLIPS PET	445.	648.	0.0006	0.0631	0.1414	0.0195	0.1648	0.2240
RALSTON PUR.	81.	191.	0.0001	0.0523	0.1513	−.1217	0.1080	0.2071
RJ REYNOLDS	391.	475.	0.0007	0.0719	0.1611	0.0334	0.0584	0.0668
ROCKWELL INT	1322.	1436.	0.0334	0.1364	0.1903	0.0468	0.2013	0.2703
SHELL OIL	715.	942.	0.0001	0.0374	0.1082	0.0440	0.0445	0.0446
SIGNAL COS.	388.	322.	0.0185	0.1187	0.1793	−.2992	−.1130	0.0126
SPERRY	424.	618.	0.0080	0.0938	0.1563	0.0265	0.1549	0.2050
STD OIL CAL	607.	584.	0.0001	0.0388	0.1121	0.0383	0.0549	0.0605
STD OIL IND	848.	585.	0.0006	0.0630	0.1411	0.0684	0.1225	0.1413
STD OIL OHIO	494.	516.	0.0001	0.0388	0.1120	0.0601	0.1999	0.2602
SUN CO	486.	524.	0.0028	0.0787	0.1488	−.0885	0.1320	0.2543
TEXACO	541.	632.	0.0007	0.0722	0.1618	0.0529	0.1547	0.1930
TEXAS INST	81.	258.	0.0	0.0238	0.1104	−.4265	−.0581	0.1220
TENNECO	374.	322.	0.0005	0.0580	0.1299	0.0416	0.0482	0.0503
TRW	586.	550.	0.0179	0.1149	0.1735	−.3646	−.1672	−.0340
UNION CARB	945.	787.	0.0173	0.1107	0.1672	0.0420	0.0714	0.0813
UNION OIL CA	325.	389.	0.0006	0.0701	0.1571	0.0299	0.1924	0.2650
UNION PACIFI	107.	112.	0.0	0.0222	0.1030	0.0643	0.1655	0.2043
UNITED BRAND	136.	79.	0.1249	0.1922	0.2173	−.5156	−.4857	−.4623
US STEEL	5003.	2236.	0.1451	0.1951	0.2129	0.0581	0.0876	0.0976
UNITED TECH	1205.	1650.	0.0180	0.1153	0.1742	−.6196	−.3786	−.2161
WARNER COMM	26.	38.	0.0	0.0	0.0074	0.0088	0.1667	0.2328
WESTINGHOUSE	1832.	883.	0.0684	0.1529	0.1889	0.0627	0.0980	0.1099
WEYERHAEUSER	296.	175.	0.0006	0.0676	0.1514	0.0542	0.0878	0.0991
XEROX	557.	386.	0.0077	0.0908	0.1513	0.1090	0.2246	0.2744

Table 3.3b **PBGC Insurance Values** (Nominal interest rate = 10%)

			Insurance Value as a Fraction of Vested Benefits					
			Voluntary Termination			Bankruptcy-Only		
	Vested	Over-	Sensitivity to Funding			Sensitivity to Debt		
Company	Benefits	Funding	−.10	0	.10	.10	.20	.30
ALLIED	703.	107.	0.1480	0.1991	0.2173	0.2087	0.3105	0.3603
ALCOA	1344.	31.	0.1446	0.1945	0.2122	0.2358	0.3262	0.3705
AMER HESS	47.	60.	0.0	0.0229	0.1062	−.0715	0.1488	0.2627
AM BRANDS	306.	30.	0.0081	0.0957	0.1595	0.1190	0.1618	0.1772
AM CAN	836.	66.	0.2294	0.2350	0.2363	0.2301	0.3445	0.4080
AN-BUSCH	190.	124.	0.0007	0.0711	0.1594	0.1030	0.1893	0.2229
ARMCO	1074.	96.	0.2027	0.2236	0.2302	0.2219	0.3182	0.3653
ASHLAND OIL	172.	168.	0.0191	0.1224	0.1849	−.0562	0.1536	0.2699
ARCO	1121.	392.	0.0026	0.0742	0.1402	0.1562	0.2484	0.2882
BETH STEEL	3154.	−830.	0.4387	0.3387	0.3049	0.3553	0.4378	0.4876
BOEING	1455.	946.	0.0487	0.1418	0.1852	0.0487	0.0786	0.0886
BORDEN	192.	50.	0.0077	0.0905	0.1509	0.1399	0.2059	0.2316
BURROUGHS	445.	126.	0.0518	0.1507	0.1968	0.1652	0.2530	0.2908
CATERPILLAR	1608.	385.	0.0943	0.1712	0.2017	0.1902	0.2998	0.3534
CHRYSLER	2906.	−958.	0.4383	0.3384	0.3046	0.3832	0.4582	0.5036
COASTAL	48.	60.	0.0031	0.0887	0.1676	−.5534	−.3237	−.1688
COCA-COLA	178.	57.	0.0	0.0198	0.0918	0.0479	0.0609	0.0652
COLG-PALMOL	269.	216.	0.0082	0.0961	0.1601	0.0592	0.1130	0.1315
CONS FOODS	78.	63.	0.0001	0.0363	0.1049	0.0745	0.1482	0.1750
CONTL GROUP	783.	135.	0.1472	0.1979	0.2160	0.2069	0.3208	0.3798
CONTROL DATA	154.	123.	0.0029	0.0810	0.1531	−.2828	−.0994	0.0243
CPC INTL	174.	−21.	0.0028	0.0787	0.1487	0.0707	0.0866	0.0920
DEERE	726.	388.	0.0495	0.1440	0.1881	0.1306	0.2355	0.2807
DIGITAL EQ	28.	145.	0.0	0.0	0.0179	0.0426	0.0440	0.0444
DOW CHEM	835.	333.	0.0184	0.1179	0.1781	0.1567	0.2804	0.3408
DRESSER	416.	201.	0.0308	0.1258	0.1755	0.1053	0.1714	0.1960
DU PONT	4576.	3067.	0.0470	0.1368	0.1787	0.0995	0.2472	0.3193
EAST KODAK	1628.	1114.	0.0028	0.0778	0.1469	0.0442	0.0445	0.0447
EXXON	2474.	1771.	0.0031	0.0875	0.1654	0.0690	0.1261	0.1460
FIRESTONE	951.	50.	0.1995	0.2201	0.2266	0.1771	0.2394	0.2648
FORD	5640.	1580.	0.1808	0.2180	0.2306	0.1726	0.2674	0.3097
GEN DYNAMICS	726.	569.	0.0298	0.1219	0.1700	0.0441	0.0445	0.0447
GEN ELEC	5370.	3312.	0.0333	0.1362	0.1900	0.0432	0.0476	0.0491
GEN FOODS	507.	425.	0.0176	0.1125	0.1699	0.0872	0.1893	0.2299
GEN MILLS	282.	41.	0.0085	0.1006	0.1677	0.0813	0.1113	0.1215
GEN MOTORS	16837.	−2405.	0.2545	0.2454	0.2417	0.1470	0.1834	0.1968
GEORGIA PAC	124.	95.	0.0001	0.0431	0.1245	0.0916	0.2340	0.3002
GETTY OIL	295.	192.	0.0006	0.0611	0.1368	0.1031	0.1894	0.2230
GOODYEAR	1254.	319.	0.0930	0.1689	0.1990	0.1618	0.2414	0.2747
WR GRACE	139.	210.	0.0006	0.0704	0.1577	−.0772	0.1446	0.2530
GREYHOUND	837.	145.	0.1704	0.2054	0.2174	0.1806	0.2607	0.2953
GULF OIL	1362.	561.	0.0302	0.1236	0.1724	0.1423	0.2319	0.2693
GULF&WEST	312.	65.	0.0503	0.1464	0.1912	0.1573	0.2926	0.3743
HEWLETT-PACK	294.	206.	0.0	0.0165	0.0765	0.0442	0.0445	0.0447

Table 3.3b (continued)

			Insurance Value as a Fraction of Vested Benefits					
			Voluntary Termination			Bankruptcy-Only		
	Vested	Over-	Sensitivity to Funding			Sensitivity to Debt		
Company	Benefits	Funding	−.10	0	.10	.10	.20	.30
IC INDUS	222.	55.	0.0929	0.1686	0.1987	0.0462	0.1734	0.2592
IBM	3711.	4679.	0.0001	0.0365	0.1054	0.0391	0.0476	0.0505
INTL PAPER	511.	450.	0.0188	0.1208	0.1824	0.0845	0.2037	0.2535
ITT	1326.	338.	0.0735	0.1643	0.2030	0.1870	0.2979	0.3522
J&JOHNSON	187.	177.	0.0	0.0254	0.1179	0.0441	0.0445	0.0446
KERR-MCGEE	53.	73.	0.0	0.0165	0.0765	−.0200	0.1726	0.2622
LITTON INDUS	370.	209.	0.0185	0.1186	0.1792	0.0534	0.0845	0.0949
LOCKHEED	1567.	957.	0.0924	0.1677	0.1976	0.1191	0.2491	0.3096
LTV	1701.	−253.	0.3862	0.3121	0.2868	0.2836	0.3702	0.4287
MCDERMOTT	397.	184.	0.0682	0.1525	0.1884	0.1440	0.2523	0.3008
MCDONNELL DO	1211.	790.	0.0688	0.1539	0.1901	0.0438	0.0453	0.0458
3M	421.	312.	0.0001	0.0423	0.1222	0.0441	0.0445	0.0447
MOBIL	1678.	1280.	0.0082	0.0964	0.1606	0.0991	0.2196	0.2715
MONSANTO	1024.	673.	0.0303	0.1237	0.1726	0.1024	0.1890	0.2228
MOTOROLA	55.	134.	0.0	0.0	0.0059	−.0092	0.0660	0.0913
NABISCO	333.	5.	0.0186	0.1189	0.1796	0.1415	0.1868	0.2037
PEPSICO	141.	142.	0.0001	0.0476	0.1376	0.0651	0.1544	0.1876
PHILIP MORRI	249.	242.	0.0	0.0166	0.0770	0.0647	0.2107	0.2759
PHILLIPS PET	568.	525.	0.0029	0.0823	0.1555	0.0791	0.1931	0.2395
RALSTON PUR.	103.	169.	0.0001	0.0410	0.1185	−.0213	0.1587	0.2364
RJ REYNOLDS	499.	367.	0.0005	0.0563	0.1263	0.0418	0.0615	0.0680
ROCKWELL INT	1686.	1072.	0.0743	0.1660	0.2051	0.1172	0.2383	0.2924
SHELL OIL	912.	745.	0.0006	0.0605	0.1355	0.0441	0.0445	0.0446
SIGNAL COS.	495.	215.	0.0298	0.1219	0.1701	−.0635	0.0824	0.1809
SPERRY	542.	500.	0.0325	0.1327	0.1852	0.0770	0.1776	0.2168
STD OIL CAL	774.	417.	0.0006	0.0626	0.1403	0.0440	0.0570	0.0613
STD OIL IND	1082.	351.	0.0029	0.0821	0.1552	0.0875	0.1300	0.1447
STD OIL OHIO	630.	380.	0.0006	0.0626	0.1403	0.1212	0.2307	0.2780
SUN CO	620.	390.	0.0078	0.0914	0.1524	0.0547	0.2276	0.3234
TEXACO	690.	483.	0.0005	0.0566	0.1268	0.0917	0.1714	0.2014
TEXAS INST	104.	235.	0.0	0.0187	0.0865	−.2367	0.0521	0.1932
TENNECO	477.	219.	0.0027	0.0756	0.1429	0.0438	0.0490	0.0507
TRW	747.	389.	0.0512	0.1491	0.1947	−.1148	0.0399	0.1443
UNION CARB	1206.	526.	0.0494	0.1436	0.1876	0.0520	0.0751	0.0828
UNION OIL CA	415.	299.	0.0005	0.0549	0.1231	0.1040	0.2313	0.2882
UNION PACIFI	136.	83.	0.0	0.0174	0.0807	0.1034	0.1827	0.2131
UNITED BRAND	174.	41.	0.2080	0.2294	0.2362	−.1929	−.1694	−.1511
US STEEL	6384.	855.	0.2618	0.2525	0.2487	0.0682	0.0914	0.0992
UNITED TECH	1538.	1317.	0.0514	0.1496	0.1954	−.3146	−.1258	0.0016
WARNER COMM	33.	31.	0.0	0.0	0.0058	0.0755	0.1993	0.2511
WESTINGHOUSE	2338.	377.	0.1473	0.1981	0.2162	0.0748	0.1025	0.1118
WEYERHAEUSER	377.	94.	0.0031	0.0881	0.1665	0.0657	0.0920	0.1009
XEROX	711.	232.	0.0314	0.1285	0.1792	0.1595	0.2501	0.2891

Table 3.4 Insurance Value Summary Statistics (Voluntary Termination Model)

Insurance Value as a Fraction of Vested Benefits	$C_1 = -.1$		$C_1 = 0$		$C_1 = .1$	
	$r = .1276$	$r = .10$	$r = .1276$	$r = .10$	$r = .1276$	$r = .10$
A. Frequency Distribution						
0–.01	51	42	4	3	3	2
.01–.025	13	6	8	7	1	1
.025–.05	5	12	10	7	0	0
.05–.075	7	8	17	10	2	0
.075–.10	2	4	11	14	4	6
.10 –.15	4	4	22	22	26	19
.15 –.25	2	6	13	20	49	56
.25+	3	5	2	4	2	3
B. Summary Statistics						
Maximum value	.338	.439	.288	.389	.270	.305
Mean value	.033	.058	.095	.117	.153	.166
Median value	.003	.018	.082	.118	.156	.170
Weighted average[a]	.084	.134	.145	.175	.187	.204
Total value ($ billion)	6.7	13.6	11.5	17.7	14.8	20.6

[a]Weights = value of vested benefits.

for the different cases considered in the table 3.4. These values compare with PBGC reserves for insured future benefits of only $1.14 billion (PBGC *Annual Report,* fiscal year 1982). Therefore, if the option to terminate voluntarily is to be taken seriously, the PBGC reserve calculations are wildly optimistic. Keep in mind that the total insurance values presented in tables 3.4 and 3.5 are summed only over the 87 firms in the sample. The PBGC liabilities for all insured firms must be significantly greater.

The insurance values for individual firms also differ from the traditional measure of underfunding ($A - F - .3E$) by wide margins, and highlight the pitfalls of ignoring the option component of pension insurance in assessing PBGC liabilities. In fact, even ignoring the firm's contingent liability of $.3E$, the total underfunding of all the underfunded plans in the sample is only $0.48 billion for benefits calculated using a 12.76% interest rate and $4.47 billion using a 10% rate. These values are small fractions of the values derived from the voluntary termination model.

Table 3.5 presents summary statistics for the bankruptcy-only termination model. These results are similar to those presented in table 3.4. The same sensitivity to the interest rate and even greater sensitivity to the firm's funding behavior is evidenced. Interestingly, the values

Table 3.5 Insurance Value Summary Statistics (Bankruptcy-Only Termination Model)

Insurance Value as a Fraction of Vested Benefits	C_1 = .10		C_1 = .20		C_1 = .30	
	r = .1276	r = .10	r = .1276	r = .10	r = .1276	r = .10
A. Frequency Distribution						
−.6–0	16	13	7	4	5	2
0–.1	51	39	26	24	22	22
.1–.2	18	27	31	26	17	14
.2–.3	1	6	21	25	32	31
.3–.4	1	2	1	6	9	14
.4–.5	0	0	1	2	2	3
.5+	0	0	0	0	0	1
B Summary Statistics						
Maximum value	.306	.383	.401	.452	.459	.504
Mean value	.003	.073	.115	.161	.169	.203
Median value	.044	.079	.155	.173	.199	.223
Weighted average[a]	.070	.119	.155	.186	.196	.217
Total value ($ billion)	5.6	12.0	12.3	18.8	15.5	22.0

[a]Weights = value of vested benefits.

for total dollar liabilities of the PBGC are quite similar in the two models, despite the disparities in assumed funding behavior and plan-termination conditions.

The value of PBGC insurance for some firms in the bankruptcy-only model is negative. This reflects the two assumptions that (1) the PBGC would inherit the surplus of an overfunded plan if the firm were to bankrupt and that (2) there is a limit on the rate at which the firm can drain funds from the plan as bankruptcy approaches. (See eq. [13].) The firms with negative PBGC liabilities tend to be extremely overfunded. A nonlinear version of equation (13) that allowed plan disfunding to increase without bound as D/V neared 1.0 would eliminate the negative values. However, it is not clear that the latter assumption is superior to the one embodied in (13). The ability of insurance values to be negative makes the distribution of values in table 3.5 more symmetric than in table 3.4. The mean, median, and weighted average of pension insurance values are all of similar magnitudes.

3.3 Conclusion

This paper derives the value of PBGC pension insurance liabilities under two scenarios of interest. The first allows for voluntary plan termination, which appears to be legal under current statutes. The second is a termination-only-at-bankruptcy rule that has been suggested as a reform to current law. Optimal pension fund financing decisions are examined; extreme pension funding policies are shown to be optimal in both settings. This result corroborates and generalizes those of earlier authors. Finally, empirical estimates of PBGC liabilities are derived. These show that a small number of funds account for a large fraction of total prospective PBGC liabilities, and that those total liabilities far exceed current reserves for plan termination.

The empirical results support several conclusions. First, the ability of firms to voluntarily terminate pension plans is a potentially important option, the value of which can be substantially underestimated by the simple measure max(0, $A - S$). Second, even without the ability to terminate, discretionary pension-funding policy can lead to equally large PBGC liabilities. Even fully funded plans can impose contingent liabilities with present value more than 25% of vested benefits. This result implies that so-called spinoff/terminations, which effectively allow firms to recapture the surplus assets in a pension plan, impose significant costs on the PBGC, in the sense that the present value of PBGC liabilities increases substantially as surplus assets are siphoned out of funds. Moreover, these liabilities are extremely sensitive to small changes in ongoing funding policy. These results again call into question the common practice of measuring PBGC liabilities as max(0, $A - S$). Finally, the estimates of PBGC liabilities support the view that the PBGC's reserves for future terminations are far below the present value of its contingent liabilities.

Notes

1. A put option gives its owner the right to sell to the issuer of the option share of stock at a prespecified price (the exercise price) regardless of the actual price of the stock. Thus, if the stock price, S, falls below the exercise price, X, exercise of the option yields a profit of $X - S$. Similarly, PBGC insurance gives firms the right to "sell" the assets of the plan plus 30% of net worth to the PBGC at a "price" equal to the present value of pension liabilities. The gain to the firm equals the pension liabilities it transfers to the PBGC less the assets the PBGC acquires.

2. Bulow cites Chrysler as an example of a firm for which the potential costs of a termination could be large if it affected the government's willingness to participate in a bail-out scheme for the company. Such extreme examples are probably rare, however.

3. A related issue pertains to so-called spinoff/terminations that allow firms with overfunded plans to recover the surplus assets and then continue to offer a defined benefit plan with a reduced level of funding. This option obviously affects the value of PBGC insurance since firms should be expected to recapture

periodically the surplus assets that otherwise would offer a cushion against adverse investment experience. However, this option may soon be eliminated. The Labor and Treasury Departments and the PBGC are all attempting to restrict such terminations (Chernoff 1983), and Congress is expected to consider restrictions on terminations during 1984 (Chernoff 1984).

4. If the fund is overfunded, eqq. (1) and (2) imply that the firm receives $F - A$. This might be unrealistic: Bulow and Scholes (1982) cite an example of a terminating fund in which the surplus was split between the firm and its employees. However, this issue is of limited relevance for this paper. The PBGC is unconcerned with termination of overfunded plans and presumably would not block the establishment of a new fund. Overfunded plans are not terminated in order to escape liabilities and so fall outside of the scope of this paper.

5. I will treat σ_S in eq. (3) as a constant. This treatment is appropriate when the firm has no debt outstanding other than its pension liabilities (Geske 1979). Thus, this specification is suitable for the voluntary termination model but would need to be modified for the more general case in which the firm can go bankrupt. I will assume that no dividends are paid out by the firm, and that all dividends received by the pension fund are reinvested in the fund, so that α_S may be equated with the expected rate of return on the assets backing the pension liabilities.

6. This condition does not necessarily imply that the firm's goal is to maximize the value of the pension option. It implies only that conditional on other decisions, the termination rule is option value maximizing. For example, in some situations, tax considerations may lead a firm to pursue pension funding policies that reduce the value of the pension put. Nevertheless, the termination rule must maximize the value of the put given that funding policy.

7. The insurance policy could have infinite value in this case. For example, for large C_A and $C_S = 0$, the option would provide a claim on a payoff that would be growing faster than the rate of interest. The value would be infinite although the option would never be exercised. Obviously, one would not observe values of (constant) C_A and C_S leading to these singular cases.

8. Using a variance rate for S of .05 (which approximates the historical variance of the S&P 500), a variance rate for A of .01 and a correlation coefficient of .1 yields $\sigma^2 = .05 + .01 - 2(.1)(.0005)^{1/2} = .055$. I rounded down to account for the fact that pension funds hold some debt in their portfolios. The entries in table 3.1 were not extremely sensitive to changes in σ.

9. For the numerical solutions a maximum time-to-termination of 75 years was assumed. Because the option is no longer of perpetual maturity, the term P_t must be added to the left-hand side of eqq. (5) and (11).

10. Negative values for θ or ϕ would indicate nonfinite values for the insurance.

11. The variance and covariance rates used to solve (11) are set forth in table 3.2 and discussed in section 3.2.1. A time horizon of 75 years was used in the solution. For values of parameters that allow closed form solutions, the numerical and analytic solutions differed by less than 1%.

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Comment William F. Sharpe

Milton Friedman has taught us not to question assumptions, but rather to consider the consistency of implications with the facts. This applies, however, to positive theories with testable implications. This paper has relatively few such implications. Instead, it attempts to estimate values that cannot be measured directly. Thus testing is difficult, if not impossible, and it is reasonable to examine the assumptions seriously.

The subject of the paper is, in effect, the value of the PBGC's liability under different assumptions about (1) the types of behavior allowed the firm, and (2) the type of behavior chosen by the firm within those constraints. The paper examines two major policies that the PBGC might choose and attempts to determine the resultant liabilities. The implications suggest that either of the two policies could be disastrous. It is not clear whether the current policy (whatever it may be) is better or worse.

The paper has the great advantage of dealing explicitly with the true multiperiod nature of this problem. Former models had finessed or ignored this aspect, and it is gratifying to see it taken into account. On the other hand, a multiperiod problem of this sort is very difficult, and many simplifying assumptions must be made.

Technically, the paper models the process as a diffusion. This allows analytic solutions in special cases, but it is important to note that the "interesting" cases require numeric solutions. Such cases are evaluated here with a finite-period model (using 75 periods) in which difference equations are used instead of differential equations.

One of the problems with this type of formulation is the difficulty of insuring that all relevant cash flows have been included. Prior to termination, the firm contributes money to the fund and pays retired benefits. At termination, the firm either recovers the amount overfunded or pays in the shortfall, up to 30% of its equity. After termination, the firm either is bankrupt (the second major case) or institutes a defined contribution plan. In the latter case, new accruals are paid, but previously accrued benefits are covered by the PBGC.

It is less than clear that maximizing the value of the put option, as defined here, is equivalent to maximizing the present value of the firm. The benefit payments are not included, nor are the values of the tax shields, which are lost after termination. The contribution includes 30% of the equity, which is not a cash flow. It is thus possible that the optimal termination decision for a firm wishing to maximize the present

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value of cash flows might differ from that found here, and with it the implied value of the PBGC liability.

The tax aspect is important. In all probability, only the IRS has saved the PBGC. The tax advantage of overfunding has probably dominated the maximization of the put value for the vast majority of funds.

It is interesting to note that the value of the PBGC liability will be sensitive to the coefficient of adjustment of contributions to funding status (c_1 in this model). According to the author, total adjustment to full funding requires that c_1 equal infinity, since this is a continuous time model. Most actuarial methods lead to adjustments of 5%–10% per year. The procedure currently proposed by the Financial Accounting Standard Board (officially for reporting purposes, but widely believed to be likely to be used for funding as well) would increase this to 20% for a typical plan. It would be interesting to estimate the impact of such "socially responsible" behavior on the PBGC liability.

Another interesting issue concerns the correlation between the fund assets and the value of the accrued benefits. Since 30% of the firm's equity is included in the former, the correlation might be higher than the value (.1) used in the paper. Since the results depend significantly on the value of σ, and since it is clearly the standard deviation of $(A - S)$, the extent to which a fund's assets "hedge" its liabilities will greatly affect the value of the PBGC liability.

The similarity of the magnitudes of the liabilities in the two cases (voluntary termination and bankruptcy) should not be surprising. The firm approaching bankruptcy is allowed to shortchange or even raid the pension fund. This is, in effect, a form of voluntary termination. Presumably, the PBGC should have some control over such activities.

If numeric methods must be used to cover interesting cases, it may be worthwhile to consider an alternative to the procedures employed here. The state variables can be assumed to follow binomial jump processes. It is a simple matter to program complex decision rules in this type of regime and to insure that all relevant ingredients for valuation have been included. The mapping between continuous-time and discrete-time formulations is not unique, however. For example, one way to model the voluntary termination case would allow four states of the world in each discrete time period. Accrued benefits (A) could go to either of two states, as could the assets backing the liabilities (S). Since the two variables are not perfectly correlated, four states would result. To compute the present value of cash flows in this model, four state-contingent claim prices would be needed for each time period. To determine them, four marketed instruments would be required (to span the space). Here, however, we have only three (A, S, and the riskless asset). Other discrete-jump processes might be adopted, or the value of some fourth asset might be introduced.

While the paper is not primarily an exercise in positive economics, it does have an important testable implication. Like previous papers that assume value maximization by the firm, it obtains the "razor's edge" conclusion that firms will adopt corner (extreme) strategies concerning funding and asset allocation. Almost any model that uses complete-market (or "complete enough" market) assumptions is likely to obtain such results. The observation of few such situations indicates either (1) that the implicit contracts with the PBGC and the IRS are more constraining (and more complex) than usually assumed or (2) that firms use a maximand that involves a utility function. If the latter is the case, models such as this predicated on value maximization may be inappropriate.

In sum, the paper provides a major start on the very difficult task of building multiperiod models of implicit contracts between government agencies (the PBGC and the IRS) and firms with pension plans, when the latter can "game" against the former. Not surprisingly, there is more to be done.