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Volume Title: Exchange Rate Theory and Practice

Volume Author/Editor: John F. O. Bilson and Richard C. Marston, eds.

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-05096-3

Volume URL: <http://www.nber.org/books/bils84-1>

Publication Date: 1984

Chapter Title: The Real Exchange Rate, the Current Account, and the Speed of Adjustment

Chapter Author: Francesco Giavazzi, Charles Wyplosz

Chapter URL: <http://www.nber.org/chapters/c6841>

Chapter pages in book: (p. 335 - 356)

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# 10                    The Real Exchange Rate, the                           Current Account, and the Speed                           of Adjustment

Francesco Giavazzi and Charles Wyplosz

## 10.1 Introduction

A stylized recent model of exchange rate determination would include the following features: high capital mobility, rational expectations, and continuous clearing of asset markets. Such a model would exhibit saddle-path stability and, in order to solve it, one would typically first determine its long-run (steady state) equilibrium following a disturbance and then identify the unique path along which convergence would be obtained.<sup>1</sup> In this paper we present and discuss a class of models for which the steady state equilibrium seems to admit a priori an infinity of solutions so that there would appear to exist an infinity of convergence paths.<sup>2</sup> It will be shown that this indeterminacy is only apparent: the long-run equilibrium, and the path that leads to it, are uniquely determined by the dynamic characteristics of the model. In other words, the parameters which set the speed of adjustment of the model have a permanent effect on the evolution of the economy.

This interesting property is obtained in two-country models with infinite intertemporal optimization where agents typically consume their permanent income, which, in the stationary state, coincides with their actual income. Consequently, under assumptions to be specified later, the requirement that the current account be in equilibrium vanishes, opening up the possibility of

We gratefully acknowledge the benefit of remarks by Jeff Sachs and Joshua Aizenman. This work has been partially supported by a grant from the Délégation Générale à la Recherche Scientifique at Technique.

1. For a representative sample of these studies, see Dornbusch (1976), Wilson (1979), Dornbusch and Fischer (1980), Mussa (1980), and Kouri (1981).

2. This indeterminacy should not be confused with the well-known problem associated with saddle-path stability, according to which one needs additional conditions, such as ruling out explosive solutions, to identify a unique convergence path. On this problem, see for example Blanchard (1979).

an indeterminacy of the real exchange rate. Models of real exchange rate determination with intertemporal optimization have received considerable attention recently, especially in Dornbusch (1981), Obstfeld (1981, 1982), Svensson and Razin (1981), and Sachs (1983). Actually there are at least two reasons why such models are interesting. First, Dornbusch and Fischer (1980), Mussa (1980), and Rodriguez (1980) have emphasized the role of present and future current account imbalances in driving the exchange rate, following the earlier contribution of Kouri (1976). An important implication of the reemergence of the current account is a renewed interest in the intertemporal allocation of resources and spending among countries which is implied by such surpluses or deficits, and therefore the need to model this process carefully. Another reason is related to the widespread use of the rational expectations assumption. As Muth (1961) pointed out in his original contribution, if one models optimizing agents, one has to assume also that they use all available information in forming their expectations. But then, if they incorporate future anticipated events into their rational expectations, it seems natural to replace static by dynamic optimization.

It has become a standard property of exchange rate models under perfect foresight that the impact effect of an exogenous disturbance is a function of the speed at which some slow-moving variables are able to adjust, as exemplified in Dornbusch (1976). But this effect does not concern the stationary state to which the model converges, which typically remains uniquely defined and easily characterized. The class of models discussed in this paper opens up new interesting possibilities. For example, we show that the degree of flexibility of wages, or the rate at which capital is accumulated, have permanent effects on such variables as the real exchange rate and a country's external indebtedness.

The point made here will seem intuitively clear and is but a special case of the general treatment of linear models under perfect foresight by Blanchard and Kahn (1980) and Buiter (1981). Still, it does not seem to have been directly addressed in the exchange rate literature, although Obstfeld (1982) and Sachs (1983) have signaled its existence. In a completely different setup, Drazen (1980) obtains the same property and argues, as we do, that it presents attractive economic implications.

The problem at hand is illustrated through an example in the next section. The analytical solution is presented in section 10.3, and put to work in section 10.4, where another example shows the role of the labor market in determining the stationary state value of the real exchange rate through its effect on cumulated current account imbalances. Section 10.5 offers some concluding remarks.

## 10.2 The Nature of the Problem: A First Example

The model presented in table 10.1 assumes perfect foresight and intertemporal optimization. It describes two countries which trade goods and secu-

Table 10.1

(1) $y = y_0 K^\alpha$	$y^* = y_0^* K^{*\alpha}, 0 < \alpha < 1$
(2) $C = a\delta A, a > 1/2,$	$C^* = (1 - a^*)\delta A^*, a^* < 1/2$
(3) $\lambda C_m = (1 - a)\delta A$	$\lambda^{-1} C_m^* = a^*\delta A^*$
(4) $A = q(K - Z)$	$A^* = q^* K^* + \lambda^{-1} qZ$
(5) $\dot{q} = rq - D/K$	$\dot{q}^* = r^* q^* - D^*/K^*$
(6) $D = y - I + q\dot{K}$	$D^* = y^* - I^* + q^*\dot{K}^*$
(7) $I = \dot{K} \left( 1 + \frac{\phi}{2} \frac{\dot{K}}{K} \right)$	$I^* = \dot{K}^* \left( 1 + \frac{\phi^*}{2} \frac{\dot{K}^*}{K^*} \right)$
(8) $\dot{K} = K(q - 1)/\phi$	$\dot{K}^* = K^*(q^* - 1)/\phi^*$
(9)	$r = r^* + \dot{\lambda}/\lambda$
(10)	$q\dot{Z} = \lambda C_m - C_m^* + DZ/K$
(11) $y = C + C_m^* + I$	$y^* = C^* + C_m^* + I^*$

Note: The asterisk denotes foreign country's variables.

rities with each other. Each country produces one good, using capital as the sole factor of production. A symmetrical model with labor instead of capital is presented in section 10.4 below. A model with both labor and capital is too large to be solved analytically and has been simulated in Giavazzi, Odekon, and Wyplosz (1982) and in Sachs (1983). The production technology is identical in both countries and exhibits decreasing return to scale (equation [1]). Each good is used for private consumption at home and abroad, and for domestic capital formation. The two goods are imperfect substitutes in consumption, and the demand equations (2) and (3) are derived in Appendix 1 from the intertemporal optimization of an instantaneous Cobb-Douglas utility function.<sup>3</sup> The variable  $\lambda = eP^*/P$  is the real exchange rate, with  $e$  the nominal exchange rate and  $P$  and  $P^*$ , respectively, the prices of domestic and foreign goods. With this specification, total real consumption,  $C + \lambda C_m$ , in each period, is a constant share of real wealth  $A$ , the constant being the rate of time preference  $\delta$ . The assumption that  $\delta$  is constant and identical across countries is crucial and will be discussed later. Consumption of each good is, by virtue of the Cobb-Douglas assumption, a constant share of total consumption and, in each country, a larger share of consumption falls on the locally produced good ([2] and [3]).

Equity claims on the domestic and foreign capital stocks are the only assets and are taken as perfect substitutes. Consequently, the assumption,

3. The essential result does not depend upon the specification of the utility function, because, in the stationary state, total spending  $E = C + \lambda C_m$  is always equal to  $\delta A$ . In order to obtain  $E = \delta A$  at any point in time, i.e., also outside the stationary state, we need the utility function to be the logarithm of a linear homogeneous function of  $C$  and  $C_m$ . See Appendix 1 for proofs and a discussion.

implicit in the definition of wealth (4), that only domestic claims are traded is innocuous, and  $Z \geq 0$  represents the volume of domestic equities held abroad. The variable  $q$  in (4) is the market value of installed capital, that is, Tobin's  $q$ . It is given in differential form in (5), where the dividends  $D$  are defined in (6). The definition of dividends assumes that all capital outlays are financed through issues of equities, so that dividends include the proceeds of the issue of new stocks less spending on investment,  $I$ . The investment function (7), in turn, follows the cost of investment literature,<sup>4</sup> in assuming that total investment expenditures exceed the value of actually installed capital  $K$ , this cost being here a simple linear function of  $K$ . The optimal rate of investment (8) is derived in Appendix 1,<sup>5</sup> and shows the role of the cost of investment,  $\phi$ . Equation (5) is the arbitrage condition which follows from the assumption of perfect asset substitutability, so that expected real returns, adjusted for expected real exchange rate changes, are equalized. With perfect foresight there is no distinction between expected and actual variables. Finally, in (10), current account deficits at home, the sum of the trade deficit and of dividend payments, are matched by changes in the foreign ownership of domestic stocks, as we assume that these are the only traded assets. The model is closed with the conditions (11) that both goods markets are in equilibrium.

### 10.2.1 The Stationary State

Assuming away growth, technological changes, and depreciation of capital, stationarity requires that all variables become constant. With  $\dot{\lambda} = 0$ , real interest rates are equalized. With  $\dot{K} = \dot{K}^* = 0$  we need to have  $\bar{q} = \bar{q}^* = 1$ . Then with  $\dot{q} = \dot{q}^* = 0$  and  $I = I^* = 0$ , (5), (6), and (7) imply that  $\bar{y} = \bar{r}\bar{K}$  and  $\bar{y}^* = \bar{r}^*\bar{K}^*$ , which, together with (1), define uniquely  $\bar{K}$  and  $\bar{K}^*$  as functions of  $\bar{r} = \bar{r}^*$ . Next, we consider the two-goods market equilibrium conditions (11). One of them can be replaced by the requirement that world spending equals world income:

$$y + \lambda y^* = (C + \lambda C_m) + (\lambda C^* + C_m^*) = \delta(A + \lambda A^*),$$

which, given the above stationary state conditions, implies

$$(12) \quad \bar{r}(\bar{K} + \lambda \bar{K}^*) = \delta(\bar{K} + \lambda \bar{K}^*).$$

Clearly, then, the two interest rates must equal the rate of time preference.

4. This investment function is described in Abel (1979) and used in Blanchard (1980).

5. In the presence of decreasing returns, one should distinguish the shadow price of investment, Tobin's marginal  $q$ , from the present value of installed capital, Tobin's average  $q$ . Doing so would increase the order of the dynamic system, making it intractable, so that we approximate marginal  $q$  by its average (observable) value. The exact values of the two  $q$ 's are given in Appendix 1. For a discussion of the issue, see Hayashi (1982). For a simulated version of the model allowing for the two  $q$ 's, see Giavazzi, Odekon, and Wyplosz (1982). Also, note that the interest rate is not equal to the marginal productivity of capital. This is because, with only one factor of production and decreasing returns to scale, stockholders enjoy a rent which is implicitly redistributed as part of dividend payments so that all earnings are accounted for.

Otherwise, we would have permanent world net saving (when  $r > \delta$ ) or dissaving (when  $r < \delta$ ).

We then consider the current account condition (10). With goods markets in equilibrium, the current account in each country is the excess of income over spending, so that  $\dot{Z} = 0$  implies<sup>6</sup>

$$(13) \quad (\bar{r} - \delta)(\bar{K} - \bar{Z}) = 0.$$

This is where the indeterminacy appears: with  $\bar{r} = \delta$ , the current account balance condition is always satisfied, so that it is not an active condition. As a consequence, we lose one equation to find the stationary state values of the two variables yet to be determined,  $\lambda$  and  $Z$ . The only remaining available condition is any of the two goods market equilibria (11), any one of which gives

$$(14) \quad \bar{\lambda} = \frac{(1 - a)\bar{K} + (a - a^*)\bar{Z}}{a^*\bar{K}^*},$$

so that any pair of values  $(\bar{\lambda}, \bar{Z})$  which satisfy (14) is a priori compatible with the stationary state requirement: the distribution of wealth  $\bar{Z}$ , and the real exchange rate  $\bar{\lambda}$  can take an infinity of values.<sup>7</sup>

The economic reason for this apparent indeterminacy can be made intuitive by considering a transfer of wealth from domestic to foreign residents (an increase in  $\bar{Z}$ ), starting from a stationary state situation. Such a transfer, given perfect asset substitutability, does not affect investment/saving decisions and does not upset world equilibrium as seen in (12). Its only effect is to shift world demand toward foreign goods (when  $a > a^*$ ) and it only requires a real depreciation to restore equilibrium in *both* goods markets.<sup>8</sup>

This indeterminacy of the stationary state is merely apparent; following a disturbance, the economy will converge to a unique stable equilibrium, and the resulting values of  $\bar{Z}$  and  $\bar{\lambda}$  will be a function of its dynamic characteristics. Unfortunately, these values cannot be found without first spelling out the complete dynamic solution.<sup>9</sup> Although we do not present such a solution

6. Thus, at home, income is  $y - rZ$ , spending is  $\delta A = \delta(K - Z)$ , and  $\dot{Z}$  measures the current account deficit.

7. With  $r = r^* = \delta$ , (1), (5), and (6) imply  $\delta\bar{K} = y_0\bar{K}^\alpha$ , so that  $\bar{K}$  (and  $\bar{K}^*$ ) are uniquely determined.

8. When  $a = a^*$ , the transfer has no effect on relative demand for domestic and foreign goods;  $\bar{\lambda}$  is determined but  $\bar{Z}$  is irrelevant for any other variable: we actually have only one consumer. Also note that Branson (1979) has emphasized that a current account deficit will require a permanent real exchange rate depreciation in order to generate the trade surplus needed to pay for the increased foreign debt. Equation (14) seems to confirm this result when  $a > a^*$ , but for a totally different reason. The debt effect vanishes in (13) as domestic residents recognize that their wealth is reduced and lower their spending accordingly. Here the effect on the real exchange rate is entirely due to the shift in relative demand for domestic and foreign goods, as discussed in the transfer example, and with  $a < a^*$  a current account deficit implies a long-run real appreciation.

9. Thus, the general analytical solution provided by Blanchard and Kahn (1980) remains valid in this case and will provide the unique stationary state values. Yet Blanchard and Kahn have not drawn the important consequences of the singularity of the transition matrix, as we shall discuss in the next section.

for this model, it appears that the parameters describing the cost of investment,  $\phi$  and  $\phi^*$  in (7), will influence, not only the adjustment path, but also the ultimate values of  $Z$  and  $\lambda$ , and therefore the distribution of spending between the two countries. A higher cost of investment at home will slow down the accumulation (or decumulation) of  $K$  toward its optimal value, thus hampering the adjustment of domestic output and, usually, worsening, *ceteris paribus*, the current account and its total cumulated value as measured by  $Z$ . This, in turn, will require a corresponding real exchange depreciation.

### 10.2.2 How General Is the Problem?

The property shown in the previous example follows from the fact that, with intertemporal optimization, zero savings is an implication of the stationary state, achieved when the interest rate equals the rate of time preference. We now address the question whether this property is truly general or whether it follows from some special assumptions introduced into the model. The answer is that there are several ways of eliminating this property. We now discuss some of them and argue that the assumptions they entail are not obviously superior to those of the above model.

A first possibility is to do away with the perfect assets substitutability hypothesis, which is equivalent to assuming different rates of time preference in each country, since in the stationary state we will still need  $\bar{r} = \delta$ ,  $\bar{r}^* = \delta^*$ , and we now want  $r \neq r^*$ . To understand why the indeterminacy is removed, consider again a transfer of wealth  $\Delta Z$  to the foreign country. Foreign spending increases by  $\delta^* \cdot \Delta Z$  while domestic spending falls by  $\delta \Delta Z$ ; the world equilibrium is disturbed, interest rates will have to adjust, and the process will generate current account disturbances leading back to the initial distribution of wealth. Thus the nonuniqueness property is removed. But this solution has some unattractive features. Either it implies a corner solution, where one country has continuously dissaved to the point of selling away all its wealth so that the other country owns the whole world and consumes all output, or else it implies no holding of foreign assets in the stationary state, since such holdings would have spending out of these assets proportional to the holding country rate of time preference, while earnings would be proportional to the issuing country's rate.

Another possibility is to allow for each country to have variable and endogenous rates of time preference. Obstfeld (1981) has introduced such a rate, function of utility. In the stationary state, with perfect asset substitutability, we will still have identical rates of time preference in both countries and consumption is still proportional to wealth,  $\delta$  being the coefficient of proportionality. But the equalization of the rates of time preference effectively imposes a further condition which eliminates the nonuniqueness property. The reason is that a transfer of wealth—for example, from the domestic to the foreign economy—would reduce wealth and therefore consumption

at home, with the opposite effect abroad. This, then, would lower domestic utility, increase foreign utility, and result in different rates of time preference, prompting current account imbalances until the initial situation is restored. In this case, there is a unique distribution of wealth, and a unique real exchange rate, compatible with the stationary state. But the solution of the problem has a cost, because such endogenous rates of time preference are hard to justify: Should the rate of time preference be an increasing or a decreasing function of utility?<sup>10</sup>

A third possibility would be to introduce wealth into the utility function, so that transfer would alter the spending behavior, generating a Metzler-type behavior, and prompting current account adjustments until the unique stationary state distribution of wealth is reached. The question, of course, is whether wealth belongs to the utility function.

The model discussed in the previous section does not include labor as a factor of production. In the following section labor is introduced, and it will be seen that the indeterminacy remains. But could it be removed if leisure were an argument of the utility function?<sup>11</sup> In this case, the stationary state requires that real wages be equal to both the marginal productivity of labor and the marginal utility of leisure. If the utility function is not additive in leisure and consumption but assumes substitutability, a transfer of wealth abroad will reduce domestic consumption and increase the marginal utility of leisure, resulting in a reduction of labor supply. In the corresponding stationary state, the capital stock would be lower at home, higher abroad. Yet, it still is the case that equality between the interest rates in each country and the rate of time preference will guarantee balanced current accounts, so that the nonuniqueness property is preserved. But with labor and capital now depending upon wealth, the nonuniqueness spreads as it also affects these variables, as well as output levels.

Summing up, two-country models with intertemporal optimization are quite likely to exhibit the property that the stationary state is not uniquely determined or, more precisely, that it will be related to some of their dynamic characteristics. The assumptions required to eliminate the property are not necessarily superior, while the indeterminacy may prove to yield interesting and intuitive results. Of course, once we leave the general optimizing framework, the property disappears. It is, of course, the case of ad hoc

10. This issue has been recently revived by Lucas and Stokey (1982). Koopmans, Diamond, and Williamson (1964) had derived a set of postulates conveying the concept of time impatience and characterized the utility functions which satisfy these postulates. They came up with two examples, one with a constant rate of time preference, one with time preference an increasing function of utility. Lucas and Stokey build on Koopmans, Diamond, and Williamson to study the optimum equilibrium allocation in a many-agents growth model. Interestingly, they argue against a constant rate of time preference precisely because any distribution of utility is compatible with the stationary state, that is, they reach the same indeterminacy property but reject it.

11. This case is treated in a simulation context by Lipton and Sachs (1983).



Keynesian consumption functions and models where consumers are facing quantity or liquidity constraints. It should also be the case of models where optimization is carried over a finite period of time, or of models with overlapping generations, unless bequests exist and enter the utility function, although this point is just a conjecture at this time.

### 10.3 Analysis and Solution for Linear Models

In this section we present briefly the results derived in Giavazzi and Wyplosz (1983). We deal with the general case of a system of linear difference equations, characterize the mathematical aspects of the problem described in the previous section, and sketch its solution. The reader uninterested in these technical aspects can proceed directly to section 10.4 without loss of continuity.

The general form of a system of linear differential equations is:

$$(15) \quad \dot{x} = Ax - z,$$

where  $x$  is an  $n$ -vector of endogenous variables,  $z$  is an  $n$ -vector of (or combination of) exogenous variables, and  $A$  is an  $n \times n$  matrix. The solution to (15) under perfect foresight is given in Blanchard and Kahn (1980) and in Buiter (1981). They show that the system is stable when  $A$  admits as many strictly positive eigenvalues as there exist nonpredetermined variables in  $x$ . We assume here this stability condition and consider the special case where  $A$  is singular, admitting at least one zero eigenvalue. Denoting steady state levels with a bar, the long run is characterized by:

$$(16) \quad A\bar{x} = \bar{z}$$

With  $A$  singular, (16) normally admits an infinity of solutions<sup>12</sup> so that the stationary state of the model seems not to be unique. Yet, (15) can be solved and shown to converge to a unique stationary state. The procedure is exactly as in Blanchard and Kahn (1980) and in Buiter (1981): diagonalize the system,<sup>13</sup> integrate each of the  $n$  differential equations, and compute the  $n$  arbitrary constants of integration by using either the initial condition for the  $p$  predetermined variables or the transversality condition for the  $n-p$  nonpredetermined variables. Once we thus obtain  $x(t)$  we can take the limiting case when  $t$  goes to infinity. The resulting value of  $\bar{x}$  is as follows:

$$(17) \quad \bar{x} = V\Lambda^*V^{-1}\bar{z} + (VEJ)(PVJ)^{-1}(Px(0) - PV\Lambda^*V^{-1}\bar{z}),$$

where  $\Lambda^*$  is a diagonal matrix, with its nonzero elements being the inverse

12. More precisely, (16) admits either no solution or an infinity of solutions. The latter is obtained when the standard rank condition is satisfied, namely that  $\bar{z}$  be orthogonal to the eigenvector(s) associated with the zero eigenvalue(s). The case of no solution is uninteresting as it would result from a model ill-specified from the economic point of view.

13. If  $A$  cannot be diagonalized, the solution is possible by using the Jordan canonical transformation instead; see Blanchard and Kahn (1980).

of the eigenvalues, the first term(s) corresponding to the zero eigenvalue(s) and being null. ( $\Lambda^*$  is the generalized inverse of  $\Lambda$ .)  $V$  is the matrix of eigenvectors ordered conformably.  $E$  is an  $n \times n$  matrix whose elements are all zero except for the first diagonal term(s) set at unity for each zero eigenvalue(s).  $P$  is a  $p \times n$  matrix where the first  $p$  columns form the identity matrix, the remaining terms being all null, when  $x$  is ordered so that its first  $p$  elements are the predetermined variables. Then  $Px(0)$  represents the initial values of these predetermined variables. Finally  $J$  is an  $n \times p$  matrix with the first  $p$  rows forming the identity matrix, the other terms being zero.

In order to understand (17) consider first the case where  $A$  is nonsingular. Then  $\Lambda^* = \Lambda^{-1}$  and  $E = 0$  so that the second term vanishes and we obtain the standard result  $\bar{x} = A^{-1}\bar{z}$ . Thus, the effect of the singularity of  $A$  is captured by the second term. Inspection of this second term shows that the stationary state will depend upon the initial conditions (as captured by  $Px(0)$ ), the so-called hysteresis property. Furthermore, the effect of  $Px(0)$  on  $\bar{x}$  will depend on the dynamic characteristics of the system in a nontrivial way which is illustrated in the following section.

## 10.4 Second Example: Model with Labor Only

### 10.4.1 Presentation and the Stationary State

In this section, we present a model very similar in spirit to that discussed in section 10.2 but which turns out to reduce to a smaller dimension and allows for an easier analytical solution.<sup>14</sup> This model is presented in table 10.2 below. The difference is that production is now carried out with labor as the only factor of production, instead of capital ( $1'$ ). The crucial speed of adjustment will be that of the labor market, which functions as follows. Labor supply is infinitely elastic at the going real wage rate  $w$ , so that actual employment  $L$  can differ from the "natural" level  $\bar{L}$ . Excess demand for labor (respectively excess supply) in turn brings about an increase (respectively a decrease) in the real wage: the speed at which this adjustment proceeds to reestablish full employment is captured in (7') by the parameter  $\gamma$ . Demand for labor follows from the optimizing choice of the firm, so that in (6') the real wage rate is equal to the marginal productivity of labor. Total domestic wealth  $A$  is defined in (4') as the present value of domestic output:

$$x(t) = \int_t^{\infty} e^{-\int_t^s \rho(v)dv} y(s) ds,$$

14. We have been able, so far, to obtain analytical solutions for the model of section 10.2 only in cases where the dynamics is uninteresting and does not lead to current account imbalances, because of the simplifying assumptions which make it tractable.

or

$$\dot{X} = rX - y, \text{ as in (5')},$$

less domestic indebtedness  $Z$ , where  $Z$  can be positive or negative. Trade in assets takes the form of indexed bonds, that is, claims to units of output of the issuing country, and (9) ensures that the yields of such bonds are the same, irrespective of which country issues them. Equation (10') describes the current account and (11') represents the two goods markets' equilibrium conditions.

**Table 10.2**

(1')	$y = y_0 L^\alpha$	$y^* = y_0^* L^{*\alpha}$
(2)	$C = a\delta A$	$C^* = (1 - a^*)\delta A^*$
(3)	$\lambda C_m = (1 - a)\delta A$	$\lambda^{-1} C_m^* = a^*\delta A^*$
(4')	$A = X - Z$	$A^* = X^* + \lambda^{-1} Z$
(5')	$\dot{X} = rX - y$	$\dot{X}^* = r^*X^* - y^*$
(6')	$wL = \alpha y$	$w^*L^* = \alpha y^*$
(7')	$\dot{w} = \gamma(L - \bar{L})$	$\dot{w}^* = \gamma^*(L^* - \bar{L}^*)$
(9)		$r = r^* + \dot{\lambda}/\lambda$
(10')		$\dot{Z} = \lambda C_m - C_m^* + rZ$
(11')	$y = C + C_m^*$	$y^* = C^* + C_m$

As in section 10.2, the stationary state implies  $\bar{r} = \bar{r}^* = \delta$ , and the two-goods markets equilibrium then reduces to

$$(18) \quad \bar{\lambda} = \frac{(1 - a)\bar{X} + (a - a^*)\bar{Z}}{a^*\bar{X}^*}.$$

As  $\bar{X} = \bar{y}/\delta = y_0\bar{L}^\alpha/\delta$  and  $\bar{X}^* = y_0^*L^{*\alpha}/\delta$ ,  $\bar{X}$  and  $\bar{X}^*$  are clearly defined and we have, again, a relationship linking  $\bar{\lambda}$  and  $\bar{Z}$ , leaving these two variables a priori undetermined.

We will consider a change in domestic productivity  $\hat{y}_0 = dy_0/y_0$ , which occurs unexpectedly in period  $t = 0$ . We know that in the new stationary state  $\bar{w} = (y_0 + dy_0)\bar{L}^{-(1-\alpha)}$  and  $\bar{w}^*$  is unchanged, so that domestic wealth will change proportionately to the productivity gain with no long-run effect on foreign human wealth.

### 10.4.2 Solution

We note that the interest rate variables are merely definitional and can be eliminated through (5'), (6'), and (9) so as to obtain

$$(9') \quad \dot{\mu}/\mu = y/X - y^*/X^*,$$

where  $\mu = \lambda X^*/X$  is the relative value of foreign and domestic gross wealths. The model is then driven by the four equations (7'), (9'), and (10'), together with the goods market equilibrium conditions, which allows us to eliminate  $X$  and  $X^*$ . The relative value of wealths,  $\mu$ , is a nonpredetermined variable, while  $w$ ,  $w^*$ , and  $Z$  are predetermined.

For the purpose of this example, computations can be greatly reduced by a careful choice of parameters and initial values. Specifically, we assume<sup>15</sup> for  $t < 0$ ,  $X = X^* = 1$ ,  $r = r^* = \delta$ ,  $w = w^* = 1$ ,  $y = y^* = \delta$ ,  $\lambda = \mu = 1$ ,  $Z = 0$ . The system is linearized and solved around this initial position in appendix 2. The resulting laws of motions of the four driving variables are  $w(t) = 1 + \hat{y}_0(1 - e^{-\gamma_1 t})$ ,  $w^*(t) = 1$ ,

$$(19) \quad Z(t) = \frac{1 - a - a^*}{a + a^*} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{\delta}{\gamma_1 + \delta} \hat{y}_0 (1 - e^{-\gamma_1 t}),$$

and

$$(20) \quad \mu(t) - 1 = \frac{1 - a - a^*}{a + a^*} \hat{y}_0 \left( 1 + \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \right) + 2 \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \hat{y}_0 (e^{-\gamma_1 t} - 1),$$

where  $\gamma_1 = \bar{\gamma} / (1 - \alpha)$  is a measure of the speed of adjustment in the domestic labor market.

From these formulae, it is easy to obtain the stationary state values for  $Z$  and  $\mu$ :

$$\bar{Z} = \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \cdot \frac{\delta}{\gamma_1 + \delta} \hat{y}_0$$

$$\bar{\mu} - 1 = \frac{1 - a - a^*}{a + a^*} \hat{y}_0 \left( \frac{a + a^*}{a^*} + \frac{a - a^*}{a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \right).$$

It appears that the sign of  $(1 - a - a^*)$  plays an important role in the evolution of the system: in the following, we discuss the case where the home country captures less additional sales than the foreign country when world wealth increases, that is,  $1 - a - a^* > 0$ . We also assume  $a > a^*$ , a ‘‘preferred habitat’’ in consumption.

In order to interpret the solution described by (19) and (20), we turn to figure 10.1. The line  $LR$  represents the indeterminacy problem: a priori, in the stationary state,  $\mu$  and  $Z$  can be anywhere along this line which is de-

15. Yet we do not assume that the model was resting in a stationary state since, with  $Z = 0$  and  $X = X^*$ , (22) would imply  $a + a^* = 1$ . In this case we obtain a trivial solution where  $\lambda$  jumps to its new stationary state value, with  $Z(t) = 0, \forall t$ , and no dynamics at all. The reason will appear clearly in the following discussion where we show the role of the assumption  $a + a^* \neq 1$ .

rived from (18):  $a^*\mu\bar{X} = (1 - a)\bar{X} + (a - a^*)\bar{Z}$ . We have assumed that, prior to the disturbance, the economy was at point A, with  $Z = 0$  and  $\mu = 1$ . The slope of the line  $LR$  increases with  $\hat{y}_0$ , the disturbance. On impact,  $Z$  cannot change instantaneously, but  $\mu$  is free to jump. As in other models with perfect foresight, the magnitude of the jump is a function of the speed of adjustment of the economy: the more slowly the labor market reacts to a disequilibrium—that is, the smaller  $\gamma$ —the larger the impact increase in  $\mu$ . What is novel here is that, wherever  $\mu$  jumps to, there will be a convergence path leading to a stationary state position along  $LR$ , as shown on figure 10.1 by the two impact positions  $B$  and  $C$ , and the corresponding long-run points  $B'$  and  $C'$ .

In order to understand how this happens, we consider first the long-run effects of the disturbance. We note that foreign output, employment, and the wage rate stay constant. In the stationary state, therefore, world wealth will have increased proportionately to domestic output, making for equal augmentations of world spending and domestic output. With spending directed to both domestic and foreign goods, a real exchange rate depreciation is needed for goods markets to be in equilibrium. Now consider  $\mu = \lambda X^*/X$ . If we had  $a + a^* = 1$ , the increase in world wealth per se would not affect the relative demand for domestic and foreign goods so that there would be no need for  $\mu$  to change; with  $X^*$  constant, the increase in  $\lambda$ , proportional to the increase in  $X$ , would be enough to maintain both goods markets in equilibrium. If, however,  $a + a^* < 1$ , as world wealth increases, relative demand tilts toward foreign goods, which requires a further depreciation and an increase in  $\mu$ ; the relative value of foreign wealth, expressed in domestic goods units, must increase in order to eliminate the excess supply of domestic goods. This explains the stationary state value of  $\mu$  in figure 10.1.

The impact effect of the increase in  $y_0$  is in many respects similar to the

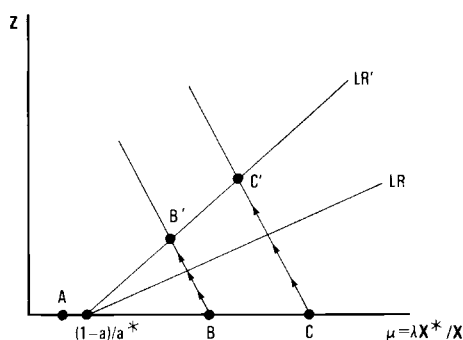


Fig. 10.1

long-run case just described. Domestic output increases but attracts only a fraction  $a + a^*$  of the increase in world wealth so that when  $a + a^* < 1$ ,  $\lambda$  and  $\mu$  increase on impact.

Over time, the domestic labor market adjusts to the increased demand for labor generated by the productivity gain. As the real wage rate increases, demand for labor and domestic output decrease, which requires a real exchange appreciation in order to reduce demand for domestic goods. We thus obtain an overshooting for  $\lambda$  (and  $\mu$ ).<sup>16</sup> This appreciation being correctly anticipated is accompanied, because of (9), by an interest rate differential so that for  $t > 0$ ,  $r < \delta$ , and  $r^* > \delta$ . This interest rate effect is important because it leads to a drop in  $X^*$ , the present value of the constant flow of foreign output; as a consequence, the foreign current account turns into a surplus as foreign spending is reduced, and this is matched by a domestic deficit.

We can now discuss the role of  $\gamma$ , the speed of adjustment of the domestic labor market. With a high speed of adjustment, the current account imbalances are eliminated faster, thus making for a smaller cumulated debt of the home country and therefore requiring a smaller real exchange rate appreciation.<sup>17</sup> On figure 10.1, the adjustment path  $BB'$  describes the response of the economy for a higher  $\gamma$  than along  $CC'$ .

#### 10.4.3 Welfare Implications

As the consumption behavior is derived from the optimization of Cobb-Douglas intertemporal utility functions, it is easy to draw implications concerning welfare in the new stationary state. This requires computing the values of total domestic and foreign wealth. As shown in Appendix 2, foreign wealth  $A^* = X^* + Z$  has to increase in the long run as  $X^*$  goes back to its initial value while  $Z$  rises. However,  $A^*$  initially drops as  $X^*$  is reduced on impact, and  $Z$  increases only over time. Domestic wealth  $A = X - Z$  increases in the stationary state if the loss in wealth  $Z$  through cumulated deficits does not offset the gain in  $X$ . The possibility that a productivity gain proves to be “immiserizing” augments when the speed of adjustment is small, as current account deficits are more prolonged. If  $U$  and  $U^*$  are the domestic and foreign Cobb-Douglas welfare functions, respectively, then we have  $U = A\lambda^{-(1-a)}$  and  $U^* = A^*\lambda^{a^*}$ . While  $U^*$  increases unambiguously through both its wealth and its terms of trade arguments, chances that  $U$  decreases grow as  $\gamma$  decreases, since it not only reduces

16. The overshooting in  $\mu$  (and in  $\lambda$ ) is now a familiar feature in exchange rate models, since Dornbusch (1976) and Black (1977). Here it follows from the stickiness of wages and the corresponding difference in speeds of adjustments on labor and assets markets.

17. While the domestic current account is more quickly eliminated with a high speed of adjustment, its initial size is larger. With a high  $\gamma$ , the exchange rate appreciation, following the depreciation on impact, is faster, pushing  $r$  further down and thus leading to larger domestic wealth and spending. Yet the accumulated debt is unambiguously smaller, as shown by (19).

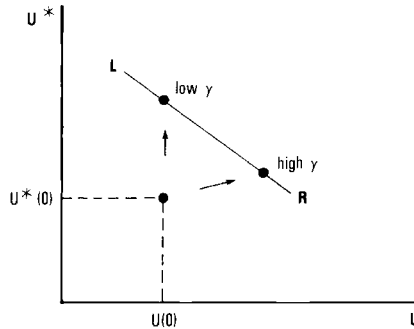


Fig. 10.2

wealth gains but also worsens the domestic terms of trade.<sup>18</sup> The role of the speed of adjustment is illustrated in figure 10.2. The line  $LR$  shows all the possibilities for the stationary state values of  $U$  and  $U^*$ . The exact position along  $LR$  is a priori unknown. With a high speed of adjustment  $\gamma$ , both countries' welfare improves. With a low  $\gamma$ , the gain at home is lower and can even be negative, while the gain abroad is enhanced.

### 10.5 Conclusion

We believe that the class of models in which the initial conditions and the speed of adjustment parameters have permanent influences on the path of the economy after a disturbance is a large and important one. There are certainly several ways of making different assumptions which eliminate the a priori indeterminacy of the stationary state in these models. We have discussed some of them and argued that they do not necessarily seem more appealing than ours. We think that choosing these assumptions simply because they solve the problem discussed in the paper amounts to discarding what appears to be an intuitively interesting property. It is unnecessary since it turns out that the usual stationarity conditions remain sufficient to pin down a unique and stable long-run equilibrium. The example, which has been solved, shows results which seem to match what one would expect to find.

It is not clear how broad is the potential applicability of this approach. In this paper, the property hinges on the fact that we have two distinct groups of consumers who trade in goods and assets.<sup>19</sup> This is why it has natural

18. At this point, it is worth reemphasizing that the foregoing discussion assumes  $1 - a - a^* > 0$ . Taking  $1 - a - a^* < 0$  would reverse this result and put the burden of potentially decreasing wealth and welfare on the foreign economy.

19. The production part of the models presented here is not required to obtain the result. We have assumed that firms optimize for the sake of coherence only.

applications in macroeconomics for two-country models. It could as well be used in a Kaldorian economy with two classes of consumers who have different spending patterns.

But the same property might also obtain in a one-small-country model, provided its spending is, again, derived from infinite horizon intertemporal optimization, somehow leaving the rest of the world unspecified. The fact that Drazen (1980) reports a similar property arising in the production side is intriguing. His model has heterogeneous capital and labor, both susceptible of "investments," so that the indeterminacy stems from the possibility of adjusting labor to the existing structure of capital, or of adjusting capital to the existing structure of labor. The interesting aspect of this is that investments in capital and in labor (i.e., job training) are sluggish, so that the final stationary state will be uniquely related to the speed of adjustment. There seems to be a scope for a generalization of the mechanisms brought up by Drazen and in the present paper.

## Appendix 1: Optimization

### 1. The Consumer's Problem

The consumer maximizes  $\int_0^{\infty} e^{-\delta t} U(t) dt$  subject to the constraint that total spending  $E = C + \lambda C_m$  exhausts, in present value, his wealth  $A$ , that is,  $A = \int_0^{\infty} e^{-\int_0^t r(s) ds} E(t) dt$ , or, equivalently,  $\dot{A} = rA - E$ . We consider the special case where

$$U(C, C_m) = \ln[u(C, C_m)]$$

and where  $u(C, C_m)$  is a function homogeneous of degree 1. The first-order conditions are:

$$(A1) \quad \partial u / \partial C = \phi u, \quad \partial u / \partial C_m = \phi \lambda u$$

$$(A2) \quad \dot{\phi} = (\delta - r)\phi,$$

where  $\phi$  is the Lagrange multiplier. Using the homogeneity of  $u$  through Euler equation, (A1) is reduced to  $E\phi = 1$ . Differentiating this relationship logarithmically, we then eliminate  $r$  to obtain  $(\dot{A}/E) = \delta(A/E) - 1$ , which, when integrated forward, gives  $E = \delta A$ . If  $u(C, C_m)$  is further specified as a Cobb-Douglas function, (A1) gives (2) and (3) in the text. Note that, in the stationary state, we have  $\dot{\phi} = 0$  and  $\dot{A} = 0$ , so that, given the constraint and (A2), we must have  $r = \delta$  and  $E = \delta A$  irrespective of the functional form of the utility function  $U(C, C_m)$ . The reason why the simple formula-



tion  $E = \delta A$  also holds outside the stationary state is that the definition of  $U(C, C_m)$  as  $\ln[u(C, C_m)]$  renders this function Cobb-Douglas over time, thus yielding the usual constant share property.

## 2. The Firm's Problem

The firm maximizes its present value  $\int_0^\infty (y - I)e^{-rt} dt$  given the cost of investment  $I = \dot{K}[1 + (\phi/2)(\dot{K}/K)]$ . Introducing the notation  $\dot{K} = J$ , the Hamiltonian is

$$H = \{y_0 K^\alpha - J[1 + (\phi/2)(J/K)] + q^m J\}e^{-rt},$$

where  $q^m$  is the marginal cost of investment. The first conditions are:

$$(A3) \quad \partial H/\partial J = 0,$$

so  $\dot{K} = J = K(q^m - 1)/\phi$ ;

$$(A4) \quad -\partial H/\partial K = e^{-rt}(\dot{q}^m - rq^m),$$

so  $\dot{q} = rq^m - (\alpha y - I + q^m \dot{K})/K$ . The average value of installed capital at time  $t$ ,  $q^a$  is the present value of the firm's earnings, the objective function in the previous optimization problem:

$$q^a(t) \cdot K(t) = \int_t^\infty [y(s) - I(s)]e^{-r(s-t)} ds,$$

which after differentiation and dropping the time parameter gives

$$(A5) \quad \dot{q}^a = rq^a - (y - I + q^a \dot{K})/K.$$

Thus (5) and (6) in the text define  $q$  to be  $q^a$  as specified in (A5), while (8) is (A3) where  $q^m$  has been replaced by  $q^a$ . Comparison of (A4) and (A5) shows the nature of this approximation, discussed in note 4.

## Appendix 2: Solution of the Model

We first linearize the model around its initial position, characterized by  $X = X^* = 1$ ,  $\mu = \lambda = 1$ ,  $y = y^* = \delta$ ,  $Z = 0$ , and  $w = w^* = 1$ . The wage adjustment equations (7'), after substitution of (6') into (1') and then plugging  $L$  and  $L^*$  into (7'), give for a given disturbance  $\hat{y}_0 = \Delta y_0/y_0$  in home productivity,

$$(B1) \quad \dot{w} = -\gamma_1(w - 1) + \gamma_1 \hat{y}_0,$$

where  $\gamma_1 = \gamma \bar{L}/(1 - \alpha)$ ,

$$(B2) \quad \dot{w}^* = -\gamma_1^*(w^* - 1),$$

where  $\gamma_1^* = \gamma^* \bar{L}^*/(1 - \alpha)$ .

Clearly, the only solution for (B2) which admits  $w^*(0) = 1$  as assumed is  $w^* = \bar{w}^* = 1$ , a constant. Thus, there will be no departure from full employment abroad. We use, in the following, the fact that  $w^*$  remains constant throughout.

From (B1), it is also clear that the stationary state value of  $w$  is a priori uniquely determined:  $\bar{w} = 1 + \hat{y}_0$ . The goods markets' equilibrium conditions (11') are solved for  $X$  and  $X^*$  after substitution of  $\mu = \lambda X^*/X$ . Actually, it is easier first to write that world income is equal to world spending,  $y + \lambda y^* = \delta A + \delta \lambda A^*$ , which gives

$$(B3) \quad X + X^* = -\alpha/(1 - \alpha)(w - w^*) + \hat{y}_0/(1 - \alpha).$$

Then the domestic goods market condition is solved for  $X$ :

$$(B4) \quad (a + a^*)(X - 1) = -[\alpha/(1 - \alpha)](w - 1) - a^*(\mu - 1) + (a - a^*)Z + \hat{y}_0/(1 - \alpha).$$

The current account equation, when linearized and after substitution of (B3) and (B4), gives

$$(B5) \quad (a + a^*)\dot{Z}/\delta = -(1 - a - a^*)[\alpha/(1 - \alpha)](w - 1) - a^*(\mu - 1) + (a - a^*)Z + (1 - a - a^*)\hat{y}_0/(1 - \alpha).$$

The asset arbitrage condition (9'), similarly, yields

$$(B6) \quad (a + a^*)\dot{\mu} = 2\delta(1 - a - a^*)[\alpha/(1 - \alpha)](w - 1) + 2\delta a^*(\mu - 1) - 2\delta(a - a^*)Z - 2\delta(1 - a - a^*)\hat{y}_0/(1 - \alpha).$$

The system is reduced to the three equations (B1), (B5), and (B6), and rewritten in matrix form as

$$\begin{bmatrix} \dot{w} \\ \dot{Z} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -\gamma_1 & 0 & 0 \\ -\frac{1 - a - a^*}{a + a^*} \frac{\delta\alpha}{1 - \alpha} & \delta \frac{a - a^*}{a + a^*} & -\delta \frac{a^*}{a + a^*} \\ 2\frac{1 - a - a^*}{a + a^*} \frac{\delta\alpha}{1 - \alpha} & -2\delta \frac{a - a^*}{a + a^*} & 2\delta \frac{a^*}{a + a^*} \end{bmatrix} \begin{bmatrix} w \\ Z \\ \mu \end{bmatrix} + \begin{bmatrix} \gamma_1 \bar{w} \\ u \\ 2u \end{bmatrix},$$

where  $u = -\frac{1 - a - a^*}{a + a^*} \frac{\delta\alpha}{1 - \alpha} + \frac{\delta a^*}{a + a^*} + \frac{1 - a - a^*}{a + a^*} \frac{\delta \hat{y}_0}{1 - \alpha}$ .

It can be checked immediately that the last two columns of the transition matrix are linearly dependent, so that the matrix is singular and we cannot find, a priori,  $\bar{Z}$  and  $\bar{\mu}$ . The eigenvalues are  $\lambda_1 = 0$ ,  $\lambda_2 = -\gamma_1$ ,  $\lambda_3 = \delta$ , so, with one positive root and one nonpredetermined variable, the model is stable under perfect foresight. The corresponding matrix of eigenvectors is

$$V = \begin{bmatrix} 0 & \frac{a + a^*}{1 - a - a^*} & 0 \\ \frac{a^*}{a - a^*} & \frac{\delta}{\gamma_1 + \delta} \frac{\alpha}{1 - \alpha} & -1 \\ 1 & -2 \frac{\delta}{\gamma_1 + \delta} \cdot \frac{\alpha}{1 - \alpha} & 2 \end{bmatrix},$$

which contains terms with  $\gamma_1$ , the speed of adjustment.

We can solve for  $w$ ,  $Z$ , and  $\mu$ :

$$(B7) \quad w(t) = \bar{w} + (1 - \bar{w})e^{-\gamma_1 t}$$

where, again,  $\bar{w} = 1 + \hat{y}_0$ ;

$$(B8) \quad Z(t) = \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \hat{y}_0 (1 - e^{-\gamma_1 t});$$

and

$$(B9) \quad \mu(t) - 1 = \frac{1 - a - a^*}{a^*} \left( 1 + \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \right) \hat{y}_0 \\ + 2 \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \hat{y}_0 (e^{-\gamma_1 t} - 1).$$

The stationary state values for  $Z$  and  $\mu$  immediately follow:

$$\bar{Z} = \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \hat{y}_0;$$

$$\bar{\mu} - 1 = \frac{1 - a - a^*}{a + a^*} \left( \frac{a + a^*}{a^*} + \frac{a - a^*}{a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \right) \hat{y}_0.$$

Also, note from (B9) that the initial jump of  $\mu$  at time zero will also be a function of  $\gamma_1$ .

Using (B3), (B4), and the linearized version of  $\mu = \lambda X^*/X$ , we can now compute

$$X(t) - 1 = \hat{y}_0 + \frac{1}{a + a^*} \frac{\alpha}{1 - \alpha} \hat{y}_0 \\ \left[ 1 - (1 - a - a^*) \frac{\delta}{\gamma_1 + \delta} \right] e^{-\gamma_1 t};$$

$$X^*(t) - 1 = -\frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\gamma_1}{\gamma_1 + \delta} \hat{y}_0 e^{-\gamma_1 t};$$

$$\lambda(t) - 1 = \frac{1 - a}{a^*} \hat{y}_0 + \left[ \frac{(1 - a - a^*)(a - a^*)}{a^*(a + a^*)} \frac{\delta}{\gamma_1 + \delta} + \frac{2 - a - a^*}{a + a^*} e^{-\gamma_1 t} \right] \frac{\alpha}{1 - \alpha} \hat{y}_0.$$

Finally, if the domestic welfare function is  $U = kC^a C_m^{1-a}$  with  $C = a\delta A$ ,  $\lambda C_m = (1 - a)\delta A$ , we obtain  $U = A\lambda^{-(1-a)}$  where  $k = \delta^{-1} a^{-a} (1 - a)^{-(1-a)}$ . Similarly, the foreign welfare function is  $U^* = A^* \lambda^{a^*}$ . Linearizing and computing the stationary state values gives

$$\frac{\bar{U} - U(0)}{U(0)} = \left[ 1 - \frac{(1 - a)^2}{a^*} - \frac{a}{a^*} \frac{(1 - a - a^*)(1 - a + a^*)}{a + a^*} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{\delta}{\gamma_1 + \delta} \right] \hat{y}_0$$

$$\frac{\bar{U}^* - U^*(0)}{U^*(0)} = (1 - a)\hat{y}_0 + [a + (1 - a^*)] \frac{1 - a - a^*}{a + a^*} \frac{\alpha}{1 - \alpha} \frac{\delta}{\gamma_1 + \delta} \hat{y}_0.$$

**Comment** Paul Krugman

Giavazzi and Wyplosz have given us an interesting and clear exposition of some consequences of a property which is common to many recent dynamic models: the existence of many possible steady states, and the dependence of the long run on the adjustment path. In my comment, I want to focus on this property, ask how robust it is, and suggest some “realistic” qualifications. I will then argue that what happens in the steady state may not be very important.

The first point to make is that the existence of a continuum of steady states is a characteristic of any model with (i) infinitely lived consumers who (ii) have utility functions which are separable over time, (iii) have the same

rate of time preference, and (iv) face perfect capital markets. The reason is simple: in the steady state, the real interest rate will equal the common rate of time preference, so that all individuals will set consumption exactly equal to the interest earnings on their wealth. The result is that *any* distribution of wealth will be self-replicating. It could be, as in Giavazzi and Wyplosz, the distribution of wealth between countries; or it could be the distribution of wealth between groups within a country. The result is the same. And if the different groups have different consumption preferences, relative prices will vary across steady states.

How might we undermine this result? Giavazzi and Wyplosz consider abandoning either assumption ii or assumption iii—that is, replacing a “Ramsey” utility function with an “Uzawa” one, or letting rates of time preference differ, leading to a corner solution. Rightly, in my view, they regard these as unsatisfactory.

In principle, however, we could also drop one of the other assumptions. We could, for example, take a life-cycle approach in which individuals have finite lifetimes. As we know, this leads to a determinate steady-state ratio of wealth to labor income—which is enough to tie down the steady state. Alternatively, we could introduce an imperfection in capital markets; say, a debt ceiling. Add some uncertainty, and we will have a precautionary motive for holding assets which will lead in aggregate to something like a target wealth level, and again tie down the steady state. In either case, the result will to some extent be “as if” wealth were in the utility function—which is the third alternative the paper proposes.

The problem with these alternatives, however, is that they are *hard*. They lead to very intractable models, while the Ramsey world is very clean and straightforward. And is it not clear what one gains for the extra difficulty. Once we understand what the multiplicity of steady states actually means—in particular, once we understand that it does not imply any actual indeterminacy—the existence of zero eigenvalues should not bother us. Infinitely lived consumers and perfect capital markets are not realistic assumptions, but if they clear the ground for more understanding of other issues, the simplification will have been justified.

Turning briefly to substance, the authors are of course right in their point that adjustment speeds affect the steady state. One wonders, however, if they are not making too much of this, because of their focus on *steady-state* utility as a welfare criterion. This is clearly not right: we should use the lifetime utility of the agents in the model. But as soon as we do this, the question of uniqueness of the steady state becomes much less interesting. Even if the steady state were unique, the transition path to that steady state would still affect the lifetime utility levels of the countries—which is a modern dynamic modeler’s way of saying that in the long run we are all dead.

An example of how exclusive focus on the steady state can be misleading is Giavazzi and Wyplosz’s discussion in section 10.4, and particularly their

figure 10.2. Here it seems that faster adjustment at home necessarily makes the foreign country worse off—as it does, in the steady state, for the steady-state utility possibility frontier is independent of the rate of adjustment. But the true world utility possibility frontier is surely expanded by a higher speed of domestic adjustment, so faster adjustment at home might actually make *both* countries better off.

In sum, this paper is valuable in clearing up a technical issue which has caused some confusion. However, the substantive conclusion that adjustment matters is not something which hinges in any crucial way on whether or not the steady state is unique.

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