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10 Medical Care, Medical Insurance, and Survival Probability: The True Cost of Living

Theodore C. Bergstrom

In this paper we construct a much simplified model of private and public social decision-making related to health care and health insurance. The model is designed to clarify the logical relationships among health insurance plans, life insurance and annuities, consumption, medical care and nursing care. The paper characterizes an efficient allocation of health care and describes decentralized insurance markets that would sustain an efficient allocation. The analysis is similar in spirit to papers by Arrow (1976) and Nordquist and Wu (1976) presented at an earlier NBER conference on health economics.

Individual Preferences and Plans

Imagine an economy with a large number of consumers, all with identical tastes. There are two commodities, bread and medical care. Tomorrow each consumer will receive a free medical check-up. There are n possible diagnoses, $d_1, \ldots, d_n$. Suppose that today the individuals' probability distributions over diagnoses are independent and identically distributed. Let π_i be the probability that any particular consumer will be diagnosed as being in condition i. We assume a simple medical technology. Regardless of the diagnosis, there are at most two things that can happen. The patient can painlessly be restored to perfect health or he can die. Let the conditional probability that someone with diagnosis i who receives m_i units of medical care will survive be $\theta_i(m_i)$.

Suppose that the amount of medical care that a person receives can be made to depend on the diagnosis of his condition. Suppose further that the amount of bread that he (or his heirs) receive can depend both on his

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diagnosis at the check-up and on whether he survives after being given the chosen medical care. As we shall later see, provision of these contingent commodities could be arranged either through a centrally imposed national plan or in a decentralized way through health insurance and life insurance contracts. Possible medical histories can be denoted d_i^0 and d_i^1, where d_i^0 denotes the event that one receives diagnosis i and proceeds to die and d_i^1 denotes the event that one survives after having diagnosis i. A *consumption strategy* is a vector (M,B) where $M=(m_1, \ldots, m_n)$ specifies the amount m_i of medical care that the consumer will receive if his diagnosis is i and where $B=(b_1^0, \ldots, b_n^0, b_1^1, \ldots, b_n^1)$ specifies the amounts b_i^0 and b_i^1 of bread that the consumer will consume if he has diagnosis i and dies or lives, respectively. We will sometimes speak of M as the consumer's "medical strategy" and B as his "bread-consumption strategy."

In the model discussed, the probability that the consumer has a particular medical history is determined by the medical strategy M. The probability of medical history d_i^x given strategy M will be denoted $\pi(d_i^x \mid M)$. In particular, our assumptions imply that

$$\pi(d_i^1 \mid M)=\pi_i \theta_i(m_i) \tag{1}$$

and

$$\pi(d_i^0 \mid M)=\pi_i(1-\theta_i(m_i)). \tag{2}$$

The von Neuman-Morgenstern expected utility of a consumer with medical strategy M and consumption strategy B will take the form:

$$V(M,B)=\sum_{x=0,1}\sum_{i=1}^{n}\pi(d_i^x \mid m_i)u(d_i^x,m_i,b_i^x). \tag{3}$$

Recall that we have assumed that medical care does not affect utility directly, and that after medical treatment one is either restored to perfect health or one is dead. Thus the utility function takes the special form

$$u(d_i^x,m,b_i^x)\equiv u_x(b_i^x) \tag{4}$$

where $u_1(b)$ can be viewed as the utility of the prospect of surviving and consuming b units of bread and $u_0(b)$ as the utility of the prospect of dying and leaving b units of bread to one's heirs. From equations (1) – (4) we see that (3) could be written equivalently as:

$$\begin{aligned} V(M,B)=&\sum_{i=1}^{n}\pi_i\theta_i(m_i)u_1(b_i^1)\\ &+\sum_{i=1}^{n}\pi_i(1-\theta_i(m_i))u_0(b_i^0) \end{aligned} \tag{5}$$

In most of the remaining discussion we will assume that the functions $u_0(\cdot)$, $u_1(\cdot)$ and $\theta_i(\cdot)$ are nondecreasing and concave. Assuming concavity of $u_1(\cdot)$ and $u_0(\cdot)$ is equivalent to assuming that the consumer is either risk-averse or risk-neutral with respect to bets that leave his survival probability unchanged. Concavity of the θ_i's means diminishing marginal returns to medical care.

An Optimal Centrally Imposed Health Plan

Having examined individual preferences for medical care and bread, we now consider the options available to the economy as a whole. We begin by considering a hypothetical central authority that seeks to impose a national health care plan. We define an "allocation of consumption strategies" to be a list, $(M^1, B^1, \ldots, M^K, B^K)$, of the consumption strategies, (M^k, B^k), of each consumer k. Since all consumers have identical preferences and the same prospects before their medical check-ups, it is of special interest to consider those allocations that offer all consumers the same consumption strategy. Such an allocation plan will be called a "uniform national plan."

Let $[M, B]$ be a uniform national plan that offers each consumer the consumption plan, (M, B). If the number of consumers is K, then the total number of persons with diagnosis i will be $K\pi_i$ and total consumption of medical care by these persons will be $K\pi_i m_i$.[1] Therefore average per capita consumption of medical care in the economy is certain to be

$$\bar{M}(M) \equiv \sum_{i=1}^{n} \pi_i m_i \tag{6}$$

if the national plan is (M, B).

If the national health plan is $[M, B]$, then the proportions of the population with medical histories d_i^1 and d_i^0 are $\pi_i\theta_i(m_i)$ and $\pi_i(1 - \theta_i(m_i))$ respectively. Therefore average per capita bread consumption in the economy will be

$$\bar{B}(M, B) \equiv \left[\sum_{i=1}^{n} \pi_i\theta_i(m_i) b_i^1 + \sum_{i=1}^{n} \pi_i(1 - \theta_i(m_i)) b_i^0\right] \tag{7}$$

The feasibility of a national plan depends on whether the economy can supply the total outputs $K\bar{M}$ and $K\bar{B}$. We develop here a very simple model of the productive capacity of the economy which is sufficient to illustrate the relevant issues. Suppose that it is technically possible to convert one unit of bread into $\frac{1}{p}$ units of medical care (regardless of how many units are produced). Suppose also that there is an initial endowment of $\bar{b}$ units of bread per consumer and that each consumer who survives produces h units of bread (or equivalently $\frac{h}{p}$ units of medical care). Consumers who do not survive produce nothing.

The proportion of the population that survives is determined by M and can be written as:

$$\Pi_1(M) \equiv \sum_{i=1}^{n} \pi_i\theta_i(m_i). \tag{8}$$

Therefore the total output of the economy measured in terms of bread must be $K[\tilde{b} + \Pi_1(M)h]$ and the constraint on the feasibility of a national plan, (M,B) is:

$$K[\bar{B}(M,B) + p\bar{M}(M)] = K[\tilde{b} + \Pi_1(M)h]. \tag{9}$$

Since consumers are assumed to have identical preferences, we don't need a deep welfare economic analysis to arrive at a criterion for an optimal national plan. We simply seek a national plan that maximizes the utility of a representative consumer on the set of feasible plans. We define an "optimal uniform national plan" to be a uniform national plan (M,B) that maximizes $V(M,B)$, as defined in equation (5) subject to the feasibility constraint expressed in equation (9). Although an optimal uniform national plan is, by definition, not dominated by any feasible allocation that treats all consumers in exactly the same way, there might conceivably be feasible allocations that are Pareto superior to an optimal uniform national plan but treat some consumers differently from others. The following proposition, which is proved in the Appendix to this paper establishes conditions under which there are no such allocations.

Proposition 1

For an economy, let the set of feasible allocations of consumption strategies be those such that:

$$\sum_{k=1}^{K} [\bar{B}(M^k,B^k) + p\bar{M}(M^k,B^k)] = K\tilde{b} + \sum_{k=1}^{K} \Pi_1(M^k)h\ .$$

If $u_0(\cdot)$ and $u_1(\cdot)$ are concave functions, then an optimal uniform national plan is Pareto optimal.

Proposition 1 enables us to restrict our search for an equalitarian Pareto optimal allocation strategy to the set of uniform national plans. Proposition 2 allows us to further limit the domain of search.

Proposition 2

Let $V(M,B)$ be the expected utility function defined in equation (5). If $u_0(\cdot)$ and $u_1(\cdot)$ are concave functions, and if (M,B) satisfies the feasibility constraint (9), then there exists a consumption plan, $(M,\bar{B})$ such that $V(M,\bar{B}) \geqq V(M,B)$, $(M,\bar{B})$ satisfies the budget equation (24), and $\bar{B} = (\bar{b}^1, \ldots, \bar{b}^1, \bar{b}^0, \ldots, \bar{b}^0)$ gives the consumer or his heirs a bread consumption that depends only on whether he lives or dies and not on the diagnosis he receives.

Proof:

If (M,B) satisfies (24), then so does $(M,\bar{B})$ where $\bar{B} = (\bar{b}^0, \ldots, \bar{b}^0, \bar{b}^1, \ldots, \bar{b}^1)$

where $$\bar{b}_1 = \left(\frac{1}{\Pi_1(M)}\right)\sum_{i=1}^{n} \pi_i\theta_i(m_i) \text{ and } \bar{b}_0 = \left(\frac{1}{1-\Pi_1(M)}\right)\sum_{i=1}^{n} \pi_i(1-\theta_i(m_i)).$$

Furthermore, since $u^1(\cdot)$ and $u^0(\cdot)$ are concave functions, it must be that

$$u_1(\bar{b}^1) \geqq \left(\frac{1}{\Pi_1(M)}\right)\sum_{i=1}^{n} \pi_i\theta_i(m_i)u_1(b_i^1) \text{ and } u_0(\bar{b}^0) \geqq \left(\frac{1}{1-\Pi_1(M)}\right)\sum_{i=1}^{n} \pi_i(1-\theta_i(m_i))u_0(b_i^0).$$

It follows that $V(M,\bar{B}) \geq V(M,B)$.
Q.E.D.

Proposition 2 enables us to confine our search for an optimal uniform national plan to those plans in which consumption strategies, (M,B), have the property that

(10) $$b_1^1 = b_2^1 = \ldots = b_n^1 \equiv b^1$$

and

(11) $$b_1^0 = b_2^0 = \ldots = b_n^0 \equiv b^0.$$

We will frequently denote such strategies by (M,b^0,b^1) and their expected utilities by

(12) $$V(M,b^0,b^1) \equiv \Pi_1(M)u_1(b^1) + (1-\Pi_1(M))u_0(b^0).$$

The definition (7) of $\bar{B}(M,B)$ reduces to

(13) $$\bar{\mathrm{B}}(M,b^0,b^1) = [\Pi_1(M)b^1 + (1-\Pi_1(M))b^0]$$

Thus the constraint in (9) can be written

(14) $$K[\Pi_1(M)b^1 + (1-\Pi_1(M))b^0 + p\bar{M}(M)] = K[\tilde{b} + \Pi_1(M)h].$$

or equivalently:

(15) $$\Pi_1(M)(b^1-h) + (1-\Pi_1(M))b^0 + p\bar{M}(M) = \tilde{b}\cdot$$

Assuming that the derivatives, $u_0'(\cdot), u_1'(\cdot)$, and $\theta_i'(\cdot)$, exist everywhere, the first-order necessary conditions for an interior solution to the maximization of (12) subject to (15) can be written:

$$u_1'(b^1) = u_0'(b^0) \tag{16}$$

and for $i = 1, \ldots, n$:

$$\frac{\partial \Pi_1(M)}{\partial m_i}\left[\frac{u_1(b^1) - u_0(b^0)}{u_1'(b^1)}\right] = p\frac{\partial \bar{M}(M)}{\partial m_i} + [b^1 - h - b^0]\,\frac{\partial \Pi_1(M)}{\partial m_i} \tag{17}$$

From the definitions of $\Pi_1(M)$ and $\bar{M}(M)$, it follows that (17) is equivalent to

$$\theta_i'(m_i)\left[\frac{u_1(b^1) - u_0(b^0)}{u_1'(b^1)} + h + b^0 - b^1\right] = p. \tag{18}$$

Equations (16) and (17) have simple and rather interesting interpretations. Equation (16) require that the marginal rate of substitution between bread contingent on being alive and bread in one's estate should be unity. Notice that this is true regardless of the probability distribution of medical histories. Typically one would expect the functions $u_1(\cdot)$ and $u_0(\cdot)$ to have the property that if $u_1'(b^1) = u_0'(b^0)$ then $u_1(b^1) > u_0(b^0)$. Operationally this means that at an optimal solution, consumers would prefer a higher survival probability to a lower one.

On the left side of (17) the rate of change of survival probability due to an increment in m_i is multiplied by an individual's marginal rate of substitution between survival probability and bread. This expresses the rate at which an individual would be willing to make a small exchange of bread for an increase in the amount of medical care he would receive if he had diagnosis i. On the right side of (17) the term $p\frac{\partial \bar{M}}{\partial m_i}$ is the direct resource cost of increasing m_i while the term $\left[b^1 - h - b^0\right]$ $\frac{\partial \Pi_1(M)}{\partial m_i}$ represents the per capita effect on net bread requirements due to the fact that increasing m_i also increases the proportion of the population that is consuming b^1 and producing h units of bread, and reduces the proportion of the population whose heirs must be given b^0 units of bread.

From equation (18) it follows that:

$$(19) \qquad \theta_1'(m_i) = \ldots = \theta_n'(m_n) = p \div \left[\frac{u_1(b^1) - u_0(b^0)}{u_1'(b^1)} + h + b^0 - b^1 \right].$$

Thus we see that an optimal health plan has the property that the marginal contribution of medical care to the conditional probability that one survives contingent on a diagnosis is equalized for all diagnoses.

Equation (19) emphasizes a fact that, on reflection, should have been obvious from the start. An optimal health plan will exclude some technically possible medical treatments on the grounds that they are too expensive. Equalizing θ_i' across diagnoses certainly need not imply equalizing θ_i across diagnoses. In fact, in an optimal medical plan there may be diagnoses for which only a small amount of medical care is given and from which recovery is then unlikely even though there exists a medical cure which, though very expensive, would ensure that persons with this diagnosis survive.

A Decentralized Economy with Actuarially Fair Insurance

We now consider provision of medical services and bread by means of private markets. We will show that an optimal national plan could also be reached as a competitive equilibrium with appropriate insurance markets. Let each consumer own an initial endowment of $\bar{b}$ units of bread. If and only if he survives, he will produce an additional h units of bread. As before we assume that one unit of bread can always be costlessly converted into one unit of medical care.

In this paper, a "health insurance plan" is a contract that specifies a vector $\mu = (\mu_1, \ldots, \mu_n)$ where a positive μ_i is the net amount, measured in units of bread, that the insurance company will pay a consumer enrolled in the plan if the consumer has diagnosis i. If μ_i is a negative number, then the consumer will pay the insurance company a net amount μ_i if the consumer's diagnosis is i. The expected value of payments to the consumer under health insurance plan μ is then $\sum_{i=1}^{n} \pi_i \mu_i$.

If transactions costs for the insurance company can be ignored and if the number of identical consumers is large enough so that the variance in the proportion of the population with a given diagnosis is negligible, then, to a close approximation, in competitive equilibrium, insurance companies must be willing to offer any actuarially fair health insurance plan. Thus an equilibrium health insurance plan must satisfy

$$(20) \qquad \sum_{i=1}^{n} \pi_i \mu_i = 0$$

Here we will treat insurance plans that are commonly called "life insurance" and "annuities" as two different kinds of bets that could be made between a consumer and a life insurance company. For us, a life insurance-annuity plan is a contract that states payments (measured in units of bread) to be made from the insurance company to the consumer (or vice versa) where the direction of net payments depends on whether the consumer lives or dies. Any life insurance-annuity plan is described by a vector (α^0, α^1). If α^0 is positive and α^1 is negative, the plan is called "life insurance". If the signs are reversed, the plan is called an "annuity". If (α^0, α^1) is life insurance, then the consumer's estate receives a net payment (measured in units of bread) of α^0 if he dies, while the consumer pays the insurance company α^1 if he survives. If (α^0, α^1) is an annuity, then the consumer pays the insurance company a net amount α^0 if he dies and receives a net amount α^1 if he survives.

As we did with health insurance, we assume away transactions costs and asymmetries of information, and assume that statistical variation in the proportion of the population having any particular life history is negligible. Therefore the supply side conditions for competitive equilibrium require that life insurance-annuity plans be actuarially fair. In this case, the condition for actuarially fair insurance is a bit more complicated than in the case of health insurance. One's survival probability depends on how much medical care he would obtain in the event of each possible diagnosis. Therefore actuarially fair life insurance-annuity plans must in general have rates that depend on the amount of medical care one will purchase in each contingency. On the face of it, it would seem unreasonably difficult to enforce a contract, signed between the insurance company and the consumer before the physical check-ups are made, requiring the consumer to purchase no less or more medical care in the event of each contingency than is specified in the life insurance or annuity contract.

As it turns out, a consumer who has chosen an insurance plan that gives him the best ex ante prospects possible with an actuarially fair plan will not in the event of any realized diagnosis be able to afford a combination of bread and medical care that he prefers ex post to that provided by the consumption strategy chosen ex ante. This is true even if medical insurance takes the form of lump sum payments contingent on one's diagnosis and not tied to any particular level of purchases of medical care. Therefore an insurance company can offer actuarially fair rates simply by setting its rates as a function of one's health insurance plan.

A "consumer's consumption strategy" is a vector (M,B) where $M = (m_1, \ldots, m_n)$ states the amount, m_i, of medical care that the individual plans to consume if he has diagnosis i and where $B = (b_1^0, \ldots, b_n^0, b_1^1, \ldots, b_n^1)$ states the amount b_i^0 of bread that he plans to consume if he has diagnosis i and dies and the amount b_n^1 that he plans to consume if he has diagnosis i and lives. We call M his medical care strategy and B his bread consumption strategy. A consumer's "insurance plan" consists of a health insurance plan and a life insurance-annuity plan. A "health insurance" plan is a vector $\mu = (\mu_1, \ldots, \mu_n)$ where μ_i represents net payments (possibly negative) measured in units of bread from a health insurance company to a consumer in the event that he has diagnosis i. A "life insurance-annuity" is a vector (α^0, α^1) where α^0 and α^1 represent net payments (possibly negative) from a life insurance company to a consumer respectively if he dies or lives. A health insurance plan, μ, is "actuarially fair" if it satisfies equation (20). A life insurance-annuity plan is actuarially fair *contingent on M* if

$$\Pi_1(M)\alpha^1 + (1 - \Pi_1(M))\alpha^0 = 0 \tag{21}$$

The insurance plan $(\mu, \alpha^0, \alpha^1)$ is said to be actuarially fair *with respect to M* if it is actuarially fair, and (α^0, α^1) is actuarially fair with respect to M.

Suppose a consumer chooses the insurance plan $(\mu, \alpha^0, \alpha^1)$. If he then has diagnosis i and survives, his net receipts from the insurance companies will be $\mu_i + \alpha^1$. He has an initial allotment of $\bar{b}$ units of bread and he produces an additional h units. Thus he has a total number of $\mu_i + \alpha^1 + \bar{b} + h$ units of bread to be spent on medical care and bread. His purchases in this event must therefore satisfy

$$pm_i + b_i^1 = \mu_i + \alpha^1 + \bar{b} + h \tag{22}$$

If a consumer has diagnosis i and dies, he and his estate receive $\mu_i + \alpha^0$ from the insurance companies. He has an initial endowment of $\bar{b}$ and produces no additional bread. Therefore his purchases in the event of this medical history must satisfy

$$pm_i + b_i^0 = \mu_i + \alpha^0 + \bar{b}\ . \tag{23}$$

An insurance plan μ, (α^0, α^1) is said to "sustain" the consumption plan (M, B) if equations (22) and (23) are satisfied for $i = 1, \ldots, n$.

All of the propositions to be developed here assume implicitly the special structure of our model. However, each of them can be extended in a fairly transparent way to much more general models.

Proposition 3

If the consumption plan (M, B) can be sustained by an insurance plan, $(\mu, \alpha^0, \alpha^1)$ that is actuarially fair with respect to M, then (M, B) must satisfy the following budget constraint:

$$\begin{aligned}\sum_{i=1}^{n} \pi_i[pm_i &+ \theta_i(m_i)(b_i^1 - h) \\ &+ (1 - \theta_i(m_i))b_i^0] = \bar{b}\end{aligned} \tag{24}$$

Proof:

For each i, multiply both sides of equation (22) by $\pi_i\theta_i(m_i)$ and both sides of equation (23) by $\pi_i(1 - \theta_i(m_i))$ and add the resulting $2n$ equations. This yields:

$$\sum_{i=1}^{n} \pi_i[pm_i + \theta_i(m_i)b_i^1 + (1 - \theta_i(m_i))b_i^0] \tag{25}$$
$$= \sum_{i=1}^{n} \pi_i[\mu_i + \theta_i(m_i)\alpha^1 + (1 - \theta_i(m_i))\alpha^0 + \theta_i(m_i)h] + \bar{b}$$

Since the insurance plans are assumed to be actuarially fair, equations (20) and (21) apply. Using (20) and (21) and slightly rearranging terms one obtains (24) from (25).
Q.E.D.
From Propositions 2 and 3, we arrive at the following result.

Proposition 4

Let (M, b^0, b^1) be a consumption plan that maximizes

$$V(M,b^0,b^1) \equiv \Pi_1(M)u_1(b^1) + (1 - \Pi_1(M))u_0(b^0) \tag{26}$$

over all $(M, b^0, b^1) \geqq 0$ such that:

$$\bar{M}(M) + \Pi_1(M)(b^1 - h) + (1 - \Pi_1(M))b^0 = b. \tag{27}$$

Then $V(M,B) \geqq V(M',B')$ if (M',B') can be sustained by an insurance plan that is actuarially fair with respect to M'.

Proposition 5

If (M, b^0, b^1) satisfies equation (27), then there exists an insurance plan, $(\mu, \alpha^0, \alpha^1)$ that sustains (M, b^0, b^1) and is actuarially fair with respect to M.

Proof:

We prove Proposition 5 by exhibiting the claimed insurance plan. Given M and (b^0, b^1), let:

$$\mu_i = p[m_i - \sum_{i=1}^{n} \pi_i m_i] \quad \text{for } i = 1, \ldots, n \tag{28}$$

$$\alpha_1 = p\sum_{i=1}^{n} \pi_i m_i + b^1 - h - \bar{b} \tag{29}$$

$$\alpha_0 = p\sum_{i=1}^{n} \pi_i m_i + b^0 - \bar{b} \tag{30}$$

It is easily verified that the insurance plans defined in (28), (29), and (30) satisfy equations (20), (22), and (23). Furthermore, if equation (27) is satisfied, it is a matter of straightforward verification to show that equation (21) is satisfied. Therefore (M, B) is sustained by the insurance plan $(\mu, \alpha^0, \alpha^1)$, the health insurance plan, μ, is actuarially fair, and the life insurance-annuity plan, (α^0, α^1), is actuarially fair contingent on M. Q.E.D.

From Propositions 4 and 5 we deduce:

Proposition 6

If (M^*, b^{0*}, b^{1*}) solves the constrained maximization problem posed in Proposition 5, then (M^*, b^{0*}, b^{1*}) maximizes $V(M, B)$ on the set of consumption strategies (M, B) that can be sustained by an insurance plan that is actuarially fair contingent on M.

Supply considerations require that a competitive equilibrium insurance plan be actuarially fair. Demand conditions require that an equilibrium insurance plan sustain a consumption strategy that consumers like at least as well as any consumption plan sustainable by another insurance plan. These considerations, together with Proposition 1, suggest the appropriateness of the following definitions.

We define a "competitive consumption strategy" for a consumer to be a strategy (M^*, b^{0*}, b^{1*}) that solves the constrained maximization problem posed in Proposition 4. A "competitive equilibrium insurance plan" is defined to be an insurance plan, $(\mu^*, \alpha^{0*}, \alpha^{1*})$ that sustains the competitive consumption strategy (M^*, B^*) and is actuarially fair with respect to M^*.

Proposition 7

If preferences are continuous and if $\Pi_1(M)$ is bounded away from 0 and from 1 for all M, then there exists a competitive equilibrium insurance plan.

Proof:

If $\Pi_1(M)$ is bounded away from 0 and 1, it is easy to see that the set of consumption plans $(M, b^0, b^1) \geqq 0$ satisfying (27) is closed and bounded. Continuity of preferences implies continuity of the function, $V(M, B)$. Since continuous functions take maxima on compact sets, there exists a competitive consumption strategy, (M^*, B^*). According to Proposition 5, there exists an actuarially fair insurance plan, $(\mu^*, \alpha^{0*}, \alpha^{1*})$ that sustains (M^*, B^*).
Q.E.D.

Observe that the constrained maximization problem that defines a competitive consumption strategy is formally the same as the maximization problem that defines an optimal uniform national plan. As a consequence of Proposition 7, we therefore have the following result.

Proposition 8

An allocation of consumption strategies in which each consumer has a competitive consumption strategy is Pareto optimal.

Propositions 7 and 8 show that an allocation of competitive consumption strategies exists and is Pareto optimal. Proposition 6 gives us reason to think that competitive consumption strategies deserve the title "competitive" since they are, in a sense, the best strategies a consumer can accomplish by means of a competitive insurance plan.

There remains some room for doubt. Even if an insurance plan leads to the best achievable strategy, ex ante, can we be sure that in the event of an announced diagnosis, the consumer might not wish to and have the ability to purchase a different amount of medical care than the optimal plan specifies? Thus we might wonder whether a consumer who has a positive initial endowment of bread and who receives an additional amount of bread μ_i after diagnosis i is announced would indeed choose the amount of medical care that was anticipated in the competitive plan. If, say, the diagnosis were that he is almost certain to die if he does not buy a very large amount of medical care and if the competitive plan does not provide a very large amount, might he not then try to spend more on medical care than the competitive plan provides? (Even if he can not raise a large enough amount of bread to pay for a cure, he could possibly bet whatever bread he has in a lottery, such that if he wins the lottery he could afford a cure.) If this were the case, then in order for a competitive life insurance-annuity contract to be workable, not only would it have to include a provision specifying the exact amount of medical care the consumer is to purchase in the event of each diagnosis, but that contract would sometimes have to be enforced, after the medical check-ups, against consumers who may be able to and wish to spend more (or less) than the contracted amount on medical care. Proposition 9, however, establishes that this is not a problem. In fact, from Proposition 9, we see that a competitive equilibrium health insurance plan could take the form of a lump sum payment (positive or negative) the size of which is contingent on the diagnosis. Even if all of the consumer's assets, including expected earnings if he survives and the expected value of his life insurance or annuity plans, could be converted freely at market value after the diagnosis to buy alternative bundles of medical care and bread, the consumer will be best off holding to the competitive consumption strategy that was sustained by the original competitive health insurance plan. A formalization of this result follows.

Proposition 9

Let (M^*, b^{0*}, b^{1*}) be a competitive consumption strategy that is sustained by the insurance plan, (μ^*, b^{*0}, b^{*1}). Then for every event i, a consumer's ex post utility function:

$$\theta_i(m_i)u_l(b^1) + (1 - \theta_i(m_i))u_0(b^0) \tag{31}$$

is maximized at (m_i^*, b^{1*}, b^{0*}) subject to the constraint:

$$\begin{aligned} &\theta_i(m_i)(b^1 - h) + (1 - \theta_i(m_i))b^0 + pm_i \\ &\leqq \mu_i + \bar{b} + \theta_i(m_i)\alpha^1 + (1 - \theta_i(m_i))\alpha^0. \end{aligned} \tag{32}$$

Proof:

A competitive consumption strategy (M^*, B^*) maximizes

$$\sum_{i=1}^{n} \pi_i \left[\theta_i(m_i)u_1(b_i^1) + (1 - \theta_i(m_i))u_0(b_i^0)\right]$$

over all (M, B) such that:

$$\sum_{i=1}^{n} \pi_i[pm_i + \theta_i(m_i)(b_i^1 - h) + (1 - \theta_i(m_i))b_i^0] \leqq \bar{b}.$$

Therefore, if

$$\begin{aligned} &\theta_i(m_i)u_1(b_i^1) + (1 - \theta_i(m_i))u_0(b_i^0) \\ &> \theta_i(m_i^*)u_1(b^{1*}) + (1 - \theta_i(m_i^*))u_0(b_0{}^*)\ , \end{aligned}$$

it must be that

$$\begin{aligned} &pm_i + \theta_i(m_i)(b_i^1 - h) + (1 - \theta_i(m_i))b_i^0 \\ &> pm_i^* - \theta_i(m_i)(b^{1*} - h) + (1 - \theta_i(m_i))b^{0*}. \end{aligned}$$

Equations (22) and (23) imply that

$$\begin{aligned} &pm_i^* + \theta_i(m_i)(b^{1*} - h) + (1 - \theta_i(m_i))b^{0*} \\ &= \mu_i + \theta_i(m_i)\alpha^{1*} + (1 - \theta_i(m_i))\alpha^{0*}. \end{aligned}$$

Substituting from this equation into the last inequality yields the conclusion of Proposition 9.

Q.E.D.

Expression (31) is the expected utility function for a consumer who knows he has diagnosis i. Equation (32) describes the budget that would be available to him if he could make any actuarially fair revision of his bets. One might argue that it is not realistic to suppose all such bets to be available to him. But if this is the case, our interpretation of the result remains intact. If the consumer can not improve on his existing contracts when all actuarially fair rearrangements are possible, then certainly he can not improve his prospects if only some of them are available.

Applications of the Analysis

A health insurance plan, as modelled in this paper, consists of a payment between the insurance company and the consumer, the size of which depends only on the consumer's diagnosed condition. In fact, as we showed in the proof of Proposition 5, if $M^* = (m_1^*, \ldots, m_n^*)$ is an optimal medical care plan, then M^* is sustained by an insurance plan in which the net payment between the insurance company and the consumer in the event of diagnosis i is $m_i^* - \bar{m}^*$ where $\bar{m}^* \equiv \sum_{i=1}^{n} \pi_i m_i^*$ is the expected cost of medical care in an optimal medical strategy. Such a plan amounts to having the consumer pay an insurance premium equal to $\bar{m}^*$ regardless of his health state. In return the consumer receives, contingent on diagnosis i, the amount m_i^* which is the full cost of the efficient level of medical care for someone with diagnosis i.

In most existing medical plans the payment to a consumer is made contingent, not on the diagnosis of his health, but rather on the amount he actually spends on medical care.[2] If consumers were to be reimbursed for the entire cost of any level of medical care they chose to purchase, one would expect that they would want to purchase more than the amount that is efficient. Thus some health insurance plans reimburse essentially all medical costs up to a predetermined maximum amount that depends on the nature of the ailment that is treated. If that maximum were approximately equal to the optimal m_i^* for each diagnosis, then a plan of this type would be equivalent to the efficient diagnosis-specific insurance we have modelled. Even where an insurance plan does not specifically limit the amount to be spent on particular illnesses, it seems likely that restraints are placed on the amount of care by current practice of physicians and hospitals. Thus if doctors were to prescribe the efficient amount, m_i^*, to patients with illness i and not offer them any other alternatives, then a health insurance plan offering full coverage would be efficient.

Many existing health care plans are characterized by coinsurance, where the insurance company pays some fixed fraction of actual medical expenditures. Since the consumer shares in the cost of medical care, he has some incentive to economize on its use. On the other hand, if the consumer does not have full coverage then he is left to bear some residual risk. Coinsurance is analyzed formally by Arrow (1976), who displays necessary conditions for an optimal rate of coinsurance.

For a model of the type we have studied, however, an optimal level of coinsurance is only second best. In Arrow's model, payments from the insurance company to the consumer are required to depend only on the amount of medical care purchased and not on the patient's diagnosis. The second best optimal rate of coinsurance results in too much medical care being purchased and too little risk-pooling relative to the optimum for the model studied here. To see this we notice that with coinsurance, the consumer is able to choose his quantity of medical care while paying less than its full marginal cost. Furthermore, because the full cost is not paid, his consumption level is not fully insured.

In the model presented above, health insurance in which payments are conditional on diagnoses does better than coinsurance. As we showed, not only are consumers left with an incentive to purchase efficient amounts of medical care, they also are able to achieve full risk-pooling.

Before we attribute practical significance to this result, it is appropriate to ask whether the case is prejudiced by the very special structure of our model. Conspicuously missing from this model are costs of information, moral hazard, adverse selection, deception, and fraud. It is reasonable to ask whether, in a model with imperfect information, coinsurance offers advantages that are not apparent in our special model. In a world where information is costly, it might be that medical expenditures are more readily observed and measured than diagnoses. Still, before a physician decides what medical treatment to perform, he has to make a diagnosis. Thus diagnosis-specific insurance should not require the acquisition of knowledge that wasn't all ready available to the doctor. There also may be greater opportunities for fraud in the case of diagnosis-specific insurance, although fraudulent reporting of diagnosis would appear to require cooperation of the patient and physician. However, particularly if second opinions are required, it is not clear that the possibilities for fraudulent reporting of diagnoses are greater than the possibilities for fraudulent reporting of expenditures.

The model we have discussed treats only a single time period. If we were to extend the model to realistically treat the passage of time, the list of possible diagnoses becomes extremely long and complicated, since each time path of medical and diagnostic history would have to be treated as a distinct event. An insurance plan that determines what happens in each case would have to be extremely elaborate. Consumers may find decision-making about such complicated alternatives too difficult to handle intelligently. Coinsurance presents a very easily stated rule determining the payments in each possibility. However, even if an insurance policy that pays different amounts for each possible diagnosis were too complicated to deal with, it should be possible to find simplified approximations to such plans that would not be unreasonably difficult to understand or administer.

A remaining question is whether diagnostic-specific health insurance is better than coinsurance in a world where there is moral hazard and adverse selection. This is an issue that deserves much more careful attention than we have time or space to deal with in this paper.

A separate issue from the question of coinsurance versus diagnostic-specific insurance is whether private competitive health plans would perform as well as governmentally imposed plans. If, for example, it is thought that private consumers typically do not think intelligently and objectively about health-related matters then a case exists for imposed solutions. Where there is moral hazard and adverse selection, little is known about the existence and welfare economic properties of competitive insurance market equilibrium. Thus it might be that in more realistic models, the case for private insurance is less good than in the model presented above.

There is a widespread view that existing private medical insurance plans, as well as medicare and medicaid, are deficient in their provision of "catastrophic health insurance". Private insurance plans typically place a ceiling on the total amount of payments they will make in a year. Private plans, medicare, and medicaid also typically limit the number of days of hospitalization for which they will pay. It seems intuitively appealing to think that health insurance plans that will not pay for the very expensive medical care that would accompany a catastrophic illness are not fulfilling the main function of insurance, namely pooling the risks of big losses for risk-averse consumers. Perhaps the main selling point of the national health insurance bills that have in recent times appeared in Congress, such as the Long-Ribicoff bill and the Kennedy-Mills bill, is the fact that both provide essentially unlimited coverage in the event of catastrophic illness.

The model presented here suggests that a case can be made that an efficient national health insurance plan should put some limits on the amount of medical care offered, even in the event of catastrophic illness. In fact, it is hard to resist making some wild guesses about how high such a ceiling might be. In particular, imagine an illness that one might get with some very small probability, π_i. Suppose that there are two feasible treatments of this illness. One treatment costs a negligible amount and leaves only a small probability that the patient will survive. A second treatment costs $\$c$ and is sure to cure the patient. How large can c be if the optimal medical strategy is to choose the second treatment? The per capita cost of providing all consumers with the second treatment rather than the first will be approximately $\$\pi_i c$. The gain in ex ante survival probability to each consumer from using the second plan rather than the first is just π_i. Thus the per capita cost per unit of survival probability added by choosing the second treatment is just $\$\pi_i c \div \pi_i = \$c$. An efficient medical strategy would choose the more expensive treatment if and only if c is less than the marginal cost at which survival probability can be increased by other means, such as improving the construction of highways, or reducing occupational hazard. These numbers should, in turn, be equal to individual willingness to pay for increased survival probability. Rosen and Thaler (1975) have estimated, from the wage premiums paid for occupational risk of death, a marginal rate of substitution between survival probability and wealth that amounted to between twenty and forty times the annual wage of the population studied. Thus according to their estimates, someone with an annual income of $20,000 should be willing to buy actuarially fair insurance against a relatively unlikely illness that costs $400,000 or possibly up to $800,000 to cure. The benefit ceilings on the existing medical plans that I have heard described are much lower than that. Of course, most medical treatments are not nearly as effective as the one we have just modelled. Typically the treatment will make only a small change in the conditional probability that one survives, and furthermore it may restore one not to full health but to some relatively unpleasant form of invalidism.

One feature of catastrophic illness that is not well described by the analysis so far is the fact that medical care not only takes the form of reducing the probability of death, but may take the form of easing the discomfort of prolonged convalescence or permanent disability. Treatment in such cases may be extremely expensive, and the absence of treatment may be extremely unpleasant. For illnesses of this kind, one might reasonably wonder whether, say, the feature of the current medicare plan that limits the number of days of hospitalization that will be paid for to 150 days is appropriate. The next section deals with this issue in the context of a more general model which allows other states of health besides "healthy" and "dead."

Invalidism and Nursing Care

In this section we consider a model with several different possible conditions of physical well-being. These include various conditions of survival with impaired health as well as death and full health. Again, we suppose that before medical treatments there are several possible medical diagnoses. The probability that someone with a particular diagnosis will reach a given condition of health depends on how much medical treatment he purchases. Formally, we let the possible states of health be denoted by $j = 0, 1, \ldots, \ell$, where states 0 and 1 are death and full health respectively. We let possible diagnoses be denoted by $i = 1, \ldots, n$. The probability that someone receives diagnosis i is π_i and the conditional probability that someone with diagnosis i and the amount, m_i, of medical care will arrive in health state j is $\theta_i^j(m_i)$. Therefore if someone adopts a medical strategy $M = (m_1, \ldots, m_n)$, the probability that he will arrive in health state j is

$$\Pi^j(M) \equiv \sum_{i=1}^{n} \pi_i \theta_i^j(m_i). \tag{33}$$

Suppose that one's utility depends on the state of his health and the amount of bread that he consumes. Let a bread consumption strategy consist of a vector $b = (b^0, b^1, \ldots, b^n)$ specifying the amount b^j of bread that an individual will consume if he arrives in each health condition j. The expected utility of a consumer with the consumption strategy (M, b) is then:

$$V(M,b) \equiv \sum_{j=0}^{\ell} \Pi_j(M) u_j(b^j). \tag{34}$$

As in our earlier discussion we can find necessary conditions for an optimal uniform national plan. Suppose that there is an initial endowment of $\bar{b}$ units of bread per capita and that an individual in health state j can produce h^j units of bread. Then the feasibility constraint for a uniform national plan is

$$\sum_j \Pi^j(M)(b^j - h^j) + p(\bar{M}(M)) = \bar{b} \tag{35}$$

(where $\bar{M}(M)$ is defined in (6).) Maximizing (34) subject to the constraint, (35), yields the following conditions:

$$u_j'(b^j) = u_1'(b^1) \text{ for } j = 1, \ldots, n \tag{36}$$

and

$$\sum_{j=0}^{\ell} \left[\left(\frac{u_j(b^j)}{u_1'(b^1)} \right) + h^j - b^j \right] \frac{d\theta_i^j}{dm_i} = p. \tag{37}$$

Since the conditional probabilities of the various outcomes must sum to one, it follow that:

$$\sum_{j=0}^{\ell} \frac{d\theta_i^j}{dm_i} = 0 \tag{38}$$

for each i.

Therefore (37) can be written equivalently as:

$$\sum_{j=1}^{\ell} \left[\frac{u_j(b^j) - u_0(b^0)}{u_1'(b^1)} (b^j - h^j - b^0 + h^0) \right] \frac{d\theta_i^j}{dm_i} = p. \tag{39}$$

for all i.

Where h^0 is assumed to be zero, it is clear that (39) generalizes (18).

Equations (36) and (39) can be given reasonable economic interpretations without a great deal of difficulty. Furthermore, results analogous to those in the previous sections on the sustainability of an optimum by actuarially fair insurance plans can be found. I think, however, that more insight can be gained by looking carefully at a very special case. In particular, suppose that we let the expected utility representation take the following form:

(40) $$u^0(b^0) = f(\beta_0 + b^0): \text{ where } \beta_0 > 0.$$

(41) $$u^1(b^1) = \alpha^1 + f(b^1): \text{ where } \alpha_1 > 0$$

(42) $$u^2(b^2) = \alpha^2 + f(b^2 - \beta_2) \text{ if } b^2 > \beta_2 \text{ and}$$

$$u^2(b^2) = f(0) \text{ if } b^2 \leqq \beta_2: \text{ where } \beta_2 > 0 \text{ and } \alpha^2 < \alpha^1.$$

These functional forms were chosen to crudely depict the following features of preferences. The α's represent a pure preference over health states, independent of consumption. The presence of the parameter β_0 in $u^0(\cdot)$ represents the notion that the needs of one's family for consumption goods are reduced if one dies. The particular representation chosen here is entirely for analytic convenience. The parameter β_2 in $u^2(\cdot)$ represents a cost of nursing care that is not needed by healthy people, but without which invalids would be extremely miserable. Once the amount of nursing care, β_2, is provided, the individual enjoys the same *marginal* (but not *total*) utility schedule for consumption as he would if he were healthy.

Assuming that $f''(b) < 0$ for all $b \geqq 0$, the first-order conditions in (36) together with the functional specifications in (40)–(42) imply that for the optimal consumption plan:

(43) $$b^0 = b^1 - \beta_0 \quad \text{and}$$

(44) $$b^2 = b^1 + \beta^2.$$

Then in an optimal plan it follows that

(45) $$u(b^0) = u(b^1) - \alpha_1 = u(b^2) - \alpha_2.$$

Thus the optimal plan provides the most consumption goods (including nursing care) in the event that the consumer becomes an invalid and the least in the event that he dies.

In the previous section, we offered the suggestion that it may be socially efficient to provide no health insurance for people who have ailments that are extremely expensive to cure. In such cases an efficient plan might allow those who contract such diseases to expire without expensive treatment. The critical level of costs in this case is the amount that the society is willing to spend on saving a life by other means.

The analysis of nursing care is rather different in character. In our model, nursing care has no effect on the probability distribution of health states; it simply makes the state of invalidism less unpleasant. So long as an individual can not choose immediate and painless death as an alternative to invalidism, the amount, β_2, which is efficient for him to spend on nursing care could quite possibly exceed the maximum amount that an efficient plan would spend on curing an illness. Thus the size of efficient "catastrophe insurance" benefits for nursing care might possibly be larger than is appropriate for medical care devoted to "curing".

Returning to our formal analysis, we find another, perhaps surprising, effect. Let us also suppose that the individual will have earnings, h, if he is healthy and zero otherwise. Let there be only one diagnosis and let $\theta^i(m)$ be the probability that one reaches health state i if he spends m on medical treatment. The condition in (39) then reduces to

$$\left[\frac{\alpha_1}{u_1'(b^1)} + (h - \beta_0)\right]\frac{d\theta^1}{dm} + \left[\frac{\alpha_2}{u_1'(b^1)} - (\beta_0 + \beta_2)\right]\frac{d\theta^2}{dm} = p \tag{46}$$

The left side of (46) is the marginal contribution of a unit of medical treatment to utility. This contribution comes in part from its effect on the probability of being healthy and in part from its effect on the probability of being an invalid. Each of these effects in turn consists of two components, a direct effect on utility and a "budget effect". The budget effects register the influence of a change in the probability distribution of health state on the expected value of consumption net of earnings. Typically one would expect the sign of the first term in brackets to be positive, indicating that when both direct and budget effects are accounted for one would prefer to have a higher probability of being healthy and a lower probability of being dead. The sign of the second bracketed term could reasonably be either positive or negative, depending on how unpleasant and how costly it is to be an invalid. It is interesting to notice that even where $\alpha_2 > 0$ so that one prefers the prospect of being an invalid (with the consumption assigned by an optimal plan) to the prospect of being dead, the second bracketed term in (46) could be negative. If this were the case, a medical treatment that increased the probability of being an invalid and lowered the probability of dying, without changing the probability of being healthy, would be socially undesirable even if it were free. The reason, of course, is that in this model being an invalid and being cared for at the efficient level is much more expensive than being dead. Thus if the extra pleasure from the prospect of being a well-cared-for invalid rather than being dead is not large, an investment in achieving this status may not be worthwhile.

Appendix

Proof of Proposition 1

We will prove Proposition 1 with the aid of two lemmas which are in themselves of some general interest. We will need just a bit of additional definitional structure. A "convex symmetric economy" is defined as follows. Let there be m consumers and a set F of feasible allocations. Where $(x_1, \ldots, x_m) \varepsilon F$ is an allocation, x_i is the consumption bundle consumed by consumer i. All consumers, i, have identical concave utility functions $u(x_i)$. The set F is convex and symmetric, where by symmetric we mean that if $(x_1, \ldots, x_n) \varepsilon F$, then $(x_1', \ldots, x_n') \varepsilon F$ if $(x_1', \ldots, x_n')$ can be obtained from $(x_1, \ldots, x_n)$ by reassigning the same commodity bundles to different individuals. (More formally, if $(x_1, \ldots, x_n) \varepsilon F$ and $x_i' = x_{\pi(i)}$ for all i where $\pi(\cdot)$ is a permutation on the set of consumers, then $(x_1', \ldots, x_n') \varepsilon F$.)

An "optimal uniform allocation" is an allocation $(\bar{x}, \bar{x}, \ldots, \bar{x}) \varepsilon F$ that maximizes $u(x)$ subject to the constraint $(x, x, \ldots, x) \varepsilon F$. A "Pareto optimal allocation" is an allocation $(x_1, \ldots, x_n)$ such that there exists no $(x_1', \ldots, x_n') \varepsilon F$ such that $u(x_i') \geqq u(x_i)$ for all i with strict inequality for some i.

Lemma 1

In a convex, symmetric economy, an optimal uniform allocation is Pareto optimal.

Proof:

Let $(\bar{x}, \ldots, \bar{x})$ be an optimal uniform allocation and suppose that for $(x_1^*, \ldots, x_n^*) \varepsilon F$, $u(x_i^*) \geqq u(x_i)$ for all i with strict inequality for some i. Define $\bar{x}^* \equiv \frac{1}{m} \sum_{i=1}^{n} x_i^*$. Since F is symmetric and convex, it follows that $(\bar{x}^*, \ldots, \bar{x}^*) \varepsilon F$. But since $u(\cdot)$ is a concave function, $u(\bar{x}^*) \geqq \frac{1}{n} \sum_{i=1}^{n} u(x_i^*) > u(\bar{x})$. But this contradicts the hypothesis that $(\bar{x}, \ldots, \bar{x})$ is an optimal uniform allocation. It follows that $(\bar{x}, \ldots, \bar{x})$ is Pareto optimal.
Q.E.D.

If the utility function $V(M, B)$ in the text of our paper were concave and the set of feasible allocations were symmetric and convex, then Proposition 1 would be immediate from Lemma 1. Fortunately (for those who like tricky proofs) matters are not quite this simple. The function $V(M,B)$ is generally not concave under reasonable assumptions, nor is the set of feasible allocations as described in Proposition 1 a convex set. However, by a judicious transformation of variables one can demonstrate the equivalence of the economy of Proposition 1 to a convex, symmetric economy. Using this equivalence, we show that Proposition 1 follows from Lemma 1. In order to accomplish this program we will also need the following result.

Lemma 2

Let $u(x)$ be a concave function with domain the nonnegative orthant in Euclidean n space. For any positive scalar y and any vector x in the domain of $u(\cdot)$, define $V(y,x) \equiv yu(\frac{x}{y})$. Then $V(y,x)$ is a concave function on its domain.

Proof:

For λ between zero and one,

$$\begin{aligned}
&V(\lambda y + (1-\lambda)y', \lambda x + (1-\lambda)x') \\
&= (\lambda y + (1-\lambda)y')u\left(\frac{\lambda x + (1-\lambda)x'}{\lambda y + (1-\lambda)y')}\right) \\
&= (\lambda y + (1-\lambda)y')u\left(\frac{\lambda y(\frac{x}{y}) + (1-\lambda)\ y'\ (\frac{x'}{y'})}{\lambda y + (1-\lambda)y'}\right) \\
&\geqq (\lambda y + (1-\lambda)y')\left[\left(\frac{\lambda y}{\lambda y + (1-\lambda)y'}\right)u(\frac{x}{y}) \right.\\
&\left. + \frac{(1-\lambda)y'}{\lambda y + (1-\lambda)y'}\, u\,(\frac{x'}{y'})\right] \\
&= \lambda yu\,(\frac{x}{y}) + (1-\lambda)y'u\,(\frac{x'}{y'}) \\
&= \lambda V(y,x) + (1-\lambda)V(y',x').
\end{aligned}$$

The resulting inequality proves the lemma.
Q.E.D.

Proof of Proposition 1

We make a change of variables by defining a one-to-one transformation T that maps the set of feasible consumption strategies for each individual into its image as follows:

$$T(M,B) = (Y,Z)$$

where

$$(Y,Z) \equiv (y_1, \ldots, y_n, z_1^0, \ldots, z_n^0, z_1^1, \ldots, z_n^1)$$

with:

(A.1) $$y_i \equiv \pi_i \theta_i(m_i)$$

(A.2) $$z_i^0 \equiv \pi_i(1 - \theta_i(m_i))b_i^0$$

and

(A.3) $$z_i^1 \equiv \pi_i \theta_i(m_i)(b_i^1 - h).$$

To see that T is one-to-one we notice that (for predetermined $(\pi_1, \ldots, \pi_n)$ and h) the equations A.1–A.3 can be inverted to solve uniquely for the m's and b's in terms of the y's and z's. In particular, we have:

(A.4) $$m_i = \theta_i^{-1}\left(\frac{y_i}{\pi_i}\right)$$

(A.5) $$b_i^0 = \frac{z_i^0}{\pi_i - y_i}$$

(A.6) $$b_i^1 = \frac{z_i}{y_i} + h$$

Our objective is now to show that where we describe the economy constructed in the text of the paper in terms of the transformed variables, we have a convex, symmetric economy. Using equations (5) of the text and A.4, A.5 and A.6, we can write:

(A.7) $$V^*(Y,Z) \equiv V(T^{-1}(Y,Z)) = \sum_{i=1}^{n} y_i u_1\left(\frac{z_i^1}{y_i} + h\right) + \sum_{i=1}^{n} (\pi_i - y_i) u_0\left(\frac{z_i^0}{\pi_i - y_i}\right).$$

From the assumption that u_1 and u_0 are concave functions, from Lemma 2, and from the fact that the sum of concave functions is concave, it follows that $V^*(Y,Z)$ is a concave function.

We next rewrite the feasibility constraint of Proposition 1 in terms of the variables (Y,Z). Using (A.4)–(A.6) and slightly rearranging terms we can describe the feasible set as follows. A typical allocation is denoted $(Y^1,Z^1, \ldots Y^m,Z^m)$ where consumer k's utility is $V^*(Y^k,Z^k)$. The set of feasible allocations is:

(A.8)
$$F^* = \{(Y^1,Z^1, \ldots, Y^m,Z^m) \mid \sum_{k=1}^{m} \sum_{i=1}^{n} \left(z_i^1 + z_i^0 + p\pi_i\theta_i^{-1}\left(\frac{y_i}{\pi_i}\right)\right) \leqq m\bar{b}\}.$$

Clearly F^* is a symmetric set. Since $\theta_i(\cdot)$ is assumed a concave function for each i, $\theta_i^{-1}(\cdot)$ must be a convex function. It is then easy to show that F^* is a convex set.

The economy in which consumers have utility functions $V^*(Y,Z)$ and in which the set of feasible allocations is described by A.8 must therefore be a convex, symmetric economy which we will call the "derived economy." If $(\bar{M},\bar{B})$ is an optimal uniform national plan, then $V(\bar{M},\bar{B}) \geqq V(M',B')$ if the allocation in which all consumers have M', B' is feasible. Where $(\bar{Y},\bar{Z}) = T(\bar{M},\bar{B})$ it follows from our definitions that $(\bar{Y},\bar{Z})$ is an optimal uniform allocation in the derived economy. From Lemma 1 it follows that $(\bar{Y},\bar{Z})$ is also Pareto optimal for the derived economy. But it is then easy to show that $(\bar{M},\bar{B}) = T^{-1}(\bar{Y},\bar{Z})$ must be Pareto optimal for the original economy.
Q.E.D.

Notes

1. Here and subsequently we proceed as if the proportion of the population having each diagnosis takes on exactly its expected value. Of course the "law of large numbers" tells us only that this proportion comes arbitrarily close to its expected value with arbitrarily high probability for a large enough population. If we were to pursue this more accurate representation carefully, we would find that for large populations, the residual social risk can be shared, so as to have negligible effects on individual utilities. Thus our shortcut has a negligible effect on the results, but much eases exposition.

2. Useful discussions of a variety of existing and proposed private and public health plans can be found in Davis (1975) and Reed and Cass (1970).

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