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Volume Author/Editor: Frederick R. Macaulay

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CHAPTER V

BOND YIELDS, ECONOMIC 'DRIFT', AND THE PRICES OF COMMON STOCKS

In the preceding chapter we discussed the relation of the movements of bond yields to the grade of the bonds. At one stage of that discussion we introduced a scatter diagram containing not only bond yields but also the yields of preferred stocks. To go one step further and bring into the picture common stocks, the most junior securities of all, leads to some interesting results. But, before presenting those results, it is desirable to refresh the reader's memory of the nature of a corporation and the relation of bondholders and stockholders to the corporation and to each other.

A corporation is a juristic person; in the words of the Dartmouth College Case it is "an artificial being, invisible, intangible and existing only in contemplation of law". It is capable of acting in many but not in all respects as though it were a natural person. As a legal entity, it takes and holds property, and conveys the same; it contracts obligations, and it sues and is sued, in its corporate name, in the same manner as a natural person. For such purposes, the stockholders or 'members' of the corporation are disregarded. They compose the corporation, but they are not the corporation.[1]

Though the stockholders may be thought of as owning the corporation, they do not own and cannot convey the corporate property.[2] But

[1] We must not obscure our present discussion by overloading it with exceptions and qualifications. For example, it is, of course, true that the concept of the corporation as a legal entity, separate and distinct from the members who compose it, is a mere legal fiction, introduced for the convenience of the corporation and of those who do business with it; and that, under certain circumstances, the fiction will be disregarded, and the fact that the corporation is really a collection of individuals be recognized in equity and even at law.

[2] This is technically true even when all the stock is owned by one person. While the

they have a definite beneficial interest in the corporate property, as may be seen from the fact that the courts have held that interest insurable. It is true that, upon dissolution, the legal title to the corporate property does not vest in the stockholders, but they still retain the beneficial interest therein, and, if the legislature has made no provision by which they can reach the property, and enforce their rights, they may come into a court of equity, and obtain relief. Such a court has jurisdiction, unless it has been taken away by statute, to reach the property of the defunct corporation, to cause the debts due it to be collected, and to distribute the assets, after payment of the creditors, to the beneficial owners, that is, to the members or stockholders.

The stockholders' beneficial interest in the corporate property extends to the profits of the corporation. A stockholder has the right not only to participate in dividends when they are declared but also to maintain a suit in equity to compel the directors to declare and pay a dividend if it is wrongfully withheld.

The bondholder is a creditor of the corporation. His claims are prior to those of the stockholder but they are limited and he has no vote nor, so long as the corporation is living up to its promises, has he any means of controlling the management. But, aside from the voting privilege, the interests of the bondholder and the stockholder in the corporation differ fundamentally only with respect to priority and limitation. The corporation is a cow from which each hopes to obtain milk.

And priority and limitation are not confined to bonds. Though the preferred stockholder may be classed by the law with the common stockholder he will tend to be classed by the economist with the holder of an income bond. And even the law has been known to waver in its discussion of preferred stocks and preferred stockholders. For example, some courts have held that a corporation has the power to create and issue preferred stock on the ground that such a transaction is virtually a borrowing of money, and that corporations have the power to borrow money, and may do it in this way.

The common stock is a blood brother of the preferred stock and the bond. Investments as a class constitute one family. They each originate in an exchange of present money for an expectation of future

(Footnote [2] concluded)
corporation exists, he is a mere stockholder of it, and nothing else. Cf. Button v. Hoffman, 61 Wisconsin.

money. If it were not for such an expectation they would have no exchange value. And they lose that value as the expectation dies out. The demand that comes from the possibility of buying them and later selling at a profit may exist for a time in a sort of economic vacuum, but it is essentially a derived demand and, in the absence of any (warranted or unwarranted) expectation of future returns, it sooner or later disappears.

Because the good that the common stock offers to its purchaser is an expectation of future money payments, the relation of its present-money price to its future-money payments is as unmistakably an interest phenomenon as is the relation of the present-money price of a bond to its future-money payments. In the fullness of time the stock will have a 'realized' or 'actual' yield just as will the bond. And, though the stock makes no 'promise', as does the bond, and therefore has no 'promised' or 'hypothetical' yield, its price discounts *estimated* future payments as truly as does the price of the bond.[3] It is the absence of promises and the high degree of uncertainty as to what the stock will pay, with the resulting inadequate forecasting, that obscures the interest relation. The fundamental difference between an ultra high grade extremely long term bond and a low grade common stock is that the future-money returns of the bond can be forecast with more assurance than can those of the stock. That an individual investor may prefer the one type of security to the other, because he believes that their market prices do not correctly measure the differences in their respective future probabilities, is a matter that concerns individual *forecasting* of future payments but neither individual nor market *discounting* of those payments.[4]

The promises contained in the bond are not merely assurances that, if possible, certain sums will be paid. They are also maximum values

[3] For discussion of the terms *assumption of payment, promised or hypothetical yield*, and *realized or actual yield*, see Chapter II. In this book the term *yield*, unless the context makes it apparent that *realized or actual yield* is referred to, means *promised or hypothetical yield*.

[4] For example, that bonds are almost inevitably a poorer investment than common stocks on the eve of a great currency inflation is evidence merely of bad market forecasting of how the inflation will affect the future money payments of the stocks. The rates at which future money is, at such a time, being discounted may be high or low, but, because they apply to the *anticipated* money payments of *both* the stocks and the bonds, they cannot explain why one is a better investment than the other. It is the error in the *anticipation* which explains that.

that cannot be exceeded. This is, of course, true of preferred stocks also. If the corporation is earning enough, the maximum values will year after year be the realized values. And this limitation leads to stability—the stability of a toy balloon pressing against the ceiling of a room. If the gas begins to escape, the balloon may drift down to the floor. But, only if it were outdoors could it ever soar to the clouds. There are upper and lower limits to the return from a bond. The return from a common stock has no necessary upper limit. Dividends may soar to the sky, but their only permanent level of stability is on the ground—with the corporation bankrupt.

The 'assumption of payment', which must be made before the promised or 'hypothetical' yield of a bond can be calculated (or obtained from a bond table), may, as we have seen, be a mere mathematical fiction for all except the highest grade of bonds. But, for common stocks it is not only a mathematical fiction but also an economic absurdity. Even if the chance that the promises contained in a bond will be kept is so negligibly small that the promises are little more than mere words, they are at least *definite* words and, as such, can stand the strain of mathematical manipulation.

But the common stock contains no promises. It is, of course, true that after sale, in other words *after the event,* the realized or 'actual' yield may be calculated for the period that the stock was held *and for the ex-holder* with the same assurance that it could have been calculated had the stock been a bond,[5] but the promised or 'hypothetical' yield completely eludes definite determination unless assumptions are introduced that are so grossly conjectural as to be virtually supposititious.

And even small variations in the assumptions affect greatly the hypothetical yields. The assumptions must extend far into the future. Theoretically, they must specify *exactly* the amounts and dates of *all* future payments. Of course, practically it is not necessary to take into account those payments that are to occur in the far distant future. But the future period that may be neglected is much more distant than the reader might imagine had he not carefully considered the matter. If it be assumed that a share of common stock selling for $100 is to return $4 per annum *forever,* it may be thought of as having a promised or 'hypothetical' yield of 4 per cent per annum. But, if the pay-

[5] *This* realized yield is, of course, not to be confused with the realized yield on a bond *that has been held to maturity—or permanent default.*

ments are to cease at the end of sixty years, the hypothetical yield must be less than 3½ per cent per annum.[6] If they are to cease at the end of 46 years, the yield must be less than 3 per cent per annum. If at the end of 35 years, the yield must be less than 2 per cent per annum. If they continue just 25 years, the yield will be exactly zero per cent per annum. With still shorter periods, the yields are negative.

If such an assumption were made as that the dividend payments were to increase in geometric progression, the future that could be neglected would be still more distant. One of the strangest rationalizations of unending price rise that appeared in the months immediately preceding the stock market culmination of 1929 was evolved by a Wall Street economist. He presented to the directors of the investment trust with which he was associated statistical evidence that the wealth of the country increased in the long run about 3 per cent per annum. He then argued that corporations as a class should be expected to share in this growth at this rate and hence that their dividends should be expected, over the long run, to increase at least 3 per cent per annum; that is to say in such a series as \$4.12, \$4.24, \$4.37, etc., or $\$4(1.03)$, $\$4(1.03)^2$, $\$4(1.03)^3$, etc. He then suggested that, with increasing financial stabilization of the country, these future dividends would eventually be discounted at a rate that would not exceed 3 per cent per annum. But, he continued, if distant enough payments were assumed, discounting them at this rate would give very high prices for the stocks. The suggestion was even made that, as there seemed to be no necessary time limit to the 3 per cent rate of growth in wealth, there should logically be no 'ceiling' whatever for stock prices.[7] The phantasy

[6] The present values of the future payments, discounted at 3½ per cent per annum, are $\$\frac{4}{1.035}$, $\$\frac{4}{(1.035)^2}$, $\$\frac{4}{(1.035)^3}$, etc. Now, from any ordinary compound interest and annuity table we may find that the sum of 60 terms of this series equals \$4(24.9447) or about \$99.78. Therefore, if the stock is selling at \$100, the yield is a little less than 3½ per cent per annum.

[7] If the dividends were $\$4(1.03)$, $\$4(1.03)^2$, $\$4(1.03)^3$, etc., as in the illustration of the text, and if these dividends were discounted at 3 per cent per annum, the price of a share of the stock that was to pay the dividends should be just four times the *number of payments* that were to be made; in other words, four times the *number of years* that the succession of dividends was to continue, even if nothing whatever was to be paid thereafter. The *present value* of each future dividend payment is \$4.

was strangely reminiscent of the Petersburg Paradox in the mathematical theory of probability.

On the other hand, it is of course true that, in pricing stocks, the market undoubtedly attempts to estimate their earnings and dividend probabilities for at least the near future. To the investor, past earnings and dividends, except in so far as they offer clues to the future, are mere 'water over the dam'. Dividing last year's dividends by the present selling price of a stock will produce a function that, for certain very restricted purposes, may be of some interest to the student of economic history and the business cycle; but it is highly undesirable to call the function a 'yield' and thus, by the use of terms, insidiously to suggest that it is of the same nature as the ('hypothetical') yield of a bond.

Though the terms of a bond's promise of future money payments change with the passage of time (as the maturity date approaches), they change in a slow and definite mathematical manner. Unless the time to maturity be very short, or the dates for which comparisons are made be very far apart, no great change in the yield of a bond can occur without a change in price. For short periods, the yield of a long term bond is virtually a function of *one* variable—the price of the bond. But the 'yield' of a stock is essentially a function of *two* variables. This difference is clearly apparent if we compare the 'yields' of stocks with the yields of perpetual bonds. The (hypothetical) yield per cent of Canadian Pacific debenture 4's (bonds that are perpetuities) will always necessarily be 400 divided by the price of a $100 bond. But the 'yield' per cent of Canadian Pacific common stock, whether it be taken as (100 times) the preceding year's total dividends divided by the present price of the stock or as some more complicated expression, will always be essentially a function of *two* variables—a fluctuating price and a fluctuating dividend rate. Sudden and great changes in the calculated 'yields' of a stock occur not only because of changes in price but also because of changes in dividend rate.

To compute either bond yields or stock yields, assumptions must be made concerning future payments. The computer of stock yields usually assumes that payments in the future will be at the same annual rate as they were in the immediate past. Having made this totally unwarranted assumption, he plunges into refinements. If an 'extra' or unexpected dividend be paid, he labors like the mountain to decide whether it is truly an 'extra' or should be considered as normal and 'regular'. If his decision whether it will or will not be regularly repeated turns out to be wrong, he 'revises' his preceding year's index of yields.[8] And if the yields, as he calculates them, seem abnormally high or abnormally low, he tacitly suggests that future 'revisions' will probably correct his figures. He speaks of 'real' yields, meaning by the term what his yields will be when the future is known and all his 'revisions' completed. And all most seriously.[9]

The market's valuation of both second grade bonds and common stocks may, for the purpose of throwing light on our present problem, be thought of as though it were a process of consciously forecasting interest or dividend payments and then discounting them at some particular rate of interest.[10] The (hypothetical) yield of a second grade bond is then high, not because the promised future interest payments are discounted at a high rate but because of the low degree of expectation that the payments that will actually be made will be as great as the promised payments. It is as though the expected future payments, which for a high grade bond are taken as identical or virtually identical with the promised payments, are for a second grade bond taken as only

[8] One of the largest financial 'services' in the United States writes: "On occasional instances, a consistent handling of the situation is impractical and arbitrary decisions must be substituted. This may at times necessitate revising a part of the recent data."

[9] Sometimes the 'yields' of stocks seem, to their computers, abnormally low or abnormally high on almost any of the various bases used for calculating them; for example, in September 1929. However, even at that time the fact that prices were unusually large multiples of immediately preceding dividends was accepted only as evidence and not as proof that 'real' yields were abnormally low. The mooted question was whether earnings could increase sufficiently in the future to carry such extremely high prices. If the immediate future of earnings had been known, it would undoubtedly have been generally realized that stock prices were relatively even higher than their 'yields' suggested. Dividends had been increasing, and the fall in 'yields' was therefore not as great as the rise in prices. If 'yields' could have been calculated on the basis of the dividends to be paid in the not too distant future instead of those paid in the immediate past, they would have fallen even more than prices rose.

[10] What a business is earning is, of course, more fundamentally important than what it is currently paying. If a corporation be earning much more than it is paying out in dividends, the stock will tend to sell on a relatively low 'yield' basis. When conditions are reversed, the stock will tend to sell on a high 'yield' basis. The ratio of price per share to earnings per share is a function that has been calculated and presented in much the same manner as the stock 'yield' concept. It has almost all the technical drawbacks of the yields concept with the additional one that earnings are never as accurately known as dividends.

some proper fraction of what is promised. The yield of the second grade bond is high because of the difference between the expected and the promised payments. But in so far as the analogy can be carried through at all, the 'yield' of a common stock is high because of the difference between the expected payments and the rate at which payments have recently been made. But recent rates of payment, though they may in exceptional circumstances turn out to be the same as future rates, are not the same as and cannot be substituted for promises.

If such an illegitimate substitution be made, we are faced with the difficulty that, when the 'promises' are actually broken, the 'yield' will *fall* violently; and, even after adjustment to the new conditions, may be no higher than before the promises were broken. If, at the time that the dividend payments on a stock paying $8 per annum are about to be cut to $4 per annum, the stock be selling for $100 a share, its yield will *fall* immediately after the cut—unless the price instantly declines to $50 a share. And, unless the price falls *below* $50 a share, the new 'yield' will be no higher than the 'yield' before the dividend cut. When dividends are completely eliminated, 'yields' immediately fall to zero. Though the 'yield' of a stock may have been, before the elimination of dividends, as high as the yields of very low grade bonds, after the elimination it becomes lower than the yields of bonds of even the very highest grade. It seems clear that, whatever else it may be, a (hypothetical) stock 'yield' is not an animal of the same species, or even genus, as a (hypothetical) bond yield.

Not only economists and statisticians but also investors and business men, when comparing the market's valuation of two bonds, tend to express that valuation in terms of the yields of the bonds rather than in terms of their conventional 'prices'. Though conventional 'price' is a price and the yield is only a function of a price, the price of which the yield is a function is, to the extent that there is warranty for 'the assumption of payment', a more expressive, enlightening, and pertinent price than the merely conventional price. Indeed, the conventional price of even a high grade bond is, *singly and by itself*, almost destitute of meaning. The quantity of the 'good' to which the dollars of the price are related is not adequately specified. The price is, by convention, the price per $100 of 'face value'. But 'face value' tells us only the amount of the last payment. It says nothing about when that payment is to be made or about the amount and timing of other payments that are to be made in the interim. And these facts are essential. As a unit of the good, $100 of 'face value' not merely is inadequate but also may be completely meaningless. Its *reductio ad absurdum* occurs with perpetuities—in which 'face value' is not even promised.

It is true that the price of which the yield is most simply and directly expressible as a function is analogous to the price of money in terms of a commodity rather than the price of a commodity in terms of money. But this inverse relationship leads to no difficulties. To state that the yield of a bond is 5 per cent per annum is to state that the relation between the *conventional* price of the bond and the promised future payments is *as it would be if* a promised payment of $105 due any time before maturity were worth $100 payable one year earlier. As a corollary, we have the less general conclusion that the price which, minus unity, equals the yield of the bond is a price of a unit of *present* money in terms of promised money due one year hence.[11]

For comparing the market's valuation of *two* bonds, conventional prices are, *by themselves,* quite useless. But they may give us some information if the comparison is between the market's valuation of a bond at one date and its valuation of *the same bond* at another date. No necessary conclusion can be drawn from the fact that, even on the same date, one bond sells for 90 and another bond for 110. But if, at one time, a bond sells for 90 and, at another time, for 110, we know at least that the yield was lower at the 110 price than it was at the 90 price. A change in yield must occur whenever a bond crosses par. Also, a change in yield must occur whenever, from an earlier to a later date, the plus or minus deviation of the price of a bond from par increases absolutely (not algebraically). And, if the bond be a perpetuity, we are not restricted to such vague and special non-quantitative conclusions.

Because the importance of the final or 'face value' payment decreases with an increase in time to maturity, comparison of the market's valuation of *two* bonds by means of their mere conventional prices (per $100 of 'face value') reaches its *reductio ad absurdum,* as we have noticed, when there is to be no final payment, in other words when the bonds are perpetuities. However, if the comparison is of the market's valua-

[11] This is all, of course, 'as if'. Unless we make the arbitrary and unreal assumption of uniformity in the rates of discount used in each future compounding interval, yield must be considered a mere average (see Ch. II).

tion of a perpetuity at one date and its valuation of *the same perpetuity* at another date, conventional price tells exactly the same story as yield. If yield halves, price doubles, etc. The yield of a perpetuity is merely a constant multiple of the reciprocal of the conventional price of the perpetuity.

The advantage of knowing and using not only the conventional price but also this multiplier, in other words the advantage of using yield, when comparing the market's valuation of the same perpetuity at different times, is that the yield gives an indication of *grade* and the conventional price (by itself) gives none. Our discussion (Ch. IV) of the relation of movement of yield to yield itself (as an index of grade) would have been impossible in terms of mere conventional prices, even if all our bonds had been perpetuities. But conventional prices, though they would not have related movements to grades, would, if all the bonds had been perpetuities, have told us just as much about the movements themselves as did yields.

Strictly speaking, common stocks have no *grade* in the sense in which the term is used of bonds, the sense in which, other things being equal, highness of grade shows itself in lowness of yield. The 'grade' of a bond depends upon the capacity of the issuing corporation to fulfill the promises contained in the bond. Yield may be low and grade high because of the greatness of the corporation's capacity or because of the smallness of its promises. If the most senior issue of a corporation be small enough, the bonds may remain 'first grade' throughout a receivership. But common stocks are most commonly described as 'high grade' when their dividends have been large and regular or even increasing, and low grade when they have been nil, small and irregular or decreasing. The 'yield' of the stock has little or no relation to this concept of grade. Price is its real indicator. The poorest grade of common stocks will have the lowest 'yields'—namely zero; and the 'yields' of all common stocks will be related to the market's estimates of the rosiness of their dividend possibilities rather than to the degree of assurance with which those dividends can be forecast. A prospect of increasingly large dividends will lower the 'yield' of a common stock, but a bond whose coupons called not for uniform but for increasing payments would not, because of that fact, sell on a lower yield basis.

But, though common stocks cannot be classified as to grade in the same way that bonds can be, their place in the investment family, to which bonds and preferred and common stocks all belong, may be described in terms of a concept that is at least related to the concept of bond grade. This concept is the concept of *priority*. If there be two bonds of the same corporation that differ only as to their *seniority*, the more senior bond will be of a higher 'grade' than the less senior, because its claim to payment will be *prior* to that of the less senior bond. And, if the corporation have preferred stock outstanding, that stock will (except in very unusual circumstances) be of a lower 'grade' than any of the corporation's bonds. In this sense, the common stock will be of a still lower grade. The claims of all the bonds will be *prior* to those of the preferred stock and the claims of the latter *prior* to the claims of the common stock.

Of course, such a classification by *priority*, embryonic as it is, is strictly possible only with the securities of a single corporation. But the fact that common stocks are junior to the other securities of their own corporations and hence, as a group, more uncertain as to their future payments than bonds suggests that the fluctuations in the valuations placed upon them by the market should be expected to be more like those of low grade than those of high grade bonds. And they are so. There are, of course, exceptions to this generalization. Some common stocks act more like high grade than really low grade bonds. But the generalization holds of common stocks *as a group*. Our future discussion therefore runs in terms of index members and not in terms of individual stocks. And the index numbers are, of course, index numbers of stock *prices*, and not index numbers of stock 'yields'. The concept of stock 'yield' is, as we have seen, quite useless for our purposes.

The stocks whose prices are used in this book are all American railroad stocks. Our decision to restrict ourselves to railroad stocks was not made merely because of the relationship between railroad stocks and railroad bonds. It is true that, in recent years industrial and public utility stocks have been relatively more important in the American speculative and investment markets than railroad stocks. But railroad stock prices present not merely a much more homogeneous but also a much longer series.[12] Our railroad stock price indexes go back monthly

[12] Cf. Wesley C. Mitchell, *Business Cycles*, pp. 170, 171:

"The number of industrial stocks regularly bought and sold on the market in every year since 1890 is too small to make significant averages."

While it would have been possible, from Boston quotations, to construct index to January 1857—a period of seventy-nine years. Until about 1909 the railroad stock market was the American security market par excellence. Only in the last twenty years or so has its relative importance seriously declined.

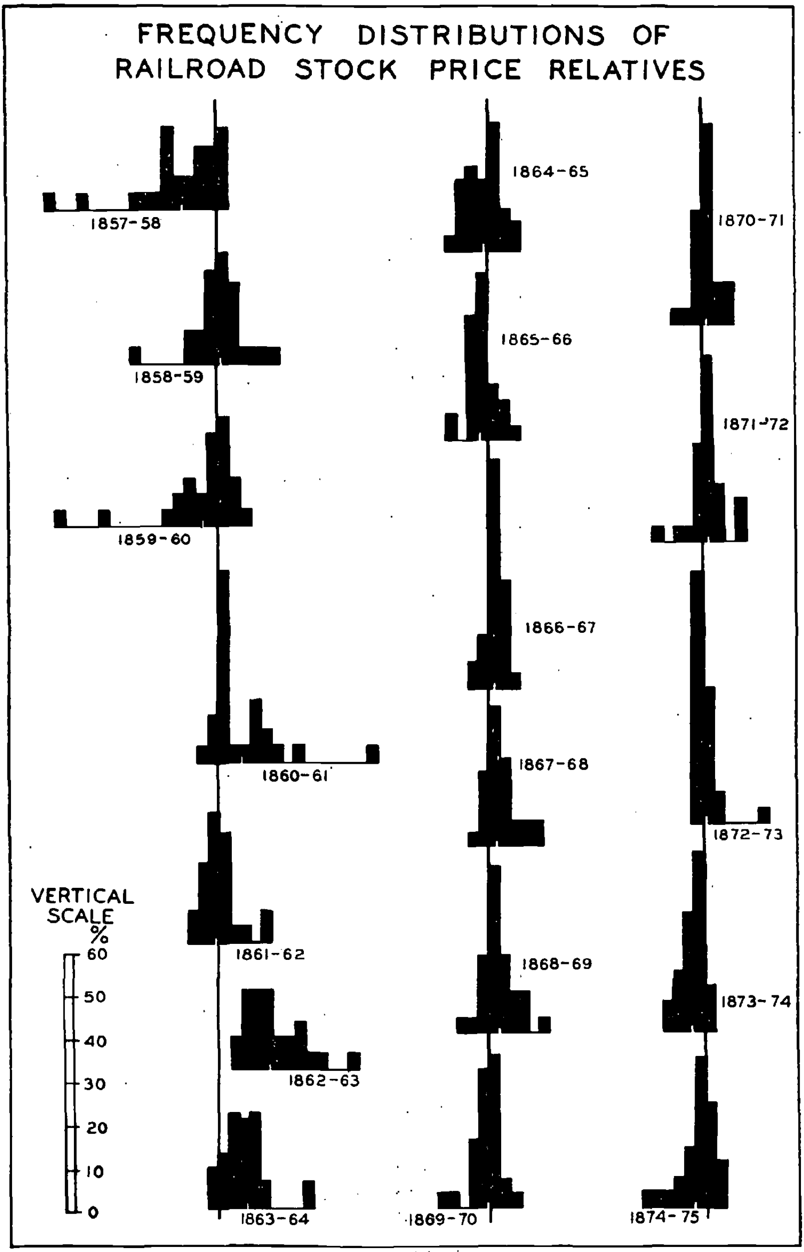
We have not used the prices of preferred stocks. The number of railroad preferred stocks paying dividends regularly and having their prices quoted has always been so small as to prevent an index of their prices having any broad general significance. And, whether they paid dividends or not, it seemed undesirable to mix them with the common stocks.

The list of stocks used includes, at all times, virtually all the railroad common stocks whose prices were being regularly quoted on any of the great exchanges. The names of the railroads and the periods during which their stocks were used is graphically exhibited in Chart 32. Most of the changes in the list were made necessary or desirable by consolidations of one kind or another. For example, stocks No. 21 (New York Central) and No. 22 (Hudson River) were used from January 1857 to January 1870, and stock No. 23 (New York Central and Hudson River) from January 1870 to January 1936. However, if a stock became so inactive as to lack quotations for many months at a time, it was usually dropped from the list. In some instances stocks were not used while the railroad was in the hands of a receiver and undergoing reorganization. In a very few instances they were not used during a period in which exceptional circumstances only negligibly related to their investment values were violently disturbing their prices. Thus Northern Pacific was not used during the year of the corner (1901). Sometimes it was possible to bring a stock back into the list. For example, Northern Pacific was brought back in January 1906.

(Footnote [12] concluded)
numbers of the prices of copper mining stocks or cotton mill stocks back as far as January 1857—the date we begin our railroad indexes—few persons would claim that such indexes would have anything approaching the general economic significance of an index of prices of the railroad stocks. And their relation to the yields of railroad bonds—which are the bonds we have used—would be difficult to interpret.

The monthly 'index of industrial stock prices' of the New York Federal Reserve Bank runs from January 1872 to date. But it is, in its early years, composed entirely of transportation, communication and mining stocks. The stocks used in 1872 were: Adams Express, American Express, Consolidated Coal, Delaware and Hudson Canal, Maryland Coal, Pacific Mail, Quicksilver Mining, Quicksilver Mining, pfd., United States Express, Wells Fargo Express, Western Union.

CHART 12


FREQUENCY DISTRIBUTIONS OF
RAILROAD STOCK PRICE RELATIVES
1857-58
1858-59
1859-60
1860-61
1861-62
1862-63
1863-64
1864-65
1865-66
1866-67
1867-68
1868-69
1869-70
1870-71
1871-72
1872-73
1873-74
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VERTICAL
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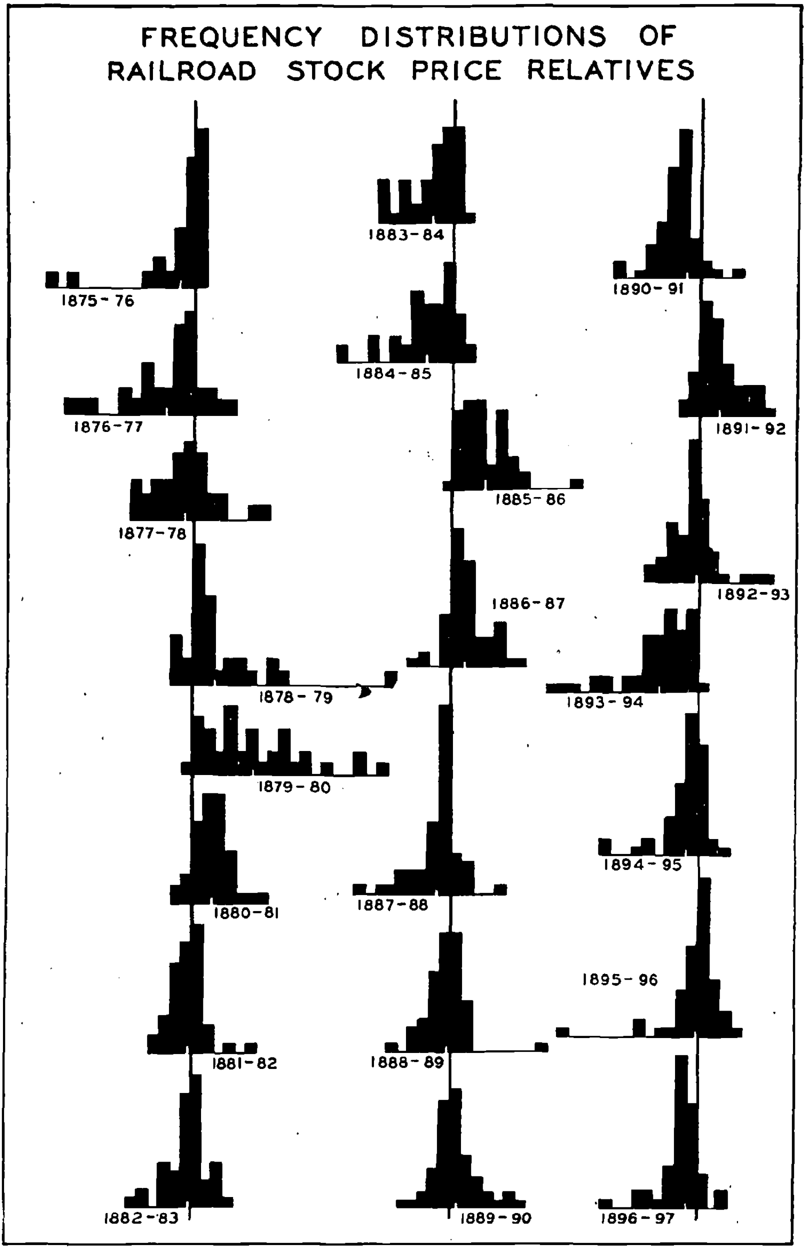
CHART 12-CONTINUED



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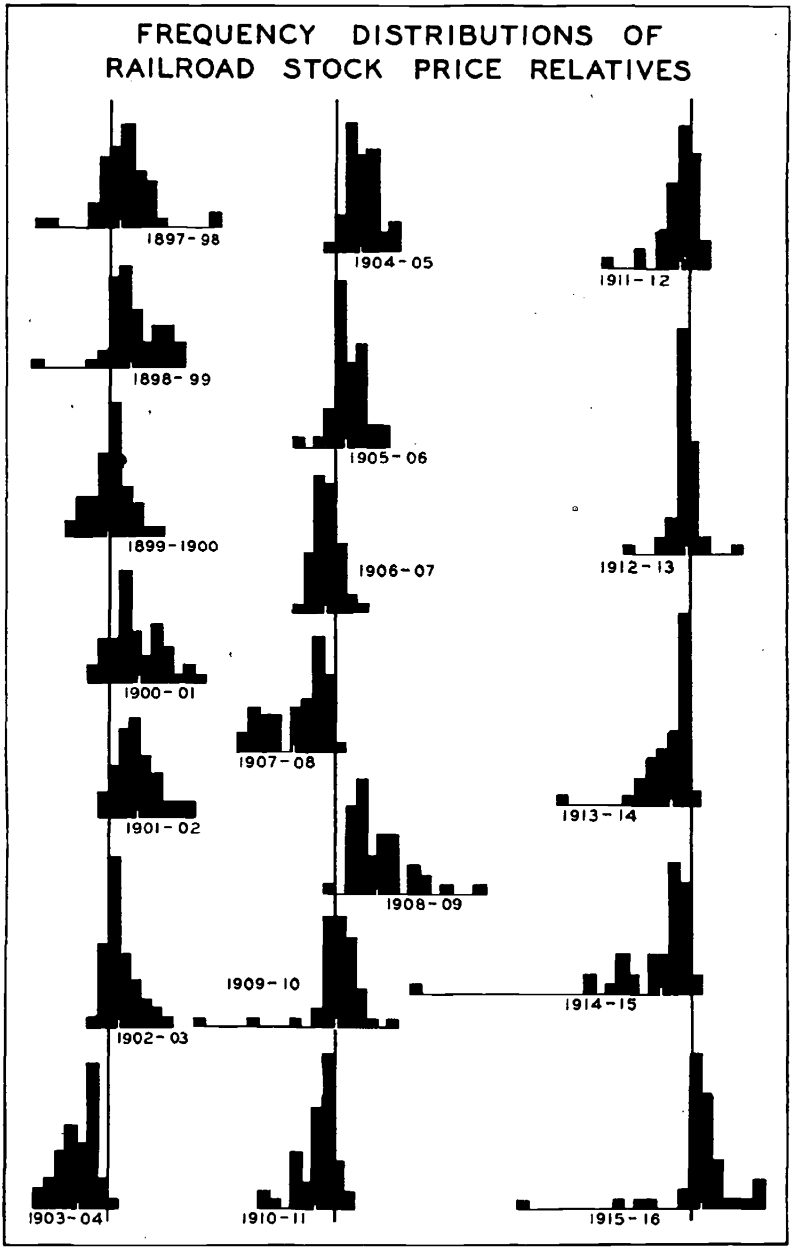
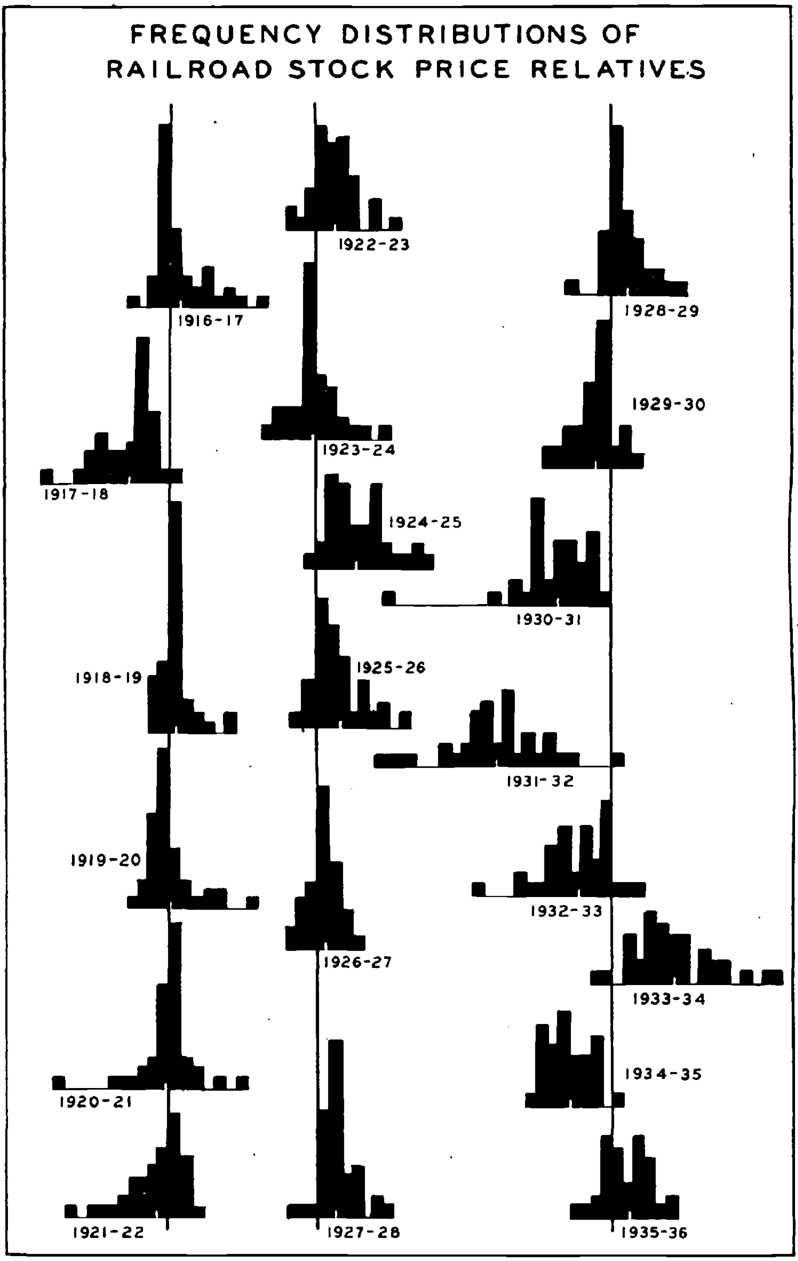


CHART 12-CONCLUDED



But, as may be seen from Chart 32, the instances in which a stock was dropped and later reinstated are few indeed.

Before discussing methods of measuring changes in the general level of railroad common stock prices, it is desirable to point out some characteristics of the price movements of the individual stocks. We must remember that, though their 'grades' may not be definitely measurable, the stocks are of different grades. Some are more speculative—less like high grade bonds—than others. Though all are 'common' stocks, they vary in their degrees of 'commonness'.

Appendix A, Table 14 gives the percentage that the price of each stock, in any particular January, was of the price of the same stock in the preceding January. Chart 12 presents 79 frequency distributions based on this table—one for each year from $\frac{\text{January 1858}}{\text{January 1857}}$ to $\frac{\text{January 1936}}{\text{January 1935}}$. This chart shows that most of these distributions are skewed in the direction of the general price change. In a year of rising prices they are skewed to the right; in a year of falling prices to the left.

The poorest stocks—those of the most overbonded roads—tend to show the greatest percentage price movements, both during periods of collapse and during periods of recovery. Now, though it is not logically necessary, the poorer stocks usually sell for fewer dollars *per share* than do the more conservative and less speculative stocks. The *lower priced* stocks, therefore, tend to show greater percentage fluctuations in price than do the higher priced stocks.[13] But the relation does not exist merely because the lower priced stocks tend to be the lower grade stocks. It is partly psychological. Speculators seem to prefer to operate in relatively low priced stocks. If a sound and conservatively managed corporation, whose stock sells at a high price per share and shows only small per-

[13] But their dollar or 'point' movements are generally *smaller* than those of the higher priced stocks. During a bull market, a stock that begins by selling for $100 a share will probably rise in price a smaller percentage but a larger number of dollars or 'points' than will a stock that begins by selling for $16 a share.

A curious empirical formula that describes the apparent tendency of price movement fairly well is that stock prices move equal increments on their square roots. Thus, if in a bull market stocks that sold for $100 per share rise in price to about $144 a share, stocks that sold for $16 a share will tend to rise to about $36 a share.

centage fluctuations, reduces greatly the market price of the stock per share by paying a large stock dividend, the new and lower priced stock will almost immediately begin to show larger percentage price fluctuations than did the old and higher priced stock.

From such strange material many types of index number could be constructed. The questions answered by some would be important, those answered by others would be trivial or even bizarre. Perhaps the first conclusion that one approaching the subject would come to would be that there was little if any importance attached to 'one share' as a unit of measurement. An index number based on totals of the prices of one share of each stock would be of the same type as an index number of commodity prices that was based on totals of the prices of one pound of each commodity. Any aggregate of actual prices calls for some weighting. The price movements of Pennsylvania stock are more important than the price movements of Western Maryland.

With almost no exceptions, the index numbers of stock prices that are currently published fall into one of three groups. All three are based upon arithmetic averages of actual prices. The first and largest group contains the 'unweighted' indexes. These are arithmetic averages of the prices of one share of each stock.[14] The second group contains index numbers in which each stock is weighted by the number of shares outstanding. The monthly index numbers (see Appendix A, Table 10, column 6 and Table 17) presented in this chapter fall in this group (for weights, see Appendix A, Table 15).[15] The third group contains index numbers in which the prices of the various stocks are weighted in proportion to their 'activity'—the number of shares that are bought and sold.[16]

The purpose of weighting is to make movements of the prices of the more 'important' stocks influence movements of the index num-

[14] Often a set of weights is gradually introduced because of stock dividends. For example, the *New York Times* index of the prices of 25 industrial stocks is constructed by multiplying the present price of each stock by the number of shares that now correspond to one share at the time that the stock was introduced into the index. Though the divisor is still 25, the total number of shares is now much greater than 25.

[15] As do the various weekly and monthly index numbers of stock prices published by the Standard Statistics Company of New York. Its index numbers extend back to January 1918.

[16] Professors Irving Fisher and W. I. Crum have each published index numbers of stock prices weighted by activity.

bers more than do movements of the prices of the less important stocks —if possible, *in proportion* to their importance. The question becomes one of deciding what shall be meant by 'importance'? Important for what? From the standpoint of an individual holder, the 'important' stocks are those that he owns, and the *relative* importance of their (arithmetic) price movement is in the absence of other factors in proportion to the number of shares that he owns of each. But all shares outstanding are owned by someone. May not the relative *social* importance of (arithmetic) fluctuations in the price per share of the stock of a particular corporation, therefore, well be thought of as a function of the number of shares that the corporation has outstanding; and, in the absence of the disturbing effects of other independent variables such as the extent to which the stock of the corporation is used as collateral for loans, etc., may not the relative social importance of (arithmetic) fluctuations in the price well be thought of as *varying directly* with the number of shares outstanding?

It is of course true that, if the price at which a share of stock is sold be multiplied by the total number of shares outstanding, the resulting figure may easily be quite unrepresentative of any intelligent valuation of the entire enterprise. Dr. Wesley C. Mitchell drew attention to this fact in his *Business Cycles* (p. 171):

"Whether the market prices of stocks in 100-share lots may be interpreted as showing accurately changes in the prices of the business enterprises concerned is highly questionable. If 1,000 shares in a railway which has 100,000 shares outstanding be sold at $80 per share on a given day, it does not necessarily follow that the whole proprietary interest could be sold (or bought) for $8,000,000. Indeed, it is seldom safe to infer the price for the total supply of any kind of goods from the current market price per unit . . . . that cannot be known except in the rare cases when such sales are actually made and the terms published. Hence we must content ourselves with taking the figures for what they are—prices of *shares* in business enterprises."

We must remember, however, when considering the above remarks of Dr. Mitchell, that, though a multiplication of price per share by total number of shares outstanding does not necessarily show what "the whole proprietary interest could be sold (or bought) for", it does determine the valuation that will be put on each and every one of the separate parts or 'shares' of the proprietary interest by the individual owners of those parts and by their creditors—such as banks. Loans are made and are called on the basis of the market prices per share of the collateral behind the loans.

For most economic purposes the 'importance' of the market price of a share of the stock of a corporation depends upon the number of shares outstanding. The economic importance of two shares of stock is not necessarily greater because they represent interests in two corporations than it would be if they represented an interest in only one corporation. To weight prices per share by the number of shares outstanding would seem a simple and logical way of allowing each separate *individual* share an importance not dependent upon the size of the corporation to which it belongs. If prices are not weighted by the number of shares outstanding, the prices of the shares of small corporations are given an importance they do not merit. If Corporation A has 1,000,000 shares outstanding and Corporation B only 100,000, a five dollar rise in the price *per share* of the stock of Corporation A tends to be ten times as important as a five dollar rise in the price *per share* of the stock of Corporation B. The price of each separate share in Corporation A is, we may assume, as important as the price of each separate share in Corporation B, and there are ten times as many of them.

The usefulness of index numbers of stock prices in which the prices of the various stocks are weighted in proportion to their market activity is very restricted. For some theoretical purposes connected with the 'equation of exchange' and for some practical purposes connected with the forecasting of stock prices by watching changes in the internal 'technical' condition of the market, the relative 'importance' of the prices of stocks may be thought of as varying directly with the number of shares sold. During a period in which a speculative football, such as Auburn Motors once was, is traded in much more actively than the stock of a much larger and economically more important company, such as General Motors, there is a sense in which fluctuations in the price of Auburn are more 'important' than fluctuations in the price of General Motors. But it is a narrow and technical sense.[17]

Index numbers of stock prices that are weighted by 'activity' are sub-

[17] Of course, when a stock becomes extremely inactive (*or abnormally active*), its price may become a much less reliable measure of careful and significant valuation by the market than it would be in more normal circumstances. But this is a matter that concerns the adequacy of the pricing process and not the importance of the commodity.

ject to a quasi mathematical drift. In periods of advancing prices there is a pronounced tendency for individual stocks to be abnormally active when they are advancing in price more rapidly than the general market. The opposite tendency—to be abnormally active when they are declining more rapidly than the general market during a period of general decline—is very much less pronounced. The reasons for these conditions are partly technical and partly economic; but, if individual prices are weighted in proportion to the changing turnover of the individual stocks, the net mathematical result is a pronounced upward drift in the index number during rising markets, accompanied by a much less pronounced downward drift during falling markets. If a stock moves from 100 to 120 during a short period of great activity (compared with the activity of other stocks) and falls back to 100 during a long period of relatively small activity, it affects the index number more while rising than it does while falling. It is theoretically possible for the price of each stock in the index number to be the same at the end of a period as it was at the beginning, and yet the index number show a pronounced movement.

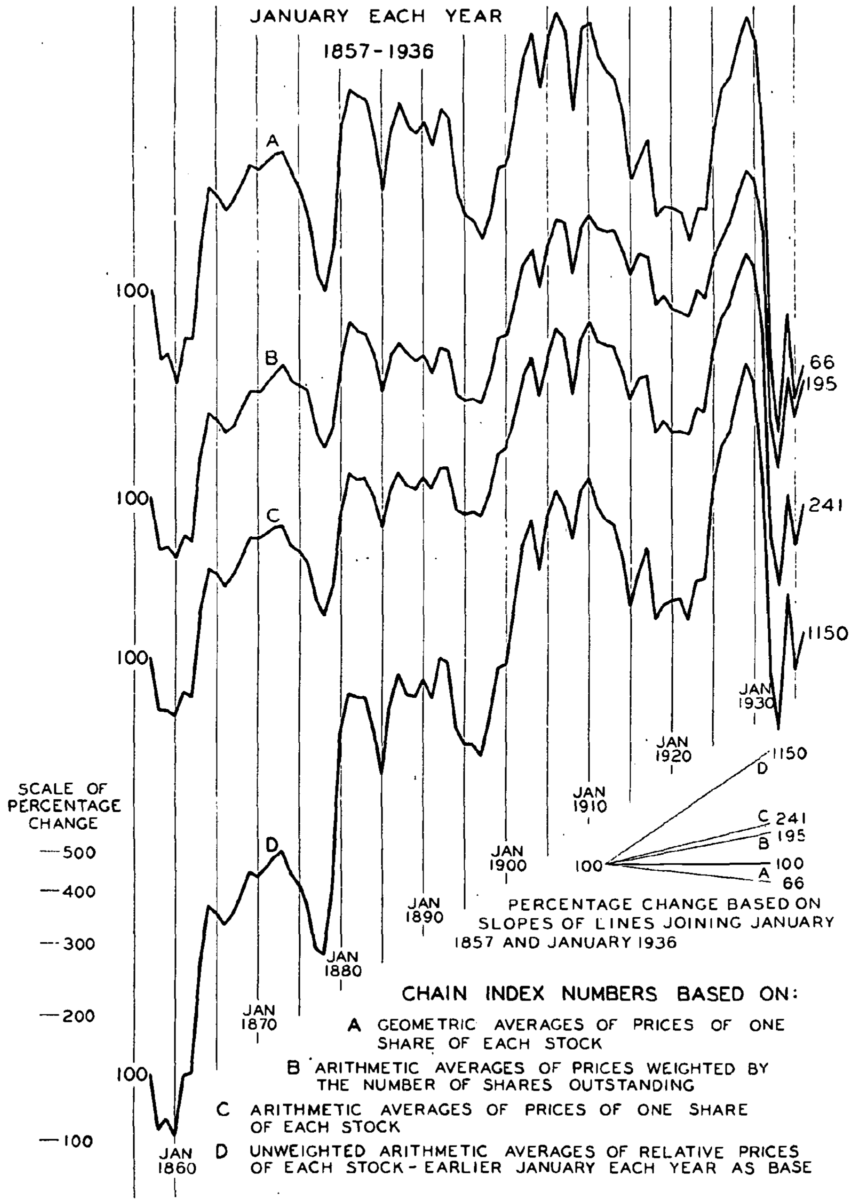
All existing index numbers of the prices of common stocks contain 'economic drift'. This is not merely almost inevitable but also desirable and necessary if the indexes are to present faithful pictures of what actually occurs. Economic drift is the essential characteristic of the movements of common stock prices. Those prices are the prices of radically junior securities and economic drift is the very badge of their 'juniority'. To eliminate it from an index, if that were feasible, would be nothing short of emasculating the index.

But *mathematical* drift is a merely disturbing influence, whether it occurs in a pure or in a disguised and quasi form. Charts 3 and 4 were introduced to illustrate types of mathematical drift in index numbers of bond yields. Chart 13 performs a similar service for stock prices. Four index numbers of railroad stock prices are there shown.[18] Each index is a 'chain' number, constructed by chaining together 79 separate index numbers, each extending from one January to the next. The indexes presented by the upper three lines on the chart (A, B and C) vary greatly in their movements, but are free from purely mathematical drift. The index represented by the lowest line on the chart (D) is subject to violent mathematical drift.

[18] For the figures, see Table 16.

CHART 13

INDEX NUMBERS OF THE PRICES OF AMERICAN RAILROAD STOCKS



Indexes A, B and C are each based on averages of actual prices. The differences in their movements result from differences in the types of average used and from differences in weighting. The averages of index A are geometric, those of B and C arithmetic. The individual prices of A and C are unweighted, those of B are weighted each year by the number of shares outstanding.

From January 1857 to January 1936 index A shows a *fall* of 34 per cent, index C a *rise* of 141 per cent, but this drift apart is not a purely mathematical drift in the sense in which we have been using the term. It is not, for example, a drift whose direction could be predicted for unknown chance material. Neither the figures of index A nor the figures of index C would be changed if the time order of the data were reversed. If the 1936 prices of individual stocks had occurred in 1857, the 1857 prices in 1936, and the prices of the intervening years had been correspondingly reversed as to time order, index A would have advanced from 66 to 100 instead of declining from 100 to 66, and index C would have declined from 241 to 100 instead of advancing from 100 to 241.

In the absence of substitutions, any quasi mathematical drift found in the relation between geometric and arithmetic indexes occurs primarily because of differences in *the economic drifts of the individual prices*. It could not, of course, occur if it were not for a difference in the mathematical treatment of the data; but its direction and very existence depend, at all times, on differences in the *individual* drifts, and whenever these individual drifts are large and important, they are of economic rather than chance origin.

The ratio of the arithmetic average of n positive quantities to the geometric average of the same quantities equals $\frac{1}{n}$th of the sum (i.e., the arithmetic average) of the ratios of the individual quantities to the geometric average.[19] The product and hence the geometric average

[19] Let A represent the arithmetic average and G the geometric average. Then the n individual quantities may be represented by Gx_1, Gx_2, Gx_3, . . . Gx_n where each x represents the ratio of a particular individual quantity to the geometric average. Now A, the arithmetic average of the n quantities, of course equals $\frac{G(x_1 + x_2 + x_3 + \ldots + x_n)}{n}$. Hence the ratio of the arithmetic average to the

of these individual ratios necessarily equals unity, but their arithmetic average varies with and constitutes a species of measure of the 'scatter' of the original quantities. With both stock prices and bond prices (in the sense of the reciprocals of their yields), this scatter tends to decrease with the passage of time; though it sometimes increases for fairly long periods. But, whether it decreases or increases depends upon economic characteristics of the data rather than upon mathematical characteristics of the index numbers.

The economic characteristics of stock price data are such that changes and substitutions among the stocks tend to affect index A much more seriously than they do index C. Stocks may be removed from an index because the road has gone into bankruptcy; they are never removed (if sales continue to be regularly made) because of the excessive prosperity of the corporation. Now index A is much more affected by a specified percentage change in the price of a low priced stock than is index C. And the percentage changes of really low priced stocks are, or at least have been (on the way down), gigantic.

The effect on index A of carrying a stock into bankruptcy and then removing it from the index may be almost impossible ever to overcome, though the effect on index C be relatively small. If, in the final year that a stock was in the indexes, its price began the year at 4 and ended it at 1, the effect on index A would be no more than merely offset by a later rise in the price of another stock from 25 to 100. And, even if the low priced stock was not removed from the index but was carried through a reorganization of the road, the mere process of reorganization would render improbable a percentage recovery in the new stock sufficiently great to offset a previous decline of the old stock into the region of zero.

Stock prices are unlike commodity prices in that commodity prices, as handled by the maker of index numbers, are the prices of new and not of second-hand or worn-out commodities. Cost of production prevents the prices of new commodities dropping to zero, or even close to zero, *and remaining there*. Though an index based on unweighted geometric averages fulfills the 'circular test' and, in that sense, has no strictly mathematical drift, it develops a quasi mathematical drift when

(Footnote 19 concluded)

geometric average, or $\frac{A}{G}$ equals $\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$.

it is constructed from data, such as stock prices, in which the size of the *ratio* fluctuations tends to increase as the price declines, approaching infinity as zero prices are approached, and in which there exists the very real possibility of permanently zero prices.

It is of general and fundamental importance to realize that the applicability, to any specific data, of no type of averaging or weighing can be determined *a priori*—by mere mathematical analysis—without considering the characteristics of the data and the purposes of the averaging. This view of the subject takes 'averaging' out of the vacuum in which it is so often discussed.

Even the index number represented by the lowest line (D) of Chart 13, subject as it is to the most violent mathematical drift, is perfectly adapted to answer certain practical questions. It is exactly the index number required to measure the changing fortune of an investor who began, in any January, by investing *equal amounts of money* in each stock and, in each following January, rearranged his portfolio so that once again the market values of his individual holdings would be identical.

To the extent that index numbers based upon arithmetic averages of 'relatives' tend to show a definite upward drift even if constructed from mere chance data, their drift is purely mathematical. But the drift in index D is greater than would occur with chance data. This is largely the result of the fact that, during the four greatest upward surges of the railroad stock market, the lower priced and more speculative stocks enjoyed a much greater percentage advance than did the higher priced and more conservative stocks. And the lower the price of the stock the greater the number of shares used in index D. As the price advances the number of shares is reduced, new low priced stocks receiving the excessive weighting.

But, for the purpose of comparing the movements of stock prices with the movements of bond yields, this quasi mathematical element in the drift is as disturbing as is the purely mathematical element. It is as undesirable to overweight low priced stocks as it would be to overweight high priced stocks. We are even less interested in how an individual investor would have fared if he had always kept an equal amount of money invested in each security than how he would have fared had he always held the same number of shares of each security. Our problem is a social and not a merely individualistic one. We are concerned with railroad common stocks as a type of security. We are therefore interested in what happened to railroad common stockholders as a class, but not in what would have happened to an individual if he had played the market in this way or that. We are interested in the changing value of the entire railroad system of the country—in so far as market prices can be used to measure changes in that value—rather than in changes in the price of arbitrary and insignificant or fluctuating and misleading units.

And, for such a purpose, there is only one index number. Indeed, were it not for changes in the number of shares outstanding and for the occurrence of amalgamations, consolidations and reorganizations with the attendant necessity of substitutions and changes in the list of stocks used, no question would ever arise. The price per share of each stock would, without discussion, be multiplied by the number of shares outstanding in order to obtain a figure for the total 'equity' value of each corporation—its worth to its stockholders. And these totals would then be added together. But, with not only changes in number of shares outstanding but also changes in capital structure that alter or even destroy the significance of the price of 'one share', 'chain' index numbers become absolutely necessary. Index B of Chart 13 is such a number.

The movements of index B are naturally more like those of index C than they are like those of index D. There is always some tendency for the stocks of the larger roads to be more conservative investments and therefore less violent in their price movements than the stocks of the smaller roads. Index B gives the greatest weight to the prices per share of the larger roads. It weights the various stocks in proportion to number of shares outstanding. Index C reduces this logical and desirable disparity of weighting by using only one share of each road. The larger roads receive no more weight than do the smaller. But index D goes so much further in this direction that it tends to introduce a system of *inverse* weighting. In so far as the stocks of the large roads are conservative and high priced and the stocks of the small roads speculative and low priced, the system of weighting is a complete reversal of common sense. The large companies are weighted the least heavily, the small companies the most heavily.

Index B is the only stock price index for which we present monthly figures. In the calculation of each link in the chain number (from one January to the next), prices were adjusted for all stock dividends, rights, etc., that occurred during the thirteen months. After this had been done, the adjusted prices per share were multiplied by the number of shares outstanding in the earlier January. We used total number of shares *outstanding* rather than number of shares in the hands of investors other than railroads. The difference in the two types of index numbers would undoubtedly be microscopic, and the calculation of the weights, under the non-railroad investor assumption, would have been a piece of foolish labor.

Even if there were good reasons for eliminating all intercompany holdings when determining weights, it would mean only that our index tended to weight slightly more heavily than it should—but not in any such extreme manner as 'one share each' indexes—the stocks of the smaller roads. Little railroads do not hold the stocks of big ones to anything like the extent that big roads hold the stocks of little ones. Railroads much more usually buy stocks for control or for a voice in the management than they do for a mere share in the profits with other investors.

Chart 14 presents the monthly movements of the railroad stock price index number, in which the prices of the individual stocks are weighted by the number of shares outstanding, and also the monthly movements of the 'adjusted' index number of the yields of high grade railroad bonds. The bond yield index is inverted and its scale is double that of the stock price index. As both scales are logarithmic, an upward movement of the bond yield index resulting from a *halving* of yields would, therefore, be of the same size as an upward movement of the stock index resulting from a *quadrupling* of prices.

It is at once apparent that the dissimilarities of the major movements of the two lines are much more striking than the similarities. The major long term movements of the bond yield line (*inverted* yields) are: (1) the irregular but violent up-swing from 1857 into 1864, followed by the short but violent down-swing into 1866; (2) the great upward swing from 1869 into 1899; (3) the great downward swing from 1899 to 1920; (4) the great, though extremely irregular, upward swing from 1920 to the date of writing. There was a fair degree of similarity between the *trend* movement of stock prices and the *trend* movement of the reciprocals of bond yields (as in the inverted line of Chart 14) from 1857 to 1864. There was, however, in railroad stock prices, no comparable movement to the violent down-swing of the bond yield recipro-

CHART 14
AMERICAN RAILROAD STOCK PRICES AND AMERICAN RAILROAD BOND YIELDS
(LOGARITHMIC SCALES)

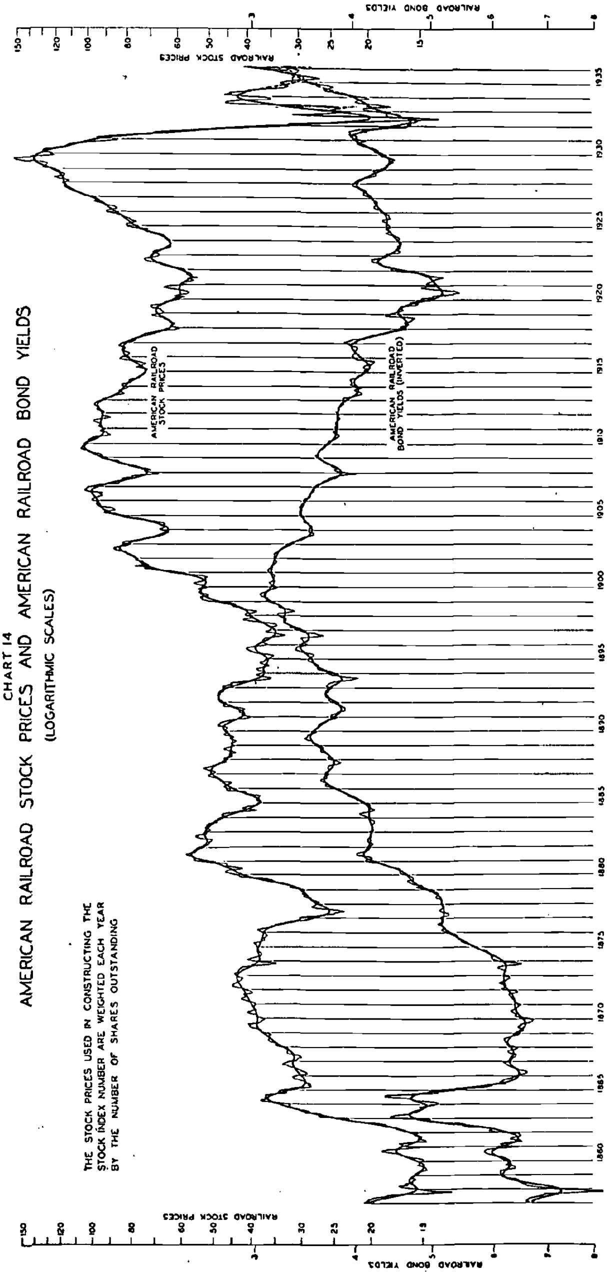


CHART 14
AMERICAN RAILROAD STOCK PRICES AND AMERICAN RAILROAD BOND YIELDS
(LOGARITHMIC SCALES)
THE STOCK PRICES USED IN CONSTRUCTING THE STOCK INDEX NUMBER ARE WEIGHTED EACH YEAR BY THE NUMBER OF SHARES OUTSTANDING
AMERICAN RAILROAD STOCK PRICES
AMERICAN RAILROAD BOND YIELDS (INVERTED)
RAILROAD STOCK PRICES
RAILROAD BOND YIELDS
1860
1865
1870
1875
1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935

cals into 1866. During the great swing of the bond yield index from a high of 6.72 per cent in December 1869 to a low of 3.07 per cent in June 1899, stock prices were see-sawing up and down on a plateau. There was a drop definitely below the plateau into 1877 and a recovery definitely above the plateau into 1881, but in December 1869 the stock price index number stood at 37.52 and as late as June 1897 at no more than 36.45.[20] During the first ten years of the great downward swing of the bond yield line from 1899 to 1920, stock prices doubled (from 51.14 in June 1899 to 105.83 in August 1909). It is true that the two lines move down together from 1909 to 1920 and up together from 1921 to 1927, and that, in the recovery after 1932, the major trends of both lines are upward. But the movements of the bond yield line, during the great 1929–32 collapse in stock prices, are little more than mere irregularities. There is no real similarity between the long term trends of the two series.

The 'cyclical' movements of the two series are much more closely related than are the long term trends. Lines D and E of Chart 21 represent the *deviations* of the bond yield and stock price mathematical graduations (presented, with the data, in Chart 14) from cycle-eliminating mathematical trends. Though there appear serious variations in the lags and even, at times, extra 'cycles' in one or other of these two deviations lines, there is, over the whole period, a real and rather striking similarity between them.

But the reader must remember that the disturbing effects of the dissimilarities of the long term trends do not exist in these deviations series. The short swings of the data, *as they actually occur,* show a much less uniform similarity between the two series than do the deviations. And even the similarity between the two sets of deviations is, as we have said, by no means uniformly close. The elimination of long term trend creates maxima and minima in the deviations series that do not exist in the original series. The deviations curve tends to pass through the zero line where mere points of inflection occur in the original series. For example, the stock price collapse that began in 1876 is accompanied by only a levelling off of the movement of the bond yield line of Chart 14. But in Chart 21 the deviations lines are strikingly similar in their movements through 1873, '74, '75, and '76. Simi-

[20] For the figures from which Chart 14 was constructed see columns 5 and 6 of Table 10.

larly, the stock price rise into the sharp peak of 1902 is accompanied by a *fall* in the (inverted) bond yield line of Chart 14. The stock price collapse of the next year (1903) is accompanied, not by a reversal of direction after a rise, but only by an increased rate of decline of the (inverted) bond yield line of Chart 14. But, in Chart 21, the two series each show a definite minimum [21] in 1902.

In exceptional instances, the deviations curves of Chart 21 require very careful interpretation if they are not to be misleading. And, in at least one instance—when they suggest a stock price maximum (*minimum* on the inverted deviations line of Chart 21) *at the end of 1930*—one naturally feels that the mathematical rigidity of the 'cycle-eliminating' trend curve has introduced a palpable absurdity into the data. The collapse of (raw) stock prices that began in the autumn of 1929 was so sudden and the downward movement so precipitate and long-continued that the trend curve, in its efforts (if we may be permitted to use such an expression) to handle 1930, 1931 and 1932, turned down even more steeply than the seasonal-eliminating graduation of the data.

That the short term or cyclical movements of stock prices and bond prices (as reciprocals of yields) should be more alike than the long term trends, if not as to the amplitude at least as to the timing of the movements, is as one would expect. The down swing of the economic cycle is a period in which the general demand is not primarily for more secure investments, but for more cash. It is a period of 'liquidation'—a period in which an extraordinarily large percentage of debtors are being forced to pay off or reduce their debts, a period in which they are attempting to sell rather than merely to improve the grade of the securities in their portfolios. A reverse process goes on during recovery. As the community emerges from the vicious circle of compulsory debt payment or bankruptcy, with its inevitably depressing effects on potential investors who fear further declines, *all* types of security soon tend to advance.

The cyclical fluctuations of common stock prices are greater than the cyclical fluctuations of the prices of high grade bonds not merely because the expectation of future payment from the stocks fluctuates while that from the high grade bonds remains virtually constant but also because the stocks are, to a much greater extent than are bonds,

[21] *Stock prices,* but not bond yields, are inverted in Chart 21.

carried on loans. But the cyclical movements of the two types of security have essentially the same relation to the business cycle, the cycle of confidence, the cycle of debt contraction and debt expansion.[22]

But there are no such simple reasons for expecting the *long term trends* of the two series to be similar. The compulsory liquidation on the cyclical decline and the plethora of funds for investment on the cyclical advance are each, in a sense, reactions from conditions that had become, both economically and psychologically, thoroughly extreme. A cyclical decline in stock prices is primarily a result of pressure rather than of a change in anticipated earnings—important as such a change may be. But this is not true of long continued movements of stock prices. Those movements are primarily the result of changes in anticipated earnings. But the prices of high grade bonds move with stock prices in business cycles, not because of the earnings factor but because of the pressure factor. For a changing list of bonds that are always of superlative grade, changes in the earnings factor may be assumed to be negligible.

And finally, the demonstration that the long term trends of the two series should not be expected to be necessarily the same may be put into the form of a *reductio ad absurdum*. If we assume that the long term movements of the prices (or yields) of bonds of superlative grade are much the same whether the bonds be railroad, industrial or public utility bonds, we realize that there is no inherent reason why the prices of superlative *railroad* bonds should not move up and down on their long time trends with the prices of industrial or public utility stocks *as closely as with the prices of railroad stocks*. But, while the cyclical movements of the three types of *stock* have usually been timed almost the same, their long term trends have often been quite different. For example, while railroad stock prices were declining from 1909 to 1920, the secular trend of industrial stock prices was definitely upward, from 1910 to 1919 strongly so.

Of course, if we were to compare stock price movements with the movements of the prices of *low grade* long term bonds, we should expect to find a greater similarity than if the comparison were with bonds of superlative grade. But the increase in similarity, though definite, would not necessarily be great. We must remember that a bond, with

[22] It is because it is a cycle of *confidence* that the movements of stock prices tend to lag behind those of bonds.

its maximum possible return, should be expected to act quite like a common stock only if that maximum return were so far above both the actual and the expected returns as to constitute no real 'damper' on upward movements. But the problems presented by bonds that are making no 'interest' payments or at best only reduced payments are, unless the bonds be income bonds, entirely different from those of common stocks. For example, the absence of dividend payments does not evoke the spectre of bankruptcy.

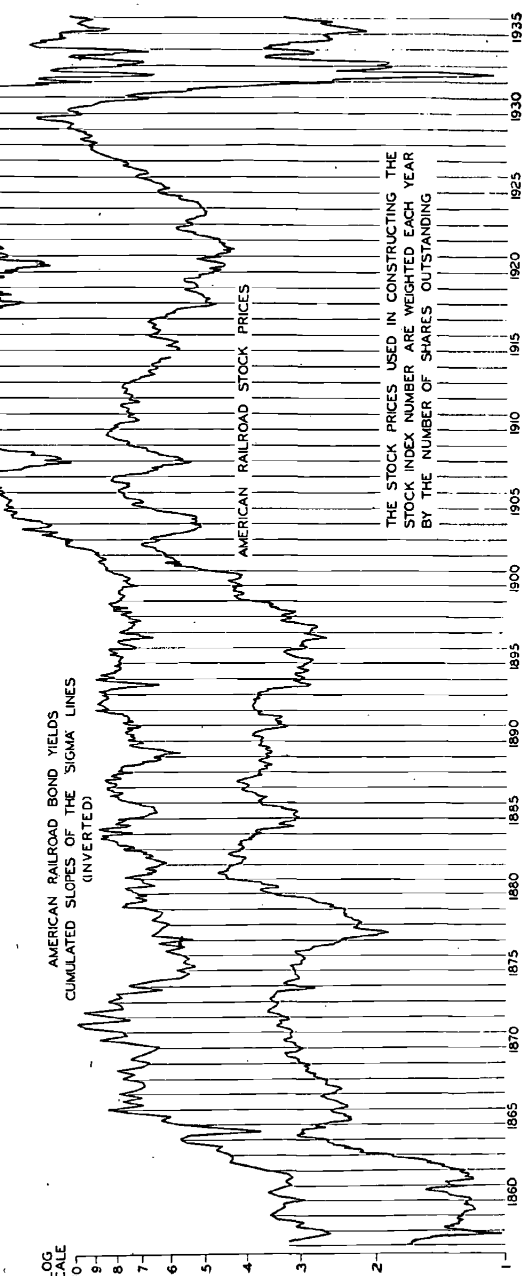
If, however, instead of collecting prices for a number of extremely low grade bonds—whose 'lowness' of grade would be undetermined—and comparing the movements of their prices (or yields?) with the movements of stock prices, we compare the movements of stock prices with a cumulation of the grade 'factor' given by the *slopes* of the 'sigma' lines, the results are rather interesting.

Chart 15 presents such a comparison. The reader will remember that, when the 'slope' of a sigma line is less than unity, the higher yield lower grade or in general more 'junior' bonds are acting better than the lower yield, higher grade bonds. They are either decreasing in yield more rapidly or increasing in yield less rapidly than the higher grade bonds or, in exceptional instances, decreasing in yield while the lower yield bonds are increasing. Also, the reader will remember that the mathematical equations represented by the sigma lines are of such a nature that, if the product of the slopes of consecutive sigma lines be cumulated, the result (in the absence of substitutions) will be the slope of the sigma line relating *directly* the yields of the bonds at the earliest and latest dates. Assuming the slopes of the sigma lines to be indexes of the degree of improvement or decline in railroad prosperity, or, more accurately, indexes of improvement or decline in the market's valuation of junior securities when compared with senior securities, we naturally feel that a comparison of stock prices with a cumulation of the sigma 'slopes' would surely be interesting.

The most cursory comparison of Charts 14 and 15 will convince anyone that, as we should expect, the *major* movements of stock prices follow much more closely and consistently the major movements of the cumulated sigma slopes (Chart 15) than they do the major movements of the yields of high grade bonds (Chart 14). But the short term movements of stock prices, whether separated from their long term trend as in Chart 21 or unseparated as in Chart 14, are more like the corre-

CHART 15

AMERICAN RAILROAD STOCK PRICES AND A CUMULATED PRODUCT OF THE SLOPES OF THE 'SIGMA' LINES FITTED TO THE YIELDS OF AMERICAN RAILROAD BONDS



sponding short term movements of the (inverted) yields than they are like the short term movements of the cumulated sigma slopes (Chart 15).

Of course, this appearance of similarity in the major movements of railroad common stock prices and the cumulated sigma slopes of the bond yields and its frequent absence in the merely cyclical movements of the two series may both, to some extent, be statistical accidents. Though our index of railroad stock prices is an almost all inclusive index and is, for most purposes, logically weighted, it is theoretically perhaps, for the purposes of our present comparisons, too all inclusive and not quite logically weighted. Though the movements of the yields of railroad bonds of the highest grade should theoretically be virtually as closely correlated with the course of industrial stock prices as with the course of railroad stock prices—if either correlation were logically called for—the sigma lines derived from the movements of the yields of railroad bonds *of various grades* have no such necessary relation to the prices of industrial stocks as they have to the prices of railroad stocks. And, to carry the argument one step further, no such necessary relation to the prices of railroad stocks in general as they should be expected to have to the prices of the stocks of *those particular railroads whose bonds were used in obtaining the sigma equations.*

There are both practical and theoretical difficulties that make the construction of a stock price index number that would be more logically adapted to our immediate problem extremely difficult; and the general usefulness of such an index would be much less than that of the one we present. Whether it would increase or decrease the similarity of movement of the two lines presented in Chart 14 is hard to say.

For we must remember that the other function—the cumulated sigma slopes—is also anything but perfect from either a theoretical or a statistical standpoint. For example, the absence from the cumulated sigma slopes of a dip in 1903 and a recovery in 1904 is definitely explained by Chart 8 on which the non-linearity of the scatter outside the range of the bonds we used from January 1903 to January 1904 is clearly shown. The slope of the asymptote of the hyperbola that extends into the region of high yields is definitely greater than unity while the slope of the sigma line is less than unity.

And finally, though it would have been interesting and enlightening, if we had had the ability to do so, to have presented such functions of stock prices and of bond yields and bond yield 'drift' as would have furnished an almost perfect correlation between the series, it would have been interesting as evidence only of the nature of the relationship, not of its existence. Chart 15 must be thought of as a test of how adequately the sigma system can represent a relationship that, from *a priori* considerations, may be assumed to exist, rather than as a demonstration of the existence of such a relationship.