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METHODS OF DESCRIPTION AND ANALYSIS

Price-Quantity Changes in Business Cycles

LL the measures here employed are based on 'reference cycle relatives', which define the average standing of each economic series at each of nine stages of business cycles. Stage I is the initial trough, stage V the peak, and stage IX the terminal trough; stages II, III, and IV mark off successive thirds of the phase of expansion in reference cycles, and stages VI, VII, and VIII mark off successive thirds of contraction. The turning points in general business activity defined by stages I, V, and IX are determined on the basis of extensive study of statistical records of economic processes of all sorts, supplemented by examination of contemporary appraisals of the state of business by financial writers, economists, and other students of business conditions.¹ The 'reference cycles' thus established provide a standard framework into which each of the many economic series to be studied is fitted. The original monthly data making up a given series, after correction for seasonal fluctuations (if these are present), are broken into segments, each covering a reference cycle.² The monthly entries making up a given segment are then expressed as percentages of their average for that reference cycle. Thus for the business cycle that extends from the trough of March 1933 through the peak of May 1937, to the

¹ The troughs and peaks of successive business cycles in the United States, over the period covered by the most extensive of the commodity records used in this study, are given below.

ugh	Pe	ak	Tro	ough		Tre	ough	Ре	ak	Tre	ough
1858	Oct.	1860	June	1861		Dec.	1900	Sep.	1902	Aug.	1904
1861	Apr.	1865	Dec.	1867		Aug.	1904	May	1907	June	1908
1867	June	1869	Dec.	1870		June	1908	Jan.	1910	Jan.	1912
1870	Oct.	1873	Mar.	1879		Jan.	1912	Jan.	1913	Dec.	1914
1879	Mar.	1882	May	1885		Dec.	1914	Aug.	1918	Apr.	1919
1885	Mar.	1887	Apr.	1888		Apr.	1919	Jan.	1920	Sep.	1921
1888	July	1890	May	1891		Sep.	1921	May	1923	July	1924
1891	Jan.	1893	June	1894		July	1924	Oct.	1926	Dec.	1927
1894	Dec.	1895	June	1897		Dec.	1927	June	1929	Mar.	1933
1897	June	1899	Dec.	1900		Mar.	1933	May	1937	May	1938
	ugh 1858 1861 1867 1870 1879 1885 1888 1891 1894 1897	ugh Pe 1858 Oct. 1861 Apr. 1867 June 1870 Oct. 1879 Mar. 1885 Mar. 1888 July 1891 Jan. 1894 Dec. 1897 June	ugh Peak 1858 Oct. 1860 1861 Apr. 1865 1867 June 1869 1870 Oct. 1873 1879 Mar. 1882 1885 Mar. 1887 1888 July 1890 1891 Jan. 1893 1894 Dec. 1895 1897 June 1899	ugh Peak Tro 1858 Oct. 1860 June 1861 Apr. 1865 Dec. 1867 June 1869 Dec. 1870 Oct. 1873 Mar. 1879 Mar. 1882 May 1885 Mar. 1887 Apr. 1888 July 1890 May 1891 Jan. 1893 June 1894 Dec. 1895 June 1897 June 1899 Dec.	Peak Trough 1858 Oct. 1860 June 1861 1861 Apr. 1865 Dec. 1867 1861 Apr. 1865 Dec. 1867 1867 June 1869 Dec. 1870 1870 Oct. 1873 Mar. 1879 1879 Mar. 1882 May 1885 1885 Mar. 1887 Apr. 1888 1888 July 1890 May 1891 1891 Jan. 1893 June 1894 1894 Dec. 1895 June 1897 1897 June 1899 Dec. 1900	ugh Peak Trough 1858 Oct. 1860 June 1861 1861 Apr. 1865 Dec. 1867 1861 Apr. 1865 Dec. 1867 1867 June 1869 Dec. 1870 1870 Oct. 1873 Mar. 1879 1879 Mar. 1882 May 1885 1885 Mar. 1887 Apr. 1888 1888 July 1890 May 1891 1891 Jan. 1893 June 1897 1894 Dec. 1895 June 1897 1897 June 1897 June 1897	ugh Peak Trough Trough 1858 Oct. 1860 June 1861 Dec. 1861 Apr. 1865 Dec. 1867 Aug. 1867 June 1869 Dec. 1870 June 1870 Oct. 1873 Mar. 1879 Jan. 1870 Oct. 1873 Mar. 1879 Jan. 1879 Mar. 1882 May 1885 Dec. 1885 Mar. 1887 Apr. 1888 Apr. 1888 July 1890 May 1891 Sep. 1891 Jan. 1893 June 1894 July 1894 Dec. 1895 Dec. 1900 Mar. 1897 June 1899 Dec. 1900 Mar.	ugh Peak Trough Trough 1858 Oct. 1860 June 1861 Dec. 1900 1861 Apr. 1865 Dec. 1867 Aug. 1904 1867 June 1869 Dec. 1870 June 1908 1870 Oct. 1873 Mar. 1879 Jan. 1912 1879 Mar. 1882 May 1885 Dec. 1914 1885 Mar. 1887 Apr. 1888 Apr. 1919 1888 July 1890 May 1891 Sep. 1921 1891 Jan. 1893 June 1894 July 1924 1894 Dec. 1895 June 1897 Dec. 1927 1897 June 1899 Dec. 1900 Mar. 1933	ughPeakTroughTroughPe1858Oct.1860June1861Dec.1900Sep.1861Apr.1865Dec.1867Aug.1904May1867June1869Dec.1870June1908Jan.1870Oct.1873Mar.1879Jan.1912Jan.1879Mar.1882May1885Dec.1914Aug.1885Mar.1887Apr.1888Apr.1919Jan.1888July1890May1891Sep.1921May1891Jan.1893June1894July1924Oct.1894Dec.1895June1897Dec.1927June1897June1899Dec.1900Mar.1933May	ughPeakTroughTroughPeak1858Oct.1860June1861Dec.1900Sep.19021861Apr.1865Dec.1867Aug.1904May19071867June1869Dec.1870June1908Jan.19101870Oct.1873Mar.1879Jan.1912Jan.19131879Mar.1882May1885Dec.1914Aug.19181885Mar.1887Apr.1888Apr.1919Jan.19201888July1890May1891Sep.1921May19231891Jan.1893June1894July1924Oct.19261894Dec.1895June1897Dec.1927June19291897June1899Dec.1900Mar.1933May1937	ugh Peak Trough Trough Peak Aug. 1902 Aug. 1902 Aug. 1902 Aug. 1907 June 1807 June 1807 June 1908 Jan. 1910 Jan. 1910 Jan. 1913 Dec. 1918 Dec. 1918 Apr. 1885 Mar. 1887 Apr. 1888 Apr. 1919 Jan. 1920 Sep. 1921 May 1923 July 1885 Mar. 1887 Apr. 1888 Apr. 1920 Mar. 1920 Sep.

² Appropriately modified procedures are used for annual and quarterly data.

terminal trough of May 1938, the average monthly amount of merchant pig iron produced was 317,000 tons. (This is an average of seasonally corrected monthly entries.) Production in March 1933 (seasonally corrected) was 75,000 tons. The March reference cycle relative is $100 \times (75,000/317,000)$, or 24. A similar figure is computed for each of the other months between March 1933 and May 1938. These reference cycle relatives, averaged by reference cycle stages, constitute the materials of the present investigation.

The special methods and measurements used in this study may be exemplified with reference to the price of pig iron and the quantity and value of merchant pig iron produced in the United States during the nine business cycles that ran their course between August 1904 and May 1938. Stage averages of reference cycle relatives, computed from monthly measurements for these nine cycles, are given in Table 2, for these series. The three patterns are shown graphically in Chart 1.

TABLE 2

Average Movements of Unit Price, Physical Volume, and Aggregate Value of Merchant Pig Iron Produced Nine Business Cycles, 1904-1938

				STAG	E MEA	SURES			
	Reference Cycle Stages								
	I	II	III	IV	v	٧Ĩ	VII	VIII	IX
Price	85	88	97	112	118	114	105	94	90
Quantity	73	86	102	120	133	132	106	76	Ğ5
Value	63	75	95	130	151	145	109	72	59
	PHASE N	1EASUR	ES, AVE	RAGE CH	ANGE	FULL CYCLE MEASURE			E
	Expansi		Contraction			Average Amplitude			
Price	+33		28			+ 61			
Quantity	+60		68			+128			
Value	+88		92			+180			

The average patterns of pig iron prices and the value of merchant production show advances between reference stages I and V, and declines between stages V and IX. These movements of the averages accord perfectly with the expansions and recessions of business cycles. Merchant production behaves similarly, except that it remains virtually stable between stages V and VI.





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Thereafter it drops more sharply than prices. The three patterns differ notably in the amplitudes of cyclical swings (Table 2). In terms of the indexes there given the amplitude of fluctuations in quantity is approximately twice that of the fluctuations in price,³ and the amplitude of fluctuations in value is approximately three times that of fluctuations in price.

Relatives defining the average standing at nine reference cycle stages of comparable price and quantity series, treated as coordinate observations on related variables, are admirably adapted to the study of the cyclical adjustments effected through the price-quantity mechanism. The procedures here employed make possible the massing of evidence relating to concurrent shifts in market prices and in quantities produced (or sold, or consumed) during expansion and contraction phases of business cycles. These procedures are briefly described and explained.

Correlative cyclical changes in prices and in quantities may be graphically portrayed by superimposing the average patterns of reference cycle changes in these factors. Pig iron prices and merchant pig iron production illustrate the procedure (Chart 1). A common framework (defined by the dates of initial trough, peak, terminal trough, and intervening stages) has been imposed on the price and production series, for each of the nine reference cycles covered. The relatives for the two series are thus comparable in respect of the time factor. As we have seen, the production series is marked on the average by wider amplitude of cyclical swings, sharper advance in the early stages of expansion, and tardier but more violent response to the forces

⁸ In this and subsequent tables the 'war cycles' are included. ('War cycles' are here defined as those occurring between 1861 and 1867, and between 1914 and 1921.) Since these cycles were marked by exceptionally violent price fluctuations, the effect of their inclusion is to give price amplitude averages greater than those that would be obtained from peacetime performance alone. The degree to which certain of our basic measures are modified by the exclusion of data for war cycles is indicated at various points in the following account, but our present concern is with the inclusive record.

In comparing price and quantity amplitudes for pig iron and certain other commodities we must bear in mind that some part of the total output may change hands at the peak of prosperity at prices above those currently quoted in trade papers, and that at the lowest point of depression some part of the output may be sold at prices below current quotations. The resulting bias may be wholly or partly offset by sales on long-term contracts, at prices much more stable than those currently quoted. of recession. These movements can be traced with precision, and in detail, on the superimposed graphs.

A different and in some respects more illuminating representation of the correlated movements of prices and quantities during business cycles is obtained when the average reference cycle relatives of the price and production series are plotted as coordinates (Chart 2, Fig. 1). The point marking the average standing of prices and production at reference cycle stage I is defined by the abscissa 73 (production) and the ordinate 85

CHART 2 Patterns of Related Price-Quantity Movements in Business Cycles Three Commodities Averages by Cycle Stages



(price). Similar points define the standings of the two variables at the other cyclical stages. The full pattern of concurrent price and production movements, as averaged for nine reference cycles, is traced on the diagram.⁴

This mode of presentation is effective in conveying the impression of coordinated changes in prices and production, during expansion and contraction phases of business cycles, and in indicating the relative importance of changes in each, in the various cyclical stages. Consideration of the forms this 'pricequantity pattern' would take under different conditions will make the meaning of a given type of pattern clearer. Thus if, for a given commodity, prices were the sole active factor in business cycles, with production stable, the diagram would take the form of a vertical line. An approach to this situation is represented by Chart 2, Fig. 2, showing correlated changes in the average price and production of crude petroleum in the Appalachian field from stage to stage of the business cycle that ran its course between April 1919 and September 1921. In the reverse case, when all cyclical adjustments are effected through quantity changes, prices remaining stable, the pattern would appear as a horizontal line. The price-production pattern for steel rails during the 1912-14 reference cycle exemplifies perfectly this type of behavior (Chart 2, Fig. 3). With equal changes in the two variables (i.e., equal in terms of reference cycle relatives), and with similar vertical and horizontal scales, the plotted points marking the standing of price and quantity components at reference cycle stages would all fall on a straight line inclined at 45 degrees to the horizontal. Chart 2, Fig. 1, roughly approximates this condition, though the inclination is somewhat less than 45 degrees because fluctuations in merchant pig iron production are greater than those in price. The inclination of such a diagram as a whole, it is clear, provides an indication of the relative roles of price and quantity changes in the ad-

⁴ For a somewhat similar presentation, based on logarithms of annual production figures and of average annual prices, see W. W. Leontief, Price-Quantity Variations in Business Cycles, *Review of Economic Statistics*, May 1935. Leontief's study embodies the first formal work of this sort with which I am familiar. Our procedures were developed independently of Leontief's, but in the application of these methods we have benefited from Leontief's suggestive and original work.

justments that accompany business cycles. Moreover, the slopes of the lines connecting pairs of points in the diagram may be interpreted in the same way, with reference to changes between any two cyclical stages.

Join't Cyclical Variability and Its Components

A measure of cyclical amplitude was described in a preceding paragraph. In the study of joint variability an index of combined amplitude is needed—a measure of the amplitude of the movements of the price-quantity points that define the pattern of price-quantity behavior in reference cycles. Such a measure is given in Table 3, col. 2. The method employed may be ex-

TABLE 3

Joint Variability of Prices and Output of Merchant Pig Iron Nine Business Cycles, 1904-1938

	JOINT	PER Full (CENTAGI Cycle	CONTRIBUTION TO Expansion			JOINT VARIABILITY Contraction		
	VARIABILITY	р	q	р	q	p+q	р	q	p+q
(1) Av. reference	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	. (10)
cycle pattern	6,477	19	81	11	29	40	8	52	60

plained with reference to the figure 6,477, defining the joint variability of pig iron prices and merchant pig iron production in the average reference cycle pattern plotted in Chart 2, Fig. 1. This is a measure of the combined variability of the price and quantity factors about the point of averages of the pattern there shown. Thus for the nine points in Chart 2, Fig. 1, the point of averages is defined by the coordinates 99, 100 (i.e., 99 on the quantity axis, 100 on the price axis).⁵ The point for stage I has the coordinates 73, 85. The deviation of point I from the point of averages is -26 on the x-axis, -15 on the y-axis; the sum of the squares of these two figures is 901. A similar figure may be computed for each of the other eight points marking out the average pattern of price-quantity behavior in reference cycles.

⁵ These are the unweighted averages of the coordinates for the nine reference cycle stages. Were each stage measurement weighted according to the number of months represented by it—an unnecessary refinement for the present purpose—the averages of the coordinates for prices and quantities would be exactly 100, 100.

The total of the nine figures, equally weighted, is 6,477. This sum measures the magnitude of the pattern, or the amplitude of joint price-quantity movements in business cycles. It takes on significance, of course, when used for comparison of different commodities, or for studying chronological changes in the pattern of price-quantity behavior for a single commodity.

The measure of joint variability is composed of two elements —a series of price components and a series of quantity components. Thus, for the average reference cycle pattern, the sum of the price components (the squared y-deviations) for the nine points in Chart 2, Fig. 1, is 1,203; the sum of the quantity components (the squared x-deviations) for the same nine points is 5,274. As percentages of the total joint variability (6,477) these are, respectively, 19 and 81 (Table 3, col. 3 and 4). They imdicate that, on the average, for merchant pig iron, price contributed 19 percent of the joint cyclical variability of prices and quantities; production contributed 81 percent.⁶ These percentages provide valuable indications of the relative importance of price and quantity fluctuations in the cyclical adjustments effected through the price-quantity mechanism.

The total amplitude of movements in the prices and output of merchant pig iron, as manifest in the average reference cycle pattern, may be broken into components of another sort. The entries in Table 3, col. 7 and 10, indicate that 40 percent of the combined movements occur during the expansion phase of reference cycles, 60 percent during contraction.⁷ This means

⁶ The difference between these two figures is materially greater than that between the amplitude measures (61 for price, 128 for quantity) given in Table 2. The measures of joint variability take account of the positions of nine points, not merely of the low and high values. They are derived, moreover, from squared deviations. (The use of squared measurements is necessary for consistency among the components of the total variability.)

In the calculation of percentages for prices and quantities (19 and 81) no account is taken of the technical 'interaction' between the price and quantity components resulting from possible correlation between their cyclical movements. The present measures relate to the joint cyclical variability of prices and quantities when their separate variations are combined in the manner indicated. There is no analysis of variance (of pig iron values) in the technical sense.

⁷ In this computation entries for stages I to V, inclusive, are considered to define the expansion phase; entries for stages V to IX are taken to define the contraction phase. The squared deviation for stage V is divided equally between the expansion and contraction phases.

that the combined deviations of price and quantity from their average cyclical standings are greater during business contractions than during business expansions. The impact of contraction on the price-quantity mechanism, for merchant pig iron, is heavier than that of expansion in the sense that joint pricequantity variability is greater during contraction. Further analysis shows that the 40 percent figure for expansion was composed of an 11 percent price contribution and a 29 percent production contribution, while the 60 percent figure for contraction was composed of an 8 percent price and a 52 percent production contribution (all these are percentages of the total joint variability of the average reference cycle pattern). Price is the smaller component during both expansion and recession. The deviations of pig iron prices from their cyclical average are wider and more sustained during reference expansion than during contraction; the reverse is true of merchant pig iron production, and in substantially greater degree.

Elasticity of Quantities and Flexibility of Prices in Business Cycles

The coordinate presentation of price and quantity changes, as in Chart 2, Fig. 1, suggests immediately the use of the customary measures of elasticity and flexibility.⁸ Conceptually, coefficients of elasticity and flexibility are derived from equations of pricequantity relationships that are inverse and essentially timeless. Implicit in them is an assumption of causality. If the reference is to elasticity, quantifies demanded are assumed to change because unit prices change; if the reference is to flexibility, prices

⁸ The coefficient of elasticity of demand is given by the expression $\eta = y/x \cdot dx/dy$, where x is quantity and y is price. This coefficient is the ratio of a relative change in quantity (usually quantity demanded) to the corresponding relative change in unit price, the relative changes being infinitesimal. The quantity factor is the dependent variable. A given coefficient is assumed to relate to a moment of time or to a period within which tastes, income distribution, and other circumstances affecting individual demand schedules may be treated as fixed.

The coefficient of flexibility of price as given by Henry L. Moore is defined by the expression $\phi = x/y \cdot dy/dx$. This is the ratio of a relative change in price per unit of commodity to the corresponding relative change in quantity, the relative changes being infinitesimal. Unit price is here assumed to be the dependent variable. are assumed to change because larger or smaller quantities are placed upon the market. It is conventional to speak of demand as elastic if η is greater than 1, inelastic if it is less than 1, and to speak of price as flexible if ϕ is greater than 1, inflexible if ϕ is less than 1.

When we follow the associated movements of prices and quantities in the framework of reference cycles we are dealing, obviously, with relations in the dimension of time, not with the relations assumed in the conventional diagrams. From one stage to another of business cycles we have a complex of changes in the many circumstances that affect individual and group demand schedules. With reference to the Cournot-Marshall diagrams, price movements from stage to stage of business cycles may reflect changes in the shape and location of demand curves, as well as movements along demand curves (interacting, of course, with similarly complex changes on the supply side). It is such changes as these that lie back of the shifts represented in the diagrams of Chart 2. Although the changes affecting quantity and price are due to diverse influences, there is reason to expect some orderliness in the behavior of the factors concerned, when studied in the framework of reference cycles, and there is justification for a systematic attempt to define these changes. Procedures deriving from those used by Cournot and Marshall may be employed, with frank recognition that we are dealing with changes in the dimension of time and that the resulting shifts reflect the play of varied dynamic forces in addition to movements along timeless curves of demand and supply. In the choice of symbols to define the resulting coefficients and of terms to describe price and quantity behavior, care should be taken to avoid confusion with the conventional meanings.

For the purposes of this study we shall use the term *elasticity*, but with qualifying adjectives or adjectival nouns to indicate the temporal reference. Thus we may speak of *stage elasticity* when we are referring to the relative responsiveness of physical quantity to price changes and to other economic forces acting between any two cyclical stages.⁹ We may speak of *phase*

⁹ The term 'responsiveness' is subject to qualifications similar to those that attach to 'adjustments' (see Ch. 1, note 3). Its use suggests that physical volume

elasticity when the reference is to responsiveness of quantity to price changes and to general economic forces (cyclical and other) during cyclical phases of expansion or contraction. We may speak of *full cycle elasticity* when an average of measurements for the phases of expansion and contraction is in question.¹⁰ Other appropriate terms may be used when the behavior of physical quantity is being studied in other frames of reference.

We have referred to coefficients of cyclical elasticity as measures of the relative responsiveness of physical volume to price changes and to other economic forces operating in the framework of business cycles. For most commodities the 'other forces' are usually more potent than the price changes. Where this is true quantity and price are in fact both dependent variables, changing under the impact of major cyclical forces. Under these circumstances the coefficient of cyclical elasticity may be thought of as a measure of the differential responsiveness of physical volume, the coefficient of cyclical flexibility as a measure of the differential responsiveness of unit prices. In each case the change in the other factor (e.g., the change in price, if elasticity of quantities is in question) is to be thought of as a standard of reference rather than as a primary causal influence. (On the supply side, of course, rising prices are a stimulus to increased output, falling prices a stimulus to output reduction. Here the prices are a link in a chain of factors influencing output.) The traditional coefficient of elasticity of demand is conceived of as

is playing a dependent role, price and other economic forces being the independent variables in the cyclical processes under discussion. There is, in fact, mutual interaction. However, since the 'dependent' variable is the physical quantity of a single commodity, while the price of that commodity and all other economic forces are cast in the role of independent variables, the usage here employed seems justifiable.

A stage coefficient, we should note, is computed from the reference cycle relatives for two successive stages (see Table 2). For interstage period I-II, for example, the coefficient of elasticity is the ratio of the relative rates of changequantity to price—at the midpoint of the line joining the quantity and price observations for stages I and II. So derived, it is identical with a coefficient of arc elasticity.

¹⁰ An alternative 'full cycle' measure of the relative responsiveness of quantities and prices to the forces of business cycles could be obtained from the ratios of the cyclical amplitudes of these two factors.

a measure of the responsiveness of the quantity factor to a change in price, all other factors being assumed constant. In practice the complete constancy of other factors can never be realized. Indeed, when price and quantity movements are being studied in the framework of business cycles these other factors are generally dominant. Hence the concept of differential responsiveness is more appropriate than is that of direct functional dependence of quantity on price, or of price on quantity.

sponsiveness is more appropriate than is that of direct functional dependence of quantity on price, or of price on quantity. When elasticity is being measured in a temporal framework a symbol other than the conventional η should be employed. We use e ($e = y/x \cdot dx/dy$, where x represents quantity and y price). When e is greater than 1 quantity is *elastic* (i.e., more responsive than price to the impinging forces); when e is less than 1 quantity is *inelastic*. The use of this conventional distinction is justified when the context makes clear that the reference is to stage elasticity, phase elasticity, or to elasticity in some other temporal framework.

One further distinction is necessary. The general relations between price and quantity changes, over time, with which we deal may be *positive* (prices and quantities increase, concurrently, or decrease, concurrently) or *inverse* (prices and quantities move in opposite directions). When we seek to define quantity behavior, we may have behavior that is elastic and positive or inverse, or inelastic and positive or inverse. As the formula indicates, quantity behavior is always defined with reference to corresponding changes in unit prices.

corresponding changes in unit prices. In the definition of price behavior, in relation to corresponding quantity changes, we use the term *flexibility*, and the coefficient f ($f = x/y \cdot dy/dx$, where y represents price and x quantity). In doing so we are using the term first employed by Moore, but giving it the wider meaning that goes with the inclusion of changes over time. Here, as with elasticity, we qualify the term to indicate the temporal framework within which the responsiveness of prices is being studied, saying stage flexibility, phase flexibility, or full cycle flexibility. We shall regard prices as *flexible* when f is greater than 1, *inflexible* when f is less than 1, flexibility as *positive* when price and quantity movements are in the same direction, *inverse* when the movements are in opposite directions.

This extension of certain traditional concepts relating to coordinated price and quantity movements makes possible a wider application of these tools of analysis. It is necessary, however, to define precisely the conditions under which price-quantity relations are being studied in any specific case. The central feature of the procedure is that time is expressly introduced and that we are studying the behavior of quantity and unit prices in a complex of economic changes, including among others those represented by conventional demand and supply schedules. In particular, we emphasize the need of defining the temporal frame within which related price and quantity movements are being studied. Thus we might be concerned with concurrent seasonal movements of prices and quantities, with concurrent movements of prices and quantities during cycles in general business, with changes occurring in longer cycles, or with concurrent secular movements of prices and quantities. It is clear that the meaning to be attached to a statement such as 'The price of commodity A is inflexible, positively' or 'The output of commodity A is elastic, inversely' depends upon the framework within which the relative movements are being studied. The price of a commodity might be inflexible, inversely, when studied in a seasonal framework; it might be flexible, positively, when studied in the reference cycle framework; it might be flexible, inversely, when movements over several decades were studied. In each case, it will be understood, the reference is to the behavior of prices relative to corresponding movements of quantities.

With these explanations in mind we turn to measures describing the movements of merchant pig iron production and the behavior of pig iron prices in the framework of reference cycles. The coefficients in Table 4 are derived from the average pattern of pig iron price and quantity movements during nine cycles in general business occurring between 1904 and 1938 (see Chart 2, Fig. 1).

The over-all measures indicate that merchant pig iron production is highly responsive to the forces of business cycles.

DESCRIPTION AND ANALYSIS

TABLE 4

Flexibil	ity o	f Pig I	ron Pr	rices, N	ine B	usiness	Cycles,	1904-1	938
				5 т	AGE I Intersta	measui ge Perio	a e s d		
		I- II	II- III	III- IV	IV- V	v- VI	VI- VII	VII- VIII	VIII- IX
Elasticity (6 Flexibility	?) (f)	+4.71 +0.21	+1.75 +0.57	+1.13 +0.88	+1.97 +0.51	+0.22 +4.54	+2.66 +0.38	+2.99 +0.33	+3.59 +0.28
		Exp: ([]]	PHASE ansion I-V)	MEASURE Contra (V-I)	s ction K)	Ft Av. of ex	ILL CYCLI pansion	E MEASUR and cont	e raction
Elasticity (Flexibility	e) (f)	+ +	•1.79 •0.56	+2.5 +0.3	5 9		+2 +0	.17 .46	

Coefficients of Elasticity of Merchant Pig Iron Production and Flexibility of Pig Iron Prices, Nine Business Cycles, 1904-1938

The coefficient of full cycle elasticity (+2.17) shows that both quantities and prices move, on the average, in the same direction during cycles in general business, rates of change in production being more than twice as great as concurrent rates of change in prices (i.e., for every change of 1 percent in pig iron prices there is on the average a change of 2.17 percent in merchant pig iron production, in the same direction). The corresponding coefficient of full cycle price flexibility is +0.46. The separate entries for expansion and contraction indicate that quantity is elastic, positively, and that prices are inflexible, positively, in both cyclical phases, but that the phase elasticity of quantities is greater and the phase flexibility of prices is less during contraction than during expansion.

The measures for the separate interstage periods reveal high elasticity of production (and relatively inflexible prices) in the first period of reference expansion (i.e., between stages I and II). Thereafter, during general business expansion, the degree of elasticity of production declines (the decline being checked, however, between stages IV and V) and the degree of flexibility of prices increases, but output shows relatively greater rates of change than price during all periods of expansion, up to the peak. Between stages V and VI merchant pig iron production declines only slightly and the coefficient e has a very low value (+0.22). During the remaining interstage periods of reference contraction pig iron production is highly, and in accelerating

degree, elastic in response to cyclical forces, and the stage flexibility of prices steadily declines. The resistances to price rise (as here measured in relation to quantity changes) are strongest during the early stages of general business expansion; the resistances to pig iron price declines (relative to quantity reductions) increase in strength as contraction develops.

Outlays of Buyers and Revenues of Sellers in Business Cycles

We have referred in Chapter 1 to the procedure by which measures of the aggregate value of a given commodity can be estimated from data on the number of units of the commodity produced during a stated period and its average unit price.¹¹ Cyclical changes in such values are of clear significance in studying business cycles. They measure the volume of payments, the monetary counterpart of the stream of physical goods and services. These payments may be thought of as constituting the outlays of buyers, or the revenues of sellers. We may study the interaction of prices and quantities as components of changes in outlays and in revenues, therefore, and thus throw new light on the patterns of joint behavior described in preceding sections.

In tracing changes in the aggregate value of a commodity and in its two components, we use average rates of change per month, from stage to stage of reference cycles. These monthly rate-of-change figures for merchant pig iron, derived from the averages of reference cycle relatives in Table 2, are given in Table 5.

The record for pig iron values shows a series rising fairly rapidly during the initial period of expansion (at 2.7 per month, in terms of reference cycle relatives), with some retardation after stage II, and quite rapid increases between stages III and V. In the last period of expansion, before the peak of the reference cycle, the maximum rate of advance is reached. This is 4.5 per month—a figure that may be interpreted as 4.5 percent,

¹¹ In this study values have been computed from monthly price and quantity records for individual commodities. The derived value series have then, been analyzed in the reference cycle framework.

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TABLE 5

Average Monthly Interstage Changes in Prices, Quantities, and Values of Merchant Pig Iron, Nine Business Cycles, 1904-1938

INTERSTAGE PERIOD	PRICE	QUANTITY	VALUE
I-II	+0.6	+2.8	+2.7
II-III	+1.1	+1.8	+2.4
III-IV	+1.8	+2.3	+4.2
IV-V	+1.3	+2.6	+4.5
V-VI		0.2	1.5
VI-VII	-1.5	-4.1	5.9
VII-VIII		5.0	6.1
VIII-IX	1.2	-3.0	3.8

Since each of these changes is computed on the reference cycle average as base and not on the initial stage of each period as base, apparent numerical discrepancies among the rates for price, quantity, and value may occur.

if we remember that the base of the percentage for a given reference cycle is the average of all the monthly figures for pig iron values for that cycle. Recession sets in relatively slowly, values declining at a rate of 1.5 per month between stages V and VI. The next two periods of contraction bring sharp declines, reaching a rate of 6.1 between stages VII and VIII. There is a notable diminution of the rate of decline from stage VIII to the terminal trough at IX.

Examination of the figures for prices and quantities that parallel the value entries indicates the role of each component, in the expansion and contraction of pig iron values. Increases in quantities dominate heavily, in the first period of expansion in values, price advances playing a minor part. Between stages II and III there is appreciable retardation in the rate of increase in output; the price rise is accelerated. The quantity factor is dominant in pushing up monetary values throughout the phase of expansion, but prices become increasingly important as a boosting factor up to stage IV of reference expansion. In the final period of expansion the rate of price advance is checked; output continues to accelerate.

On recession prices exert the main pressure in the first slow decline in the monetary value of merchant pig iron produced. Production is only slightly reduced between stages V and VI. After contraction is well started (i.e., after stage VI of the

reference cycle) the production series again takes charge, and forces monetary values down steadily, and at a high monthly rate, for the three succeeding periods of general business contraction. The maximum rates of production decline, it is to be noted, are substantially greater than the maximum rates of increase during the phase of expansion. Prices decline also, during reference contraction, but at a monthly rate that is fairly steady and close to the rates prevailing during the later stages of expansion. With recovery quantity changes provide the major stimulus to value advances, contributing rather more than unit price advances to swelling values in business expansion. But their strongest push on values is felt during the three final periods of contraction when they are almost three times as important as prices in causing aggregate values of merchant pig iron to tumble.