Volume Title: Asset Prices and Monetary Policy

Volume Author/Editor: John Y. Campbell, editor

Volume Publisher: University of Chicago Press

Volume ISBN: 0-226-09211-9

Volume URL: http://www.nber.org/books/camp06-1

Conference Date: May 5-6, 2006

Publication Date: September 2008

Chapter Title: Learning, Macroeconomic Dynamics and the Term Structure of Interest Rates

Chapter Author: Hans Dewachter, Marco Lyrio

Chapter URL: http://www.nber.org/chapters/c5372

Chapter pages in book: (p. 191 - 245)

5

Learning, Macroeconomic Dynamics, and the Term Structure of Interest Rates

Hans Dewachter and Marco Lyrio

5.1 Introduction

Since the seminal papers by Vasicek (1977) and Cox, Ingersoll, and Ross (1985), there is a consensus in the finance literature that term structure models should respond to three requirements: absence of arbitrage opportunities and both econometric and numerical tractability. Models designed to meet these criteria can be useful, for instance, in the pricing of fixed income derivatives and in the assessment of the risks implied by fixed income portfolios. More recently, however, a number of requirements have been added to the modeling of the yield curve dynamics. Satisfactory models should also (1) be able to identify the economic forces behind movements in the yield curve, (2) take into account the way central banks implement their monetary policies, and (3) have a macroeconomic framework consistent with the stochastic discount factor implied by the model. In this chapter, we present a model that fulfills all of the preceding requirements and, in addition, integrates learning dynamics within this macro-finance framework.

The model presented in this chapter builds on recent developments (phases) in the *affine term structure* literature. The first phase is character-

Hans Dewachter is a professor in international economics at the Catholic University of Leuven. Marco Lyrio is an assistant professor of finance at Warwick Business School.

We are grateful for financial support from the FWO-Vlaanderen (Project No. G.0332.01). We thank the discussant, Jordi Galí; the organizer, John Campbell; and the participants at the National Bureau of Economic Research (NBER) Conference on Asset Prices and Monetary Policy for very helpful comments. We also thank Konstantijn Maes; Raf Wouters; and seminar participants at the National Bank of Belgium; 2004 Conference on Computing in Economics and Finance; Heriot-Watt University; the Catholic University of Leuven; Tilburg University; the University of Amsterdam; the European Central Bank; and the University of Warwick for useful discussions and comments on a previous version of this chapter. The authors are responsible for remaining errors.

ized by the use of latent or unobservable factors, as defined in Duffie and Kan (1996) and summarized in Dai and Singleton (2000). Although this framework excludes arbitrage opportunities and is reasonably tractable, the factors derived from such models do not have a direct economic meaning and are simply labeled according to their effect on the yield curve (i.e., as a "level," a "slope," and a "curvature" factor).

The second phase involves the inclusion of macroeconomic variables as factors in the standard affine term structure model. Ang and Piazzesi (2003) show that such an inclusion improves the forecasting performance of vector autoregression (VAR) models in which no-arbitrage restrictions are imposed.[1] Their model, nevertheless, still includes unobservable factors without an explicit macroeconomic interpretation. Kozicki and Tinsley (2001, 2002) indicate the importance of long-run inflation expectations in modeling the yield curve and connect the level factor in affine term structure models to these long-run inflation expectations. This interpretation of the level factor is confirmed by Dewachter and Lyrio (2006), who estimate an affine term structure model based only on factors with a well-specified macroeconomic interpretation. The mentioned papers do not attempt, however, to propose a macroeconomic framework consistent with the pricing kernel implied by their models.

The third and most recent phase in this line of research is marked by the use of structural macro relations together with the standard affine term structure model. The structural macro model replaces the unrestricted VAR setup adopted in previous research and has commonly been based on a New Keynesian framework. Hördahl, Tristani, and Vestin (2006) find that the forecasting performance of such a model is comparable to that of standard latent factor models. They are also able to explain part of the empirical failure of the expectations hypothesis. A similar approach is adopted by Rudebusch and Wu (2003). Bekaert, Cho, and Moreno (2006) go one step further and impose consistency between the pricing kernel and the macro model.

The success of macro-finance models is remarkable given the well-documented dynamic inconsistencies between the long-run implications of the macroeconomic models and the term structure of interest rates.[2] In particular, standard macroeconomic models fail to generate sufficient persistence to account for the time variation at the long end of the yield curve. The success of macro-finance models in fitting jointly the term structure and the macroeconomic dynamics, in fact, crucially hinges on the introduction of additional inert and latent factors with a macroeconomic inter-

1. Other papers using this approach include Dewachter, Lyrio, and Maes (2006) and Diebold, Rudebusch, and Aruoba (2006).

2. For instance, Gürkaynak, Sack, and Swanson (2005) and Ellingsen and Söderström (2001) show that standard macroeconomic models cannot account for the sensitivity of long-run forward rates to standard macroeconomic shocks.

pretation. For instance, Bekaert, Cho, and Moreno (2006), Dewachter and Lyrio (2006), and Hördahl, Tristani, and Vestin (2006), among others, introduce a time-varying (partly) exogenous implicit inflation target of the central bank and show that it accounts for the time variation in long-maturity yields.

The main goal of this chapter is to build and estimate macro-finance models that generate these additional factors endogenously from a macroeconomic framework. To this end, we introduce learning into the framework of standard macro-finance models. Extending macro-finance models with learning dynamics seems a promising route to model jointly the macroeconomic and term structure dynamics for two reasons. First, learning generates endogenously additional and potentially persistent factors in the form of subjective expectations.[3] Second, learning, especially constant gain learning, introduces sufficient persistence in the perceived macroeconomic dynamics to generate a level factor in the term structure of interest rates. Such a level factor is crucial to account for the time variation in the long end of the yield curve.[4]

Our approach connects the macro-finance models of the term structure to the learning literature. Links between learning and the term structure of interest rates are also actively analyzed in the learning literature. For example, Cogley (2005) uses a time-varying Bayesian VAR to account for the joint dynamics of macroeconomic variables and the term structure of interest rates. Kozicki and Tinsley (2005) use a reduced-form VAR in macroeconomic and term structure variables and assume agents have imperfect information with respect to the inflation target. They find that subjective long-run inflation expectations are crucial in fitting movements in long-maturity yields and inflation expectations and report a substantial difference between the central bank's inflation target and the subjective expectations of the inflation target. Orphanides and Williams (2005a) introduce long-run inflation expectations in the structural macroeconomic models by substituting expectations by and calibrating the learning parameters on observed survey data.[5] This chapter complements this recent and rapidly growing literature. First, we do not rely on reduced-form VAR dynamics. Instead, we use a standard New Keynesian model to describe the macroeconomic dimension and impose consistency of the pricing kernel for the term structure and the macroeconomic dynamics. Second, following Sargent and Williams (2005), we generate the subjective expectations based on a learning technology that is optimal given the structural equations and the

3. Milani (2007) finds that the persistence in the learning dynamics is sufficiently strong to capture much of the inertia of the macroeconomic series.

4. Orphanides and Williams (2005a,b), using a calibrated learning model, show that learning affects the long end of the term structure.

5. Other papers using survey expectations as proxies for the theoretical expectations include Roberts (1997) and Rudebusch (2002).

priors of the agents. Third, we estimate jointly the deep parameters of the structural equations and the learning parameters. The term structure of interest rates and surveys of inflation expectations are included as additional information variables in the measurement equation. We find that the proposed model generates sufficiently volatile subjective long-run expectations of macroeconomic variables to account for most of the time variation in long-maturity yields and surveys of inflation expectations. This is achieved without reference to additional latent factors and, hence, offers an alternative approach to the current macro-finance literature.

The remainder of the chapter is divided in four sections. In section 5.2, we present the macroeconomic framework, which is based on a standard New Keynesian macro model. We introduce imperfect information with respect to the endpoints of macroeconomic variables, discuss the respective priors, and derive the optimal learning rule. The perceived and actual laws of motion are derived together with the conditions for stability of the macroeconomic dynamics. The perceived law of motion forms the basis to generate the implied term structures of interest rates and inflation expectations. The estimation methodology is presented in section 5.3. Both the yield curve and surveys of inflation expectations are used as additional information variables to identify subjective expectations. In section 5.4, we present the estimation results and compare the performance of the estimated models in fitting the term structure of interest rates. We show that macro-finance models, built on structural equations and learning, explain a substantial part of the time variation of long-maturity yields and inflation expectations. We conclude in section 5.5 by summarizing the main findings of the chapter.

5.2 Macroeconomic Dynamics

We use the standard monetary three-equation New Keynesian framework as presented in, for instance, Hördahl, Tristani, and Vestin (2006), Bekaert, Cho, and Moreno (2006), and Cho and Moreno (2006). These models can be considered as minimal versions of a fully structural model (e.g., Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2003) and commonly represent the benchmark model in the literature linking macroeconomic dynamics and the term structure. We follow the standard Euler equation procedure employed in the learning literature (Bullard and Mitra 2002; Evans and Honkapohja 2001) and replace the rational expectations operator by a subjective expectations operator.[6] In the following model, subjective expectations differ from rational expectations because we assume that agents do not observe the inflation target of the central

6. An alternative micro-founded approach to learning has been developed by Preston (2005). We leave this extension for future research.

bank nor the equilibrium output-neutral real interest rate. Finally, in section 5.2.3 we solve for the macroeconomic dynamics, that is, the actual law of motion. The solution is given in the form of a reduced VAR(I) model in an extended state space.

5.2.1 Structural Equations

The structural model used is a parsimonious three-equation representation of the underlying macroeconomic structure, containing aggregate supply (AS) and investment saving (IS) equations and a monetary policy rule identifying the riskless nominal interest rate. To account for the persistence in inflation, the output gap, and the policy rate, we add inflation indexation, habit formation, and interest rate smoothing to the standard model.

The design of the AS equation is motivated by the sticky price models based on Calvo (1983). In line with the standard Calvo price-setting theory, we assume a world where only a fraction of the firms updates prices at any given date, while the nonoptimizing firms are assumed to use some rule of thumb (indexation scheme) in adjusting their prices (e.g., Galí and Gertler 1999). This setting leads to a positive relation between (transitory) inflation on the one hand and real marginal costs on the other. Specific assumptions are made with respect to the marginal costs and the indexation scheme of the nonoptimizing agents. First, we assume that marginal costs are proportional to the output gap and an additional cost-push shock, ε_π. Second, nonoptimizing firms are assumed to adjust prices according to an indexation scheme based on past inflation rates. The degree of indexation is measured by the parameter δ_π, and the indexation scheme at time t is given by $\pi^* + \delta_\pi(\pi_{t-1} - \pi^*)$ with π^* the inflation target and π_{t-1} the previous period inflation rate. Following these assumptions, the standard AS curve is given by:

$$\pi_t = c_\pi + \mu_{\pi,1} E_t \pi_{t+1} + \mu_{\pi,2}\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi \varepsilon_{\pi,t}, \tag{1}$$

$$c_\pi = \left[1 - \frac{\delta_\pi}{(1+\psi\delta_\pi)} - \frac{\psi}{(1+\psi\delta_\pi)}\right]\pi^*, \tag{2}$$

$$\mu_{\pi,1} = \frac{\psi}{(1+\psi\delta_\pi)},\ \mu_{\pi,2} = \frac{\delta_\pi}{(1+\psi\delta_\pi)},$$

where ψ represents the discount factor, and κ_π measures the sensitivity of inflation to the output gap. Given the assumed proportionality between marginal costs and output gap, κ_π is a rescaled parameter of the sensitivity of inflation to the real marginal cost. Endogenous inflation persistence, $\mu_{\pi,2} > 0$, arises as a consequence of the assumption that nonoptimizing agents use past inflation in their indexation scheme. Finally, we impose long-run neutrality of output with respect to inflation. Given the setup of the model, this amounts to setting the discount factor (ψ) to one. Long-run neutrality is characterized by inflation parameters in the AS equation adding up to one, implying that $\mu_{\pi,1} = (1 - \mu_{\pi,2})$.

The IS curve is recovered from the Euler equation on private consumption. Following the recent strand of literature incorporating external habit formation in the utility function (e.g., Cho and Moreno 2006), and imposing the standard market clearing condition, we obtain the following IS equation:

$$(3) \qquad y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} + \phi(i_t - E_t \pi_{t+1} - r) + \sigma_y \varepsilon_{y,t},$$

where the parameters μ_y and ϕ are functions of the utility parameters related to the agent's level of relative risk aversion, σ, and (external) habit formation, h:[7]

$$(4) \qquad \mu_y = \frac{\sigma}{\sigma + h(\sigma - 1)}, \quad \phi = -\frac{1}{\sigma + h(\sigma - 1)}.$$

Habit formation is introduced as a means to generate additional output gap persistence. Without habits, that is, $h = 0$, the purely forward-looking IS curve is recovered. The demand shock $\varepsilon_{y,t}$ refers to (independent) shocks in preferences.[8] Equation (3) establishes the interpretation of r as an output-neutral real interest rate. Other things equal, ex ante real interest rate levels $(i_t - E_t\pi_{t+1})$ above r reduce output (and inflation), while for ex ante real interest rates below r, output (and inflation) increases. Although we could allow for time variation in this output-neutral real interest rate, we restrain from doing so in order to avoid additional complexities in the estimation arising from the fact that this variable is unobservable.

We close the model by specifying a monetary policy in terms of a Taylor rule. Following Clarida, Galí, and Gertler (1999), we use a policy rule accounting for both interest rate smoothing and idiosyncratic policy shocks, $\varepsilon_{i,t}$. The monetary policy rate equation is given by:

$$(5) \qquad i_t = (1 - \gamma_{i-1}) i_t^T + \gamma_{i-1} i_{t-1} + \sigma_i \varepsilon_{i,t}.$$

We model the central bank's targeted interest rate, i_t^T, by means of a Taylor rule in the output gap, y_t, and inflation gap, $\pi_t - \pi^*$:

7. We assume the following utility function:

$$U(C_t) = (1 - \sigma)^{-1} G_t \left(\frac{C_t}{H_t} \right)^{1-\sigma},$$

with G_t an independent stochastic preference factor and an external habit level, H_t, specified as $H_t = C_{t-1}^h$. Note that in order to have a well-defined steady state, the habit persistence needs to be restricted, $0 \leq h \leq 1$, as explained in Fuhrer (2000).

8. Note that only by linearly detrending output we obtain a one-to-one relation between the shock in the IS equation and preference (demand) shocks. In general, the interpretation of ε_y as a demand shock is at least partially flawed, given the fact that it might also contain shocks to permanent output.

$$i_t^T = r + E_t\pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t, \tag{6}$$

where π^* denotes the inflation target of the central bank. This policy rule differs from the standard formulation of Taylor rules as we assign a weight of one to the expected inflation term. By imposing this condition, we model explicitly the idea that the central bank is actually targeting an ex ante real interest rate in function of the macroeconomic state, that is, $\pi_t - \pi^*$ and y_t.

The model can be summarized in a standard matrix notation by defining the state space by a vector of macroeconomic variables, $X_t = [\pi_t, y_t, i_t]'$ and a vector of structural shocks, $\boldsymbol{\varepsilon}_t = [\varepsilon_{\pi,t}, \varepsilon_{y,t}, \varepsilon_{i,t}]'$. Using a vector **C** and matrices **A**, **B**, **D**, and **S** of appropriate dimensions, we write the structural equations as:

$$\mathbf{A}\mathbf{X}_t = \mathbf{C} + \mathbf{B}E_t\mathbf{X}_{t+1} + \mathbf{D}X_{t-1} + \mathbf{S}\varepsilon_t. \tag{7}$$

5.2.2 Perceived Law of Motion

The structural model (eq. [7]) is solved under two sets of expectations operators. First, we solve the model under the assumption of rational expectations. The rational expectations solution builds on perfect information of agents with respect to the structural parameters and results in a time-invariant structural VAR representation for the perceived law of motion. Next, we relax the perfect information assumption and extend the perceived law of motion by introducing uncertainty with respect to the endpoints of the macroeconomic variables. This alternative implies a perceived law of motion described in terms of a VECM. Both versions can be seen as special cases of a generic model for expectations, composed of (1) a set of prior beliefs of the agents and (2) an optimal learning rule. In this section, we first describe the priors of the generic model; subsequently, we solve for the optimal learning rule under both rational expectations and the extended set of beliefs. Finally, we discuss the implications of the perceived law of motion for the term structure of interest rates and the term structure of inflation expectations.

Priors and Learning

The beliefs of agents are summarized in terms of a generic model for the macroeconomic dynamics. More specifically, denoting the perceived stochastic trends by ζ_t^P and observable macroeconomic variables by $\mathbf{X}_t$ the priors take the form:

$$\begin{aligned}
\mathbf{X}_t &= \xi_t^P + \boldsymbol{\Phi}^P(\mathbf{X}_{t-1} - \xi_t^P) + \boldsymbol{\Sigma}^P \varepsilon_t \\
\xi_t^P &= V^P\zeta_t^P \\
\zeta_t^P &= \zeta_{t-1}^P + \Sigma_\zeta \upsilon_{\zeta,t}.
\end{aligned} \tag{8}$$

Macroeconomic dynamics can be decomposed into a transitory and permanent component. The permanent component of the dynamics is generated by a set of stochastic trends ζ_t^P. The stochastic trends are generated by a set of permanent shocks, $\upsilon_{\zeta,t}$ with standard deviation:

(9)
$$\Sigma_\zeta = \begin{bmatrix} \sigma_{\zeta,\pi} & 0 & 0 \\ 0 & \sigma_{\zeta,y} & 0 \\ 0 & 0 & \sigma_{\zeta,r} \end{bmatrix}.$$

The stochastic trends determine a set of perceived stochastic endpoints ξ_t^P, $\xi_t^P = V^P \zeta_t^P$, identifying the long-run expectations of the macroeconomic variables, $\mathbf{X}_t$ (see Kozicki and Tinsley 2001):

(10)
$$\xi_t^P = \lim_{s \to \infty} E_t^P X_{t+s}.$$

The perceived transitory dynamics, that is, dynamics relative to the long-run expectations, are modeled in terms of a standard VAR(I) model. More in particular, transitory dynamics are described by the matrix $\mathbf{\Phi}^P$, modeling the inertia and interaction in the transitory dynamics, and $\mathbf{\Sigma}^P$ the impact matrix of the transitory shocks, ε_t. Finally, the priors can be restated in extended state space $\tilde{X}_t = [X_t', \xi_t'^P]$ by defining $\eta_t = [\varepsilon_t', \upsilon_{\zeta,t}']'$ as:

(11)
$$\tilde{X}_t = \tilde{\mathbf{\Phi}}^P \tilde{X}_{t-1} + \tilde{\mathbf{\Sigma}}^P \eta_t$$

or equivalently:

(12)
$$\begin{bmatrix} \mathbf{X}_t \\ \xi_t^P \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}^P & (I - \mathbf{\Phi}^P) \\ 0 & I \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \xi_{t-1}^P \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}^P & (I - \mathbf{\Phi}^P) V \Sigma_\zeta \\ 0 & V \Sigma_\zeta \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \upsilon_{\zeta,t} \end{bmatrix}.$$

In general, we assume that the stochastic endpoints are not observed. Agents, therefore, face an inference problem for the stochastic endpoints ξ_t^P, which is solved by means of a mean squared error (MSE) optimal Kalman filter learning rule. Denoting the inferred values for the stochastic endpoints by $\xi_{t|t}^P$, the learning algorithm becomes:

(13)
$$\xi_{t|t}^P = \xi_{t-1|t-1}^P + K(\mathbf{X}_t - E_{t-1}^P \mathbf{X}_t),$$

where K is obtained as the steady-state solution to the Kalman filtering equations:

(14)
$$\begin{aligned} K_t &= P_{t|t-1}(I - \mathbf{\Phi}^P)' F_t^{-1} \\ F_t &= (I - \mathbf{\Phi}^P) P_{t|t-1} (I - \mathbf{\Phi}^P)' + \mathbf{\Sigma}^P \Sigma^{P\prime} \\ P_{t+1|t} &= P_{t|t-1} - P_{t|t-1}(I - \mathbf{\Phi}^P)' F_t^{-1} (I - \mathbf{\Phi}^P) P_{t|t-1} + \Sigma_\zeta \Sigma_\zeta'. \end{aligned}$$

This perceived law of motion embeds various forms of expectational assumptions. We distinguish between rational expectations and models incorporating imperfect information or credibility with respect to the inflation target and output-neutral real interest rate.

Rational Expectations

Rational expectations are obtained as a specific case of the perceived law of motion. More in particular, rational expectations generated by the preceding structural equations are recovered by two sets of informational assumptions. First, agents believe in deterministic endpoints. Within the context of the preceding structural model endpoints are deterministic, that is, $\Sigma_\zeta = 0$, and are identified by solving the structural model for the steady state. Under the restriction that in the long run no trade-off exists between the output gap and the monetary policy, that is, $\mu_{\pi,1} = (1 - \mu_{\pi,2})$, the steady state of the model is determined by the level of the inflation target, π^*, the steady state of the output gap, y^* (fixed to zero), and the output-neutral real interest rate level, $r = r^*$:

$$\text{(15)} \qquad \xi_t^P = \lim_{s\to\infty} E_t \begin{bmatrix} \pi_{t+s} \\ y_{t+s} \\ i_{t+s} \end{bmatrix} = V \begin{bmatrix} \pi^* \\ y^* \\ r^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi^* \\ y^* \\ r^* \end{bmatrix},$$

where E_t denotes the rational expectations operator. The mapping V is determined by the structural equations. Under rational expectations, the inflation target determines the long-run inflation expectations. The long-run expectations for the output gap are fixed at $y^* = 0$, and the long-run expectations for the nominal interest rate are determined by the Fisher hypothesis, linking the endpoint of the interest rate to the sum of the real interest rate and inflation expectations. Next, rational expectations imply that agents know the structural parameters, such that transitory dynamics, that is, the matrices $\mathbf{\Phi}^P = \mathbf{\Phi}^{re}$ and $\mathbf{\Sigma}^P = \mathbf{\Sigma}^{re}$ are determined by the standard rational expectations conditions:

$$\text{(16)} \qquad \begin{aligned} \mathbf{\Phi}^{re} &= (A - B\mathbf{\Phi}^{re})^{-1}\mathbf{D} \\ \mathbf{\Sigma}^{re} &= (A - B\mathbf{\Phi}^{re})^{-1}\,\mathbf{S}. \end{aligned}$$

The perceived law of motion under rational expectations can be restated in an extended state space as:[9]

$$\text{(17)} \qquad \begin{bmatrix} \boldsymbol{X}_t \\ \xi_t^P \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}^{re} & (I - \mathbf{\Phi}^{re}) \\ 0 & I \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{t-1} \\ \xi_{t-1}^P \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}^{re} \\ 0 \end{bmatrix} \varepsilon_t,$$

with initial condition ξ_0^P

9. Note that the solution can also be written more concisely in structural VAR form as:

$$\boldsymbol{X}_t = C^{re} + \mathbf{\Phi}^{re}\boldsymbol{X}_{t-1} + \Sigma^{re}\varepsilon_t,$$

with $C^{re} = (I - \Phi^{re})\xi_t^{re}$.

$$\xi_0^P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi^* \\ y^* \\ r^* \end{bmatrix}. \tag{18}$$

Finally, note that under rational expectations with deterministic endpoints and perfect information, agents do not face an inference problem. This perfect information assumption (i.e., $\Sigma_\zeta = 0$) generates a Kalman gain $K = 0$ (see eq. [14]). Rational expectations are, therefore, a limiting case of the learning model.

Stochastic Endpoints and Learning

Next to rational expectations, we introduce an alternative set of priors implying stochastic endpoints for the macroeconomic variables. Within the context of the preceding structural model, stochastic endpoints ξ_t^P arise as the consequence of perceived underlying stochastic trends in the economy, $\zeta_t^P = [\pi_t^{*P}, y_t^{*P}, r_t^{*P}]$, representing the vector containing the perceived inflation target, π_t^{*P}, the perceived long-run output gap, y_t^{*P} (fixed to zero), and the perceived long-run output-neutral real interest rate, r_t^{*P}. The size of the perceived time variation in the stochastic trends is measured by Σ_ζ. Denoting the expectations operator under this set of priors by E_t^P, it can be verified that:

$$\lim_{s\to\infty} E_t^P X_{t+s} = \xi_t^P = V\zeta_t^P \tag{19}$$

or equivalently

$$\lim_{s\to\infty} E_t^P \begin{bmatrix} \pi_{t+s} \\ y_{t+s} \\ i_{t+s} \end{bmatrix} = V \begin{bmatrix} \pi_t^{*P} \\ y_t^{*P} \\ r_t^{*P} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^{*P} \\ y_t^{*P} \\ r_t^{*P} \end{bmatrix}. \tag{20}$$

The priors about the transitory dynamics, that is, the dynamics relative to the stochastic endpoints, are assumed to coincide with the ones implied by the rational expectations model. This implies that the matrices $\mathbf{\Phi}^P$ and $\mathbf{\Sigma}^P$ are identical to their rational expectations equivalents: $\mathbf{\Phi}^P = \mathbf{\Phi}^{re}$ and $\mathbf{\Sigma}^P = \mathbf{\Sigma}^{re}$. By equating the perceived transitory dynamics to those implied by the rational expectations model, we obtain a perceived law of motion that differs from the rational expectations solution only due to the introduction of stochastic endpoints. The final perceived law of motion (PLM) can be written in extended state space as:

$$\begin{bmatrix} \mathbf{X}_t \\ \xi_t^P \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}^P & (I - \mathbf{\Phi}^P) \\ 0 & I \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \xi_{t-1}^P \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}^P & (\mathrm{I} - \mathbf{\Phi}^P)V\Sigma_\zeta \\ 0 & V\Sigma_\zeta \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \upsilon_{\zeta,t} \end{bmatrix}. \tag{21}$$

Finally, under this prior, agents face an inference problem and, hence, resort to learning the stochastic endpoints by means of a Kalman filter. The optimal learning rule for this prior is within the class of stochastic gradient rules with the gain defined by the Kalman filter. This gain depends on the specificities of the prior, that is, the specific values for Σ_ζ, $\mathbf{\Sigma}^P$, and $\mathbf{\Phi}^P$. As in Orphanides and Williams (2005a), we assume that agents substitute the unknown stochastic endpoints by the ones inferred by the learning rule.

The Term Structure of Interest Rates

Standard no-arbitrage conditions are used to generate bond prices consistent with the perceived law of motion. Imposing no-arbitrage under the PLM reflects the view that bond prices are set by the private sector and should, therefore, be consistent with the perceived dynamics and information set of these agents. Within the context of default-free, zero-coupon bonds, no-arbitrage implies a pricing equation of the form:

$$P_t(\tau) = E_t^P\,[M_{t+1}P_{t+1}(\tau - 1)], \tag{22}$$

where E^P denotes the subjective expectations operator generated by the PLM (see eq. [11]), $P(\tau)$ denotes the price of a default-free, zero-coupon bond with maturity τ, and M_t denotes the pricing kernel consistent with the PLM. We follow Bekaert, Cho, and Moreno (2006) in using the utility function implied by the macroeconomic framework to identify the prices of risk. While this approach has the advantage of guaranteeing consistency of the pricing kernel, it comes at the cost of loss of flexibility in modeling the prices of risk.[10] The (log) pricing kernel consistent with the PLM is the homoskedastic (log) pricing kernel:

$$m_{t+1} = -i_t - \frac{1}{2}\sigma_m^2 - \Lambda\eta_{t+1}, \tag{23}$$

where the prices of risk, Λ, are determined by the structural parameters

$$\Lambda = \sigma e_y\tilde{\Sigma}^P + e_\pi\tilde{\Sigma}^P - \sigma_y e_y, \tag{24}$$

where $\mathbf{e}_x$ denotes a vector selecting the elements of the x-equation, that is, e_y selects the row of $\tilde{\Sigma}^P$ related to the y-equation. No-arbitrage restrictions imposed on conditional Gaussian and linear state space dynamics generate exponentially affine bond pricing models (see, for instance, Ang and Piazzesi 2003):

$$P(\tau) = \exp\,[a(\tau) + b(\tau)\tilde{\mathbf{X}}_{t|t}], \tag{25}$$

10. The standard approach in modeling the term structure is to assume a essentially affine term structure representation. As shown by Duffee (2002), such representations do not restrict the prices of risk to be constant.

where $\tilde{\mathbf{X}}_{t|t}$ denotes the inferred state vector, obtained by replacing ξ_t^P by its inferred value $\xi_{t|t}^P$, $\tilde{\mathbf{X}}_{t|t} = (X_t', \xi_{t|t}^{P\prime})'$. The factor loadings $a(\tau)$ and $b(\tau)$ can be obtained by solving difference equations representing the set of nonlinear restrictions imposed by the no-arbitrage conditions:

$$(26) \quad a(\tau) = -\delta_0 + a(\tau - 1) - [b(\tau - 1)]\tilde{\Sigma}^P\Lambda' + \frac{1}{2}b(\tau - 1)\tilde{\Sigma}^P\tilde{\Sigma}^{P\prime}b(\tau - 1)'$$

$$b(\tau) = b(\tau - 1)\tilde{\Phi}^P - \delta_1',$$

with $\delta_0 = 0$, and δ_1 implicitly defined by the identity $i_t = \delta_1'\tilde{\mathbf{X}}_{t|t}$. The system has a particular solution given the initial conditions $a(0) = 0$ and $b(0) = 0$.

Exponentially affine bond price models lead to affine yield curve models. Defining the yield of a bond with maturity τ_1 by $y(\tau_1) = -ln[P_t(\tau_1)]/\tau_1$ and the vector of yields spanning the term structure by $\mathbf{Y}_t = [y_t(\tau_1), \ldots, y_t(\tau_n)]'$, the term structure can be written as an affine function of the extended state space variables:

$$(27) \qquad \mathbf{Y}_t = \mathbf{A}_y + \mathbf{B}_y\tilde{\mathbf{X}}_{t|t} + \upsilon_{y,t},$$

where $\mathbf{A}_y$ and $\mathbf{B}_y$ denote matrices containing the maturity-specific factor loadings for the yield curve $\{\mathbf{A}_y = [-a(\tau_1)/\tau_1, \ldots, -a(\tau_n)/\tau_n]'$ and $\mathbf{B}_y = [-b(\tau_1)'/\tau_1, \ldots, -b(\tau_n)'/\tau_n]'\}$, and $\upsilon_{y,t}$ contains maturity-specific measurement errors.

The Term Structure of Inflation Expectations

The representation of the term structure of inflation expectations is obtained by iterating the PLM (eq. [11]) forward. It is straightforward to show that the linearity of the PLM generates an affine representation for the term structure of inflation expectations in the extended state space, $\tilde{\mathbf{X}}_{t|t}$. The term structure of average inflation expectations is described by

$$(28) \qquad E_t^P\overline{\pi}(\tau) = \frac{1}{\tau}\sum_{i=0}^{\tau-1}E_t^P(\pi_{t+i}) = e_\pi[a_s(\tau) + b_s(\tau)\tilde{\mathbf{X}}_{t|t}],$$

where $E_t^P\overline{\pi}(\tau)$ denotes the time t average inflation expectation over the horizon τ, e_π denotes a vector selecting π_t out of the vector $\tilde{\mathbf{X}}_{t|t}$, and $a_e(\tau)$, $b_e(\tau)$, $a_s(\tau)$ and $b_s(\tau)$ are maturity-dependent functions generated by the system:

$$(29) \qquad a_e(\tau) = 0,\ b_e(\tau) = b_e(\tau - 1)\tilde{\Phi}^P$$

$$a_s(\tau) = \frac{1}{\tau}\sum_{i=0}^{\tau-1}a_e(i) = 0, \text{ and } b_s(\tau) = \frac{1}{\tau}\sum_{i=0}^{\tau-1}b_e(i)$$

solved under the initial conditions $a_e(0) = 0$ and $b_e(0) = I$. Equation (28), applied over varying horizons τ, forms the model-implied term structure of average inflation expectations. The term structure of inflation expectations, unlike the term structure of interest rates, is not observable. We use surveys of average inflation expectations for different maturities as a proxy for the term structure of inflation expectations. We relate these surveys,

$s(\tau)$, to the model-implied average inflation expectations by allowing for idiosyncratic measurement errors, $\upsilon_{s,t}(\tau)$, in the survey responses:

$$s_t(\tau) = e_\pi a_s(\tau) + e_\pi b_s(\tau)\tilde{\mathbf{X}}_{t|t} + \upsilon_{s,t}(\tau), \tag{30}$$

where $s_t(\tau)$ denotes the time t survey response concerning the average inflation expectations over the horizon τ. Finally, denoting the vector containing a set of surveys of inflation expectations for different horizons by $\mathbf{S}_t = [s_t(\tau_1), \ldots, s_t(\tau_m)]'$, and defining $A_s = 0$ and $B_s = [[e_\pi b_s(\tau_1)]', \ldots, [e_\pi b_s(\tau_m)]']'$, equation (30) can be restated as:

$$\mathbf{S}_t = A_s + B_s\tilde{X}_{t|t} + \upsilon_{s,t}. \tag{31}$$

5.2.3 Actual Law of Motion

The actual law of motion (ALM), describing the observed dynamics of macroeconomic variables, is obtained by substituting the subjective expectations (eq. [11]) into the structural equations (eq. [7]). Because the subjective expectations are formed on the basis of the inferred stochastic endpoints, $\xi^P_{t|t}$ and on observable macroeconomic data, the relevant space of the ALM coincides with that of the PLM, that is, $\tilde{\mathbf{X}}_{t|t}$. Due to the simplicity of the learning algorithm, the ALM can be solved in closed form. In the appendix, we show that the ALM reduces to a standard VAR(I) in the extended state space:

$$\tilde{\mathbf{X}}_{t|t} = \tilde{C}^A + \tilde{\Phi}^A\tilde{\mathbf{X}}_{t-1|t-1} + \tilde{\Sigma}^A\varepsilon_t, \tag{32}$$

with

$$\tilde{C}^A = \begin{Bmatrix} [A - B(\mathbf{\Phi}^P + K_\Phi)]^{-1}C \\ K[A - B(\Phi^P + K_\Phi)]^{-1}C \end{Bmatrix} \tag{33}$$

$$\tilde{\Phi}^A = \begin{pmatrix} \Phi^P & [A - B(\mathbf{\Phi}^P + K_\Phi)]^{-1}B(I - K_\Phi)(I - \Phi^P) \\ 0 & I - K\{I - [A - B(\mathbf{\Phi}^P + K_\Phi)]^{-1}B(I - K_\Phi)\}(I - \mathbf{\Phi}^P) \end{pmatrix}$$

$$\tilde{\Sigma}^A = \begin{Bmatrix} [A - B(\mathbf{\Phi}^P + K_\Phi)]^{-1}S \\ K[A - B(\mathbf{\Phi}^P + K_\Phi)]^{-1}S \end{Bmatrix},$$

and $K_\Phi = (I - \Phi^P)\,K$, A, B and S and $\mathbf{\Phi}^P$ determined by the parameters of the structural equations, and $\mathbf{K}$ the constant gain matrix implied by the agents' priors. The closed form solution can be used to highlight some of the properties of the ALM. First, subjective beliefs about the stochastic endpoints, $\xi^P_{t|t}$, only affect the actual macroeconomic dynamics if an expectation channel exists, that is, $B \neq 0$ in equation (7). One aspect in which macroeconomic dynamics may be affected by subjective beliefs concerns the modeling of persistence. Under rational expectations, persistence is driven by inflation indexation, habit persistence, and interest rate smoothing affecting the roots of the $\mathbf{\Phi}^{re} = \mathbf{\Phi}^{P}$ matrix. Learning introduces an additional source of persistence in the form of the persistence in the subjective expectations, $\xi^{P}_{t|t}$. Persistence in the beliefs follows itself from the inertia in the learning rule, that is, the updating procedure. Milani (2007) shows in a different context that persistence due to learning is important and (partly) takes over the role of inflation indexation and habit formation. In the empirical section, we find similar results, especially for inflation persistence and interest rate smoothing.

Second, the rational expectations model is nested within the learning framework. By imposing the priors consistent with rational expectations, that is, $\Sigma_{\zeta} = 0$ (implying $K_{\Phi} = 0$) and $\xi^{P}_{t|t} = V[\pi^{*}, 0, r]'$, it can be verified that the ALM simplifies to the rational expectations reduced form, equation (17). Third, the nonstationarity of the PLM does not necessarily carry over to the ALM. The eigenvalues of the matrix $\tilde{\Phi}^{A}$ depend both on the structural parameters contained in A, B, Φ^{P} and on the learning parameters K.[11] Finally, if the ALM is stationary, the unconditional distribution of the extended state space vector $\tilde{\mathbf{X}}_{t|t}$ is identified. Conditional on the maintained assumption of normality of the structural shocks, ε_t, this distribution is given by:

$$\tilde{X}_{t|t} \sim N(E\tilde{\mathbf{X}}_{t|t}, \Omega_{\tilde{X}}), \tag{34}$$

with

$$E\tilde{\mathbf{X}}_{t|t} = \begin{bmatrix} (I - \mathbf{\Phi}^{re})^{-1}C^{re} \\ (I - \mathbf{\Phi}^{re})^{-1}C^{re} \end{bmatrix}$$

$$\mathrm{vec}\,(\Omega_{X}) = (I - \tilde{\mathbf{\Phi}}^{A} \otimes \tilde{\mathbf{\Phi}}^{A})^{-1}\,\mathrm{vec}\,(\tilde{\Sigma}^{A}\tilde{\Sigma}^{A\prime}).$$

Equation (34) represents the unconditional distribution for the extended state under learning. This distribution is characterized by two properties. First, as far as unconditional means are concerned, the ALM and the rational expectations model are observationally equivalent. The unconditional mean of the rational expectations model, that is, $(I - \mathbf{\Phi}^{re})^{-1}C^{re}$, coincides with the unconditional mean under the ALM for both the observable macroeconomic variables (inflation, output gap, and policy rate) and the perceived long-run expectations of the agents. The rational expectations model thus serves as a benchmark in mean for the model under learning. Second, in line with the literature on constant gain learning (e.g., Evans

11. Note that the stationarity of the ALM is inconsistent with the nonstationarity of the PLM under learning. This inconsistency arises from the fact that the ALM is solved under the assumption of a time-invariant inflation target of the central bank. In the empirical section, we allow for time variation in the central bank inflation target. More specifically, we allow for the inflation target to be chairman-specific. This extension renders the ALM nonstationary and partially reconciles the ALM and the PLM dynamics.

and Honkapohja 2001), the unconditional variance of the stochastic endpoints, $\xi^{P}_{t|t}$, is in general positive, implying nonconvergence of the stochastic endpoints to the true values implied by the rational expectations equilibrium, $[\pi^*, 0, r + \pi^*]'$.

5.3 Estimation Methodology

The actual law of motion for both macroeconomic variables and the inferred stochastic endpoints is used to estimate both the structural and the learning parameters. In order to identify the subjective beliefs, we use information variables directly related to the PLM, that is, the term structure of interest rates and inflation expectations. In section 5.3.1, we discuss the details of the estimation procedure. Subsequently, in section 5.3.2, we explain the different versions of the model that are estimated.

5.3.1 Maximum Likelihood Estimation

The model is estimated by means of log-likelihood in the extended state space with the ALM dynamics serving as the transition equation:

$$\tilde{\mathbf{X}}_{t|t} = \tilde{C}^A + \tilde{\mathbf{\Phi}}^A \tilde{X}_{t-1|t-1} + \tilde{\Sigma}^A \varepsilon_t \tag{35}$$

and a measurement equation, relating the extended state to observable economic variables. The observable variables included in the measurement equation consist of macroeconomic variables, X_t (inflation, output gap, and policy rate), a sample of yields spanning the term structure of interest rates, Y_t (one-, two-, three-, four-, five-, and ten-year yields), and a sample of the term structure of inflation expectations, proxied by survey data on inflation expectations, S_t (one- and ten-year average inflation expectations).[12] The observable variables are collected in the vector $Z_t = [X_t', Y_t', S_t']'$. Using the affine representation of each of these variables in the extended state space, as discussed in section 5.2.2, the measurement equation becomes:

$$\mathbf{Z}_t = A_m + B_m \tilde{X}_{t|t} + \upsilon_{z,t}, \tag{36}$$

where $\upsilon_{z,t}$ denotes idiosyncratic measurement errors with variance-covariance matrix $\mathbf{\Psi}$, and A_m and B_m represent the derived affine representations of the respective subsets of observable variables X_t, Y_t and S_t (B_x is defined as $X_t = B_x\tilde{X}_{t|t}$, i.e., $B_x = [I_{3\times3}, 0_{3\times3}]$, A_y, A_s, B_y and B_s are defined in equations [27] and [31], respectively):

12. Survey expectations are increasingly used in the empirical literature. Roberts (1997) shows that models including survey expectations can account for some of the (unexplained) inflation inertia. Survey expectations are also starting to be used in the bond pricing literature. Kim and Orphanides (2005) use survey expectations on short-term interest rate movements as an additional input in a otherwise standard Vasicek model. Also, Chun (2005) uses several survey expectations as additional inputs in a two-factor term structure model. Finally, Bekaert, Cho, and Moreno (2006) show the empirical relevance of surveys (on inflation) by showing that surveys help to forecast inflation better than any rational expectations model.

$$A_m = \begin{bmatrix} 0 \\ A_y \\ A_s \end{bmatrix}, B_m = \begin{bmatrix} B_x \\ B_y \\ B_s \end{bmatrix} \text{ and } \mathbf{\Psi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{\Psi}_y & 0 \\ 0 & 0 & \mathbf{\Psi}_s \end{bmatrix}.$$

Prediction errors, $\mathbf{Z}_t - E^A_{t-1}Z_t$, and their corresponding log-likelihood value $l(\mathbf{Z}_t - E^A_{t-1}\mathbf{Z}_t; \theta)$, where E^A_{t-1} denotes the expectations operator based on the ALM, are functions of both the structural macroeconomic shocks and the measurement errors:

$$\text{(37)} \qquad \mathbf{Z}_t - E^A_{t-1}Z_t = B_m(\tilde{\mathbf{X}}_{t|t} - E^A_{t-1}\tilde{\mathbf{X}}_{t|t}) + \upsilon_{z,t} = B_m(\tilde{\Sigma}^A\varepsilon_t) + \upsilon_{z,t}$$

$$l(\mathbf{Z}_t - E^A_{t-1}\mathbf{Z}_t; \theta) = -\frac{1}{2}|\Omega_Z| - \frac{1}{2}(\mathbf{Z}_t - E^A_{t-1}\mathbf{Z}_t)'\Omega_Z^{-1}(\mathbf{Z}_t - E^A_{t-1}\mathbf{Z}_t)$$

$$\Omega_Z = B_m\tilde{\Sigma}^A\tilde{\Sigma}^{A\prime}B_m' + \mathbf{\Psi}.$$

One contribution of this chapter is that the deep parameters of the structural equations and the parameters of the learning procedure are estimated jointly based on a wide variety of information variables, that is, macroeconomic variables, term structure variables, and surveys of inflation expectations.[13] The parameters to be estimated are collected in the parameter vector θ, containing the deep parameters of the structural equations (δ_π, κ_π, σ, h, r, π^*, γ_π, γ_y, γ_{i-1}, σ_π, σ_y, σ_i), the learning parameters (priors on the volatility of the stochastic trends $\sigma_{\zeta,\pi}$, $\sigma_{\zeta,r}$, and initial values $\zeta_{0|0}$), and the variances of the measurement errors [$diag(\mathbf{\Psi})$]:

$$\text{(38)} \qquad \boldsymbol{\theta} = [\delta_\pi, \kappa_\pi, \sigma, h, r, \pi^*, \gamma_\pi, \gamma_y, \gamma_{i-1}, \sigma_\pi, \sigma_y, \sigma_i, \sigma_{\zeta,\pi}, \sigma_{\zeta,r}, \zeta_{0|0}, \text{diag}(\mathbf{\Psi})].$$

Not all deep parameters and learning parameters are estimated. We follow Hördahl, Tristani, and Vestin (2006) and Bekaert, Cho, and Moreno (2006) by fixing the discount factor to one, $\psi = 1$. Also, throughout the estimation the prior on the uncertainty of the long-run value for the output gap is restricted to zero, $\sigma_{\zeta,y} = 0$. This restriction guarantees that the long-run expected output gap is fixed to zero under the PLM. Furthermore, we impose the theoretical constraints $\sigma_{\zeta,\pi}, \sigma_{\zeta,r} \geq 0$ and $0 \leq h \leq 1$. Finally, parameter estimates are constrained to satisfy two conditions. First, parameter estimates must be consistent with the existence of a unique rational expectations solution. Second, under learning, parameter estimates should imply eigenvalues of $\tilde{\mathbf{\Phi}}^A$ strictly smaller than one in absolute value in order

13. Other research estimating learning parameters include Orphanides and Williams (2005a) and Milani (2007). Orphanides and Williams (2005a) estimate the constant gain by minimizing the distance between the model-implied inflation expectations and those reported in the survey of professional forecasters. Milani (2007) estimates jointly, using Bayesian methods, the constant gain and the deep parameters of a structural macroeconomic model. We complement their analyses by including more information in the measurement equation, notably term structure of interest rates.

to guarantee stability of the ALM. The model is estimated using a Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.

5.3.2 Estimated Versions of the Model

We estimate a total of eight models. Model versions differ depending on (1) the type of information included in the measurement equation, (2) the assumptions concerning the learning procedure, (3) the time variation in the inflation target, and (4) the prices of risk. Four versions are based on the baseline model presented in the previous section, two versions are extensions allowing for heterogeneity in the monetary policy, and the final two versions allow for more flexibility in the prices of risk.

Regarding the information included in the measurement equation, we distinguish between the macro and the general versions of the model. In the macro version, we restrict the measurement equation to incorporate only macroeconomic information, while in the general version, we include all available information. The macro version of the model is motivated by the concern that including term structure and survey information in the measurement equation may bias the estimates of the deep and learning parameters in order to fit the term structure and the survey expectations. To avoid this problem, a two-step procedure is employed. In the first step, the deep and learning parameters are estimated while restricting the measurement equation to contain only macroeconomic variables. In the second step, we fix the parameter estimates for the deep and learning parameters obtained in the first step and optimize the likelihood based on the full measurement equation over the remaining parameters, $diag(\boldsymbol{\Psi})$. In the general versions of the model, the estimation of all parameters is performed in one step on the basis of the most general measurement equation.

We estimate both rational expectations and learning versions of the model. The learning versions of the model include four additional parameters describing the priors of the agents, $\sigma_{\zeta,\pi}$, $\sigma_{\zeta,r}$, and the starting values for the stochastic trends, $\zeta_{0|0}$. The distinction between rational expectations and learning models identifies the contribution of learning to the overall fit of the respective series. The four baseline models can be summarized as follows:

- Rational Expectations Macro: The rational expectations version is estimated using a two-step procedure ensuring that the deep parameters are based only on macroeconomic information.
- Rational Expectations I: The rational expectations version is estimated using a one-step procedure based on the general measurement equation.
- Learning Macro: The learning version is estimated using a two-step procedure ensuring that the deep parameters are based only on macroeconomic information.

- Learning I: The learning version is estimated using a one-step procedure based on the general measurement equation.

In addition to the four baseline models, we estimate two extensions to allow for heterogeneity in the monetary policy rule and in the agents' priors. The heterogeneity is modeled by means of chairman-specific policy rules and priors.[14] Specifically, the time-invariant policy rule parameters π^*, γ_π, γ_y and γ_{i-1} of the baseline models are replaced by chairman-specific parameters π^*_j, $\gamma_{\pi,j}$, $\gamma_{y,j}$ and $\gamma_{i-1,j}$, where j denotes the presiding chairman.[15] The heterogeneity in priors is modeled analogously by replacing the learning parameters $\sigma_{\zeta,\pi}$ and $\sigma_{\zeta,r}$ by their chairman-specific equivalents, $\sigma_{\zeta,\pi,j}$ and $\sigma_{\zeta,r,j}$. We estimate both the rational expectations version of this model, labeled Rational Expectations II, and the learning version of the model, labeled Learning II. The model versions Rational Expectations I and Learning I, implying time-invariant policy rules and beliefs, are nested in the respective extensions and, hence, identify the contribution of allowing for policy heterogeneity in the overall fit.[16] Finally, the last two models, Rational Expectations III and Learning III, extend the models Rational Expectations II and Learning II by allowing for time variation in the prices of risk. In these versions, we disregard consistency of the princing kernel with the IS curve and posit an affine function for the prices of risk: $\Lambda_t = \Lambda_0 + \Lambda_1 \tilde{\mathbf{X}}_{t|t}$.[17]

5.4 Estimation Results

5.4.1 Data

We estimate the proposed models using quarterly data for the United States. The data covers the period from 1963:Q4 until 2003:Q4 (161 quarterly observations). The data set contains three series of macroeconomic

14. This procedure differs from other research that allows for time variation in the inflation target. For instance, Dewachter and Lyrio (2006), Kozicki and Tinsley (2005), and Hördahl, Tristani, and Vestin (2006) allow for variation in the inflation target of the central bank by modeling the inflation target as a inert autoregressive process. This approach results in quite variable inflation target dynamics. In contrast, this chapter allows for discrete jumps in the inflation target at prespecified dates. Beyond these dates, the inflation target is constant.

15. The chairmen included in the analysis are Martin (1951–1970), Burns (1970–1978), Miller (1978–1979), Volcker (1979–1987), and Greenspan (1987–2006). We divide the Volcker period in two subperiods in order to account for the well-documented change in monetary policy that took place during this term, that is, the change from monetary targeting to a more convential monetary policy. The first Volcker period ends in 1982:Q3.

16. For an analysis of regime changes on monetary policy, see Schorfheide (2005) or Sims and Zha (2006). Both papers make use of Markov switching techniques identifying the regime breaks endogeneously. We, in contrast, fix the dates of the breaks to the moments of a change in the Fed chairman.

17. Note that allowing for time-varying prices of risk adds 42 parameters to be estimated. In order to keep the estimation tractable, we restrict Λ_1 to be diagonal.

observations: quarter-by-quarter inflation (based on the gross domestic product [GDP] deflator and collected from the National Income and Product Accounts), the output gap (constructed as the log of GDP minus the log of the natural output level, based on Congressional Budget Office data), and the Federal funds rate, representing the policy rate. Next to the macroeconomic variables, the data set includes six yields with maturities of one, two, three, four, five, and ten years. The data for yields up to five years are from the Center for Research in Security Prices (CRSP) database.[18] The ten-year yields were obtained from the Federal Reserve. Finally, we also use survey data on short- and long-run inflation expectations. More specifically, we include the one- and ten-year average inflation forecast, as reported by the Federal Reserve Bank of Philadelphia in the Survey of Professional Forecasters.

Table 5.1 presents descriptive statistics on the data set described in the preceding, which are depicted in figure 5.1. These statistics point to the usual observations: the average term structure is upward sloping; the volatility of yields decreases with maturity; normality is rejected for all series (based on Jarque-Bera [JB] statistics); and all variables display significant inertia, with a first-order autocorrelation coefficient typically higher than 0.90. Inflation displays a somewhat lower inertia, that is, an autocorrelation coefficient of 0.76.

Table 5.1 also presents the correlation structure of the data. Three data features can be highlighted. First, the yields are extremely correlated across the maturity spectrum. This points to the well-known fact that a limited number of factors account for the comovement of the yields. Second, there is a strong correlation between the term structure and the macroeconomic variables, with significant positive correlations between inflation and the term structure and significant negative correlations between the term structure and the output gap. These correlation patterns are an indication of common factors driving macroeconomic and yield curve dynamics. Finally, we observe a substantial and positive correlation between the surveys of inflation expectations and both the macroeconomic variables (especially inflation and the Federal funds rate) and the yield curve. Again, this suggests that the factors affecting the yield curve and macroeconomic variables also drive movements in the surveys of inflation expectations.

5.4.2 Parameter Estimates

Tables 5.2–5.4 report the estimation results for the rational expectations versions of the model. Our estimates for the macro model (table 5.2) are broadly in line with the literature. We observe a mild domination of the forward looking terms for both the AS and IS curves ($\mu_{\pi,1} = 0.524$ and

18. We thank Geert Bekaert, Seonghoon Cho, and Antonio Moreno for sharing the data set.

Table 5.1 Summary of data statistics (United States, 1963:Q4–2003:Q4, 161 observations)

	π	y	i	$\bar{y}_{1y}$	$\bar{y}_{2y}$	$\bar{y}_{3y}$	$\bar{y}_{4y}$	$\bar{y}_{5y}$	$\bar{y}_{10y}$	S_{1y}	S_{10y}
Mean (%)	4.49**	–1.11**	6.63**	6.52**	6.73**	6.90**	7.03**	7.11**	7.42**	4.18**	4.01**
SD (%)	2.83**	2.59**	3.28**	2.73**	2.67**	2.58**	2.53**	2.48**	2.47**	1.98**	1.49**
Auto	0.76**	0.95**	0.91**	0.93**	0.94**	0.95**	0.95**	0.96**	0.96**	0.98**	0.96**
Skew	1.49**	–0.47**	1.21**	0.78**	0.81**	0.85**	0.90**	0.88**	0.99**	0.83**	1.14**
Kurt	5.51**	3.51**	5.20**	4.06**	3.96**	3.92**	3.95**	3.67**	3.66**	2.79**	3.72**
JB	101.85	7.61	71.59	23.84	23.88	24.84	27.92	23.63	29.07	15.88	23.02
	(0.00)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
					Correlation matrix						
π	1.00										
y	–0.25**	1.00									
i	0.72**	–0.25**	1.00								
$\bar{y}_{1y}$	0.67**	–0.31**	0.95**	1.00							
$\bar{y}_{2y}$	0.65**	–0.39**	0.93**	0.99**	1.00						
$\bar{y}_{3y}$	0.62**	–0.44**	0.90**	0.98**	0.99**	1.00					
$\bar{y}_{4y}$	0.61**	–0.48**	0.89**	0.96**	0.99**	0.99**	1.00				
$\bar{y}_{5y}$	0.59**	–0.51**	0.87**	0.95**	0.98**	0.99**	0.99**	1.00			
$\bar{y}_{10y}$	0.59	–0.57	0.84	0.92**	0.96**	0.98**	0.99**	0.99**	1.00		
S_{1y}	0.83**	–0.46**	0.77**	0.77**	0.76**	0.74**	0.75**	0.73**	0.73**	1.00	
S_{10y}	0.80**	–0.63**	0.86**	0.86**	0.87**	0.870**	0.88**	0.88**	0.90**	0.98**	1.00

Notes: Inflation (π) is expressed in annual terms and is constructed by taking the quarterly percentage change in the GDP deflator (collected from the National Income and Product Accounts). The output gap (y) series is constructed from data provided by the Congressional Budget Office (CBO). The Fed rate is used as the short-term interest rate or the policy rate (1). Bond yield data concern month-end yields on zero-coupon U.S. Treasury bonds, expressed in annual terms. Data on yields up to five years are based on CRSP data and on ten-year yields based on the Federal Reserve. Mean = the sample arithmetic average in percentage p.a.; SD = standard deviation; Auto = the first order quarterly autocorrelation; Skew = skewness; Kurt = kurtosis; and JB = the Jarque-Bera normality test statistic with the significance level at which the null of normality may be rejected underneath it.

**Significant at the 5 percent confidence level.

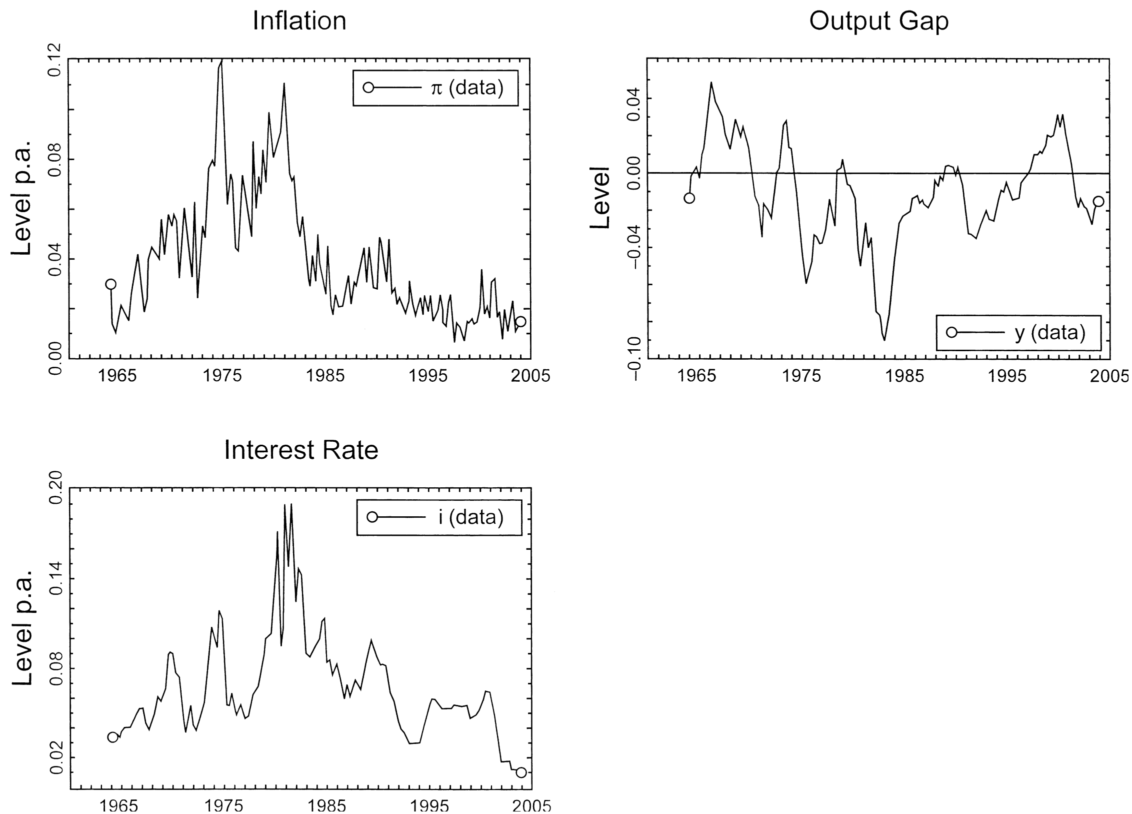

Fig. 5.1 Data, United States, 1963:Q4–2003:Q4 (161 observations)

Table 5.2 **Parameter estimates: Rational Expectations Macro and Rational Expectations I**

$$\pi_t = \mu_{\pi,1} E_t \pi_{t+1} + (1 - \mu_{\pi,1})\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi \varepsilon_{\pi,t}$$
$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} + \phi(i_t - E_t \pi_{t+1} - r) + \sigma_y \varepsilon_{y,t}$$
$$i_t = (1 - \gamma_{i-1})[r + E_t \pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t] + \gamma_{i-1} i_{t-1} + \sigma_i \varepsilon_{i,t}$$

		Rat. Exp. Macro		Rat. Exp. I	
π-eq.	$\mu_{\pi,1}$	0.524**	(0.019)	0.527**	(0.007)
	$\kappa_\pi(\times 10^2)$	0.055	(0.278)	0.582**	(0.236)
y-eq.	μ_y	0.509**	(0.013)	0.580**	(0.018)
	ϕ	–0.019*	(0.011)	–0.012**	(0.005)
i-eq.	γ_{i-1}	0.862**	(0.036)	0.934**	(0.004)
	γ_π	0.674*	(0.356)	0.100	(0.165)
	γ_y	0.569	(0.504)	0.010	(0.172)
	r	0.025**	(0.010)	0.028**	(0.002)
	π^*	0.032**	(0.011)	0.044**	(0.002)
SD	σ_π	0.0063**	(0.0004)	0.0069**	(0.0004)
	σ_y	0.0043**	(0.0003)	0.0070**	(0.0009)
	σ_i	0.0133**	(0.0004)	0.0134**	(0.0005)
Struct	δ_π	0.908**	(0.069)	0.895**	(0.025)
	h	1.000	—	0.738**	(0.055)
	$\sigma(\times 10^{-2})$	0.274	(0.170)	0.496**	(0.201)

Notes: SD = standard deviation; Struct = structural parameters. Maximum likelihood estimates with standard errors in parentheses. Dash indicates standard deviation was not computed.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

$\mu_y = 0.509$, respectively). The deviation from the purely forward-looking model ($\mu_{\pi,1} = 1$ and $\mu_y = 1$) is explained by the relatively high values for the inflation indexation parameter, δ_π, and the habit persistence, h, estimated at 0.908 and 1, respectively. Both estimates for the inflation sensitivity to the output gap, κ_π, and the output gap sensitivity to the real interest rate, ϕ, are small, 0.00055 and –0.019, respectively. Although these values are smaller than the ones typically used in calibration-based studies, they are commonly found in empirical studies using generalized method of moments (GMM) or full information maximum likelihood (FIMI) methods. Our estimates imply an active monetary policy rule. The ex ante real interest rate reacts positively to both inflation and the output gap, $\gamma_\pi = 0.674$ and $\gamma_y = 0.569$. Significant interest rate smoothing is also observed in the policy rule ($\gamma_{i-1} = 0.862$). As often found in the literature, some of the estimated parameters are not statistically significant. Similar results have been reported, for instance, by Cho and Moreno (2006).

Table 5.3 Parameter estimates: Rational Expectations II

$$\pi_t = \mu_{\pi,1}E_t\pi_{t+1} + (1-\mu_{\pi,1})\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi\varepsilon_{\pi,t}$$
$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y)y_{t-1} + \phi(i_t - E_t\pi_{t+1} - r) + \sigma_y\varepsilon_{y,t}$$
$$i_t = (1-\gamma_{i-1})\,[r + E_t\pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t] + \gamma_{i-1}i_{t-1} + \sigma_i\varepsilon_{i,t}$$

		Rational Expectations II					
π-eq.	$\mu_{\pi,1}$	0.598**	(0.010)				
	$\kappa_\pi(\times 10^2)$	0.627**	(0.256)				
y-eq.	μ_y	0.589**	(0.018)				
	ϕ	–0.020**	(0.008)				
	r	0.029**	(0.002)				
		Martins		Burns		Miller	
i-eq.	γ_{i-1}	0.863**	(0.033)	0.612**	(0.032)	0.782**	(0.515)
	γ_π	0.745	(0.537)	0.244**	(0.109)	0.374	(1.605)
	γ_y	0.397	(0.425)	0.735**	(0.181)	1.131	(2.044)
	π^*	0.018**	(0.003)	0.038**	(0.002)	0.055**	(0.012)
		Volcker (a)		Volcker (b)		Greenspan	
i-eq.	γ_{i-1}	0.840**	(0.009)	0.959**	(0.022)	0.951**	(0.011)
	γ_π	0.541**	(0.212)	2.301	(2.091)	1.676**	(0.847)
	γ_y	0.531**	(0.222)	–1.189**	(0.505)	1.674**	(0.717)
	π^*	0.082**	(0.002)	0.052**	(0.001)	0.032**	(0.001)
SD	σ_π	0.0071**	(0.0004)				
	σ_y	0.0069**	(0.0008)				
	σ_i	0.0194**	(0.0017)				
Struct	δ_π	0.673**	(0.029)				
	h	0.721**	(0.052)				
	$\sigma(\times 10^{-2})$	0.294**	(0.115)				

Note: See table 5.2 notes.
**Significant at the 5 percent level.

Extending the standard macro model by (1) including yield curve and inflation survey data in the measurement equation (Rational Expectations I, II, and III); (2) allowing for chairman-specific monetary policy (Rational Expectations II and III); and (3) introducing time variation in the prices of risk (Rational Expectations III) affects the parameter estimates significantly. First, the estimated persistence decreases, as shown by the decrease in the indexation parameter δ_π, which takes a value of 0.67 and 0.57 in the Rational Expectations II and III models, respectively, and by the decrease in the habit persistence, h, for the Rational Expectations I, II, and III models to 0.738, 0.721, and 0.750, respectively. As a result of the drop in the indexation or the habit persistence, the forward-looking components

Table 5.4 Parameter estimates: Rational Expectations III

$$\pi_t = \mu_{\pi,1} E_t \pi_{t+1} + (1 - \mu_{\pi,1})\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi \varepsilon_{\pi,t}$$
$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} + \phi(i_t - E_t \pi_{t+1} - r) + \sigma_y \varepsilon_{y,t}$$
$$i_t = (1 - \gamma_{i-1})[r + E_t \pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t] + \gamma_{i-1} i_{t-1} + \sigma_i \varepsilon_{i,t}$$

		Rational Expectations III					
π-eq.	$\mu_{\pi,1}$	0.638**	(0.017)				
	$\kappa_\pi(\times 10^2)$	0.925*	(0.549)				
y-eq.	μ_y	0.582**	(0.013)				
	ϕ	–0.025**	(0.012)				
	r	0.020**	(0.006)				
		Martins		Burns		Miller	
i-eq.	γ_{i-1}	0.845**	(0.029)	0.482**	(0.041)	0.654**	(0.289)
	γ_π	0.511	(0.681)	0.176	(0.170)	0.233	(1.391)
	γ_y	0.363	(0.346)	0.516**	(0.248)	0.846	(0.991)
	π^*	0.018**	(0.005)	0.047**	(0.002)	0.060**	(0.006)
		Volcker (a)		Volcker (b)		Greenspan	
i-eq.	γ_{i-1}	0.779**	(0.029)	0.943**	(0.034)	0.928**	(0.026)
	γ_π	0.317	(0.308)	1.657	(1.759)	1.338	(0.949)
	γ_y	0.395*	(0.221)	–0.239	(0.396)	1.092	(0.692)
	π^*	0.078**	(0.003)	0.049**	(0.002)	0.032**	(0.001)
SD	σ_π	0.0081**	(0.0006)				
	σ_y	0.0053**	(0.0004)				
	σ_i	0.0136**	(0.0008)				
Struct	δ_π	0.567**	(0.042)				
	h	0.750**	(0.044)				
	$\sigma(\times 10^{-2})$	0.233**	(0.107)				
Prices of risk	$\Lambda_{0,\pi}$	–0.974	(0.612)				
	$\Lambda_{0,y}$	–0.271	(0.389)				
	$\Lambda_{0,i}$	0.045	(0.136)				
	$\Lambda_{1,\pi,\pi}$	0.067	(7.131)				
	$\Lambda_{1,y,y}$	–0.573	(14.905)				
	$\Lambda_{1,i,i}$	3.010	(2.152)				

Notes: See table 5.2 notes.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

($\mu_{\pi,1}$ and μ_y) in the AS and IS equation increase. The estimates of monetary policy rule indicate for all versions of the model that (1) monetary policy is relatively inert, and (2) the Taylor principle is satisfied because the ex ante real interest rate tends to increase with both the inflation gap and the output gap. Nevertheless, the estimated inflation and output gap responses vary across the alternative versions of the model. Based on the results in table 5.3 and 5.4, we find, as do Clarida, Galí, and Gertler (2000), a strong increase in the responsiveness to the inflation gap during the Volcker and Greenspan periods.

Tables 5.5 to 5.7 report the estimation results for the versions where learning is introduced. The central parameters in the analysis, distinguishing learning models from rational expectations models, are the standard deviations of the perceived stochastic trends ζ_t^P, $\sigma_{\zeta,\pi}$, and $\sigma_{\zeta,r}$.[19] Our estimates for these parameters are statistically significant, indicating a rejection of rational expectations models. This finding holds irrespective of the version of the learning model and indicates the importance of the learning specification in modeling the joint dynamics of the macroeconomic variables, the yield curve, and the survey expectations. One interpretation of the parameters $\sigma_{\zeta,\pi}$ and $\sigma_{\zeta,r}$ is in terms of the perceived uncertainty with respect to the endpoints of the macroeconomic state. The estimates of the parameters $\sigma_{\zeta,\pi}$ and $\sigma_{\zeta,r}$ in the Learning II and III versions of the model indicate substantial time variation in the uncertainty with respect to the inflation and real interest rate endpoints. Interestingly, we find uncertainty for both inflation and real interest rate endpoint to be significantly lower during the Greenspan term than under previous chairmen. The introduction of learning dynamics affects significantly the estimates of the deep parameters relative to those obtained for the rational expectations counterparts. First, across learning models, we find that the forward-looking component in the AS equation ($\mu_{\pi,1}$) increases substantially and significantly relative to the rational expectations versions of the model. This increase is explained by the decrease in the inflation indexation parameter.[20] The interest rate smoothing parameter drops significantly to values on average around 0.8 in the learning cases, which are more in line with Rudebusch (2002). A second effect of learning is the increase in the inflation sensitivity to the output gap. We estimate κ_π levels of 0.05, 0.012, and 0.009 in the Learning I, II, and III models, respectively. Finally, note that one problematic feature of the estimation across learning specifications is the identification of the inflation targets, π_j^*, and the real interest rate r, which present large standard errors. This drop in significance can be attributed to the fact that the stochastic endpoints take over the role of these parameters in the expectation formation process.

Figures 5.2 to 5.4 plot the macro variables and their endpoints for each model. Endpoints, representing long-run (subjective) expectations, are de-

19. Note that the parameter $\sigma_{\zeta,y}$ was fixed to zero for consistency with the assumption of long-run neutrality of output (see section 5.2).

20. The decrease in the inflation indexation as a consequence of the introduction of learning is also found in other studies. For instance, Milani (2007), introducing constant gain learning in a New Keynesian macroeconomic model, finds an even stronger effect, with the inflation indexation parameter close to zero after the introduction of learning.

Table 5.5 **Parameter estimates: Learning Macro and Learning I**

$$\pi_t = \mu_{\pi,1}E_t\pi_{t+1} + (1-\mu_{\pi,1})\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi\varepsilon_{\pi,t}$$
$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y)y_{t-1} + \phi(i_t - E_t\pi_{t+1} - r) + \sigma_y\varepsilon_{y,t}$$
$$i_t = (1-\gamma_{i-1})[r + E_t\pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t] + \gamma_{i-1}i_{t-1} + \sigma_i\varepsilon_{i,t}$$

		Learning Macro		Learning I	
π-eq.	$\mu_{\pi,1}$	0.672**	(0.056)	0.759**	(0.023)
	$\kappa_\pi(\times 10^2)$	0.431	(0.504)	4.912	(3.184)
y-eq.	μ_y	0.504**	(0.028)	0.541**	(0.007)
	ϕ	–0.008	(0.023)	–0.038**	(0.016)
i-eq.	γ_{i-1}	0.833**	(0.039)	0.671**	(0.009)
	γ_π	0.401	(0.300)	0.149	(0.097)
	γ_y	0.504	(0.416)	0.363**	(0.046)
	r	0.028	(0.265)	0.030	(0.053)
	π^*	0.031	(0.669)	0.036	(0.360)
SD	σ_π	0.0062**	(0.0005)	0.0087**	(0.0006)
	σ_y	0.0043**	(0.0003)	0.0050**	(0.0003)
	σ_i	0.0132**	(0.0004)	0.0126**	(0.0004)
Struct	δ_π	0.489**	(0.123)	0.318**	(0.039)
	h	1.000	—	0.913**	(0.036)
	$\sigma(\times 10^{-2})$	0.618	(1.778)	0.142**	(0.058)
Learning	$\sigma_{\zeta,\pi}$	0.044**	(0.013)	0.015**	(0.001)
	$\sigma_{\zeta,y}$	0.000	—	0.000	—
	$\sigma_{\zeta,r}$	0.043	(0.141)	0.023**	(0.001)
Initial points	$\xi_{0,\pi}$	0.018	(0.022)	0.012**	(0.005)
	$\xi_{0,y}$	0.000	—	0.000	—
	$\xi_{0,i}$	0.014	(0.022)	0.042**	(0.004)

Notes: See table 5.2 notes.
**Significant at the 5 percent level.

terministic in the rational expectations models and stochastic in the learning cases. As figure 5.2 shows, in the presence of learning long-run inflation expectations are time varying and therefore different from the constant central bank's inflation target (around 3 to 4 percent per year). These endpoints are also remarkably similar across model specifications. Allowing for chairman-specific policy rules (models II and III) leads to significantly different inflation targets across rational expectations and learning models. In the Rational Expectations models II and III, the estimated inflation targets show a gradual increase over the seventies until the end of the Volcker experiment, subsequently decreasing over time (around 5 percent in the second Volcker period and 3.2 percent in the Greenspan term). This gradual decline in inflation targets seems unrealistic given the strong deflationary

Table 5.6 **Parameter estimates: Learning II**

$$\pi_t = \mu_{\pi,1} E_t \pi_{t+1} + (1 - \mu_{\pi,1})\pi_{t-1} + \kappa_\pi y_t + \sigma_\pi \varepsilon_{\pi,t}$$
$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} + \phi(i_t - E_t \pi_{t+1} - r) + \sigma_y \varepsilon_{y,t}$$
$$i_t = (1 - \gamma_{i-1})\,[r + E_t \pi_{t+1} + \gamma_\pi(\pi_t - \pi^*) + \gamma_y y_t] + \gamma_{i-1} i_{t-1} + \sigma_i \varepsilon_{i,t}$$

		Learning II					
π-eq.	$\mu_{\pi,1}$	0.728**	(0.027)				
	$\kappa_\pi(\times 10^2)$	1.182**	(0.323)				
y-eq.	μ_y	0.528**	(0.008)				
	ϕ	–0.022**	(0.008)				
	r	0.026	(0.100)				
		Martins		Burns		Miller	
i-eq.	γ_{t-1}	0.804**	(0.046)	0.268**	(0.047)	0.637**	(0.388)
	γ_π	0.406	(1.150)	0.161	(0.111)	0.244	(1.014)
	γ_y	0.012	(0.293)	0.513**	(0.060)	0.310	(0.315)
	π^*	0.028	(0.265)	0.087	(0.621)	0.053	(0.478)
Learning	$\sigma_{\zeta,\pi}$	0.018**	(0.003)	0.014**	(0.002)	0.019**	(0.006)
	$\sigma_{\zeta,y}$	0.000	—	0.000	—	0.000	—
	$\sigma_{\zeta,r}$	0.007	(0.008)	0.016**	(0.002)	0.019	(0.040)
		Volcker (a)		Volcker (b)		Greenspan	
i-eq.	γ_{i-1}	0.185**	(0.049)	0.795**	(0.022)	0.850**	(0.018)
	γ_π	0.564**	(0.095)	0.353	(0.290)	0.405	(0.630)
	γ_y	0.109	(0.079)	0.369**	(0.122)	0.224	(0.210)
	π^*	0.003	(0.177)	0.010	(0.285)	0.025	(0.266)
Learning	$\sigma_{\zeta,\pi}$	0.004	(0.004)	0.018**	(0.002)	0.008**	(0.002)
	$\sigma_{\zeta,y}$	0.000	—	0.000	—	0.000	—
	$\sigma_{\zeta,r}$	0.031**	(0.002)	0.049**	(0.003)	0.016**	(0.002)
SD	σ_π	0.0088**	(0.0006)				
	σ_y	0.0048**	(0.0004)				
	σ_i	0.0105**	(0.0006)				
Struct	δ_π	0.374**	(0.051)				
	h	0.935**	(0.035)				
	$\sigma(\times 10^{-2})$	0.235**	(0.079)				
Initial points	$\xi_{0,\pi}$	0.009	(0.008)				
	$\xi_{0,y}$	0.000	—				
	$\xi_{0,i}$	0.033**	(0.004)				

Notes: See table 5.2 notes.

**Significant at the 5 percent level.

Table 5.7 **Parameter estimates: Learning III**

		Learning III					
π-eq.	$\mu_{\pi,1}$	0.702**	(0.023)				
	$\kappa_\pi(\times 10^2)$	0.883**	(0.417)				
y-eq.	μ_y	0.523**	(0.016)				
	ϕ	–0.023**	(0.010)				
	r	0.016	(0.094)				
		Martins		Burns		Miller	
i-eq.	γ_{i-1}	0.735**	(0.088)	0.198**	(0.077)	0.352	(0.932)
	γ_π	0.394	(0.628)	0.124	(0.129)	0.925	(0.765)
	γ_y	0.032	(0.239)	0.350**	(0.068)	0.223	(0.858)
	π^*	0.015	(0.258)	0.041	(0.753)	0.067	(0.106)
Learning	$\sigma_{\zeta,\pi}$	0.018**	(0.004)	0.013**	(0.002)	0.019**	(0.006)
	$\sigma_{\zeta,y}$	0.000	—	0.000	—	0.000	—
	$\sigma_{\zeta,r}$	0.005	(0.009)	0.013**	(0.003)	0.013	(0.074)
		Volcker (a)		Volcker (b)		Greenspan	
i-eq.	γ_{i-1}	0.284**	(0.047)	0.796**	(0.033)	0.846**	(0.024)
	γ_π	0.659**	(0.223)	0.275	(0.366)	0.558	(0.601)
	γ_y	0.042	(0.174)	0.198	(0.145)	0.040	(0.176)
	π^*	0.002	(0.148)	0.009	(0.376)	0.016	(0.189)
Learning	$\sigma_{\zeta,\pi}$	0.005	(0.005)	0.018**	(0.002)	0.009**	(0.002)
	$\sigma_{\zeta,y}$	0.000	—	0.000	—	0.000	—
	$\sigma_{\zeta,r}$	0.027**	(0.003)	0.054**	(0.006)	0.008**	(0.002)
SD	σ_π	0.0087**	(0.0007)				
	σ_y	0.0045**	(0.0004)				
	σ_i	0.0117**	(0.0010)				
Struct	δ_π	0.424**	(0.047)				
	h	0.956**	(0.073)				
	$\sigma(\times 10^{-2})$	0.223**	(0.0947)				
Initial points	$\xi_{0,\pi}$	0.009	(0.012)				
	$\xi_{0,y}$	0.000	—				
	$\xi_{0,i}$	0.033**	(0.007)				
Prices of risk	$\Lambda_{0,\pi}$	0.156	(0.256)				
	$\Lambda_{0,y}$	0.132	(0.227)				
	$\Lambda_{0,i}$	–0.235**	(0.104)				
	$\Lambda_{1,\pi,\pi}$	0.145	(3.750)				
	$\Lambda_{1,y,y}$	–2.667	(5.194)				
	$\Lambda_{1,i,i}$	2.197**	(0.972)				
	$\Lambda_{1,\xi_\pi,\xi_\pi}$	–0.937	(4.538)				
	Λ_{1,ξ_y,ξ_y}	0.000	—				
	Λ_{1,ξ_i,ξ_i}	0.029	(1.068)				

Notes: See table 5.2 notes.
**Significant at the 5 percent level.

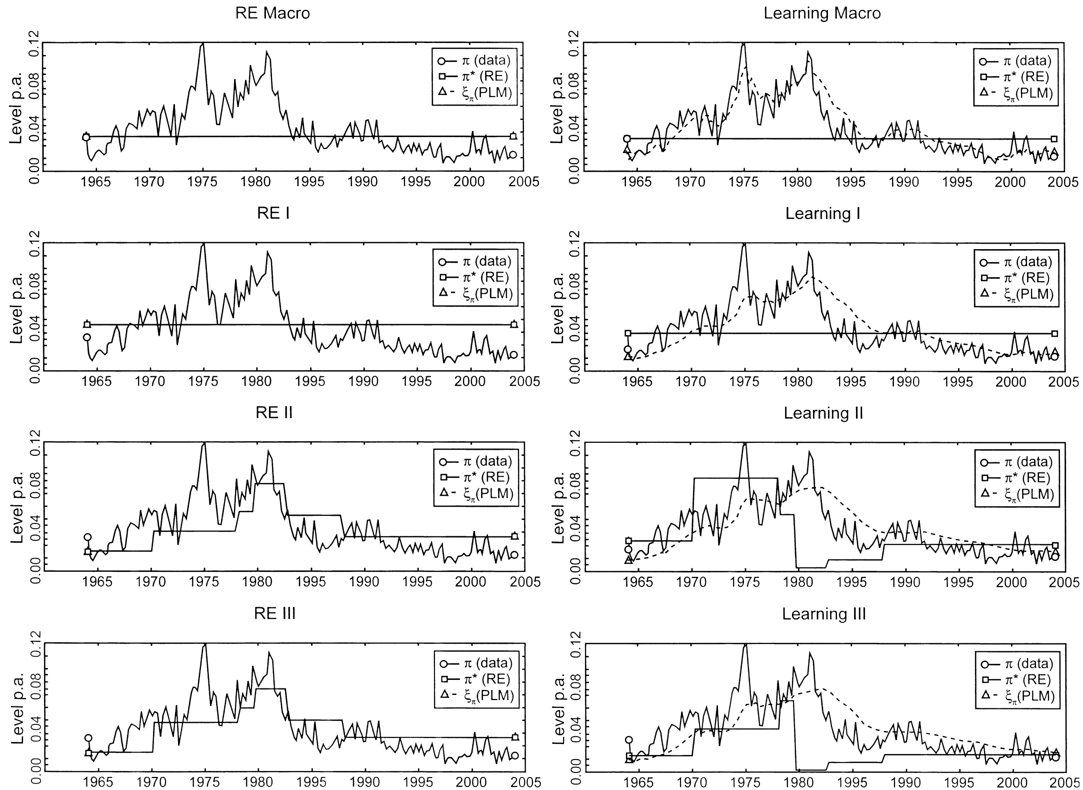

Fig. 5.2 Inflation

policy conducted by Volcker.[21] Under learning, the estimated chairman-specific inflation targets seem more in line with the historical record of U.S. monetary policy. Estimates of time-varying inflation targets, in line with our results, can be found in Kozicki and Tinsley (2005) and Milani (2007).

Figure 5.3 shows the differences between the long-run real interest rate expectations under learning and the values implied by rational expectations models. This difference is less pronounced than in the inflation case and is also similar across learning models. Figure 5.4 presents equivalent graphs for the long-run expectations regarding the short-run policy rate. We observe again sizable differences between the implied rational expectations endpoints and the subjective long-run expectations under learning. The variability in the long-run expectation for the nominal interest rate is dominated by variation in the inflation endpoint.

5.4.3 Comparing Learning and Rational Expectations Models

Bayesian Information Criterion (BIC) and Likelihood Decomposition

We use the Bayesian Information Criterion (BIC) for an overall evaluation of the performance across models. Although this criterion does not constitute a formal statistical test, it takes into account (1) the use of different procedures in the estimation of the models (i.e., Macro and general versions) and (2) the fact that, although rational expectations and learning models are nested, standard likelihood ratio tests are not appropriate as the parameter restrictions of the rational expectations models are on the boundary of the admissible parameter space, that is, $\sigma_{\zeta,\pi} = 0$ and $\sigma_{\zeta,r} = 0$. Next to the BIC, we also compare the performance of the different models through a likelihood decomposition, showing the contribution of the macro variables, the yield curve, and the surveys of inflation expectations.

The results are presented in table 5.8. According to the BIC, learning models outperform their rational expectations counterparts. More strongly, Learning I models outperform any of the estimated rational expectations models. According to this criterion, Learning III models present the best specification, incorporating learning dynamics, heterogeneity in monetary policy rules and priors, and time variation in the prices of

21. One explanation for the observed time series of inflation targets is that inflation targets adapt so as to fit the surveys of inflation expectations. Because under rational expectations long-run expectations coincide with the inflation targets, inflation targets need to track the survey of inflation expectations. Some evidence in favor of this interpretation can be found in table 5.8. Comparing the macro part of the likelihood, one observes a drop from the Rational Expectations I to the Rational Expectations II model, indicating that allowing for chairman-specific inflation targets worsened the macroeconomic fit. This drop in likelihood is more than compensated by the increase in likelihood in the term structure of interest rates and survey parts of the likelihood.

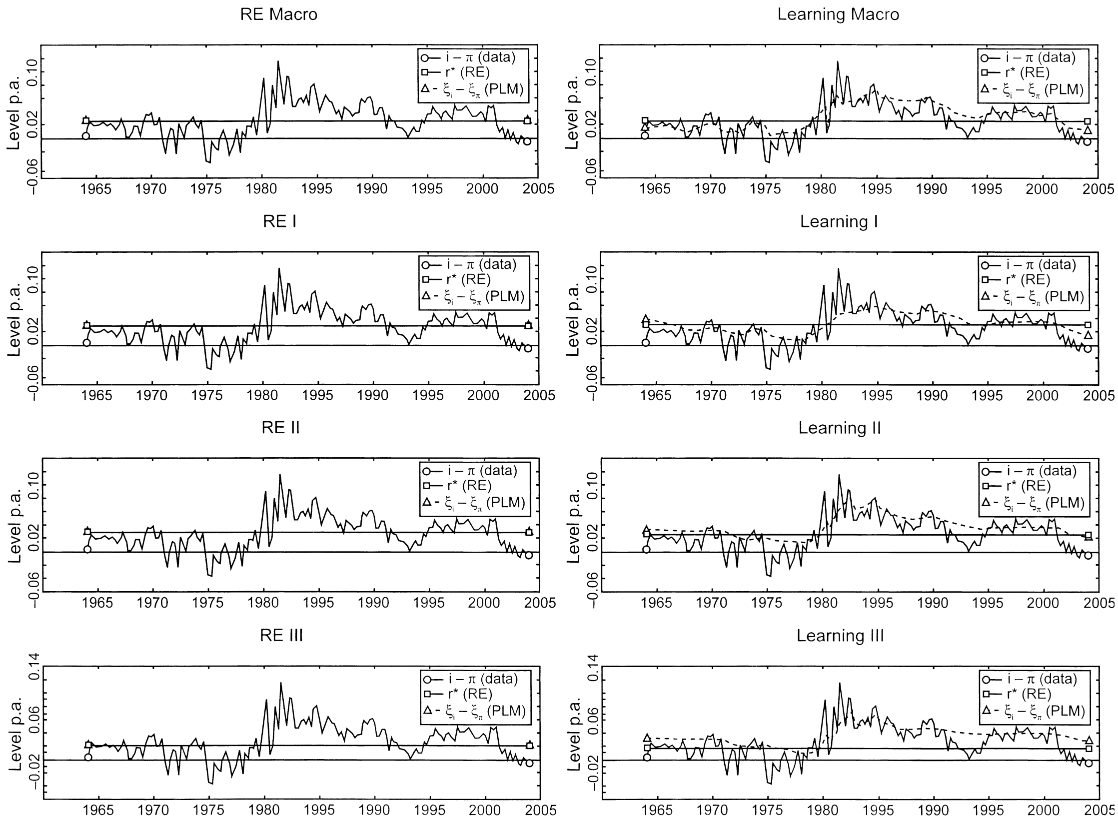

Fig. 5.3 Real interest rate

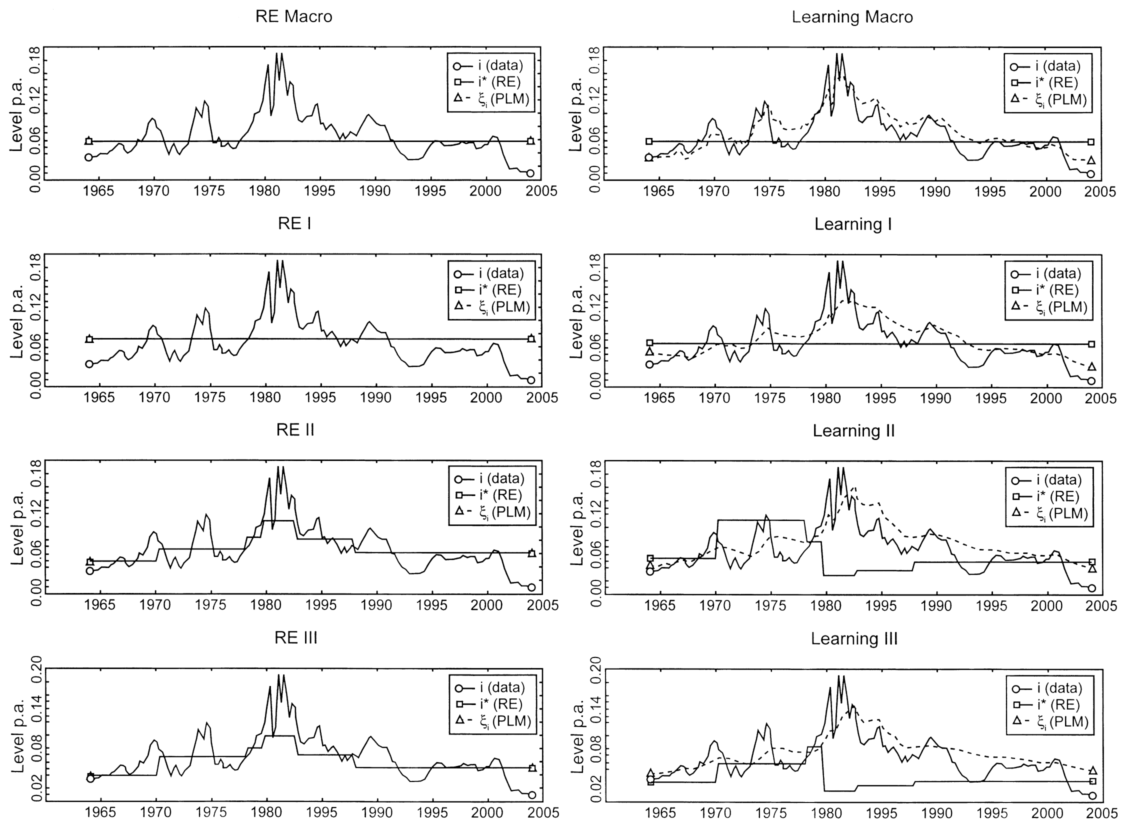

Fig. 5.4 Policy interest rate

Table 5.8 **Likelihood decomposition**

	Components				
	Macroeconomy	Yield curve	Inflation expectation	Total	BIC
Rational Expectations Macro	12.07	20.98	5.61	38.66	–76.70
Rational Expectations I	11.75	23.23	5.55	40.53	–80.42
Rational Expectations II	11.57	25.07	6.21	42.85	–84.44
Rational Expectations III	11.71	25.25	6.22	43.18	–85.00
Learning Macro	12.08	22.08	6.07	40.23	–79.72
Learning I	11.81	26.05	6.50	44.36	–87.97
Learning II	12.03	27.74	6.74	46.51	–91.32
Learning III	12.02	27.93	6.78	46.73	–91.50

risk.[22] The likelihood decomposition shows that the superior performance of learning models is accounted for in each of its components. There seems to be, however, a trade off in fitting those components. From a macro perspective, the Learning Macro model presents the best performance (12.08 as average log-likelihood). From a yield curve and inflation expectations perspective, the Learning III model provides the best fit. The inclusion of this information in the measurement equation, therefore, slightly biases the model toward fitting yield curve and survey data at the expense of the macroeconomic part.

Prediction Errors

Table 5.9 presents summary statistics for the prediction errors of all variables in the alternative model specifications. In all cases, we find evidence of model misspecification due to the significant means and autocorrelation coefficients of the prediction errors. Therefore, none of the models is accepted as a completely satisfactory representation of the joint dynamics of the macroeconomy, the yield curve, and surveys of inflation expectations.[23] There is, however, a clear distinction between learning and rational expectations models. In most cases, learning models outperform their rational expectations counterpart. Introducing learning typically leads to an in-

22. The findings of the BIC are confirmed by approximative likelihood ratio tests. We reestimated the learning models fixing the learning parameters to small values, $\sigma_{\zeta,\pi} = \sigma_{\zeta,r} = 0.0001$, and $\zeta_{0|0} = [\pi^*, 0, r]$. Likelihood ratio (LR) tests performed using the latter models as the null hypothesis reject the proxy models at 1 percent significance levels. Also, note that the Rational Expectations I, II, and III and Learning I, II, and III are nested. Likelihood ratio tests indicate that both the Learning I and II and the Rational Expectations I and II models are rejected against the alternatives, Rational Expectations III and Learning III.

23. The rejection of the overall model is common in the macro-finance literature (e.g., Bekaert, Cho, and Moreno 2006 and Cho and Moreno 2006). In the pure finance literature, it has also been shown to be difficult to find affine term structure representations that are not rejected by the data.

Table 5.9 **Summary statistics of prediction errors of macroeconomic variables, yield curve, and survey of inflation expectations**

	π	y	i	y_{1y}	y_{3y}	y_{5y}	y_{10y}	S_{1y}	S_{10y}
				A. Rational Expectations Macro					
R^2	0.77	0.91	0.83	0.71	0.49	0.35	0.21	0.75	0.36
Mean (%)	0.07	–0.04	0.05	0.02	0.66**	1.08**	1.64**	0.31**	0.51**
SD (%)	1.20	0.78	1.36	1.48	1.83	2.00	2.19	0.92	1.02
Auto	–0.25**	0.21**	–0.10	0.38**	0.75**	0.86**	0.93**	0.58**	0.97**
				B. Rational Expectations I					
R^2	0.76	0.87	0.82	0.76	0.74	0.68	0.50	0.65	–0.08
Mean (%)	–0.04	–0.35**	–0.03	–0.25**	–0.08	0.00	0.14	0.03	–0.10
SD (%)	1.21	0.94	1.38	1.34	1.32	1.41	1.74	1.08	1.22
Auto	–0.20**	0.53**	–0.12	0.19**	0.51**	0.72**	0.89**	0.74**	0.98**
				C. Rational Expectations II					
R^2	0.74	0.84	0.83	0.82	0.84	0.83	0.79	0.75	0.72
Mean (%)	0.05	–0.36**	–0.03	–0.25**	–0.03	0.07	0.25	0.01	–0.08
SD (%)	1.27	1.04	1.34	1.16	1.03	1.03	1.12	0.91	0.62
Auto	0.09	0.63**	–0.12	0.10	0.40**	0.59**	0.75**	0.80**	0.79**
				D. Rational Expectations III					
R^2	0.74	0.85	0.83	0.84	0.84	0.82	0.78	0.74	0.76
Mean (%)	0.06	–0.27**	0.13	–0.15	0.03	0.10	0.23**	–0.02	0.01
SD (%)	1.28	1.02	1.33	1.11	1.03	1.04	1.15	0.93	0.57
Auto	0.19	0.61**	–0.06	0.10	0.42**	0.59**	0.74**	0.85**	0.81**
				E. Learning Macro					
R^2	0.79	0.90	0.83	0.77	0.75	0.74	0.79	0.83	0.66
Mean (%)	0.00	–0.07	–0.01	–0.03	0.63**	1.20**	2.83**	–0.02	0.29**
SD (%)	1.15	0.82	1.34	1.30	1.30	1.26	1.14	0.74	0.72
Auto	–0.08	0.26**	–0.10	0.22**	0.48**	0.57**	0.58**	0.72**	0.88**
				F. Learning I					
R^2	0.70	0.89	0.83	0.84	0.86	0.88	0.88	0.88	0.66
Mean (%)	0.02	–0.22**	–0.03	–0.24**	–0.02	0.14**	0.52**	–0.11**	0.10
SD (%)	1.36	0.88	1.34	1.09	0.96	0.87	0.84	0.64	0.68
Auto	0.42**	0.43**	0.07	0.24**	0.46**	0.52**	0.59**	0.85**	0.97**
				G. Learning II					
R^2	0.72	0.89	0.87	0.88	0.90	0.91	0.92	0.90	0.81
Mean (%)	0.03	–0.19	0.04	–0.18**	–0.14**	–0.09	0.19**	–0.09	0.04
SD (%)	1.32	0.84	1.18	0.96	0.82	0.75	0.70	0.59	0.51
Auto	0.36**	0.34**	0.11	0.21**	0.32**	0.38**	0.41**	0.83**	0.94**
				H. Learning III					
R^2	0.72	0.90	0.86	0.87	0.90	0.91	0.91	0.89	0.83
Mean (%)	0.07	–0.16**	0.17	0.00	0.01	–0.03	0.05	–0.04	0.03
SD (%)	1.32	0.83	1.23	0.99	0.83	0.76	0.72	0.60	0.48
Auto	0.34**	0.31*	0.12	0.21**	0.30**	0.35**	0.38**	0.84**	0.94**

Notes: Mean = the sample average in percentage per year; SD = the standard deviation in percentage per year; and Auto = the first order quarterly autocorrelation.

**Significant at the 5 percent level.

*Significant at the 10 percent level.

crease in the in-sample predictive power of all variables, except inflation, and to a decrease in the standard deviation and the autocorrelation of the prediction errors. The inclusion of chairman-specific policies seems to have a considerable positive effect in the fit of the yield curve and surveys of inflation expectations. For learning models, although the inclusion of time-varying prices of risk decreases the mean of the forecast errors, overall, it does not seem to improve the results in a significant way.

5.4.4 Learning Dynamics, Inflation Expectations, and Bond Markets

Do macroeconomic models including learning fit the term structure of interest rates and inflation expectations? To answer this question, we analyze the fitting errors of the respective models. As shown in table 5.10, learning models with chairman-specific policy rules (Learning II and III) explain 95 percent of the variation in the yield curve and more than 85 percent of the variation in the surveys of inflation expectations. Furthermore, the mean fitting errors for the yield curve are low, ranging from 6 to 20 basis points for the Learning II model, and from 2 to 8 basis points for the Learning III model. These results are comparable to studies using latent factor models (e.g., de Jong 2000). This can also be seen in figures 5.5 and 5.6, which show the fit for the one- and the ten-year yields across models. The difference in performance across models is especially pronounced for the ten-year yield. The performance across models regarding the fit of survey of ten-year average inflation expectations can be seen in figure 5.7. In general terms, Learning II and III models fit both the yield curve and surveys of inflation expectations relatively well.

To identify the contribution of learning in the mentioned performance, we compare the Rational Expectations II and Learning II models (analogous results are obtained for Rational Expectations III and Learning III models). We observe an increase in fit due to learning between 4 percent (one-year yield) and 14 percent (ten-year yield). Furthermore, we observe a significant reduction in the remaining autocorrelation in the fitting errors. To identify the contribution of chairman-specific monetary policy rules and priors, we compare the Learning I and II models. Learning II models show an increase in the explained variation in the yield curve between 2 and 4 percent and in the survey of inflation expectations between 1 and 14 percent. We observe also a general decrease in the remaining autocorrelation in the fitting errors.

Why do learning models outperform their rational expectations counterparts? To answer this question, we analyze the affine term structure representations of rational expectations and learning models. More specifically, we look at the affine representations for the term structure of interest rates and inflation expectations in a transformed state space, decomposing the observed macroeconomic variables in perceived permanent and temporary components. This decomposition is achieved by the rotation matrix **T**:

Table 5.10 **Summary statistics of fitting errors of yield curve and survey of inflation expectations**

	y_{1y}	y_{3y}	y_{5y}	y_{10y}	S_{1y}	S_{10y}
			A. Rational Expectations Macro			
R^2	0.84	0.54	0.38	0.23	0.70	0.38
Mean (%)	–0.04	0.61**	1.04**	1.62**	0.26**	0.52**
SD (%)	1.10	1.75	1.96	2.16	1.00	1.01
Auto	0.64**	0.85**	0.91**	0.95**	0.57**	0.97**
			B. Rational Expectations I			
R^2	0.87	0.78	0.70	0.52	0.65	–0.05
Mean (%)	–0.22	–0.05	0.02	0.16	0.06	–0.08
SD (%)	0.97	1.22	1.37	1.70	1.07	1.20
Auto	0.51**	0.72**	0.82**	0.94**	0.66**	0.97**
			C. Rational Expectations II			
R^2	0.91	0.88	0.85	0.81	0.82	0.73
Mean (%)	–0.23**	0.01	0.11	0.27**	0.02	–0.08
SD (%)	0.83	0.90	0.95	1.07	0.78	0.61
Auto	0.43**	0.62**	0.72**	0.82**	0.75**	0.82**
			D. Rational Expectations III			
R^2	0.92	0.89	0.86	0.81	0.79	0.77
Mean (%)	–0.24**	0.00	0.10	0.23**	0.00	0.02
SD (%)	0.78	0.86	0.92	1.07	0.83	0.56
Auto	0.40**	0.60**	0.70**	0.80**	0.83**	0.83**
			E. Learning Macro			
R^2	0.88	0.78	0.75	0.80	0.81	0.71
Mean (%)	–0.02	0.66**	1.22**	2.85**	0.01	0.33**
SD (%)	0.96	1.21	1.24	1.12	0.80	0.69
Auto	0.53**	0.70**	0.74**	0.79**	0.70**	0.85**
			F. Learning I			
R^2	0.93	0.91	0.91	0.91	0.91	0.71
Mean (%)	–0.20**	0.02	0.17**	0.53**	–0.07	0.14**
SD (%)	0.73	0.78	0.74	0.74	0.54	0.64
Auto	0.39**	0.56**	0.60**	0.71**	0.77**	0.96**
			G. Learning II			
R^2	0.95	0.95	0.95	0.95	0.92	0.85
Mean (%)	–0.16**	–0.10**	–0.06	0.20**	–0.05	0.09
SD (%)	0.59	0.59	0.57	0.55	0.51	0.46
Auto	0.23**	0.33**	0.39**	0.52**	0.71**	0.92**
			H. Learning III			
R^2	0.95	0.95	0.95	0.95	0.92	0.86
Mean (%)	–0.08	–0.02	–0.05	0.03	0.00	0.07*
SD (%)	0.60	0.58	0.56	0.54	0.51	0.44
Auto	0.22**	0.31**	0.35**	0.49**	0.69**	0.92**

Note: See table 5.9 note.

**Significant at the 5 percent level.

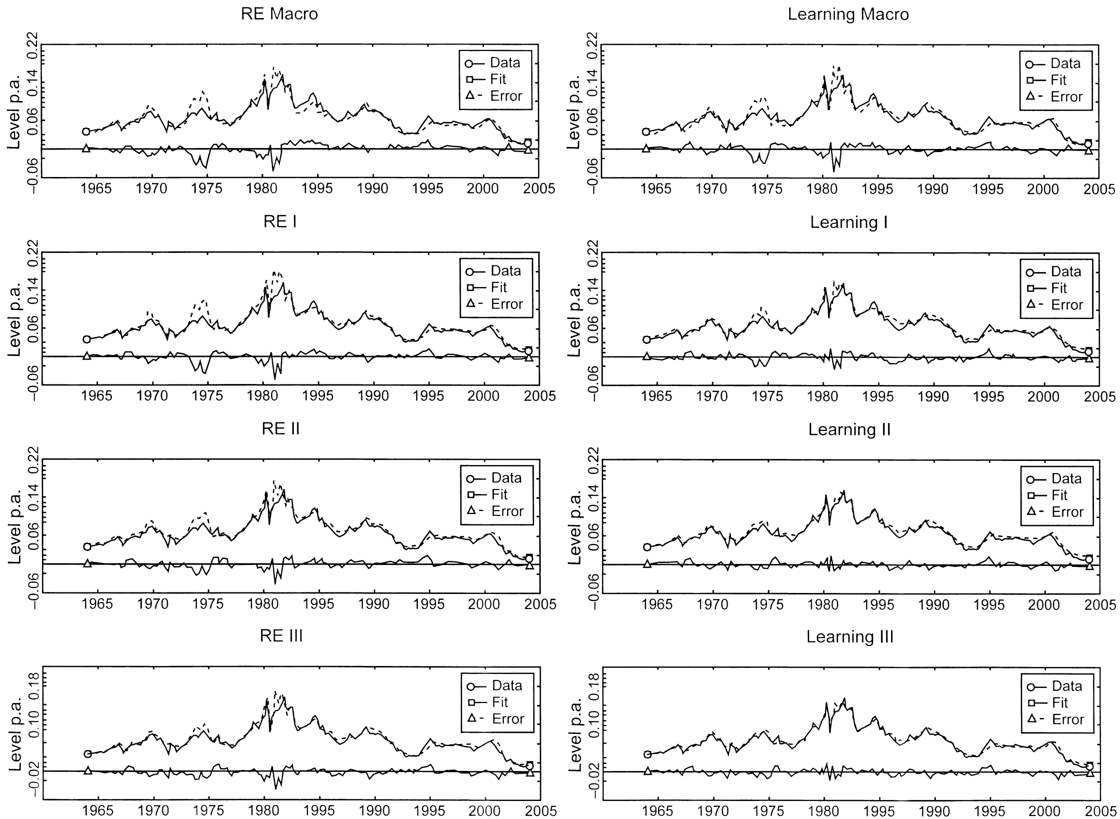

Fig. 5.5 **Term structure fit across models, one-year yield**

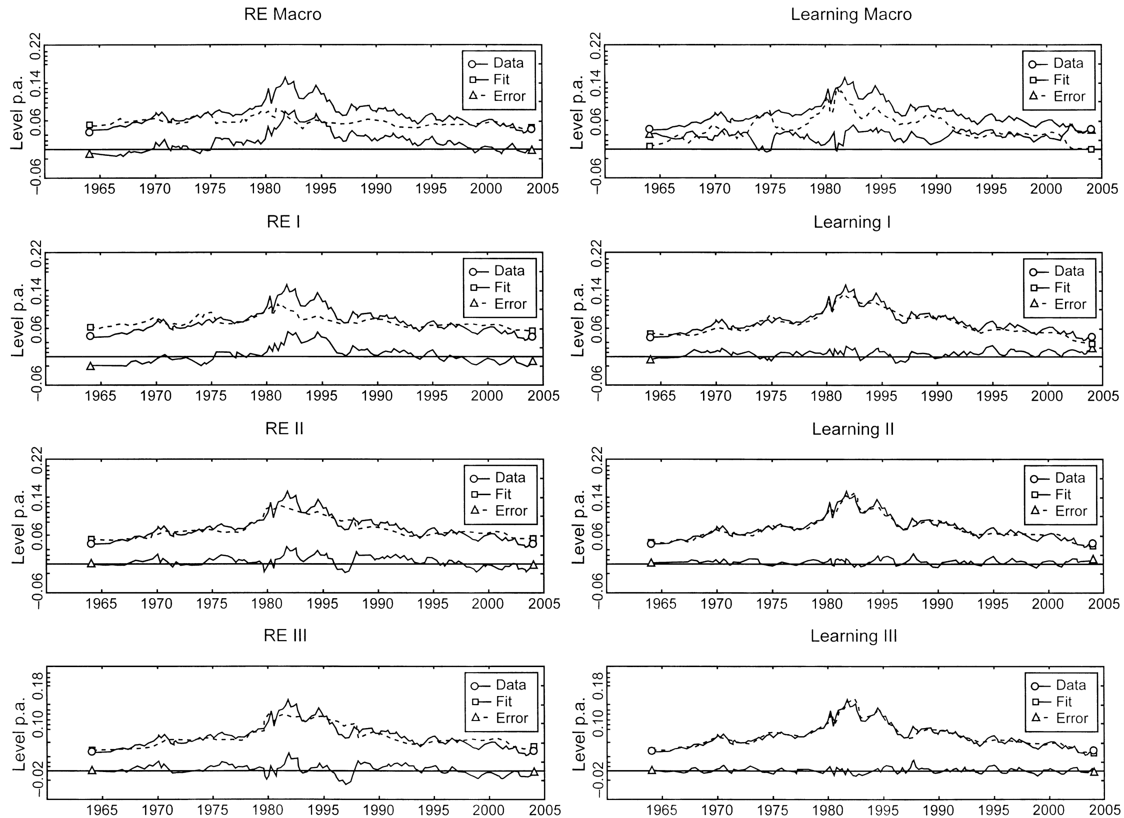

Fig. 5.6 Term structure fit across models, ten-year yield

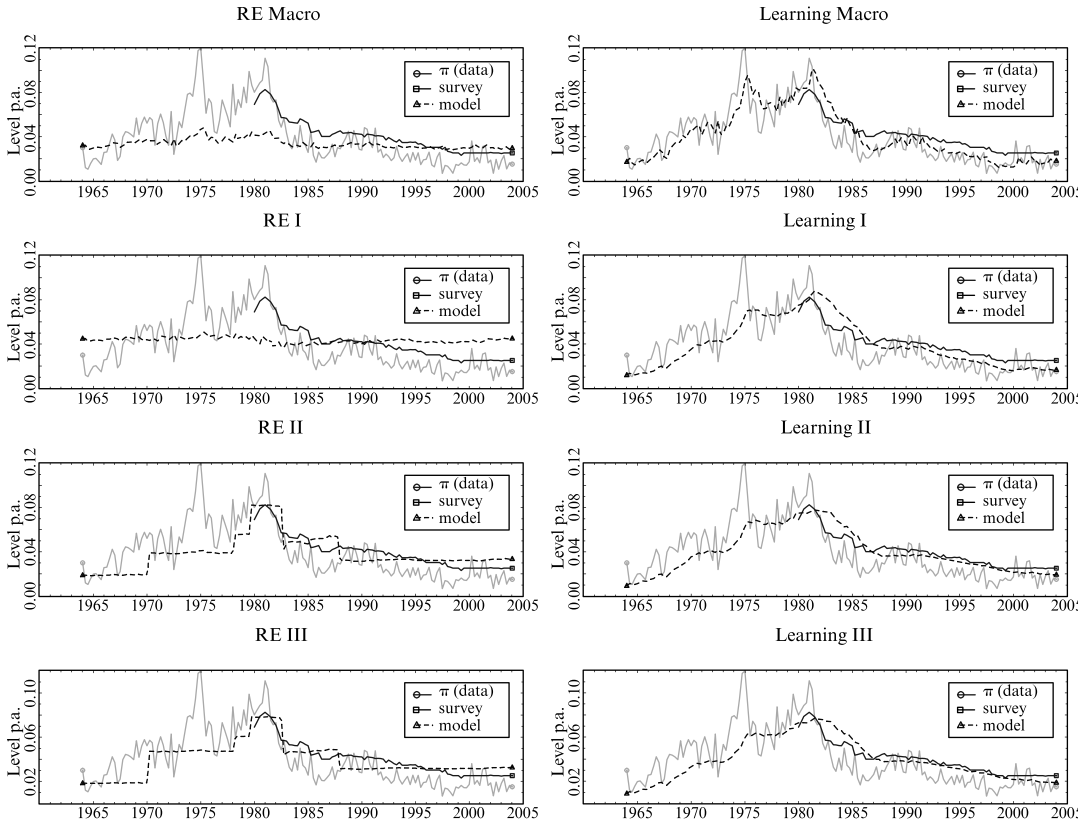

Fig. 5.7 Fit of survey of ten-year average inflation expectations across models

(39)
$$\mathbf{T} = \begin{bmatrix} I_3 & -I_3 \\ 0 & I_3 \end{bmatrix},$$

which generates the decomposition:

(40)
$$\tilde{\mathbf{X}}^T_{t|t} = \begin{bmatrix} \mathbf{X}_t - \xi^P_{t|t} \\ \xi^P_{t|t} \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \mathbf{X}_t \\ \xi^P_{t|t} \end{bmatrix}.$$

The affine representation of the term structure of interest rates and inflation expectations can be restated in this state space as:

(41)
$$\begin{aligned} Y_t &= A_y + B_y \tilde{\mathbf{X}}_{t|t} + \upsilon_{y,\,t} = A_y + B_y \mathbf{T}^{-1} \mathbf{T} \tilde{\mathbf{X}}_{t|t} + \upsilon_{y,t} \\ &= A_y + B^T_y \tilde{\mathbf{X}}^T_{t|t} + \upsilon_{y,\,t} \end{aligned}$$

and

(42)
$$\begin{aligned} S_t &= A_s + B_s \tilde{\mathbf{X}}_{t|t} + \upsilon_{s,t} = A_s + B_s \mathbf{T}^{-1} \mathbf{T} \tilde{\mathbf{X}}_{t|t} + \upsilon_{s,t} \\ &= A_s + B^T_s \tilde{\mathbf{X}}^T_{t|t} + \upsilon_{s,\,t}. \end{aligned}$$

Figure 5.8 shows the transformed yield curve loadings for each of the models.[24] We identify one slope factor driving the yield spread, represented by the perceived transitory interest rate component, and two curvature factors, that is, the perceived output gap and the perceived inflation gap. The curvature factors affect primarily but marginally the intermediate maturity yields. We also obtain a level factor, exerting its influence equally over the entire yield curve. This factor is driven only by changes in the perceived stochastic endpoint for the policy rate. While both rational expectations and learning models share a level factor in the transformed state space, the implications of this factor differ across models. Rational expectations models imply a deterministic endpoint for the policy rate, that is, $\xi_{i^*,t} = r + \pi^*$.[25] The level factor is, therefore, constant and cannot explain the time variation in long-maturity yields. Learning models generate endogenous stochastic endpoints for the policy rate, which seem to be sufficient ly volatile to account for the time variation in the long end of the yield curve.

24. Note that in the versions II and III of both rational expectations and learning models, yield curve and inflation expectations loadings also depend on the policy rule parameters. Given that we identify six policy regimes, we have six sets of loadings. For reasons of brevity, we only present the loadings implied by the Greenspan policy rules.

25. Note that to the extent that one allows for time-varying inflation targets within the rational expectations framework, one can generate exogenously volatility in the endpoints. This is the approach followed in the standard macro-finance literature. The Rational Expectations II and III panels in figures 5.2 and 5.4 are examples of this approach. The main advantage of learning is that there is no need to refer to exogenous shocks (i.e., in the inflation target) to account for the time variation in the long end of the yield curve. The stochastic endpoints are generated endogenously in the model.

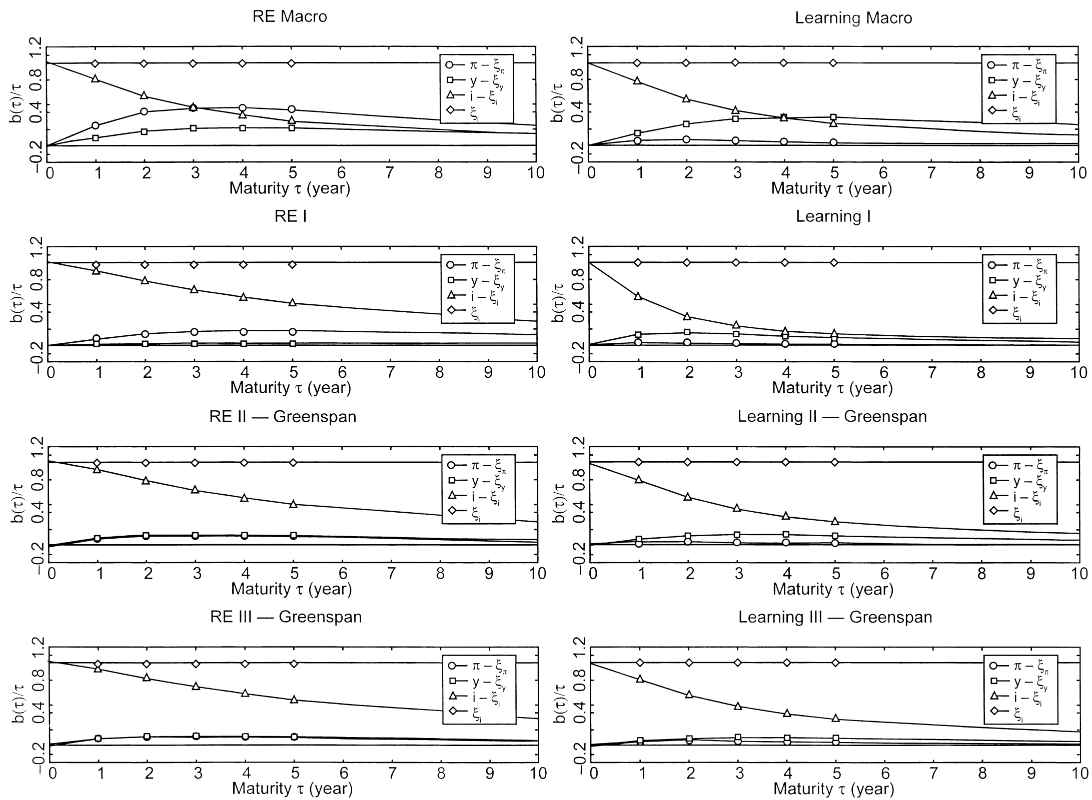

Fig. 5.8 Loading—Term structure of interest rates across models

5.5 Conclusions

In this chapter we built and estimated a macroeconomic model including learning. Learning was introduced in the model by assuming that agents do not believe in time-invariant inflation targets nor in constant equilibrium real rates. Given these priors, the optimal learning rule was derived in terms of a Kalman gain updating rule. We estimated the model including in the measurement equation, next to the standard macroeconomic variables, bond yields, and surveys of inflation expectations. The structural and learning parameters were estimated jointly. The findings of the chapter can be summarized as follows. First, including learning improves the fit of the model independently of the type of information included in the measurement equation. Although learning models improve on the rational expectations models, they are not fully satisfactory. Autocorrelation in the errors was found to be significant. Finally, we found that introducing learning in a standard New Keynesian model generated sufficiently volatile stochastic endpoints to fit the variation in long-maturity yields and in surveys of inflation expectations. The learning model, therefore, complements the current macro-finance literature linking macroeconomic and term structure dynamics.

Appendix

ALM Dynamics

In this appendix, we derive a closed form solution for the actual law of motion (ALM). The derivation follows the standard approach in the learning literature by substituting subjective expectations, that is, the PLM, into the structural equations. The structural equations are described in equation (7), which is repeated here as:

$$AX_t = C + BE_tX_{t+1} + DX_{t-1} + S\varepsilon_t, \tag{43}$$

while the PLM is described by means of a vector error correction model (VECM) in the inferred stochastic endpoints:

$$X_t = (I - \Phi^P)\xi^P_{t|t} + \Phi^P X_{t-1} + \Sigma^P \varepsilon_t \tag{44}$$

and a learning rule based on the Kalman filter updating rule:

$$\xi^P_{t|t} = \xi^P_{t-1|t-1} + K(X_t - E^P_{t-1}X_t). \tag{45}$$

Deriving the Actual Law of Motion

A first step in obtaining the actual law of motion (ALM) consists of deriving the expectations implied by the PLM, equations (44) and (45). Under the PLM, the one-step ahead prediction, $E^P_t X_{t+1}$, is given by:

$$E_t^P X_{t+1} = (I - \Phi^P) E_t^P \xi_{t+1|t+1}^P + \Phi^P X_t. \tag{46}$$

Under the PLM dynamics, the stochastic endpoints $\xi_{t|t}^P$ are random walks, that is, $E_{t-1}^P(X_t - E_{t-1}^P X_t) = 0$, such that $E_t^P \xi_{t+1|t+1}^P = \xi_{t|t}^P$. The one-step ahead expectations are given by:

$$E_t^P X_{t+1} = (I - \Phi^P)\xi_{t|t}^P + \Phi^P X_t. \tag{47}$$

Substituting the learning rule, equation (45), for $\xi_{t|t}^P$, we obtain a description for the expectations as:

$$E_t^P X_{t+1} = (I - \Phi^P)[\xi_{t-1|t-1}^P + K(X_t - E_{t-1}^P X_t)] + \Phi^P X_t \tag{48}$$

or equivalently, by lagging equation (47) one period giving a closed form expression for $E_{t-1}^P X_t = (I - \Phi^P)\xi_{t-1|t-1}^P + \Phi^P X_{t-1}$:

$$\begin{aligned} E_t^P X_{t+1} = (I - \Phi^P)\{\xi_{t-1|t-1}^P + K[X_t - (I - \Phi^P)\xi_{t-1|t-1}^P - \Phi^P X_{t-1}]\} \\ + \Phi^P X_t. \end{aligned} \tag{49}$$

This expression can also be written as:

$$\begin{aligned} E_t^P X_{t+1} = {} & [I - (I - \Phi^P)K](I - \Phi^P)\xi_{t-1|t-1}^P \\ & + [\Phi^P + (I - \Phi^P)K]X_t - (I - \Phi^P)K\Phi^P X_{t-1}. \end{aligned} \tag{50}$$

Denoting the matrix $(I - \Phi^P)K$ by $\mathbf{K}_\Phi$, we obtain the final expression for the one-step ahead expectation as:

$$E_t^P X_{t+1} = (I - \mathbf{K}_\Phi)(I - \Phi^P)\xi_{t-1|t-1}^P + (\Phi^P + \mathbf{K}_\Phi)X_t - \mathbf{K}_\Phi \Phi^P X_{t-1}. \tag{51}$$

The second step in deriving the ALM dynamics consists of inserting the subjective expectations, equation (51), into the structural equations, that is, equation (43):

$$\begin{aligned} AX_t = C + B[(I - \mathbf{K}_\Phi)(I - \Phi^P)\xi_{t-1|t-1}^P + (\Phi^P + \mathbf{K}_\Phi)X_t - \boldsymbol{K}_\Phi \Phi^P X_{t-1}] \\ + DX_{t-1} + S\varepsilon_t. \end{aligned} \tag{52}$$

Solving for X_t, we obtain:

$$\begin{aligned} X_t = {} & [A - B(\Phi^P + K_\Phi)]^{-1}C + [A - B(\Phi^P + K_\Phi)]^{-1} \\ & \cdot B(I - K_\Phi)(I - \Phi^P)\xi_{t-1|t-1}^P [A - B(\Phi^P + K_\Phi)]^{-1}(D - BK_\Phi \Phi^P)X_{t-1} \\ & + [A - B(\Phi^P + K_\Phi)]^{-1}S\varepsilon_t. \end{aligned} \tag{53}$$

Note that if the rational expectations solution is unique, and if $\Phi^P = \Phi^{re}$, the expression $[A - B(\Phi^P + K_\Phi)]^{-1}(D - BK_\Phi \Phi^P)$ equals Φ^P, which allows us to rewrite the preceding dynamics as:

$$\begin{aligned} X_t = {} & [A - B(\Phi^P + K_\Phi)]^{-1}C + [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi) \\ & \cdot (I - \Phi^P)\xi_{t-1|t-1}^P \Phi^P X_{t-1} + [A - B(\Phi^P + K_\Phi)]^{-1}S\varepsilon_t. \end{aligned} \tag{54}$$

Equation (54) describes the actual law of motion for the observable macroeconomic variables as a function of the previous state, X_{t-1}, the inferred stochastic endpoints, $\xi^P_{t-1|t-1}$, and the structural shocks, ε_t. This description is only a partial description of the ALM because the dynamics of the stochastic endpoints are not taken into account. In order to obtain a complete characterization of the ALM, we add the learning rule, that is, equation (45). The joint dynamics of the observable macroeconomic variables, X_t, and the inferred stochastic endpoints, $\xi^P_{t|t}$ is given by:

$$\begin{bmatrix} I & 0 \\ -K & I \end{bmatrix}\begin{bmatrix} X_t \\ \xi^P_{t|t} \end{bmatrix} = \begin{Bmatrix} [A - B(\Phi^P + K_\Phi)]^{-1}C \\ 0 \end{Bmatrix}$$

$$+ \begin{Bmatrix} \Phi^P & [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi)(I - \Phi^P) \\ -K\Phi^P & [I - K(I - \Phi^P)] \end{Bmatrix}\begin{bmatrix} X_{t-1} \\ \xi^P_{t-1|t-1} \end{bmatrix}$$

$$+ \begin{Bmatrix} [A - B(\Phi^P + K_\Phi)]^{-1}S \\ 0 \end{Bmatrix}\varepsilon_t,$$

where the dynamics for $\xi^P_{t|t}$ are given by equation (45). Finally, premultiplying by

$$\begin{bmatrix} I & 0 \\ -K & I \end{bmatrix}^{-1} = \begin{bmatrix} I & 0 \\ K & I \end{bmatrix} \tag{55}$$

yields a complete description of the ALM:

$$\begin{bmatrix} X_t \\ \xi^P_{t|t} \end{bmatrix} = \begin{Bmatrix} [A - B(\Phi^P + K_\Phi)]^{-1}C \\ K[A - B(\Phi^P + K_\Phi)]^{-1}C \end{Bmatrix}$$

$$+ \begin{pmatrix} \Phi^P & [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi)(I - \Phi^P) \\ 0 & I - K\{I - [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi)\}(I - \Phi^P) \end{pmatrix}\begin{bmatrix} X_{t-1} \\ \xi^P_{t-1|t-1} \end{bmatrix}$$

$$+ \begin{Bmatrix} [A - B(\Phi^P + K_\Phi)]^{-1}S \\ K[A - B(\Phi^P + K_\Phi)]^{-1}S \end{Bmatrix}\varepsilon_t.$$

This ALM is represented in extended state space, $\tilde{X}_{t|t} = [X'_t, \xi^{P\prime}_{t|t}]'$ by

$$\tilde{X}_{t|t} = \tilde{C}^A + \tilde{\Phi}^A \tilde{X}_{t-1|t-1} + \tilde{\Sigma}^A \varepsilon_t, \tag{56}$$

with

$$(57)\qquad \tilde{C}^A = \begin{Bmatrix} [A - B(\Phi^P + K_\Phi)]^{-} C \\ K[A - B(\Phi^P + K_\Phi)]^{-1} C \end{Bmatrix}$$

$$\tilde{\Phi}^A = \begin{pmatrix} \Phi^P & [A - B(\Phi^P + K_\Phi)]^{-1} B(I - K_\Phi)(I - \Phi^P) \\ 0 & I - K\{I - [A - B(\Phi^P + K_\Phi)]^{-1} B(I - K_\Phi)\}(I - \Phi^P) \end{pmatrix}$$

$$\tilde{\Sigma}^A = \begin{Bmatrix} [A - B\,(\Phi^P + K_\Phi)]^{-1} S \\ K[A - B(\Phi^P + K_\Phi)]^{-1} S \end{Bmatrix}.$$

Properties of the Actual Law of Motion

Based on the final representation of the ALM as stated in equation (56), some properties of the ALM can be described in more detail. A first property is that the unconditional mean of the ALM coincides with the unconditional mean of the rational expectations model. Denoting the expectations operators under rational expectations and under the ALM by, respectively, E^{re} and E^A, the equivalence between unconditional expectations can be formalized as:

$$(58)\qquad \begin{aligned} E^A X_t &= E^{re} X_t = (I - \Phi^{re})^{-1} C^{re}, \\ E^A \xi^P_{t|t} &= E^{re} X_t = (I - \Phi^{re})^{-1} C^{re}. \end{aligned}$$

We show this property by showing that $X_t = (I - \Phi^{re})^{-1} C^{re} = \xi_{t|t}$ is a steady state under the ALM. In the derivation we make extensive use of the properties of the rational expectations solution. More specifically, the unconditional mean for X_t based on the rational expectations model is given by:

$$(59)\qquad E^{re}(X_t) = (I - \Phi^{re})^{-1} C^{re},$$

where the values for Φ^{re} and C^{re} satisfy the rational expectations conditions:

$$(60)\qquad \begin{aligned} C^{re} &= (A - B\,\Phi^{re})^{-1} C + (A - B\,\Phi^{re})^{-1} B C^{re} \\ \Phi^{re} &= (A - B\,\Phi^{re})^{-1}\, D \\ \Sigma^{re} &= (A - B\,\Phi^{re})^{-1} S. \end{aligned}$$

We now show that the unconditional mean of X_t under the ALM, denoted by $E^A_t X_t$ coincides with the unconditional mean of the rational expectations model:

$$(61)\qquad E^A_t X_t = E^{re} X_t = (I - \Phi^{re})^{-1} C^{re}.$$

In order to show this equivalence, we show that the point $X_t = (I - \Phi^{re})^{-1} C^{re}$ and $\xi_{t|t} = (I - \Phi^{re})^{-1} C^{re}$ are a steady state for the ALM. Substituting this particular point in the ALM, we obtain that this point is a steady state if it solves:

$$(I - \Phi^{re})^{-1}C^{re} = [A - B(\Phi^P + K_\Phi)]^{-1}C + \Phi^P(I - \Phi^{re})^{-1}C^{re}$$
$$+ [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi)(I - \Phi^P)(I - \Phi^{re})^{-1}C^{re}.$$

Noting that $\Phi^{re} = \Phi^P$ we can rewrite the equation by subtracting from both sides $\Phi^P(I - \Phi^{re})^{-1}C^{re}$, resulting in the equality:

$$(I - \Phi^P)(I - \Phi^{re})^{-1}C^{re} = [A - B(\Phi^P + K_\Phi)]^{-1}C$$
$$+ [A - B(\Phi^P + K_\Phi)]^{-1}B(I - K_\Phi)(I - \Phi^P)(I - \Phi^{re})^{-1}C^{re}.$$

Premultiplying by $[A - B(\Phi^P + K_\Phi)]^{-1}$,

$$[A - B(\Phi^P + K_\Phi)]C^{re} = C + B(I - K_\Phi)C^{re}.$$

Finally, this condition holds whenever a rational expectations equilibrium exists, that is, adding $B\mathbf{K}_\Phi C^{re}$ to both sides, the preceding condition reduces to the rational expectations condition for C^{re}:

$$(A - B\Phi^P)C^{re} = C + BC^{re}.$$

The preceding derivation thus implies that if a rational expectations equilibrium exists, then the unconditional expectations of the rational expectations equilibrium coincides with the steady state of the ALM. If we assume, moreover, that all of the eigenvalues of $\tilde{\Phi}^A$ are strictly smaller than 1 in absolute value, the steady state of the ALM is attracting and defines the unconditional mean of the observable variables X_t. The second equality, that is, $E^A\xi^P_{t|t} = (I - \Phi^{re})^{-1}C^{re}$, can be shown analogously.

A second property is the unconditional normality of the extended state vector $\tilde{\mathbf{X}}_{t|t}$ under the ALM. Assuming a standard normal distribution for the structural shocks, ε_t, it is well known that the linearity of the state space dynamics and the assumed stability of the ALM (all eigenvalues of $\tilde{\Phi}^A$ are assumed to be strictly smaller than 1) implies that the unconditional distribution for $\tilde{\mathbf{X}}_{t|t}$ is:

$$\tilde{\boldsymbol{X}}_{t|t} \sim N(E^A\tilde{\boldsymbol{X}}_{t|t}, \Omega_{\tilde{X}}),$$

with

$$E^A\tilde{\boldsymbol{X}}_{t|t} = \iota_{2\times 1} \otimes (I - \Phi^{re})^{-1}C^{re}$$
$$\mathrm{vec}(\Omega_{\tilde{X}}) = (I - \tilde{\Phi}^A \otimes \tilde{\Phi}^A)^{-1}\mathrm{vec}(\tilde{\Sigma}^A\tilde{\Sigma}^{A\prime}).$$

References

Ang, A., and M. Piazzesi. 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics* 50:745–87.

Bekaert, G., S. Cho, and A. Moreno. 2006. New-Keynesian macroeconomics and the term structure. Columbia University, Working Paper.

Bullard, J., and K. Mitra. 2002. Learning about monetary policy rules. *Journal of Monetary Economics* 49 (6): 1105–29.

Calvo, G. 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12:383–98.

Cho, S., and A. Moreno. 2006. A small-sample study of the New-Keynesian macro model. *Journal of Money, Credit, and Banking* 38 (6): 1461–81.

Christiano, L., M. Eichenbaum, and C. Evans. 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113 (1): 1–45.

Chun, A. 2005. Expectations, bond yields and monetary policy. Stanford University, Working Paper.

Clarida, R., J. Galí, and M. Gertler. 1999. The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature* 37:1661–1707.

———. 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics* 115 (February): 147–80.

Cogley, T. 2005. Changing beliefs and the term structure of interest rates: Cross-equation restrictions with drifting parameters. *Review of Economic Dynamics* 8:420–51.

Cox, J., J. Ingersoll, and S. Ross. 1985. A theory of the term structure of interest rates. *Econometrica* 53:385–408.

Dai, Q., and K. Singleton. 2000. Specification analysis of affine term structure models. *Journal of Finance* 55 (5): 1943–78.

De Jong, F. 2000. Time series and cross-section information in affine term-structure models. *Journal of Business and Economic Statistics* 18 (3): 300–314.

Dewachter, H., and M. Lyrio. 2006. Macro factors and the term structure of interest rates. *Journal of Money, Credit, and Banking* 38 (1): 119–40.

Dewachter, H., M. Lyrio, and K. Maes. 2006. A joint model for the term structure of interest rates and the macroeconomy. *Journal of Applied Econometrics* 21 (4): 439–62.

Diebold, F., G. Rudebusch, and S. Aruoba. 2006. The macroeconomy and the yield curve: A dynamic latent factor approach. *Journal of Econometrics* 131 (1–2): 309–38.

Duffee, G. 2002. Term premia and interest rate forecasts in affine models. *Journal of Finance* 57:405–43.

Duffie, D., and R. Kan. 1996. A yield-factor model of interest rates. *Mathematical Finance* 6:379–406.

Ellingsen, T., and U. Söderström. 2001. Monetary policy and market interest rates. *American Economic Review* 91 (5): 1594–1607.

Evans, G., and S. Honkapohja. 2001. *Learning and expectations in macroeconomics*. Princeton, NJ: Princeton University Press.

Fuhrer, J. 2000. Habit formation in consumption and its implications for monetary-policy models. *American Economic Review* 90 (3): 367–90.

Galí, J., and M. Gertler. 1999. Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44:195–222.

Gürkaynak, R., B. Sack, and E. Swanson. 2005. The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models. *American Economic Review* 95 (1): 425–36.

Hördahl, P., O. Tristani, and D. Vestin. 2006. A joint econometric model of macroeconomic and term-structure dynamics. *Journal of Econometrics* 131 (1–2): 405–44.

Kim, D., and A. Orphanides. 2005. Term structure estimation with survey data on interest rate forecasts. CEPR Discussion Paper Series no. 5341. London: Center for Economic Policy Research.
Kozicki, S., and P. A. Tinsley. 2001. Shifting endpoints in the term structure of interest rates. *Journal of Monetary Economics* 47:613–52.
———. 2002. Dynamic specifications in optimizing trend-deviation macro models. *Journal of Economic Dynamics and Control* 26:1585–1611.
Kozicki, S., and P. A. Tinsley 2005. Permanent and transitory policy shocks in an empirical macro model with asymmetric information. *Journal of Economic Dynamics and Control* 29:1985–2015.
Milani, F. 2007. Expectations, learning and macroeconomic persistence. *Jounral of Monetary Economics* 54:2065–82.
Orphanides, A., and J. Williams. 2005a. The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations. *Journal of Economic Dynamics and Control* 29:1927–50.
———. 2005b. Inflation scares and forecast-based monetary policy. *Review of Economic Dynamics* 8:498–527.
Preston, B. 2005. Learning about monetary policy rules when long-horizon expectations matter. *Journal of Central Banking* 1 (2): 81–126.
Roberts, J. 1997. Is inflation sticky? *Journal of Monetary Economics* 39:173–96.
Rudebusch, G. 2002. Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Economics* 49:1161–87.
Rudebusch, G., and T. Wu. 2003. A macro-finance model of the term structure, monetary policy, and the economy. *Economic Journal*, forthcoming.
Sargent, T., and N. Williams. 2005. Impacts of priors on convergence and escapes from Nash inflation. *Review of Economic Dynamics* 8:360–91.
Schorfheide, F. 2005. Learning and monetary policy shifts. *Review of Economic Dynamics* 8 (2): 392–419.
Sims, C., and T. Zha. 2006. Were there regime switches in U.S. monetary policy? *American Economic Review* 96 (1): 54–81.
Smets, F., and R. Wouters. 2003. Monetary policy in an estimated stochastic dynamic general equilibrium model of the euro area. *Journal of the European Economic Association* 1 (5): 1123–75.
Vasicek, O. 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics* 5:177–88.

Comment Jordi Galí

The present chapter by Dewachter and Lyrio is part of a small but growing literature that seeks to understand the yield curve and its evolution over time by combining two different modeling approaches: the arbitrage-free relations familiar from the finance literature and the dynamic general equilibrium approach of modern macroeconomic theory. Dewachter and

Jordi Galí is director and a senior researcher of the Centre de Recerca en Economia Internacional (CREI), and a professor of economics, all at the Universitat Pompeu Fabra, and a research associate of the National Bureau of Economic Research.

Lyrio's specific objective (and that of other recent papers cited by the authors) is to reconcile the observed behavior of the term structure with a fully articulated model of inflation, monetary policy, and economic activity. As discussed by Dewachter and Lyrio, an important requirement in order to achieve that objective is the introduction of a "level factor," that is, variations in long-term expectations of short-term rates, that can account for the high volatility of the long-term yield. The main novelty of the present Dewachter and Lyrio chapter lies in the endogenous modelling of that level factor, whose variations result from the evolving perceptions by private agents on the endpoint short-term rate (or, more precisely, its two components: the real rate and inflation), brought about by the assumed learning dynamics.

A Simple Model of the Term Structure

In order to illustrate the basic point of the Dewachter and Lyrio chapter, consider the following model of the term structure, generally referred to as the expectations hypothesis (EH) model

$$i_t^{(n)} = \frac{1}{n}\sum_{k=0}^{n-1} E_t\{i_{t+k}\}, \tag{1}$$

where $i_t^{(n)}$ is the yield on an n-period bond, and i_t denotes the short-term nominal rate on a (nominally) riskless one-period bond held between period t and $t + 1$. Let me assume the following exogenous stationary process for the short rate:

$$i_t - i^* = \phi(i_{t-1} - i^*) + \varepsilon_t, \tag{2}$$

where i^* is the unconditional mean of the short term rate, and $\phi \in (0, 1)$. Then, under rational expectations we have

$$i_t^{(n)} = (1 - \Theta_n)i^* + \Theta_n i_t, \tag{3}$$

where $\Theta_n \equiv (1/n)(1 - \phi^n)/(1 - \phi)$.

An Empirical Puzzle

As equation (3) makes clear, the EH model implies a very tight relation between short-term and long-term rates, one which is clearly violated in U.S. data. In particular, the EH model implies that long-term rates should be much less volatile than they actually are. To see this, note that applying ordinary least squares (OLS) to equation (2) using quarterly data on the three-month Treasury Bill (TB) rate over the sample period 1954:Q1 to 2005:Q4 yields an estimate $\hat{\phi} = 0.96$. The latter, in turn, implies a value $\Theta_{40} \simeq 0.5$, where $n = 40$ corresponding to a ten-year maturity. Hence, the model predicts that the yield on a ten-year bond should have a standard deviation roughly half the size the standard deviation of the TB rate. That prediction is clearly rejected by the data: the ratio of standard deviations is approximately 0.9 rather than 0.5. In other words, the long-term rate appears to be excessively volatile relative to the predictions of the EH model.

A Proposed Solution: Endpoint Learning

Let me define, following Dewachter and Lyrio, the *perceived endpoint* for the short rate as the subjective long-run expectation

$$i_t^{*P} \equiv \lim_{k\to\infty} E_t^P\{i_{t+k}\},$$

where E_t^P is the subjective expectations operator. Agents' *perceived law of motion* for that endpoint is assumed to be given by the random walk model

$$i_t^{*P} = i_{t-1}^{*P} + v_t. \tag{4}$$

Deviations from the endpoint are assumed to follow a stationary AR(1) process analogous to the rational expectations model described in the preceding:

$$i_t - i_t^{*P} = \phi(i_{t-1} - i_{t-1}^{*P}) + \varepsilon_t$$

given that the perceived endpoint is not observed, agents estimate it using the Kalman filter learning algorithm:

$$i_{t|t}^{*P} = i_{t-1|t-1}^{*P} + K[i_t - E_{t-1}^P(i_t)],$$

where $K \in (0, 1)$.

Note that, while agents in this economy believe the endpoint for the short-term rate to vary over time, we assume that the short-term rate fluctuates around a constant mean value i^* according to the process

$$i_t - i^* = \phi(i_{t-1} - i^*) + \varepsilon_t.$$

By combining the previous equations, one can show that, in equilibrium, agents' estimate of the endpoint follows the stationary AR(1) process

$$i_{t|t}^{*P} - i^* = [1 - K(1-\phi)](i_{t-1|t-1}^{*P} - i^*) + K\varepsilon_t, \tag{5}$$

Finally, one can combine the previous equation with the EH model of the term structure (1) to yield the following expression for the n-period bond yield under learning:

$$\begin{aligned} i_t^{(n)} &= \frac{1}{n}\sum_{k=0}^{n-1} E_t^P\{i_{t+k}\} \\ &= (1-\Theta_n) i_{t|t}^{*P} + \Theta_n\, i_t. \end{aligned}$$

A comparison of equation (6) to equation (3) makes clear that variations in the estimated endpoint $i^{*P}_{t|t}$ in the model under learning provide an additional source of volatility for long-term yields, and one whose relative importance rises with the maturity on the bond (as Θ_n is decreasing in n). Furthermore, because the time series properties of $i^{*P}_{t|t}$ depend on some unobservables (e.g., the variance of υ_t in the preceding model—which measures the extent of the departure from rational expectations), the model with learning gives the researcher some room to improve on the fit of its rational expectations counterpart.

Dewachter and Lyrio's Contribution

The simple learning model of the previous subsection conveys the essence of Dewachter and Lyrio's proposed framework for understanding the term structure dynamics. Needless to say, Dewachter and Lyrio's model is richer in several dimensions, some of which are likely to be important. First, and most noticeably, Dewachter and Lyrio's model is a general equilibrium one. Thus, and in contrast with the preceding framework, the short-term rate does not follow an exogenous process but instead is determined according to a Taylor-type rule that has the output gap and inflation as arguments. The output gap and inflation are, in turn, determined (simultaneously with the short-term rate) by a hybrid New Keynesian Phillips curve and a dynamic IS equation, which, in combination with the interest rate rule, constitute the macro block of Dewachter and Lyrio's model.

Secondly, Dewachter and Lyrio use a pricing kernel consistent with the macro model in order to derive an affine model for the yield curve. This is in contrast with the simple (though pedagogically useful) expectations hypothesis model shown in the preceding. As a result, the yields for different maturities are not only a function of the current short-term rate and its perceived endpoint, but also of inflation, the output gap, as well as agents' current estimates of all those variables' endpoints.

The different models estimated by Dewachter and Lyrio (four versions of the rational expectations model and four of the learning model) and their implied fit of the time series for bond yields of different maturities lead a number of interesting insights, many of which are discussed in detail in Dewachter and Lyrio's chapter. Most importantly, given the chapter's objectives, and as summarized graphically by figure 5.6 in that chapter, Dewachter and Lyrio's findings point to a potentially large explanatory role of learning dynamics as a source of the low frequency movement in long-term yields. While the estimated versions of the rational expectations (RE) model that allow for chairman-specific interest rate rules and time-varying price of risk (RE II and RE III) do a much better job than the simple bare-bones RE model (RE macro), they fall well short of the learning model once term structure data are used to estimate the latter (as in Learning I through III). Furthermore, much of the improvement in fit is due to a "level factor" generated by variations in the estimated inflation endpoint, which is reflected one for one in variations in the short-term rate endpoint. That feature of Dewachter and Lyrio's learning model is shown to be largely consistent with the observed evolution of survey-based long-term inflation expectations (which, in turn, display more variation than any model with chairman-specific inflation targets—but no learning about the latter—is bound to entail).

Of particular interest to monetary economists (even to those who may not care so much about the term structure) are the implications for the estimated deep parameters of Dewachter and Lyrio's "macro block" resulting from the need to fit the term structure data, as well as the allowance for learning dynamics. Two findings are worth emphasizing. First, the importance of the backward-looking component of the hybrid New Keynesian Phillips curve goes down substantially when learning dynamics are allowed for. Second, the variances of the innovations in the perceived inflation and real rate endpoints tend to be smaller under Greenspan than under previous Fed chairmen, possibly suggesting an enhanced transparency of monetary policy over the past two decades (because the true endpoints are indeed constant during each chairman's tenure).

Open Issues and Caveats

The present chapter by Dewachter and Lyrio constitutes an important contribution to the macro-finance literature on term structure dynamics. It is well written, and it contains a careful and extensive empirical analysis. Naturally, the chapter leaves a number of issues unexplained. It also relies on a number of assumptions that are not fully appealing. Let me turn to those briefly.

Do We Need a Full-Fledged DSGE Model to Explain the Term Structure Dynamics?

A simpler alternative to the full-fledged macro model developed and analyzed by Dewachter and Lyrio would consist of a partial equilibrium model of the term structure (e.g., the affine model used by Dewachter and Lyrio) that takes as given the joint process for the short-term rate i_t and the price kernel m_t.(e.g., a reduced form vector autoregression [VAR]). That process could be augmented with a perceived law of motion for the short-term endpoint, as well as with a learning algorithm similar to the one proposed by Dewachter and Lyrio. The use of a full-fledged model may impose unnecessary structure for the purpose at hand.

On the other hand, one can think of a possible justification for the dynamic stochastic general equilibrium (DSGE) approach pursued in the Dewachter and Lyrio chapter: to explore the macroeconomic implications of versions of a framework whose structure (including the embedded endpoint learning model) and estimated parameters are successful at fitting the term structure data. Among the questions one could ask based on that framework are the following: How does endpoint learning affect the transmission of monetary policy shocks? How does endpoint learning affect the desirability of alternative monetary policy rules? These are interesting and possibly important questions, but ones that fall beyond the scope of the Dewachter and Lyrio chapter.

A Strong Departure from Rational Expectations

A persistent gap between the perceived dynamics for some macro variables (or driving forces) and the actual equilibrium dynamics is a natural feature of models with constant-gain learning. Dewachter and Lyrio's framework is no exception in that regard. Yet, in Dewachter and Lyrio's model, the gap between the perceived law of motion and the actual law of motion is particularly large. In particular, Dewachter and Lyrio's assumptions imply that agents believe the law of motion for the inflation and real rate endpoints corresponds to two independent random walks. By contrast, in all the equilibriums considered by Dewachter and Lyrio, the estimated endpoints follow a stationary process, with an unconditional mean that corresponds to the deterministic steady state of the rational expectations equilibrium. A similar gap emerges in the simple model of the term structure analyzed in the preceding, as a comparison of equations (4) and (5) reveals. In my opinion, a perceived law of motion that shares with the actual law of motion the latter's order of integration would seem to be among the desiderata to be fulfilled by nonrational expectations models.

Two Competing Models

One can think of two alternative competing models that are likely to account for the observed behavior of bond yields equally well. The first class of models, exemplified by the present chapter, takes the "true" inflation and real rate endpoints to be constant, while letting agents learn about those endpoints using some constant-gain learning algorithm. The second class of models, exemplified by Hördahl, Tristani, and Vestin (2006), among others, assumes rational expectations, combined with time-varying endpoints for the real rate or inflation. The latter could, in turn, be justified by changes in the central bank's inflation target or changes in trend productivity growth. Sorting out the empirical merits of both families of models is likely to be nontrivial and is a task that also falls beyond the scope of Dewachter and Lyrio's present chapter. Yet the use of information on survey-based long-run expectations, as done in the present chapter, may be useful in achieving that objective.

Reference

Hördahl, P., O. Tristani, and D. Vestin. 2006. A joint econometric model of macroeconomic and term-structure dynamics. *Journal of Econometrics* 131 (1–2): 405–44.

Discussion Summary

Hans Dewachter responded to Jordi Galí that the authors had estimated a model of rational expectations and time-varying endpoints, but had found their estimates to be implausible in several respects. For example, they had found a considerable degree of variation in the inflation target, on the order of 2 percent per year. One of the aims of this chapter was to avoid this.

John Y. Campbell suggested that it might be interesting to examine the endpoints for the real rate and inflation separately. He found it plausible that there was considerable variation in the case of inflation, but puzzling—although, in the light of long-term Treasury inflation-protected securities (TIPS) rates, perhaps empirically plausible—that there should be much variation in the case of the real rate. Second, he thought that allowing for regime changes when a new chairman was appointed was a nice idea, but that it seemed inconsistent with the assumption of constant gain learning. Plausibly, there was more uncertainty following the appointment of a new chairman. At such times, one might have expected to see more volatility and higher risk premiums.

Glenn D. Rudebusch said that the aim of this literature was to unify the macro and finance approaches. But he was unhappy at the assumption that the price of risk was constant. In combination with homoskedasticity, this assumption led to constant risk premiums in the model. He suggested that the finance literature attributed some of the variation in the long bond rate to changing risk premiums. In this sense, the chapter did not fall into the mainstream macro-finance literature. Peter Westaway agreed that the Bank of England had also found that much of the variation in long interest rates could be attributed to a changing risk premium.

In response to Galí's question of why a dynamic stochastic general equilibrium (DSGE) model was needed, *Thomas Laubach* suggested that one answer was that DGSE models offer the possibility of calibrating macro parameters.

Michael Woodford agreed with Galí that the benchmark model should assume rational expectations. Regarding the fact that the inflation target has to move "too much" in an estimated rational expectations model, he asked whether the authors were uncomfortable believing this themselves or believing that other people could believe it.

On the topic of different endpoints for the real rate and for inflation, *Brian Sack* pointed out that for much of the last thirty years, the "level" factor has been the dominant influence on movements of the yield curve (that is, the yield curve has moved mostly in parallel). Over the last five years, however, the "slope" factor has been much more important (that is, the short end of the curve has moved, while the long end has remained stable). A shift to more stable inflation expectations may explain this. Sack also argued that the authors' model should be confronted with the evidence that macroeconomic forecasts respond strongly to data announcements.