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Expectations, Asset Prices, and Monetary Policy
The Role of Learning

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2.1 Introduction

Recent studies on asset prices and monetary policy consider the benefits of allowing the monetary authority to respond to asset prices in a monetary policy rule.[1] These studies frequently rely on two key assumptions: (1) asset price movements create distortions in economic activity through their effect on the ability of managers to finance investment; and (2) there exist exogenous "bubbles" or nonfundamental asset price movements.[2] In such environments, nonfundamental increases in asset prices cause investment booms, an increase in output above potential, and rising rates of inflation. In this framework, a monetary policy that responds strongly to inflation is frequently found to be sufficient in suppressing the undesirable consequences of these asset price fluctuations. In other words, there is no

1. Bernanke and Gertler (1999, 2001), Cecchetti et al. (2000), Gilchrist and Leahy (2002), and Tetlow (2005) provide recent examples.

2. Mishkin and White (2003) provide recent discussions of the evidence on stock market bubbles and their role in monetary policy for the U.S. economy, while Okina, Shirakawa, and Shiratsuka (2001) describe the Japanese stock market boom of the late 1980s and assess the conduct of monetary policy during this episode. Borio and Lowe (2002) discuss the relationship between financial imbalances and monetary policy.

need to respond to asset prices above and beyond what is implied by their ability to forecast inflation.

The notion that adopting a policy of responding strongly to inflation is a sufficient response to bubbles rests in part on the assumption that bubbles distort the economy by increasing managers' ability to invest without distorting their perceptions of the value of new investment. As Dupor (2005) emphasizes, these conclusions are tempered to the extent that bubbles directly influence managerial valuations of capital. More generally, nonfundamental movements in asset prices cause distortions in aggregate demand through their influence on markups and, hence, inflation and distort the consumption/investment decision by influencing the cost of capital. A monetary policymaker with one instrument—the nominal interest rate— faces a trade-off between reducing distortions owing to variation in the markup and distortions owing to variations in the return on capital. In such an environment, the policymaker may find monetary policy rules that respond to asset prices to be beneficial.

While much of the literature has focused on nonfundamental movements in asset prices, it is often recognized that asset price booms occur in conjunction with changes in the underlying economic fundamentals (Beaudry and Portier 2004). A case in point is the late 1990s run-up in U.S. stock prices that was closely tied to perceived changes in trend productivity growth. Thus, a key question in the literature is whether the monetary authority can identify the source of movements in asset prices in an environment of technological change. As emphasized by Edge, Laubach, and Williams (2004), it is plausible to believe that the underlying trend growth in productivity is unknown and that both the private sector and the policymaker learn over time about the true state of the economy. In this case, the benefits of allowing the monetary authority to respond to asset prices may depend on both the information structure of the economy and the extent to which asset price movements distort economic activity through the financing mechanism described in the preceding.

To address these issues, we reconsider the design of monetary policy rules in an environment where asset prices reflect expectations about underlying changes in the trend growth rate of technology. Our economy is a standard New Keynesian framework augmented to include financial market imperfections through the financial accelerator mechanism described in Bernanke, Gertler, and Gilchrist (1999). In our framework, the private sector and the policymaker are uncertain about the trend growth rate of technology but gradually learn over time. This learning process is reflected in asset price movements. Revisions to expectations owing to learning influence asset prices and entrepreneurial net worth. Such revisions feed back into investment demand and are magnified through the financial accelerator mechanism.

Our findings reinforce previous results in the literature. In the absence of

financial frictions, a policy of responding strongly to inflation is sufficient, even in situations where the private sector is uncertain about the true state of technology growth. In the absence of financial frictions, our economy shows essentially one distortion, owing to variations in the markup, which influences input choices. Suppressing inflation stabilizes the markup. Adding asset prices to the monetary policy rule is unlikely to provide further benefits, even in situations where the private sector is uninformed about the economy's true state of growth.

In the presence of financial market imperfections, a policy that responds strongly to inflation eliminates much of the distortionary effect of asset price movements on economic activity. Nonetheless, with inflation stabilized, the economy still exhibits significant deviations of output from potential. By giving weight to asset prices in the monetary policy rule, the monetary authority can improve upon these outcomes. Stabilizing output relative to potential comes at the cost of increased volatility of inflation, however. Thus, as in Dupor (2005), the monetary authority faces a trade-off owing to its desire to eliminate two distortions with one instrument.

Our policy analysis emphasizes the benefits to responding to an asset price gap—the gap between the observed asset prices and the potential level of asset prices that arises in a flexible-price economy without financial market imperfections. Computing such a gap requires the policymaker to make inferences regarding the true state of technology growth. We can thus distinguish between situations where the monetary authority has full information regarding the underlying state of technology growth and situations where the policymaker is learning about it over time. We can similarly distinguish between environments where the private sector is fully informed or is learning over time.

Our results imply that the benefits to responding to the asset price gap depend on the information structure of the economy. The benefits of responding to the asset price gap are greatest when the private sector is uninformed about the economy's true state of growth, but the policymaker is informed. At the other extreme, responding to the asset price gap may be detrimental when the private sector is informed and the policymaker is uninformed. In this case, the policymaker is responding to the "wrong" asset price gap.

We also consider alternative monetary policy rules that do not require the policymaker to infer the state of growth of the economy. These include responding to either asset price growth or output growth. Our findings suggest that both of these policies are likely to do well in our environment. On the other hand, we find that responding to the level of asset prices, as considered in much of the previous literature, is a particularly bad policy. Thus, the destabilizing effects of responding to asset price movements emphasized in previous studies may in part reflect the assumption that the monetary authority responds to the level of asset prices rather than their

deviation from the potential level. If the latter is unobservable, responding to changes in asset prices is better than responding to the level itself.

2.1.1   Related Literature

Bernanke and Gertler (1999, 2001), Cecchetti et al. (2000), Gilchrist and Leahy (2002), and Tetlow (2005) introduce nonfundamental bubbles into an economy and study the benefits of allowing the monetary authority to respond to asset prices. According to Bernanke and Gertler (1999, 2001), a policy that implies a strong response to inflation stabilizes the economy, and asset prices are only useful to the extent that they provide information about inflation and the output gap. In this environment, bubbles are exogenous and affect the economy by increasing aggregate demand through a financial accelerator mechanism. A policy that responds strongly to inflation is sufficient to suppress this aggregate demand channel. Cecchetti et al. (2000) argue that there may be some benefit to responding to asset prices in such environments although it is likely to be small. This literature suggests that adopting a monetary policy rule that implies a strong policy response to inflation is sufficient even under two situations in which asset prices may contain a relatively large amount of information about the state of the economy: an economy with financial frictions and an economy with shocks that have a persistent impact on technology growth (Gilchrist and Leahy 2002).

Our framework differs from this analysis in two fundamental ways. First, in our economy, deviations between asset prices and underlying cash flows occur because agents do not know the true state of technology growth but instead are learning about it over time. Recent studies by French (2001), Roberts (2001), and Kahn and Rich (2003) emphasize the distinction between transitory and persistent movements in the growth rate of technology. Edge, Laubach, and Williams (2004) study the effect of learning about transitory and persistent movements in technology growth in a model-based environment. As an example of such learning, they document that the productivity growth forecasts of professional forecasters and policymakers did not change until 1999 although the trend had shifted in the mid-1990s. They also demonstrate that a constant-gain Kalman filter tracks well the actual forecasts of trend productivity in the 1970s and in the 1990s made by forecasters and policymakers. Pakko (2002) and Edge, Laubach, and Williams (2004) introduce learning with a Kalman filter to a real business cycle (RBC) model to understand the effect of changes in the trend growth rate of technology on economic activity. Our chapter is also related to Tambalotti (2003), who considers the role of learning in a dynamic stochastic general equilibrium model with price rigidities but no capital accumulation, and Dupor (2005), who considers an environment where agents learn about fundamental and nonfundamental shocks to the return on capital.

Our framework is closely related to Edge, Laubach, and Williams (2005), who allow for learning about the trend growth rate of technology in a dynamic stochastic general equilibrium model with price rigidities and capital accumulation. We extend their framework by allowing both the private sector and the policymaker to learn about the true state of technology growth. We do so in an environment where learning influences asset values, which feed back into real economic activity through the net worth channel emphasized by Bernanke, Gertler, and Gilchrist (1999). We show that this financial accelerator mechanism may be enhanced in the presence of learning. This stronger feedback mechanism raises the benefit to responding to asset prices, even in an environment where the policymaker is itself uninformed about the true state of technology growth.

Second, much of the previous literature focuses on the benefits of responding to the level of asset prices. In our framework, asset price movements would occur in the absence of frictions in either price-setting or financial markets. Thus, we emphasize the importance of the monetary authority's response to the asset price gap—the gap between the observed asset prices and the underlying potential level of asset prices. Our finding that responding to the growth rate of asset prices is also beneficial is related to Tetlow (2005), who compares the benefit of responding to the growth rate of asset prices relative to the level of asset prices in a robust control framework.

Our emphasis on asset price movements that are tied to fundamental changes in the underlying trend growth rate of the economy is related to the recent literature on the response of asset prices to news about future economic fundamentals. Barsky and DeLong (1993) and Kiyotaki (1990) study the effects of learning about the transitory and persistent components of dividend growth on asset prices in a partial equilibrium model. When the transitory and persistent shocks to dividend growth are not observed separately, investors extrapolate a transitory movement in dividend growth into the future, generating a large response in asset prices. The interest rate is fixed in these partial equilibrium models, which helps to generate large movements in asset prices. Kiley (2000) provides a comparison of the asset pricing implications of partial and general equilibrium models. Asset prices may fall in response to increases in the growth rate of technology, as real interest rates rise in general equilibrium.

In an RBC framework that allows for capital accumulation, a persistent increase in the growth rate of technology leads to a rise in the real interest rate and decreases in investment and asset prices. Consumption rises by a large amount due to a large wealth effect of expectations of future technology improvements (Barro and King 1984; Campbell 1994; Cochrane 1994). Using a New Keynesian model, Gilchrist and Leahy (2002) show that asset prices may rise rather than fall in response to a persistent increase in the growth rate of technology. This positive response in asset

prices relies on an accommodative monetary policy that responds weakly to inflation. More recently, Christiano, Motto, and Rostagno (2005) emphasize the role of monetary policy in generating an asset price boom in a model with habit formation and adjustment costs to investment growth. In their model, favorable news about future technology tends to lower current inflation. As the monetary authority responds by lowering interest rates, asset prices rise. Jaimovich and Rebelo (2006) consider RBC environments that may produce asset price booms following favorable news about future technology. In our framework, as in Gilchrist and Leahy (2002), asset prices are more likely to rise in response to favorable news about future technology in the presence of accommodative monetary policy, and movements in asset prices are amplified in the presence of the financial accelerator mechanism.

Finally, there is a rich literature emphasizing the welfare benefits of monetary policy rules in environments with imperfect information and environments that allow for financial frictions. Dupor (2005) and Edge, Laubach, and Williams (2005) solve a Ramsey problem to study the characteristics of the optimal monetary policy, while Tambalotti (2003) uses a second-order approximation to the utility function in a model without capital. More closely related to our work, Faia and Monacelli (2006) use a second-order approximation to the policy function in the Bernanke, Gertler, and Gilchrist (1999) framework and find that including the level of asset prices in the interest rate rule with a modest coefficient is beneficial to welfare when the coefficient on inflation is relatively small. When the coefficient on inflation is sufficiently large, including asset prices in the policy rule does not improve welfare. Although we focus on a quadratic loss function rather than formal welfare analysis, our results imply modest benefits of allowing the monetary authority to respond to the asset price gap, even when the monetary authority is responding strongly to inflation. This difference in results may be partially attributable to our emphasis on asset price gaps rather than asset price levels as the variable in the policy rule.[3]

2.2   Model

The model is a dynamic stochastic general equilibrium model with a financial accelerator mechanism (Bernanke, Gertler, and Gilchrist 1999).[4]

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3. Our finding that a policy that responds to the growth rate of asset prices or the growth rate of output performs well when the policymaker has imperfect information about the state of technology growth is related to Orphanides and Williams (2002), who find that in environments where the natural rates are unobservable, an interest rate rule that includes changes in economic activity (which does not require information on the natural rates) performs well.

4. The description of the model closely follows Bernanke, Gertler, and Gilchrist (1999) and Gertler, Gilchrist, and Natalucci (2006).

The financial accelerator mechanism links the relative price of capital (interpretable as asset prices), balance-sheet conditions of borrowers, the external finance premium defined as the cost of external funds relative to the cost of internal funds, and investment spending. Specifically, an unexpected increase in asset prices—as a result of a favorable shock to productivity of the economy, for example—increases the net worth of borrowers, decreases the external finance premium, and increases the capital expenditures of these borrowers. In general equilibrium, the increase in capital expenditures leads to a further increase in asset prices and magnifies the mechanism just described. To clarify the role of the financial accelerator mechanism in the relationship between asset prices and monetary policy, the following sections also consider a model in which the financial accelerator mechanism is absent.

2.2.1   Structure of the Economy

We first describe the structure of the economy, including the specification of monetary policy rules and the information structure. We consider the problems of households, entrepreneurs, capital producers, and retailers in turn.

Households

Households consume, hold money, save in the form of a one-period riskless bond whose nominal rate of return is known at the time of the purchase, and supply labor to the entrepreneurs who manage the production of wholesale goods.

Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t, \frac{M_t}{P_t}\right),$$

with

$$u\left(C_t, H_t, \frac{M_t}{P_t}\right) = \ln C_t - \theta \frac{H_t^{1+\gamma}}{t+\gamma} + \xi \ln \frac{M_t}{P_t},$$

where C_t is consumption, H_t is hours worked, M_t/P_t is real balances acquired in period t and carried into period $t + 1$, and γ, θ, and ξ are positive parameters.

The budget constraint is given by

$$C_t = \frac{W_t}{P_t} H_t + \Pi_t - T_t - \frac{M_t - M_{t-1}}{P_t} - \frac{B_{t+1} - R_t^n B_t}{P_t},$$

where W_t is the nominal wage for the household labor, Π_t is the real dividends from ownership of retail firms, T_t is lump-sum taxes, B_{t+1} is a riskless bond held between period t and period $t + 1$, and R_t^n is the nominal rate of return on the riskless bond held between period $t - 1$ and period t.

The first-order conditions for the household's optimization problem include

(1)
$$\frac{1}{C_t} = \beta E_t \left(\frac{1}{C_{t+1}} R^n_{t+1} \frac{P_t}{P_{t+1}} \right),$$

and

(2)
$$\frac{1}{C_t} \frac{W_t}{P_t} = \theta H_t^{\gamma}.$$

Entrepreneurs

Entrepreneurs manage the production of wholesale goods. The production of wholesale goods uses capital constructed by capital producers and labor supplied by both households and entrepreneurs. Entrepreneurs purchase capital from capital goods producers and finance the expenditures on capital with both entrepreneurial net worth (internal finance) and debt (external finance). We introduce financial market imperfections that make the cost of external funds depend on the entrepreneur's balance-sheet condition.

Entrepreneurs are risk neutral. To ensure that entrepreneurs do not accumulate enough funds to finance their expenditures on capital entirely with net worth, we assume that they have a finite lifetime. In particular, we assume that each entrepreneur survives until the next period with probability η. New entrepreneurs enter to replace those who exit. To ensure that new entrepreneurs have some funds available when starting out, each entrepreneur is endowed with H_t^e units of labor that are supplied inelastically as a managerial input to the wholesale-good production at nominal entrepreneurial wage W_t^e.

The entrepreneur starts any period t with capital K_t purchased from capital producers at the end of period $t-1$ and produces wholesale goods Y_t with labor and capital. Labor L_t is a composite of household labor H_t and entrepreneurial labor H_t^e:

$$L_t = H_t^{1-\Omega} (H_t^e)^{\Omega}.$$

The entrepreneur's project is subject to an idiosyncratic shock ω_t, which affects both the production of wholesale goods and the effective quantity of capital held by the entrepreneur. We assume that ω_t is independently and identically distributed (*i.i.d.*) across entrepreneurs and time, satisfying $E(\omega_t) = 1$. The production function for the wholesale goods is given by

(3)
$$Y_t = \omega_t (A_t L_t)^{\alpha} K_t^{1-\alpha},$$

where A_t is exogenous technology common to all the entrepreneurs. Let $P_{W,t}$ denote the nominal price of wholesale goods, Q_t the price of capital relative to the aggregate price P_t to be defined later, and δ the depreciation

rate. The entrepreneur's real revenue in period t is the sum of the production revenues and the real value of the undepreciated capital:

$$\omega_t \left[\frac{P_{W,t}}{P_t} (A_t L)^\alpha K_t^{1-\alpha} + Q_t(1-\delta)K_t \right].$$

In any period t, the entrepreneur chooses the demand for both household labor and entrepreneurial labor to maximize profits given capital K_t acquired in the previous period. The first-order conditions are

(4)
$$\alpha(1-\Omega)\frac{Y_t}{H_t} = \frac{W_t}{P_{W,t}},$$

and

(5)
$$\alpha\Omega\frac{Y_t}{H_t^e} = \frac{W_t^e}{P_{W,t}}.$$

At the end of period t, after the production of wholesale goods, the entrepreneur purchases capital K_{t+1} from capital producers at price Q_t. The capital is used as an input to the production of wholesale goods in period $t+1$. The entrepreneur finances the purchase of capital $Q_t K_{t+1}$ partly with net worth N_{t+1} and partly by issuing nominal debt B_{t+1}:

$$Q_t K_{t+1} = N_{t+1} + \frac{B_{t+1}}{P_t}.$$

The entrepreneur's capital purchase decision depends on the expected rate of return on capital and the expected marginal cost of finance. The real rate of return on capital between period t and period $t+1$, R_{t+1}^k, depends on the marginal profit from the production of wholesale goods and the capital gain:

(6)
$$R_{t+1}^k = \frac{\omega_{t+1}\left[\frac{P_{W,t+1}}{P_{t+1}}(1-\alpha)\frac{\overline{Y}_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1} \right]}{Q_t},$$

where $\overline{Y}_{t+1}$ is the average wholesale good production per entrepreneur ($Y_{t+1} = \omega_{t+1}\overline{Y}_{t+1}$). Under our assumption of $E_t\omega_{t+1} = 1$, the expected real rate of return on capital, $E_t R_{t+1}^k$, is given by

(7)
$$E_t R_{t+1}^k = E_t \left[\frac{\frac{P_{W,t+1}}{P_{t+1}}(1-\alpha)\frac{\overline{Y}_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t} \right].$$

In the presence of financial market imperfections, the marginal cost of external funds depends on the entrepreneur's balance-sheet condition. As

in Bernanke, Gertler, and Gilchrist (1999), we assume asymmetric information between borrowers (entrepreneurs) and lenders and a costly state verification. Specifically, the idiosyncratic shock to entrepreneurs, ω_t, is private information for the entrepreneur. To observe this, the lender must pay an auditing cost that is a fixed proportion μ_b of the realized gross return to capital held by the entrepreneur: $\mu_b R_{t+1}^k Q_t K_{t+1}$. The entrepreneur and the lender negotiate a financial contract that induces the entrepreneur to not misrepresent her earnings and minimizes the expected auditing costs incurred by the lender. We restrict attention to financial contracts that are negotiated one period at a time and offer lenders a payoff that is independent of aggregate risk. Under these assumptions, the optimal contract is a standard debt with costly bankruptcy: if the entrepreneur does not default, the lender receives a fixed payment independent of the realization of the idiosyncratic shock ω_t; and if the entrepreneur defaults, the lender audits and seizes whatever it finds.

In equilibrium, the cost of external funds between period t and period $t + 1$ is equated to the expected real rate of return on capital (7). We define the external finance premium s_t as the ratio of the entrepreneur's cost of external funds to the cost of internal funds, where the latter is equated to the cost of funds in the absence of financial market imperfections $E_t[R_{t+1}^n(P_t/P_{t+1})]$:

(8)
$$s_t \equiv \frac{E_t R_{t+1}^k}{E_t\left(R_{t+1}^n \dfrac{P_t}{P_{t+1}}\right)}.$$

In the absence of financial market imperfections, there is no external finance premium ($s_t = 1$).

The agency problem implies that the cost of external funds depends on the financial position of the borrowers. In particular, the external finance premium increases when a smaller fraction of capital expenditures is financed by the entrepreneur's net worth:

(9)
$$s_t = s\left(\frac{Q_t K_{t+1}}{N_{t+1}}\right),$$

where $s(\cdot)$ is an increasing function for $N_{t+1} < Q_t K_{t+1}$. The specific form of the function $s(\cdot)$ depends on the primitive parameters of the costly state verification problem, including the bankruptcy cost parameter μ_b and the distribution of the idiosyncratic shock ω_t. We specify a parametric form for the function $s(\cdot)$ in the next section.

The aggregate net worth of entrepreneurs at the end of period t is the sum of the equity held by entrepreneurs who survive from period $t-1$ and the aggregate entrepreneurial wage, which consists of the wage earned by the entrepreneurs surviving from period $t-1$ and the wage earned by newly emerged entrepreneurs in period t:

(10) $$N_{t+1} = \eta \left(R_t^k Q_{t-1} K_t - E_{t-1} R_t^k \cdot \frac{B_t}{P_{t-1}} \right) + \frac{W_t^e}{P_t}$$

$$= \eta [R_t^k Q_{t-1} K_t - E_{t-1} R_t^k (Q_{t-1} K_t - N_t)] + \frac{W_t^e}{P_t},$$

where the second line used the relation $Q_{t-1} K_t = N_t + B_t / P_{t-1}$.

Unexpected changes in asset prices are the main source of changes in the entrepreneurial net worth and, hence, the external finance premium. Equations (6) and (7) suggest that unexpected changes in asset prices are the main source of unexpected changes in the real rate of return on capital—the difference between the realized rate of return on capital in period t, R_t^k, and the rate of return on capital anticipated in the previous period, $E_{t-1} R_t^k$, where the latter is the marginal cost of external funds between period $t-1$ and t. Equation (10), in turn, suggests that the main source of changes in the entrepreneurial net worth is unexpected movements in the real rate of return on capital, under the calibration that the entrepreneurial wage is small.[5] Finally, equation (9) implies that changes in the entrepreneurial net worth are the main source of changes in the external finance premium. Thus, movements in asset prices play a key role in the financial accelerator mechanism.

Entrepreneurs going out of business in period t consume the residual equity:

(11) $$C_t^e = (1 - \eta) \left(R_t^k Q_{t-1} K_t - E_{t-1} R_t^k \cdot \frac{B_t}{P_{t-1}} \right),$$

where C_t^e is the aggregate consumption of the entrepreneurs who exit in period t.

Overall, the financial accelerator mechanism implies that an unexpected increase in asset prices increases the net worth of entrepreneurs and improves their balance-sheet conditions. This, in turn, reduces the external finance premium and increases the demand for capital by these entrepreneurs. In equilibrium, the price of capital increases further, and capital producers increase the production of new capital. This additional increase in asset prices strengthens the mechanism just described. Thus, the countercyclical movement in the external finance premium implied by the financial market imperfections magnifies the effects of shocks to the economy.

Capital Producers

Capital producers use both final goods I_t and existing capital K_t to construct new capital K_{t+1}. They lease existing capital from the entrepreneurs.

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5. In the calibration below, we set $\Omega = 0$, which makes the effects of changes in the entrepreneurial wage on net worth negligible.

Each capital producer operates a constant returns-to-scale technology for capital production $\phi(I_t/K_t)K_t$, where the function $\phi(\cdot)$ is increasing and concave, capturing the increasing marginal costs of capital production. The aggregate capital accumulation equation is given by

$$(12) \qquad K_{t+1} = (1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t.$$

Taking the relative price of capital Q_t as given, capital producers choose inputs I_t and K_t to maximize profits from the formation of new capital. The following first-order condition for the capital producer's problem implies that investment (the demand for final goods by capital producers) and the quantity of new capital increase as the relative price of capital—interpretable as asset prices—increases:

$$(13) \qquad Q_t = \frac{1}{\phi'\left(\dfrac{I_t}{K_t}\right)}.$$

Retailers

There is a continuum of monopolistically competitive retailers of measure unity. Retailers buy wholesale goods from entrepreneurs in a competitive manner and then differentiate the product slightly at zero resource cost.

Let $Y_t(z)$ be the retail goods sold by retailer z, and let $P_t(z)$ be its nominal price. Final goods, Y_t, are the composite of individual retail goods

$$Y_t = \left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon}\,dz\right]^{\varepsilon/(\varepsilon-1)},$$

and the corresponding price index, P_t, is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon}\,dz\right]^{1/(1-\varepsilon)}.$$

Households, capital producers, and the government demand the final goods.

Each retailer faces an isoelastic demand curve given by

$$(14) \qquad Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\varepsilon} Y_t.$$

As in Calvo (1983), each retailer resets price with probability $(1 - \upsilon)$, independently of the time elapsed since the last price adjustment. Thus, in each period, a fraction $(1 - \upsilon)$ of retailers reset their prices, while the remaining fraction υ keeps their prices unchanged. The real marginal cost to the retailers of producing a unit of retail goods is the price of wholesale goods,

relative to the price of final goods ($P_{W,t}/P_t$). Each retailer takes the demand curve in equation (14) and the price of wholesale goods as given and sets the retail price $P_t(z)$. All retailers given a chance to reset their prices in period t choose the same price P_t^* given by

$$(15) \qquad P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{i=0}^{\infty} \upsilon^i \Lambda_{t,i} P_{t+i}^W Y_{t+i} \left(\dfrac{1}{P_{t+i}}\right)^{1-\varepsilon}}{E_t \sum_{i=0}^{\infty} \upsilon^i \Lambda_{t,i} Y_{t+i} \left(\dfrac{1}{P_{t+i}}\right)^{1-\varepsilon}},$$

where $\Lambda_{t,i} \equiv \beta^i C_t/C_{t+i}$ is the stochastic discount factor that the retailers take as given.

The aggregate price evolves according to

$$(16) \qquad P_t = [\upsilon P_{t-1}^{1-\varepsilon} + (1 - \upsilon)(P_t^*)^{1-\varepsilon}]^{1/(1-\varepsilon)}.$$

Combining equations (15) and (16) yields an expression that relates the current inflation to the current real marginal cost and the expected inflation, as described in the appendix.

Aggregate Resource Constraint

The aggregate resource constraint for final goods is

$$(17) \qquad Y_t = C_t + C_t^e + I_t + G_t,$$

where G_t is the government expenditures that we assume to be exogenous.[6]

Government

Exogenous government expenditures G_t are financed by lump-sum taxes T_t and money creation:

$$(18) \qquad G_t = \frac{M_t - M_{t-1}}{P_t} + T_t.$$

The money stock is adjusted to support the interest rate rule specified in the following. Lump-sum taxes adjust to satisfy the government budget constraint.

Technology Shock Process

The growth rate of technology has both transitory and persistent components:

$$(19) \qquad \ln A_t - \ln A_{t-1} = \mu_t + \varepsilon_t.$$

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6. In the following calibration, we assume that actual resource costs to bankruptcy are negligible.

The persistent component of technology growth in deviation from the mean growth rate of technology, $(\mu_t - \mu)$, follows an AR(1) process:

$$(20) \qquad (\mu_t - \mu) = \rho_d(\mu_{t-1} - \mu) + \upsilon_t.$$

Shocks to the transitory and persistent components of technology growth are

$$(21) \qquad \varepsilon_t \sim i.i.d.\, N(0, \sigma_\varepsilon^2),$$

and

$$(22) \qquad \upsilon_t \sim i.i.d.\, N(0, \sigma_\upsilon^2).$$

Information Structure

Our technology process allows for two sources of variation: shocks to the transitory and persistent components of technology growth. We consider both the case of full information where agents observe both shocks separately and the case of imperfect information where agents observe the technology series, A_t, but cannot decompose movements in technology growth into their respective sources.

Monetary Policy Rules

The monetary authority conducts monetary policy using interest rate rules. We consider the following types of interest rate rules.

Policy Rule with Inflation Only. The first rule we consider is the one with current inflation only:

$$(23) \qquad R_{t+1}^n = R^n \, \pi_t^{\phi_\pi},$$

where $\pi_t \equiv P_t/P_{t-1}$ is inflation, and R^n is the steady-state nominal interest rate on the one-period bond. We assume that the policymaker targets 0 percent inflation. Bernanke and Gertler (1999, 2001) show that this rule with a large coefficient ϕ_π performs well in the economy with shocks to the bubble component of asset prices as well as shocks to technology.

Policy Rule with the Asset Price Gap. In the second rule that we consider, the monetary authority adjusts interest rates based on current inflation and the gap between the observed asset prices Q_t and the inferred potential level of asset prices Q_t^*:

$$(24) \qquad R_{t+1}^n = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_t^*} \right)^{\phi_Q},$$

where Q_t^* is the equilibrium level of asset prices in the economy without pricing and financial frictions.

The potential level of asset prices is computed under the information available to the policymaker. When the policymaker has full information,

we use $Q^*_{\text{full},t}$, which is obtained by solving a flexible-price model without financial frictions under full information. When the policymaker has imperfect information, we use $Q^*_{\text{imp},t}$, which is obtained by solving a flexible-price model without financial frictions under imperfect information.

There are two ways to construct Q^*_t from the model. In the first, one could use the hypothetical levels of the state variables in the frictionless economy to compute Q^*_t. In the second, one may use the levels of the state variables in the model with both pricing and financial market frictions combined with the decision rule for the frictionless economy to compute Q^*_t. Neiss and Nelson (2003) follow the first approach, and Woodford (2003) argues that the second approach is more realistic. We adopt the first procedure because it is somewhat easier to work with.

Policy Rule with the Natural Rate and the Asset Price Gap. We also consider a policy rule that allows the policymaker to respond to movements in the natural rate of interest:

$$(25) \qquad R^n_{t+1} = R^*_{t+1}\, \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q^*_t} \right)^{\phi_Q},$$

where R^*_{t+1} is the natural rate of interest that prevails between period t and period $t+1$. We define the natural rate of interest as the real interest rate that supports the efficient allocation in the economy without pricing and financial frictions. It is computed based on the information available to the policymaker.

Policy Rule with Asset Price Growth or Output Growth. The policy rule with the asset price gap requires the policymaker to compute Q^*_t—the level of asset prices in the flexible-price economy without financial frictions. An alternative would be to allow the policymaker to respond to the growth rate of observed asset prices:

$$(26) \qquad R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_{t-1}} \right)^{\phi_Q}.$$

This rule is considered in Tetlow (2005).

For comparison purposes, we also consider a monetary policy rule that includes a policy response to the growth rate of output:

$$(27) \qquad R^n_{t+1} = R^n \pi_t^{\phi_\pi} \left[\frac{Y_t}{\exp(\mu)\, Y_{t-1}} \right]^{\phi_Y},$$

where μ is the mean growth rate of technology.

Policy Rule with the Level of Asset Prices. As another rule that does not require the policymaker to infer the unobserved shocks and thus the potential level of asset prices, we consider a policy rule that includes a response

to the level of asset prices in deviation from the nonstochastic steady-state level:

$$(28) \qquad R_{t+1}^{n} = R^{n} \pi_{t}^{\phi_{\pi}} \left(\frac{Q_{t}}{Q} \right)^{\phi_{Q}},$$

where Q is the nonstochastic steady-state level of asset prices. This rule is considered in Bernanke and Gertler (1999, 2001) and in Faia and Monacelli (2006). This rule does not take into account variation in the potential level of asset prices.

2.2.2   Filtering under Imperfect Information

Let $Z_{t} \equiv A_{t}/A_{t-1}$ denote technology growth, $\tilde{z}_{t} \equiv (\ln Z_{t} - \mu)$ the percentage deviation of technology growth from the mean, and $\tilde{d}_{t} \equiv (\mu_{t} - \mu)$ the percentage deviation of the persistent component of technology growth from the mean. Then we can write the technology process in equations (19) and (20) as

$$(29) \qquad \tilde{z} = \tilde{d}_{t} + \varepsilon_{t},$$

and

$$(30) \qquad \tilde{d}_{t} = \rho_{d}\tilde{d}_{t-1} + v_{t}.$$

Under full information, agents observe both the shock to the transitory component of technology growth, ε_{t}, and the shock to the persistent component of technology growth, v_{t}. Under imperfect information, agents observe $\tilde{z}_{t}$, or the sum of two components, $(\tilde{d}_{t} + \varepsilon_{t})$ but do not observe the two shocks separately.

Let $E[\tilde{d}_{t}|\tilde{z}_{t}, \tilde{z}_{t-1}, \ldots] \equiv \tilde{d}_{t|t}$ denote the inference of agents about the current state of the persistent component of technology growth based on the observations of current and past technology growth. We assume that agents update inferences based on the steady-state Kalman filter:

$$(31) \qquad \tilde{d}_{t|t} = \lambda\tilde{z}_{t} + (1 - \lambda) \rho_{d}\tilde{d}_{t-1|t-1},$$

where the gain, λ, is given by

$$(32) \qquad \lambda \equiv \frac{\phi - (1 - \rho_{d}^{2}) + \phi\sqrt{(1 - \rho_{d}^{2})^{2}\frac{1}{\phi^{2}} + 1 + \frac{2}{\phi} + 2\rho_{d}^{2}\frac{1}{\phi}}}{2 + \phi - (1 - \rho_{d}^{2}) + \phi\sqrt{(1 - \rho_{d}^{2})^{2}\frac{1}{\phi^{2}} + 1 + \frac{2}{\phi} + 2\rho_{d}^{2}\frac{1}{\phi}}},$$

and ϕ measures the signal-to-noise ratio:

$$(33) \qquad \phi \equiv \frac{\sigma_{v}^{2}}{\sigma_{\varepsilon}^{2}}.$$

It is straightforward to show that the gain, λ, is monotonically increasing in both the signal-to-noise ratio, ϕ, and the AR(1) coefficient on the persistent component of technology growth, ρ_d.

Given $\tilde{d}_{t|t}$, the inference about the shock to the transitory component of technology growth, $\varepsilon_{t|t} \equiv E[\varepsilon_t | \tilde{z}_t, \tilde{z}_{t-1}, \ldots]$, is given by

$$(34) \qquad\qquad \varepsilon_{t|t} = \tilde{z}_t - \tilde{d}_{t|t},$$

and the inference about the shock to the persistent component of technology growth, $\upsilon_{t|t} \equiv E[\upsilon_t | \tilde{z}_t, \tilde{z}_{t-1}, \ldots]$, is given by

$$(35) \qquad\qquad \upsilon_{t|t} = \tilde{d}_{t|t} - \rho_d \tilde{d}_{t-1|t-1}.$$

Properties of the Inference under Imperfect Information

We now illustrate the properties of the inference of agents about the state of technology growth. We consider how each of the shocks to the transitory and persistent components of technology growth affects the inference of agents.[7]

Figure 2.1 presents the response to a 1 percent increase in the transitory component of technology growth. The dashed line is the actual persistent component of technology growth in deviation from the mean technology growth rate, $\tilde{d}_t \equiv (\mu_t - \mu)$. The solid line is the inferred persistent component of technology growth in deviation from the mean growth rate, $\tilde{d}_{t|t}$. Although the shock considered here has no impact on the persistent component of technology growth, agents initially interpret part of the observed changes in technology growth to be persistent. Over time, they gradually learn that the shock was to the transitory component of technology growth.

Figure 2.2 presents the effect of a 1 percent increase in the persistent component of technology growth on both the actual and the inferred persistent component of technology growth, $\tilde{d}_t$ and $\tilde{d}_{t|t}$. Although the shock considered here changes the persistent component of technology growth, agents initially interpret most of the observed increase in technology growth to be transitory. Over time, as agents accumulate more observations of technology growth, they gradually revise their inferences.

Difference in Information between the Private Sector and the Policymaker

Our framework allows us to consider the case where the policymaker has different information from the private sector. The case where the policymaker and the private sector have the same information about the aggregate shocks to the economy is arguably more realistic than the case where

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7. The parameter values related to the shock process used in these experiments are described in the following section.
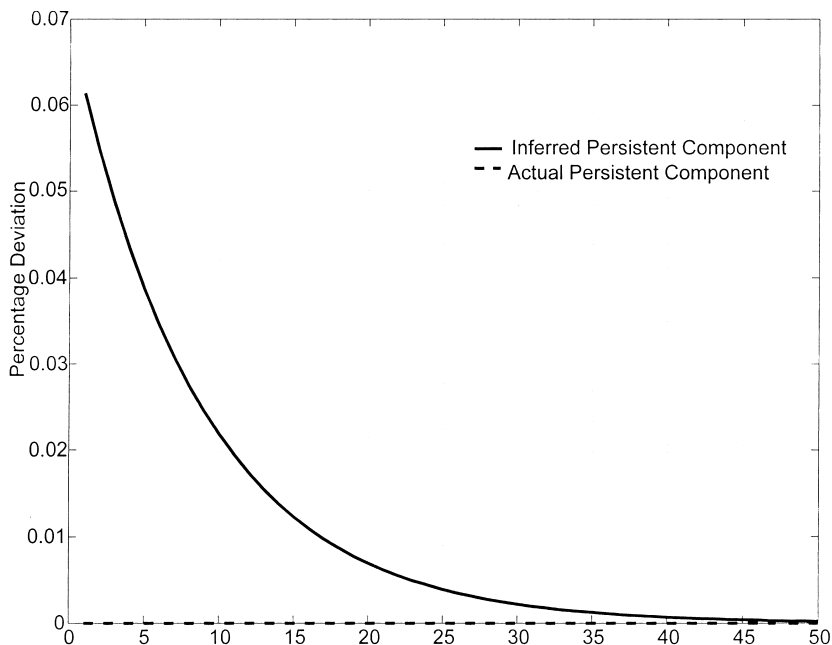
Figure 2.1   Belief response to a transitory shock to technology growth

Note: The dashed line is the realization of the persistent component of technology growth in percentage deviation from the mean technology growth rate: $\tilde{d}_t \equiv (\mu_t - \mu)$. The straight line is the inference about the persistent component of technology growth in percentage deviation from the mean technology growth rate: $E[\tilde{d}_t | \tilde{z}_t, \tilde{z}_{t-1}, ...] \equiv \tilde{d}_{t|t}$.

they have different information. Considering the cases where they have different information is useful for our analysis because in these cases the benefits or the losses from allowing a policy response to the asset price gap or the natural rate of interest are the greatest. Specifically, as we see in later sections, the gains from allowing the policymaker to respond to movements in the natural rate of interest or the asset price gap are greatest when the policymaker has full information and the private sector has imperfect information.[8] Allowing the policymaker to respond to the natural rate of interest or the asset price gap is most harmful when the policymaker has imperfect information and the private sector has full information.

In the case where the policymaker has full information and the private sector has imperfect information, we preclude the possibility that the latter learns more about the realizations of the shocks to the transitory and persistent components of technology growth by observing the former's be-

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8. As described in the following, we assess the benefits of adopting various interest rate rules based on the variance of inflation and the output gap.
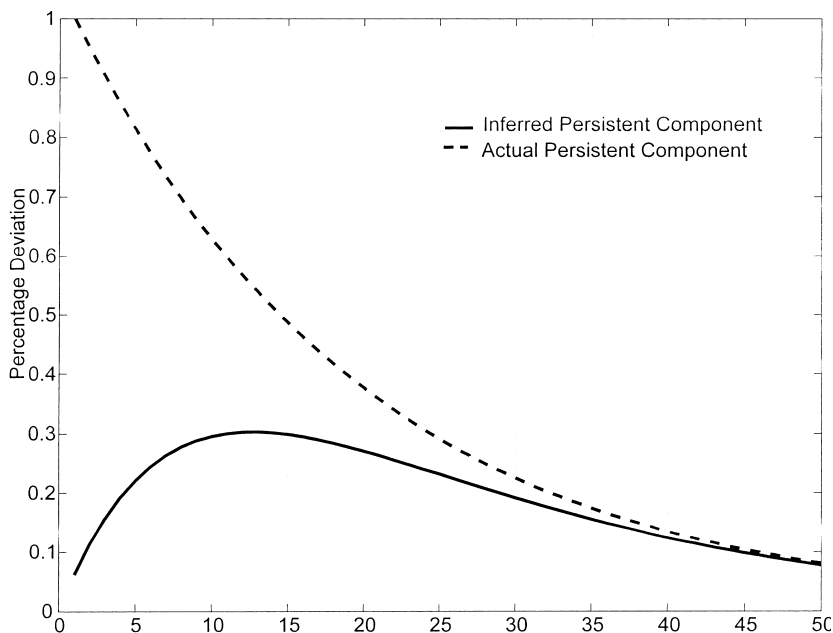
Figure 2.2   Belief response to a persistent shock to technology growth

Note: The dashed line is the realization of the persistent component of technology growth in percentage deviation from the mean technology growth rate: $\tilde{d}_t \equiv (\mu_t - \mu)$. The straight line is the inference about the persistent component of technology growth in percentage deviation from the mean technology growth rate: $E[\tilde{d}_t | \tilde{z}_t, \tilde{z}_{t-1}, ...] \equiv \tilde{d}_{t|t}$.

havior.[9] Because the policymaker's setting of the interest rate is affected by the information it possesses, the policymaker's information indirectly affects the behavior of the private sector through movements in the interest rate that is set, however. Thus, the policymaker's information affects the private sector's incentives but not the inferences regarding the state of technology growth. Likewise, in the case where the policymaker has imperfect information and the private sector has full information, we preclude the possibility that the former learns about the unobserved shocks to technology growth from the latter's behavior. Thus, when considering the case of different information between the private sector and the policymaker, we

9. Specifically, we assume that, when the private sector solves its optimization problem, it does not internalize the fact that the potential level of asset prices Q_t^* in the policy rule in equations (24) and (25) and the natural rate of interest R_{t+1}^* in the policy rule in equations (25) are functions of the realizations of the shocks μ_t and ε_t and capital stock, where those functions are obtained by solving for the efficient allocation in the frictionless economy. Note also that the variables about which the private sector learns—the realizations of the shocks to the transitory and persistent components of technology growth—are exogenous and independent of the policymaker's behavior.

view our results as providing a useful benchmark to assess the best- and worst-case scenarios relative to the more realistic situation where the private sector and the policymaker have the same information or may learn from each other's actions. Allowing for learning between the private sector and the policymaker is an interesting avenue for future research.

2.3   Calibration

We adopt a fairly standard calibration of preferences, technology, and the price-setting structure. The financial sector is calibrated to conform to a simplified version of Bernanke, Gertler, and Gilchrist (1999). These simplifications allow us to focus on the main distortion that is introduced by financial market imperfections—the introduction of a countercyclical premium on external funds that drives a wedge between the cost of external funds and the cost of internal funds.

2.3.1   Preferences, Technology, and Price-Setting

A period in the model is a quarter. The discount factor is $\beta = 0.984$. The labor share of income is $\alpha = 2/3$. Setting $\gamma = 0.8$ implies that the labor supply elasticity is $1/\gamma = 1.25$. The depreciation rate is $\delta = 0.025$. The elasticity of asset prices with respect to the investment-capital ratio is $\eta_k \equiv -[\phi''(i/k)Z(i/k)Z]/\phi'[(i/k)Z] = 0.25$, the same as in Bernanke, Gertler, and Gilchrist (1999) and Bernanke and Gertler (1999).[10] For the price setting, the steady-state markup is $\varepsilon/(\varepsilon - 1) = 1.1$, while the probability that a producer does not adjust prices in a given quarter is $\upsilon = 0.75$.

2.3.2   Financial Market Imperfections

When log-linearizing the model, we adopt a number of simplifications to the original financial sector specified in Bernanke, Gertler, and Gilchrist (1999). These simplifications allow us to focus on the primary distortion associated with financial market imperfections—namely, that it introduces a time-varying countercyclical wedge between the rate of return on capital and the rate of return on the riskless bond held by households. We assume that variations in entrepreneurial consumption and the entrepreneurial wage are negligible and can be ignored. We further assume that actual resource costs to bankruptcy are also negligible. Model simulations conducted under the original Bernanke, Gertler, and Gilchrist (1999) framework imply that these simplifications are reasonable.

The log-linearized model then implies that there are two key financial parameters to choose—the steady-state leverage ratio and the elasticity of the external finance premium with respect to leverage. The steady-state ratio of

10. Tetlow (2005) uses a value of 0.5641, and Faia and Monacelli (2005) use a value of 0.5 for the parameter η_k.

the real value of the capital stock to the entrepreneur's net worth is chosen so that the steady-state leverage ratio is 80 percent or $(QK - N)/N = 0.8$, which implies $(QK)/N = 1.8$. We also adopt a simplified functional form for the determination of the external finance premium in equation (9):

$$(36) \qquad s_t = \left(\frac{Q_t K_{t+1}}{N_{t+1}} \right)^x.$$

Financial market imperfections imply that the external finance premium increases when the leverage of the borrowers increases ($\chi > 0$). In line with the calibration adopted by Bernanke, Gertler, and Gilchrist (1999), the elasticity of the external finance premium with respect to leverage is set to 5 percent: $\chi = 0.05$. These parameterizations imply that the nonstochastic steady-state level of the external finance premium is $s = (QK/N)^x = 1.0298$. Increasing the level of the steady-state leverage ratio or the size of the sensitivity parameter χ strengthens the financial accelerator mechanism. In the case of no financial market imperfections, $\chi = 0$. In this case, balance-sheet conditions of the entrepreneurs are irrelevant for the cost of external funds and thus for their capital expenditure decisions.

2.3.3 Shock Process and Filtering

We set the mean technology growth rate at the average quarterly growth rate of total factor productivity in the United States between 1959 and 2002: $\mu = 0.00427$. We set the standard deviation of the shock to the transitory component of technology growth at $\sigma_\varepsilon = 0.01$, the standard deviation of the shock to the persistent component of technology growth at $\sigma_v = 0.001$, and the AR(1) coefficient on the persistent component of technology growth at $\rho_d = 0.95$. These parameter choices imply that the signal-to-noise ratio in equation (33) is

$$\phi = 0.01.$$

The Kalman gain parameter in equation (32) consistent with these shock parameters is[11]

$$\lambda = 0.06138.$$

2.4 Impulse Responses

In this section, we report impulse response functions to technology shocks to explore the roles of imperfect information and financial market imperfections and their effects on output, inflation, asset prices, and the ex-

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11. This is within the range of values used in the literature. Edge, Laubach, and Williams (2005) use $\lambda = 0.025$ together with $\rho_d = 0.95$. Erceg, Guerrieri, and Gust (2005) use $\lambda = 0.1$ together with $\rho_d = 0.975$. Tambalotti (2003) uses $\rho_d = 0.93$ together with $\sigma_v/\sigma_\varepsilon = 0.08$ or $\phi \equiv \sigma_v^2/\sigma_\varepsilon^2 = 0.0064$, implying $\lambda = 0.0369$.

ternal finance premium. We explore the potential benefits of various monetary policy rules within this framework.

2.4.1   Transitory Shock to Technology Growth

We begin by examining the response of output, inflation, asset prices, and the external finance premium to a transitory increase in the growth rate of technology. We consider the model with and without the financial accelerator and also report the response of the flexible-price economy without the financial accelerator. This economy is undistorted and corresponds to our notion of the potential. We first consider a situation where both the private sector and the policymaker are fully informed regarding the state of technology growth. We then consider a situation where they both have imperfect information but learn over time according to the Kalman filter specified in the preceding.

For each model, we consider three monetary policy rules: a policy of responding weakly to inflation ($\ln R_{t+1}^n = \ln R^n + 1.1 \ln \pi_t$), a policy of responding strongly to inflation ($\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t$), and a policy rule that allows a policy response to the asset price gap in addition to a strong response to inflation [$\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + 1.5(\ln Q_t - \ln Q_t^*)$]. In the case of imperfect information for the private sector, we assume that the monetary authority also has imperfect information so that the interest rate rule with the asset price gap is now $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + 1.5(\ln Q_t - \ln Q_{\text{imp},t}^*)$, where $Q_{\text{imp},t}^*$ is the level of asset prices in the frictionless economy under imperfect information.

Full Information for Both the Private Sector and the Policymaker

Figure 2.3 plots the response of the economy without the financial accelerator to a transitory increase in the growth rate of technology, when both the private sector and the policymaker have full information. The transitory shock to technology growth causes immediate increases in output, asset prices, and inflation. Along the path, output continues to rise owing to capital accumulation, while inflation and asset prices return to their initial steady-state levels. With no financial frictions, the external finance premium is constant at zero.

The strength of the response of output, inflation, and asset prices depends on the conduct of monetary policy. Under the policy of responding weakly to inflation, expected real interest rates are low relative to those implied by the flexible-price economy. As a result, asset prices are high, and output is above potential. In addition, inflation is above its target level of zero. The policy of responding strongly to inflation provides substantial improvement. Expected real interest rates rise sufficiently so that asset prices and output track their potential levels implied by the frictionless economy. In addition, the inflation response is dampened considerably. Because the asset price gap is essentially zero under the policy of responding
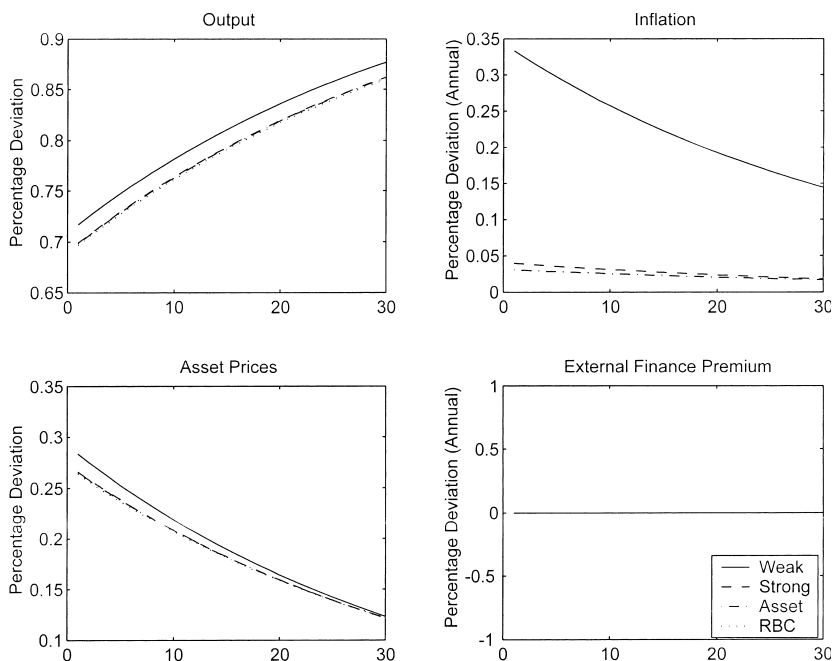
Figure 2.3   Response to a transitory shock to technology growth (full information, no financial accelerator

Note: Weak: $\ln R_{t+1}^{n} = \ln R^{n} + 1.1 \ln \pi_{t}$, Strong: $\ln R_{t+1}^{n} = \ln R^{n} + 2.0 \ln \pi_{t}$, Asset: $\ln R_{t+1}^{n} = \ln R^{n} + 2.0 \ln \pi_{t} = 1.5(\ln Q_{t} - \ln Q_{t}^{*})$, RBC: Flexible-price model with full information and no financial market imperfections.

strongly to inflation, adding the asset price gap to the monetary policy rule produces no change in the path of output and has a negligible effect on inflation. Thus, with full information and no financial accelerator, there is little, if any, gain to allowing the monetary authority to respond to the asset price gap.

Figure 2.4 plots the response of the economy with the financial accelerator to the same transitory shock to technology growth. The financial accelerator mechanism amplifies the response of output and inflation because a favorable shock to technology raises asset prices and reduces the external finance premium. This amplified response represents distortions in the resource allocation induced by financial market imperfections. Asset prices and investment—variables that are closely linked to the financial accelerator mechanism—deviate from their efficient levels by a larger amount in the presence of financial market imperfections.

In the economy with the financial accelerator, adopting a policy rule that implies a strong response to inflation brings the path of inflation close to the target. It also reduces the response of the external finance premium and
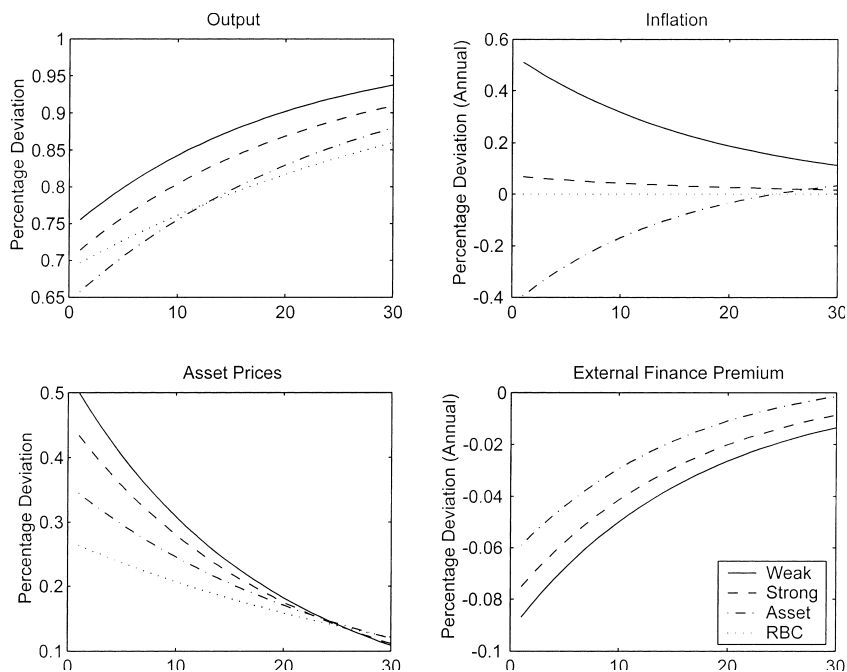
Figure 2.4   Response to a transitory shock to technology growth (full information, financial accelerator)

Note: Weak: $\ln R_{t+1}^n = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t$, $1.5(\ln Q_t - \ln Q_t^*)$, RBC: Flexible-price model with full information and no financial market imperfections.

reduces the amount of overinvestment that occurs. Nonetheless, there are still large deviations in output, asset prices, and investment from their potential levels. A policy of responding strongly to inflation is successful in decreasing the distortions arising from price rigidities, but is not sufficient to eliminate the distortions arising from financial market imperfections. Allowing the policymaker to respond to the asset price gap further reduces the investment distortion owing to the financial accelerator. As a result, output tracks potential more closely. This comes at the cost of producing deflation and increasing inflation variability, however.

Imperfect Information for Both the Private Sector and the Policymaker

Figure 2.5 plots the response of the economy without the financial accelerator to a transitory shock to technology growth, when both the private sector and the policymaker have imperfect information. For comparison purposes, the figure also shows the path of the frictionless economy under full information (the path labeled "RBC"). With imperfect information,
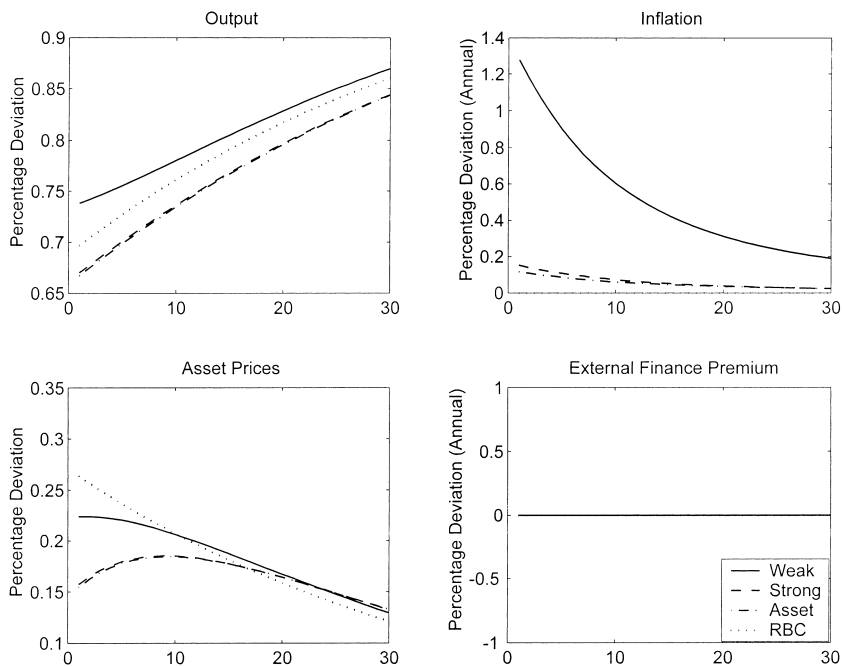
Figure 2.5   Response to a transitory shock to technology growth (imperfect information, no financial accelerator)

Note: Weak: $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t \, 1.5(\ln Q_t - \ln Q^*_t)$, RBC: Flexible-price model with full information and no financial market imperfections.

agents initially give some weight to the possibility that the observed increase in technology growth is persistent. An additional wealth effect owning to a perception of future technology improvements raises desired level of current consumption relative to the case of full information. Also, such a perception steepens the desired consumption profile. In the frictionless economy with imperfect information (not shown in the figure), this change in the desired consumption profile is supported by a higher expected real interest rate, and we observe a smaller response of asset prices and investment relative to the case of full information.

With the policy that implies a weak response to inflation, the rise in expected real interest rates is smaller than that in the frictionless economy, and consumption rises sharply without inducing an offsetting fall in investment. These combined effects imply a larger increase in output than what is observed in the case of full information. The inflation response is also much larger in the case of imperfect information. A policy of responding strongly to inflation implies an output path below the potential

under full information, but substantially smaller response in inflation.[12] In the model with imperfect information but no financial accelerator, adding the asset price gap to the monetary policy rule again has no effect on the output path and only a negligible effect on inflation.

In the economy with the financial accelerator (figure 2.6), the policy of responding strongly to inflation is again beneficial, leading to reductions in the response of both the markup and the external finance premium. The model still implies distortions owing to the financial accelerator, however, and as a result, there are benefits to responding to the asset price gap. Allowing the monetary authority to respond to the asset price gap stabilizes the external finance premium and largely eliminates the overinvestment that occurs due to the financial accelerator. Output tracks potential more closely, but this once again occurs at the cost of increasing inflation variability.

Overall, the financial accelerator has effects on the external finance premium under imperfect information that are similar to those under full information. In response to a transitory shock, the primary effect of imperfect information is to cause a consumption boom that leads to increases in output and inflation. Although such a consumption boom can also influence asset prices and investment demand, imperfect information leads to an offsetting impulse to wait to invest in response to a perceived persistent increase in the growth rate of technology. As a result, with a policy that responds weakly to inflation, the investment distortions owing to the financial accelerator are only slightly larger under imperfect information than under full information.[13] Under both full and imperfect information, we find that there are benefits to adopting a policy rule that implies a strong response to inflation. In both cases, allowing the monetary authority to respond to the asset price gap reduces the overinvestment that occurs because of the decline in the external finance premium. Because responding to the asset price gap also produces deflation, the overall benefits will depend on the relative importance of output gap stability and inflation stability.

2.4.2   Persistent Shock to Technology Growth

We now consider the effect of a persistent increase in the growth rate of technology. We begin with the case in which both the private sector and the policymaker have full information and then report the results obtained un-

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12. With imperfect information, a policy of responding strongly to inflation implies an output path that tracks the "potential output" path consistent with the policymaker's belief under imperfect information rather than that consistent with the true state of technology growth. The former (not shown in the figure) is below the latter (the path labeled "RBC") in this case.
13. This can be seen by comparing the movements in asset prices and the external finance premium labeled "Weak" in figure 2.4 and figure 2.6.
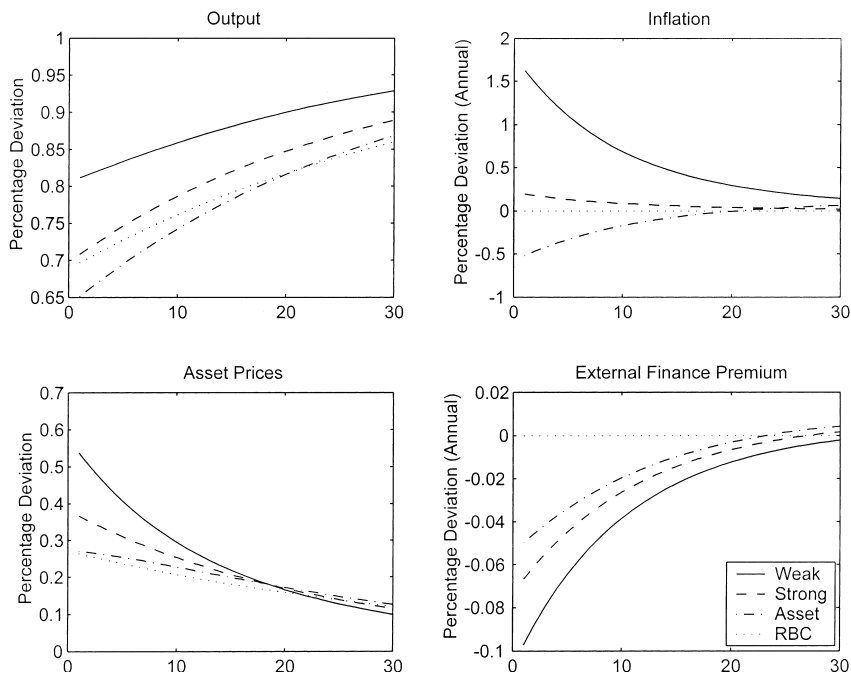
Figure 2.6   Response to a transitory shock to technology growth (imperfect information, financial accelerator)

Note: Weak: $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t\, 1.5(\ln Q_t - \ln Q^*_t)$, RBC: Flexible-price model with full information and no financial market imperfections.

der imperfect information. We again consider policy rules that include a weak response to inflation, a strong response to inflation, and a rule that allows the monetary authority to respond to the asset price gap. We also report the response of the frictionless economy under full information, which corresponds to our notion of potential when we assess economic outcomes under alternative monetary policy rules.

Full Information for Both the Private Sector and the Policymaker

Figure 2.7 plots the response of the economy without the financial accelerator to a persistent increase in technology growth, when both the private sector and the policymaker have full information. With no distortions (the path labeled "RBC"), a persistent increase in technology growth implies a boom in consumption, but an initial fall in investment and asset prices. Over time, investment and asset prices rise as the process of capital accumulation takes place.

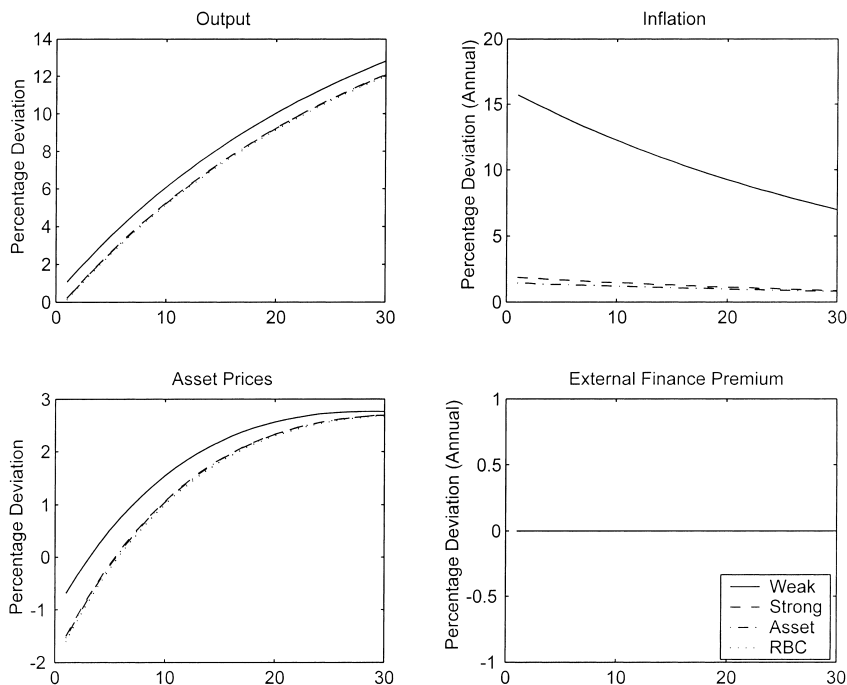In the sticky-price model, the response of the economy again depends on

Figure 2.7   Response to a persistent shock to technology growth (full information, no financial accelerator)

Note: Weak: $\ln R_{t+1}^n = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t\, 1.5(\ln Q_t - \ln Q_t^*)$, RBC: Flexible-price model with full information and no financial market imperfections.

the conduct of monetary policy. Under the policy of responding weakly to inflation, the model generates less of an initial reduction in investment and a stronger output response. Inflation rises by 16 percentage points in this case. The policy of responding strongly to inflation succeeds in dampening inflation and brings output in line with potential. Investment and asset prices now fall upon impact, which eliminates the asset price gap. Without the financial accelerator, there is essentially no difference between the economy's response with and without the asset price gap in the monetary policy rule.

In the economy with the financial accelerator (figure 2.8), the persistent increase in technology growth combined with the policy of responding weakly to inflation causes a sharp drop in the external finance premium, a positive response of investment, and a substantial increase in asset prices. Asset prices rise rather than fall at the onset of a persistent increase in technology growth in the presence of the financial accelerator and accommodative monetary policy. The initial inflation response is also larger

Figure 2.8   Response to a persistent shock to technology growth (full information, financial accelerator)

Note: Weak: $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$ 1.5($\ln Q_t - \ln Q^*_t$), RBC: Flexible-price model with full information and no financial market imperfections.

now—on the order of 20 percentage points. The policy of responding strongly to inflation provides substantial benefits in terms of the output gap and inflation stabilization. We still observe movements in the external finance premium and, hence, some distortions in asset prices and investment, however. Allowing the monetary authority to respond to the asset price gap provides modest benefits in terms of further reducing the distortion in investment spending owing to the financial accelerator. This policy once again produces deflation.

Imperfect Information for Both the Private Sector and the Policymaker

Under imperfect information, the private sector initially gives a relatively large weight to the possibility that the observed increase in technology growth is transitory. The initial response is thus closer to what we would observe in the case of a transitory shock to technology growth under full information. Over time, the private sector learns that the increase in technology growth is persistent, and the economic outcomes become

more similar to those obtained in the case of a persistent shock to technology growth under full information.

In the economy without the financial accelerator (figure 2.9), the persistent increase in technology growth combined with the policy of responding weakly to inflation again implies a large, albeit delayed, increase in inflation. In addition, output is more procyclical with sticky prices than would be the case under flexible prices. A policy of responding strongly to inflation dampens movements in the markup and eliminates most of the movements in inflation. In this case, output is above true potential but tracks the output level that would occur in the frictionless economy with imperfect information.[14]

With the financial accelerator (figure 2.10), the persistent increase in technology growth again produces a countercyclical movement in the external finance premium that implies a large distortion in investment spending relative to the frictionless RBC outcome. A policy of responding strongly to inflation reduces the size of asset price movements and reduces but does not eliminate movements in the external finance premium. Allowing the monetary authority to respond to the asset price gap is again beneficial. Such a policy further dampens asset price movements as well as the movements in the external finance premium. Once again, such a policy produces benefits in terms of stabilizing output gap but comes at the cost of destabilizing inflation.

Imperfect information magnifies the movements in the external finance premium in response to persistent shocks to the growth rate of technology. These magnification effects are sizeable. For example, with a policy that responds strongly to inflation, the movement in the external finance premium is twice as large in the case of imperfect information (figure 2.10), relative to the case of full information (figure 2.8). Because the private sector gives a relatively low initial weight to the probability that the increase in technology growth is persistent, imperfect information implies a series of positive shocks to expectations regarding future economic fundamentals. Such positive surprises raise the *ex post* realized rate of return on capital, relative to the anticipated rate of return, and enhance entrepreneurial net worth. These procyclical movements in net worth imply a strong hump-shaped countercyclical response in the external finance premium as well as a greater degree of procyclicality in asset prices than would be the case under full information. Because the financial accelerator mechanism is strengthened by imperfect information and learning on the part of the private sector, we expect that the benefits of allowing the monetary authority to respond to asset prices, particularly in the form of reduction in the volatility of the output gap, to be greater in the case of imperfect infor-

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14. The path labeled "RBC" in figure 2.9 is computed under full information.
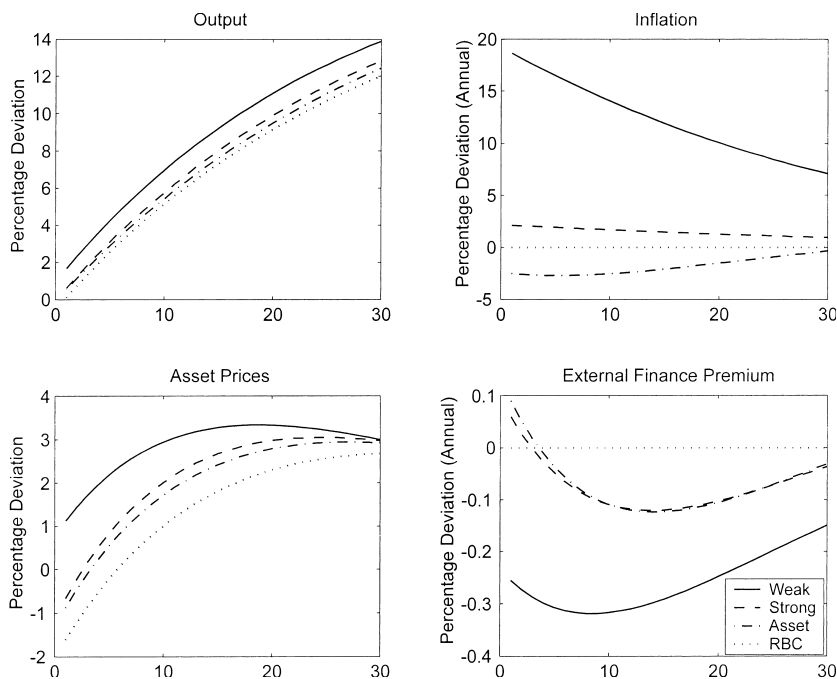
Figure 2.9   Response to a persistent shock to technology growth (imperfect information, no financial accelerator)

Note: Weak: $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, $1.5(\ln Q_t - \ln Q^*_t)$, RBC: Flexible-price model with full information and no financial market imperfections.

mation than in the case of full information.[15] We now turn to stochastic simulations to explore this issue further.

2.5   Stochastic Simulations

The previous section computed impulse response functions to technology shocks under alternative monetary policy rules. These results suggest potential benefits to adopting a policy that implies a strong response to inflation as well as to allowing the monetary authority to respond to the asset price gap—the gap between the observed asset prices and the potential

15. To be precise, the validity of this statement depends on the relative importance of the two types of shocks to technology growth. As we saw in this section, in response to a persistent shock to technology growth, the financial accelerator mechanism is strengthened by imperfect information on the part of the private sector. As we saw in the second subsection of section 2.4.1, in response to a transitory shock to technology growth, the effect of information structures on the strength of the financial accelerator mechanism is relatively small.
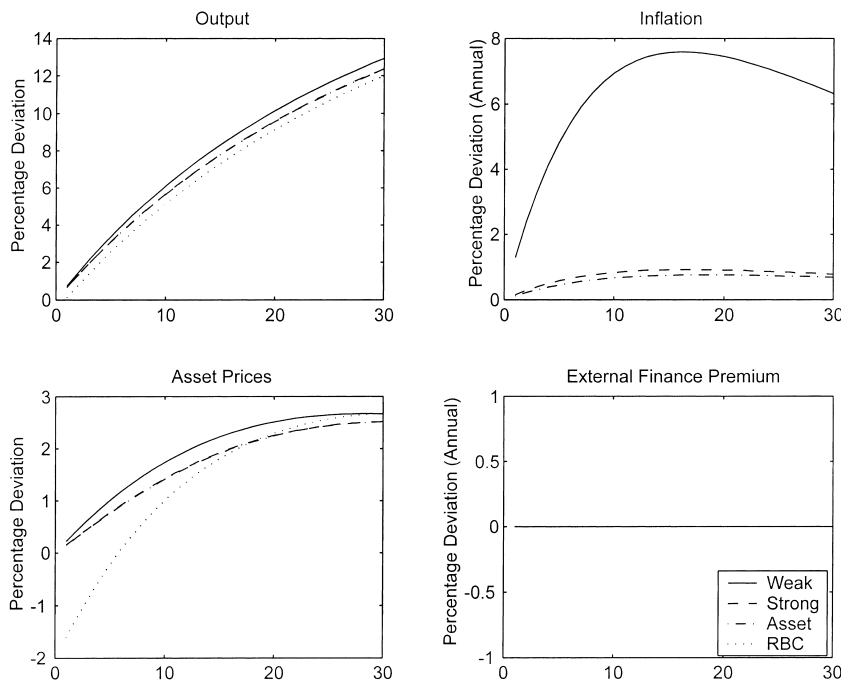
Figure 2.10   Response to a persistent shock to technology growth (imperfect information, financial accelerator)

Note: Weak: $\ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t$, Strong: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t$, Asset: $\ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t \, 1.5(\ln Q_t - \ln Q^*_t)$, RBC: Flexible-price model with full information and no financial market imperfections.

level of asset prices that would occur in the flexible-price economy without financial market imperfections. The extent of these benefits depends on the degree of financial market imperfections and the information structure of the economy. To further explore these issues, we now conduct stochastic simulations of the various models considered. The stochastic simulations depend on the combined effect of both transitory and persistent shocks to technology growth. When conducting such simulations, we parameterize the technology shock process in the manner described in our calibration.

2.5.1   Benefits of Responding Strongly to Inflation

We first consider the benefits to adopting a policy that responds strongly to inflation. As Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) have emphasized, most of the destabilizing effects of asset price fluctuations on the aggregate activity can be eliminated using such a rule. The results emphasized in Bernanke and Gertler (1999, 2001) are derived in an environment where exogenous movements in asset prices

(bubbles) provide an additional source of fluctuations in net worth. These bubbles do not alter entrepreneurs' perceptions regarding the value of new investment in their framework, however.

In our environment, misperceptions regarding the future technology growth cause fluctuations in asset values. These misperceptions also influence investment demand. We wish to consider whether the policy prescription of responding strongly to inflation is robust to the information environment that we consider. To do so, we compare economic outcomes under the two alternative monetary policy rules—a policy rule that implies a weak response to inflation:

$$(37) \qquad \ln R^n_{t+1} = \ln R^n + 1.1 \ln \pi_t,$$

and a policy rule that implies a strong response to inflation:

$$(38) \qquad \ln R^n_{t+1} = \ln R^n + 2.0 \ln \pi_t.$$

To compute the benefits of various policy rules, we use stochastic model simulations to compute the variance of both the output gap and inflation, where the potential level of output, Y^*_{full}, is defined as the level of output that would prevail in the flexible-price economy without financial market imperfections but with full information about the shocks to technology growth. We also compute a loss function based on a weighted average of the variance of the output gap and the variance of inflation:[16]

$$(39) \qquad Loss = 0.5var(\ln Y - \ln Y^*_{full}) + 0.5var(\pi).$$

We report the results of these simulations in table 2.1.

The first two rows of table 2.1 consider an environment where the private sector has full information regarding the state of technology growth. For comparison purposes, we provide results for the model without the financial accelerator as well as the model with the financial accelerator. The variance of the output gap and inflation are reported in percentage points on a quarterly basis.

Responding strongly to inflation provides substantial benefits in both the economy with and without the financial accelerator. Without the financial accelerator, moving from a weak to strong response to inflation implies large reductions in the variance of both the output gap and inflation. In fact, under the policy of responding strongly to inflation, the variance of the output gap is very close to zero (0.006). The variance of inflation is also very small (0.044). This result is consistent with our observation from the impulse response experiments that, in the absence of the financial accelerator, the sticky-price model under the policy of responding strongly to inflation comes very close to reproducing the frictionless RBC outcome.

In the economy with the financial accelerator, we also see substantial

16. For simplicity, we report the results only for the equal-weighted loss.

Table 2.1   **Benefits of responding strongly to inflation**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
	Full information for the private sector					
$\phi_\pi = 1.1$	0.431	2.811	1.621	1.923	3.022	2.473
$\phi_\pi = 2.0$	0.006	0.044	0.025	0.470	0.056	0.263
	Imperfect information for the private sector					
$\phi_\pi = 1.1$	0.579	2.103	1.341	2.247	2.265	2.256
$\phi_\pi = 2.0$	0.099	0.028	0.063	0.870	0.045	0.458

Notes: The policy rule is $\ln R_{t+1}^n = \ln R^n + \phi_\pi \ln \pi_t$. Y gap is defined as ($\ln Y - \ln Y_{full}^*$), where Y_{full}^* is the flexible-price equilibrium level of output in the absence of financial frictions and under full information. The loss is defined as $0.5\text{var}(Y$ gap$) + 0.5\text{var}(\ln \pi)$.

benefits to a policy that responds strongly to inflation. Both the output gap and inflation volatility are reduced with such a policy. Nonetheless, with the financial accelerator, output gap volatility is still significant (0.470) relative to the baseline sticky-price model (0.006). This finding reinforces the intuition that the model with the financial accelerator has two distortions—one on the markup and one on the return on capital. A policy of responding strongly to inflation does well at reducing the distortion owing to variation in the markup, but does not eliminate the distortion on the return on capital. The presence of this distortion causes an increase in output gap volatility.

We now consider the role of imperfect information regarding the state of technology growth. These results are reported in the second two rows of table 2.1. Imperfect information implies an increase in the variance of the output gap and a reduction in the variance of inflation.[17] Under the policy of responding weakly to inflation, the equal-weighted loss is actually lower with imperfect information than under full information. Because the pol-

17. The result that the variance of the output gap is larger under imperfect information than under full information on the part of the private sector can be explained by the fact that, when computing the variance of the output gap, we define the potential level of output as the level of output in the frictionless economy with full information. Under imperfect information on the part of the private sector, the equilibrium level of output deviates from such a full-information level. The result that the variance of inflation is smaller under imperfect information can be understood by considering the strength of the wealth effect of shocks to technology growth on consumption, which constitutes a large component of the aggregate demand. Under full information, wealth effect on consumption is larger when a movement in technology growth is persistent than when it is transitory. Under imperfect information, our calibration of the Kalman gain parameter ($\lambda = 0.06138$) implies that the private sector initially infers that observed movements in technology growth is mostly transitory, even when these movements are, in fact, generated by a shock to the persistent component of technology growth. The overall wealth effect of technology growth movements on consumption, including the effects of both transitory and persistent shocks (which occur with the same frequency), is thus smaller under imperfect information than under full information.

icy of responding strongly to inflation is clearly the dominant policy, it provides the more relevant comparison, however.

With the monetary policy that responds strongly to inflation, in the model without the financial accelerator, the presence of imperfect information has only a small effect on the variance of the output gap and inflation. In the model with the financial accelerator, imperfect information leads to a large increase in output gap volatility with very little reduction in the variance of inflation. As a result, with the financial accelerator, the loss is substantially higher under imperfect information (0.458) than under full information (0.263).

2.5.2   Benefits of Responding to the Asset Price Gap

We now consider whether a monetary policy that allows the nominal interest rate to respond to the asset price gap can improve upon a policy that responds to inflation only.[18]

Because we have already shown that a policy of responding strongly to inflation is beneficial, we restrict our attention to the case where the monetary authority responds strongly to inflation and then consider the additional gains from responding to the asset price gap:

$$(40) \qquad \ln R_{t+1}^n = \ln R^n + 2.0\ln \pi_t + \phi_Q(\ln Q_t - \ln Q_t^*).$$

We report results varying the coefficient on the asset price gap, ϕ_Q, from 0.1 to 2.0.

An important question in this analysis is how to gauge the benefits of one policy relative to another. Because there is a consensus in the literature that there are substantial gains to conducting a policy that responds strongly to inflation, we use these gains as the relevant benchmark. In particular, table 2.2 reports the difference between outcomes obtained from pursuing policy in equation (40) versus the policy of responding weakly to inflation in equation (37), divided by the difference between outcomes obtained from pursuing the policy of responding strongly to inflation in equation (38) versus the policy of responding weakly to inflation. For example, when computing the relative gain of adopting Policy Rule x in terms of the equal-weighted loss, we compute

$$(41) \quad \text{Relative gain(Policy Rule } x)$$

$$= \frac{\text{Loss(weak inflation response)} - \text{Loss(Policy Rule } x)}{\text{Loss(weak inflation response)} - \text{Loss(strong inflation response)}}.$$

18. Although not reported here, the output gap serves a similar role as the asset price gap: in the presence of financial market imperfections, allowing the monetary authority to respond to changes in the output gap in addition to responding strongly to inflation is beneficial, especially when the policymaker has full information. An interesting future direction of this line of research is to study economic environments in which the asset price gap plays a different role from the output gap.

Table 2.2   **Benefits of responding to the asset price gap (full information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
Full information for the policymaker						
$\phi_Q = 0.1$	1.00	1.00	1.00	1.03	1.01	1.01
$\phi_Q = 0.5$	1.00	1.00	1.00	1.10	1.01	1.04
$\phi_Q = 1.0$	1.00	1.00	1.00	1.13	0.98	1.03
$\phi_Q = 1.5$	1.01	1.00	1.00	1.17	0.95	1.02
$\phi_Q = 2.0$	1.01	1.00	1.00	1.22	0.92	1.02
Imperfect information for the policymaker						
$\phi_Q = 0.1$	0.98	1.00	1.00	1.02	1.01	1.01
$\phi_Q = 0.5$	0.85	1.00	0.98	0.97	1.00	0.99
$\phi_Q = 1.0$	0.59	0.99	0.94	0.93	0.98	0.97
$\phi_Q = 1.5$	0.31	1.00	0.91	0.94	0.94	0.94
$\phi_Q = 2.0$	0.21	1.00	0.90	0.79	0.88	0.85

Notes: The policy rule is ln R_{t+1}^n = ln R^n + 2.0 ln π_t + ϕ_Q(ln Q_t − ln Q_t^*). Y gap is defined as (ln Y − ln Y_{full}^*), where Y_{full}^* is the flexible-price equilibrium level of output in the absence of financial frictions and under full information. The loss is defined as 0.5var(Y gap) + 0.5var(ln π). A value of larger than one implies that the policy is better than the policy that responds strongly to inflation. A negative value implies that the policy is worse than the policy that responds weakly to inflation.

We compute the relative gain for the reduction in output gap variance and inflation variance in an analogous manner. Doing so enables us to easily summarize the results: if the relative gain is above one, the policy in question provides gains relative to the policy of responding strongly to inflation. If the relative gain is negative, the policy in question provides outcomes that are strictly worse than those under the policy of responding weakly to inflation.[19]

In imperfect information environments, the policymaker may not have sufficient information to correctly compute the potential level of asset prices Q_t^*. We thus distinguish between cases where the policymaker can correctly assess the state of technology growth ($Q_t^* = Q_{full,t}^*$) and the case where the policymaker infers it based on current and past observations of technology growth ($Q_t^* = Q_{imp,t}^*$).

When considering the benefits of such rules, we distinguish between environments where the private sector has full and imperfect information.

19. Note that we cannot directly compare the numbers for the relative gain in the case of imperfect information for the private sector and in the case of full information for the private sector because the gain from moving from the policy of responding weakly to inflation to the policy of responding strongly to inflation (the denominator in the formula to calculate the relative gain in equation [41]) differs depending on the information structure for the private sector.

Thus, our information structure allows for four cases: (1) full information on the part of both the private sector and the policymaker; (2) full information for the private sector but imperfect information for the policymaker; (3) imperfect information for the private sector and full information for the policymaker; and (4) imperfect information for both. Within these four cases, we report results for the model with and without the financial accelerator.

Full Information for the Private Sector

We first consider the case of full information on the part of the private sector (table 2.2). The top rows of table 2.2 consider the case where the policymaker also has full information. In the sticky-price model without the financial accelerator, the relative gain is approximately unity.[20] Thus, there are almost no gains to allowing the monetary authority to respond to the asset price gap relative to the policy that responds strongly to inflation. By responding strongly to inflation, the monetary authority succeeds in stabilizing the markup, which is the only distortion in the economy. With the markup stabilized, the actual path for asset prices is nearly identical to the path under flexible prices, so giving weight to the asset price gap provides almost no gain.

In contrast, in the model with the financial accelerator, responding to the asset price gap provides clear gains in terms of output gap stabilization—on the order of 22 percent when the coefficient on the asset price gap is relatively large, with $\phi_Q = 2.0$. Although the policy that responds strongly to inflation stabilizes the markup, it does not eliminate the distortion due to financial market imperfections, which is reflected in the deviations of asset prices from the potential level that arises in the economy without pricing and financial frictions. Thus, responding to the asset price gap helps reduce distortions due to financial market imperfections. As the coefficient on the asset price gap is increased, the variance of the output gap falls but the variance of inflation rises. Based on the loss function in equation (39), which gives equal weight on the output gap and inflation, our parameterization implies a modest gain to responding to the asset price gap, with a coefficient on the asset price gap $0.1 < \phi_Q < 1.0$ minimizing this loss.

We now consider the case where the private sector has full information, but the policymaker has imperfect information. These results are reported

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20. To deemphasize small differences in simulation results that may reflect sensitivity to a numerical solution or a simulation error, we report the relative gains rounded to the second decimal place. Our actual results suggest that the model exhibits an extremely small but positive gain to allowing the monetary authority to respond to the asset price gap in the case of full information and no financial accelerator. The relative gains are always less than 1.005, however, implying that to a first approximation the absolute gains to allowing the policymaker to respond to the asset price gap are zero.

in the bottom rows of table 2.2. In the sticky-price model without the financial accelerator, responding to the asset price gap is a strictly inferior policy, which leads to large increases in the variance of the output gap and inflation. In this environment, the potential level of asset prices measured by the monetary authority is no longer correct, and putting weight on the asset price gap pushes the economy away from the frictionless RBC outcome that is attainable under the policy of responding strongly to inflation. With the financial accelerator, there is a small gain to allowing a very weak policy response to the asset price gap ($\phi_Q = 0.1$), but a deterioration in terms of the variance of output gap and inflation for larger coefficients. When the monetary authority has imperfect information, it responds to the wrong measure of the asset price gap, which offsets any potential gains to be achieved relative to the policy that responds strongly to inflation only.

Imperfect Information for the Private Sector

We now consider the case where the private sector has imperfect information (table 2.3). We again begin with the case where the policymaker has full information. In the sticky-price model without the financial accelerator, allowing the monetary authority to respond to the asset price gap produces a small gain in terms of reducing the variance of the output gap. These gains are no longer present when the monetary authority also has imperfect information, however. In the absence of financial frictions, there are unlikely to be significant gains to allowing the monetary authority to respond to the asset price gap, even in the case where the private sector has imperfect information.

In the model with the financial accelerator, the gains to responding to the asset price gap are substantial. If the policymaker has full information, adopting a rule that responds to the asset price gap produces an incremental reduction in the variance of the output gap of 50 percent when $\phi_Q = 1.0$. Allowing the monetary authority to respond to the asset price gap reduces the variance of the output gap, but increases the variance of inflation. Overall, we see an improvement as measured by the equal-weighted loss, however.

When the policymaker has imperfect information, the gains obtained from responding to the asset price gap are somewhat smaller than the case where it has full information. Nonetheless, the gains are still positive and economically interesting. When the private sector has imperfect information, output gap volatility is increased relative to the case of full information (table 2.1). Because allowing the monetary authority to respond to the asset price gap reduces distortions arising from financial market imperfections and thus reduces the output variability, the overall gains from responding to the asset price gap are larger when the private sector has imperfect information relative to the case where it has full information. These larger gains offset the loss associated with the fact that the policymaker is

Table 2.3   **Benefits of responding to the asset price gap (imperfect information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
	Full information for the policymaker					
$\phi_Q = 0.1$	1.02	1.00	1.00	1.09	1.00	1.04
$\phi_Q = 0.5$	1.11	0.99	1.01	1.36	1.00	1.14
$\phi_Q = 1.0$	1.12	0.99	1.01	1.50	0.98	1.18
$\phi_Q = 1.5$	1.06	0.99	1.00	1.51	0.93	1.16
$\phi_Q = 2.0$	0.97	0.99	0.99	1.53	0.86	1.12
	Imperfect information for the policymaker					
$\phi_Q = 0.1$	0.92	1.00	0.98	1.20	1.01	1.08
$\phi_Q = 0.5$	0.94	1.00	0.99	1.22	1.01	1.09
$\phi_Q = 1.0$	0.96	1.00	0.99	1.38	0.97	1.12
$\phi_Q = 1.5$	0.98	1.00	1.00	1.44	0.93	1.12
$\phi_Q = 2.0$	0.96	1.00	1.00	1.42	0.87	1.08

Notes: See table 2.2 notes.

responding to the "wrong" asset price gap. As a result, when the private sector has imperfect information, allowing the policymaker to respond to the asset price gap can be beneficial even when the policymaker also has imperfect information.

In summary, the results from tables 2.2 and 2.3 imply that there are gains associated with responding to the asset price gap in the presence of distortions in the return on capital caused by financial market imperfections. These gains are greatest when the private sector has imperfect information and the policymaker is fully informed about future economic fundamentals. Nonetheless, there are also gains from responding to the asset price gap when both the private sector and the policymaker have imperfect information. Finally, when choosing how to respond, the policymaker faces a trade off—increasing the coefficient on the asset price gap in the monetary policy rule reduces output gap volatility but increases inflation volatility.

2.5.3 Effects of Allowing a Policy Response to the Natural Rate

We now consider the robustness of the results in the previous subsection to allowing the policymaker to respond to movements in the natural rate of interest.[21] We consider the following interest rate rule:

$$(42) \qquad \ln R_{t+1}^n = \ln R_{t+1}^* + 2.0 \ln \pi_t + \phi_Q (\ln Q_t - \ln Q_t^*),$$

where R_{t+1}^* is the natural rate of interest that prevails between period t and period $t + 1$. The natural rate of interest is defined here as the real interest

21. We thank Michael Woodford for suggesting this line of extension.

rate that supports the efficient allocation in the economy in the absence of both the pricing and financial frictions. It is computed based on the information available to the policymaker. We fix the coefficient on inflation in the policy rule at 2.0 and consider various values for the coefficient on the asset price gap, ϕ_Q.

Tables 2.4 and 2.5 summarize the results. Table 2.4 considers the case of full information for the private sector, and table 2.5 considers the case of imperfect information for the private sector. As in tables 2.2 and 2.3, we report the gains from adopting a policy that implies a response to the natural rate of interest and the asset price gap as well as a strong response to inflation, relative to the gains from adopting a policy of responding strongly to inflation only.

When the policymaker has full information, allowing the policymaker to respond to movements in the natural rate of interest reduces the variability of both inflation and the output gap because the policymaker in this case computes the natural rate of interest correctly.

The effects of allowing the monetary authority to respond to movements in the natural rate of interest differ greatly depending on whether the financial accelerator is present. Without financial market imperfections, al-

Table 2.4   **Effects of allowing a policy response to the natural rate (full information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
	Full information for the policymaker					
$\phi_Q = 0$	1.02	1.02	1.02	1.09	1.02	1.04
$\phi_Q = 0.1$	1.02	1.02	1.02	1.11	1.02	1.05
$\phi_Q = 0.5$	1.02	1.02	1.02	1.16	1.01	1.05
$\phi_Q = 1.0$	1.02	1.02	1.02	1.20	0.98	1.05
$\phi_Q = 1.5$	1.02	1.02	1.02	1.23	0.94	1.03
$\phi_Q = 2.0$	1.02	1.02	1.02	1.25	0.88	1.00
	Imperfect information for the policymaker					
$\phi_Q = 0$	1.01	1.01	1.01	1.08	0.97	1.03
$\phi_Q = 0.1$	1.00	1.01	1.01	1.05	1.01	1.02
$\phi_Q = 0.5$	0.87	1.00	0.98	1.04	1.00	1.01
$\phi_Q = 1.0$	0.61	1.00	0.95	1.00	0.97	0.98
$\phi_Q = 1.5$	0.40	1.00	0.92	0.96	0.93	0.94
$\phi_Q = 2.0$	0.12	1.01	0.89	0.88	0.86	0.87

Notes: The policy rule is ln R_{t+1}^n = ln R_{t+1}^* + 2.0 ln π_t + ϕ_Q(ln Q_t – ln Q_t^*). Y gap is defined as (ln Y – ln Y_{full}^*), where Y_{full}^* is the flexible-price equilibrium level of output in the absence of financial frictions and under full information. The loss is defined as 0.5var(Y gap) + 0.5var(ln π). A value of larger than one implies that the policy is better than the policy that responds strongly to inflation. A negative value implies that the policy is worse than the policy that responds weakly to inflation.

Table 2.5   **Effects of allowing a policy response to the natural rate (imperfect information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
Full information for the policymaker						
$\phi_Q = 0$	1.04	1.01	1.01	1.27	1.01	1.11
$\phi_Q = 0.1$	1.10	1.00	1.02	1.32	1.01	1.13
$\phi_Q = 0.5$	1.16	0.99	1.03	1.44	0.99	1.16
$\phi_Q = 1.0$	1.13	0.99	1.02	1.49	0.95	1.16
$\phi_Q = 1.5$	1.04	0.99	1.00	1.51	0.89	1.13
$\phi_Q = 2.0$	0.96	1.00	1.00	1.51	0.83	1.09
Imperfect information for the policymaker						
$\phi_Q = 0$	1.01	1.01	1.01	1.19	1.02	1.09
$\phi_Q = 0.1$	1.03	1.01	1.02	1.26	1.02	1.11
$\phi_Q = 0.5$	0.98	1.01	1.01	1.39	1.00	1.15
$\phi_Q = 1.0$	0.98	1.01	1.01	1.39	0.95	1.12
$\phi_Q = 1.5$	0.99	1.01	1.01	1.42	0.90	1.10
$\phi_Q = 2.0$	0.99	1.01	1.01	1.43	0.86	1.08

Notes: See table 2.4 notes.

lowing the monetary authority with full information to respond to movements in the natural rate of interest almost completely eliminates the only distortion in the economy arising from the pricing frictions. In this situation, allowing the policymaker to respond to the asset price gap provides little gain. In the presence of the financial accelerator, allowing the monetary authority to respond to movements in the natural rate of interest tends to reduce distortions arising from both pricing and financial frictions. This is because the natural rate of interest is defined as the rate of interest that arises in the absence of both pricing and financial frictions.[22] In this situation, we still observe gains from allowing the monetary authority to respond to the asset price gap, but those gains are smaller relative to the case where the policy rule does not include a response to the natural rate of interest.

2.5.4   Policy Rules That Do Not Require Inferences

Monetary policy rules that allow the policymaker to respond to the asset price gap require inferences regarding the true state of technology growth. Because these policies are not necessarily robust to incorrect inference on the part of the policymaker, it is also useful to consider mone-

22. A different definition of the natural rate of interest would lead to somewhat different conclusions. For instance, if one defines the natural rate of interest as the interest rate in the absence of pricing frictions but in the presence financial frictions, allowing the monetary authority to respond to movements in the natural rate would have a smaller impact on the distortions arising from financial frictions.

tary policy rules that do not require the monetary authority to make inferences. We consider three such rules:

1. Policy rule with output growth:

(43) $\ln R_{t+1}^{n} = \ln R^{n} + 2.0\ln \pi_{t} + \phi_{Y}(\ln Y_{t} - \ln Y_{t-1} - \mu).$

2. Policy rule with asset price growth:

(44) $\ln R_{t+1}^{n} = \ln R^{n} + 2.0\ln \pi_{t} + \phi_{Q}(\ln Q_{t} - \ln Q_{t-1}).$

3. Policy rule with the level of asset prices:

(45) $\ln R_{t+1}^{n} = \ln R^{n} + 2.0\ln \pi_{t} + \phi_{Q}(\ln Q_{t} - \ln Q),$

where Q is the nonstochastic steady-state level of asset prices ($Q = 1$ under our specification of the capital adjustment cost function).

Table 2.6 and table 2.7 report the relative gains from adopting these policy rules in the case where the private sector has full and imperfect information, respectively.

Table 2.6   **Policy rules that do not require inferences (full information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var(ln π)	Loss	var(Y gap)	var(ln π)	Loss
Policy with output growth: $\ln R_{t+1}^{n} = \ln R^{n} + 2.0 \ln \pi_{t} + \phi_{Y}(\ln Y_{t} - \ln Y_{t-1} - \mu)$						
$\phi_{Y} = 0.1$	0.99	1.00	1.00	1.03	1.01	1.01
$\phi_{Y} = 0.5$	0.85	1.01	0.99	1.04	1.01	1.02
$\phi_{Y} = 1.0$	0.57	1.00	0.95	1.04	1.01	1.02
$\phi_{Y} = 1.5$	0.23	0.98	0.88	0.97	0.99	0.98
$\phi_{Y} = 2.0$	−0.05	0.94	0.81	0.83	0.95	0.91
Policy with asset price growth: $\ln R_{t+1}^{n} = \ln R^{n} + 2.0 \ln \pi_{t} + \phi_{Q}(\ln Q_{t} - \ln Q_{t-1})$						
$\phi_{Q} = 0.1$	1.00	1.00	1.00	1.07	1.00	1.02
$\phi_{Q} = 0.5$	0.96	1.00	1.00	1.05	1.00	1.02
$\phi_{Q} = 1.0$	0.87	1.00	0.99	1.04	1.00	1.02
$\phi_{Q} = 1.5$	0.78	1.00	0.97	1.02	1.00	1.01
$\phi_{Q} = 2.0$	0.69	1.00	0.96	0.96	1.00	0.99
Policy with the level of asset prices: $\ln R_{t+1}^{n} = \ln R^{n} + 2.0 \ln \pi_{t} + \phi_{Q}(\ln Q_{t} - \ln Q)$						
$\phi_{Q} = 0.1$	0.99	1.01	1.00	1.01	1.01	1.01
$\phi_{Q} = 0.5$	0.71	0.70	0.70	1.10	0.73	0.85
$\phi_{Q} = 1.0$	0.13	−0.01	0.00	1.05	−0.31	0.13
$\phi_{Q} = 1.5$	−0.78	−1.57	−1.46	0.91	−1.98	−1.03
$\phi_{Q} = 2.0$	−2.16	−3.60	−3.41	0.71	−4.18	−2.57

Notes: Y gap is defined as ($\ln Y - \ln Y_{\text{full}}^{*}$), where Y_{full}^{*} is the flexible-price equilibrium level of output in the absence of financial frictions and under full information. The loss is defined as $0.5\text{var}(Y \text{ gap}) + 0.5\text{var}(\ln \pi)$. A value of larger than one implies that the policy is better than the policy that responds strongly to inflation. A negative value implies that the policy is worse than the policy that responds weakly to inflation.

Table 2.7   **Policy rules that do not require inferences (imperfect information for the private sector)**

	No financial accelerator			Financial accelerator		
	var(Y gap)	var($\ln \pi$)	Loss	var(Y gap)	var($\ln \pi$)	Loss
Policy with output growth: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + \phi_Y(\ln Y_t - \ln Y_{t-1} - \mu)$						
$\phi_Y = 0.1$	0.97	1.00	1.00	1.20	1.01	1.08
$\phi_Y = 0.5$	0.90	1.01	0.99	1.40	1.02	1.16
$\phi_Y = 1.0$	0.74	1.00	0.95	1.40	1.01	1.16
$\phi_Y = 1.5$	0.54	0.96	0.88	1.40	0.98	1.14
$\phi_Y = 2.0$	0.33	0.90	0.79	1.37	0.92	1.24
Policy with asset price growth: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + \phi_Q(\ln Q_t - \ln Q_{t-1})$						
$\phi_Q = 0.1$	0.95	1.00	0.99	1.11	1.00	1.05
$\phi_Q = 0.5$	0.93	1.00	0.99	1.15	1.00	1.06
$\phi_Q = 1.0$	0.96	1.00	0.99	1.33	1.00	1.13
$\phi_Q = 1.5$	0.92	1.00	0.99	1.31	1.00	1.12
$\phi_Q = 2.0$	0.98	1.00	0.99	1.39	1.00	1.15
Policy with the level of asset prices: $\ln R_{t+1}^n = \ln R^n + 2.0 \ln \pi_t + \phi_Q(\ln Q_t - \ln Q)$						
$\phi_Q = 0.1$	0.96	1.01	1.00	1.21	1.02	1.09
$\phi_Q = 0.5$	0.91	0.61	0.66	1.44	0.61	0.93
$\phi_Q = 1.0$	0.49	−0.65	−0.44	1.52	−0.80	0.09
$\phi_Q = 1.5$	0.13	−1.86	−1.49	1.48	−2.49	−0.97
$\phi_Q = 2.0$	−0.78	−4.54	−3.83	1.30	−5.42	−2.85

Notes: See table 2.6 notes.

In the absence of financial market imperfections, none of these policies provide substantial gains relative to the policy of responding strongly to inflation. Policies that respond to either output growth or asset price growth lead to an increase in the variance of the output gap, but have little impact on the variance of inflation. This is true under either full or imperfect information on the part of the private sector. In the absence of financial frictions, the policy of responding strongly to inflation does well at reducing variation in the markup, which is the only source of distortions. As a consequence, there is little to be gained from adding additional variables to interest rate rules.

In the presence of financial market imperfections, policies based on either output growth or asset price growth provide benefits relative to the policy of responding strongly to inflation. In relative terms, these benefits are much larger when the private sector has imperfect information regarding the state of technology growth. Depending on the coefficient values, these policies can do as well as policies based on the asset price gap. Because these policies do not require the policymaker to make inferences regarding the underlying potential of the economy, they are arguably more robust than policies based on the asset price gap.

Finally, we consider the policy rule that includes the level of the asset prices. This policy rule has been considered in the previous literature, but

studies such as Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) have argued against it. Here we confirm their results, albeit for somewhat different reasons. When the private sector has imperfect information, allowing a policy response to the level of asset prices provides clear benefits in terms of reducing output gap volatility in the model with the financial accelerator. It also leads, however, to a large increase in inflation volatility. For coefficients on the level of asset prices above 0.5, the inflation outcome is actually worse than what is obtained under the policy of responding weakly to inflation. This policy does not allow the monetary authority to adjust its policy owing to movements in asset prices that reflect changes in the desired level of investment spending in the frictionless economy. Because asset prices are procyclical on average in the frictionless economy, responding to the observed level of asset prices itself implies a strongly countercyclical policy that leads to significant deflation in expansionary environments. This deflationary response can be limited by adopting a policy that responds to either the asset price gap or the growth rate of asset prices.

2.6   Conclusion

This chapter considers the design of monetary policy rule in an environment where both the private sector and the monetary authority learn about the trend growth rate of technology. In the presence of financial market imperfections resulting from asymmetric information between lenders and borrowers, shocks to the economy that cause increases in asset prices improve the balance-sheet conditions of borrowers, reduce the external finance premium, and amplify the response of real economic activity. This amplification mechanism—the financial accelerator mechanism—represents a distortion in underlying economic activity that can only partially be eliminated by a policy of responding strongly to inflation. Such a policy stabilizes inflation but leaves a relatively large variability in the output gap. In this environment, because fluctuations in asset prices are closely linked to the financial accelerator mechanism, allowing the monetary authority to respond to the asset price gap—the gap between the observed asset prices and the potential level of asset prices that arises in the frictionless economy with flexible prices and no financial market imperfections—stabilizes the output gap but tends to increase the variability in inflation.

We also show that the overall gains from allowing the monetary authority to respond to the asset price gap are greatest when the monetary authority can correctly identify the true state of technology growth, while the private sector must infer it from past observations of technology growth. These gains are reduced to the extent that the monetary authority is also imperfectly informed about the state of technology growth. We further show that policy rules that respond to either the growth rate of asset prices or the growth rate of output provide most of the benefits associated with

including the asset price gap in the monetary policy rule. Because it is efficient that asset prices fluctuate in the presence of shocks to technology growth, monetary policies that respond to the observed level of asset prices itself, and hence do not take into account changes in the potential level of asset prices, are particularly detrimental, however.

This chapter focuses on a quadratic loss function rather than formal welfare analysis in evaluating economic outcomes under different monetary policy rules. Thus, future work should be oriented toward assessing the robustness of our conclusions for welfare calculations. In addition, although learning combined with the financial accelerator mechanism increases the procyclicality in asset prices as well as the extent to which asset prices deviate from the potential level, our underlying frictionless economy implies a fall in asset prices in response to a persistent increase in technology growth. We are, therefore, also interested in exploring the robustness of our conclusions to alternative mechanisms that may provide a more realistic characterization of the link between asset prices and changes in expectations or news regarding future economic fundamentals.

Appendix

Equilibrium Conditions in Normalized Variables

This section lists the equilibrium conditions for the model in terms of the normalized, stationary variables.

We normalize the levels of consumption, investment, output, capital stock, and net worth by the level of technology so that these real quantities are stationary:

$$c_t \equiv \frac{C_t}{A_t}, \, i_t \equiv \frac{I_t}{A_t}, \, y_t \equiv \frac{Y_t}{A_t}, \, k_t \equiv \frac{K_t}{A_{t-1}}, \text{ and } n_t \equiv \frac{N_t}{A_{t-1}}.$$

K_t and N_t are determined in period $t - 1$, and we normalize these variables by the level of technology in period $t - 1$. Also, let

$$Z_t \equiv \frac{A_t}{A_{t-1}}$$

denote technology growth.

The equilibrium conditions in terms of the normalized variables are as follows.

Consumption savings:

(A1)
$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \frac{1}{Z_{t+1}} R^n_{t+1} \frac{P_t}{P_{t+1}} \right).$$

Expected real rate of return on capital:

$$(A2) \qquad E_t R_{t+1}^k \equiv \frac{E_t \left[(1 - \alpha)\frac{y_{t+1}}{k_{t+1}} Z_{t+1} mc_{t+1} + (1 - \delta)Q_{t+1} \right]}{Q_t},$$

where $mc_t \equiv P_t^W/P_t$ is the real marginal cost.

Definition of the external finance premium:

The external finance premium is defined as the ratio of the expected real rate of return on capital (which is equal to the cost of external funds in equilibrium) to the expected real rate of return on the riskless bond (which is interpreted as the cost of internal funds):

$$(A3) \qquad s_t \equiv \frac{E_t R_{t+1}^k}{E_t \left(R_{t+1}^n \frac{P_t}{P_{t+1}} \right)}.$$

Determination of the external finance premium:

$$(A4) \qquad s_t = \left(\frac{Q_t k_{t+1}}{n_{t+1}} \right)^\chi.$$

Evolution of net worth:

Under our calibration of $\Omega = 0$,

$$n_{t+1} = \eta \left[R_t^k Q_{t-1} k_t \frac{1}{Z_t} - E_{t-1} R_t^k \left(Q_{t-1} k_t \frac{1}{Z_t} - n_t \frac{1}{Z_t} \right) \right].$$

Or, using the definition of the external finance premium $E_{t-1} R_t^k = s_{t-1} E_{t-1} [R_t^n (P_{t-1}/P_t)]$,

$$(A5) \quad n_{t+1} = \eta \left[R_t^k Q_{t-1} k_t \frac{1}{Z_t} - s_{t-1} E_{t-1} \left(R_t^n \frac{P_{t-1}}{P_t} \right) \left(Q_{t-1} k_t \frac{1}{Z_t} - n_t \frac{1}{Z_t} \right) \right].$$

Investment-Q relationship:

$$(A6) \qquad Q_t = \frac{1}{\phi' \left(\dfrac{i_t}{k_t Z_t} \right)}.$$

Aggregate resource constraint:

Under our calibration of $C_t^e = 0$ and $G_t = 0$,

$$(A7) \qquad y_t = c_t + i_t.$$

Production function:

Under our calibration of $\Omega = 0$,

$$(A8) \qquad y_t = H_t^\alpha k_t^{1-\alpha} \frac{1}{Z_t^{1-\alpha}}.$$

Labor market equilibrium condition:

(A9)
$$\theta H_t^\gamma = \frac{1}{c_t}\alpha\frac{y_t}{H_t}mc_t.$$

Price setting:

(A10) $$P_t^* = \frac{\varepsilon}{\varepsilon - 1}\; \frac{E_t\sum_{i=0}^{\infty} \upsilon^i\beta^i\left(\frac{c_{t+i}}{c_t}\frac{1}{A_t}\right)^{-1} MC_{t+i}y_{t+i}\left(\frac{1}{P_{t+i}}\right)^{1-\varepsilon}}{E_t\sum_{i=0}^{\infty} \upsilon^i\beta^i\left(\frac{c_{t+i}}{c_t}\frac{1}{A_t}\right)^{-1} y_{t+i}\left(\frac{1}{P_{t+i}}\right)^{1-\varepsilon}},$$

where $MC_t \equiv P_t mc_t = P_t^W$ is the nominal marginal cost of retail goods production.

Price index:

(A11)
$$P_t = [\upsilon P_{t-1}^{1-\varepsilon} + (1 - \upsilon)(P_t^*)^{1-\varepsilon}]^{1/(1-\varepsilon)}.$$

Capital accumulation:

(A12)
$$k_{t+1} = k_t\frac{1}{Z_t}(1 - \delta) + \phi\left(\frac{i_t}{k_t}Z_t\right)k_t\frac{1}{Z_t}.$$

Policy rule with inflation only:

$$R_{t+1}^n = R^n\pi_t^{\phi_\pi}.$$

Policy rule with the asset price gap:

$$R_{t+1}^n = R^n\pi_t^{\phi_\pi}\left(\frac{Q_t}{Q_t^*}\right)^{\phi_Q},$$

where Q_t^* is the flexible-price equilibrium level of asset prices in the absence of financial frictions. Q_t^* is computed under the information available to the policymaker.

Policy rule with the natural rate of interest and the asset price gap:

$$R_{t+1}^n = R_{t+1}^*\pi_t^{\phi_\pi}\left(\frac{Q_t}{Q_t^*}\right)^{\phi_Q},$$

where R_{t+1}^* is the natural rate of interest that is defined as the real interest rate that supports the efficient allocation in the economy without pricing and financial frictions. R_{t+1}^* is computed under the information available to the policymaker.

Policy rule with output growth:

$$R_{t+1}^n = R^n\pi_t^{\phi_\pi}\left(\frac{\frac{y_t}{y_{t-1}}Z_t}{\exp(\mu)}\right)^{\phi_Y}.$$

Policy rule with asset price growth:

$$R_{t+1}^n = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q_{t-1}} \right)^{\phi_Q}.$$

Policy rule with the level of asset prices:

$$R_{t+1}^n = R^n \pi_t^{\phi_\pi} \left(\frac{Q_t}{Q} \right)^{\phi_Q},$$

where Q is the nonstochastic steady-state level of asset prices.

Technology shock process:

$$\ln Z_t = \mu_t + \varepsilon_t,$$

and

$$(\mu_t - \mu) = \rho_d(\mu_{t-1} - \mu) + \upsilon_t,$$

with $\varepsilon_t \sim i.i.d.\, N(0, \sigma_\varepsilon^2)$ and $\upsilon_t \sim i.i.d.\, N(0, \sigma_\upsilon^2)$.

Nonstochastic Steady State

This section lists the conditions for the nonstochastic steady state of the economy in terms of the normalized variables.

Let

$$\pi_t \equiv \frac{P_t}{P_{t-1}}$$

denote inflation.

Normalize the steady-state inflation at 0 percent.

$$\pi = 1.$$

We specify the capital adjustment cost function such that in the nonstochastic steady state, we have

$$Q = 1.$$

From equation (A10),

$$m_c = \frac{\varepsilon - 1}{\varepsilon}.$$

From equation (A1) and $\pi = 1$,

$$R^n = \frac{Z}{\beta}.$$

Using equations (A3) and (A4) and $Q = 1$, the nonstochastic steady-state level of the external finance premium, s, is given by

$$s = \frac{R^k}{R^n} = \left(\frac{k}{n}\right)^\chi,$$

where the parameter χ and the steady-state ratio of capital to net worth, k/n, are calibrated as described in the text.

From equation (A5),

$$\eta R^k \frac{1}{Z} = 1.$$

Note that R^k must also satisfy the preceding condition, $R^k/R^n = (k/n)^\chi$.

From equation (A2),

$$\frac{y}{k} = \frac{1}{(1-\alpha)\, Z \cdot mc}\, [R^k - (1-\delta)].$$

From equation (A12),

$$\frac{i}{k} = 1 - \frac{1}{Z}(1-\delta).$$

From equation (A7),

$$\frac{c}{k} = \frac{y}{k} - \frac{i}{k}.$$

We also have

$$\frac{y}{c} = \frac{\dfrac{y}{k}}{\dfrac{c}{k}}.$$

From equation (A9),

$$H = \left[\frac{\alpha}{\theta}\frac{y}{c}\, mc\right]^{1/(1+\gamma)}.$$

From equation (A8),

$$k = \frac{H}{\left(\dfrac{y}{k}\right)^{1/\alpha} Z^{(1-\alpha)/\alpha}}.$$

Then,

$$c = \frac{c}{k}k, \quad i = \frac{i}{k}k, \quad y = \frac{y}{k}k.$$

Log-Linearized Equilibrium Conditions

This section lists the equilibrium conditions in terms of log deviations in the normalized variables from the nonstochastic steady state.

Let $\tilde{z}_t$ denote the percentage deviation in technology growth from the mean:

$$\tilde{z}_t \equiv \ln Z_t - \mu.$$

Consumption-savings:

$$-\tilde{c}_t = -E_t\tilde{c}_{t+1} - E_t\tilde{z}_{t+1} + \tilde{r}^n_{t+1} - E_t\tilde{\pi}_{t+1}.$$

Expected real rate of return on capital:

$$E_t\tilde{r}^k_{t+1} \equiv \frac{mc(1-\alpha)\dfrac{y}{k}Z}{mc(1-\alpha)\dfrac{y}{k}Z + (1-\delta)} (E_t\tilde{y}_{t+1} - \tilde{k}_{t+1} + E_t\tilde{z}_{t+1} + E_t\widetilde{mc}_{t+1})$$

$$+ \frac{1-\delta}{mc(1-\alpha)\dfrac{y}{k}Z + (1-\delta)} E_t\tilde{q}_{t+1} - \tilde{q}_t.$$

Definition of the external finance premium:

$$\tilde{s}_t \equiv E_t\tilde{r}^k_{t+1} - (\tilde{r}^n_{t+1} - E_t\tilde{\pi}_{t+1}).$$

Determination of the external finance premium:

$$\tilde{s}_t = \chi(\tilde{q}_t + \tilde{k}_{t+1} - \tilde{n}_{t+1}).$$

Evolution of net worth:

Using the steady-state condition $\eta R^k/Z = 1$, net worth evolves according to

$$\tilde{n}_{t+1} = \frac{k}{n}\tilde{r}^k_t - \left(\frac{k}{n} - 1\right)E_{t-1}\tilde{r}^k_t + \tilde{n}_t - \tilde{z}_t.$$

Or, using the definition of the external finance premium, $E_{t-1}\tilde{r}^k_t \equiv \tilde{s}_{t-1} + (\tilde{r}^n_t - E_{t-1}\tilde{\pi}_t)$, we have

$$\tilde{n}_{t+1} = \frac{k}{n}\tilde{r}^k_t - \left(\frac{k}{n} - 1\right)(\tilde{s}_{t-1} + \tilde{r}^n_t - E_{t-1}\tilde{\pi}_t) + \tilde{n}_t - \tilde{z}_t.$$

Investment-Q relationship:

$$\tilde{q}_t = \eta_k(\tilde{i}_t - \tilde{k}_t + \tilde{z}_t),$$

where

$$\eta_k \equiv \frac{-\left[\Phi''\left(\frac{i}{k}Z\right)\frac{i}{k}Z\right]}{\Phi'\left(\frac{i}{k}Z\right)} = \frac{-\{\Phi''[Z - (1 - \delta)] \cdot [Z - (1 - \delta)]\}}{\Phi'[Z - (1 - \delta)]}.$$

Aggregate resource constraint:

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{i}{y}\tilde{i}_t.$$

Production function:

$$\tilde{y}_t = \alpha\tilde{h}_t + (1 - \alpha)\tilde{k}_t - (1 - \alpha)\tilde{z}_t.$$

Labor market equilibrium condition:

$$\tilde{y}_t + \widetilde{mc}_t - \tilde{c}_t = (1 + \gamma)\tilde{h}_t.$$

Inflation:

$$\tilde{\pi}_t = \kappa\widetilde{mc}_t + \beta E_t\tilde{\pi}_{t+1},$$

where $\kappa \equiv (1 - \upsilon)(1 - \beta\upsilon)/\upsilon$.

Capital accumulation:

$$\tilde{k}_{t+1} = \frac{1 - \delta}{Z}(\tilde{k}_t - \tilde{z}_t) + \left(1 - \frac{1 - \delta}{Z}\right)\tilde{i}_t.$$

Policy rule with inflation only:

$$\tilde{r}^n_{t+1} = \phi_\pi\tilde{\pi}_t.$$

Policy rule with the asset price gap:

$$\tilde{r}^n_{t+1} = \phi_\pi\tilde{\pi}_t + \phi_Q(\tilde{q}_t - \tilde{q}^*_t).$$

Policy rule with the natural rate of interest and the asset price gap:

$$\tilde{r}^n_{t+1} = \tilde{r}^*_{t+1} + \phi_\pi\tilde{\pi}_t + \phi_Q(\tilde{q}_t - \tilde{q}^*_t).$$

Policy rule with output growth:

$$\tilde{r}^n_{t+1} = \phi_\pi\tilde{\pi}_t + \phi_Y(\tilde{y}_t - \tilde{y}_{t-1} + \tilde{z}_t).$$

Policy rule with asset price growth:

$$\tilde{r}^n_{t+1} = \phi_\pi\tilde{\pi}_t + \phi_Q(\tilde{q}_t - \tilde{q}_{t-1}).$$

Policy rule with the level of asset prices:

$$\tilde{r}^n_{t+1} = \phi_\pi\tilde{\pi}_t + \phi_Q\tilde{q}_t.$$

Technology shock process:

$$\tilde{z}_t = \tilde{d}_t + \varepsilon_t,$$

and

$$\tilde{d}_t = \rho_d \tilde{d}_{t-1} + v_t,$$

where $\tilde{d}_t$ is defined as

$$\tilde{d}_t \equiv (\mu_t - \mu).$$

Solution to the Model

This section describes the solution to the model.

When the Monetary Policy Rule Does Not Include the Asset Price Gap and the Natural Rate of Interest

When the interest rate rule does not include the asset price gap and the natural rate of interest, we do not need to compute the equilibrium in the frictionless economy to characterize the equilibrium in the economy with both pricing and financial frictions.

When the Private Sector Has Full Information

The solution to the model takes the form:

(A13) $X_t = B_1 X_{t-1} + B_2 u_t,$

where

$$X_t \equiv [\tilde{c}_t; \tilde{y}_t; \tilde{h}_t; \tilde{i}_t; \tilde{k}_{t+1}; \tilde{n}_{t+1}; \tilde{r}_t^k; \tilde{s}_t; \tilde{r}_{t+1}^n; \tilde{q}_t; \widetilde{mc}_t; \tilde{\pi}_t; \tilde{d}_t],$$

and

$$u_t \equiv [v_t; \varepsilon_t].$$

When the Private Sector Has Imperfect Information

We assume certainty equivalence. The solution under imperfect information is characterized by the same coefficients, B_1 and B_2, as in the case of full information. We replace the unobserved variables ($\tilde{d}_{t-1}$, v_t, ε_t) on the right-hand side of the solution system in equation (A13) with inferences ($\tilde{d}_{t-1|t-1}$, $v_{t|t}$, $\varepsilon_{t|t}$) that are determined by the following four equations. The first specifies the process of the persistent component of technology growth:

(A14) $\tilde{d}_t = \rho_d \tilde{d}_{t-1} + v_t.$

The second links the observed technology growth, $\tilde{z}_t = (\tilde{d}_t + \varepsilon_t)$, to the inference about the persistent component of technology growth, $\tilde{d}_{t|t}$:

(A15) $\tilde{d}_{t|t} = \lambda_1 \tilde{z}_t + (1 - \lambda_1) \rho_d \tilde{d}_{t-1|t-1}$

$\qquad\qquad = \lambda_1 (\tilde{d}_t + \varepsilon_t) + (1 - \lambda_1) \rho_d \tilde{d}_{t-1|t-1},$

where λ_1 is the Kalman gain that the private sector uses.

The third defines the inference of the private sector about the realization of the shock to the persistent component of technology growth, $v_{t|t}$:

(A16) $$v_{t|t} = \tilde{d}_{t|t} - \rho_d \tilde{d}_{t-1|t-1}.$$

The fourth defines the inference of the private sector about the realization of the shock to the transitory component of technology growth, $\varepsilon_{t|t}$:

(A17) $$\varepsilon_{t|t} = \tilde{z}_t - \tilde{d}_{t|t}$$
$$= (\tilde{d}_t + \varepsilon_t) - \tilde{d}_{t|t}.$$

When the Monetary Policy Rule Includes the
Asset Price Gap Or the Natural Rate of Interest

The solution described in the following concerns the case where the interest rate rule includes the natural rate of interest or the asset price gap.

When Both the Private Sector and the Policymaker Have Full Information

The solution to the model takes the form:

(A18) $$X_t = B_3 X_{t-1} + B_4 u_t,$$

where

$$X_t \equiv [\tilde{c}_t; \tilde{y}_t; \tilde{h}_t; \tilde{i}_t; \tilde{k}_{t+1}; \tilde{n}_{t+1}; \tilde{r}_t^k; \tilde{s}_t; \tilde{r}_{t+1}^n; \tilde{q}_t; \widetilde{mc}_t; \tilde{\pi}_t; \tilde{d}_t;$$
$$\tilde{c}_t^*; \tilde{y}_t^*; \tilde{h}_t^*; \tilde{i}_t^*; \tilde{k}_{t+1}^*; \tilde{n}_{t+1}^*; \tilde{r}_t^{k*}; \tilde{s}_t^*; \tilde{r}_{t+1}^{n*}; \tilde{q}_t^*; \widetilde{mc}_t^*; \tilde{\pi}_t^*; \tilde{d}_t^*],$$

and

$$u_t \equiv [v_t; \varepsilon_t; v_t^*; \varepsilon_t^*].$$

The variables with * denote those in the model without pricing and financial frictions, and the variables without * denote those in the model with both frictions.[23]

*When the Private Sector Has Full Information and
the Policymaker Has Imperfect Information*

The solution is characterized by the same coefficients, B_3 and B_4, as in the case where both the private sector and the policymaker have full information. We replace the unobserved variables ($\tilde{d}_{t-1}^*$, v_t^*, ε_t^*) on the right-hand side of the solution system with the inferences of the policymaker ($\tilde{d}_{t-1|t-1}^*$, $v_{t|t}^*$, $\varepsilon_{t|t}^*$) that are determined by the following four equations:

23. When we compute the impulse response or conduct stochastic simulations, the shocks are common across the model with frictions and the model without frictions: $v_t = v_t^*$ and $\varepsilon_t = \varepsilon_t^*$ for any period t.

(A19) $$\tilde{d}_t^* = \rho_d \tilde{d}_{t-1}^* + v_t^*,$$

and

(A20) $$\tilde{d}_{t|t}^* = \lambda_2 \tilde{z}_t + (1 - \lambda_2)\rho_d \tilde{d}_{t-1|t-1}^*$$
$$= \lambda_2(\tilde{d}_t^* + \varepsilon_t^*) + (1 - \lambda_2)\rho_d \tilde{d}_{t-1|t-1}^*,$$

where λ_2 is the Kalman gain that the policymaker uses, and

(A21) $$v_{t|t}^* = \tilde{d}_{t|t}^* - \rho_d \tilde{d}_{t-1|t-1}^*,$$

and

(A22) $$\varepsilon_{t|t}^* = \tilde{z}_t - \tilde{d}_{t|t}^*$$
$$= (\tilde{d}_t^* + \varepsilon_t^*) - \tilde{d}_{t|t}^*.$$

When Both the Private Sector and the Policymaker Have Imperfect Information

The solution is characterized by the same coefficients, B_3 and B_4, as in the case where both the private sector and the policymaker have full information. We replace the unobserved variables $(\tilde{d}_{t-1}, v_t, \varepsilon_t, \tilde{d}_{t-1}^*, v_t^*, \varepsilon_t^*)$ on the right-hand side of the solution system in equation (A18) with the inferences of the private sector and the policymaker $(\tilde{d}_{t-1|t-1}, v_{t|t}, \varepsilon_{t|t}, \tilde{d}_{t-1|t-1}^*, v_{t|t}^*, \varepsilon_{t|t}^*)$ that are determined by the eight equations (A14) to (A17) and (A19) to (A22). We assume that the private sector and the policymaker use the same Kalman gain ($\lambda_1 = \lambda_2$).

When the Private Sector Has Imperfect Information and the Policymaker Has Full Information

The solution is characterized by the same coefficients (B_3, B_4) as in the case where both the private sector and the policymaker have full information. We replace the unobserved variables $(\tilde{d}_{t-1}, v_t, \varepsilon_t)$ on the right-hand side of the solution system in equation (A18) with the inferences of the private sector $(\tilde{d}_{t-1|t-1}, v_{t|t}, \varepsilon_{t|t})$ that are determined by the four equations (A14) to (A17).

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Discussion Summary

Lars E.O. Svensson suggested that as a general methodological point, it would be natural to specify a loss function, and then to solve for the optimal policy, resulting in an optimal reaction function. He argued that this should be feasible because the model had a linear-quadratic structure. The next question would then be how one could implement this optimal policy. Starting from ad hoc rules, without checking whether they are close to optimal, approaches the problem from the wrong direction. Gilchrist responded that the authors appreciated the merits of this approach, but had sought to work within the context of the previous literature, in particular, the influential work of Bernanke and Gertler.

Stephen G. Cecchetti emphasized the importance of distinguishing between the loss function, which defines the target variables, and the reaction

function, which defines the response of the policy interest rate in terms of variables that need not themselves be target variables. For example, if asset prices were included in the reaction function, this did not necessarily imply that "asset price targeting" was taking place. Rather, it could reflect the fact that responding to asset prices helped in targeting other variables that entered the loss function.

Martin Schneider pointed out that in a linear model, one could always solve for asset prices as a linear function of state variables. In principle, therefore, the reaction function could be rewritten in terms of asset prices. Although this was legitimate if the model was correctly specified, he said it would be interesting to know the costs of doing so if the model was misspecified. Gilchrist responded that ideally one would have a model in which asset prices were forward-looking and, therefore, captured information that was not available elsewhere.

Tommaso Monacelli said that he had investigated the possibility of targeting the leverage ratio with a coauthor. It did not matter whether the reaction function was defined in terms of the leverage ratio, or asset prices, or inflation. The best outcome was obtained in all cases because the measures were all positively correlated. Gilchrist responded that in the framework of his chapter with Saito, the objective function includes both stability of inflation and stability of output. Because the model has two distortions, to the markup and the return on capital, two instruments are helpful to maximize the objective function. Inflation targeting enables policymakers to stabilize the markup, but targeting asset prices allows them to stabilize the return on capital and, hence, to reduce the variance of output.

Donald L. Kohn said that informational assumptions are critical and that policymakers learn about technological change from private-sector behavior. In the 1990s, he pointed out, models of the economy used by Federal Reserve Bank (FRB) staff had failed and predicted inflation; he was, therefore, skeptical that central banks have an informational advantage over the private sector. He said that the Fed would have been wrong to act on the claim that there was "irrational exuberance" in the stock market in 1996 and that it was extremely hard to measure the potential asset price Q^*.

Kohn also suggested that the role of leverage in the economy is complicated. It was popularly suggested that central banks should look at credit, but there had been a huge amount of financial innovation and, hence, credit expansion over the last twenty-five years. It would have been wrong to respond to this. As a general point, he argued that financial frictions are diminishing over time and that asymmetric information is, therefore, becoming less of a problem. Gilchrist responded that although he agreed that financial markets are handling information asymmetries in a more sophisticated fashion, the fact that people are taking on more leverage means that there could be severe consequences if the financial system were placed under strain.

Andrew Levin was also skeptical about the assumption that central bankers had more information than the private sector. He said that it was extremely hard to project total factor productivity and that his view was that the Fed's task was to try to infer total factor productivity (TFP) from information produced in the private sector. It was people in the private sector who saw TFP in their own sector of the economy—in the form, say, of patents that had just been obtained or the latest marketing information on a new product. Gilchrist said that he and Saito did not intend to promote the view that the Fed knows more than the private sector.

Jordi Galí commented that it was important to find variables that would indicate the state of the economy in a manner that did not depend on the details of the assumed model or the nature and dynamic properties of the underlying shocks hitting the economy. It would be appropriate for policymakers to respond to variables of this sort, whereas no consistently welfare-improving response would be possible if it required detailed knowledge of the economy's structure and the dynamic process of shocks to the economy.