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Chapter Author: Anthony J. Curley, Jack M. Guttentag

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ANTHONY J. CURLEY
Pennsylvania State University

JACK M. GUTTENTAG
Wharton School of Commerce and Finance,
University of Pennsylvania

The Yield on Insured Residential Mortgages

ABSTRACT: This paper develops a procedure for calculating the internal yield on a discounted mortgage. The procedure uses an estimate of the entire distribution of termination probabilities relevant to a mortgage of given characteristics. This contrasts with the conventional practice of basing yield on an estimate of average life. The preferred method developed in the paper for estimating the distribution of termination probabilities is a regression model in which termination rates depend on maturity, policy year, and the relationship between the contract rate on the mortgage and market yield in the specified policy year. This procedure allows us to specify alternative market yield patterns over the life of a mortgage cohort. ¶ The conventional practice of basing the yield calculation on an estimate of average life imparts a downward bias to the yield estimate because mortgages prepaid prior to average life have a yield substantially higher than the yield at average life while mortgages prepaid later than average life have yields only slightly lower. An additional downward bias arises from the practice of calculating mortgage yields on a "nominal" basis, that is, on the assumption that mortgage payments are received annually whereas in fact they are received monthly. The extent of the downward bias from both sources depends on the size of discounts, on the general level of yield, and on the average life assumed in the yield calculation. Using the assumption employed by the Federal Housing

Administration and the Board of Governors of the Federal Reserve System that termination occurs after a period equal to half the face maturity, the bias was generally more than 35 basis points during 1966–72. ¶ Our procedure also was used to calculate the yield on foreclosed FHA loans, taking account of special costs to the lender associated with foreclosure. Despite these costs, the yield realized on a foreclosed loan is not necessarily lower than the yield expected at the time the loan was acquired. Foreclosed loans have a relatively short life, and this tends to raise their yields if they were acquired at a discount. In general, if discounts exceed 3–4½ points, the realized yield on a foreclosed insured loan is likely to be higher than the original "expected" yield.

INTRODUCTION

This paper arose out of earlier work on mortgage yields carried out as part of the National Bureau's investigation of interest rates.[1] Intermittently, our efforts to construct reliable time series of mortgage yields and to explain the determinants of yield were interrupted by the nagging question, "How should mortgage yields be calculated?" As we came to understand this question better, it became increasingly evident that the answer was not simple, that misconceptions were widespread, and that measurement errors were common. We resolved, therefore, to make a frontal assault on the question, and this study is the result.

The study was financed by the National Bureau, but extensive computer time was contributed by the University of Pennsylvania Computer Center and the Center for Research of the College of Business Administration of the Pennsylvania State University. The Bureau staff reading committee, consisting of Phillip Cagan, Stanley Diller, Robert Moore Fisher, and John Wetmore made many useful comments and suggestions on an early draft; and Robert Lipsey was very helpful on a later draft. Richard Parli was an industrious and capable research assistant. In addition, we are indebted to Allan Thornton and Mortimer Kaplan of the Federal Housing Administration for providing the basic data used in the study.

The format of the paper is as follows. In the first section we develop an analytically correct method of calculating yield on discounted mortgages. In the second section we begin making the methodology operational by estimating relevant parameters. Next, in the third section, we consider the bias in existing yield calculations by official agencies and private investors. In the next section we apply our basic methodology to the problem of calculating yield on foreclosed FHA mortgages. Section V is a summary of the major conclusions and points up some of the policy implications of our findings.

I. CALCULATING RESIDENTIAL MORTGAGE YIELDS

Definition

Mortgage yield is defined as that rate of discount which equates the present value of the stream of principal and interest payments received by the mortgage lender with the net amount disbursed. This is the standard "internal rate of return" concept that is applied to all types of financial and real assets. In general terms,

$$\text{Net Amount Disbursed} = \frac{\text{Int. + Prin. Rec'd. 1st Yr.}}{1 + \text{Yield}} + \frac{\text{Int. + Prin. Rec'd. 2nd Yr.}}{(1 + \text{Yield})^2} + \dots + \frac{\text{Int. + Prin. Rec'd. Final Yr. } (n)}{(1 + \text{Yield})^n}$$

In the case of residential mortgages the net amount disbursed can be expressed as $L(1 - d)$, where L is the face amount of the loan and d is the number of discount points per $100 loaned.[2] On the assumption followed in this paper that we are measuring yield return to the lender (rather than yield cost to the borrower), the discount (d) should be measured net of the variable costs of placing a loan on the books.

Return flows of interest and principal to the lender are of two types. R is defined as the periodic payment of principal and interest, and in accordance with conventional practice it is calculated as a uniform amount that will completely retire the loan after a period equal to the face maturity; such a loan is said to be "fully amortizing."[3] R depends on the face amount of the loan (L), the face maturity (n), and the contract rate (r).

$$(1) \quad R = L\left[\frac{r}{1 - (1 + r)^{-n}}\right]$$

The second type of return flow is the balance of the loan at the time the borrower prepays it in full, which we denote by B_p. Most home mortgages (90 per cent or more) are prepaid in full before maturity.[4] We assume that this occurs after a period equal to p years, termed the "prepayment period." The loan balance at that time depends on the original loan amount (L), the contract rate (r), maturity (n), and prepayment period (p).

$$(2) \quad B_p = L\left[\frac{1 - (1 + r)^{-(n-p)}}{1 - (1 + r)^{-n}}\right]$$

Equation 2 abstracts from two factors that in practice could affect the final payment to the lender. One is prepayment penalties, which we ignore because it complicates the yield calculation without adding anything of substance. The second is the possibility of foreclosure, which could make the final payment fall short of B_p. We temporarily assume away the possibility of foreclosure, which is considered in Section IV.

Denoting the yield on a mortgage prepaid in p years as Y_p, we can write the basic mortgage yield equation as follows:[5]

$$(3)\quad L(1-d) = \frac{R}{(1+y_p)} + \frac{R}{(1+y_p)^2} + \ldots + \frac{R}{(1+y_p)^p} + \frac{B_p}{(1+y_p)^p}$$

Inserting equations 1 and 2 into equation 3, dividing both sides by L, and simplifying, we get:

$$(3a)\quad 1-d = \frac{r}{1-(1+r)^{-n}} \sum_{t=1}^{p} (1+y_p)^{-t} + \frac{1-(1+r)^{-(n-p)}}{1-(1+r)^{-n}} (1+y_p)^{-p}$$

Since mortgages are amortized monthly, the maturity, contract rate, and periodic payment shown in equation 3a should be expressed in monthly values; the resulting yield is termed the "effective yield." It is customary in the trade, however, to calculate yield in annual values, which we shall refer to as the "nominal yield."

Mortgage Yield Properties

Mortgage yields share three well-known properties of internal rates of return. First, yield must be computed by trial and error. With the aid of computers, however, this is not a serious problem.

Second, calculations of internal yield may result in multiple solutions. Equation 3a is a p^{th} degree polynomial and there are p roots (y_p values) that satisfy the equality. Only positive real roots have economic relevance, however, and if there is only one such root it is defined as "the" mortgage yield.

Mortgages that are prepaid in full (or run to maturity) can have only one positive real solution because they never shift from positive to negative cash flows.[6] Foreclosed mortgages are subject to multiple yield solutions in principle but probably not in practice because cash flow reversals are not large enough.[7]

Third, calculation of internal rate of return assumes that interim cash flows (R in the case of mortgages) are reinvested when received at the calculated yield (y_p). This amounts to assuming that the market yield remains unchanged over the life of the instrument, which is patently unrealistic. If future rate levels were known, the internal rate would be biased and would not be used. Instead, analysts would calculate the future value of the investment stream at p, taking account of reinvestment at the varying rates prevailing over the period until p, and then they would calculate that discount rate which equated present and future value. This rate would be higher (lower) than the internal yield if market yields on balance had been rising (falling) over the life of the instrument. The justification of the internal yield measure is that, since future yield levels are unknown, the best assumption is that prevailing yield levels will continue. We will not defend this assumption here, but portfolio strategies are available for dampening the effects of future variations in market yields on the yield realized from investments made today.[8]

Most of the interesting problems associated with the mortgage yield concept presuppose the existence of discounts. (If there are no discounts, yield is always equal to the contract rate, appropriately adjusted to take account of compounding.) The impact of a discount on yield depends on other transaction characteristics. A discount of given size will raise yield above contract rate by a larger amount the shorter the face maturity (n), the higher the contract rate (r), and the shorter the prepayment period (p).

The most important relationship involves discount, yield, and prepayment period, because the yield is very sensitive to changes in the other two variables within empirically relevant ranges of these variables and the prepayment period is not known at the time the mortgage is written (all the other variables in equation 3a are known when the yield is calculated). Figure 1 shows the relationship between yield and prepayment period for a 30-year, 8½ per cent mortgage at discounts of 2 points and 10 points. It is evident that the larger discount has a much greater impact on yield if the prepayment period is short than if it is long. Also, variability in the prepayment period has a much greater effect on yield if the discount is large.

Methods of Estimating Mortgage Yield

The prepayment date for any individual mortgage loan is unknown ex ante, which introduces uncertainty into any estimate of ex ante yield. This uncertainty can be reduced, however, by using information on past terminations of mortgages with similar characteristics. The appropriate analogy is to mortality experience used by life insurance companies. Given other loan characteristics, the yield is estimated from termination experience covering similar mortgages, just as a required life insurance premium is estimated from mortality experience covering similar persons. In both cases the calculated value is unlikely to be accurate for the individual mortgage or life insurance policyholder. It is meant to be accurate for a large group with similar characteristics.

FIGURE 1 Effective Yield on a 30-year, 8½ per cent Mortgage

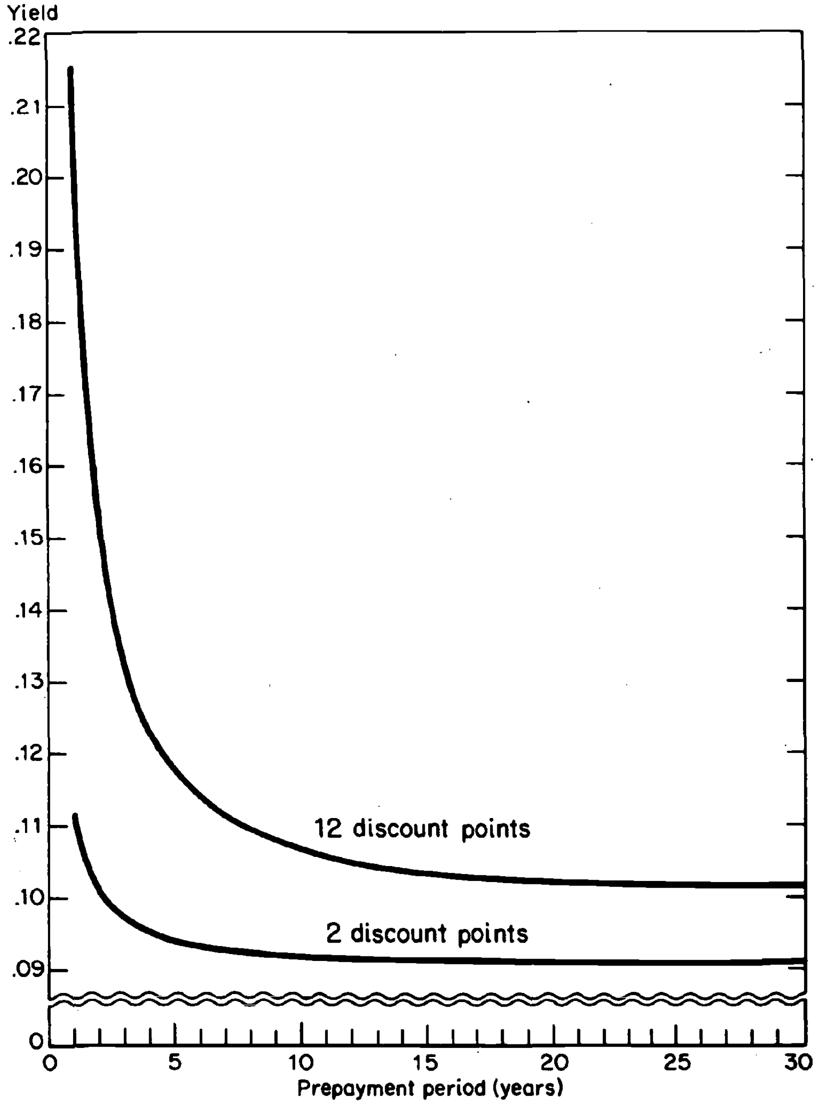
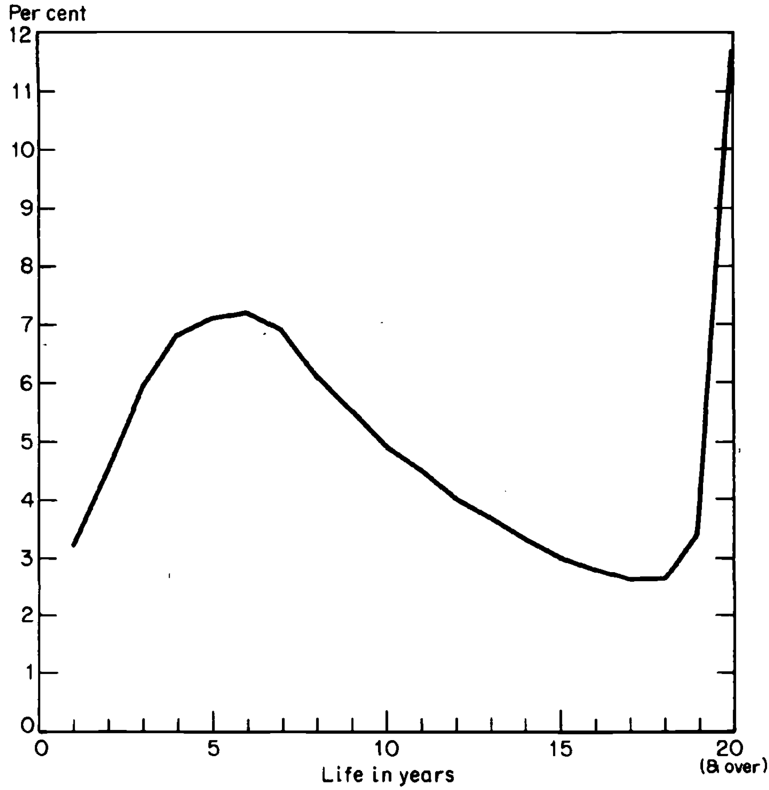


Figure 2 provides some insight regarding the pattern of mortgage termination probabilities; it portrays the aggregate empirical distribution of lives of FHA mortgages with maturities of 18 to 22 years during the period 1935–65.[9] Dispersion in annual termination rates is wide, ranging from less than 3 per cent in later years to more than 7 per cent in the fifth and sixth years. The distribution is skewed positively, with about two-fifths of the

FIGURE 2 Percentage Distribution of Lives, FHA Home Mortgages, Maturities of 18–22 Years (1935–65 experience)



SOURCE: U.S. Department of Housing and Urban Development, *Annual Report, 1967.*

mortgages terminated during the first 6 years. Although this particular distribution has little claim to current relevance—it reflects experience over a very long period when conditions affecting termination experience shifted markedly—the general shape of the distribution is characteristic of most of those we have examined.

For the moment, let us assume that the distribution of termination probabilities is known with certainty. What is the correct way to estimate yield? One approach is to assume that p is equal to the mean of the distribution. This procedure, however, will lead to a biased estimate, which can be illustrated by a simple example. Assume that a lender is about to acquire three 8½ per cent, 30-year mortgages of the same amount at a six point discount; mean expected life is known to be 10 years. If all three loans are paid at the end of 10 years, the effective yield on each would be 9.89 per cent. Suppose, on the other hand, that one mortgage is paid off after 10 years, one after 2 years, and one after 18 years, the average life remaining at 10 years. The percentage yields would be, respectively, 9.89 (10 years), 12.58 (2 years), and 9.64 (18 years). The combined yield for all three mortgages is clearly higher than the yield at average life. The reason is that mortgages prepaid early have a yield much higher than the average, whereas mortgages prepaid late have yields only slightly lower than the average. This fact is evident from Figure 1.

Because of the non-linear relationship between yield and mortgage life, the entire distribution of termination probabilities should be used in estimating yield. This is done by assuming that yield on the individual mortgage is identical to the yield on a portfolio of similar mortgages for which termination experience is available. Yield is calculated from the cash flows generated by this portfolio using a revised (enlarged) version of equation 3a,

$$(4)\quad 1 - d = \pi_1 \left[\frac{R}{1+\bar{y}} + \frac{B_1}{(1+\bar{y})} \right] + \pi_2 \left[\frac{R}{1+\bar{y}} + \frac{R}{(1+\bar{y})^2} + \frac{B_2}{(1+\bar{y})^2} \right] + \ldots + \pi_n \left[\frac{R}{1+\bar{y}} + \frac{R}{(1+\bar{y})^2} + \ldots + \frac{R}{(1+\bar{y})^{n-1}} + \frac{R}{(1+\bar{y})^n} \right]$$

Equation 4 defines the cash flow received by the lender as the flow generated by a mortgage prepaid after one period weighted by the one-period termination probability (π_1) (proportion of mortgages in the portfolio prepaid after one period) plus the flow generated by a mortgage prepaid after two periods weighted by the two-period termination probability (π_2), etc. This procedure implies that all flows are reinvested at the average portfolio yield ($\bar{y}$), which is tantamount to assuming that all flows are reinvested in the entire portfolio. We will term the yield obtained in this fashion the "true internal yield." If the contract rate, maturity, and payment are expressed as monthly values, it is the "effective true internal yield."

It is important to point out that the yield generated by equation 4 cannot be estimated by calculating the weighted average of periodic yield (y_p) where termination probabilities are used as weights. This procedure implicitly assumes that interim and terminal flows are reinvested at the yield on the particular mortgage generating the flow, rather than at the average portfolio yield. In the three-mortgage example given above, for example, the weighted average yield of

$$\frac{9.89(1) + 12.58(1) + 9.64(1)}{3} = 10.70\%$$

implies that at the end of 2 years the balance outstanding on the mortgage prepaid at that time is reinvested in another 2-year mortgage. This assumption is invalid and biases the yield upward. The termination probabilities on reinvested cash flows are independent of the life of the specific mortgages generating these flows.

Equation 4 must be solved by trial and error, and the computation is far more difficult than the computation required for equation 3. Although hand calculation is neither practical nor economical, the yield can be determined easily by computer, and the authors have developed an efficient program that does this.

There is, of course, some assumed mortgage life that, when used as the single prepayment assumption, will produce the same yield as equation 4. We term this the "equalizing prepayment"; it will be less than average life if the distribution of termination probabilities is rectangular, normal, or skewed to the right.

II. ESTIMATING THE DISTRIBUTION OF TERMINATION PROBABILITIES

To this point we have been concerned with the correct method of calculating mortgage yields on the assumption that the relevant distribution must be estimated, and we now turn to the several methods available for performing this estimation.

Determinants of Termination Rates

Termination rates on a group of mortgages written in a given year (a "cohort") are affected by characteristics of the group, by policy year, and by exogenous developments during the cohort's lifetime.

Face maturity is an important determinant of termination rates, with shorter-maturity mortgages carrying higher rates in the early years. This is illustrated by Table 1, which shows early-year termination rates calculated from experience covering ten cohorts divided into three maturity groups.[10] The major reason for this relationship, presumably, is that buyers of existing homes are less likely to assume loans with shorter maturities because loan balances are smaller. This reflects both a faster paying down of principal on short maturity loans and a positive correlation, on loans made during any period, between maturity and ratio of loan to property value. The termination data covering FHA mortgages used in this study are classified by face maturity. Although termination patterns are no doubt affected by other mortgage and borrower characteristics, data are not available to analyze such factors. As we shall see, however, we are able to explain almost 90 per cent of the variability in termination rates with the data available.

TABLE 1 Termination Rates on FHA Mortgages during Each of the First 10 Years of Life, by Maturity (terminations as percentage of mortgages originated in base year)

Age of Mortgage	Maturity (Years) 18–22	23–25	26–30
1	.0157	.0095	.0073
2	.0336	.0247	.0215
3	.0468	.0357	.0316
4	.0553	.0439	.0385
5	.0579	.0481	.0423
6	.0580	.0502	.0459
7	.0587	.0503	.0469
8	.0538	.0488	.0452
9	.0499	.0465	.0426
10	.0460	.0432	.0389

NOTE: FHA experience during 1955–65 on Section 203 mortgages only. Termination rates are unweighted averages of annual figures. Age of mortgage at termination is determined by the policy year in which it is terminated, with the n^{th} policy year for the cohort of mortgages insured in calendar year t defined as running from July 1 of year $t+n-1$ to June 30 of year $t+n$. (For example, the second policy year for the cohort of mortgages insured in calendar year 1952 is July 1, 1953–June 30, 1954.) Thus, the age of a mortgage at termination is assumed to be n years.

SOURCE: Calculated from data provided by the FHA.

Termination rates vary considerably by policy year, as shown in Figure 2. Termination rates tend to rise sharply in the early policy years, then decline slowly until maturity, when they jump sharply again. The rise in termination rates during the early years largely reflects the fact that home sales (which usually lead to mortgage terminations), as well as mortgage refinancings, become increasingly probable the older the mortgage. Home sales become increasingly probable because changes in the conditions underlying the initial purchase become increasingly probable.[11] Similarly, the older the mortgage, the lower its balance and the greater the incentive to refinance. For these reasons, when terminations are expressed as a percentage of mortgages outstanding in the current year, rather than in the base year, termination rates rise progressively until the final year of life. Base year termination rates reverse direction after a period, however, as the effect of progressive erosion of the base tends to offset the increasing probability of termination by those remaining in the base.

The third important factor influencing termination rates is the market discount prevailing at the time the mortgage cohort is originated. The larger the discount, the less the incentive for the original borrower to refinance at a later date (since he would be obliged to pay the discount again), and the greater the incentive for any future purchaser of his property to assume the existing mortgage rather than take out a new one.

Hence, mortgages written at large discounts will tend to have relatively low early-year termination rates.

The fourth factor influencing termination rates is changes in market yields over the life of the cohort. The reasoning here is simply an extension of the point made in the immediately preceding paragraph. If contract rates or discounts rise, refinancing is discouraged, mortgage assumptions by homebuyers are encouraged, and termination rates are therefore lower. The influence of changes in market yields is clearly evident in year-to-year fluctuations in termination rates for mortgages of given age. Table 2 illustrates this influence for selected years of "ease" and "restraint" in mortgage markets. Thus, in every mortgage age category for which a comparison is possible, termination rates were higher in years of ease than in the immediately prior year of restraint.

Of the four determinants of termination rates, the influence of policy year and maturity can readily be taken account of by appropriate classification of termination data. The influence of changes in market conditions is more troublesome to deal with, as we shall see later.

Data and Methods

To properly estimate yield, as mentioned above, we need the complete distribution of termination rates for the cohort of mortgages to which a new mortgage belongs. If it is a 30-year mortgage, for example, we need to know the percentage of mortgages in the cohort terminated in every one of the 30 years of the cohort's life. Even if our data cover a full 30 years, the termination rate for the thirtieth year would be based on only one observation, the termination rate for the twenty-ninth year would be based on only two observations, and so on. The data requirements thus appear to be quite formidable.

In fact, the problem is not quite so monumental as the preceding paragraph suggests. Notice from Figure 1 that even at large discounts early-year termination rates exert the predominant influence on yield. It makes a great deal of difference to the yield whether a mortgage is paid off in the second as opposed to the seventh year. But it makes very little difference whether it is paid off in the fifteenth as opposed to the twentieth year. Thus, data covering a period equal, say, to one-half of a cohort's face maturity are more than adequate for the intended purpose. Projections covering the remaining life of the cohort will involve a very small error at worst. Nevertheless, the period for which data are needed remains a long one and this limits the available options.

The data available to us are of two types:

1. Published FHA data, classified by maturity, which are available every year (beginning in 1949) and which cover the period from 1935 to

TABLE 2 Termination Rates on FHA 25-Year Mortgages during Selected Periods of Ease (E) and Restraint (R), by Age of Mortgage (terminations as percentages of mortgages originated in base year)

Age of Mortgage	Year of Termination 1953(R)	1954(E)	1956(R)	1958(E)	1959(R)	1964(E)
1	.006	.010	.007	.008	.005	.010
2	.017	.027	.018	.024	.015	.028
3	.029	.053	.029	.036	.027	.046
4		.054	.034	.044	.036	.058
5			.044	.051	.039	.067
6			.052	.055	.045	.070
7				.053	.049	.068
8				.053	.044	.064
9					.047	.058

NOTE: The calendar year of termination is assumed to be $t + n - 1$, where t is the year a loan is insured and n is the policy year in which it is terminated (see the note to Table 1). The terms "ease" and "restraint" refer to interest rate levels relative to the prior period specified; e.g., rates were lower in 1954(E) than in 1953(R), higher in 1956(R) than in 1954(E), etc.
SOURCE: Calculated from data provided by the FHA.

the current year. These tabulations thus cover the periods 1935–49, 1935–50, . . . , 1935–72.

Because of the period coverage, these data are of limited usefulness (pre-World War II experience cannot be broken out). In addition, the implicit weighting scheme involved in these tabulations is arbitrary.[12]

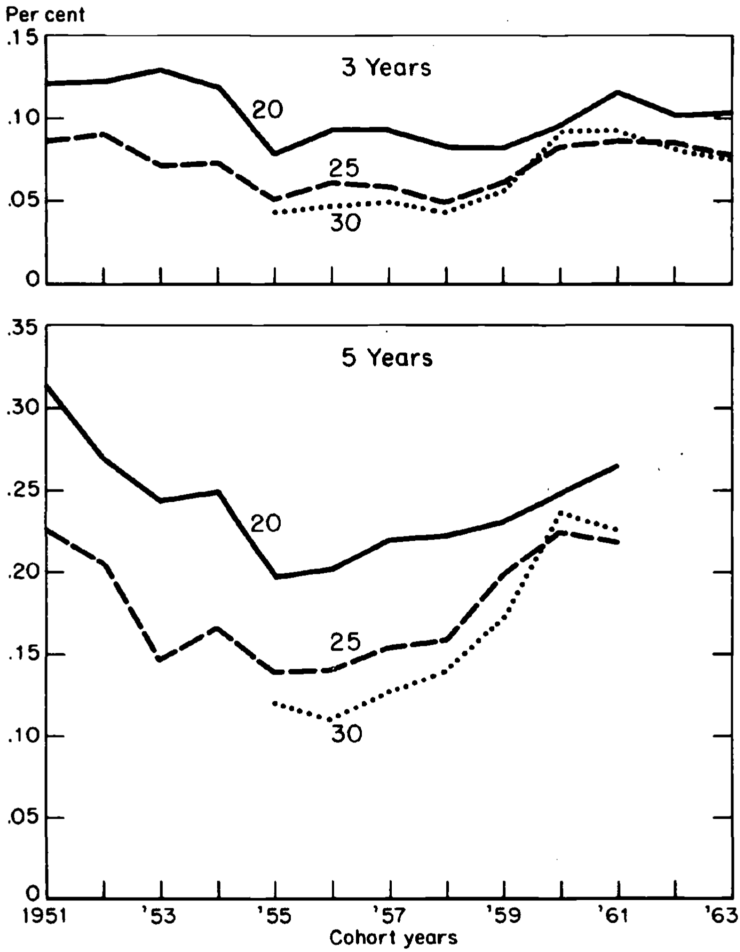
2. The FHA kindly provided us with annual data covering the period 1951–65, for 20-, 25-, and 30-year mortgages. These data, which underlie the tabulations shown in tables 1 and 2, were used to compile a complete distribution of termination probabilities in two different ways.

1951–65 Average Our first approach was to compile a distribution representing average experience during the entire period 1951–65. (In this distribution the first-year termination rate was an average of 15 rates, the fifteenth-year termination rate was based on one observation, and the termination rates for the years 16–30 were projected.)

We might have chosen a somewhat shorter period within the years 1951–65, but found no reason to do so because there was no evidence of trend in termination patterns within the 1951–65 period. Figure 3, which shows the percentage of mortgages terminated in both the first 3 years and the first 5 years,[13] indicates some variation in termination patterns within the period but no trend.[14]

Marked changes in maturity composition during 1951–65 present a problem. As shown in Table 3, 20-year loans predominated during the early part of the period, whereas 30-year loans predominated toward the

FIGURE 3 FHA-Insured Mortgages: Percentage of Mortgages Terminated within the First 3 and 5 Years of Life, by Maturity and Cohort Year



SOURCE: Calculated from data provided by the FHA.

end of the period. In view of this pattern, we decided to combine the data across maturity groups. Since termination rates in a policy year vary inversely with loan maturity, we normalized this relationship by expressing policy year as a percentage of maturity. Thus, the first calendar year was transformed into 5 percent of maturity on 20-year mortgages and 4 percent of maturity on 25-year mortgages. The relationship between cumulative termination rates[15] for each maturity group and this transformed measure is presented in Figure 4. The broken line is a weighted average of the termination rates for all three maturities. This average was taken as the most appropriate distribution for all three maturity groups. In calculating

TABLE 3 Number of FHA Insured Home Mortgages Originated, 1951–65, by Maturity

Year Insured	20-Year	25-Year	30-Year	Total
		Maturity		
1951	136,570	91,447	5,222	233,239
1952	160,044	41,584	273	201,901
1953	158,362	60,124	200	218,686
1954	101,218	61,085	3,590	165,893
1955	91,680	145,793	46,107	283,580
1956	70,059	126,093	28,773	224,925
1957	52,507	93,048	25,933	171,488
1958	64,795	164,216	116,226	345,237
1959	61,166	180,796	209,792	451,754
1960	36,537	95,681	195,901	328,119
1961	31,808	88,306	213,949	334,063
1962	27,773	77,608	221,756	327,137
1963	23,862	64,199	228,969	317,030
1964	23,352	63,968	271,314	358,634
1965	23,689	65,873	317,906	407,468
	1,063,422	1,419,821	1,885,911	4,369,154

SOURCE: FHA.

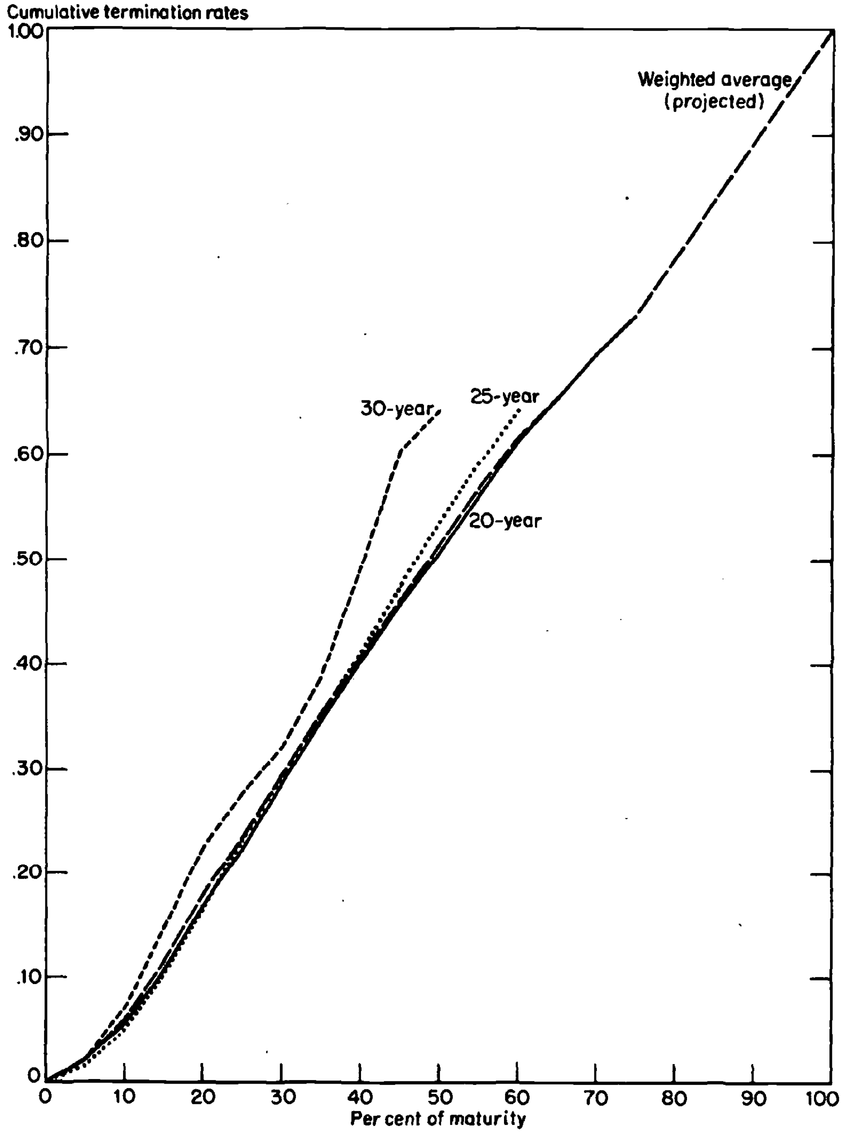
the yield for each group, the average was reconverted to a policy-year basis. The termination rates used are shown in Table 4.

The normalization procedure underlying the weighted average termination rates is somewhat arbitrary and is defensible only in terms of results. Although it succeeds very well in equalizing termination rates between 20- and 25-year mortgages, as Figure 4 shows, it does less well in closing the gap with 30-year mortgages. The weighted average rates were nevertheless regarded as preferable to rates calculated separately for each maturity group because of the small number of observations available for both 20-year and 30-year mortgages during part of the period covered, and the associated statistical noise in the separate distributions. When the weighted average termination rates are used to calculate yields on 30-year mortgages, any bias in the procedure is conservative—it leads to lower estimated yields than would be the case if a 30-year pattern alone were used.

The 1951–65 average distribution has two major problems. First, it reflects all the peculiarities of the period, which may or may not be applicable to the future. Most important, interest rates trended upward during 1951–65. If this distribution is used to calculate the yield on new mortgages, the implicit assumption is that interest rates will on balance behave similarly in the future. This gives the distribution a downward bias.

The second problem is the implicit assumption underlying the construc-

FIGURE 4 Averaged Cumulative Termination Rates, by Original Maturity



tion of this distribution that termination rates are independent of the level of discounts prevailing at the time the mortgage is written. This is inherently implausible for reasons already given.

Hence, the 1951–65 distribution probably has a generally conservative bias because of the upward trend in market yields during that period, but

TABLE 4 Termination Rates Employed in Calculating Yields, Based on 1951–65 Experience

Year after Insurance	20 Years	25 Years	30 Years
1	.01797	.01438	.01198
2	.04060	.02795	.01952
3	.05786	.03938	.02707
4	.06225	.04717	.03857
5	.05714	.04980	.03589
6	.05601	.04571	.04565
7	.05837	.04503	.03809
8	.05563	.04575	.03772
9	.05843	.04615	.03734
10	.05026	.04451	.03891
11	.05217	.04674	.03800
12	.04925	.04185	.03709
13	.03716	.04097	.03895
14	.04240	.04115	.03623
15	.03580	.03940	.03351
16	.05390	.02973	.03478
17	.05380	.03287	.03381
18	.05370	.03128	.03283
19	.05370	.03220	.02477
20	.05360	.04296	.02652
21		.04296	.02827
22		.04302	.02387
23		.04296	.02983
24		.04296	.03580
25		.04312	.03580
26			.03583
27			.03587
28			.03573
29			.03584
30			.03593
	1.00000	1.00000	1.00000
Average Life	9.7	12.1	14.6

the size of the bias would be inversely correlated with the number of discount points on new mortgages.

Some indication of the sensitivity of internal yield to different termination patterns is provided in Table 5. The first two columns indicate that the termination pattern covering the period 1951–65 generated a much lower

TABLE 5 Internal Effective Yield on 6 Per Cent, 20-Year Mortgages, Using Various Distributions of Termination Probabilities

Discount Points	1951–65	1935–49	Rectangular
2	6.55	6.61	6.55
4	6.93	7.08	6.94
6	7.33	7.56	7.35
8	7.75	8.06	7.78
10	8.18	8.57	8.23
12	8.63	9.11	8.68
14	9.09	9.68	9.16

yield on mortgages of equivalent maturity than the pattern for 1935–49. Early-year termination rates were much lower in the more recent period.[16] Yields based on the 1951–65 distribution do not differ greatly from those derived from a rectangular distribution, which assumes an equal number of mortgages terminated every month.

Regression Model To increase our flexibility and control, while avoiding the inherent biases in the 1951–65 distribution, we employed another approach. We used the same basic data source (extended, however, to cover the period 1965–67) to build a regression model in which termination rates would depend on policy year, maturity, the relationship between the contract rate on the mortgages in the cohort and the contract rate in the specified policy year, and the discount in the specified year.[17]

We experimented at length with respect to the form in which the independent variables were expressed. As an example, we tested extensively to determine whether market conditions could better be represented by a single variable (market yield less the contract rate on the mortgages in the cohort) or by the two market variables noted in the paragraph above. Similarly, we experimented with many different ways of expressing maturity and policy year, both as separate variables and as a single combined variable. In addition, we explored many different ways of specifying the regression by policy year coverage. We calculated regressions for each policy year separately and for many combinations of policy year (such as years 2 to 12, 3 to 13, 2 to 13, etc.).

We used a number of criteria in evaluating different equations. Our objectives were to (1) minimize standard errors; (2) avoid negative predicted termination rates; (3) in regressions covering more than one policy year, minimize correlation between residuals and policy year; and (4) in equations in which market conditions were represented by two variables, minimize both the degree of collinearity between the variables and the degree of variability in yield associated with changes in yield components.[18]

The best results were obtained by a single equation covering policy years 1 through 17. It generates a relatively high R^2, none of the calculated termination rates is negative, orthogonality is high, collinearity is modest, and calculated yield is relatively insensitive to changes in the components of market yield. The equation is as follows:

$\text{Log } TR_t =$	$-.56178 +$	$.90249 \text{ Log } (P/M)$	$- .10580(C_t - C)$	$- .02179D_t$
S.E.	.01720	.01603	.00678	.00365
Student t	−32.67	56.29	15.61	5.97

$R^2 = .867$
$F = 1135.66$
TR_t = annual termination rate in year t
P = policy year
M = maturity
C_t = contract rate prevailing in year t
D_t = discount prevailing in year t
C = contract rate on mortgages in cohort

In using the regression procedure we must specify the market discount and contract rate prevailing in every year of the cohort's life. The power of the procedure is that it allows us to specify alternative patterns of market conditions over the life of the mortgage cohort. Each pattern implies a unique set of termination rates. This is illustrated in Table 6, which shows the yield on an 8½ per cent, 30-year mortgage at varying current dis-

TABLE 6 True Internal Yield on 8½ Per Cent, 30-Year Mortgages, Using Various Distributions of Termination Probabilities

	Calculated from Regression			
Current Discount (Points)	Stable Yields (a) (1)	Falling Yields (b) (2)	Rising Yields (c) (3)	1951–65 Average (4)
2	9.18	9.24	9.14	9.15
4	9.51	9.64	9.44	9.48
6	9.84	10.00	9.75	9.82
8	10.17	10.37	10.06	10.17
10	10.50	10.73	10.38	10.53
12	10.84	11.08	10.70	10.91

(a) Assumes market contract rate and discount stable throughout life of cohort.
(b) Assumes market yields fall by .5 per cent in each of the first 3 years of the cohort's life.
(c) Assumes market yields rise by .5 per cent in each of the first 3 years of the cohort's life.

counts, using three sets of assumptions regarding future market conditions. The assumption underlying the yield in Col. (1) is that market conditions remain unchanged over the 30-year period. (This means that the contract rate stays at 8½ per cent and the discount at the level specified.) Underlying each of the six yields shown in Col. (1) is a unique set of termination rates (shown in Appendix Table 1). As the discount rises, the set of termination rates becomes more conservative—i.e., early-year termination rates are lower.

The yield shown in Col. (2) assumes that market yields fall by .5 per cent in each of the first 3 years and then level off, with the fall accounted for entirely by a decline in market discounts from the assumed initial level. (The calculated yield, as noted earlier, is not very sensitive to changes in the composition of market yield.) Similarly, the yield in Col. (3) assumes that discounts increase by amounts required to raise market yields by .5 per cent in each of the first 3 years.

It may be noted that assumed changes in future market yields have a much greater effect on the internal yield when they occur in the early years of a cohort's life. Thus, the pattern of market yields shown in Table 6, wherein market yields rise by 0.5 per cent in each of the first 3 years, generates a lower internal yield than a pattern wherein market yields increase by 0.1 per cent every year for 30 years, even though the average increase in the market yield over the entire 30 years would be much higher in the second case.

For comparative purposes we also show yields calculated from the average 1951–65 termination rates, which of course reflect the actual pattern of market yield change (mainly upward) during that period. Up to 8 discount points, the yields based on the assumption of stable market yields are slightly higher than the yields based on the average 1951–65 distribution, reflecting the depressive effect on termination rates of the upward trend in interest rates during 1951–65. At higher discounts, however, the yields based on the average 1951–65 distribution are higher because of the failure of that distribution to incorporate the depressive effect of higher discounts on termination rates.

Obviously, there is no one "correct" set of termination rates. The power of the regression procedure is that it can be used to explore the implications for internal yield of any assumed pattern of future yield change one wishes to make. In comparing the yields obtained with our procedure with the yield calculated by others using different procedures, we shall use the four sets of termination rates that underlie the yields shown in Table 6. However, we view the rates drawn from the regression on the assumption of stable market yields as the most appropriate for general use.

III. EXISTING METHODS OF CALCULATING MORTGAGE YIELDS

The Prevailing Method

Official agencies and investors usually obtain their estimate of yield from the *Prepayment Mortgage Yield Tables for Monthly Payment Mortgages,* which shows the yield at varying contract rates, maturities, discounts, and lives. A sample page is shown in Figure 5.[19]

Although the yield book allows the investor to choose from a range of prepayments, the general practice is to base the prepayment assumption on the estimated average life. This practice is followed by the FHA, the Board of Governors of the Federal Reserve System, and all private investors to whom the question was put. For convenience we refer to the yield obtained from the *Prepayment Mortgage Yield for Monthly Payment Mortgages* as the "yield book yield," using an estimate of average life as the prepayment assumption.

There are two sources of error involved in the yield book yield. The first, which was noted earlier, involves the use of the mean of the distribution of termination probabilities rather than the entire distribution. The magnitude of the error depends on the number of discount points, and on the particular set of termination probabilities employed.[20] These points are illustrated in Table 7.

The error involved in using average life is invariably to understate the yield, because yield is a sharply declining (negative) function of mortgage life, as illustrated in Figure 1. Hence, if the distribution of termination probabilities was symmetric, the use of average life would bias the yield downward (see the arithmetical example on page 121). In addition, distributions of termination probabilities tend to be skewed to the right, for reasons pointed out earlier. Right skewness is shown by the average distributions calculated for the periods 1951–65 and 1935–49, by distributions covering individual year cohorts within the 1951–65 period, by all the distributions calculated from our regression equations, and by distributions covering individual states (1958 and 1959 cohorts only).[21] We have not seen any distributions that showed left skewness.

The figures shown in Table 7 do not tell us anything about actual errors in yield estimates of official agencies or investors. These figures assume that the average life used to calculate yield is drawn from the same distribution of termination probabilities as is used to calculate the true internal yield. In practice, use of an inappropriate methodology may be accompanied by choice of an inappropriate distribution of terminations, or by an erroneous estimate of the average life of a given distribution. We return to this problem later.

FIGURE 5 Sample Mortgage Yield Table

8½% Mortgage Rate — **28** Years

Price	Prepaid In 5 Years	8 Years	10 Years	12 Years	15 Years	18 Years	20 Years	To Maturity
80	14.24	12.63	12.13	11.81	11.52	11.36	11.29	11.20
½	14.07	12.51	12.02	11.71	11.43	11.27	11.21	11.12
81	13.91	12.40	11.92	11.61	11.34	11.19	11.12	11.04
½	13.75	12.28	11.81	11.52	11.25	11.10	11.04	10.96
82	13.59	12.16	11.71	11.43	11.17	11.02	10.96	10.88
½	13.43	12.05	11.61	11.33	11.08	10.94	10.88	10.80
83	13.27	11.93	11.51	11.24	11.00	10.86	10.80	10.73
½	13.12	11.82	11.41	11.15	10.91	10.78	10.72	10.65
84	12.96	11.71	11.31	11.06	10.83	10.70	10.65	10.58
½	12.81	11.59	11.21	10.97	10.75	10.62	10.57	10.50
85	12.66	11.48	11.11	10.88	10.67	10.54	10.49	10.43
½	12.50	11.37	11.02	10.79	10.58	10.47	10.42	10.35
86	12.35	11.26	10.92	10.70	10.50	10.39	10.34	10.28
½	12.20	11.16	10.82	10.62	10.42	10.32	10.27	10.21
87	12.05	11.05	10.73	10.53	10.34	10.24	10.20	10.14
½	11.91	10.94	10.64	10.44	10.27	10.17	10.12	10.07
88	11.76	10.84	10.54	10.36	10.19	10.09	10.05	10.00
½	11.61	10.73	10.45	10.27	10.11	10.02	9.98	9.93
89	11.47	10.63	10.36	10.19	10.04	9.95	9.91	9.86
½	11.32	10.52	10.27	10.11	9.96	9.88	9.84	9.79
90	11.18	10.42	10.18	10.02	9.88	9.80	9.77	9.73
½	11.04	10.32	10.09	9.94	9.81	9.73	9.70	9.66
91	10.90	10.22	10.00	9.86	9.74	9.66	9.63	9.59
½	10.76	10.11	9.91	9.78	9.66	9.59	9.57	9.53
92	10.62	10.01	9.82	9.70	9.59	9.53	9.50	9.46
½	10.48	9.91	9.74	9.62	9.52	9.46	9.43	9.40
93	10.34	9.82	9.65	9.54	9.45	9.39	9.37	9.33
½	10.20	9.72	9.56	9.47	9.38	9.32	9.30	9.27
94	10.07	9.62	9.48	9.39	9.30	9.26	9.24	9.21
½	9.93	9.52	9.39	9.31	9.24	9.19	9.17	9.15
95	9.80	9.43	9.31	9.23	9.17	9.13	9.11	9.09
½	9.67	9.33	9.23	9.16	9.10	9.06	9.05	9.03
96	9.53	9.24	9.14	9.08	9.03	9.00	8.98	8.96
½	9.40	9.14	9.06	9.01	8.96	8.93	8.92	8.91
97	9.27	9.05	8.98	8.93	8.89	8.87	8.86	8.85
½	9.14	8.96	8.90	8.86	8.83	8.81	8.80	8.79
98	9.01	8.86	8.82	8.79	8.76	8.74	8.74	8.73
½	8.88	8.77	8.74	8.72	8.69	8.68	8.68	8.67
99	8.75	8.68	8.66	8.64	8.63	8.62	8.62	8.61
½	8.63	8.59	8.58	8.57	8.56	8.56	8.56	8.56
100	8.50	8.50	8.50	8.50	8.50	8.50	8.50	8.50
½	8.37	8.41	8.42	8.43	8.44	8.44	8.44	8.44
101	8.25	8.32	8.34	8.36	8.37	8.38	8.38	8.39
½	8.13	8.23	8.27	8.29	8.31	8.32	8.33	8.33
102	8.00	8.15	8.19	8.22	8.25	8.26	8.27	8.28
½	7.88	8.06	8.12	8.15	8.19	8.21	8.21	8.23
103	7.76	7.97	8.04	8.08	8.12	8.15	8.16	8.17
½	7.64	7.89	7.96	8.02	8.06	8.09	8.10	8.12
104	7.52	7.80	7.89	7.95	8.00	8.03	8.05	8.07
½	7.40	7.71	7.82	7.88	7.94	7.98	7.99	8.01
105	7.28	7.63	7.74	7.81	7.88	7.92	7.94	7.96
½	7.16	7.55	7.67	7.75	7.82	7.86	7.88	7.91
106	7.04	7.46	7.60	7.68	7.76	7.81	7.83	7.86
½	6.92	7.38	7.52	7.62	7.70	7.75	7.78	7.81
107	6.81	7.30	7.45	7.55	7.65	7.70	7.72	7.76

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8½% Mortgage Rate — **29** Years

Price	Prepaid In 5 Years	8 Years	10 Years	12 Years	15 Years	18 Years	20 Years	To Maturity
80	14.23	12.62	12.11	11.79	11.49	11.33	11.26	11.16
½	14.06	12.50	12.00	11.69	11.41	11.24	11.18	11.08
81	13.90	12.38	11.90	11.60	11.32	11.16	11.10	11.00
½	13.74	12.26	11.80	11.50	11.23	11.08	11.01	10.92
82	13.58	12.15	11.69	11.41	11.15	11.00	10.94	10.84
½	13.42	12.03	11.59	11.32	11.06	10.92	10.86	10.77
83	13.26	11.92	11.49	11.22	10.98	10.84	10.78	10.69
½	13.11	11.81	11.39	11.13	10.89	10.76	10.70	10.62
84	12.95	11.69	11.30	11.04	10.81	10.68	10.62	10.54
½	12.80	11.58	11.20	10.95	10.73	10.60	10.55	10.47
85	12.65	11.47	11.10	10.86	10.65	10.52	10.47	10.39
½	12.49	11.36	11.00	10.78	10.57	10.45	10.40	10.32
86	12.34	11.25	10.91	10.69	10.49	10.37	10.32	10.25
½	12.19	11.15	10.81	10.60	10.41	10.30	10.25	10.18
87	12.05	11.04	10.72	10.52	10.33	10.22	10.18	10.11
½	11.90	10.93	10.63	10.43	10.25	10.15	10.11	10.04
88	11.75	10.83	10.53	10.35	10.18	10.08	10.04	9.97
½	11.61	10.72	10.44	10.26	10.10	10.00	9.96	9.90
89	11.46	10.62	10.35	10.18	10.02	9.93	9.89	9.84
½	11.32	10.51	10.26	10.10	9.95	9.86	9.83	9.77
90	11.17	10.41	10.17	10.02	9.87	9.79	9.76	9.70
½	11.03	10.31	10.08	9.93	9.80	9.72	9.69	9.64
91	10.89	10.21	9.99	9.85	9.73	9.65	9.62	9.57
½	10.75	10.11	9.90	9.77	9.65	9.58	9.55	9.51
92	10.61	10.01	9.82	9.69	9.58	9.52	9.49	9.45
½	10.47	9.91	9.73	9.62	9.51	9.45	9.42	9.38
93	10.34	9.81	9.64	9.54	9.44	9.38	9.36	9.32
½	10.20	9.71	9.56	9.46	9.37	9.32	9.29	9.26
94	10.06	9.62	9.47	9.38	9.30	9.25	9.23	9.20
½	9.93	9.52	9.39	9.31	9.23	9.18	9.17	9.14
95	9.80	9.42	9.31	9.23	9.16	9.12	9.10	9.08
½	9.66	9.33	9.22	9.16	9.09	9.06	9.04	9.02
96	9.53	9.23	9.14	9.08	9.02	8.99	8.98	8.96
½	9.40	9.14	9.06	9.01	8.96	8.93	8.92	8.90
97	9.27	9.05	8.98	8.93	8.89	8.87	8.86	8.84
½	9.14	8.95	8.90	8.86	8.82	8.80	8.80	8.78
98	9.01	8.86	8.82	8.79	8.76	8.74	8.74	8.72
½	8.88	8.77	8.74	8.71	8.69	8.68	8.68	8.67
99	8.75	8.68	8.66	8.64	8.63	8.62	8.62	8.61
½	8.63	8.59	8.58	8.57	8.56	8.56	8.56	8.56
100	8.50	8.50	8.50	8.50	8.50	8.50	8.50	8.50
½	8.37	8.41	8.42	8.43	8.44	8.44	8.44	8.45
101	8.25	8.32	8.35	8.36	8.37	8.38	8.39	8.39
½	8.13	8.23	8.27	8.29	8.31	8.32	8.33	8.34
102	8.00	8.15	8.19	8.22	8.25	8.27	8.27	8.28
½	7.88	8.06	8.12	8.15	8.19	8.21	8.22	8.23
103	7.76	7.97	8.04	8.09	8.13	8.15	8.16	8.18
½	7.64	7.89	7.97	8.02	8.07	8.09	8.11	8.12
104	7.52	7.80	7.89	7.95	8.01	8.04	8.05	8.07
½	7.40	7.72	7.82	7.88	7.95	7.98	8.00	8.02
105	7.28	7.63	7.75	7.82	7.89	7.93	7.94	7.97
½	7.16	7.55	7.67	7.75	7.83	7.87	7.89	7.92
106	7.04	7.47	7.60	7.69	7.77	7.82	7.84	7.87
½	6.93	7.38	7.53	7.62	7.71	7.76	7.78	7.82
107	6.81	7.30	7.46	7.56	7.65	7.71	7.73	7.77

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SOURCE: *Prepayment Mortgage Yield Tables for Monthly Payment Mortgages,* the Financial Publishing Company, Boston.

The second source of error is that yields calculated by investors and official agencies are "nominal"; that is, they are based on annual rather than monthly mortgage payments. *The Prepayment Mortgage Yield Table for Monthly Payment Mortgages* shows nominal yield. (It shows, for example, that a 6 per cent contract rate mortgage acquired at par yields 6 per cent, whereas, in fact, the "effective yield" calculated to take account of monthly compounding is 6.17 per cent.) This source of error also biases in one direction, to understate yield.

TABLE 7 Error In Calculating Internal Yield on 8½ Per Cent, 30-Year Mortgages, Using Average Life as the Prepayment Assumption

	Distribution of Termination Probabilities Calculated from 1951–67 Regression			
Discount Points	Stable Rates (1)	Falling Rates (2)	Rising Rates (3)	1951–65 Average (4)
2	.02	.01	.03	.03
4	.05	.03	.05	.06
6	.08	.06	.08	.10
8	.11	.09	.11	.13
10	.14	.11	.14	.18
12	.17	.15	.18	.22

NOTE: Data drawn from Appendix Table 2, Col. (2) less Col. (1).

The difference between nominal yield and effective yield increases sharply as yield increases, as Table 8 shows. The marked rise in mortgage yield levels between 1965 and 1970, from less than 6 per cent to more than 9 per cent, roughly tripled the margin between nominal and effective yields.[22]

TABLE 8 Percentage Difference between Nominal and Effective Yield on Monthly Payment Mortgage

Nominal	Effective	Effective Less Nominal
5.00	5.12	.12
5.50	5.64	.14
6.00	6.17	.17
6.50	6.70	.20
7.00	7.23	.23
7.50	7.76	.26
8.00	8.30	.30
8.50	8.84	.34
9.00	9.38	.38
9.50	9.92	.42
10.00	10.47	.47

Thus, the yield book yield has a downward bias from two sources. We turn next to the question of how large the bias may be in the official yield estimates reported by the Board of Governors of the Federal Reserve System and the Federal Housing Administration.

Bias in Official Yield Estimates

The Board of Governors and the FHA report the yield on FHA 30-year mortgages as the yield book yield, assuming prepayment in 15 years. The 15-year assumption is based on aggregate termination statistics reported by the FHA and VA that suggest that average life tends to be about one-half of face maturity (see Table 9).[23]

The error in the officially reported yield depends on the particular distribution of termination probabilities that is viewed as correct or appropriate, and on the prevailing level of discounts. Table 10 shows the error on 8½ per cent, 30-year mortgages at varying discounts, using the same four distributions described earlier. The error is partitioned between the part due to absence of monthly compounding in the calculation of the yield book yield (the "compounding component"), and the part due to the use of average life as the prepayment assumption rather than the entire distribution of lives (the "average life component"). The latter is much more sensitive to differences in termination rates than is the former.

TABLE 9 Estimated Life Expectancy of FHA and VA Mortgages, by Maturity (years)

	FHA—Maturity				
Period Covered	Less than 13 Years	13 through 17 Years	18 through 22 Years	23 through 25 Years	26 through 30 Years
1935–65	5.9	7.6	9.7	11.9	13.5
1935–68	5.9	7.7	9.7	12.2	14.8

	VA—Maturity			
	Less than 15 Years	20 Years	25 Years	30 Years
1944–60	8.3	10.9	13.8	16.6

Note: FHA experience on Section 203 mortgages only; VA experience on primary home loans only.
Source: 1967 and 1970 *Statistical Yearbook of the Department of Housing and Urban Development*; *Probable Life Expectancy of GI Home Mortgages* (mimeograph), Veterans Administration.

TABLE 10 Yield Differences on 8½ per cent, 30-Year Mortgages: Yield Book Yield Assuming Prepayment in 15 Years Subtracted from True Effective Yield Calculated from Various Distributions of Termination Probabilities (basis points)

Distribution of Termination Probabilities and Components of "Error"	Discount Points 2	4	6	8	10	12
1951–67 Regression (Stable Yields)						
Total Error	42	49	55	60	64	68
Compounding	37	39	41	44	47	51
Half Life	5	10	14	16	17	17
1951–67 Regression (Rising Yields)						
Total Error	38	42	46	49	52	54
Compounding	36	38	41	43	47	49
Half Life	2	4	5	6	5	5
1951–67 Regression (Falling Yields)						
Total Error	48	62	71	80	87	92
Compounding	37	41	43	46	50	53
Half Life	11	21	28	34	37	39
1951–65 Average						
Total Error	39	46	53	60	67	75
Compounding	36	37	42	45	47	51
Half Life	3	9	11	15	20	24

The error using the stable rates distribution is sizable, ranging from 42 basis points at 2 discount points to 68 basis points at 12 discount points. The error using the rising yields distribution, which can be viewed as "conservative," is not much smaller at small discounts but is considerably less at large discounts, because the average life component carries heavy weight when discounts are large. Even so, the range of error using the rising yields distribution is an impressive 38–54 basis points.

The error in the official yield estimate rises with the contract rate, reflecting in the main increases in the compounding component. The average life component of the error is little affected by changes in contract rate. Extension of the maturity increases the error, mainly because of a larger average life component. These points are illustrated in Table 11.

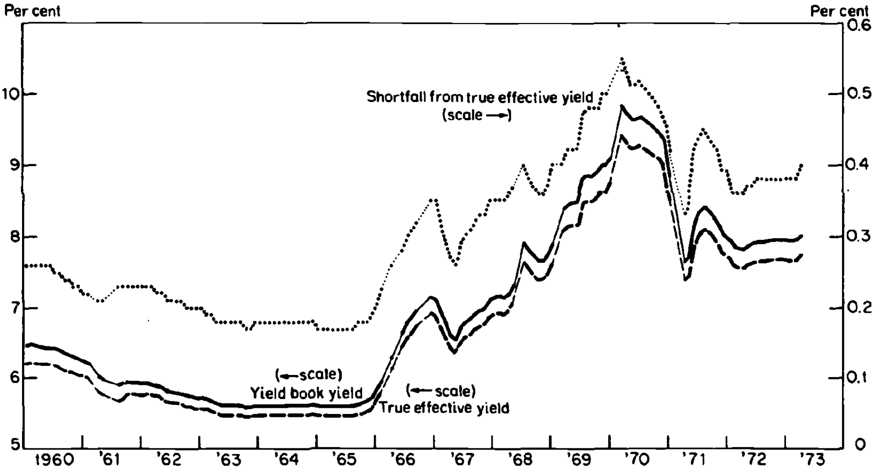
Time Series on Mortgage Yields Recalculated

It is useful to examine the impact on mortgage yield series of our new method of calculation. For this purpose we have taken monthly data on secondary market prices of FHA mortgages, which are reported with some gaps by the FHA,[24] and have calculated the true effective yield assuming stable rates and the yield book yield assuming prepayment at half maturity.[25] Figure 6 shows the yield book yield and the shortfall from true internal yield over the period 1960–72. The shortfall was generally less than 25 basis points during 1960–65 and generally more than 35 basis points during 1966–72, with pronounced cyclical swings. For example, between May 1967 and January 1970 the shortfall rose from 25 to 51 basis points, or by 26 basis points.

TABLE 11 Differences Between True Effective Yield Calculated on the Assumption of Stable Yields and Yield Book Yield Assuming Prepayment at Half Maturity (basis points)

Contract Rate and Maturity	Discount Points 2	4	6	8	10	12
6½%, 30 Years						
Total Error	27	33	38	41	44	46
Compounding	22	23	25	27	29	31
Half Life	5	10	13	14	15	15
8½%, 30 Years						
Total Error	42	49	55	60	64	68
Compounding	37	39	41	44	47	51
Half Life	5	10	14	16	17	17
10½%, 30 Years						
Total Error	61	70	77	83	88	93
Compounding	55	59	63	66	70	74
Half Life	6	11	14	17	18	19
8½%, 20 Years						
Total Error	41	47	52	55	58	60
Compounding	37	40	44	47	51	54
Half Life	4	7	8	8	7	6

FIGURE 6 Time Series on Mortgage Yields: Yield Book Yield Assuming Prepayment at Half Maturity and True Effective Yield Assuming the Stable Yields Distribution of Termination Rates



Cyclical changes in mortgage yield levels are highly correlated with swings in the supply of residential mortgage credit and the level of residential building. Supply-induced cyclical swings in residential construction levels have become increasingly severe over the last decade and are a source of major policy concern. A question arises whether the miscalculation of yields on mortgages could be implicated in these swings. The error in reported yields is highest at the very time that investors are shifting out of residential mortgages. This raises a question regarding the behavior of private investors in residential mortgages and whether or not they may make the same mistake as the official agencies.

Behavior of Private Investors

Whether individual investors tend to underestimate mortgage yield depends on the assumptions they make with respect to average life. Our inquiries among private investors indicate that the prepayment assumption that is most widely used (for 30-year FHA mortgages) is 12 years. This assumption has been given official sanction by the Government National Mortgage Association (GNMA), which has authorized its use in connection with GNMA-guaranteed mortgage bonds issued against FHA and VA mortgages. The 12-year assumption generates a somewhat smaller shortfall than the one shown in Figure 6 (wherein yield book yield is calculated on the assumption of prepayment at half life). The relative magnitudes involved can be illustrated by comparing the shortfall at cyclical yield peaks and troughs, when yield book yield is calculated on the assumption of prepayment at 15 years and 12 years, respectively. These are shown in the following table.

Shortfall between True Effective Yield Assuming Stable Market Yields and Yield Book Yield Assuming Prepayment in 15 and 12 Years (basis points)

Cyclical Peak (*P*) or Trough (*T*)	Yield Book Yield Calculated at 15 Years	12 Years
T Aug. 1965	17	14
P Dec. 1966	35	25
T May 1967	25	22
P Jan. 1970	51	38
T Apr. 1971	33	29
P Aug. 1971	45	33

NOTE: The August 1965 observation applies to a 25-year mortgage, with assumed prepayment at 12.5 years and 10 years.

The assumption of prepayment in 12 years thus generates a somewhat smaller downward bias than the 15-year assumption employed by the official agencies, and the same cyclical pattern in the shortfall is evident.

The Equalizing Prepayment

Earlier we stated that there is always some assumed average life, shorter than the actual average of the relevant distribution, that provides a yield book yield equal to the true internal yield. We have termed this the "equalizing prepayment," or EPP. Why wouldn't investors discover the EPP over time through an error-learning process? We might expect that as realized yields consistently exceeded expected yields, the assumed average life would be scaled down and investors would eventually estimate the correct yield, using the incorrect methodology.

To some degree this may indeed have happened. The prevailing prepayment assumption in 1972 was probably less conservative than in the 1950s. For the most part, however, whatever adjustment occurred probably reflects reevaluation of termination experience rather then yield experience. Realized yields are typically calculated on a portfolio basis, wherein total interest income received during the period, less expenses and losses, is taken as a percentage of the average portfolio during that period. In such calculations, discount income is amortized on a straight line basis over some arbitrary period (e.g., 10 years). Investors could calculate yields in this fashion for an indefinite period without ever realizing that the ex ante calculation for some mortgages in their portfolios was substantially biased.

What, then, is the correct EPP? The answer, of course, depends on which set of termination rates are considered "right," but even if one uses the conservative rising yields distribution it is clear from Table 12 that the 12-year assumption is too high. The table also reveals that, unfortunately, no single assumption will do. The EPP varies markedly with the level of discounts and is also significantly affected by the contract rate level. The smaller the discount and the higher the contract rate, the shorter the EPP.[26]

Hence, there is no simple rule of thumb that enables an investor to obtain the right answer using the wrong method. The practical remedy is to compile a new yield book showing true effective yield for mortgages carrying different contract rates, maturities, and discounts. In lieu of the various prepayment assumptions shown in the existing yield book, the new book could show true effective yield on various assumptions regarding future interest rates.

IV. THE YIELD ON FORECLOSED FHA MORTGAGES

The true internal yield described in this paper can be viewed as an "expected yield." When a mortgage goes into foreclosure, expectations are not borne out and the realized yield differs from the expected yield.[27] In this section we consider the yield on foreclosed FHA mortgages.

TABLE 12 Equalizing Prepayments at Varying Contract Rates, Maturities, and Discounts (months)

Contract Rate and Maturity	Discount Points 2	4	6	8	10	12
	True Effective Yield at Constant Rates					
6½%, 30 Years	58	79	90	99	106	112
8½%, 30 Years	42	61	73	82	89	95
10½%, 30 Years	31	48	59	68	74	80
8½%, 20 Years	38	53	63	70	74	79
	True Effective Yield at Rising Rates					
6½%, 30 Years	65	89	103	113	120	127
8½%, 30 Years	45	67	81	91	98	104
10½%, 30 Years	33	51	64	73	80	86
8½%, 20 Years	40	57	68	75	80	85

Calculating the Yield

Calculating yield on foreclosed loans simply requires modifying equation 4 to take account of the net reduction in cash flow associated with foreclosure. When an FHA loan is foreclosed the lender is reimbursed for the unpaid principal, but usually not for 6 months to a year. (The period between receipt of the last payment from the borrower and the settlement date when the final payment is received from the FHA we call the "settlement period.") Interest is paid beginning 2 months after the last payment received from the borrower, at the FHA debenture rate that is below the contract rate on the mortgage. In addition, one-third of the legal expenses of the lender are not reimbursed. Other expenses are reimbursed, but not until the settlement date, with interest paid at the debenture rate.

Because of these costs the yield on a foreclosed loan is always less than the yield on a loan prepaid in full on the settlement date. The difference between these two yields can be termed the "foreclosure cost" expressed in terms of basis points of yield. In Figure 7 a foreclosed loan with a life of (n) years has a yield of (f) and foreclosure cost of (af). The higher the legal expenses, the greater the differential between the contract rate and the debenture rate, the longer the settlement period, and the shorter the total life or "duration" of the loan (period from acquisition to settlement), the higher the foreclosure cost.

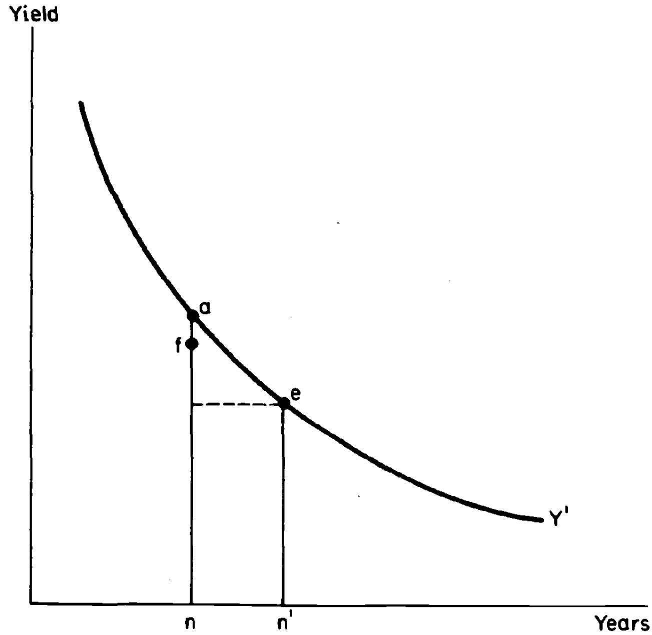
If the foreclosed loan is acquired at a discount, however, its yield is not necessarily less than the expected yield. The foreclosed loan typically has a short life, so that the investor can "ride" the yield curve leftward. In Figure 7 the foreclosure yield (f) is higher than the expected yield (e), because the equalizing prepayment (n') implied by the expected yield substantially exceeds the life of the foreclosed loan (n).

Thus the yield on a foreclosed loan can be higher or lower than the expected yield. Foreclosure costs tend to reduce the relative yield on foreclosed loans but short life and high discounts tend to raise it. We term the difference between the yield on a foreclosed loan and the expected yield the "net foreclosure gain or loss" (NFGL).

The NFGL on a Sample of Foreclosed Loans

A large lending institution provided us with complete data on 106 foreclosed FHA mortgages on which claims were settled during the period November 1967 through January 1968. In most cases foreclosure proceedings were initiated in 1967. The actual yield on each foreclosed loan was calculated taking account of the various foreclosure costs noted above, and actual yield was compared to the expected yield on the same mortgage.[28] The distribution of foreclosure gain or loss is shown in Table 13.

FIGURE 7 Yield on a Foreclosed FHA Loan Compared to Expected Yield



NOTE: Y^1 assumes a specified contract rate, discount, and Maturity.

The NFGL ranged from a gain of 396 basis points to a loss of 101 points, with about one-third of the sample showing gains and two-thirds losses. Although this is suggestive of wide diversity, the results are heavily influenced by the size of discounts at which the mortgages in the sample were acquired; discounts change with market conditions. In order to generalize about foreclosure gain or loss, we must quantify the influence of each of the factors that affect it; to this end, a multiple regression model was specified and tested.

Determinants of NFGL

The factors examined are the following (simple coefficients of correlation with NFGL are shown in parentheses):

X_2–Discount points (.59)
X_3–Settlement period (−.39)
X_4–Legal expense (−.38)
X_5–Duration (−.19)
X_6–Mortgage contract rate, net of service costs (.11)
X_7–Contract rate-debenture rate differential (−.04)
X_8–Maturity (−.02)

TABLE 13 Net Foreclosure Gain or Loss for Mortgages in Sample (in basis points)

Net Foreclosure Gain (or Loss)	Number
Over 00	3
51–100	3
41–50	1
31–40	2
21–30	1
11–20	6
0–10	13
(1)–(10)	25
(11)–(20)	16
(21)–(30)	9
(31)–(40)	8
(41)–(50)	5
(51)–(100)	13
Over 100	1
	106

The most troublesome of these variables is duration, the period from acquisition to settlement, since it is highly variable and its relationship to NFGL is not linear.[29] As an aid in properly specifying this relationship, we simulated NFGL for a batch of hypothetical mortgages of varying duration, discount points and total expenses.[30] The results are shown in Table 14.

As we would expect, NFGL is linearly related to expense. A given absolute increase in expense reduces NFGL by a constant number of basis points when duration is held constant. (Variation in discount points, for any given duration, did not appreciably change these yield decrements.) Since expense is not correlated with duration, it can enter the regression in linear form.

The non-linear relationship between NFGL and duration is clearly evident from the simulation. At a 4-point discount, NFGL falls by 38, 16 and 9 basis points, respectively, as duration increases from 2 to 4, 4 to 6, and 6 to 8 years. Sensitivity of NFGL to variations in discounts is of course markedly affected by the duration.

To obtain a correct specification of the duration, we first deducted foreclosure cost from NFGL to obtain the figures in the last three columns of Table 14. These figures are independent of variation in expense and roughly linear with respect to the discount in each duration classification. We then divided each value by the discount to obtain a series independent of both expenses and discounts. This series approximates a geometric progression that solves to a value of $52p(1/2)^{(d-1)/2}$, where (d) is the

TABLE 14 Simulated Gain (or Loss) on Hypothetical Foreclosed Mortgages (in basis points)

Duration, Period from Acquisition to Settlement, in Years	Discount Points	Foreclosure Cost Total Expense			Net Foreclosure Gain (or Loss) Total Expense			Difference Total Expense		
		$200	$600	$1,000	$200	$600	$1,000	$200	$600	$1,000
2	.01	(46)	(69)	(91)	(20)	(43)	(65)	26	26	26
	.02	(46)	(69)	(90)	6	(16)	(39)	52	53	51
	.04	(45)	(68)	(90)	61	38	16	106	106	106
	.06	(44)	(67)	(90)	117	94	71	161	161	161
	.12	(42)	(65)	(89)	295	271	247	337	336	336
4	.01	(26)	(39)	(51)	(14)	(27)	(40)	12	12	11
	.02	(26)	(39)	(51)	(2)	(15)	(28)	24	24	23
	.04	(25)	(38)	(51)	23	10	(3)	48	48	48
	.06	(25)	(38)	(51)	48	35	22	73	73	73
	.12	(24)	(37)	(51)	126	113	100	150	150	151
6	.01	(17)	(26)	(35)	(12)	(20)	(29)	5	6	6
	.02	(17)	(26)	(35)	(6)	(14)	(23)	11	12	12
	.04	(17)	(26)	(34)	7	(2)	(11)	24	24	23
	.06	(17)	(26)	(34)	19	10	2	36	36	36
	.12	(17)	(25)	(34)	57	48	39	74	73	73
8	.01	(13)	(19)	(25)	(10)	(17)	(23)	3	2	2
	.02	(13)	(19)	(25)	(8)	(14)	(20)	5	5	5
	.04	(12)	(19)	(25)	(2)	(8)	(15)	10	11	10
	.06	(12)	(19)	(25)	4	(3)	(9)	16	16	16
	.12	(12)	(18)	(25)	19	13	7	31	31	32

duration in years and (p) is the number of discount points. This transformation term (X_1) explained 92 percent of the variance in hypothetical NFGL when tested by means of linear regression; defining X_1 to be $p(1/2)^{(d-1)/2}$ generated parameter estimates strikingly close to the assumed value of 52.

This specification was then applied to the sample of 106 foreclosed mortgages. Because of the importance of the duration transformation term, it is important that the sample include a wide range of durations and a large number of small duration values, since the transformation term decreases at a decreasing rate as duration increases. The sample is adequate in both respects; duration values ranged from 9 to 147 months, with a median value of only 53 months.

The very high explanatory value of the transformation term in the regression on simulated data cannot be expected when the model is applied to real data. The simulated values were generated through manipulation of only three variables, whereas actual values of NFGL are subject to the other influences. In addition, the sample contains unavoidable measurement errors.[31]

The regression results are shown in Table 15. Despite the measurement error, and notwithstanding the effect of purely stochastic influences, the transformation term explained more then half of the variance in NFGL and did much better than the discount and duration entered separately (compare equation 1 and equation 2). The duration added more to the explanatory value of the regression than the discount when the transformation term was included; this is evident from a comparison of equations 5 and 6. (Inclusion of both transformation term and discount points results in serious multicollinearity.)

The legal expense, settlement period, and contract rate-debenture rate differential are all significant in runs that include both the transformation term and duration (see equations 4, 5, and 7). Parameter estimates suggest that the investor loses approximately 6 basis points of yield for every $100 of legal expense (assuming that two-thirds is reimbursed), about 2 basis points for every month required to complete settlement, and 38 basis points for every 100 basis points differential between the contract rate and the debenture rate. The maturity and contract rate we found to be generally insignificant and excluded them from the regressions shown in Table 15.

Equation 7 explains about 80 percent of the variance in NFGL, which is extremely high for cross-sectional data. Equations 8 and 9, in which legal expenses are measured as a percentage of original loan amount and as a percentage of balance at foreclosure, respectively, improve the fit slightly. Equation 8 is particularly useful for calculating the "equalizing discount," a statistic of material importance to mortgage lenders.

TABLE 15 Regression of Net Foreclosure Gain (or loss), Actual Sample (t values in parentheses)

Equation	Regression Constant	X_1 Trans. Term	X_2 Disc. Pts.	X_3 Sett. Period	X_4 Legal Exp.	X_5 Dur.	X_7 Rate Diff.	X_9 Legal Exp. %	R^2
1	− 51.07	47.83							.559
	(10.17)[a]	(11.58)[a]							
2	− 51.45		28.05			−.48			.407
	(4.39)[a]		(8.20)[a]			(3.37)[a]			
3	−122.32	68.57				.94			.704
	(11.44)[a]	(15.45)[a]				(7.22)[a]			
4	− 78.82	64.69			−9.34	.81			.777
	(6.64)[a]	(16.54)[a]			(5.88)[a]	(7.04)[a]			
5	− 66.31	64.35		−2.68	−5.30	.83			.787
	(5.24)[a]	(16.84)[a]		(2.46)[c]	(2.34)[c]	(7.34)[a]			
6	− 25.67	37.94	11.86	−1.53	−8.68				.727
	(2.22)[c]	(10.13)[a]	(4.41)[a]	(1.23)	(3.43)[a]				
7	− 53.48	67.12		−2.52	−5.71	1.12	−.38		.796
	(3.93)[a]	(17.08)[a]		(2.35)[c]	(2.57)[b]	(6.65)[a]	(2.30)[c]		
8	− 51.71	67.03		−2.79		1.14	−.38	−7.38	.798
	(3.80)[a]	(17.13)[a]		(2.89)[b]		(6.76)[a]	(2.34)[c]	(2.75)[b]	
9	− 56.73	67.51		−2.43		1.19	−.38	−6.90	.803
	(4.28)[a]	(17.52)[a]		(2.51)[c]		(7.14)[a]	(2.34)[c]	(3.25)[a]	

[a] Significant at .001 level; [b] Significant at .01 level; [c] Significant at .05 level

NOTE: X_3 and X_5 are in months, X_4 in units of $100, X_7 in decimals, and D in X_1 is in years. X_9 is legal expenses as a percentage of loan amount in equation 8, and as a percentage of balance at foreclosure in equation 9.

The Equalizing Discount

The equalizing discount is that discount at which the yield on a foreclosed mortgage is equal to the expected yield at the time the mortgage was acquired; i.e., NFGL is zero. We solve for P (number of points) in the following equation:

$$67.03P(1/2)^{(D-1)/2} - 2.79S - 738E + 1.14(12D) - .38R - 51.71 = 0$$

where

P = discount points
D = duration from acquisition to settlement, in years
S = default settlement period, in months
E = legal expense as a percentage of loan amount, in decimals
R = debenture-contract rate differential in basis points.

Table 16 shows values of the equalizing discount at various values for duration, legal expense, and loan size. The settlement period is assumed to be fixed at 9 months,[32] and the contract rate-debenture rate differential is fixed at 150 basis points.[33]

The table may be interpreted as follows. If a $20,000 loan acquired at a discount of 3.4 points is foreclosed and settled after 3 years, and if legal expense is $450, the realized yield will be equal to the expected yield. If the same loan had been acquired at a discount above 3.4 points, the lender would benefit if it went to foreclosure. The equalizing discount increases as legal expense and duration increase, and as loan size declines.

The range of legal expenses shown in the table is drawn from a study of foreclosure costs by state prepared by the American Bar Association.[34] The figures shown are, respectively, the lowest, first quartile, median, third quartile, and highest. They included some non-legal costs that are fully reimbursable by FHA, and are therefore probably on the high side. Durations are shown only up to 6 years because few foreclosed loans are more than 6 years old; indeed, Table 17 suggests that most are less than 4 years old.[35]

The "typical" foreclosed FHA mortgage would thus carry legal expenses of about $450 and be about 3 to 4 years old; although we are not sure of its amount, it probably would be between $15,000 and $20,000. The equalizing discount on this mortgage would be 3 to 4½ points. (On VA loans the equalizing discount probably would be smaller because foreclosure costs are generally lower.) Over the last decade the average discount on FHA and VA loans usually has been more than 4½ points —sometimes substantially more.

Because the equalizing discount is very sensitive to the age of foreclosed loans, a question arises regarding the stability of age patterns such as those

TABLE 16 Equalizing Discounts at Various Durations, Legal Expenses, and Loan Amounts (discount points)

Legal Expenses	Duration in Years 1	2	3	4	5	6
			Loan = $20,000			
$ 75	1.8	2.3	2.9	3.5	4.1	4.6
350	2.0	2.5	3.2	3.9	4.7	5.5
450	2.0	2.6	3.4	4.0	4.9	5.8
820	2.2	2.9	3.8	4.6	5.7	6.9
2,550	3.2	4.2	5.7	7.3	9.5	12.3
			Loan = $10,000			
$ 75	1.9	2.4	3.0	3.6	4.2	4.8
350	2.2	2.8	3.6	4.4	5.5	6.5
450	2.3	3.0	3.9	4.7	5.9	7.2
820	2.7	3.5	4.7	5.9	7.5	9.5
2,550	4.6	6.2	8.6	11.3	15.1	20.2
			Loan = $40,000			
$ 75	1.8	2.3	2.9	3.4	4.0	4.5
350	1.9	2.4	3.1	3.6	4.3	4.9
450	1.9	2.4	3.1	3.7	4.4	5.1
820	2.0	2.6	3.3	4.0	4.8	5.6
2,550	2.5	3.2	4.3	5.3	6.7	8.3

shown in Table 17. One possible cause of change in age patterns is shifts in the characteristics of mortgages, such as increases in loan-value ratios or extensions of maturities. Von Furstenberg found, however, that the age-of-mortgage pattern of foreclosure was not significantly affected by such changes.[36]

A second possible cause of changes in age patterns is exogenous developments that affect the entire structure of foreclosure rates, such as a substantial rise in unemployment and/or a decline in property values. The age of foreclosed loans probably is inversely correlated with changes in the foreclosure rate from such sources. When foreclosure rates are low, a large proportion of foreclosures are associated with household dissolution from various causes, which can occur anytime during the life of a loan. When foreclosure rates are high, economic forces are involved in most foreclosures and new loans are most vulnerable. Thus, if the overall foreclosure rate were to rise substantially in the future, the equalizing discount probably would tend to fall.

TABLE 17 Percentage Distributions of FHA Section 203 Home Properties Acquired under Foreclosure Proceedings between July 1, 1960 and March 31, 1962, by Year of Mortgage Insurance

	Percentage Distribution of Properties Acquired		
Year of Insurance	July 1, 1960–June 30, 1961	July 1, 1961–December 31, 1961	July 1, 1961–March 31, 1962
1961	.5	6.0	6.2
1960	14.8	28.5	27.3
1959	38.8	34.9	34.1
1958	26.6	19.4	20.4
1957	5.6	3.4	4.3
1956	3.4	1.9	1.8
1955	4.8	2.7	2.7
1954	1.8	1.0	1.1
1953	2.1	1.4	1.6
1952	.7	.3	.3
1951 or earlier	.9	.5	.2
Total	100.0	100.0	100.0

SOURCE: *FHA Mortgage Foreclosures,* Hearings before the Senate Committee on Banking and Currency, January 27 and 28, 1964, Washington, D.C., 1964.

V. SUMMARY AND CONCLUSIONS

1. In line with the standard concept of an "internal rate of return," mortgage yield is defined as that rate of discount which equates the present value of the stream of principal and interest payments received by the mortgage lender with the net amount disbursed. When mortgages carry discounts (the net amount disbursed is less than the face amount of the loan), the yield is a negative function of the "prepayment period"; i.e., the period between origination and the time when the remaining loan balance is paid in full. This function is not linear but declines rapidly in the early years and gradually flattens out. Since the prepayment period is not known at the time a loan is made, there is uncertainty in any estimate of expected yield. The larger the discount, the greater the effect on yield of variations in the prepayment period and the greater the range of uncertainty in the expected yield.

2. To properly calculate the yield on an individual mortgage, it is necessary to assume that the yield is identical to that on a portfolio of similar mortgages. If the distribution of termination probabilities for the portfolio is known and the correct method of calculation is employed, the

calculated yield on the individual loan will equal the average yield on the portfolio, although only by chance would this be the actual yield on the individual loan.

3. The cash flows assumed in the yield calculation are based on the distribution of termination probabilities applicable to the portfolio to which the specified loan belongs. Thus, the first period flow is that generated by a mortgage prepaid after one period weighted by the proportion of the mortgages in the portfolio prepaid after one period, and so on, for each subsequent period, with all flows assumed to be reinvested at the average portfolio yield. We term the yield obtained in this fashion the "true internal yield." If the contract rate, maturity, and payment are expressed as monthly values, it is the "effective true internal yield."

4. The yield calculation requires data on the complete distribution of termination probabilities for the portfolio of mortgages to which a new mortgage belongs. Two procedures were used in this study to estimate termination probabilities for 20-year, 25-year, and 30-year mortgages. One approach was to average annual termination rates during 1951–65 for the three maturity groups after the relationship between termination rates and maturity had been "normalized" by expressing policy year as a percentage of maturity. The disadvantage of this distribution is that it reflects all the exogenous factors influencing termination rates during the 1951–65 period, including, and most important, the upward trend in interest rates. Rising interest rates tend to depress early-year termination rates and therefore generate a *conservative* distribution (one resulting in relatively low yields). On the other hand, this approach implicitly assumes that termination rates are not affected by the level of discounts prevailing at the time the loan is made. This erroneous assumption generates an upward yield bias at large discounts relative to small discounts.

5. The second approach used to estimate the distribution of termination probabilities was to build a regression model in which termination rates depended on maturity, policy year, and the relationship between the contract rate on the mortgage and market yields in the specified policy year. This procedure allowed us to specify alternative market yield patterns over the life of a mortgage cohort. Three such patterns were specified, involving rising market yields, declining market yields, and stable yields, respectively. The distribution assuming stable market yields was viewed as the most appropriate for general use.

6. Official agencies and investors usually obtain their estimates of expected yield from the Financial Publishing Company's *Prepayment Mortgage Yield Tables for Monthly Payment Mortgages*, using an estimate of average life as the prepayment assumption; we term this the "yield book yield." There are two sources of downward bias in the yield book yield. The use of the mean of the distribution of termination probabilities rather

than the entire distribution biases the yield estimate because yield is a sharply declining negative function of mortgage life. (The yield on mortgages prepaid prior to average life is substantially higher than the yield at average life, whereas the yield on mortgages prepaid later than average life are only slightly lower.) In addition, the yields shown in *Prepayment Mortgage Yield Tables for Monthly Payment Mortgages* are nominal; that is, they assume that mortgage payments are received annually when in fact they are received monthly.

7. The extent of the downward bias in the yield book yield depends mainly on the size of discounts, on the general level of yields, on the average life assumed on determining the yield book yield, and on the distribution of termination rates that is considered appropriate in determining the true effective yield. For the period 1960–72, we calculated yield book yield on the assumption of prepayment after a period equal to half the face maturity, which is the procedure used by the Board of Governors of the Federal Reserve System and the Federal Housing Administration, and true effective yield using termination rates derived from our regression equations on the assumption of stable market yields. The monthly shortfall in yield book yield was generally less than 25 basis points during 1960–65, generally more than 35 basis points during 1966–72, and shows pronounced cyclical swings. For example, between the May 1967 cyclical trough in yields and the subsequent peak in January 1970 the shortfall rose from 25 to 51 basis points. The shortfall of yield book yield from true yield is highest when yields are high because the difference between nominal and effective yield rises with the level of yield, and because a larger proportion of the yield tends to be accounted for by discounts.

8. Most private investors have for some time used a 12-year prepayment assumption in calculating the yield book yield on 30-year mortgages. This generates a slightly smaller downward bias than the 15-year assumption, and the same cyclical pattern in the shortfall is evident. (Thus, on this basis the shortfall between May 1967 and January 1970 rose from 22 to 38 basis points.) This tendency for investors to underestimate the yield on federally underwritten mortgages carries some important implications. There is a presumption that investors are not allocating loanable funds efficiently if they are consistently underestimating the ex ante yield on an important capital market instrument. The point is most relevant to periods of monetary restraint and generally high interest rates, such as existed in 1966 and 1969, when lenders shift out of residential mortgages into bonds because "bond yields are more attractive." Reducing discounts by eliminating rate ceilings would reduce only one source of error in the reported yield.

9. The error in yield book yield could be eliminated entirely simply by compiling a new yield book to replace the *Prepayment Mortgage Yield Tables for Monthly Payment Mortgages*. Instead of showing nominal yield at varying prepayment assumptions, the book would show an estimate or estimates of true yield, based on complete distributions of termination probabilities and assuming monthly compounding. Several sets of yields could be shown, based on alternative termination patterns corresponding to varying assumptions regarding future market yields. Every 5 years or so, termination experience could be reviewed and the probabilities recomputed.

10. The yield on an FHA loan foreclosed after x years is always less than the yield on a loan prepaid in x years because foreclosure imposes some non-reimburseable costs on lenders. In general, however, foreclosed loans have a shorter life than non-foreclosed loans; and if there is a discount at acquisition, the yield on a foreclosed loan may exceed the expected yield. We term the difference between the expected yield on a mortgage and the yield realized in the event it goes into foreclosure the "net foreclosure gain or loss" (NFGL).

11. The NFGL can be explained largely by the number of discount points, the duration of the foreclosed loan from acquisition to settlement, the length of the period required to settle the foreclosure with FHA, the lender's legal expenses, and the difference between the contract rate on the mortgage and the debenture rate used by FHA to calculate interest on amounts owed to the lender. Using a regression equation relating NFGL to these factors, we have calculated the "equalizing discount"; i.e., the discount at which the yield on a foreclosed loan is equal to the expected yield. (This calculation involves setting NFGL at zero, assigning specified values to all variables except the discount, and solving for the discount.) The equalizing discount on the typical FHA loan is about 3 to 4½ points, and probably somewhat smaller on VA loans. Over the last decade the average discount on FHA and VA loans usually has been more than 4½ points—sometimes substantially more. This means that, on balance, lenders have had little real incentive to guard against foreclosure.

TABLE A-1 Termination Rates on 30-Year Mortgages Used to Calculate True Effective Yield

Year after Insurance	1951–65 Average
1	.0120
2	.0195
3	.0271
4	.0386
5	.0359
6	.0456
7	.0381
8	.0377
9	.0373
10	.0389
11	.0380
12	.0371
13	.0389
14	.0362
15	.0335
16	.0348
17	.0338
18	.0328
19	.0248
20	.0265
21	.0283
22	.0239
23	.0298
24	.0358
25	.0358
26	.0358
27	.0359
28	.0357
29	.0358
30	.0359
Total	1.000

TABLE A-1(a) Termination Rates on 30-Year Mortgages Used to Calculate True Effective Yield (assuming stable market yields)

Year after Insurance	Current Discount (Points) 2	4	6	8	10	12
1	.0115	.0104	.0094	.0085	.0077	.0070
2	.0213	.0193	.0175	.0158	.0143	.0130
3	.0300	.0273	.0247	.0224	.0203	.0184
4	.0377	.0343	.0312	.0284	.0258	.0234
5	.0443	.0405	.0370	.0337	.0307	.0280
6	.0496	.0456	.0418	.0383	.0350	.0320
7	.0537	.0496	.0458	.0421	.0387	.0355
8	.0566	.0526	.0488	.0452	.0417	.0384
9	.0582	.0545	.0509	.0474	.0440	.0408
10	.0586	.0554	.0522	.0489	.0457	.0426
11	.0580	.0554	.0526	.0497	.0467	.0438
12	.0565	.0545	.0522	.0498	.0472	.0445
13	.0541	.0528	.0511	.0492	.0470	.0447
14	.0511	.0505	.0495	.0480	.0463	.0444
15	.0476	.0477	.0473	.0464	.0452	.0437
16	.0438	.0445	.0447	.0444	.0437	.0426
17	.0397	.0410	.0417	.0420	.0418	.0412
18	.0356	.0374	.0386	.0393	.0396	.0394
19	.0315	.0337	.0354	.0365	.0372	.0375
20	.0276	.0300	.0320	.0336	.0347	.0354
21	.0239	.0265	.0288	.0306	.0321	.0331
22	.0204	.0231	.0256	.0277	.0294	.0308
23	.0173	.0200	.0226	.0248	.0268	.0284
24	.0144	.0171	.0197	.0221	.0242	.0260
25	.0120	.0145	.0170	.0195	.0217	.0237
26	.0098	.0122	.0146	.0170	.0193	.0214
27	.0079	.0101	.0124	.0148	.0171	.0192
28	.0063	.0083	.0105	.0127	.0150	.0171
29	.0050	.0068	.0087	.0109	.0130	.0152
30	.0158	.0243	.0357	.0502	.0680	.0891

TABLE A-1(b) Termination Rates on 30-Year Mortgages Used to Calculate True Effective Yield (assuming falling market yields)

Year after	Current Discount (Points)					
Insurance	2	4	6	8	10	12
1	.0140	.0126	.0114	.0102	.0092	.0083
2	.0319	.0286	.0256	.0229	.0205	.0184
3	.0555	.0511	.0443	.0396	.0352	.0314
4	.0677	.0627	.0548	.0492	.0439	.0394
5	.0766	.0714	.0630	.0570	.0512	.0462
6	.0820	.0770	.0689	.0628	.0569	.0516
7	.0840	.0797	.0723	.0667	.0610	.0558
8	.0829	.0796	.0735	.0686	.0634	.0585
9	.0792	.0771	.0727	.0687	.0643	.0600
10	.0735	.0726	.0700	.0672	.0639	.0603
11	.0663	.0667	.0659	.0644	.0621	.0594
12	.0582	.0597	.0607	.0604	.0593	.0576
13	.0499	.0522	.0548	.0557	.0557	.0549
14	.0417	.0447	.0484	.0504	.0514	.0516
15	.0340	.0374	.0420	.0448	.0468	.0478
16	.0271	.0306	.0358	.0392	.0419	.0437
17	.0211	.0245	.0299	.0337	.0370	.0394
18	.0160	.0192	.0246	.0285	.0322	.0351
19	.0119	.0148	.0198	.0238	.0276	.0308
20	.0086	.0111	.0157	.0195	.0234	.0268
21	.0061	.0082	.0122	.0158	.0195	.0230
22	.0042	.0059	.0094	.0126	.0161	.0195
23	.0029	.0042	.0070	.0098	.0131	.0164
24	.0019	.0029	.0052	.0076	.0105	.0135
25	.0012	.0020	.0038	.0058	.0083	.0111
26	.0008	.0013	.0027	.0043	.0065	.0090
27	.0005	.0008	.0019	.0032	.0050	.0072
28	.0003	.0005	.0013	.0023	.0038	.0057
29	.0002	.0003	.0009	.0017	.0029	.0045
30	.0002	.0005	.0016	.0036	.0073	.0134

TABLE A-1(c) Termination Rates on 30-Year Mortgages Used to Calculate True Effective Yield (assuming rising market yields)

Year after	Current Discount (Points)					
Insurance	2	4	6	8	10	12
1	.0095	.0087	.0079	.0072	.0066	.0059
2	.0148	.0136	.0124	.0113	.0104	.0095
3	.0177	.0164	.0151	.0140	.0128	.0118
4	.0226	.0209	.0193	.0178	.0164	.0151
5	.0270	.0250	.0231	.0214	.0197	.0182
6	.0309	.0287	.0266	.0247	.0227	.0210
7	.0343	.0319	.0297	.0276	.0255	.0236
8	.0372	.0347	.0324	.0302	.0279	.0260
9	.0395	.0370	.0347	.0324	.0301	.0280
10	.0413	.0389	.0365	.0343	.0319	.0298
11	.0426	.0403	.0380	.0358	.0335	.0314
12	.0434	.0412	.0391	.0369	.0347	.0326
13	.0437	.0417	.0397	.0377	.0356	.0336
14	.0435	.0418	.0400	.0382	.0362	.0343
15	.0429	.0415	.0399	.0383	.0365	.0347
16	.0419	.0408	.0395	.0381	.0365	.0349
17	.0407	.0398	.0388	.0377	.0363	.0349
18	.0391	.0386	.0379	.0370	.0359	.0347
19	.0373	.0371	.0367	.0361	.0352	.0342
20	.0353	.0354	.0353	.0349	.0343	.0335
21	.0332	.0336	.0337	.0336	.0333	.0327
22	.0309	.0316	.0320	.0322	.0321	.0318
23	.0287	.0296	.0302	.0306	.0308	.0307
24	.0264	.0275	.0284	.0290	.0294	.0295
25	.0241	.0254	.0265	.0273	.0279	.0282
26	.0219	.0233	.0245	.0255	.0263	.0268
27	.0198	.0213	.0226	.0238	.0247	.0254
28	.0177	.0193	.0208	.0220	.0232	.0240
29	.0158	.0175	.0190	.0203	.0216	.0225
30	.0964	.1171	.1396	.1641	.1922	.2205

NOTE: Some of the columns in tables A-1, A-1(a), A-1(b), and A-1(c) do not sum exactly to 1.0000 because of rounding errors.

TABLE A-2 True Internal Yield and Yield Book Yield on 8½ Per Cent, 30-Year Mortgages Using Various Distributions of Termination Probabilities

Discount Points	Yield Book Yield (1)	True Nominal Yield (2)	True Effective Yield (3)	Equalizing Prepayment (Mos.) (4)	Average Life (Mos.) (5)
			1951–65 Average		
2	8.76	8.79	9.15	44	175
4	9.03	9.09	9.48	64	175
6	9.30	9.40	9.82	75	175
8	9.59	9.72	10.17	82	175
10	9.88	10.06	10.53	87	175
12	10.18	10.40	10.91	91	175
			1951–67 Regression: Stable Rates		
2	8.79	8.81	9.18	42	137
4	9.07	9.12	9.51	61	145
6	9.35	9.43	9.84	73	153
8	9.62	9.73	10.17	82	162
10	9.89	10.03	10.50	89	171
12	10.16	10.33	10.84	95	180
			1951–67 Regression: Falling Rates		
2	8.86	8.87	9.24	38	97
4	9.20	9.23	9.64	53	102
6	9.51	9.57	10.00	64	111
8	9.82	9.91	10.37	70	118
10	10.12	10.23	10.73	76	126
12	10.40	10.55	11.08	82	135
			1951–67 Regression: Rising Rates		
2	8.75	8.78	9.14	45	183
4	9.01	9.06	9.44	67	192
6	9.26	9.34	9.75	81	201
8	9.52	9.63	10.06	91	210
10	9.77	9.91	10.38	98	220
12	10.03	10.21	10.70	104	231

NOTE: Yield book yield is calculated at the average life of each distribution. Yield book yields at 15 years life are as follows: 8.76, 9.02, 9.29, 9.57, 9.86, 10.16.

NOTES AND REFERENCES

1. See particularly Jack M. Guttentag and Morris Beck, *New Series on Home Mortgage Yields* (New York: NBER General Series 92, 1970). The study of interest rates was financed by a grant to the National Bureau by the Life Insurance Association of America.
2. Each discount point is 1 per cent of the face amount of the loan.
3. The great majority of home mortgages written in the United States are fully amortizing.
4. We assume that there are no partial prepayments during the life of the loan. Such payments are rare.
5. The yields referred to in this paper are gross of all other costs, including the fixed costs of maintaining a mortgage lending function and the costs of mortgage servicing. Yields can easily be measured net of servicing costs if the servicing fee is constant over the life of the loan simply by deducting the percentage servicing fee from the contract rate before calculating the yield. If the service fee changes over the life of the loan, it is more complicated to calculate precisely the net yield; but deducting of a weighted average servicing fee from the contract rate generates a net yield that is accurate enough for most purposes.
6. The necessary (but insufficient) condition for multiple rates, by Descartes' law of signs, is multiple reversal of the signs of periodic cash flows. A pattern of loan and loan repayment of (− ++ . . . ++) has but one change of sign and one positive real root. This is the pattern for mortgages that are not defaulted.
7. In the case of insured loans, unreimbursed default costs cannot possibly be large enough to produce multiple rates. Because the situation is less clear for uninsured loans, mortgage default was simulated by computer for defaults occurring at the end of the third, twelfth, and nineteenth year of a 6 per cent, 20-year mortgage. For combined default loss and default expense ranging from 2½ to 125 per cent of original principal, only one real root resulted. It can be reasonably concluded that the problem of multiple roots has no relevance to mortgage yields.
8. Fisher and Weil have developed a strategy in this respect for bondholders, with excellent empirical verification. See Lawrence Fisher and Roman L. Weil, "Coping With the Risk of Interest-Rate Fluctuations: Returns to Bondholders From Naive and Optimal Strategies," *Journal of Business*, October 1971, pp. 408–431.
9. Terminations in a given year are taken as a percentage of mortgages outstanding in the base year. As we shall see later, for purposes of explaining variability in termination rates it is desirable to express terminations as a percentage of mortgages outstanding in the current year. We refer to these measures as "fixed base" and "current base" termination rates, respectively.
10. The data in Table 1 differ from the tabulations of the same type published every year in the annual reports of the FHA. The first-year termination rate for 20-year mortgages in Table 1 is an unweighted average of the ten first-year termination rates for 1955–65 cohorts, whereas the first-year termination rate for 20-year mortgages in FHA statistics is the weighted average for all 20-year cohorts since the beginning of the program in 1935, the weights being the number of mortgages in the cohort (see footnote 12). Differences in termination rates among the maturity groups are much more likely to be affected by exogenous influences on termination rates in the weighted averages than in the unweighted averages.
11. Thus, if *A* moves his residence in year 1, and *B* moves in year 2, it is more likely that *A* than *B* will move again in year 3 because the chances are greater that the factors underlying *A*'s original move have changed.
12. The termination rates in these tabulations are weighted by the number of mortgages in the sample. This means that as the volume of FHA activity has grown, experience in more recent years weighs more heavily than experience in earlier years. A better

procedure (which we have followed) is to calculate annual termination rates for every cohort and then to average the cohort termination rates.

13. No observations are shown for 30-year mortgages during 1951–54 because loan volume during this period was too small to be meaningful.
14. This finding is not inconsistent with the progressive lengthening of average life that is observed in the aggregate FHA data described earlier. The general level of termination rates was lower during 1951–65 than earlier periods, but because of the weighting procedure employed in compiling the average life statistics, these lower termination rates have affected the average only gradually over a period of time.
15. Cumulative termination rates for mortgages in each maturity group are based on weighted averages of single-year termination rates.
16. The termination pattern for 1935–49 is based on experience for 20-year loans only, and is partly estimated.
17. For this purpose termination rates were converted to current-base rates. The termination rates computed from the model were then converted back to fixed-base rates for use in calculating yields. Data on 30-year mortgages for 1951–54 were omitted from the regression because of the small number of cases (see Table 3).
18. The rationale of this objective will be discussed below.
19. Courtesy of the Financial Publishing Company, Boston.
20. Errors are slightly smaller on shorter-maturity and lower-contract rate loans.
21. In addition, right skewness is a feature of mortgages that are terminated because of default. See George M. von Furstenberg, "Default Risk on FHA-Insured Home Mortgages as a Function of the Terms of Financing: A Quantitative Analysis," *The Journal of Finance*, June 1969, p. 465.
22. The difference between nominal and effective yield is independent of the composition of yield between contract rate and discount points. It is also independent of the distribution of termination probabilities employed to calculate yield except to the extent that the distribution affects the level of yield. Bond yields are also quoted on a nominal basis, but the difference between nominal and effective yield is smaller since bond interest is usually paid semi-annually and not monthly. Thus, a 9 per cent bond yields 9.20 per cent if interest is paid semi-annually, whereas a nominal 9 per cent mortgage amortized monthly yields 9.38 per cent.
23. Although the FHA data, which are relied on more heavily, go back to 1935, the more recent years carry heavier weight. As Table 4 indicates, our own distributions for the 1951–65 period also generate average lives close to half maturity.
24. For a description and analysis of this series, see Jack M. Guttentag and Morris Beck, *New Series on Home Mortgage Yields Since 1951* (New York: NBER, 1970), pp. 173–177.
25. The FHA specified 25 years as the maturity from January 1957 to November 1966, and 30 years thereafter.
26. It may appear strange that both the EPP and the shortfall from true effective yield rise with the discount. The reason is that smaller discounts require larger reductions in life to offset any given shortfall in yield book yield. The EPP must indeed approach zero as the discount approaches zero. (At zero discounts there is no EPP.)
27. On any single loan, of course, realized yield is unlikely to equal expected yield in any case, whereas on a portfolio of similar mortgages realized and expected yields will differ if the assumed termination probabilities do not materialize.
28. The expected yield in these calculations (and in all subsequent calculations in this section) are based on the 1951–65 average distribution of termination probabilities.
29. Duration differs from age of mortgage in cases wherein seasoned mortgages are acquired. When duration is given, age adds very little to the explanation of NFGL; i.e.,

the gain or loss is about the same for a new mortgage foreclosed after 2 years and a mortgage 6 years old when acquired that is foreclosed 2 years later.

30. All other factors influencing NFGL were held constant. We assumed a 6 per cent, 30-year mortgage, 5 per cent debenture rate, 12 months settlement period with 90 per cent of the claim settled after 11 months and the balance settled one month later, one-fifth of total expenses not reimbursed, and no interest paid for the first 2 months of the settlement period.
31. Although claim settlement may occur on any day, such payments were recorded as received on the first day of the month nearest to the settlement date. This procedure was necessary because trial-and-error solutions of equation 4 on a daily basis are inordinately costly.
32. This is close to the median of our sample. Although we have no other source of information on the settlement period, the range of variability in our sample was small, and the associated variability in equalizing discounts was small.
33. The debenture rate is tied to the market yield on intermediate-term government securities, which runs below the current FHA contract rate by varying amounts but generally by less than 150 basis points. Our assumption is, therefore, conservative in the sense of generating higher equalizing discounts than would usually be justified. The median differential in our sample was only 75 basis points, although this might have been influenced by rising market yields during the sample period, which tend to reduce the differential.
34. The average legal costs in our sample amounted to $350 instead of $450.
35. The equalizing discount equation is sensitive to the range of values for mortgage duration. Restating the equation,

$$p = \frac{51.71 + 2.795S + 738E + .38R - 13.68D}{(67.03)(1/2)^{(D-1)/2}}$$

Thus there is some (large) value of D, given values of S, E, and R, at which the numerator solves to zero. For lesser D values, p is positive (equalizing discount); for larger D values, p is negative (equalizing premium). Moreover, when D is large the denominator is small and negative p values increase exponentially as D continues to increase. The equation is useful, then, only for mortgage defaults that occur within the general range of durations in the sample (median of 53 months).

This is not a serious problem for our purposes. Von Furstenberg's study, covering the period 1957–65, shows median ages of foreclosed mortgages generally in the 3- to 4-year range (*op. cit.*, pp. 468–469), not inconsistent with the range of values in our sample. We applied the equation to mortgages in the sample and found a reasonable distribution of equalizing discounts:

Equalizing Discount	**Number**
Less than 2.0	19
2.0–2.99	44
3.0–3.99	13
4.0–4.99	12
5.0–5.99	11
6.0 and greater	7
	106

36. *Op. cit.*, p. 465.