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THE EFFECT OF THE AVAILABILITY OF FUNDS,  
AND THE TERMS THEREOF,  
ON BUSINESS INVESTMENT

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I

This paper is in the nature of a progress report rather than a final statement of results. While our ultimate goal is to embrace as many as possible of the complex and interrelated factors entering into corporate financing decisions, our approach has been to start with relatively simple models and gradually to broaden them by introducing factors which seemed essential to a more realistic analysis. At this stage we can only present some of our initial results which focus on the relative roles of debt and equity in financial decisions.

Perhaps the first problem suggested by the title of the paper is whether the availability of funds is in a somewhat different category from the terms at which funds are available. For two reasons we decided to start our work by concentrating on the cost of funds and its influence on corporate investment. Both of these reasons follow from a previous decision to deal primarily with the financial behavior of large firms, and this decision in turn was prompted by the overwhelming importance of large firms in the long-term credit markets, to which the available statistical data refer.

The first consideration is that, for these firms, absolute limitations can frequently be interpreted in terms of costs. For example, when financial executives claim they "simply cannot get" outside equity funds, the reason often is that the cost of these funds is, in some sense, prohibitive, with the result that equity funds are not even considered in formulating financing

plans. Second, terms other than costs, such as restrictions on current or quick ratios, limitations on future debt, etc., probably do not influence the financing operations of large corporations to a very great extent and, therefore, are probably not very important in explaining the movements of aggregate data. At any rate, we felt that our analysis could profit by omitting these factors, at least at the start.

The major problem, then, is to define the meaning of the cost of funds. The cost of debt funds is fairly clear: it is the effective interest rate paid by the firm to its creditors (taking into account costs of flotation and premiums and discounts). It is the same whether we are considering the firm as a whole, or the stockholders. Measuring the cost of common stock funds, however, is far more complex. To begin with, one has to face the crucial question of which group's interests are supposed to be considered in management's financial decision-making. In other words, if we are to speak of costs, costs to whom? In what follows we have decided to hold to the conventional assumption that management acts and decides primarily in the long-term interest of the "present" owners of the firm, i.e., of holders of shares at the time a financial decision is to be made. This assumption may not always be in accordance with the facts, but we feel that it is pertinent to a sufficiently large number of cases to merit its use, at least as a starting point.

In view of this assumption, we can dispose of one simple measure of costs that has occasionally been advanced, namely, the additional dividends that would have to be paid per dollar of new funds acquired through common stock flotations. A payment of dividends can only be considered a cost if the payer and the payee are different persons, which obviously is not the case, since the stockholders are the owners of the firm.

What, then, are the factors controlling the total amount of new financing and the relative use of long-term debt and equity instruments, if we assume that management acts rationally to maximize the return to existing stockholders? Because of the complexity of the problem, our presentation follows much the same lines as our research, i.e., we first analyze the problem on the basis of simple assumptions and present the tentative conclusions suggested, and then proceed to introduce more realistic conditions.

### CASE I SUBJECTIVE CERTAINTY

Consider first the case in which management is assumed to know the outcome of any decision with "subjective" certainty. This assumption yields very simple solutions to our problem which can conveniently be discussed under two headings:

## Subcase I-a

ABSENCE OF CORPORATE INCOME TAXES,  
AND CONSTANT MARKET FACTORS

Under the general assumption of subjective certainty, the simplest case is the one in which: 1) corporate income taxes are absent, and 2) the per share net proceeds from the sale of stock and the effective interest rate on debt funds are not significantly affected by the financing plan itself, i.e., in which market factors can be regarded as parameters of action. In this case certain conclusions can easily be established. In the statement of these conclusions, given below, the word "earnings" is to be understood as a shorthand expression for "anticipated, net, long-run, average, yearly earnings," and "price" as the shorthand expression for "current net proceeds per share from an issue of stock."

1) If bonds are the only available type of financial instrument, a fixed amount of new financing will be advantageous, i.e., will increase the earnings of existing stockholders if, and only if, the anticipated rate of return on new money exceeds the effective interest rate.<sup>1</sup> If the amount of new money to be raised is not a fixed sum, but a variable amount, it will pay to push new financing to the point at which the marginal rate of return equals the effective interest rate. This is, of course, a familiar conclusion; nor is it surprising, since we are proceeding thus far under traditional textbook assumptions.

2) If common stock is the only available type of financial instrument, the raising of a given sum will be advantageous, if and only if, the anticipated rate of return on new money exceeds the original-earnings/price ratio, where "original earnings" refers to earnings if no new financing is undertaken.<sup>2</sup> If we treat the amount of new money to be raised as a variable, then it will pay to push new financing to the point at which the marginal rate of return equals the earnings-price ratio of the "enlarged" firm.<sup>3</sup>

3) Even if management is free to use both junior and senior financing instruments in any desired combination, the most advantageous financial plan will still involve the use of bonds alone or stocks alone, but not both simultaneously; i.e., *only corner solutions* are possible — except in the trivial case where the choice between the two is a matter of complete indifference.

4) The condition under which stock or bond financing is more advantageous can be stated in terms of the earnings per share that would result

<sup>1</sup> See Note 1, Appendix C.

<sup>2</sup> See Note 2, Appendix C.

<sup>3</sup> See Note 2, Appendix C.

from the use of the optimum amount of bonds. If the effective rate of interest is less (greater) than the earnings-price ratio involving this earnings figure, bond (stock) financing is more advantageous. Observe that this implies that if the effective rate of interest is less than the original-earnings/price ratio, bond financing must be preferable, but if the relation between these two factors is reversed, either may be preferable.<sup>4</sup>

5) If common stock is the most advantageous instrument, the total amount of funds acquired will be greater than the amount that would have been raised through flotation of bonds, and the rate of return on capital at the margin will be less than the effective interest rate (Note 3, Appendix C).

### Subcase I-b

#### CONSIDERATION OF CORPORATE INCOME TAXES

When corporate income taxes are taken into account, the analysis and conclusions of Subcase I-a remain essentially unchanged.<sup>5</sup> If we use the word earnings to mean earnings gross of taxes, conclusions 1 through 5 in the previous section remain true as stated.

This does not mean, however, that corporate income taxes have no effect on the choice of instruments or on the amount of new financing. Since the market price of a share of stock depends, at least in part, upon the *net* earnings per share, a rise in the corporate income tax, unaccompanied by other changes in the corporate picture, will depress the price of stock. The earnings-price ratio gross of taxes will therefore rise and prospective returns on new capital will have to be greater than before for stock financing to be advantageous. For the same reason, a rise in the tax rate will also swing the balance toward bond financing and against equity financing.

As an alternative to the method described above, we may use the word earnings to mean net earnings, as is more usual in speaking of earnings-price ratios, and introduce a separate factor by which to describe the effects of corporate income taxes. In this case, conclusions 1, 3, and 5 do not change, and conclusions 2 and 4 may be "corrected" by inserting the words "a multiple of" in front of every expression referring to an earnings-price ratio. The multiplier referred to is the reciprocal of unity-minus-the-effective-corporate-income-tax-rate.

<sup>4</sup> See Note 3, Appendix C.

<sup>5</sup> The conclusions of this section have already been substantiated in previous footnotes. See the second remark on definitions, Note 1, Appendix C.

This discussion, of course, is not meant to be an exhaustive treatment of the effects of corporate income taxes. For this purpose, one should also consider the reactions of other sectors of the economy.<sup>6</sup>

Before concluding this first section, a word may be added about the relation of preferred stock financing to both bond and common stock financing. Under the condition of subjective certainty of estimates, non-voting preferred stock is very similar to bonds, and preferred dividends are very similar to interest payments that are not deductible for tax purposes. Thus, preferred stock financing increases common stockholders' returns when the rate of return on new money is greater than the tax multiplier used above times the dividend yield; and it is preferable to bonds only when this latter factor is less than the effective interest rate. When preferred stock is compared to common stock, however, it need only meet the test that the preferred yield be less than the common-stock-*net-earnings*/price ratio in order to be preferable, since there is no tax-deductibility advantage to either of these financial instruments.

Summing up, then, the analysis thus far indicates that the factor controlling financial decisions is the relation between the effective interest rate, the current effective preferred dividend yield, and the management-anticipated-*earnings*/price ratio before taxes. It also leads to the conclusion that in the optimum financing plan all money should be secured through the use of only one type of financing instrument. Furthermore, from what we know about the usual relations between market factors we should have to conclude that, under the assumptions of the present model, stock financing, especially preferred stock financing, would hardly ever be chosen.

This conclusion follows from the fact that the interest rate is usually less than the preferred dividend yield and that both of these rates are usually less than either the current-*earnings*/price ratio or any of the usual average-*earnings*/price ratios. Although none of these latter ratios is exactly comparable to the pertinent concept of our analysis, they do give a sufficiently strong indication that interest and preferred yields are usually also lower than management-anticipated-*earnings*/price ratios. Indeed, that they must usually be lower follows from the well-known fact that a premium must generally be paid to investors to compensate them for the

<sup>6</sup> See Richard Goode, "The Corporate Income Tax and the Price Level," *American Economic Review*, XXXV (March 1945). In his "Effects of Taxation on Individual Investment and Corporate Financing," *Annals of the American Academy of Political and Social Science*, CCLXVI (November 1949), Dan T. Smith considers the "global" effects of corporate income taxes in addition to examining their immediate effects on corporate financing.

additional risks involved in stock ownership. The opposite relation could therefore only hold if investor anticipations were considerably more optimistic than management anticipations.

The conclusion that stock would hardly ever be used as a vehicle of financing is only reinforced by tax considerations, as well as by considerations of control. Obviously this conclusion is not in accordance with the facts since stock financing is a source of new funds, even though a minor one.

This might imply: 1) that our assumptions as to what constitutes the stockholders' best interest are too unrealistic; and/or 2) that management is not primarily motivated by stockholder interest, or at any rate does not pursue the rational course to this goal; and/or 3) that some other elements of our model are too far from reality to be useful in explaining observed behavior. It seems reasonable not to accept the second alternative until we have explored the remaining two. One assumption that seems particularly untenable in connection with the problem at hand is that of subjective certainty of estimates. In Section II, we examine some of the implications of dropping this assumption.

## II

### CASE II SUBJECTIVE UNCERTAINTY

Our starting point will be to recognize that managements' anticipations of the effects of any action on long-run earnings will not, in general, consist of a single figure but rather of a whole range of estimated possible outcomes having varying degrees of likelihood.<sup>7</sup> To cast this concept into operational form, we may think of these anticipations as having the form of a probability distribution of possible outcomes, and for the sake of simplicity, we will characterize this subjective probability distribution by just two parameters: 1) a measure of central tendency (such as the arithmetic mean or the mode) which we will call "expected outcome," and 2) a measure of dispersion, i.e., of the extent to which the actual outcome is considered capable of differing from the expected outcome (such as the standard

<sup>7</sup> On the general problem of choices involving risks see: G. Tintner, "The Theory of Choice under Subjective Risk and Uncertainty," *Econometrica*, IX (July-October 1941); M. Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risks," *Journal of Political Economy*, LVI (August 1948); and the following articles by J. Marschak: "Money and the Theory of Assets," *Econometrica*, VI (October 1938); "Rational Behavior, Uncertain Prospects, and Measurable Utility," and "Optimal Investment of a Firm" (abstract), both found in *Econometrica*, XVIII (April 1950). See also the bibliography following the article, "Rational Behavior, Uncertain Prospects, and Measurable Utility."

deviation or some interquartile range). The latter is clearly a measure of the risk involved in a given course of action.

Further, a useful discussion of risk must involve the concept of risk aversion or preference. For if management were completely indifferent to risk and, therefore, sought only to maximize the "expected" earnings per share, the conclusions reached in Case I would remain substantially unchanged. Indeed, the only change required in the previous discussion would be to substitute the word "expected" for "anticipated" before such expressions as "rate of return on new money." In particular, it would remain true that, in any given situation, it would pay to use *either* debt or equity financing, but not both.

Of the many possible ways of introducing risk aversion into the analysis, two will be considered here.

#### Subcase II-a

##### RISK AVERSION MEASURED BY CERTAINTY EQUIVALENTS

One method that is sometimes used to take uncertainty and risk aversion into account with a minimum of complication is that of using certainty equivalents. The certainty equivalent of an uncertain outcome is defined as the "expected outcome" reduced by a "risk discount factor." This latter factor is directly proportional to the dispersion of the subjective probability distribution of outcomes, and is greater, the greater the subjective aversion to bearing risk.

For the problem at hand, this method of introducing uncertainty and risk aversion again does not substantially affect the conclusions reached under Case I. In particular, it continues to be true that the optimum financing plan involves the use of bonds alone or stocks alone, but not a combination of the two. At the same time, as one might expect, the conditions under which any type of financing is advantageous to old stockholders become more stringent, and the balance between the two shifts in favor of equity financing.<sup>8</sup>

#### Subcase II-b

##### RISK AVERSION MEASURED BY AN EXPLICIT FUNCTION

The influence of uncertainty and of subjective attitudes toward risk bearing may be analyzed more fully by explicitly introducing a "risk aversion" function into our model.

Each management recognizes that any action it decides upon may fail in its earnings objective, and consequently "worsen" the position of exist-

<sup>8</sup> See Note 4, Appendix C.

ing stockholders, either absolutely or in comparison with the results of alternative actions. It is well known that managements differ in the chances they are willing to take in providing stockholders with earnings. The "tastes" of a given management in this respect are primarily a reflection of its temperament, i.e., of its adventurousness or cautiousness. To some extent these "tastes" are also conditioned by the temperament of the stockholder group, but in view of the tendency toward "mutual natural selection" of stockholders and managements, the "tastes" of these two groups are unlikely to be very dissimilar.

The "risk aversion" function we now introduce is intended to describe these "tastes" of management. Specifically, we may assume that in choosing between alternative courses of action the decision-makers take into account both the "expected outcome" and the dispersion or risk of each alternative course, the former being a desirable, the latter an undesirable thing, and that they aim at striking a balance between the two on the basis of the opportunities open to them and of their "taste," i.e., their willingness to bear risk. As indicated above, "taste" can be described by a "risk aversion" or utility function and the endeavor to strike a balance may be translated into one of maximizing this utility function within the limitations imposed by the available opportunities.

Because of the complexity of the problem, a rigorous statement of the conditions under which positive net financing is advantageous and of the conditions describing optimum balance between debt and equity financing is best given in mathematical form.<sup>9</sup> However, the essential features of our analysis can be described in terms of the familiar graphical analysis of the theory of consumers' choice. The subjective evaluation of various combinations of profits and risk can be represented by a set of indifference or "risk aversion" curves; the combinations of these "commodities" that are within reach of management can be represented by a set of opportunity curves; and the two sets of curves can be combined to exhibit the maximizing solutions.

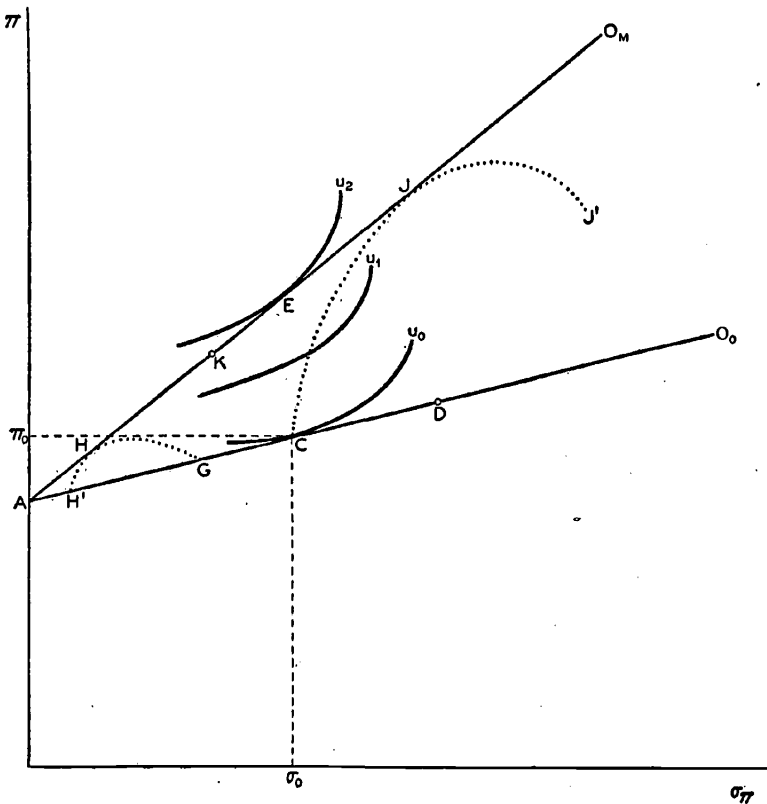
In Figure 1, on the vertical axis we measure  $\pi$ , the expected, net, long-run, average annual earnings of the old stockholders, and on the horizontal axis we measure  $\sigma_{\pi}$ , the dispersion of the subjective probability distribution of these earnings. Any point on the plane bounded by these axes, then, represents an inseparable bundle of expected earnings and corresponding risk; i.e., of a given amount of earnings and the given amount of risk that must be borne to secure these earnings.

Then, the attitude toward risk and profits is represented by "indifference" curves  $u_0, u_1, u_2, \dots$ , having the following characteristics:

<sup>9</sup> See Note 5, Appendix C.

Figure 1

## RISK AVERSION MEASURED BY AN EXPLICIT FUNCTION



- 1) As is usual, all points along any one curve exhibit combinations of  $\pi$  and  $\sigma_{\pi}$  which are equally preferable.
- 2) Each of these curves slopes upward from left to right because dispersion is considered to be undesirable. That is, as between two ventures having the same expected outcome, the one for which the dispersion is smaller is the one preferred; consequently, an increase in dispersion must be accompanied by an increase in expected earnings if the level of utility is to remain unchanged.
- 3) The slope of a line at a given point measures the amount of profit that would be necessary to induce management to accept an extra "unit" of dispersion. Therefore, the steeper the curve, the greater the risk aversion.

4) Points lying on curves higher and to the left are preferred to those lying on curves lower and to the right.

5) Each curve is convex to the horizontal axis indicating that risk aversion increases with the amount of risk already borne, but decreases with the amount of profit already enjoyed.

The next step is to represent in our diagram the "market opportunity function," that is to say, the relation between pairs of values of  $\pi$  and  $\sigma_\pi$  that can be secured by management within the limitations imposed by the given data, i.e., 1) the market factors — per share net proceeds from sale of stock and the effective interest rate on borrowed funds, assumed constant; and 2) management anticipation factors — expected gross return and the subjective dispersion of returns corresponding to varying amounts of new money invested in the firm. It turns out that under the conditions of our problem the entire set of opportunities confronting management can be represented by a family of straight lines like  $O_0, \dots, O_M, \dots$  in Figure 1, all of which intersect the vertical axis at the same point — represented by point  $A$  in our diagram.

In order to get a better understanding of this point, which is of central importance for the analysis that follows, it will be useful to think of any financial decision as consisting of two parts: 1) the decision as to the most advantageous amount (if any) of new funds to acquire, and 2) the decision as to the extent to which this amount should be secured through debt or equity financing. Given the initial debt-equity structure of the firm, this latter decision determines uniquely a new structure; if we like, we may therefore think of this second part of the decision as the choice of the most advantageous debt-equity structure.

Suppose now that point  $C$  in our diagram, with coordinates  $(\sigma_0, \pi_0)$ , represents the initial position of the firm before any financial transaction whatever is undertaken. This point indicates that, with the initial amount of total assets and the initial debt-equity structure of say  $B_0$  dollars of bonded indebtedness and  $S_0$  dollars of common stock (at market prices), expected stockholders' earnings estimated by management are  $\pi_0$  dollars, while the uncertainty attached to this estimate is measured by  $\sigma_0$ . Now, even without securing any net new funds, it will generally be possible for management to move from  $C$  to new  $\pi, \sigma_\pi$  positions — i.e., to other points in our diagram — by engaging in what we may call "pure refinancing operations": floating stocks and using the proceeds to retire outstanding bonds and vice versa.

For instance, if the effective interest rate is less than the original-expected-earnings/price ratio (as is assumed in Figure 1), then, ignoring for the moment certain institutional limitations, management can increase

the expected return per share by "trading on the equity," i.e., by refinancing part of the common stock with bonds. But this will at the same time increase the uncertainty that this expected return per share will be realized since it will decrease the number of shares over which the risk is to be spread.

Thus, if  $B_1$  dollars of funds were acquired by issuing bonds and the proceeds used to repurchase, at market price,  $S_1$  dollars of common stock (all transactions taking place at constant prices), so that the new balance sheet would show  $B_0 + B_1$  dollars of bonded debt while the market value of equity would become  $S_0 - S_1$  dollars, the new  $\pi$ ,  $\sigma_\pi$  position of the firm would be represented by a point like  $D$  in our graph, higher and to the right of  $C$ . Similarly, repaying debt through funds acquired by sale of stock would produce a shift to some other position with lower expected earnings and lower dispersion and which therefore would be represented by a point lower and to the left of  $C$ , like  $G$  in our graph.

Now if we consider the entire set of points that correspond to all combinations of  $\pi$ ,  $\sigma_\pi$  that can be achieved by pure refinancing operations of the type described, it turns out (Note 5, Appendix C) that all these points fall on a straight line through the point  $A$  and  $C$  like  $O_0$  in our graph. In other words, the increase (decrease) in expected earnings that can be secured by "pure refinancing operations" — replacing of equity funds with debt funds or vice versa, without changing the "size" of the firm — turns out to be a constant multiple of the concomitant increase (decrease) in dispersion.

Since the line  $O_0$  represents all combinations of  $\pi$ ,  $\sigma_\pi$  achievable without any net acquisition of funds, it may be thought of as the opportunity line for zero net financing. To each point on line  $O_0$  will of course correspond uniquely a value of  $B$  and  $S$  adding up to zero. These values of  $B$  and  $S$  can unfortunately not be exhibited directly in our graph but they can be immediately derived from the data of the problem.

It will now be easy to see that the opportunity function for any other fixed amount of net financing must be represented by another straight line like  $O_M$ . Suppose for example that the firm acquired  $F_M$  dollars of net new funds by means of a specific combination of junior and senior financing, say  $S'_M$  and  $B'_M$  which together add up to  $F_M$ , and suppose that as a result of this operation the  $\pi$ ,  $\sigma_\pi$  position shifts from the original position  $C$  to a new position, say point  $J$  in our diagram. Now there clearly are many other  $\pi$ ,  $\sigma_\pi$  positions that the firm can achieve by using different combinations of debt and equity financing to secure the same total sum  $F_M$ . The interesting point is that all such achievable combinations must again form a straight line through the point  $A$  and  $J$  like  $O_M$  in our graph.<sup>10</sup>

<sup>10</sup> The height of the  $Y$ -intercept is given by the product of unity-minus-the-effective-

This becomes very clear if we now think of  $J$  as an initial position just like  $C$ , only corresponding to a larger fixed corporate size. We may then think also of other  $\pi$ ,  $\sigma_\pi$  positions that can be achieved by changing the distribution of  $F_M$  between bonds and stocks as being achievable by means of pure refinancing operations starting from  $J$ . But as we have just argued, the relation between the values of  $\pi$  and  $\sigma_\pi$  that can be achieved by means of pure refinancing operations is always a linear one. Thus, the line  $O_M$  will represent the market opportunity function corresponding to the fixed amount  $F_M$  of net new financing. To each point on this line will again correspond uniquely a value of  $B$  and  $S$  adding up to  $F_M$ .

Similarly, to any other amount of net new financing,  $F$ , there will correspond a different opportunity function and this function will be represented in our graph by another straight line through  $A$ . It follows, finally, that the entire set of opportunities confronting management must consist of the family of straight lines corresponding to all possible values of  $F$ . The slope of any such line will depend of course on the initial data, as well as on the value of  $F$ .

So far, then, we have shown 1) that the entire opportunity set may be represented by a family of "curves," each of which corresponds to a different value of  $F$ ; 2) that each of these opportunity "curves" is a straight line intercepting the vertical axis at a single point, point  $A$  in Figure 1; and 3) that there is an initial line, corresponding to no net financing ( $F = 0$ ), which in our graph is line  $O_0$ , passing through points  $A$  (the common intercept) and  $C$  (the initial position).

We come now to an explanation of the relation between opportunity curves for successively larger values of  $F$ . The slope of the lines may increase as new funds are acquired or it may decrease.<sup>11</sup> Only if the slope increases as corporate size is increased from its initial level (as  $F$  increases above zero) will net positive financing be advantageous. In this case, as  $F$  continues to grow larger the opportunity lines will grow steeper until some value of  $F$ , call it  $F_M$ , has been reached, after which they will flatten out. Thus, there will, in general, be some value of  $F$ , such as  $F_M$ , to which

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corporate-income-tax-rate, the effective interest rate, and the market value of the existing shares of stock. See equation (5.11), Note 5, Appendix C. It, therefore, is independent of the form or amount of new financing and is identical for all opportunity lines.

<sup>11</sup> The condition for net positive financing is that the marginal rate of return on new capital should exceed the interest rate by more than a quantity which depends on the difference between the original-earnings/price ratio and the interest rate and on the elasticity of dispersion with respect to new funds. See equation (5.12), Note 5, Appendix C.

would correspond the steepest opportunity line represented by  $O_M$  in our graph.

Now it is easy to see that this steepest line is, in general, the "best" opportunity line and the corresponding value of  $F$  represents the optimum amount of financing to be done. Indeed, it is clear that by staying on the steepest  $F$  line, management would be in a position to secure, for any given value of  $\sigma_\pi$ , the greatest value of  $\pi$  consistent with the available opportunities. Thus, in Figure 1, the greatest value of  $\pi$ , associated with  $\sigma_\pi = \sigma_0$ , within the opportunity set, is obviously on the steepest line  $O_M$ ; and a similar result holds for any other value of  $\sigma_\pi$ .

Having thus determined the best opportunity line, the next problem is that of choosing the best point on this line, i.e., the point that maximizes the utility function. As in the familiar case of consumers' choice, this will be the point of tangency between the best opportunity line and an indifference curve, such as the point  $E$  on the line  $O_M$ . As indicated before, to this point,  $E$ , will correspond uniquely a value of net bond financing,  $B$ , and of net stock financing,  $S$ , say  $B_M$  and  $S_M$ , which together add up to the optimum amount of financing  $F_M$ .

What has been said up to this point has the following interesting and important implications: 1) The total amount of financing in the optimum financial plan will, in general, be independent of the "tastes" of the decision-makers as expressed by the risk aversion function; and 2) the optimum financial plan need not involve only one type of financial instrument, but may include a combination of both junior and senior financing (for exceptions see Section II-c below).

As has been shown, the optimum amount of new financing is the amount that corresponds to the steepest  $F$  line and the position and slope of the  $F$  lines are determined entirely by market and anticipation factors and not by the parameters of the indifference map. In other words, any decision-maker confronted with the same market and anticipation data would choose the same amount of total financing regardless of his attitude toward risk bearing. The influence of tastes spends itself in determining which point within the line  $O_M$  will be chosen; that is to say, in determining the distribution of the total amount,  $F_M$ , between stock and bond financing. Decision-makers who are inclined to take more risk than the one whose tastes are depicted in Figure 1 would be characterized by indifference lines which are flatter in the neighborhood of  $E$ . They would, therefore, choose a point on  $O_M$  to the right of  $E$ , which in turn implies having greater recourse to bonds and less to stock. More cautious decision-makers, on the other hand, would move from  $E$  toward  $A$ .

## Subcase II-c

## THE EFFECT OF INSTITUTIONAL LIMITATIONS

The conclusions reached so far may have to be somewhat restricted if we take into account certain "institutional" limitations on the values that may be taken by  $B$  and  $S$ . The solution of the problem as given in Figure 1 may well yield values of  $B$  and  $S$  that are institutionally impossible. For instance, while the optimum value of  $F$  may be positive, the corresponding value of  $B$  may be negative; this would imply raising, through stock issuance, an amount greater than is necessary to expand the firm, and using the difference to retire bonds. Now, while there is nothing impossible about this implication, in principle, it is clear that the firm cannot retire more bonds than are outstanding. Similarly, it is not usually considered feasible for a firm to issue bonds for the direct purpose of retiring its common stock; because of this restriction, the optimum value of  $S$  consistent with the institutional framework cannot be negative (at least as long as the optimum value of  $F$  is not negative).

In terms of our graph this means that certain portions of each opportunity line must be excluded from the solutions. On line  $O_0$  for instance, the portion of the line to the right of  $C$  must be excluded because of the second institutional limitation pointed out above. Similarly, if point  $G$  on  $O_0$  denotes the values of  $\pi$  and  $\sigma_\pi$  attainable when all outstanding bonds have been refinanced by stock, the portion of  $O_0$  to the left of  $G$  must be excluded because of the first factor mentioned above. For the same reasons, two segments of every other opportunity line must be excluded as "institutionally unfeasible."

It can be shown that these institutional limitations can be translated into our graph in the form of two "boundary curves" such as  $CJJ'$  and  $GHH'$ , tangent to  $O_M$ , the best opportunity line, at  $J$  and  $H$  respectively.<sup>12</sup> The first of these curves is the locus of points on successive opportunity lines showing the  $\pi, \sigma_\pi$  combinations attainable when all financing is done by bonds, so that  $S = O$ , and  $B = F$ ; the second is the locus of points on successive opportunity lines, showing the  $\pi, \sigma_\pi$  combinations attainable when  $B = -B_0$ ,  $S = F + B_0$  (where  $B_0$  is the initial outstanding amount of bonds). The shape of these two curves depends, of course, on the market and anticipation factors.

The relevant portion of the institutional opportunity set, then, consists of the area  $CGHJ$  for  $F \leq F_M$ , and the whole area between  $AO_0$  and the curve  $H'HJJ'$  if we consider values of  $F > F_M$ . Similarly the "best opportunity" line degenerates into the curve  $H'HJJ'$ . The optimum solution is now given by the point of tangency of  $H'HJJ'$  and an indifference curve.

<sup>12</sup> The tangency of these curves is shown at the end of Note 5, Appendix C.

Taking the above discussion into account, we can now describe six types of optimum financial plans that may emerge. Let  $K$  represent the point on  $O_M$  that corresponds to  $S = F_M$ ,  $B = 0$ . Case 1: If the point of tangency of the indifference curve with the "best" opportunity curve lies between  $K$  and  $J$ , the optimum financing plan will involve both stocks and bonds. Case 2: If the tangency point falls at  $H$  or between  $H$  and  $K$ , only stock will be issued, but some (all at  $H$ ) of the outstanding bonds will be retired with part of the proceeds. In both cases the same total net amount of funds will be acquired and the conclusion about the independence of tastes and the amount of net financing will hold.

In Case 3 the tangency point falls in the sector  $JJ'$ ; in Case 4 it falls in the sector  $HH'$ . We shall describe only the former case and allow the reader to apply a similar reasoning to the latter. Case 3, then, would lead to a financing plan involving the use of bonds alone, but the total amount of funds acquired would be greater than  $F_M$ , say  $F_N$ . The excess of  $F_N$  over  $F_M$  may be thought of as a substitute type of "trading on the equity" which, in the absence of restrictions, would have been done by retiring part of the common stock, thus moving to the right of  $J$  on line  $O_M$ . Furthermore, since the value of  $F_N$  depends on the specific point of tangency, the total amount of financing must depend on the shape of the indifference curve. The flatter the indifference curves in the neighborhood of  $J$ , the greater this amount would be. Whether or not such cases are empirically important depends on the specific values of the market and anticipations data and cannot be settled on theoretical grounds. It would seem, however, that  $F_N$  is unlikely to be much larger than  $F_M$  so that the independence of tastes and total amount of financing may still be approximately true.

Cases 5 and 6 are the trivial cases where the point of tangency falls exactly at  $K$  or  $J$ . In these, the plan will call, respectively, for sale of stock alone or issuance of debt instruments alone, without any refinancing. The independence of tastes and amount of financing holds for these two cases and the total net amount of funds acquired is the same as in Cases 1 and 2.

#### Subcase II-d

##### CONSIDERATION OF CORPORATE INCOME TAXES

In the previous section we have not found it necessary to have much reference to the corporate income tax rate, even though the presence of such taxes was taken into account in deriving our results. The reason is essentially the same as was discussed for Subcase I-b, namely that the effect of such taxes shows up primarily in influencing the price per share.

The optimum financial plan depends, among other things, on the relation between the earnings-price ratio gross of taxes and the interest

rate. Corporate income taxes, therefore, affect the solution of the problem by influencing the price per share which presumably depends on earnings *net* of taxes. On the basis of our model a decrease in price, other things being equal, can be shown to bring about, in general, a decline in the optimum amount of total financing.<sup>13</sup> Hence, to the extent that an increase in corporate taxes — with unchanging expectations of earnings gross of taxes — depresses the price per share, such an increase will also tend to reduce the total amount of financing. The effect such an increase would have on the distribution of new financing between stocks and bonds is not unambiguous. In general, it would seem likely to encourage the use of debt as against equity financing. But this is not necessarily true as it depends partly on the shape of the indifference map. The fall in price would tend to make the “best” *F* line steeper and this would produce a “substitution” effect toward greater bond financing.

At the same time, however, there is a deterioration in the opportunities available, and this produces an additional effect not unlike the income effect in the theory of consumers’ choice. This effect is also likely to pull in the direction of bond financing if, as seems reasonable, an over-all decline in opportunities makes people more willing to take risk for the sake of keeping up the “expected” income. However, the opposite outcome is also possible depending on the specific shape of the risk preference function.

### III

#### THE PROBLEM RESTATED

The previous discussion presents that part of the development of our model which we feel justified in exhibiting at this time. We have made several attempts to broaden the model by introducing less restrictive assumptions about market factors, and by introducing other financing alternatives besides common stock and long-term debt; but the broader model has not yet reached the stage of maturity.

In spite of the incompleteness of the conclusions, a brief summary may be useful. Traditional economic theory considered only debt financing, and hence limited its conclusion on business expansion to the well-known condition of equality between the marginal productivity of capital and the interest rate.<sup>14</sup> However, as corporation finance specialists have pointed

<sup>13</sup> See Note 6, Appendix C.

<sup>14</sup> More recently, the effect of risk on interest cost has been taken into account. See, for instance, Michal Kalecki, *Essays in the Theory of Economic Fluctuations* (London: George Allen and Unwin, Ltd., 1939), Chapter 4; and Albert G. Hart, *Anticipations, Uncertainty, and Dynamic Planning* (Chicago: University of Chicago Press, 1940).

out for some time, without drawing exact conclusions,<sup>15</sup> corporate expansion can also be financed by selling shares of ownership. When equity financing is used, a new cost element enters the picture, namely, the ratio of expected earnings per share, gross of taxes, to the effective market price of stock.

The essential quality of this type of instrument is its ability to reduce the risk borne by the owners, while at the same time forcing them to accept lower returns than could be provided if other instruments were used. It is not surprising, therefore, that only when the desire to limit risk is taken into account does the analysis lead to conclusions that are not inconsistent with observed facts. If subjective certainty of estimates is assumed, or if the adjustment for uncertainty is made by means of a risk discount factor, we obtain the unrealistic conclusion that the most advantageous financial plan involves the use of only one instrument (at least as long as market data remain approximately constant). Furthermore, given the usual market relations between costs, the former assumption would make equity financing extremely rare.

When an explicit risk aversion function is brought into the analysis, our model leads to conclusions that are not inconsistent with observed facts. Stock financing alone and "combination" financing both become definite possibilities. The composition of the optimum financing plan depends on the strength of decision-makers' willingness to bear risk for the sake of increasing expected returns, as well as on market factors and expectations of management. But while "tastes" enter into the decision as to the composition of the plan, they do not necessarily influence the total amount of money to be acquired. That is, for certain ranges of anticipation and market data, the optimum total amount of financing is determined exclusively by market and expectation factors.

The fact that, in reality, "corner solutions" predominate is also not inconsistent with our results, for they indicate that these solutions are also perfectly possible. Note, however, that according to our analysis, "corner solutions" are associated in general (though not necessarily) with situations in which the optimum amount of financing is *not* independent of tastes. Therefore, even if the remaining assumptions of our model are sufficiently realistic, the range of situations in which  $F$  is independent of tastes may not be empirically very large.

<sup>15</sup> Typical textbook discussions may be found in W. H. Husband and J. C. Dockeray, *Modern Corporation Finance* (Chicago: Richard D. Irwin, 1948); F. F. Burtchett and C. M. Hicks, *Corporation Finance* (New York: Harper and Brothers, rev. ed., 1948); H. G. Guthmann and H. E. Dougall, *Corporate Financial Policy* (New York: Prentice-Hall, 1948); N. S. Buchanan, *The Economics of Corporate Enterprise* (New York: Henry Holt, 1940); and O. P. Decker, *Corporation Finance* (New York: American Institute of Banking, 1941).

Yet it must be remembered that an optimum plan may call for sequential issuance of two types of securities, with a considerable time lag, and that, because of the costs and the nuisance value of small issues, one form of financing may be used exclusively even though the optimum plan also calls for minor use of the other. Thus, in spite of the preponderance of "corner solutions," the independence property may hold at least approximately; and, in fact, there are some indications that even in "corner solution" cases the optimum amount of new funds, according to our model, would not be too greatly influenced by "tastes."

Quite obviously, what has been presented here is hardly more than the beginning of the application of this type of analysis to an extremely complex field. Other researchers could undoubtedly suggest several alternative hypotheses for each that we have proposed, starting with the one based on motivation of management and moving down the scale. We have become especially aware of the inadequacy of the comparison of permanent equity and permanent debt funds. For industrial corporations, at least, long-term debt instruments seem to be an important source of funds that management wishes to use for a few years only; common stock, of course, cannot be used for this purpose because it cannot easily be retired. Nevertheless, once we decided to investigate this complex problem, we felt that there was some merit in making some sort of a start, rather than merely listing the various alternatives and their shortcomings. We hope that, in the process, we have suggested, or even contributed toward, some useful avenues of investigation.

#### IV

##### TESTING THE THEORETICAL MODEL

Since in the last analysis the real test of any theory is its ability to account for the observed facts, an attempt was made to carry out statistical tests of certain implications of our model. These tests will be described in the present section. To be sure, our model is far from complete at present; but if we are moving in the right direction it should be able to account for certain facts even at the present stage.

By its very nature our theory can only be tested against mass observations. Any theory based on simplifying assumptions cannot be expected to explain all the "quirks" of all the individual decisions; it is only when these idiosyncrasies have been averaged out by dealing with a substantial number of cases that the influence of the basic factors upon which the theory is based can become apparent. For this reason, and because of their availability, time series data seemed to be the appropriate type to use, in spite of many limitations.

The difficulties presented by the available data are indeed formidable. Some of them revolve around the nature of the data themselves and have been discussed at length at this conference. But the problem of deriving, from available data, statistical series that may reasonably be supposed to measure our theoretical concepts is at least equally formidable. The main difficulty centers on the problem of measuring expectation factors statistically. This hurdle, in fact, prevented us from carrying out a direct test of the full implications of our theoretical analysis. Such a test would have required obtaining a measure of the anticipated yield on various amounts of new capital; finding even an approximation to this schedule seems an impossible task at present.

However our analysis does give us some hints as to the factors controlling the distribution of new financing between debt and equity forms, and it was felt that an attempt might be made to approximate these factors, at least crudely, by means of available statistical data.

The statistical analysis that follows deals, therefore, exclusively with the factors accounting for the distribution of net corporate acquisitions of new external long-term funds between their two main sources: the common stock market and the long-term debt market. As a measure of the distribution of funds between these two sources we may conveniently use the ratio of the amount of new funds raised through issuance of common stock to the amount raised through the issuance of both common stock and long-term debt instruments. Estimates of the annual amount secured from each source from 1920 to date are available in the *Commercial and Financial Chronicle*, though these estimates undoubtedly leave much to be desired.<sup>16</sup>

One particularly serious drawback in the use of the compiled figures as presented in the *Chronicle* is that preferred and common stock issues are separated only for the aggregate of all corporations. To be sure, basic data are supplied from which individual industry estimates of new capital common stock issues could be computed. It seemed preferable, however, to see what could be learned from the over-all aggregates before undertaking the substantial amount of work involved in compiling industry estimates. Accordingly we shall deal here only with the behavior of the above-mentioned ratio for the entire corporate universe; this will be the dependent variable whose behavior we intend to "explain."

Our theoretical analysis does not directly yield an expression relating the above variable to market and expectation factors. We did find, however, that under conditions of subjective certainty the relative advantage in the use of equity as against debt funds would tend to be greater the smaller the ratio of anticipated earnings gross of taxes to stock prices, relative to the

<sup>16</sup> For further information on the series used see Appendix B, ref. [1].

interest rate. This same conclusion appeared to hold in the "uncertainty model" also, but in this case another important variable appeared to be the degree of risk, measured in our analysis by the dispersion of the subjective distribution of anticipated gross earnings (i.e. gross of both interest and taxes); *ceteris paribus*, the larger this dispersion the larger would tend to be the incentive to the use of equity financing.<sup>17</sup>

In order to carry out a statistical test of these conclusions we must face the problem of how to measure the theoretically relevant variables. No serious problem seems to arise with respect to the interest rate; we have at our disposal an estimate of the interest yield on new bonds, which ought to be a sufficiently good approximation to our theoretical concept and is accordingly made use of.<sup>18</sup>

A much more formidable problem is that of obtaining a satisfactory measure of the earnings-price ratio since, according to our theoretical analysis, earnings should refer to expected average long-term earnings, gross of taxes, as seen by management. It is extremely doubtful that this variable can be measured by the customary current-earnings/price ratio even after an adjustment for corporate income taxes. An inspection of the scatter diagram of Figure 2, which is based on this assumption, very much reinforces such doubts. In this chart our dependent variable, denoted by  $X_1$ , is measured on the vertical axis while the relation between expected earnings and the interest rate, as approximated by the quotient<sup>19</sup>

$$\frac{\text{Current Earnings/Price Ratio} \times \text{Tax Adjustment Multiplier}}{\text{Interest Yield on New Bonds}}$$

(denoted by  $Z_1$ ), is measured on the horizontal axis.<sup>20</sup> The tax adjustment factor is an estimate of the multiplier referred to in Subcase I-b, and is made necessary by the fact that the available earnings-price ratio series relate to earnings net of taxes.

On the basis of the considerations developed above we should be led to expect a negative correlation between the two variables. As a matter of fact, it will be seen that large values of  $Z_1$  were generally accompanied by relatively low values of  $X_1$ . On the other hand, although low values of  $Z_1$

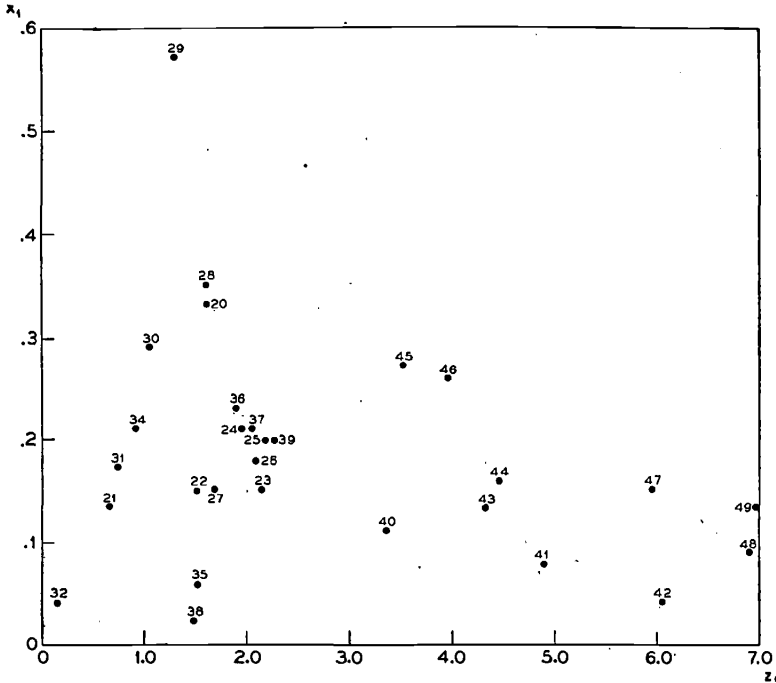
<sup>17</sup> On this point see Appendix C, Note 7.

<sup>18</sup> See Appendix B, ref. [4].

<sup>19</sup> For methods and sources used in computing this variable see the definition of the variable  $Z_1$  in Appendix A and the references quoted there.

<sup>20</sup> The year 1933 was eliminated in this graph as in all the remaining graphs and all the computations in this paper, since  $X_1$  for this year is seriously distorted as a result of the repeal of Prohibition. Because the total issues in this year were very small, the relatively large stock issues by breweries, malt manufacturers, etc., make the ratio of stock financing to total financing unrepresentative.

Figure 2

SCATTER DIAGRAM OF RELATION BETWEEN  $X_1$  AND  $Z_1$ 

were accompanied by relatively large values of  $X_1$  in such prosperous years as 1928 and 1929, equally low or even lower values of  $Z_1$  occurring in years of severe depression (e.g., 1921, 1931, 1932) were accompanied by a very low share of equity financing. The reason for this phenomenon is to be sought, we feel sure, in the unrepresentativeness of current earnings as a measure of long-term earnings expectations.<sup>21</sup> In years of exceptionally low earnings, long-term earnings expectations are sure to be above current earnings. Thus, even though stock prices may be high relative to current earnings, this need not prove an incentive to equity financing; they may still be, and are in fact likely to be, low relative to long-term earning expectations.

This suggests that a more reasonable way of deriving a measure of expected average long-term earnings from the type of data we are now considering would be to take an average of the earnings of several past years. If, however, we take a simple average and include in it a number of

<sup>21</sup> It may be noted in this connection that the removal of the tax adjustment factor from  $Z_1$  does not appreciably affect the general shape of the scatter.

years sufficiently large to make the average meaningful, the resulting measure has the undesirable quality of "trailing." For example, during the late twenties a long-period simple average tends to decline because the very high earnings of the World War I years are being dropped from the average. This, as well as other considerations, suggests that in forming such an average the most recent experience should receive extra weight. Unfortunately there is not much to guide us in the choice of a specific weighting formula. We finally chose as our measure of expected earnings a ten-year moving average of earnings gross of taxes in which the current year receives roughly a third of the total weight.<sup>22</sup> While this formula is admittedly quite arbitrary, it should be remembered that variation in the weighting system within reasonable limits does not, in general, affect the result significantly, and this was confirmed in our case by some experiments that were carried out.

We also made use of an entirely different measure of long-term earnings expectations which does not involve an arbitrary weighting system, though it suffers from shortcomings of its own: namely, the current dividend yield (adjusted for taxes).<sup>23</sup> The use of this variable is suggested by the apparently widespread tendency on the part of management to follow a policy of dividend stabilization. To the extent that this tendency exists, dividend rates would tend to be set at levels capable of being maintained by anticipated future earnings (net of taxes).

Two serious shortcomings in this measure are worth mentioning. In the first place, dividends are also affected by certain temporary factors — for example, by the undistributed profits tax of 1936 and 1937. Secondly, this measure may be expected to underestimate systematically anticipated *total* earnings, since it only measures that *portion* of anticipated earnings which management feels it is wise to distribute, on the average, over a period of years. Neither of these difficulties however should reduce the representativeness of this series below the level of usefulness. The second shortcoming in particular should still permit us to measure the *relative* movement of earnings expectations, provided the above-mentioned portion remains approximately constant in time. The absolute level and movements of this series, however, cannot properly be made use of.

With one further remark we may conclude this discussion of empirical equivalents to expectations data. Neither of our two tentative measures of the behavior of long-term profit expectations can be considered really satis-

<sup>22</sup> For details on the methods used see the definition of the variable  $Z_2$  in Appendix A and the references quoted there.

<sup>23</sup> For sources of data and the method of adjusting for taxes see the definition of the variable  $Z_3$  in Appendix A and the references quoted there.

factory, but it may be hoped that they represent reasonably good first approximations. This hope derives some slight support from the fact that the behavior of our two measures turns out to be surprisingly similar and the results of the statistical analysis were substantially the same with either variable.

Having thus somehow settled the problem of measuring the group of variables that were suggested by both the "certainty" and the "uncertainty" model, we may turn to the problem of measuring the subjective uncertainty surrounding the estimate of expected returns, which in our model was represented by the dispersion of the subjective probability distribution of anticipated outcomes. This is again a variable which obviously cannot be directly observed or measured and the best we can do is to try to secure a reasonable approximation. We felt there was some justification for attempting to measure this variable by means of the actual standard deviation of profits over the recent past.

The reasoning underlying this measure is, briefly, that management's subjective probability distribution is largely a reflection of its recent experience.<sup>24</sup> The choice as to number of years that should be included in the computation of this standard deviation is again pretty much in the nature of a shot in the dark. However, having chosen a ten-year average in connection with the problem of anticipated earnings, it appeared reasonable to use the same span of time as a basis for computing the standard deviation. Because we felt it was essential for the purpose of this computation that we use earnings gross of interest and taxes, as indicated by our theoretical analysis, these computations were based primarily on Bureau of Internal Revenue data<sup>25</sup> rather than on the type of data used in computing the expected-earnings/price ratio.<sup>26</sup>

<sup>24</sup> The statistical equivalent of this statement may clarify the concept. Where profit data for past years can reasonably be assumed to be a sample of the constant universe of annual profit data, the near future may be thought of as another sample from this universe. The problem of prediction then becomes one of estimating from the mean and dispersion of one sample (past earnings) the mean of another sample from the same population (mean earnings over some future span) and the dispersion of this mean.

<sup>25</sup> For further details on sources and methods of computation see the definition of the variable  $X_2$  in Appendix A and the references quoted there.

<sup>26</sup> In computing the latter variable it was essential that the data on earnings (or dividends) and on stock prices should refer to the very same sample. This was the reason for using the type of sample data indicated in Appendix B, ref. [2]. These data, however, refer to earnings net of both interest and taxes; while it was possible to introduce some adjustment for taxes, there did not seem to be any simple way of making an adjustment for interest payments. From B.I.R. data, on the other hand, profits gross of both interest and taxes could be easily obtained for most years.

One more problem that remains to be settled relates to the specific form in which our variables should enter into the regression equation. Considerations derived from our theoretical analysis suggested using the difference between the expected-earnings/price ratio and the interest rate as one variable and the standard deviation of past profits adjusted for the growth in outstanding equity as a second variable.<sup>27</sup> This was precisely the approach used when the expected earnings were approximated by the average of past earnings; accordingly, our dependent variable  $X_1$  was related to the two variables mentioned above, denoted respectively by  $Z_2$  and  $X_2$ .

We get very little help from our theoretical analysis as to the specific form of the relation between the three variables, except for information relating to the expected sign of the partial derivatives. However, several considerations, including examination of the data, lead us to the use of a semi-logarithmic relation, the variable  $X_1$  being related to logarithms of  $Z_2$  and  $X_2$ .

The relation between  $X_1$  and  $Z_2$  is shown graphically by the scatter diagram of Figure 3. Clearly this scatter represents a vast improvement over the corresponding scatter of Figure 2. This is confirmed by a correlation of  $-.72$  (as compared with a correlation in the order of zero for the series of Figure 2), which is quite significant when account is taken of the relatively large number of observations: twenty-nine. Unfortunately, the introduction of  $\log X_2$  as a third variable gave very disappointing results. The partial correlation of  $X_1$  with this variable was very small and definitely insignificant; furthermore, the regression coefficient was negative, contrary to what we should have expected.

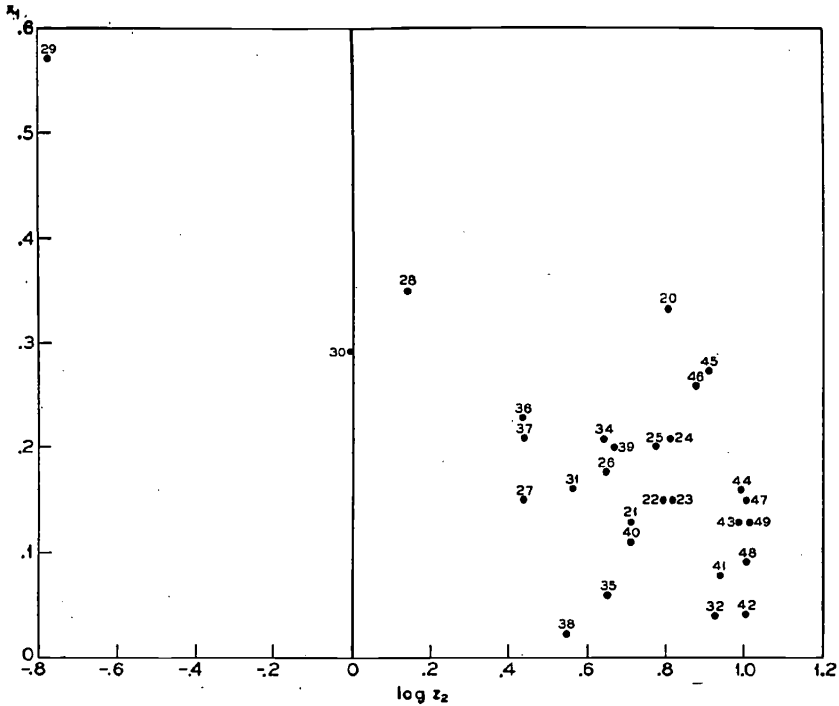
In using dividend payments as a measure of expected earnings, a modification appeared necessary in the form of the first independent variable. Because of the probable systematic downward bias in the dividend yield as a measure of earnings expectations, mentioned earlier, we did not feel justified in making use of the difference between the dividend yield and the interest rate. Instead, as a measure of the relation between the interest rate and the expected-earnings/price ratio as approximated by the dividend yield, we used the ratio of the former to the latter variable; in principle, with a ratio form, the correlation should not be too much affected by the aforementioned bias. The resulting variable is denoted hereafter by  $Z_3$ .<sup>28</sup>

Tests were carried out with linear as well as semi-logarithmic forms

<sup>28</sup> For an exact definition and for sources of data for the variable  $Z_3$  see Appendix A and the references quoted there.

<sup>27</sup> See Note 7, Appendix C.

Figure 3

SCATTER DIAGRAM OF RELATION BETWEEN  $X_1$  AND  $\log Z_2$ 

with very similar results. In the first case  $X_1$  was related to  $Z_3$  and  $X_2$ . Once more we found a significant correlation between the first two variables (.56; the correlation is now positive because of the form of the variable  $Z_3$ ), but the partial correlation with  $X_2$  was again insignificant. In the second case  $X_1$  was related to  $\log Z_3$  and to the square of this variable, which again yielded a significant correlation (.61); but no significant results were obtained for  $X_2$ .

The analysis carried out thus far, then, presents fairly clear evidence of the influence of the "cost" variable suggested by *both* the "certainty" and the "uncertainty" analysis; on the other hand, these tests fail to support our conclusion concerning the influence of the "uncertainty" variable. It is difficult, however, to tell whether this is due to a failure of our theoretical construct, or, as seems more likely, to our inability thus far to secure a valid measure of the theoretically relevant concept. It may be that management's estimate of risk is not formed on the basis of recent past experience, or that the data we have are too inaccurate actually to measure past expe-

rience. In spite of our present failure we feel that further research in this field should prove fruitful.

In addition to testing the influence of the variables directly suggested by our theoretical analysis of rational behavior, we also conducted a number of tests to ascertain the possible influence on our dependent variable of a number of factors that have been suggested by other students or which are suggested by empirical observations.

A variable that has elicited much interest recently is the debt-equity ratio.<sup>29</sup> The difficulties in accurately measuring this ratio are numerous and have been dwelt upon at great length at this conference. In spite of the inadequacy of the data, however, we decided to make some attempt at including it in our empirical tests.<sup>30</sup> Perhaps because of the inadequacy of the measure, or because management does not think of equity in terms of its book value, this variable was not of any help in explaining the distribution of new financing between debt and equity forms. Its year-to-year movements are minor and, by and large, in the wrong direction. Our debt-equity ratio is lower in the late forties, when relatively little stock financing was done, than in the late twenties; and it is higher in the mid-thirties than in the mid-twenties, though the proportion of new financing in the form of stocks is approximately the same in the two periods.

As a result of interview data, several writers have suggested that the ratio of the market value to the book value of equity is an important determining factor in bond-stock financing decisions.<sup>31</sup> An examination of

<sup>29</sup> It should be noted that the influence of the debt-equity ratio has already been taken into account in our analysis insofar as the amount of debt determines the size of prior claims on earnings, i.e., the interest burden. For, other things being equal, the larger the proportion of debt in the capital structure the smaller will tend to be the ratio of the numerator to the denominator of (7.1) or (7.2) given in Note 7, Appendix C.

<sup>30</sup> For our debt series we used E. T. Bonnell's compilation of long-term debt in *Survey of Current Business*, October 1949, p. 8. Short-term debt was not included because of its trade debt component. Rises in trade debt are offset by rises in trade receivables and we felt that it should therefore not be included in the debt-equity ratio.

Our equity series was based on the Bureau of Internal Revenue data published in the *Statistics of Income for 1944*, Part II, pp. 406-9; in Press Release S-782 (July 9, 1948), Table 3, p. 17, and S-1051 (April 21, 1949) Table 3, p. 17; and as computed and listed in Martin Taitel, *Profits, Productive Activities, and New Investment* (T.N.E.C., Monograph No. 12, 1941), Appendix I, pp. 138-42. A somewhat oversimplified, but necessary, adjustment was introduced on the basis of retained earnings and new external funds in order to make the data for years when consolidated returns were permitted comparable to the data based on unconsolidated returns. The data were extrapolated to 1947-49 on the basis of sources and uses of funds data published by the Department of Commerce.

<sup>31</sup> See, for example, the comments by Dan T. Smith delivered at this conference.

scatter diagrams utilizing our version of this ratio, however, convinced us that the weak influence this factor seems to exert would work in a direction that is contrary to the one postulated.<sup>32</sup>

A "dynamic" factor not directly suggested by our essentially static analysis, but one that may be supposed to exert some influence on the distribution of new financing between debt and equity forms did turn out to be of considerable importance. This factor was the relation of the level of current stock prices to the level prevailing in the recent past. The rationale of introducing this factor is that even when the differential between cost factors is not too favorable to stock financing, if stock prices are high relative to their recent levels, a sort of historical relative advantage may be achieved which may encourage stock financing; and vice versa. In other words, the introduction of this price variable implies an "if-I'm-ever-going-to-do-it-now-is-the-time" type of thinking on the part of management.

For purposes of statistical testing this variable was measured by the ratio of current stock prices to the average level of stock prices in the previous five years; it is denoted hereafter by  $X_3$ .<sup>33</sup> The statistical results obtained by using this variable in addition to the one measuring the relation between the earnings-price ratio and the interest rate are given below:<sup>34</sup>

I. Long-term earnings expectations measured by an average of past earnings:

$$\text{Regression Equation I: } X_1 = .14 - \underset{(.04)}{.15} (\log Z_2) + \underset{(.046)}{.13} X_3$$

$$\text{Multiple Correlation I: } .80$$

II. Long-term earnings expectations measured by current dividend payments:

$$\text{Regression Equation IIA: } X_1 = -.17 + \underset{(.05)}{.18} Z_3 + \underset{(.05)}{.20} X_3$$

$$\text{Multiple Correlation IIA: } .78$$

Regression Equation IIB:

$$X_1 = -.01 + \underset{(.13)}{.43} (\log Z_3) + \underset{(.28)}{.72} (\log Z_3)^2 + \underset{(.04)}{.19} X_3$$

$$\text{Multiple Correlation IIB: } .81$$

All computations were based on the years 1920 through 1949 but do not

<sup>32</sup> For the sources of data used to compute book value, see footnote 30 above. For the market value, see the definition of the variable  $X_2$ , Appendix A, and the sources quoted there.

<sup>33</sup> For methods and sources used in computing this variable see Appendix A and the references quoted there.

<sup>34</sup> The addition of our measure of dispersion  $X_2$  as an additional independent variable again gave insignificant results which are therefore not presented here.

include 1933.<sup>35</sup> The figures given in parentheses below the regression coefficients are the standard errors of the corresponding coefficient.

These results are fairly satisfactory. All regression coefficients have the expected signs and are quite significant by the customary statistical tests.<sup>36</sup> Furthermore, the two independent variables are seen to account for a substantial proportion of the variation of  $X_1$ . It may also be noted that there is substantial agreement between the results obtained with either of our two measures of expected profits, though the contributions of the rate of change of prices ( $X_3$ ) appears somewhat greater with the use of dividends (Equations II) than with the use of past earnings (Equation I). This similarity of the results is also confirmed by a comparison of the residuals, which are presented for Equations I and IIB in the two panels of Figure 4.

Before concluding, a few words about the over-all meaning of the relationships tested are in order. In this type of analysis it is always pertinent to ask whether the observed joint distribution of the variables represents a demand curve, a supply curve, or the intersection "points" of demand and supply curves. We have strong reasons to suppose that the relationships exhibited above do represent the demand function for funds on the part of the corporate sector, as we have assumed, since, to a decision-maker dealing only with new financing, all the elements of the cost factors can reasonably be assumed to be fixed and not influenced by his actions. The corporate income tax rate is a governmentally fixed parameter. Management's estimate of anticipated per share earnings is assumed to be primarily based on past data. The interest rate on *all* corporate bonds is strongly affected by the actions of the Treasury Department and the Federal Reserve System. However, even if the influence of these governmental agencies were weak, the interest rate on *new* bonds can still be considered to be "given" since this rate is determined in a market where new bonds constitute only a small proportion of the total amount of bonds traded. Similarly, the issue price of new stock is determined in the market for all outstanding stock, of which it forms only a minor part. Thus, the independent variables used in explaining the common stock to common-stock-plus-long-term-debt new issues variable, may reasonably be considered to be parameters of action to which financial management adjusts.

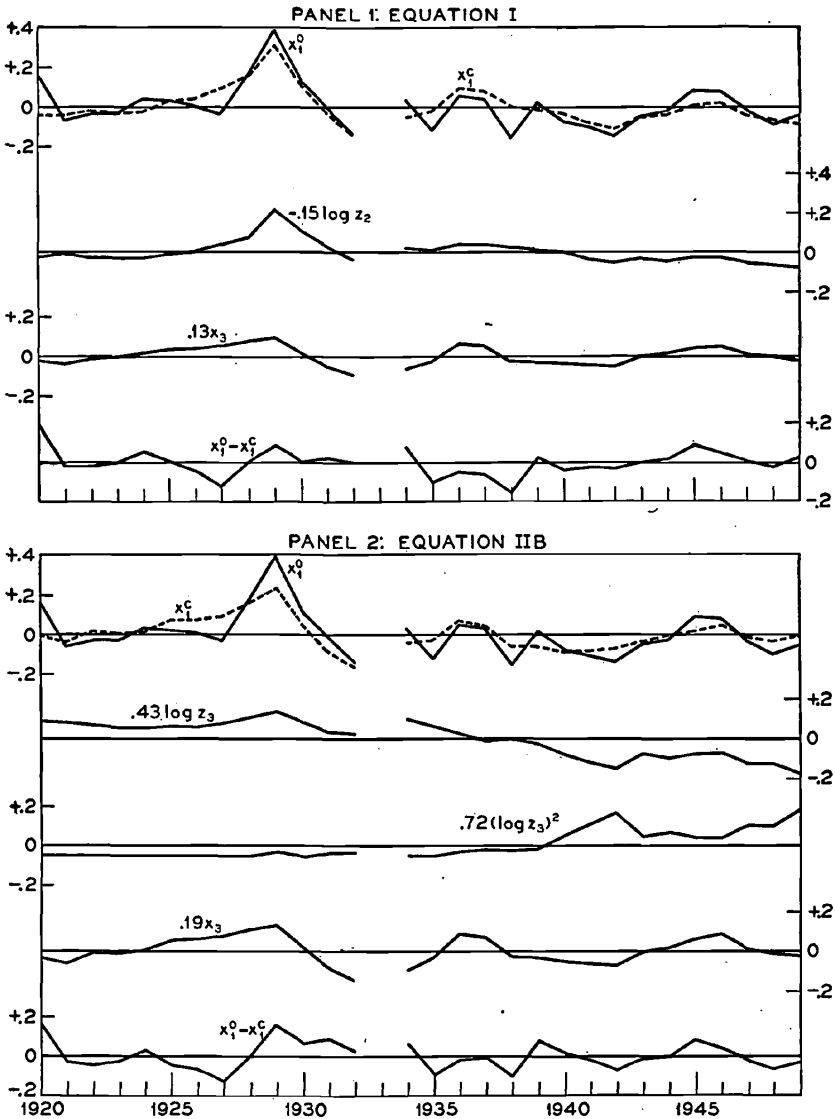
By way of conclusion we should like to stress once more that the statistical analysis presented above should be considered as a mere beginning. As such it may be regarded as fairly promising. In particular, the influence

<sup>35</sup> The reason for omitting 1933 was explained in footnote 20 above.

<sup>36</sup> All coefficients are significant at the 1 percent level or better except the coefficient of ( $\log Z_3$ ) in Equation IIB, which barely fails to reach this level.

Figure 4

THE COMPONENTS OF EQUATIONS I AND IIB AND A COMPARISON OF ORIGINAL AND COMPUTED VALUES OF THE DEPENDENT VARIABLE



$x_1^0$  denotes the original values of the dependent variable.  
 $x_1^c$  denotes the computed values of the dependent variable.  
 For definition of other variables see text.  
 All variables are expressed in terms of deviations from their respective means.

of the relation between the expected-earnings/price ratio and the interest rate on the share of equity financing, which was suggested by the theoretical analysis, receives fairly clear support. While it is true that the use of this variable, supplemented by the rate of change of prices, still failed to account for a non-negligible proportion of the fluctuations of the dependent variable, it should be remembered that, according to our tests, none of the other factors which have most frequently been suggested appear to be of any real relevance.

For further research in this field the development of more reliable and suitable basic data should be placed very high on the agenda. As such data become available it will be possible to retest our results and to improve the statistical analysis. In place of the over-all aggregates we have used it would be particularly useful to substitute more homogeneous series on new capital flotations, measuring the new financing of smaller industry groups. The work now being completed at the National Bureau will provide a more accurate series on net debt issues with which to retest our conclusions.

On the anticipated earnings side, much still remains to be done. Perhaps, eventually, a series based on questionnaires may become available. Of course, such a series would itself have to be tested for accuracy, but it should certainly be given a trial. Also, as more is learned about the formation of expectations, more accurate use of historical data may become possible.

Although, as mentioned earlier in the paper, this type of analysis can only be tested by means of mass observations, this does not bar a test that utilizes data of a cross section of firms within a given industry group over a shorter period of time. The difficulties involved in such a project are great, and the assembled data meager; nevertheless, further tests using this type of data would seem to be worth the effort involved.

On the negative side, we can only view with disappointment our abortive attempt at including a measure of uncertainty in the tests. Nevertheless, we still feel that considerations of risk are important and that further tests should attempt to include other measures of this concept. In this connection again, questionnaire data may be a useful means of discovering more reasonable measures.

We may conclude much as we began. The theoretical as well as the statistical analysis presented in this paper is really only a beginning. As the analysis is broadened to include other types of financing, and to take into account other factors, some of which have been mentioned at various places in this paper, several revisions and substantial additions will have to be made. In the process it may be hoped we will come to understand more fully the influence of financial factors on investment decisions.

## APPENDIX A

## DEFINITIONS OF VARIABLES USED IN THE STATISTICAL ANALYSIS OF SECTION IV

The number in brackets after each series refers to the number in Appendix B which precedes the source of the series. The variables are listed in the order in which they appear in the text.

$$X_1: \frac{\text{Common-Stock New Capital Issues [1]}}{\text{Common-Stock plus Long-Term-Debt New Capital Issues [1]}}$$

This series is reproduced in Column 1 of Appendix Table A.

$$Z_1: \frac{\text{Current-Earnings/Price Ratio [2]} \div (1 - \text{Marginal Tax Rate}) [3]}{\text{Interest Yield on New Bonds [4]}}$$

$$Z_2: \frac{.075 \sum_{j=0}^9 \frac{e_{t-j}}{(1-\tau_{t-j})} + .25 \frac{e_t}{1-\tau_t}}{\text{Index of Current Stock Prices [2]}} - \text{Interest Yield on New Bonds [4]}$$

$t$ : current year

$e_{t-j}$ : earnings per share [2]

$\tau_{t-j}$ : average corporate tax burden [5]

This series is reproduced in Column 2 of Appendix Table A.

$$Z_3: \frac{\text{Interest Yield on New Bonds [4]}}{\text{Current Dividend Yield [2]} \div (1 - \text{Marginal Tax Rate}) [3]}$$

This series is reproduced in Column 3 of Appendix Table A.

$$X_2: \frac{\text{Standard Deviation of Gross Corporate Earnings in the Latest Ten Years [6]}}{\text{Market Value of Shares Listed on the New York Stock Exchange in Current Year [7]}}$$

This series is reproduced in Column 4 of Appendix Table A.

$$X_3: \frac{P_t}{\frac{1}{5} \sum_{j=1}^5 P_{t-j}}$$

$t$ : current year

$P$ : annual index of common stock prices [8]

This series is reproduced in Column 5 of Appendix Table A.

Table A  
STATISTICAL SERIES USED IN SECTION IV

Year	(1) $X_1$	(2) $Z_2$	(3) $Z_3$	(4) $X_2$	(5) $X_3$
1920	.33	6.37	1.041	.116	.93
1921	.13	5.14	1.000	.153	.81
1922	.15	5.49	.952	.112	1.06
1923	.15	6.74	.901	.108	1.08
1924	.21	6.51	.893	.087	1.11
1925	.20	5.60	.962	.080	1.36
1926	.18	4.45	.909	.073	1.41
1927	.15	2.71	.971	.058	1.48
1928	.35	1.39	1.162	.051	1.67
1929	.57	.17	1.369	.046	1.79
1930	.29	.99	1.063	.052	1.16
1931	.17	3.65	.758	.082	.67
1932	.04	8.51	.741	.214	.35
1933	<sup>a</sup>	<sup>a</sup>	<sup>a</sup>	<sup>a</sup>	<sup>a</sup>
1934	.21	4.35	1.111	.161	.66
1935	.06	4.47	.885	.142	.92
1936	.23	2.71	.719	.097	1.56
1937	.21	2.77	.625	.094	1.49
1938	.02	3.50	.662	.110	.96
1939	.20	4.59	.595	.080	.98
1940	.11	5.11	.422	.090	.88
1941	.08	8.71	.339	.128	.79
1942	.04	12.04	.289	.181	.74
1943	.13	9.31	.415	.168	1.09
1944	.16	9.72	.385	.164	1.18
1945	.27	8.15	.429	.130	1.42
1946	.26	7.33	.431	.112	1.51
1947	.15	10.46	.345	.132	1.18
1948	.09	12.47	.342	.123	1.08
1949	.13	13.48	.278	.102	1.00

<sup>a</sup> 1933 has been omitted from all computations. See footnote 20 above.

## APPENDIX B

## SOURCES OF DATA USED IN THE STATISTICAL ANALYSIS OF SECTION IV

- [1] Common-Stock and Long-Term-Debt New Capital Issues:  
*Commercial and Financial Chronicle*, Table on "Capital Flotations for the Year," annual volumes. Data are annual totals and include all industries except investment trusts, holding companies, etc.
- [2] Earnings, Dividend, and Price Data:  
 1911 through 1937: Cowles Commission Monograph No. 3, *Common Stock Indexes* (Bloomington: Principia Press, 1940). The earnings-price ratios are found in Table R-1, the earnings per share series is found in Table E-1, the dividend yields are found in Table  $Y_a-1$ , and the price series is found in Table P-1.  
 1938 through 1948: Moody's Investors Service, *Industrial Manual*, 1949, p. a20. In computing the ten-year moving average of earnings per share (see numerator of first term of  $Z_2$ ) for the years 1938 through 1949 Moody's data from 1929 on were used.  
 1949: Moody's Investors Service, current releases.
- [3] Marginal Tax Rate:  
 National Industrial Conference Board, *Economic Almanac for 1949*, pp. 146-49. The highest bracket marginal tax rate was used throughout since we are dealing largely with corporations to which this rate applies.
- [4] Interest Yield on New Bonds:  
 Moody's Investors Service, *Industrial Manual*, 1949, p. a17. For 1920 and 1949 the yield on new bonds was not available; the composite yield on all bonds was used instead.
- [5] Average Corporate Income Tax Burden: \*  
 1920 through 1922: B.I.R. "Total Taxes" divided by B.I.R. "Net Income"; *S. of I.*, 1944, Part II, p. 372.  
 1923 through 1928: B.I.R. "Total Taxes" (*S. of I.*, 1944, Part II, p. 372) divided by B.I.R. "Compiled Net Profits" (*S. of I.* for 1923 through 1925, annual volumes; for other years, 1944, Part II, pp. 376 ff.).  
 1929 and 1934 through 1946: D. of C. "Corporate Tax Liability" (*S.C.B.* National Income Supplement, July 1947, p. 30, and National Income Issue, July 1950, p. 17) divided by B.I.R. "Compiled Net

\* The abbreviations used in [5] through [8] have the following meanings: B.I.R., Bureau of Internal Revenue; *S. of I.*, *Statistics of Income*; D. of C., Department of Commerce; *S.C.B.*, *Survey of Current Business*.

Profits" (*S. of I.*, 1944, Part II, pp. 376 ff.; and Press Releases S-782, July 9, 1948, and S-1051, April 21, 1949). The shift from B.I.R. to D. of C. data for the estimate of tax liability was suggested by the increasing importance of state income taxes. However, it was thought preferable to use the B.I.R. profits series as far as available as it is likely to be a better approximation to profits as seen by management.

1947 through 1949: D. of C. "Corporate Tax Liability" (*S.C.B. National Income Supplement*, July 1947, p. 30, and *National Income Issue*, July 1950, p. 17) to D. of C. "Corporate Income before Taxes" (*S.C.B.*, July 1950, p. 17).

1931 through 1933: No ratio representing the tax burden was computed for these years because of negative profits (i.e. losses). The following adjustment based on the year 1929 was used instead:

The Cowles Commission figure for earnings per share in 1929 is approximately \$1.60 less than the figure we get by correcting for taxes. According to the Department of Commerce, total tax liability in 1929 was \$1.4 billion. This indicates that a \$1 billion total tax liability is equivalent to \$1.15 tax liability per share. For each of the years 1931, 1932, and 1933, therefore, the Department of Commerce tax liability was multiplied by \$1.15 per billion dollars and this amount was added to the Cowles Commission earnings per share figure. The adjustments were: 1931, \$.56; 1932, \$.45; 1933, \$.56. Carrying this adjustment on to 1934 and 1935 would have given adjustments of \$.79 and \$1.13 respectively. Using the direct tax burden the adjustments are \$.88 and \$.90 respectively.

The same procedure was used to compute the 1931, 1932, and 1933 tax correction adjustment for use with Moody's data. The coefficient in this case was \$.478 per billion dollars of tax liability and the adjustments were: 1931, \$.24; 1932, \$.19; 1933, \$.25.

[6] Corporate Earnings Gross of Interest and Taxes:

The sum of two series: 1) Net Profits before Taxes and 2) Corporate Interest Payments.

*Net Profits before Taxes:*

1910 through 1922: The B.I.R. "Net Income" series was adjusted to make it comparable to the B.I.R. "Compiled Net Profits" series (see [5] above) on the basis of the relation between these two series for the next few subsequent years. Specifically, the "Net Income" series was divided by .9 in each of the years 1910 through 1919, by .85 in 1920 and 1921, and by .83 in 1922. This adjustment is obviously no more than a rough guess but this need not be serious in view of the relatively small size of the adjustment and of the fact that the yearly data were used only to compute a ten-year standard deviation and not individually.

1923 through 1946: B.I.R. "Compiled Net Profits" (see [5] above).

1947 through 1949: D. of C. "Corporate Income before Taxes" (see [5] above).

*Corporate Interest Payments:*

1910 through 1916: A graphical extrapolation of the B.I.R. "Interest Paid" series based on W. I. King's "Net Interest on Funded Debt Paid to Individuals," found in his *The National Income and Its Purchasing Power* (National Bureau of Economic Research, 1930) p. 186.

1917 through 1946: B.I.R. "Interest Paid" series (*S. of I.*, 1944, p. 372 and Press Releases cited above).

1947 through 1949: D. of C. "Interest Payments" for all industries.

[7] Market Value of All Shares Listed on the New York Stock Exchange:

*S.C.B.*, March 1950, p. S-20, and *Statistical Supplements*, 1932, 1938, and 1949, pp. 104, 76, and 104 respectively. This series is only available from 1925 to date. Values for the years 1920 through 1923 were estimated by graphical extrapolation, using the Cowles Commission stock indexes (Monograph No. 3, *Common Stock Indexes*, Table P-1) as follows (in billions of dollars): 1920, 22.0; 1921, 18.0; 1922, 23.5; 1923, 24.0. For 1924 the figure for January 1, 1925 was used.

[8] Average Annual Stock Prices:

1915 through 1917: Cowles Commission Price Index (see [7] above).

1918 through 1949: Standard and Poor's Index, reprinted in *S.C.B.*, March 1950, p. S-20, and *Statistical Supplements*, 1938 and 1949, pp. 77 and 104 respectively.

## APPENDIX C

### NOTE 1

For use in this paper (except as noted at the appropriate places) we define the symbols as follows:

$\pi$  = the anticipated net, long-run, average, yearly return to existing stockholders, net of interest and of corporate income taxes.

$\Pi_0$  = the anticipated net, long-run, average, yearly return to stockholders if no new capital is acquired, net of interest and of corporate income taxes.

$B$  = the amount of new capital to be raised through sale of debt instruments.

$S$  = the amount of new capital to be raised through sale of common stock.

$F = B + S$  = the total amount of new capital to be raised.

$Y = Y(F)$  = the anticipated long-run, average, yearly return on new capital, gross of interest and of corporate income taxes.

$\rho = \rho(F) = dY(F)/dF$ .

$r$  = the effective current interest rate on debt funds (includes allowances for premium or discount and costs of flotation or placement).

$P$  = the current per share net proceeds obtainable through sale of common stock (includes allowance for costs of flotation).

$n$  = the number of "old" shares of common stock outstanding.

$\tau$  = the anticipated effective corporate income tax rate. (Strictly speaking, our analysis applies only to a situation in which the expected tax rate is constant and in which the tax laws allow full carry-forward and carry-back privileges over the period of the estimate.)

$\alpha = (1 - \tau)$ .

The following remarks concerning these definitions may be made:

- 1)  $\Pi_0$  is defined net of interest and taxes because this corresponds to the form in which much of the empirical data are found.  $Y(F)$  is defined gross of these items because the amount of these charges depends on the type of financing employed. Using our previous definitions, the anticipated long-run mean return to stockholders if no new capital is acquired, net of interest, but *gross of corporate income taxes* is  $\Pi_0/\alpha$ .
- 2) To conserve space, the following proofs include corporate income tax considerations. Proofs of the conclusions set forth in Subcase I-a in the text may be obtained by merely setting  $\tau = 0$ ,  $\alpha = 1$ .
- 3) With few exceptions, which are clearly indicated, the analysis has been limited to cases where

$$(1.0) \quad F, B, S, \cong 0$$

Although negative values of  $F$ ,  $B$ ,  $S$ , can be given definite meaning, their inclusion in the analysis complicates it considerably while yielding conclusions that are strikingly similar to the ones exhibited. In particular, complications are encountered because the  $Y(F)$  function may be discontinuous at  $F = 0$ , and because the latitude to retire common stock allowed management by the courts differs so greatly in different jurisdictions.

If the only financing instrument used is bonds, acquiring and investing a "lump" sum  $\bar{B}$  will increase the "return" to existing stockholders if

$$(1.1) \quad \frac{\Pi_0 + \alpha[Y(\bar{F}) - \bar{B}r]}{n} > \frac{\Pi_0}{n} \quad \text{with } \bar{F} = \bar{B}$$

which reduces to

$$(1.2) \quad \frac{Y(\bar{F})}{\bar{F}} > r$$

Again considering bonds to be the only financial instrument used, the "return" to existing stockholders in the continuous case is given by

$$(1.3) \quad \pi = \Pi_0 + \alpha[Y(F) - Br] \quad \text{with } F = B$$

and the condition for this "return" to be maximized is given by

$$(1.4) \quad \frac{d\pi}{dB} = \alpha[\rho(F) - r] = 0$$

$$\text{i.e., } \rho(F) = r \quad \text{provided } \frac{d\rho(F)}{dF} < 0$$

The optimum value of  $B$  in this case is therefore  $F^*$ , where  $F^*$  is the value of  $F$  satisfying (1.4).  $F^*$  will be positive only if

$$(1.5) \quad \rho(0) > r$$

If condition (1.5) does not hold,  $F^*$  will be zero because of (1.0). If we drop this latter condition the optimum plan may call for no change in amount of bonds outstanding or the repayment of debt by means of a reduction in assets. The first is highly likely in view of the discontinuity of the  $Y(F)$  function at  $F = 0$ . If the second is called for, the amount of repayment will depend on the volume of bonds outstanding. In either case the optimum plan may not fulfill condition (1.4).

## NOTE 2

With the symbols defined as in Note 1, if common stock is the only financing instrument available, acquiring and investing a "lump sum" will increase the "return" to existing stockholders if:

$$(2.1) \quad \frac{\Pi_0 + \alpha Y(\bar{F})}{n + \bar{S}/P} > \frac{\Pi_0}{n} \quad \text{with } \bar{F} = \bar{S}$$

which reduces to

$$(2.2) \quad \frac{Y(\bar{F})}{\bar{F}} > \frac{\Pi_0/\alpha}{nP}$$

Again considering common stock to be the only financial instrument used, the "return" to existing stockholders in the continuous case is given by

$$(2.3) \quad \pi = \frac{n}{n + S/P} [\Pi_0 + \alpha Y(F)] \quad \text{with } F = S$$

and the condition for this return to be a maximum is given by

$$(2.4) \quad \frac{d\pi}{dS} = \frac{\alpha n}{n + S/P} \left[ \rho(F) - \frac{\Pi_0/\alpha + Y(F)}{nP + S} \right] = 0$$

i.e.,  $\rho(F) = \frac{\Pi_0/\alpha + Y(F)}{nP + S}$  provided  $\frac{d\rho(F)}{dF} < 0$

The optimum value of  $S$  in this case is therefore  $F^0$  where  $F^0$  is the value of  $F$  satisfying (2.4).  $F^0$  will be positive only if

$$(2.5) \quad \rho(0) > \frac{\Pi_0/\alpha}{nP}$$

If condition (2.5) does not hold, the optimum value of  $S$  will be zero because of (1.0). If we drop this latter condition, the optimum financing plan may call for no change in the outstanding stock or for repurchase of outstanding stock by means of a reduction in the assets. The stringency of legal rules governing repurchase of stock as well as the discontinuity in the  $Y(F)$  function at  $F = 0$  makes the former the more likely solution. In either case the optimum plan may not satisfy condition (2.4).

### NOTE 3

If both bonds and stocks are eligible financing instruments, the earnings accruing to existing stockholders are given by:

$$(3.1) \quad \pi = \frac{n}{n + \frac{S}{P}} \left\{ \Pi_0 + \alpha [Y(F) - Br] \right\}$$

and the problem is to maximize  $\pi$  subject to condition (1.0) of Note 1. The usual first order maximum conditions yield the equations:

$$(3.2) \quad \frac{\partial \pi}{\partial B} = \frac{\alpha n}{n + S/P} [\rho(F) - r] = 0 \quad \text{i.e. } \rho(F) = r$$

$$(3.3) \quad \frac{\partial \pi}{\partial S} = \frac{\alpha n}{n + \frac{S}{P}} \left[ \rho(F) - \frac{\Pi_0/\alpha + Y(F) - Br}{nP + S} \right] = 0$$

Suppose, now, equation (3.2) is satisfied by  $F = F^* > 0$  (the condition for which is given by (1.5), Note 1). Substituting in (3.3)  $F^*$  for  $F$ ,  $r$  for  $\rho(F^*)$ , and  $(F^* - S)$  for  $B$ , we obtain  $\frac{d\pi}{dS}$ , the change in existing stockholders' "profit" per unit substitution of equity money for debt money in a

financing plan to acquire a total of  $F^*$  dollars. We then find that the derivative  $\frac{d\pi}{dS} \gtrless 0$  depending on whether

$$(3.4) \quad r \begin{matrix} > \\ < \end{matrix} \frac{\Pi_0/\alpha + Y(F^*) - F^*r}{nP}$$

an expression that does not depend on  $S$ . Suppose, first, that (3.4) takes the "less than" sign; then  $\frac{d\pi}{dS}$  is negative for any value  $S$  and old stockholders' profits will be maximized when  $S$  has the smallest value consistent with our problem which by condition (1.0) is

$$(3.5) \quad S = 0, \text{ and therefore } B = F = F^*$$

In this case, then, the optimum financial plan involves only bonds, and the optimum amount of bonds is given by (3.5) and (3.2). At the same time equation (3.3) will not be satisfied since  $S$  cannot be negative.

If (3.4) takes the "greater than" sign, existing stockholders' profits increase continuously with  $S$ , within the range  $0 \leq S \leq F^*$ . The greatest value of  $\pi$  within this range must be reached when  $S = F^*$ ,  $B = 0$ . But at this point  $\frac{\partial\pi}{\partial S}$  is still positive and earnings can be further increased by increasing  $S$ . As long as  $B$  is to be non-negative (condition 1.0) this means increasing  $F$  itself beyond  $F^*$  (since if  $S > F^*$  and  $B \geq 0$  then  $F = S + B > F^*$ ). But from (3.2) we see that this implies  $\frac{\partial\pi}{\partial B} < 0$ , so that in this case profits will be maximized by making  $B$  as small as possible; because of condition (1.0) this means  $B = 0$ . The optimum value of  $S = F^0$  can then be found by maximizing (3.1) with respect to  $S$ , with  $B = 0$ , which again yields condition (2.4). The solution will have the properties:

$$(3.6) \quad S = F^0 \geq F^*, B = 0 \text{ as noted in the text.}$$

A brief word about the effects on these conclusions of relaxing condition (1.0) may be in order, in spite of the fact that negative values of  $F$ ,  $S$ ,  $B$  are unlikely to be eligible for the practical reasons of discontinuities at 0 and of institutional restrictions. It is possible that the formal optimum financing plan may involve liquidation of part of the assets of the firm and repayment of outstanding bonds and/or purchase by the corporation of part of its outstanding stock. Under other conditions, the formal optimum plan may call for an issue of stock (bonds) not only large enough to increase the size of the firm, but also to refinance part of the outstanding bonds (stock).

It is interesting to examine the relationship that the above analysis implies between the effective interest rate and the original-earnings/price

ratio. As long as condition (1.5) holds, positive financing is in order and will increase earnings per share, and therefore the right hand side of (3.4) will be larger than  $\frac{\Pi_0/\alpha}{nP}$ . This implies that if  $r < \frac{\Pi_0/\alpha}{nP}$  necessarily  $\frac{d\pi}{dS} < 0$ , but if this inequality is reversed  $\frac{d\pi}{dS}$  may still remain negative.

## NOTE 4

In Note 1, interpret all symbols having to do with "returns" as denoting expected, rather than anticipated, "returns" and in addition let:

$\sigma = \sigma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right]$  = a measure of the dispersion of the subjective probability distribution of outcomes

$\Pi^* = \gamma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right]$  =  $\Pi^*(F)$  = the certainty equivalent of an outcome having an expected value  $\frac{\Pi_0}{\alpha} + Y(F)$ , and a dispersion  $\sigma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right]$ .

In general,  $\gamma$  need not be a constant but may be considered a function of  $Y$ , which in turn is a function of  $F$ , such that

$$(4.1) \quad \gamma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right] < \frac{\Pi_0}{\alpha} + Y(F)$$

$$(4.2) \quad 0 \leq \gamma' \left[ \frac{\Pi_0}{\alpha} + Y(F) \right] = \frac{d}{dY} \gamma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right] < 1$$

$$0 \leq \Pi^{*'}(F) = \frac{d\Pi^*}{dF} < 1$$

Then the "certainty equivalent earnings" accruing to existing stockholders,  $\pi^*$ , is given by:

$$(4.3) \quad \pi^* = \frac{\alpha n}{n + S/P} \left\{ \gamma \left[ \frac{\Pi_0}{\alpha} + Y(F) \right] - Br \right\}$$

Proceeding in a manner similar to the one followed in Notes 1, 2, and 3, we find that the condition for positive bond financing is

$$(4.4) \quad \rho(0) > \frac{r}{\Pi^{*'}(0)}$$

which is more stringent than condition (1.5), Note 1, because of (4.2); and the condition for positive stock financing is

$$(4.5) \quad \rho(0) > \frac{\gamma(\Pi_0/\alpha)/(nP)}{\gamma'(\Pi_0/\alpha)}$$

which, if anticipated and expected values are equal, is more stringent than (2.5), Note 2, if as is usual the marginal risk and the marginal rate at which it is discounted are greater than the respective averages.

Let  $F^{**}$  be the value of  $F > 0$  which maximizes return to old stockholders with respect to bonds alone, i.e., let  $F^{**}$  satisfy

$$(4.6) \quad \frac{\partial \pi^*}{\partial B} = 0 \text{ or } \rho(F^{**}) = \frac{r}{\gamma' \left[ \frac{\Pi_0}{\alpha} + Y(F^{**}) \right]}$$

If both (4.4) and (4.5) hold, the choice between the two types of financing depends on whether

$$(4.7) \quad r \begin{matrix} \geq \\ < \end{matrix} \frac{\gamma \left[ \frac{\Pi_0}{\alpha} + Y(F^{**}) \right] - F^{**}r}{nP}$$

where the "greater than" sign indicates that stock financing is preferable. By comparing this condition to condition (3.4), Note 3, it can be seen that if expected value equals anticipated value, the range of market "prices" for which equity financing will be preferable is widened by taking uncertainty and risk aversion into account, since clearly

$$(4.8) \quad \frac{\gamma \left[ \frac{\Pi_0}{\alpha} + Y(F^{**}) \right] - F^{**}r}{nP} < \frac{\frac{\Pi_0}{\alpha} + Y(F^*) - F^*r}{nP}$$

#### NOTE 5

Let  $\pi$  denote the "expected," future, average, yearly, net return to existing stockholders. This may be thought of as the arithmetic mean of the distribution of anticipated actual earnings, or, as any other measure of central tendency having (at least approximately) the property that if  $K_1$  and  $K_2$  are constant, "expected"  $(K_1X + K_2Y) = K_1$ ("expected"  $X$ ) +  $K_2$ ("expected"  $Y$ ).

Let  $\sigma_\pi$  denote the dispersion of the distribution of anticipated net earnings of existing stockholders; it measures the extent to which the actual outcome is considered capable of deviating from the "expected" outcome.  $\sigma_\pi$  may be thought of as the standard deviation of the distribution, or any other measure of dispersion having the property (at least approximately) that if  $\sigma_X$  is the dispersion of  $X$ , and  $K$  is a constant, the dispersion of  $KX$  is  $K\sigma_X$ .

Similarly we denote by  $\Pi_0/\alpha$  the "expected" total earnings to the

existing stockholders (gross of taxes but net of interest) if no new financing is undertaken; by  $Y(F)$  the "expected" additional earnings before interest and taxes that would result by investing a fresh amount  $F$  in the firm; by  $\rho(F)$  the derivative  $\frac{dY(F)}{dF}$  and by  $\sigma_{II} = \sigma_{II}(F)$  the dispersion of total gross earnings (before interest and taxes).  $\sigma_{II}(0)$  is of course the dispersion of earnings if no new financing is undertaken and corresponds to the expected return  $\Pi_0/\alpha$ . The remaining symbols have the same meaning as in Note 1.

We then have by definition:

$$(5.1) \quad \pi = \frac{\alpha n}{n + \frac{S}{P}} \left[ \frac{\Pi_0}{\alpha} + Y(F) - Br \right]$$

$$(5.2) \quad \sigma_\pi = \frac{\alpha n}{n + \frac{S}{P}} \sigma_{II}(F)$$

$$(5.3) \quad F = B + S$$

For (5.1) and (5.2) to hold exactly, we must assume not only corporate income taxes proportional to income but also provision for offsetting of profits against losses, for tax purposes.

The problem is to choose  $S$  and  $B$  so as to maximize the assumed utility function

$$(5.4) \quad u = u(\pi, \sigma_\pi) \text{ with } u_1 = \frac{\partial u}{\partial \pi} > 0, u_2 = \frac{\partial u}{\partial \sigma_\pi} < 0$$

subject to restrictions (5.1) and (5.2). Introducing the Lagrangian multipliers  $\lambda$  and  $\mu$ , this is equivalent to finding the unconstrained maximum of

$$u(\pi, \sigma_\pi) - \lambda \left[ \pi - \frac{\alpha n}{n + \frac{S}{P}} \left( \frac{\Pi_0}{\alpha} + Y(F) - Br \right) \right] \\ - \mu \left[ \sigma_\pi - \frac{\alpha n}{n + \frac{S}{P}} \sigma_{II}(F) \right]$$

with respect to  $\pi$ ,  $\sigma_\pi$ ,  $B$ , and  $S$ . Differentiating and setting the first derivative equal to zero we obtain, as first order maximum conditions,

$$(5.5) \quad u_1 - \lambda = 0$$

$$(5.6) \quad u_2 - \mu = 0$$

$$(5.7) \quad \lambda[\rho(F) - r] + \mu \frac{d\sigma_{II}}{dF} = 0$$

$$(5.8) \quad \lambda \left[ (nP + S)\rho(F) - \left( \frac{\Pi_0}{\alpha} + Y(F) - Br \right) \right] \\ + \mu \left[ (nP + S) \frac{d\sigma_{II}}{dF} - \sigma_{II} \right] = 0$$

Simultaneous solution of the equations (5.1), (5.2), and (5.5) to (5.8) yields the values of  $S$ ,  $B$ ,  $\pi$ ,  $\sigma_\pi$  maximizing (5.4), provided the second order conditions are satisfied.

To bring out more clearly the implications of this solution, as stated in the text, let us make use of (5.7) to replace  $\mu \frac{d\sigma_{II}}{dF}$  in (5.8) by  $-\lambda[\rho(F) - r]$ ; simplifying, (5.8) becomes:

$$\lambda \left( nPr - \frac{\Pi_0}{\alpha} - Y(F) + Fr \right) - \mu \sigma_\pi(F) = 0$$

Eliminating the Lagrangian multipliers, the conditions (5.5) to (5.8) can now be reduced to the following two:

$$(5.9) \quad -\frac{u_2}{u_1} = \frac{\Pi_0/\alpha + Y(F) - (nP + F)r}{\sigma_{II}(F)}$$

$$(5.10) \quad \frac{\Pi_0/\alpha + Y(F) - (nP + F)r}{\sigma_{II}(F)} = \frac{\rho(F) - r}{\frac{d\sigma_{II}}{dF}}$$

It will be noted that equation (5.10) has the following properties:

(a) It does not involve  $B$  and  $S$  separately, but only their sum,  $F$ , i.e., total new financing; and since it involves no other variables but  $F$ , it is in general sufficient to determine the optimum value of this variable.

(b) It does not involve in any way the parameters of the preference function,  $u$ .

Thus (disregarding for the moment certain problems raised by boundary conditions) it is seen that the optimum amount of financing is determined independently of the specific risk preference function of management and depends exclusively on the "data" of the problem, i.e., the market data and the "expectation of return" parameters. As for equation (5.9), it may be easily shown that the right hand side expression measures the slope of the "market opportunity" relation between  $\pi$  and  $\sigma_\pi$  confronting management for any given amount of total new financing,  $F$ . To prove this we first replace  $B$  in equation (5.1) by the expression  $F - S$ , and then eliminate  $S$  between (5.1) and (5.2) which can be done very easily thanks to the form of these two equations. This yields the equation:

$$(5.11) \quad \pi = \alpha r nP + \frac{\Pi_0/\alpha + Y(F) - r(F + nP)}{\sigma_{II}(F)} \sigma_\pi$$

Equation (5.11) represents the market opportunity function corresponding to any given value of  $F$ , i.e., the set of pairs of  $\pi$  and  $\sigma_\pi$  that are open to management under the given market and expectations data. Any point on this line can be achieved by a proper distribution of  $F$  between  $B$  and  $S$  [the specific value of  $S$  and  $B$  corresponding to a given  $\pi$ ,  $\sigma_\pi$  and  $F$  can of course be found from (5.2) and (5.3) above]. It will be seen that for any given  $F$  this opportunity function is a straight line and its slope  $\frac{d\pi}{d\sigma_\pi}$  is precisely equal to the right hand side of (5.9). Furthermore, the opportunity lines corresponding to different values of  $F$  represent a system of straight lines in the  $\pi$ ,  $\sigma_\pi$  plane all going through the point  $(0, \alpha nP)$ .

Because of this property it is clear that the "best" opportunity line within this set must be the steepest one; for this line will yield the largest  $\pi$  for any  $\sigma_\pi$ . Thus the best line can be found by differentiating the slope of (5.11) with respect to  $F$  [or, which is equivalent, differentiating (5.11) partially with respect to  $F$ ] and setting the derivative equal to zero (first order condition). It is easily verified that the resulting condition is equivalent to (5.10). This helps clarify the reason why the optimum amount of net new financing is independent of the risk preference function. The risk preference function only determines which point will be chosen on this "best" opportunity line; and thus also the distribution of  $F$  between  $B$  and  $S$ . The chosen point will be a point of tangency of an indifference curve with the "best" line, as indicated by (5.9), after the variable  $F$  on its right hand side is replaced by the specific value given by the solution to (5.10).

From the derivative of the slope of the opportunity lines with respect to  $F$  at  $F = 0$ , it is possible to note the condition under which any net positive financing will be advisable. Specifically it turns out that the derivative at 0 is positive only when

$$(5.12) \quad \rho(0) > r + \left( \frac{\Pi_0/\alpha}{nP} - r \right) \frac{d\sigma_\pi}{dF} \frac{nP}{\sigma_{II}(0)}$$

The statement in the text is derived from this expression.

Equation (5.10) has other interesting properties. The right hand side measures the slope of the "bonds-only boundary curve," discussed in the text, Subcase II-c. This curve is defined as the locus of the  $\pi$ ,  $\sigma_\pi$  combinations that are achievable by using bonds only as the instrument of financing. To verify this statement it is only necessary to substitute  $B = F$ ,  $S = 0$  into (5.1) and (5.2), differentiate each resulting equation with respect to  $F$ , and form the derivative  $d\pi/d\sigma_\pi$  from  $\frac{d\pi}{dF}$  and  $\frac{d\sigma_\pi}{dF}$  in the usual manner. It follows directly from this that the bonds-only boundary curve is tangent to the "best" opportunity line.

It is also possible to show that the boundary curve representing the  $\pi, \sigma_\pi$  combinations attainable with  $B = \pm B_0, S = F \mp B_0$  (where  $B_0$  is any fixed amount of bonds) is also tangent to the "best" opportunity line. For, the value of  $F$  for which (5.10) holds, i.e., for which the slope of the "best" opportunity line is equal to the slope of the bonds-only boundary curve, is also the value of  $F$  for which the slope of the  $B = \pm B_0, S = F \mp B_0$  boundary is equal to the slope of the bonds-only boundary. To find the expression for the slope of the  $B = \pm B_0, S = F \mp B_0$  boundary the reader may substitute these values into (5.1) and (5.2) and proceed as for the slope of the bonds-only boundary. The reader may then prove the equality-of-slopes statement made above by multiplying the numerator of each side of (5.10) by  $-1$ , adding to and subtracting from the numerator on the left hand side the factor  $[\rho(F_M)(nP + F_M \mp B_0) \pm B_0r]$  (where  $F_M$  is the value of  $F$  for which (5.10) holds), and rearranging terms.

It will be noted that the solution will be a "corner" solution, if and only if the value of  $F$  being positive, the value of either  $B$  or  $S$  is zero or negative. It is clear that such solutions are possible but that they are not the only possible type of solutions.

#### NOTE 6

Since the maximizing value of  $F$  is given by equation (5.10), the effect of variation in the price on  $F$  can be found by differentiating this equation totally with respect to  $P$  and solving for  $dF/dP$ . The result is

$$(6.1) \quad \frac{dF}{dP} = \frac{-nr \frac{d\sigma_{II}}{dF}}{\sigma_{II} \frac{d\rho}{dF} - [\Pi_0/\alpha + Y(F) - (nP + F)r] \frac{d^2\sigma_{II}}{dF^2}}$$

The numerator of this expression may generally be assumed negative; as to the denominator, it may be shown that it must be negative if the second order maximum conditions are satisfied. Indeed, these conditions imply

$$(6.2) \quad \frac{d^2}{dF^2} \left( \frac{\Pi_0/\alpha + Y(F) - (nP + F)r}{\sigma_{II}(F)} \right) < 0$$

which, taking into account the first order maximum condition, reduces precisely to the condition that the denominator of (6.1) be negative. It follows that  $dF/dP$  will generally be positive, i.e., total financing will tend to move in the same direction as the price per share.

## NOTE 7

The rationale underlying the use of these variables deserves a brief explanation at this point. It is based on the notion that the relative advantage in the use of equity versus debt funds depends (at least approximately) on the slope of what we have previously called the "best opportunity line" (Section II, page 272). As was shown in Note 5, the slope of this line is represented by the right hand side of equation (5.9), after we replace  $F$  in this expression by the optimum amount of new financing. Clearly, the slope of this line depends partly on the "productivity of new capital" function,  $Y(F)$ , which, as indicated previously, we feel unable at present to approximate by means of available data. We do feel, however, that the various measures discussed in the text may represent a reasonable approximation to the relevant parameters of the distribution of anticipated returns on *existing assets*, even though they are likely to represent a rather poor measure of the anticipated productivity of *varying amounts of new capital*.

Now the slope of (5.9) depends on the parameters of the distribution of anticipated earnings on *all assets*, i.e., existing assets as well as contemplated additions to these assets. However, net new financing,  $F$ , and its anticipated earning power,  $Y(F)$ , are bound to represent a relatively small fraction of already invested capital and its earning power. Accordingly, the slope of the best opportunity line should tend to be closely approximated by the slope of the zero net financing line obtained by setting  $F = 0$  in the right hand side of (5.9). This slope then becomes:

$$(7.1) \quad \frac{\Pi_0/\alpha - nrP}{\sigma_{II}(0)}$$

which may also be rewritten as:

$$(7.2) \quad \frac{\frac{\Pi_0/\alpha}{nP} - r}{\sigma_{II}(0)/nP}$$

It will now be recognized that the two independent variables suggested in the text represent a measure respectively of the numerator and denominator of (7.2) and therefore an approximation to the numerator and denominator of (5.9). The closeness of this approximation is, of course, improved to the extent that our variables also measure anticipations relating to the productivity of new capital.

It should be noted that the expression in the denominator of (7.2) consists of the dispersion of anticipated earnings divided by the market value of the outstanding common stock. Since the dispersion of anticipated earnings was approximated by the standard deviation of past gross earn-

ings of *all* corporations, this variable should properly be divided by the aggregate market value of all common stock. Estimates of the latter variable are not available, however; as a substitute we were forced to use a variable that should be roughly proportional to it, namely, the market value of all common stock issues listed on the New York Stock Exchange. On this point see the definition of the variable  $X_2$  in Appendix A and the reference quoted there.

It may be added that the use of the variables appearing in (7.2) rather than in (7.1) was dictated by the nature of the available data. As indicated in Appendix B ref. [2] we do not have a series on earnings per share and corresponding stock prices covering the entire period and we were therefore forced to shift from one source to another in 1937. The ratios of earnings per share to stock prices from the two sources agree fairly well over the overlapping period but the actual level of the two component series is quite different since they are based on different samples. This made it advisable to make use of variables involving the ratio of earnings to prices, as in the numerator of (7.2), rather than the expression in the numerator of (7.1).

#### DISCUSSION:

LAWRENCE R. KLEIN, *National Bureau of Economic Research*

Messrs. Modigliani and Zeman have attacked the interesting problem of trying to show how corporations reach a decision on whether to issue debt or equity capital in raising funds. I find their preliminary findings instructive but would like to raise a few questions as to the completeness of their results before any definite conclusions can be drawn.

The approach of Modigliani and Zeman implies a very special type of decision process, at the theoretical level, within the corporation. Decisions about current operations (the use of manpower, raw materials, equipment, etc.), about physical capital expansion, dividend distribution, or the use of internal or external funds do not appear explicitly in their theoretical framework. They assume that debt-equity decisions are made in a compartment excluding the other types of decisions. I should prefer to see them develop a general theory of corporate decisions and then show how debt-equity decisions fit into the more complete scheme. If this were properly done, I should expect that more variables would be called for in their basic relationship.

It is hard to disagree with the proposition that the relative prices of debt and equity funds influence the debt-equity proportions of new issues. I should certainly expect the relative prices of internal and external (debt or equity) funds to be of some importance in this relation, and this raises the difficult problem of measuring the imputed costs of using internal funds.

Greater generality should be achieved not only in the development of the decision-making process of corporations but also in the network of market relations involved. Is it a good assumption to claim that corporations accept market prices of bonds and shares as given variables and then adjust to them in such a way that these market prices are not influenced by the actions of individual companies? I seriously doubt that debt-equity decisions of giant corporations have a negligible influence on the market prices of their bonds or shares.

The market structure must also be carefully examined to determine whether the observed correlation between the ratio of new issues of debt to equity securities, on the one hand, and the ratio of prices of debt to equity securities, on the other, describes the behavior of corporations, of market investors, or of neither. The price ratio is an important variable underlying corporate debt-equity decisions, but it is also important in helping the individual investor to decide whether to purchase bonds or shares. There are undoubtedly other variables influencing decisions on both sides of the securities market, and they must be introduced if the statistical calculations are to be identified as estimates of particular patterns of behavior.

Purely theoretical considerations will almost certainly not be adequate to develop a satisfactory model of behavior in the securities market. Several important institutional characteristics must also be taken into account. For example, railroads raise a significant amount of funds through the issuance of equipment trust certificates, a security that is specific to this industry. Equipment trust certificates have been important in enabling carriers to raise funds at reasonable interest rates and must have had a profound influence on debt-equity discussions in the railroad industry. This influence is, to a large extent, not reflected in market prices, but rather in legal or institutional characteristics of these securities. Similarly, railroad reorganization proceedings have achieved, by purely legal means, a complete restructuring of corporations' balance sheets and current interest payments. There can be no doubt that such legal decisions have influenced debt-equity financing by railroads. A further example of the influence of institutional characteristics in the securities market is the case of those life insurance companies that are legally limited in the types of securities they may purchase. It hardly seems possible that the full effect

of these restrictions is shown by market prices of debt and equity funds.

Modigliani and Zeman use as one variable the ratio between new issues of bonds and new issues of shares and as the other variable the ratio between the yield on new issues of bonds and the yield on new issues of shares. There is consistency in their procedure of using only new issues throughout, but I wonder whether the analysis should not more correctly be in terms of outstanding rather than new issues. The question is whether basic decisions of corporations are made in terms of stocks or flows, as far as securities are concerned. It is my feeling that the securities market should be analyzed in terms of stocks rather than of flows. It so happens that the theoretical and statistical treatment of the relationships involved is essentially different for stock and for flow analysis.

Another specific variable that may be of some importance in debt-equity decisions is the matter of corporate control. Many small corporations do not like to issue shares because that method of financing would allocate a measure of control to persons outside an inner circle, say a family. A factor of this sort may not be important for the statistical sample of Modigliani and Zeman in which large corporations dominate the scene, but it should be taken into account in a more complete analysis.

Two definite types of research possibilities seem to be open for further studies along the lines initiated by Modigliani and Zeman. There is a wealth of cross-section data available showing debt-equity structure and prices for individual companies. A careful microeconomic analysis made from these data would go far to supplement the aggregative time series analysis presented by the authors. Secondly, carefully designed surveys of business firms may provide a basis for getting at some of the matters that cannot be studied by conventional techniques applied to published data. We need to learn much more about business attitudes toward debt, equity, and other sources of funds. The personal interview technique seems well adapted to this problem. Moreover, we need to find out the mechanics of the decision-making process within corporations. This sort of information is essential to the setting up of hypotheses about debt-equity decisions, among many other types relevant to corporate operations.

Especially important for the work of Modigliani and Zeman is the possibility of getting at subjective magnitudes through survey techniques. One of their prices, the yield on shares, is really supposed to indicate the rate at which future anticipated earnings must be discounted in order to equal the present price per share of stock. The best method open to the authors was to construct some function of past and current earnings as an indicator of future anticipated earnings; but survey methods may bring us much closer to a measure of a corporation's expected earnings. Naturally

the application of survey techniques must proceed with caution in this area. The most advanced methods of sampling, interviewing, coding, editing, and general survey analysis must be used, but these will involve substantial costs. Problems arise as to whom to interview within a corporation and what type of questions to ask. It is almost certain that casual surveys using blunt, direct questionnaires will fail to elicit the correct responses.

As so many participants in this conference have indicated, decisions about debt and equity financing are in themselves interesting. I mean this especially in the sense that the analysis of these decisions presents a challenging intellectual problem; but I do not mean to imply that the ratio of debt to equity is necessarily a strategic economic magnitude. It remains to be demonstrated whether equity financing is, in some economic or social welfare sense, good, and that debt is bad. It is not at all certain that a switch from equity to debt sources will cut down the flow of investment in the economy.

I suppose that Modigliani and Zeman have, in the back of their minds, anticipated further steps to be taken in the analysis after they have satisfied themselves on the matter of explaining the debt-equity ratio. I should like to see them make quite clear the implications of various debt-equity ratios if these magnitudes are not to be thought of as ends in themselves.

GEORGE L. BACH, *Carnegie Institute of Technology*

The preliminary status of Messrs. Modigliani's and Zeman's statistical explorations makes detailed criticism of their methods and findings inappropriate. But their inability, and that of others before them, to formulate for investigation a satisfactory and testable hypothesis which specifically includes business expectations suggests the appropriateness of some observations on research methodology in this area.

Faced with this and related difficulties, researchers in business finance (and investment decision-making more generally) have turned toward questionnaires and interviews with businessmen to get at the way expectations are formed and the bases on which business investment decisions really depend. No doubt, useful information will be obtained through these efforts, but I should like to sound a note of warning. Superficial generalizations concerning businessmen's motives, and rationalizations of professional economists' hypotheses, are dangerously easy to come by through questionnaires and interviews. I have been led to a position of extreme caution concerning these approaches by a long string of informal com-

ments by businessmen, of high and low status, concerning the haphazard manner in which they provide ("cook up") questionnaire information and the naïveté which they feel most economists reveal in interviews aimed at discovering business decision bases.

Most business decisions, financing or otherwise, are simply not the one-spot careful weighing of economic (monetary) alternatives that most of our business firm models imply. It is inevitable that we as researchers will focus our questions (written or oral) on the elements that we expect to be most important. Thus one of the greatest dangers is that we will close out useful and novel insights that more fundamental and "unbiased" investigation might reveal. Partly this is a language problem between us and the businessmen. More fundamentally it is the danger that we subconsciously will want to have business behavior fit our theoretical molds and thus fail to see what else there is to be seen. Only recently have we generally introduced liquidity and control motivations along with profit as likely key considerations in business output and price decision-making, obvious as these now seem. Development and acceptance of better-rounded hypotheses in this field have a long way yet to go.

I suggest that we recognize financial decisions as merely one aspect of the whole complex of implicit and explicit considerations that make up a going business concern, and that we focus some of our analysis on the going organization *per se* as a major area of business research. I suggest that we make use of the insights the gradually growing modern literature on organization theory can give us.<sup>1</sup> This approach seems to me clearly essential for analysis of short-term (e.g., inventory) investment and financing decisions. Businessmen repeatedly are unable to separate out any few simple key economic determinants for such business decisions. Acquaintance with going business concerns suggests that such decisions are seldom really "made" by any one or two top officials. Rather, they develop sequentially and often very informally through a series of lesser decisions at lower points in the organization that gradually build up to something called "policy." Often the decisions are largely "automatic," in the sense that they arise informally out of the formal and informal control procedures of the firm (e.g., production scheduling and inventory controls) without conscious decision-making by officials. Mrs. Mack has pointed out an excellent example in the shoe industry, and Mr. Fulton has emphasized the

<sup>1</sup> I refer especially to the work of men such as Herbert Simon (*Administrative Behavior*, Macmillan, 1947), and Chester I. Barnard (*Functions of the Executive*, Harvard University Press, 1938), and of the growing circle of social scientists working in the area loosely termed "group dynamics."

same varied pattern of considerations for long-term investment and financing decisions.

Insofar as "decisions" are generated informally or formally by the going organization, rather than being "made" by any one or few individual officials, it becomes important to study in detail the organization as an operating concern if we are really to understand, for example, how business short-run financing steps are actually arrived at. In this sense, I suggest that penetrating case studies (in collaboration with other social scientists) of a few concerns as going organizations may be a more fruitful use of resources than predominant emphasis on more narrowly focussed questionnaire and interview approaches. This implies not a few hours, but days and weeks and even months, on each case study, with research time spent in the firm in continuous skilled observation and examination, in contrast to reliance on a few brief or even "depth" interviews. It implies, too, recognition of the financial side as only one aspect of the decision-reaching process — often, probably, only a very minor aspect. It implies a willingness to search far outside our typical economics for new hypotheses, insights, and analytical concepts.

Supplementing modern organization theory, the rapidly growing theory of "servo-mechanisms" and "communications," especially as developed by the electrical engineers and mathematicians, may be highly suggestive for the analysis of business firm behavior. In essence, this approach would view any organization as a system of "reporting" and "feed-back" relationships. Thus, various kinds of information are reported (formally or informally) to certain points in the organization, whereupon certain responses or controls are "fed back" to the reporting points, inducing changed behavior with correspondingly changed succeeding reports, and so on. The system of servo-mechanisms and communications chains (corresponding to electrical circuits in electrical engineering) may, of course, vary from simple to highly complex. Looking at the business firm in this light, one would thus see a variety of financial information fed into the reporting system at various points, depending on the organizational arrangements involved. These, for example, might be extensively or little utilized in the production control process that primarily adjusts the output and inventory behavior of the firm to changing information on sales prospects from outside.<sup>2</sup>

In the analysis of the financial aspects of long-run investment decisions,

<sup>2</sup> Some pilot research along these lines, though not with primary attention to business financing, is now under way at Carnegie Institute of Technology. The U.S. Air Forces have substantial work under way on the implications of servo-mechanisms and communications theory for mobilization planning and production controls on an economy-wide basis.

the case for emphasis on the "going organization" aspects of the problem is less strong. This is largely because such decisions are made less frequently, are usually of individual major importance, are given extended separate attention by top officials, and are thus less apt to be "made" informally and quasi-automatically by the ongoing organizational processes. But even here, we could profitably devote some research attention to the approach I am suggesting. Long-run investment decisions often grow out of a sequence of diverse smaller investment decisions, which at least set the stage if they do not in fact determine what the final major decision will be. Moreover, the servo-mechanism analogy may again be helpful here — e.g., in analyzing the way in which sales forecasts and financial market information are fed into the decision-making process and the channels and mechanisms the organization has for reacting to the various kinds of information fed to it.<sup>3</sup>

I do not suggest that the particular research avenue I am emphasizing should supplant the established historical-statistical approach, nor even the questionnaire-interview techniques that have become so popular recently. Case studies encompassing the whole organizational process are expensive and time-consuming. They promise no quick generalizations, nor even simple hypotheses for statistical testing. And they will appear peculiarly unattractive to many economists because they involve *actual* emphasis on other disciplines and major attention to noneconomic as well as to economic factors, rather than mere lip-service to the broader approach. But by going outside our established research channels, whose relatively narrow focus has often been remarked, we may be able at least to develop some better-rounded hypotheses on business behavior. Perhaps ultimately, with much aggregative historical-statistical testing, we may be able to get at an acceptable theory of the behavior of business firms, which seems to be so elusive when we use purely "economic" considerations. The real promise of this one facet of over-all business research is much broader

<sup>3</sup> A further note on the importance of explicitly recognizing organizational factors was suggested in Professor Domar's comments on the business growth problem, when it was stated that many concerns simply will not grow beyond certain rates because of the organizational problem of assimilating new personnel and processes and of adjusting to different levels and types of processes. Professor Domar's answer, incidentally, that this can hardly be an important long-run factor because probably all we will need is a 3 or 4 percent annual growth rate, overlooked a crucial aggregation problem. A 3 percent growth rate for the economy has hinged, and probably will continue to hinge, on much faster growth by key booming firms and industries, partially offset by stable and declining sectors of the economy. Thus, organizational restraints might be worth analysis as key restraints on the crucial growth elements in the economy, even though only 3 percent per annum is needed over all.

than the business finance area, and it may have little place in a financial research program as such. But if, in fact, financing decisions are part and parcel of broader going organization decision-making, we will not change the reality nor enhance our understanding greatly by considering business finance as though it were a neat separate compartment.