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Instructional Costs of University Outputs

I. INTRODUCTION

The purpose of this paper is to develop a simple model that evaluates the instructional costs of educating student cohorts enrolled in an institution of higher education. We have the additional objective of analyzing some of the cost implications of new operating policies and plans that modify the content, number, and type of degree programs available to these cohorts. Although the data we use is specifically adopted from sources at the University of Colorado, the University of California (Berkeley Campus), and Stanford University, the underlying model of the

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educational process and the mathematical methods that we use to evaluate these costs may be applicable to other private or public educational institutions. Since our emphasis is on finding a scheme for predicting budgets under specified policy constraints, no judgment is made or implied regarding the quality of educational programs. The assumption is that quality standards determine some of the constraints on feasible operating policies and that these may obviously differ from one institution to another.

Throughout this paper, student cohorts are identified by their status upon entering and by the sequence of educational programs which they undertake at the educational institution in question. For example, one cohort might consist of students who enter the lower division, continue into the upper division, and obtain a bachelor's degree. A second cohort might be students who enroll for the first time at the upper-division level and drop out prior to receiving a degree; still a third example is a junior transfer who receives his bachelor's degree and then continues for an additional period of time in order to receive his master's degree. Although we do not do so in this paper, it is possible to define a cohort by means of a finely divided classification which identifies such things as the student's major; his precise status, such as second quarter, third year; his educational background, two years of high school, three years of preparatory school; and even socioeconomic factors such as income and educational background of parents. We have chosen to restrict the problem and data requirements to a manageable level, but at the same time, to select aggregations that allow us to evaluate the costs of administrative and institutional policies, such as the implementation of year-round operations, the imposition of various enrollment ceilings, the adoption of new undergraduate and graduate programs or the alteration of dropout rates through selective admission policies.

As the reader will see, cohorts are defined in a way that makes it simple and straightforward to calculate the unit cost of educating a student member of that cohort. This accounting is straightforward when the life history and costs of each student cohort are independent of all others. Unfortunately, this is seldom the case, and possibly the most important feature of our model and its findings is the recognition that different degree programs have substantial interactions with one another. Changes in the unit costs of one program usually affect a large number of cohorts and hence the total costs of educating different student cohorts. This feature is particularly important when a fraction of the students being educated are themselves used as teachers.

It is common practice in many institutions to allocate a large amount of historical accounting data in such a way as to come up with a cost per student for every year that he attended the institution in question. The emphasis in this paper is quite the opposite: first of all, we are more interested in estimating and analyzing the unit costs of educating a particular student during his lifetime at the institution than we are in obtaining the unit cost of enrolling a student for a single time period. The connection between the two types of costs is not always obvious; while lifetimes of the student at the institution enter the former calculation, they are not involved in the latter. In our experience, calculations of the latter type always make it difficult to distinguish between the unit costs of those students who do, or do not, drop out, so that the effect of attendance patterns on unit costs is not explicitly made. Secondly, we are much less interested in manipulating large amounts of historical accounting data than we are in obtaining order-of-magnitude estimates which reveal the underlying structure of marginal and unit costs and the impact that new institutional policies have upon these costs. For example, if dropouts affect enrollment levels in a reasonably predictable way, we are interested in understanding how costs of educating different types of students are sensitive to policies which affect these dropouts.

In the remainder of this paper, we assume that "instructional cost" refers only to the direct salary cost of students and faculty who engage in the instruction of students at a given institution. While we do not become involved in such computations, standard accounting techniques for converting these direct labor costs into a total instructional budget that includes related items, such as expenditures for nonacademic staff and office space, do exist.

The organization of this paper is as follows: Section II discusses related work on university cost models. Notation and terminology are introduced in Section III, which we then use in Section IV to formulate a mathematical model of student flow patterns and costs on a network characterized by the degree programs available to student cohorts. The flows and costs have an apparent multi-commodity structure which can lead to interesting and nontrivial interpretations for shadow prices associated with final demands, admissions and enrollment ceilings. Section V gives source data for behavioral and institutional parameters and the unit cost data for campuses that we study in Section VI. The data and the model are used to analyze instructional costs at each institution if one were to adopt the policy recommendation made by the Carnegie Commission on Higher Education [1971] that lower-division programs be reduced from two years to one year. The paper concludes with a bibliography.

II. RELATED WORK

Some interesting papers on unit costs as they relate to productivity in education may be found in UNESCO [1967]. In this volume, the paper by de Escondrillas describes two types of aggregate unit costs that are commonly used for monitoring educational institutions: cost per student year and cost per graduate. These are computed by dividing the total annual operating cost by either the total enrollment or by the total number of students graduating per year. Observe that the latter computation has the effect of attributing the cost of all students who do not complete the curriculum (i.e. dropouts) to those who do. Moreover, de Escondrillas did not discuss specific uses for each type of unit cost. That it is important to decide a priori on the type of cost most pertinent to a given study was demonstrated in the paper by Chau who used real data to calculate both the cost per student year and the cost per departing student in the primary school systems of Cameroun and Senegal. The results showed Cameroun to be more "efficient" with respect to the first criterion, and Senegal with respect to the second. Finally, the paper by Gern analyzes various components of the cost per student year, such as the cost of teachers, capital equipment, and construction and maintenance of buildings, in order to isolate the factors that influence them. Gern also suggested a number of different unit cost comparisons which might be made for the purpose of identifying efficient alternatives, for example between similar institutions in a given country, between different countries, or between different teaching techniques.

Several cost simulation models have recently been developed for institutions of higher education to calculate costs in terms of levels of instructional activity. Although the details of these models are not generally available in the open literature, mimeographed reports, such as Weathersby [1967] and Judy [1969], have been widely circulated. Taking student enrollments to be the measure of instructional activity, these models determine instructional costs in the following manner: let $x(t) = [x_j(t)]$ be an n-dimensional column vector of student enrollments at time t (by grade level, major department, etc.) and let $y(t) = [y_i(t)]$ be an m-dimensional column vector of faculty staffing levels at time t (by rank, department, etc.). The assumption is made that $y(t)$ is linear in $x(t)$, i.e. given $x(t)$, one can compute $y(t)$ by the rule

$$y(t) = Mx(t)$$

where $M = [\mu_{ij}]$ is an $m \times n$ matrix of faculty-student ratios. Given an m-dimensional row vector of average faculty salaries, s, one obtains the total instructional cost for time period t, $C(t)$, by taking the vector product

$$C(t) = sy(t) = (sM)x(t)$$

In such a scheme, the unit (per period) cost of instruction for students in category i is expressed by the ith component of the vector (sM). We emphasize that the elements of sM are holding costs, not product costs.

This basic model was extended by Koenig et al. [1967] to include equations that describe the transitions of students as they flow through the system. These authors made the following assumptions regarding student attendance behavior: (1) each student's progress through his educational program does not depend on any other student's progress; and (2) a student's status at time $t + 1$ does not depend on his status *prior to* time t. Under these assumptions, it is reasonable to postulate the existence of a Markov-like transition matrix $P = [p_{ij}]$ whose (i,j)th element is the fraction of students in state i at the beginning of period t that will be in state j at the beginning of period $t + 1$. If $z(t) = [z_i(t)]$ is an n-dimensional column vector of new admissions during period t, student enrollments at $t + 1$ are related to the enrollments at t by the equations

$$x(t + 1) = P'x(t) + z(t)$$

where prime denotes matrix transposition. These authors were particularly interested in describing the cumulative instructional costs invested in students as they flow through the system. Thus, defining $\hat{c}_i(t)$ to be the average *cumulative* educational investment in students in state i at the end of period t and assuming that new students have accumulated the same average investment as those who entered previously, they described the conservation of money flows for each state j as follows:

$$\hat{c}_j(t + 1)x_j(t + 1) = \sum_i \hat{c}_i(t)[p_{ij}x_i(t)] + \hat{c}_j(t)z_j(t) + \left(\sum_k s_k \mu_{kj}\right)x_j(t + 1)$$

[total cumulative investment in students in state j in period $t + 1$] = [total cumulative investment prior to $t + 1$ in continuing students who are in state j in period $t + 1$]
+ [total cumulative investment in new students entering at state j at the end of period t]
+ [value added during period $t + 1$]

While their purpose was to investigate cumulative educational investments regardless of where these investments were made, we are interested only in those investments made by the given institution. Therefore, it is appropriate to delete the second term in the above equation. If we then divide both sides by $x_j(t + 1)$, we obtain a set of linear equations that describe the propagation of the unit cumulative investment in students in the various states j:

$$\hat{c}_j(t+1) = \sum_i \hat{c}_i(t)\left|\left[\frac{p_{ij}x_i(t)}{x_j(t+1)}\right]\right| + \sum_k s_k \mu_{kj}$$

Observe that in order to calculate numerical values for the $\hat{c}_j(t)$ in this recursive relationship, it is necessary that we first be given values for investments in the initial period, $\hat{c}_i(0)$.

Models of this type are cross-sectional in the sense that the elements of M and P are estimates of ratios observed at a particular point in time or are, at best, the average of a small number of time periods. They tend to suffer from the real difficulty that cross-sectional data is sensitive to historical institutional policies, and it is often difficult to examine the cost implications of new operating policies. In the Koenig model, for instance, the meaning of the quantity $\hat{c}_i(t)$, computed for a group of students undifferentiated by where they entered the system, is not clear. Nor is it clear how the $\hat{c}_i(t)$ will be affected by changes in student admission and dropout rates.

Because these models are usually formulated to include a great deal of detail (i.e. numerous categories of students and faculty), they are costly to implement. An additional drawback is that, at such levels of disaggregation, the existence of widespread substitutability between members of a university instructional staff would seem to contradict the assumption of a single-efficient-point technology in which resource inputs are always used in fixed proportions; in addition, it may be unreasonable to assume that individual instructional programs exhibit constant returns to scale. These issues are discussed at greater length in Hopkins [1971].

Sengupta and Fox [1969] formulated a linear programming model to determine optimal policies for the recruitment of new faculty and the allocation of total faculty time to various instructional and research activities over a four-year planning period. Although they were not concerned directly with the costs of educating students, these were included as debit items in their maximand. The remaining coefficients in the objective function corresponded to the value added by "producing" a graduate from a bachelor's, master's, or doctoral program, this quantity being measured as the difference in expected discounted lifetime income due to the earning of the degree. The constraints specified demands for faculty time in teaching, research, and administration; available supplies of faculty time; undergraduate and graduate student admission quotas; and various technological restrictions. A major weakness of this model is its omission of the effects of dropouts and student lifetimes, for it assumes that admissions are equivalent to degree outputs and that all students are enrolled in a given degree program for the same period of time.

Our model in this paper differs in several ways from earlier ones and offers an alternative way to calculate the cost of educating a student at a given institution. First of all, the model is longitudinal rather than cross-sectional in nature. That is to say, student cohorts and the unit cost of educating a student cohort are defined over the lifetime of the cohort in question, not at a single point in time. Attrition rates and other factors related to student attendance patterns are defined for each student cohort; again the relevant time period is the lifetime of the cohort at the institution.

Secondly, we are concerned with the interactions that occur when an institution is in equilibrium with respect to student flows, enrollments, and various parameters of student behavior. We assume that input flows, output flows, enrollments, cohort lifetimes, and dropout rates are the same in each time period and make a concerted effort to attribute costs to the actual output flow rates of the instructional process, namely various types of degree recipients and dropouts per unit time.

Finally, our model is highly aggregated in the sense that individual departmental majors are not taken into account. Thus, we are interested in the implications of new policies at the campus-wide level, not at the departmental level. In this sense, it is similar to one developed earlier by Oliver, Hopkins, and Armacost [1970] expressing the enrollments of students, the number of teaching staff, and the flow rates of students who eventually drop out in terms of the final demand for degree recipients at various degree levels.

The size of our model is such that the number of policy variables that can be identified and studied is of the order of ten or twenty, not hundreds or thousands; the amount of data that must be collected and analyzed does not obscure one's understanding of the budgetary flow process; and it is possible, with a minimal amount of computation and analysis, to identify the effects of certain proposed policies.

III. NOTATION AND TERMINOLOGY

For our purposes it is convenient to represent the educational system by a directed flow network $[N, M]$, where N denotes an unordered set of nodes and M is an ordered set of chains. In this scheme, each node is equivalent to an educational program (e.g. the successful completion of upper division, or the termination of master's studies prior to completion of the degree requirements), while a chain corresponds to the sequence of programs pursued by a specific cohort of students (e.g. entrance at the upper-division level, successful completion of upper-division followed by admission to a master's program, with termination as a master's dropout).

We define a *chain* in M as a sequence of distinct ordered nodes in N, where the first node of the chain is the *origin* node and the last node of the chain is the *destination* node. All other nodes on a chain are intermediate nodes. To uniquely specify a chain in our networks it is sufficient to list the sequence of nodes that comprise the chain.

Figure 1 illustrates the basic network used in analyses and calculations throughout this paper. Nodes 1′, 2′, 3′, 4′ are dummy nodes used to denote common origins for admissions to lower-division, upper-division, master's and doctoral degree programs, respectively. Nodes 1, 2, 3, 4 represent the programs associated with completion of the above while 5, 6, 7, 8 are the programs identified with dropouts at these same levels. Nodes 9 and 10 represent graduate students that are employed as teaching assistants during part of their career at the institution. Although the teaching assistantship is depicted in Fig. 1 as taking place at the end of a graduate student's career, we recognize that the sequence may differ in individual cases. It can be shown that the equations we obtain in the model of Section IV do not depend on the actual timing of the teaching assistantship.

There are 12 chains having origin node 1′, 11 chains with origin node 2′, 9 chains with origin node 3′, and 3 chains with origin node 4′. Three typical ones are

{1′, 5}	with destination node 5,
{3′, 3, 8}	with destination node 8,
{1′, 1, 2, 3, 4, 9}	with destination node 9.

The first chain represents the student cohort that enters at the lower division and drops out before completing the lower division. The second chain represents a student who enters the institution, obtains a master's degree and drops out from the doctoral degree program. The third chain represents the student cohort that enters at the lower division, completes the lower division, upper division, master's degree, and Ph.D., and is employed as a teaching assistant before leaving the institution.

Associated with each node are the scalar quantities L_i: the *enrollment* in the program represented by the ith node; v_i: the *lifetime* required for completing the program at the ith node; g_i: the total *flow rate* entering the program at the ith node; and c_i: the *unit* (lifetime) *cost* of the program at the ith node. Finally, for certain nodes we specify *exogenous supplies* a^i or *exogenous demands* b^i. Lifetimes, costs and enrollments at dummy nodes are zero.

Associated with each chain are the scalar quantities: h_j^k—the flow on the jth chain in the set of chains M^k having origin node k; and C_j^k—the unit cost of the jth chain having origin node k.

Associated with the network is the vector $h = (h_1^1, h_2^1, \ldots, h_j^k, \ldots)$ which specifies the *flow pattern* at the educational institution, the

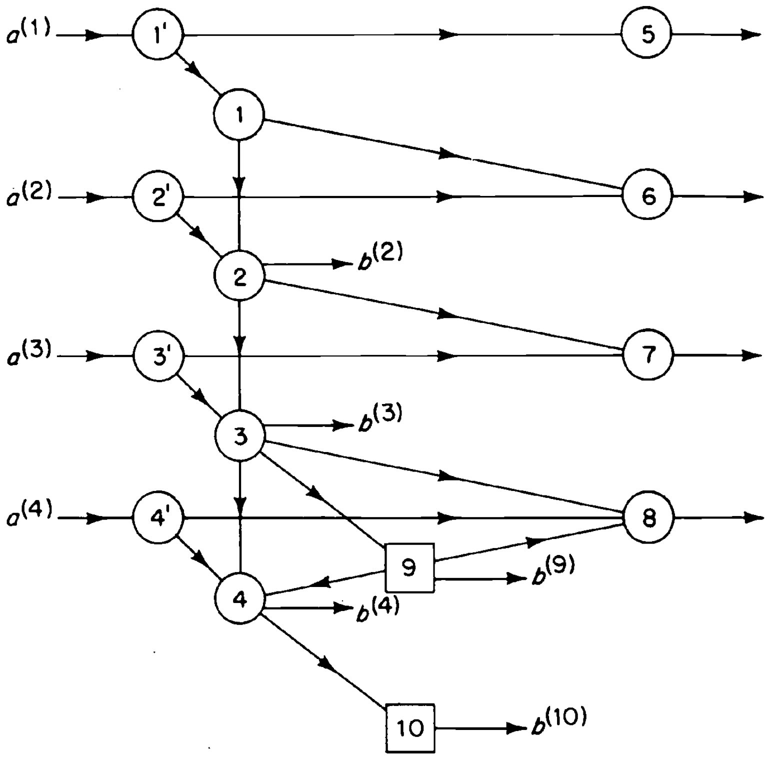

FIGURE 1 The Network of Student Cohorts and Programs

incidence matrix B that defines the network configuration of the institutional programs and flow patterns and finally the scalar C which represents the total instructional costs of educating all stúdent cohorts. These variables are summarized in Table 1.

IV. A COHORT MODEL

The assumptions of the paper are:

A1: Flows and stock levels are sufficiently large so that a deterministic analysis is reasonable.

A2: An equilibrium exists with respect to flows, enrollments, lifetimes, costs, and student dropout rates over time.

A3: Student dropouts from a given program are specified as a fractional flow rate of all students enrolling in that program and having a common origin node.

TABLE 1 Summary of Notation

1.	N:	The set of nodes in the network $[N,M]$
2.	M:	The set of chains in the network $[N,M]$
3.	i,j,k:	Indices referring to nodes, chains, origins
4.	M^k:	The set of chains originating from node k
5.	N^k:	The set of chains ending in node k
6.	c_i:	The unit (lifetime) cost of the ith node
7.	L_i:	The enrollment at the ith node
8.	v_i:	The lifetime in the ith node
9.	g_i:	The total flow entering the ith node
10.	a^i:	The exogenous flow (admissions) into the ith node
11.	b^i:	The exogenous flow (final demands) from the ith node
12.	h_j^k:	The jth chain flow with origin k
13.	C_j^k:	The unit cost of the jth chain with origin k
14.	$h = (h_1^1, h_2^1, \ldots, h_j^k, \ldots)$:	The vector of chain flows on the network, i.e. the flow pattern
15.	C:	The total network cost
16.	$B = [b_{ij}]$:	The node-chain incidence matrix

A4: Technological requirements are specified in terms of teacher-student ratios.

A5: The unit cost of a cohort equals the sum of unit costs of the sequence of programs that define the cohort.

In our model there are four types of equations and inequalities that must be satisfied by student flows. As the reader will see, all of these are linear in the chain flows h_j^k. The first two types of equations are inhomogeneous: admission equations require that chain flows with a common origin satisfy certain equalities or inequalities, whereas final demands imposed exogenously on the educational system constrain chain flows with a common destination node. The third type expresses a dropout cohort flow in terms of all other flows having the same origin node. Finally, we impose technological requirements on teaching assistants. In the last two cases, the equations in chain flows are homogeneous.

In order to conserve flows among student cohorts, it is useful to specify a node-chain incidence matrix B^k for the kth origin node of the network. The elements of each such incidence matrix are given by

(1) $$b_{ij}^k = 1 \text{ if node } i \text{ is in chain } j \text{ with origin } k$$
$$= 0 \text{ otherwise}$$

To simplify notation for those cases where we include all origins we write

(2) $$B = [B^1; B^2; \dots ; B^k; \dots]$$

Thus the augmented matrix B has columns identical with columns of B^k. Since a row corresponds to a node and a column corresponds to a chain, summing entries of B in a particular column gives the number of nodes in that chain, while summing a row of B gives the number of chains passing through a given node.

Using this notation one can write the supply and demand equations in terms of the chain flows and the flows entering or leaving each node of the network. If a^k is the total admission rate originating at the kth node, then a^k is simply the sum of all chain flows originating at k, i.e.

(3) $$a^k = \sum_{j \epsilon M^k} h_j^k \qquad k \epsilon N$$

Similarly, when N^k denotes the set of all chains with destination node k, then

(4) $$b^k = \sum_{j \epsilon N^k} h_j^l \qquad l \epsilon N$$

denotes the total final demand with destination k. In (4) we retain the convention that the index l refers to origin nodes on chains in N^k. Equations 3 and 4 represent the inhomogeneous conservation equations for admissions and final demands. In general there are as many constraints of type (3) or (4) as one chooses to impose. If no constraint is imposed, no equation is written. Furthermore, if a^k or b^k are lower or upper bounds, the appropriate inequalities are substituted for equalities.

We denote the average enrollment level at each node by the product of the lifetime at each node with the total flow into the node (Assumption A2). If we denote the total flow into node i by g_i and the expected lifetime by v_i, then

(5) $$L_i = g_i v_i \qquad i \epsilon N$$

where the total kth origin flow into i is

(6) $$g_i^k = \sum_{j \epsilon M^k} b_{ij}^k h_j^k$$

and the total flow into node i from all origins is

(7) $$g_i = \sum_{k \in N} g_i^k = \sum_k \sum_j b_{ij}^k h_j^k$$

Specification of dropouts is, by Assumption A3, simply a matter of expressing the flow rate of dropouts in terms of the total flow on chains having the same origin node and enrolling in the same program. For instance, the flow rate of students who enter as freshmen and drop out from upper division is proportional to the total freshmen admission rate. In general, if the jth chain with origin node k corresponds to a flow of dropouts we write

(8) $$h_j^k = \alpha_j^k \left[\sum_l h_l^k \right]$$

where α_j^k is the fractional dropout rate and the summation extends over chains $l \in M^k$ containing the program of interest. If, as is often the case, the term in brackets is given and fixed, as in equation 3, then the flow rate h_j^k of the dropout cohort is explicitly calculable. If, on the other hand, the chain flows within the brackets are a priori unknown, then equation 8 can be viewed as a single homogeneous equation restricting a subset of the chain flows with a common origin.

If technological requirements are specified (Assumption A4) as a ratio of teacher inventories required to instruct a given enrollment of students, say

(9) $$L_m = \mu_{mn} L_n$$

then (5) and (7) yield a homogeneous equation in chain flows as follows

(10) $$\begin{aligned} L_m - \mu_{mn} L_n &= v_m g_m - v_n \mu_{mn} g_n \\ &= \sum_k \sum_j (v_m b_{mj}^k - v_n \mu_{mn} b_{nj}^k) h_j^k = 0 \end{aligned}$$

or

(11) $$\sum_k \sum_j (b_{mj}^k - v_m^{-1} v_n \mu_{mn} b_{nj}^k) h_j^k = 0$$

Generalizations of (9) lead to essentially the same structure as that of (11). If, for example, teachers at node m instruct several student cohorts in programs at several nodes

(12) $$L_m = \sum_n \mu_{mn} L_n$$

and equation 11 becomes

(13) $$\sum_k \sum_j (b_{mj}^k - v_m^{-1} \sum_n \mu_{mn} v_n b_{nj}^k) h_j^k = 0$$

Just as a chain can be written as a sequence of distinct nodes, the *unit or average cost* of the jth chain with origin k can be written as the sum of the unit costs of the distinct nodes that comprise that chain, i.e.

(14) $$C_j^k = \sum_i c_i b_{ij}^k \qquad j \in M^k$$

where c_i is the unit (lifetime) cost of the ith node. It is now possible to write total costs of the instructional program in two ways: the first is to multiply all chain flows having a common origin node by the unit cost of the appropriate chain and then sum over origin nodes to obtain the total cost

(15) $$C = \sum_k \sum_{j \in M^k} C_j^k h_j^k$$

To formulate (15) in terms of unit node costs one makes use of (14) and (6) to obtain

(16) $$C = \sum_k \sum_{j \in M^k} \sum_i c_i b_{ij}^k h_j^k$$
$$= \sum_i \sum_k c_i g_i^k = \sum_{i \in N} c_i g_i$$

Thus, an alternative expression for total network cost is to multiply the unit cost of each node by the total flow entering each node and sum over nodes.

In the remainder of this paper, we assume that node costs are proportional to teacher salaries, s_i, node lifetimes, v_j, and teacher-student ratios, μ_{ij}; i.e.

(17) $$c_j = (\sum_i s_i \mu_{ij})\, v_j$$

in other words, the unit cost of a chain does not depend upon the chain flow. For this reason, policies which might affect enrollment levels, degree output rates, admission rates, staffing levels, and so on, *do not* affect the *unit* costs of a chain but *may* affect the total costs or budgets which have to be allocated to produce the chain flows.

The reader should note that a given node may be a member of many chains; thus, changing a node cost will, in general, affect the unit costs of many chains in the network. For example, node 4 in Figure 1 is the destination node for 7 chains and is an intermediate node for 7 additional chains, for a total of 14 chains. To put it another way, any change in the costs of educating doctoral graduates will affect the costs of 14 out of a total of 35 different student cohorts that are educated at the institution. On the other hand, node 5 is a member of only a single chain; altering the costs of lower-division dropouts will affect the costs of that particular student cohort and of no other.

Our formulation of the input-output model is expressed as a set of linear equations in unknown nonnegative chain flows h_j^k. For given right-hand sides of equations 3 and 4, i.e. given admissions and/or demands, the problem is one of finding flow patterns $h = (h_1^1, h_2^1, \ldots, h_j^k, \ldots)$ satisfying equations 3, 4, 8, and 13. Once a feasible flow pattern is found it is a simple matter to compute enrollments from equations 5 through 7. In our experience, it has always been the case that these inequalities contain a large set of feasible solutions, i.e. the number of degrees of freedom is large. Stated another way, we have not yet found an institution where administrative restrictions overconstrain the system of inequalities.

Administrative and institutional policies can affect the model structure in three ways: (1) by altering parameters such as lifetimes, v_j, dropout fractions, α_j, and teaching ratios, μ_{ij}; (2) by imposing constraints of various types, e.g., an enrollment ceiling,

$$\sum_{j \in N} L_j = \text{a constant}$$

or a budget restriction such as equation 16, or altering the types of teachers assigned to students such as equation 13; finally, (3) the cohort flows and programs in chains can be altered by choosing different network configurations which change the incidence matrix $[b_{ij}^k]$. Since the model is a multi-commodity network flow problem with each origin-destination pair serving as a single commodity, it will not, in general, be true that a feasible flow pattern is obtained by superposing cohort flows that are feasible with respect to each commodity. In general such flow patterns violate equations 3 or 4 or 13.

How are these models similar to or different from the cross-sectional models of Section II? (1) The model in this section is formulated in terms of chain *flows*, not *stock levels*. (2) Parameters such as α_j or v_j are based on *longitudinal*, not *cross-sectional* data. α_j is the fraction dropping out over the lifetime of a cohort, not the fraction of enrollments dropping out in one time period. (3) Our chain flow model is not generally linear in policy variables. By comparison, $x(t)$ in the first equation of Section II is often viewed as a policy variable, with $y(t)$ being calculated in terms of $x(t)$ and M being estimated from historical data. Once historical policies have been established, M is fixed and $y(t)$ is linear in $x(t)$. In our model, neither the enrollments nor the cohort flows are policy variables; rather, they are dependent variables which are functions of the policies discussed in the previous paragraph. Such policies not only determine some of the coefficients in the system of constraints but also possibly the set of constraints themselves. Finally, (4) adding or removing programs that constitute a chain, force particular elements of b_{ij}^k to be 0 or 1, and thus alter the coefficients and the number of cohorts in equations 3 through 16. In general, a feasible cohort flow is not a linear function of these *policy* variables.

V. SOURCES AND ANALYSIS OF INSTITUTIONAL DATA

The model described in Section IV was implemented using data for the 1969–70 academic year from Stanford University (SU) and the University of California, Berkeley campus (UCB). Several data sources at each institution were used to estimate parameters in the model for that institution. When good data were not available for estimating a parameter we relied on the judgment and intuition of persons familiar with campus operations and made no attempt to organize a large-scale data collection and analysis effort. We have used three additional assumptions. These are: (1) we allocate the entire salary cost of the faculty to instructional outputs; (2) the salary of a given faculty member is allocated to the various student levels in proportion to the formal courses that he teaches; and (3) the nontenure and tenure faculty inputs for teaching assistants are the same as those for other graduate students enrolled in the same degree program. Since none of these assumptions is crucial to the theoretical discussion of Section III, it is important that the reader bear in mind the distinction between formulation and implementation. Were it desirable to do so, one could modify or remove any of these three assumptions.

A. Stanford University

Our data for Stanford does not include students or faculty at the Stanford Medical School or at the various overseas campuses which have a total enrollment capacity of approximately 400 undergraduates.

1. Student Enrollments and Flow Rates

The enrollment, admissions, and graduation figures shown in Table 2 were obtained from sources in the Registrar's Office and the Graduate Study Office. Students classified as "Terminal Graduates" were included in graduate enrollments. Admission and degree flows during the year begin with the Summer Quarter of 1969 and end with the Spring Quarter of 1970. Separate figures for master's and doctoral admissions and enrollments were not available. Also, it should be mentioned that the total fall enrollment has been virtually constant during the past five years.

Observe that the Stanford enrollment is almost evenly divided between undergraduate and graduate programs and that most undergraduates are admitted as freshmen with only a small fraction entering as junior transfers. This contrasts with many state universities which are

TABLE 2 Stanford University 1969–70 Enrollments and Flow Rates

Student-Level	Fall 1969 Enrollment	1969–70 Admission Rate	1969–70 Degree Output Rate
Lower division	2,949	1,696	–
Upper division	2,782	167	1,515
Total undergraduate	5,731	1,863	1,515
Master's }	5,163		1,555
Doctoral }			441
Teaching assistants	465		
Total graduate	5,628	2,227[a]	1,996
Total students	11,359	4,090	3,511

[a] Of this total graduate admissions, approximately 260 students earned their bachelor's degree from Stanford.

required to draw a significant portion of their undergraduates from junior colleges within the state.

2. Student Dropout Fractions

For the dropout equations, we required estimates of thirteen distinct fractions, α_j^k. For each origin k, these correspond to dropouts that occur in each level at or above the level of admission. For example, there are four dropout cohorts and chains in Figure 1 for $k = 2'$, upper-division admissions. These correspond to dropouts at upper division, $\{2', 6\}$; master's, $\{2', 2, 7\}$; doctoral, $\{2', 2, 3, 8\}$; and doctoral following a master's teaching assistantship, $\{2', 2, 3, 9, 8\}$.

The dropout fractions estimated for Stanford appear in Table 3. The fractions α_1^1 and α_2^1 were obtained directly from a Registrar's study of successive freshman cohorts entitled "Survival of Freshmen Who Enter Autumn Quarter to Baccalaureate Degree Objective." Although the report did not state at what stage the dropouts occurred, there is much

TABLE 3 Stanford University Dropout Fractions

Origin k:	1	1	1	1	1	2	2	2	2	3	3	3	4
Chain i:	1	2	3	4	5	1	2	3	4	1	2	3	1
α_i^k:	.15	0	.05	.45	.45	.15	.05	.45	.45	.05	.45	.45	.45

evidence to indicate that almost all occur during the first two years following admission; therefore, the entire observed dropout fraction was allocated to the lower division, i.e. we set $\alpha_2^1 = 0$. The dropout fraction α_1^2 was taken directly from a Registrar's study of 1967–68 junior transfers.

Data on graduate dropouts was not differentiated according to the level at which students first entered the system. Therefore, it was assumed that the fraction who drop out at each graduate level is the same irrespective of the level at entrance. The master's dropout fraction represents an educated guess by the Head of the Graduate Study Office, while the doctoral dropout fraction was estimated from a study of doctoral students entering under the Ford Foundation Four-Year Guaranteed Assistance Program in the Humanities and Social Sciences. According to the Dean of the Graduate Division, this figure represents a low estimate of the overall fraction because it was derived with reference to a special cohort that was being provided with substantial financial aid as an incentive to complete the degree program.

Using the admissions rates from the second column of Table 2, the master's and doctoral graduation rates from the third column of Table 2, and the dropout fractions from Table 3, one can compute the steady-state output rate for bachelor's degrees. This computed value is 1,584, which seems reasonably close to the observed 1969–70 value of 1,515. In view of the many short-run fluctuations in flows and enrollments that occur even under a fixed enrollment ceiling, we judged this to be a good "fit" of model predictions and real data.

3. Student Lifetimes

The values for lifetimes in each program, v_j, were selected on the basis of discussions with persons familiar with Stanford operations. These are shown in the third column of Table 4. The only group of students for which lifetimes have actually been recorded are those receiving a Ph.D. According to a study by the Graduate Division entitled "Time Required for the Ph.D. at Stanford," the average length of attendance for all students receiving doctoral degrees in the 1967–68 academic year was 4.5 years. This figure was reduced to 4 years in our computations because, in many cases, a one-year half-time teaching assistantship (treated separately in our model) was included in the recorded data.

Observe that the estimated lifetime for lower division is actually less than the customary two years. This is primarily due to the fact that a substantial proportion of Stanford freshmen enter with advanced standing. There is a similar, yet less pronounced, effect from the group who spend their sophomore year at an overseas campus.

TABLE 4 Stanford University Student Lifetimes, Teacher-Student Ratios, and Unit Program Costs

Node j	Program	Lifetime (v_j)	Teacher-Student Ratios (Fall 1969): Teaching Assistants (μ_{1j})	Nontenure Faculty (μ_{2j})	Tenure Faculty (μ_{3j})	Unit Node Costs (c_j)
1	L. D. Grad.	1.8	.137	.009	.017	$1,279
2	U. D. Grad.	2.0	.013	.020	.037	1,909
3	Master's Grad.	1.7	.004	.016	.047	1,839
4	Doctoral Grad.	4.0	.004	.016	.047	4,328
5	L. D. D/O	1.2	.137	.009	.017	853
6	U. D. D/O	1.0	.013	.020	.037	954
7	Master's D/O	1.0	.004	.016	.047	1,082
8	Doctoral D/O	2.0	.004	.016	.047	2,164
9	T. A. (Master's)	1.0	–	.016	.047	1,073
10	T. A. (Doctoral)	1.0	–	.016	.047	1,073

If one computes steady-state flows consistent with the actual admissions rates and master's and doctoral graduation rates for 1969–70 and with the attrition rates in Table 3, and then converts these to enrollments using the lifetimes in Table 4, one obtains the figures 6,098 for undergraduate enrollment and 5,213 for graduate enrollment, exclusive of teaching assistants. The corresponding actual enrollments in the fall of 1969 were 5,731 undergraduates and 5,163 graduates. Again, the agreement between calculations and data is quite good.

4. Teacher-Student Ratios

The parameters μ_{ij} represent ratios of teacher inventories to student inventories. Instructional costs are computed in our model with reference to the following three categories of faculty: teaching assistants (i = 1), nontenure regular faculty (i = 2), and tenure regular faculty (i = 3); we did not include special temporary appointments such as lectureships and instructorships, because they are relatively small in number at the campuses we studied and policies regarding their use differ widely among institutions.

While separate data were available at each institution on the inventories of teachers by rank and students by level, it was necessary to devise a rule for allocating the total inventory of teachers of a given rank to students of each level. We chose to allocate each teacher on the basis of the classes he taught in the fall of 1969. That is, we first assigned each course taught by a given faculty member or teaching assistant to the student level represented by the majority of the students enrolled in that course (i.e., lower division, upper division, or graduate) and then allocated the individual to student levels in the same proportions as his courses. Summing up these allocations over all individuals of a given rank yielded the desired total allocations.

At Stanford, the source for our data was the Registrar's report on fall 1969 courses of instruction. These figures include persons with visiting and acting titles who were teaching at that time. We had to treat teaching assistants as a special case, because, out of a total of 465 teaching assistantships recorded in the Graduate Student Support Table for 1969–70 (Dean's Office, Graduate Division), only 99 appeared in the Registrar's report. Therefore, we allocated all 465 T. A.'s in the proportions established by the data on the smaller sample contained in the Registrar's report. Once allocated, these fall 1969 teacher inventories were divided by the appropriate fall 1969 enrollments from Table 2 to obtain the teacher-student ratios shown in Table 4.

Observe that our data did not permit us to compute separate ratios for different classes of graduate students. Thus, our figures assume that the same number of faculty of each rank are required for the instruction of a doctoral student as for a master's student. Also teaching assistants are assumed to have the same requirements for regular faculty as do other graduate students. In spite of these limitations, however, the trend in the computed ratios appears quite logical. That is, teaching assistants are used almost exclusively for instructing lower-division students, nontenure faculty are associated more with upper-division and graduate students, while tenure faculty are employed in increasing proportions as one proceeds from the lowest to the highest student level.

5. Unit Node Costs

Annual salaries of $2,100 for teaching assistants, $11,500 for nontenure faculty, and $19,000 for tenure faculty were used together with the lifetimes and teacher-student ratios from Table 4 to compute the unit node costs of Equation 17. These are displayed in the last column of Table 4. These faculty salaries were obtained from two sources. For teaching assistants, we divided the total allocation for teaching assistant salaries in the 1969–70 Instructional Budget by the total number of teaching assistants shown in the 1969–70 Graduate Student Support Table. For regular faculty, we used the mean academic salaries reported for the year 1969–70 by the Controller's Office, rounded to the nearest $500.

The results conform to a logical ordering of unit instructional costs. The unit costs of degree programs vary from $1,839 for a master's degree to $4,328 for a Ph.D., with the cost of a bachelor's degree appearing in between at $3,188 (for those who enter as freshmen). Due to shorter lifetimes, the unit costs of dropouts are around one-half the unit costs of the corresponding degree programs.

B. The University of California, Berkeley Campus

1. Student Enrollments and Flow Rates

The quantities in Table 5 were obtained directly from 1969–70 *Campus Statistics* (Office of Institutional Research), except for the teaching assistant inventory which came from the November, 1969 Payroll Accounts. Flows correspond to the period: summer 1969 through spring 1970. Again, separate figures for master's and doctoral admissions and enrollments were not available. As was the case with Stanford, the total fall enrollment at UCB has changed only slightly from its 1965 level.

TABLE 5 University of California (Berkeley) 1969–70 Enrollments and Flow Rates

Student Level	Fall 1969 Enrollment	1969–70 Admission Rate	1969–70 Degree Output Rate
Lower division	7,198	4,250	–
Upper division	10,918	2,925	5,107
Total undergraduate	18,116	7,175	5,107
Master's	8,940		2,358
Doctoral			859
Teaching assistants	1,032		
Total graduate	9,972	4,067[a]	3,217
Total students	28,088	11,242	8,324

[a] Of this total graduate admissions, approximately 800 students earned their bachelor's degree from Berkeley.

In contrast to Stanford, Berkeley is predominantly an undergraduate institution; undergraduates outnumber graduates by a ratio of nearly two to one. In addition, Berkeley accepts a significant portion of its undergraduate admissions as junior transfers.

2. Student Dropout Fractions

All dropout fractions shown in Table 6 were estimated from cohort studies performed by the Office of Institutional Research on undergraduates admitted in 1965 and graduate students admitted in 1960. Since the graduate student cohort study did not provide enough information to yield unique fractions for all dropout cohorts, the values of α_3^1, α_2^2, α_1^3, and α_1^4 in Table 6 are based partly on experience and judgment. In comparing Table 6 with Table 3, we observe that the Berkeley dropout fractions are uniformly higher than their Stanford counterparts, as one might expect.

If one uses the 1969–70 Berkeley admission rates from Table 5, the dropout fractions in Table 6, and some additional information contained

TABLE 6 University of California (Berkeley) Dropout Fractions

Origin k:	1	1	1	1	1	2	2	2	2	3	3	3	4
Chain i:	1	2	3	4	5	1	2	3	4	1	2	3	1
α_i^k:	.30	.20	.30	.55	.55	.25	.30	.55	.55	.30	.55	.55	.45

in the graduate student cohort study, one obtains the following steady-state degree output rates: bachelor's, 4,574; master's, 2,122; and Ph.D., 772. Again, the discrepancy between these computed flow rates and their actual 1969–70 values shown in the third column of Table 5 is of the order of 10 per cent.

3. Student Lifetimes

With the exceptions of v_3, v_4, v_9, and v_{10}, which were obtained directly from the graduate student cohort study, lifetimes were estimated on the basis of discussions with persons familiar with Berkeley campus operations. These are displayed in the second column of Table 7. Using these lifetimes, one can convert the steady-state flows corresponding to 1969–70 admission rates to enrollments, obtaining values of 18,161 for undergraduates and 9,247 for graduates, excluding teaching assistants. The agreement between these computed values and the actual fall 1969 enrollments shown in Table 5 is excellent.

4. Teacher-Student Ratios

Due to data limitations, the Berkeley faculty was allocated to student levels in a slightly different manner from that used at Stanford. The essential difference is that, whereas at Stanford each faculty member was allocated according to the level of students enrolled in the courses he taught, at Berkeley the total inventory of faculty in each category was allocated to lower-division, upper-division, and graduate *students* in proportion to the total number of classroom contact hours spent by members of that category in lower-division, upper-division, and graduate division *courses*. Thus, we assumed a one-to-one correspondence between student level and course level and allocated the teaching assistants and regular faculty (including visiting and acting appointments) reported in the fall 1969 Schedule of Classes on the basis of the contact hours reported in the same document. These allocations were then divided by the fall 1969 enrollments from Table 5 to yield the teacher-student ratios shown in Table 7. Observe that these ratios exhibit exactly the same trends as did those computed for Stanford.

5. Unit Node Cost

The average salary for teaching assistants was obtained in the following way: the 1969–70 full-time equivalent (FTE) salary reported by the Chancellor's Office was multiplied by the ratio of FTE teaching assistants reported in the 1969–70 Instructional Budget to head-count teaching assistants reported in the fall 1969 Payroll Accounts. The average

TABLE 7 University of California (Berkeley) Student Lifetimes, Teacher-Student Ratios, and Unit Program Costs

			Teacher-Student Ratios (Fall 1969)			
Node j	Program	Lifetime (v_j)	Teaching Assistants (μ_{1j})	Nontenure Faculty (μ_{2j})	Tenure Faculty (μ_{3j})	Unit Node Costs (c_j)
1	L. D. Grad.	2.0	.098	.013	.018	$1,642
2	U. D. Grad	2.1	.024	.018	.031	1,861
3	Master's Grad.	1.8	.007	.014	.074	2,912
4	Doctoral Grad.	4.0	.007	.014	.074	6,472
5	L. D. D/O	1.0	.098	.013	.018	821
6	U. D. D/O	1.0	.024	.018	.031	886
7	Master's D/O	1.0	.007	.014	.074	1,618
8	Doctoral D/O	2.0	.007	.014	.074	3,236
9	T. A. (Master's)	1.0	–	.014	.074	1,595
10	T. A. (Doctoral)	1.0	–	.014	.074	1,595

regular faculty salaries were supplied by the Office of the Vice President for Academic Affairs and were based upon all faculty engaged in instruction during the 1969–70 regular academic year (i.e. excluding the summer quarter). The resulting figures were \$3,300 for teaching assistants, \$11,400 for nontenure regular faculty, and \$19,400 for tenure faculty.

Using these salaries and the lifetimes and teacher-student ratios from Table 7, we computed the unit node costs displayed in the last column of that table. Observe that although they are ordered in the same way, the unit costs for degrees at Berkeley are significantly higher than at Stanford, except in the case of junior transfers earning a bachelor's degree. This finding is explained by the combination of two factors, for both the lifetimes and the tenure faculty-student ratios are generally greater at Berkeley than at Stanford. Thus, for example, a large discrepancy occurs at the doctoral level, where although the lifetimes are identical, one finds nearly 60 per cent more tenure faculty per student at Berkeley than at Stanford.

C. A Comparison of Unit Chain Costs

The unit chain costs for each institution are shown in Table 8 along with the chain descriptions. These figures were obtained by inserting the node costs from Tables 4 and 7 in equation 14 of Section IV. It is clear that they obey the following ordering scheme: each chain has a higher unit cost than all other chains made up of a subset of its nodes; moreover, the unit cost of any chain that ends in a dropout node is strictly less than the unit cost of the chain that has the same origin, passes through the sequence of nodes, and ends with the corresponding graduate node. Obviously, the least expensive way for an educational institution to meet final demands for degrees is to admit students at the highest level appropriate to the degree, and then to prohibit degree-winners from continuing further. However, several factors contribute to make this an unrealistic solution. (1) There are many reasons for preferring to admit freshmen instead of junior transfers to the undergraduate program. (2) Master's graduates are generally free to decide whether they wish to continue in the doctoral program. (3) There exist well-established teaching assistant ratios for different undergraduate cohorts. (4) Enrollment ceilings force certain cohorts to contribute to undergraduate as well as graduate enrollments. The reader may be troubled by the assumption that, for instance, the costs of a master's and doctoral program are strictly additive for those students who pursue both degrees. While this represents an abstraction from reality, we do not believe it influences our cost estimates in a significant way.

TABLE 8 Unit Chain Costs

Origin k	Chain j	Description of Cohort[a]						Unit Chain Costs (C_j^k) SU	UCB
1	1	LD	UD					$3,188	$3,503
1	2	LD	UD	M				5,027	6,415
1	3	LD	UD	M	D			9,355	12,887
1	4	LD	UD	M	D	TA		10,428	14,482
1	5	LD	UD	M	D	D/O		7,191	9,651
1	6	LD	UD	M	TA			6,100	8,010
1	7	LD	UD	M	TA	D		10,428	14,482
1	8	LD	UD	M	TA	D	TA	11,501	16,077
1	9	LD	UD	M	TA	D	D/O	8,264	11,246
1	10	LD	UD	M	D/O			4,270	5,121
1	11	LD	UD	D/O				2,233	2,528
1	12	LD	D/O					853	821
2	1	UD						1,909	1,861
2	2	UD	M					3,748	4,773
2	3	UD	M	D				8,076	11,245
2	4	UD	M	D	TA			9,149	12,840
2	5	UD	M	D	D/O			5,912	8,009
2	6	UD	M	TA				4,821	6,368
2	7	UD	M	TA	D			9,149	12,840
2	8	UD	M	TA	D	TA		10,222	14,435
2	9	UD	M	TA	D	D/O		6,985	9,604
2	10	UD	M	D/O				2,991	3,479
2	11	UD	D/O					954	886
3	1	M						1,839	2,912
3	2	M	D					6,167	9,384
3	3	M	D	TA				7,240	10,979
3	4	M	D	D/O				4,003	6,148
3	5	M	TA					2,912	4,507
3	6	M	TA	D				7,240	10,979
3	7	M	TA	D	TA			8,313	12,574
3	8	M	TA	D	D/O			5,076	7,743
3	9	M	D/O					1,082	1,618
4	1	D						4,328	6,472
4	2	D	TA					5,401	8,067
4	3	D	D/O					2,164	3,236

[a] LD = Lower-division graduate
UD = Upper-division graduate
M = Master's graduate
D = Doctoral graduate
TA = Teaching assistant
LD D/O = Lower-division dropout
UD D/O = Upper-division dropout
M D/O = Master's dropout
D D/O = Doctoral dropout

Inter-institutional comparisons are also revealing. Chain costs at Stanford are in the range \$853 to \$11,501, while those at Berkeley range from \$821 to \$16,077. In nearly all cases, the unit chain costs are substantially higher at Berkeley than at Stanford, due to the higher program (node) costs, which we discussed earlier.

VI. AN ANALYSIS OF THE CARNEGIE UNDERGRADUATE PLAN

In this section, we use the numerical data of Section V and apply the model of Section IV to obtain some preliminary estimates of the unit and total instructional costs that would result from the adoption of the Carnegie Commission recommendation [1971] that the lower division be effectively reduced from a two- to a one-year program. In all cases that we discuss, we estimate lower bounds for the resulting instructional budget.

To understand how these recommended policies can be incorporated within the model of Section IV, it may be useful to characterize the structure of solutions of constraints that we have discussed in Section IV.

Table 9 lists the coefficients of 20 equations in 35 chain flows (cohorts). There are 5 dropout equations for the lower division, 4 dropout equations for the upper division, 3 dropout equations for master's degree candidates and 1 dropout equation for doctoral students. There is 1 technological constraint on teaching assistants, 1 enrollment ceiling constraint, 1 constraint on admissions to upper division, 2 constraints on admissions to graduate programs and 2 constraints on final demand for doctoral graduates. Flows are nonnegative, which is a simple way to require that students flow in the direction of the arrows of the network in Figure 1. While these constraints are obviously not representative of all educational institutions, they seem realistic for the three campuses which we studied.

In summary there are 20 linear constraints on 35 nonnegative chain flows to describe each institution. It is interesting to note that only the technological constraint (15), the enrollment ceiling (16) and the requirements on final demands (19 and 20) prevent the system of equations from being decomposed into four independent subproblems with solutions a function only of origin-dependent parameters. The reason that the system of equations cannot be decomposed is that (1) students in lower- and upper-division programs affect the number of teaching assistants in graduate cohorts, (2) an enrollment ceiling places a constraint on the total number of students, including teaching assistants, and (3) constraints on the output flows of a particular type of student, say

TABLE 9 Stanford University Constraints (BCC)

Chain / Chain Flow	1 h_1^1	2 h_2^1	3 h_3^1	4 h_4^1	5 h_5^1	6 h_6^1	7 h_7^1	8 h_8^1	9 h_9^1	10 h_{10}^1	11 h_{11}^1	12 h_{12}^1
LD D/O	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	1
LD UD D/O											1	
LD UD M D/O		−.053	−.053	−.053	−.053	−.053	−.053	−.053	−.053	1		
LD UD M D D/O			−.818	−.818	1							
LD UD M TA D D/O							−.818	−.818	1			
UD D/O												
UD M D/O												
UD M D D/O												
UD M TA D D/O												
M D/O												
M D D/O												
M TA D D/O												
D D/O												
Technological	−.274	−.281	−.299	.701	−.290	.719	.701	1.70	.710	−.278	−.260	−.165
Enroll. ceiling	3.8	5.5	9.5	10.5	7.5	6.5	10.5	11.5	8.5	4.8	2.8	1.2
Junior transfers												
Internal grad. adm.		1	1	1	1	1	1	1	1	1		
Ext. grad. adm.												
Fin Dem PHD			1				1					
Fin Dem PHD TA				1				1				

TABLE 9 (continued)

Chain Chain Flow	13 h_1^2	14 h_2^2	15 h_3^2	16 h_4^2	17 h_5^2	18 h_6^2	19 h_7^2	20 h_8^2	21 h_9^2	22 h_{10}^2	23 h_{11}^2	24 h_1^3
LD D/O												
LD UD D/O												
LD UD M D/O												
LD UD M D D/O												
LD UD M TA D D/O												
UD D/O	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	−.177	1	
UD M D/O		−.053	−.053	−.053	−.053	−.053	−.053	−.053	−.053	1		
UD M D D/O			−.818	−.818	1							
UD M TA D D/O							−.818	−.818	1			
M D/O												−.053
M D D/O												
M TA D D/O												
D D/O												
Technological	−.026	−.034	−.051	.949	−.043	.966	.949	1.949	.957	−.031	−.013	−.007
Enroll. ceiling	2.0	3.7	7.7	8.7	5.7	4.7	8.7	9.7	6.7	3.0	1.0	1.7
Junior transfers	1	1	1	1	1	1	1	1	1	1	1	
Internal grad. adm.		1	1	1	1	1	1	1	1	1		
Ext. grad. adm.												1
Fin Dem PHD			1				1					
Fin Dem PHD TA				1				1				

TABLE 9 (concluded)

Chain / Chain Flow	25 h_2^3	26 h_3^3	27 h_4^3	28 h_5^3	29 h_6^3	30 h_7^3	31 h_8^3	32 h_9^3	33 h_1^4	34 h_2^4	35 h_3^4	*rhs*
LD D/O												0
LD UD D/O												0
LD UD M D/O												0
LD UD M D D/O												0
LD UD M TA D D/O												0
UD D/O												0
UD M D/O												0
UD M D D/O												0
UD M TA D D/O												0
M D/O	−.053	−.053	−.053	−.053	−.053	−.053	−.053	1				0
M D D/O	−.818	−.818	1									0
M TA D D/O					−.818	−.818	1					0
D D/O									−.818	−.818	1	0
Technological	−.025	.975	−.016	.993	.975	1.975	.984	−.004	−.018	.982	−.009	0
Enroll. ceiling	5.7	6.7	3.7	2.7	6.7	7.7	4.7	1	4	5	2	11,359
Junior transfers												167
Internal grad. adm.												260
Ext. grad. adm.	1	1	1	1	1	1	1	1	1	1	1	1,968
Fin Dem PHD	1				1				1			116
Fin Dem PHD TA		1				1				1		325

doctoral graduates, place restrictions on chain flows having a common destination rather than a common origin.

It is interesting to see how a lower bound for the instructional budget and a feasible solution of the system of equations in Table 8 can be generated. Consider, in the case of Stanford University, the chain costs in the first column of Table 8, and the 1969–70 graduation rates of Table 2. To estimate the cost of producing 1,515 bachelor's degrees, 1,555 master's degrees and 441 doctoral degrees, one could select the three cheapest chains ending in nodes 2, 3, 4 with costs

$C_1^2 = 1{,}909,\ C_1^3 = 1{,}839,\ C_1^4 = 4{,}328$

By forcing all students to use these three programs one obtains an unrealistically low instructional budget of

$(1{,}515)(\$1{,}909) + (1{,}555)(\$1{,}839) + (441)(\$4{,}328) = \$7{,}660{,}428.$

This estimate is approximately $3.2 million less than the actual 1969-70 budget of $10.9 million. What accounts for this large difference? With the unrealistic flow pattern we have just used, the university is divided into three independent components in which no students receiving one degree continue at the same institution to obtain a more advanced degree. There are no dropouts from any program and the total enrollment capacity of 11,359 is underutilized by approximately 4,300 students. Neither teaching assistants nor associates are involved in the instruction of undergraduate programs and no recognition is made of the value that teaching experience has upon the quality of education of a graduate student. We hasten to point out that each of these requirements costs money; the magnitude of the additional costs can also be estimated and we proceed to do so.

By requiring that no more than 167 junior transfers be admitted to Stanford, chain (1′,1,2) with a unit cost of $3,188 rather than $1,909 is introduced for the production of bachelor's degrees. By requiring that admissions to Stanford graduate schools must have a nominal flow of Stanford's undergraduates, we introduce a large number of chains with unit costs beginning at $3,748 and ending as high as $11,501.

The recognition of distinct final demands for doctoral students with or without teaching-assistant experience forces the use of chains such as (4′,4,10) with a unit cost of $5,401. Once the reader is convinced that such chain flows are desirable it is then a simple (but tedious) matter to consider all the associated dropout cohorts and their costs. Generally speaking, for every chain flow that results in some degree recipient there is a corresponding dropout flow.

By the time some minor readjustments in the flow patterns are made to meet the constraints of Table 9, one moves from the unrealistic budget estimate of $7.3 million to the more realistic estimate of $10.82 million.

A similar set of calculations for the Berkeley campus begins with a budget estimate of \$21.93 million for positive flows on the three cheapest chains and increases to an estimate of \$31.56 million when one includes the appropriate teacher-student ratios, enrollment ceilings, dropout flows and restrictions on junior transfers. In each case, the second estimate that we derive is still less than the instructional budget one obtains for actual 1969–70 fiscal operations. In the 1969–70 Stanford Budget, instructional salaries, excluding the Medical School, sabbatical leaves, and overseas campuses amount to \$10,900,000; Payroll Accounts in November of 1969 give an instructional budget at Berkeley of \$31,692,600. Both figures are in close agreement with estimates from the model.

Comparative solutions for enrollments, degrees, and cohort costs for Stanford University, the University of California at Berkeley, and the University of Colorado at Boulder are summarized in Table 10. Columns headed by BCC denote quantities predicted by our model before the Carnegie Commission recommendation is implemented while ACC denotes after Carnegie Commission. The fifth and sixth columns in the table refer to an unpublished study made by students in a graduate Operations Research course at the University of Colorado. It should be mentioned that the constraint set of the Colorado model differs from that used to analyze the Stanford and Berkeley campuses. In the first case, an enrollment ceiling is *not* imposed and the following are specified: total output rates for bachelor's, master's, and doctoral degrees, a ratio of entering junior transfers to entering freshmen and a ratio of external graduate admissions to internal graduate admissions.

Consider the following solution of the Stanford tableau in Table 9:

$$h_1^1 = 1{,}311,\ h_2^1 = 112,\ h_{10}^1 = 6,\ h_{12}^1 = 252,\ h_2^2 = 2$$
$$h_6^2 = 133, h_{10}^2 = 7, h_{11}^2 = 25, h_1^3 = 1{,}108, h_9^3 = 58$$
$$h_1^4 = 116, h_2^4 = 325, h_3^4 = 361, \text{all other}\, h_j^k = 0$$

Elements in the first column of Table 10 are derived from these flows and the chains of Table 8 in the following way: Enrollments are obtained by substituting these chain flows in equations 5 and 6 with lifetimes from Table 4. Undergraduate degrees are obtained by summing h_1^1 through h_{10}^1 and h_1^2 through h_{10}^2. Master's degrees are obtained by summing h_2^1 through h_9^1, h_2^2 through h_9^2 and h_1^3 through h_8^3. Doctoral degrees sum the flows h_3^1, h_4^1, h_7^1, h_8^1, h_3^2, h_4^2, h_7^2, h_8^2, h_2^3, h_3^3, h_6^3, h_7^3, h_1^4 and h_2^4. Cohort costs are obtained by multiplying these chain flows by their unit costs in Table 8. In those cases where a cohort receives an undergraduate degree and then enrolls in graduate programs, we allocated the program cost to the appropriate category of degree-winner or dropout. For example, the cohort flow $h_{10}^1 = 6$

TABLE 10 Enrollments, Degree Rates, Costs of Degrees and Dropouts before (BCC) and after (ACC) Carnegie Commission Recommendation

	Stanford		University of California (Berkeley)		University of Colorado (Boulder)	
	BCC	ACC	BCC	ACC	BCC	ACC
Enrollments						
Lower division	2,874	2,194	7,302	5,647	8,906	5,379
Upper division	3,166	3,930	10,994	12,771	6,574	6,574
Graduate	5,317	5,233	9,792	9,670	2,447	2,384
Total	11,357	11,357	28,088	28,088	17,927	14,338
Degrees/Year						
B.S., B.A.	1,571	1,953	4,603	5,359	2,640	2,640
M.S., M.A.	1,355	1,354	1,751	1,751	814	814
Doctoral	441	441	859	859	302	302
Total	3,367	3,748	7,213	7,969	3,756	3,756
Cohort costs (in millions of dollars)						
Undergraduate degrees	4.83	5.01	12.52	12.57	–	–
Undergraduate dropouts	.24	.30	3.22	3.38	–	–
	5.07	5.31	15.74	15.95	–	–
Graduate degrees	4.89	4.80	12.32	12.12	–	–
Graduate dropouts	.86	.86	3.50	3.50	–	–
	5.75	5.66	15.82	15.62	–	–
Total	10.82	10.97	31.56	31.57	16.78	14.33
Instructional Cost/Degree (in dollars)						
Undergraduate	3,225	2,720	3,419	2,977	–	–
Graduate	3,201	3,151	6,061	5,987	–	–
Combined	3,211	2,927	4,374	3,962	4,469	3,815

results in 6 × \$3,188 = \$19,128 for undergraduate degree costs and 6 × \$1,082 = \$6,492 for graduate dropout costs.

To see how one calculates parameters relevant to the Carnegie Commission plan, consider the lifetime of node 1, the lifetime for the lower division program. This lifetime affects (1) the technological constraint associated with teaching assistants that are required to instruct lower-division students (see equation 11 or 13 of Section IV) and (2) the total lifetimes of all chains which contain node 1; thus, the coefficients of the first eleven chain flows passing through node 1 are reduced in the enrollment ceiling constraint which is obtained by substituting chain lifetimes rather than unit costs in equation 15. Finally, (3) the unit costs, as defined by equation 14 in that section, are also modified. In summary, the reduction in lower-division lifetimes affects the total lifetimes and unit costs of 11 out of 35 possible chains, and affects a single technological constraint for teaching assistants. We display the least-cost budgets, enrollment levels, graduation rates, and costs of educating the dropout and degree-winning cohorts in the ACC columns of Table 10.

The lower-bound estimates that we have just described can be routinely calculated by using a linear programming algorithm to minimize the sum of costs of all chain flows, i.e. the total instructional cost of equation 15 in Section IV, subject to the constraints of equations 3, 4, 8, and 13 in the same section. Chain costs used are those of Table 8, with the modifications described above for calculations based on the Carnegie plan; all parameters used in the constraints are derived from the appropriate terms in Tables 2 through 7 of Section V. For example, the magnitude of the coefficient of the first chain flow in the first restriction of Table 9 is $\alpha_1^1 \, (1 - \alpha_1^1)^{-1} = .1765$.

It appears that the major impacts the Carnegie Commission recommendation would have if adopted in the long run are the following: (a) At both Stanford and Berkeley the flow rate for graduating B.S. degrees would increase substantially, from approximately 1,571 per year to 1,943 per year at Stanford, from approximately 4,603 to 5,359 per year at Berkeley. (b) There would be an increased flow rate of the number of students that drop out each year at each institution. At Stanford, the lower-division dropout flows would increase from 252 to 320 per year; at Berkeley, from 1,292 to 1,698 per year. (c) Lower-division enrollments would decrease and admissions would increase because of the reduction in lifetime to complete the lower-division programs. At Stanford, lower-division enrollments would decrease from 2,874 to 2,194; at Berkeley, from 7,302 to 5,647. (d) If enrollment ceilings were maintained at their current levels, as well as current restrictions on junior transfers, upper-division enrollments would increase from 3,166 to 3,930 at Stanford, from 10,994 to 12,771 at Berkeley. This increase is due, of course, to the increased admission rate into the lower-division program. (e) Unless different admission policies were adopted, the graduate components at both institutions would remain roughly the same, with one important exception—(f) the enrollments of teaching assistants would decrease at Berkeley from 1,042 to 921 and at Stanford from 458 to 374. In both cases, the decrease is due to the fact that there is a substitution of small teacher-student ratios at the upper division for large teacher-student ratios at the lower division which more than offsets the increased upper-division enrollments discussed in (d). (g) The total instructional costs increase slightly at both institutions—by approximately $150,000 per year at Stanford and by $25,000 per year at Berkeley; this small increase in total instructional budget is offset by (h) the very large increases in total degree rates at both institutions; from 3,367 to 3,748 at Stanford, and from 7,213 to 7,969 at Berkeley. The net result which should be of primary interest to educational administrators is (i)—unit costs of all degree recipients decreases from $3,211 to $2,927 at Stanford and from $4,374 to $3,962 at Berkeley. To state it in another way, it appears that if the total degree output rates were held constant at their current values, the total instructional budget could decrease substantially at both institutions.

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9 COMMENTS

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There is little controversy inherent in the description of the Oliver-Hopkins model. Given their assumptions, their equations appear to be correct. However, considerable controversy arises when it comes time to apply a model such as this one or any of a myriad of other mathematical models which simulate the operation of a university system. In this note, then, I wish to comment on the applicability of the Oliver-Hopkins model. Later I shall discuss the workings of the model and some of the inevitable shortcomings of a model of this size.

The Oliver-Hopkins model provides a contrast to many large-scale simulations which cost hundreds of thousands of dollars to build and require thousands of data inputs. Such simulations have been built by Weathersby [3], Koenig et al. [2], Judy [1], and others. They rely on a sequence of linear transformations and thus require that several large input-output matrices be specified. The volume of computer output can be overwhelming, although to a certain extent there can be an advantage in having information more disaggregated than in the Oliver-Hopkins model.

Input-output matrices in the large simulations are filled with data based on historical policy trends. Without guessing coefficients out of the blue, it is difficult to eliminate the effect of presently irrelevant past policies in the simulated future. This shortcoming which Oliver and Hopkins point out in the other models is also present in theirs, where presumably cost and dropout rate estimates have their basis partly in historical trends. However, the size of this model allows for much more experimentation with different assumptions.

Constant returns to scale are assumed both in this model and in the large-scale simulations. It would add unreasonable computational complications if this assumption were changed. A decision maker should be sobered by this and by many other restrictive assumptions.

All of these models are designed to assist university administrators in their decision making. To this end, they should provide outputs which are neither misleading as regards accuracy nor too voluminous. The decision maker must interpret output of these models in light of possible future trends, which cannot possibly be incorporated directly into the model.

For a moment, let us consider applying these models at a point in time ten years ago in an attempt to predict future student enrollments and various costs. We could use input data based on past enrollments, student-faculty ratios, faculty salaries, building costs, and so on, and could project certain changes in these input measures over past trends. However, could we have reliably quantified the effects of such external environmental changes as the abolition of student draft deferments, the changing public attitudes toward education and resulting budget cuts, greatly decreased demand for Ph.D.'s, mass unemployment in the aerospace field, and increased concern with various forms of environmental pollution? My impression is that there are many future influences external to the university that cannot be adequately incorporated in any present model. Such influences introduce a great deal of uncertainty into any future cost or enrollment estimates and add futility to the process of calculating volumes of "precise" estimates based on past trends and detailed disaggregated present figures. The calculations of a large-scale simulation are analogous to an engineer's painstaking calculation of one figure to seven significant digits when it must then be added to another figure with only three-digit accuracy. The size of the Oliver-Hopkins model represents a better match between input precision and detail and the predictive power of the output.

To further simplify computations, there are no time dependencies included in Oliver and Hopkins's inputs. The assumption that flows are stable from year to year allows for computing characteristics of the resulting equilibrium. Thus, rather than answering specific questions about enrollment patterns and costs in 1972, 1973, 1974, et cetera, this model examines the equilibrium which would result from continued use of a given policy and thus gives a picture of the direction in which such a policy is leading.

The most impressive features of this model then are its size and its computational simplicity. The idea of computing steady-state characteristics associated with the system, although not at all new to operations researchers, has appeal in reducing the computational burden. However, there are disadvantages arising from the fact that these computations do not comment on the feasibility of immediately implementing the policy which looks good at equilibrium. Perhaps the current state of the system prohibits the use of a policy which would eventually meet all of the constraints if allowed to run long enough.

There are a couple of puzzling features as regards the cost arguments in this paper. The authors give a lower bound on the Stanford instructional budget and then show how constraints cause more expensive chains to enter the picture. It is clear that an upper limit on junior-college transfers forces the school to give expensive four-year B.A.'s. However, the requirement of admitting some Stanford undergrads to graduate school does not increase the budget in this model. The cost of educating one student from freshman to Ph.D. does not differ from the cost of educating one student through a B.A. and another from a B.A. to a Ph.D. The costs of these alternative means of producing one B.A. and one Ph.D. could be made to differ by assuming different dropout rates in graduate school for Stanford graduates and others. Depending on these specific rates, a least-cost production policy might call for admitting either as many Stanford graduates into graduate school as possible or as few as possible. Thus a wide policy swing depends on whether Stanford graduates are more or less likely than others to drop out of graduate school.

This model allows dropout rates from a given program to depend on a student's status when he first enters the institution as well as on the program he is currently enrolled in. It is only when this former type of dependence is very real that the model is sensitive to the past history of any given student. Then the optimal policy (although not necessarily the budget figure) is very sensitive to different dropout-rate values. If, on the other hand, the dropout probability depends only on the current program of a student, the cost of any policy with the same enrollment levels in each program is the same regardless of the past history of students.

The problem of assigning instructional costs to students in various programs is a somewhat sensitive one but not nearly as difficult as the problem of measuring the value of a student's experience as a function of contact hours with faculty and faculty salaries. I was pleased to see Oliver and Hopkins stay away from that issue; my experience with UCLA data showed that educational values could be assigned to students in different ways to reach whatever pet conclusion one had in mind.

In focusing on one institution, the authors naturally find that that institution can attain a desired degree output at minimum cost by admitting as many undergrads at the junior level as possible and avoiding the costs of educating lower-division students—many of whom drop out and none of whom earn a degree while in lower division. In judging, for example, whether this is a wise policy for a public university, one must really compare the savings to costs of providing lower division schooling in other public institutions such as community colleges. It is natural also that an institution can save money by abbreviating the lower division to one year. Again, to really judge the advantages of this policy one should compare the budget savings to the value of the "year of education lost" to the student.

The constraint for junior-college transfers is expressed as an equality constraint (i.e. exactly 167 should be admitted). It might be that with different cost assumptions, the shadow price of changing that requirement to 168 would be negative. The argument that it is unlikely that the real shadow price will be more than 2 or 3 times the estimate computed by the model seems to need more justification (Stanford's shadow price is more than ten times Berkeley's).

Many different constraints could be placed on the output of an institution through this model. It appears most natural from the network diagram (Figure 1) to place lower bounds on $b^{(2)}$, $b^{(3)}$, $b^{(4)}$, $b^{(9)}$, *and* $b^{(10)}$ (or perhaps equality constraints). Yet it seems hardly more reasonable to constrain $b^{(2)}$ than to constrain the total flow out of node 2. In the first case, the university has a responsibility to send a minimum number of B.A. graduates from the graduation ceremony immediately into the real world; in the second, there is a responsibility to send a minimum number to the graduation ceremony. As the authors point out, the location of those constraints influences shadow prices.

As I have mentioned, the form of an optimal operating policy is very sensitive to the various dropout-rate values. It would be valuable to find out whether shadow prices are equally sensitive.

In conclusion, I am pleased with the size of this model. It lends itself very well to all sorts of sensitivity testing. Thus an administrator making use of it is in a good position to evaluate how big a grain of salt must be swallowed with the output. It is important that no operations research model be accepted by practitioners on blind faith; this model's simplicity and sensitivity-testing features guard against that possibility in an admirable manner. However, lacking personal computational experience, I still have some questions as to the outcome of many sensitivity tests.

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My comments on the interesting paper by Oliver and Hopkins fall into two categories. First, I shall give some general critical reactions, as an economist, to the operations research models which are much in vogue these days for experimental planning and budgeting in higher education. These reactions, which focus on some conceptual ambiguities in the definition and measurement of costs, apply to a broad class of models, including the one developed by Oliver and Hopkins. Secondly, I will make a few brief points specific to the paper under discussion.

The university is a multi-product institution and any attempt to measure unit costs must take cognizance of all the major outputs to which resources are allocated or else the resulting cost figures will be grossly distorted. Oliver and Hopkins have decided to aggregate over all departments—which produce different outputs using vastly different inputs—in order to make their problem more manageable. Changes in university costs and resource requirements generated by a varying departmental mix are therefore exogenous to, and unpredicted by, their model.

I am more concerned, however, about the ease with which they (and many others) have overlooked the fact that considerable university resources are spent on nonteaching activities such as research and administration. According to data which I have collected at Stony Brook, the faculty—the resource with which Oliver and Hopkins are primarily concerned—spends barely 50 per cent of its time on classroom instruction, and even less if one takes account of holidays, summers, and sabbaticals.[1] Thus, allocating the full faculty cost to teaching seems unwarranted.

Research and teaching activities of the faculty are not inherently tied together in fixed proportions; indeed, we find liberal arts colleges which produce undergraduate education exclusively and institutes which specialize in research. Graduate schools usually produce some research—presumably, the faculty cannot teach graduate students how to do research without doing some themselves—but the exact mix varies among institutions. Thus, research may be viewed as an output, albeit difficult to measure, or, alternatively, as an input into the graduate program. Its input into undergraduate teaching, particularly at lower-division levels, is probably much less. (I say "probably" because this is basically an empirical question about which one can only make assumptions until an operationally sound means of testing the hypothesis is devised.)

Whether this explicit treatment of research makes any difference depends, of course, on the question being asked. When estimating unit costs, ignoring research overstates the *absolute* cost of teaching in general and teaching undergraduates in particular. It does not, however, significantly alter the *relative* instructional costs of two schools with a similar teaching-research mix and a similar undergraduate-graduate mix. Stanford and Berkeley have similar mixes, so the specific comparison that Oliver and Hopkins make is probably unbiased. If, on the other hand, they had chosen to look at two institutions with different product mixes, their approach *would* have distorted these relative costs. For example, when comparing lower-division teaching costs at a university and a community college, Oliver and Hopkins would probably predict much higher figures for the former than the latter, whereas in studies that I have made, program costs were often the same or higher at the community college. I would thus view the choice between these school types by a legislature as a decision about the optimal product mix for the institution and the state, rather than a response to differential costs of undergraduate education.

Similarly, an increase in teaching loads at a given institution would reduce faculty costs for each student or degree in the Oliver-Hopkins model, whereas I would view this primarily as a shift in product mix, to a higher teaching/research ratio. Any cost-saving for a given enrollment would be attributed to a lower quantity of research and to a lower quality of graduate education—a more accurate and useful way of looking at the matter, in my opinion.[2]

Faculty spend their time, too, on administrative activities—curriculum planning, recruitment, and so forth. Oliver and Hopkins simply ignore this, implying that such activities are current costs which should be allocated among various student cohorts in the same proportion as teaching time. I suggest that much of this administrative activity may more properly be regarded as an investment in the future research and teaching functions of the university than regarded as an input into its present instructional functions. If the university were to close down next year there would be no need to introduce new courses, revise requirements, hire additional faculty, and so on. Such costs are thus relevant only when considering whether to extend the life of the institution into the future, and not when discussing its current operations. This year's research and teaching depends, of course, on past administrative inputs, but by now these are sunk costs. Furthermore, there is no reason to believe that the "depreciation" of these past administrative activities is exactly equal to, and therefore measurable by, the current activities. The distortion is particularly significant in an old, declining institution or in a young, growing university, of which we have many today.

The multi-product nature of a university comes to the fore again when dealing with the cost of teaching assistants. Teaching assistants have been variously interpreted as a "slave labor" input into the undergraduate program or as "parasites" who are paid in excess of their true marginal product. Oliver and Hopkins adopt a variant of the former, considering teaching assistants a cost of undergraduate education, without even mentioning other definitions. I prefer to value the input of teaching assistants into the undergraduate program according to the market price of equivalent resources. The difference (if any) between this figure and the total payment to teaching assistants represents a subsidy to graduates, a portion of forgone earnings which is borne by the university rather than the student, or, alternatively, a purchase by the university of the student input into its graduate program. Using the wage for moonlighters at a nearby community college and for high school teachers in the area as a proxy for market value, I found that, in general, only half of the cost of teaching assistants at Stony Brook should be allocated to the undergraduate program. The remainder should be considered a cost of, or transfer payment to, the graduate students.

I have, in effect, been arguing that the operations research models in general, and Oliver-Hopkins in particular, overstate the real undergraduate instructional costs at a university by ignoring its joint supply of multiple products.[3] The key distinction between money and real (opportunity) costs is overlooked in other ways as well. I would question, for example, whether we are justified in using annual salaries as an index of faculty services, and wage differentials as an index of real cost differences. Since faculty tends to be hired on a long-term contractual or tenured basis, the university reaches its hiring decision on the basis of lifetime wages and expected performance, and current wages are not necessarily tied to current performance. Although the university will not become a "net lender" to the professor, who is free to leave, the professor may become a "net lender" to the university, knowing that he will afterwards be compensated. Thus, wages of young people may be less, and of older people more, than their current productivity. On the other hand, tenured professors are buying insurance as well as wages with their lifetime services, so the latter is understated by looking at the wage payment alone. Similar comments regarding the complex measurement problem might be made about other university resources which are not included in the Oliver-Hopkins model and which, therefore, will not be discussed here.

It is true, of course, that the university administration may be more interested in money than in social costs. More generally, there are many levels of decision makers at a university, and costs relevant at one level may not be relevant at another. Consequently, we must distinguish not simply between social and private costs but also among private costs perceived by varying decision makers. For example, the secretarial staff may be considered a variable cost to the central campus administration, a constraint to the department chairman who is not permitted to hire additional people, and a free good to the professor as he ponders whether to have a manuscript typed. Benefits of different activities also vary among decision makers, depending on the precise consequences and objective functions involved. Therefore, in building a useful operations research model one must clearly and consistently specify the decision maker for whom it is intended, in order to ascertain the appropriate set of costs, constraints, and goals. Oliver and Hopkins score well on this point, with one exception noted below.

Returning to the paper directly at hand, I have only a few specific criticisms. These could be handled easily in theory, but sometimes at the cost of a rather more complicated model.

1. No note is taken of faculty inputs into the graduate program other than regular course work; e.g. time spent advising students and supervising theses without corresponding credit hours appears to be omitted. This may help explain why the Oliver-Hopkins differences between undergraduate and graduate costs are not as great as other sources claim.

2. Although graduate students who serve as teaching assistants are recognized to spend a longer lifetime at the institution, it is also assumed that their annual course load and faculty input are the same as for non-teaching-assistants. Thus, the cost to the institution of educating a Ph.D. who has served as a teaching assistant is considerably greater than educating one who hasn't. Empirically, I wonder whether teaching assistants tend to take fewer courses and use less faculty time per year, which would reduce somewhat the unit cost for that cohort.

3. I am troubled by the possible emphasis on destination modes, particularly for the lengthy chains which include one or more intermediate degrees. If one is interested in finding the least-cost method of obtaining a fixed number of degrees, I presume that such intermediate degrees would also count. I see no reason, for example, why Berkeley should prefer taking a

Stanford B.S. into its Ph.D. program, rather than its own (as the discussion on page 396 seems to imply), in order to meet specific degree constraints.

For the educational system as a whole, such emphasis on final rather than intermediate dèstinations might be useful (stemming from national manpower needs) but for a single institution it is difficult to justify. Conversely, a single institution may, following Oliver and Hopkins, focus on different entry points and ignore the previous investment in human capital embodied in their students, but for the system as a whole such previous investment is fully relevant. As discussed above, a model builder should determine whose viewpoint he is adopting and consistently use that same viewpoint, while contrasting it, if he desires, with the viewpoint which would be appropriate to a different decision maker.

4. In the Oliver-Hopkins model, costs of all degrees are additive and independent of where the student's earlier work was done. This strikes me as a somewhat questionable assumption, especially when dealing with M.A.'s and Ph.D.'s. In many fields, one goes directly from a B.S. to a Ph.D., sometimes picking up a master's en route at virtually no extra cost. The lifetimes associated with these two programs are then not sequential but simultaneous, and the corresponding costs should not be added together. Furthermore, students switching institutions after the master's may require greater time toward completion of the Ph.D. than those continuing on at the same school. Such interaction between program costs and points of entry or destination may be important for certain policy questions but is not explored by the Oliver-Hopkins formulation.

5. I should like to underscore the word of caution Oliver and Hopkins rightly extend about interpreting their shadow prices and other results, which depend so critically upon the particular constraints assumed. For example, their very low shadow price for a junior transfer is based on a constant enrollment figure, thereby implying fewer freshmen. If we held the number of freshmen constant and broke the enrollment ceiling instead, the shadow price on junior transfers would be much higher. Conversely, if we held degrees constant instead of enrollment, increasing junior transfers would actually have a negative shadow price, since this is the cheapest way of granting a given number of bachelor's degrees. Similarly, if one examines the effects on total instructional costs of the Carnegie Commission recommendation to compress lower-division work to one year (Table 10), one gets completely divergent results depending on whether an enrollment constraint or degree constraint is assumed. Thus, this analysis can certainly be useful, but the structure of the model and changes resulting therefrom must be clearly specified and understood.

Finally, I should emphasize that many of the above observations belong more to the domain of economists than operations research specialists, so it is hardly a surprise that they have not been dealt with in the operations research literature on education. Oliver and Hopkins have developed a promising way of applying network theory to educational planning. Every attempt should be made by economists to provide conceptually meaningful inputs for their model.

NOTES

1. I am referring here to the faculty budgeted under "departmental instruction" not "organized research." While some research is funded separately, much of it is financed out of the regular departmental budgets at most universities. My particular results for Stony Brook, as well as the broader conceptual and measurement problems outlined in this comment, are discussed by me in the following Stony Brook working papers: "Resource Allocation and Costs in Higher Education"; "Some Notes on the Faculty as a University Resource"; and "Methods of Resource Measurement and Allocation."
2. Parenthetically, I am worried when an adjustment for research is not made in other contexts as well. For example, subsidies to undergraduates at state universities are frequently overstated for this reason; community college students are subsidized at least as much, contrary to the usual impression. The social rate of return to college teaching is understated if it is based on an unadjusted calculation. Changes through time in faculty-student ratios and teaching productivity, two topics discussed at this conference, might well have looked different if an explicit correction for research costs had been attempted. I also wonder whether this measurement problem might help to account for the apparent lack of connection between level of college expenditures and quality of educational output in cross-sectional studies. The "high-spending" universities may be the research-oriented institutions, whose true teaching costs are relatively overstated, and we may be observing, in part, that research is not an important input into undergraduate education.
3. To indicate the rough order of magnitude of this effect, my own figures for undergraduates are approximately 60 per cent of those of Oliver and Hopkins. My graduate costs are also somewhat less, unless research is counted as an input into graduate study, in which case, costs of a master's or Ph.D. skyrocket by a multiple of six.