Volume Title: Economic Research: Retrospect and Prospect Vol 1: The Business Cycle Today

Volume Author/Editor: Victor Zarnowitz, ed.

Volume Publisher: UMI

Volume ISBN: 0-87014-251-8

Volume URL: http://www.nber.org/books/zarn72-1

Publication Date: 1972

Chapter Title: A Study Of Discretionary And Nondiscretionary Monetary And Fiscal Policies In The Context Of Stochastic Macroeconometric Models

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Chapter URL: http://www.nber.org/chapters/c4399

Chapter pages in book: (p. 261 - 310)

A Study of Discretionary and Nondiscretionary Monetary and Fiscal Policies in the Context of Stochastic Macroeconometric Models

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I. INTRODUCTION

In this paper we study stochastic or random versions of three quarterly representations or models of the U.S. economy: the FRB-MIT, Wharton, and Michigan econometric models.[1] For each model, we examine the results of imposing a set of macroeconomic policy rules—rules comprised of different combinations and magnitudes of monetary and fiscal responses to what is occurring in the economy. The results that flow from each of the policy rules according to each representation of the economy are obtained from sixteen-quarter simulation experiments designed to answer the following question: If the economy is represented by a particular model that is stochastic and if a

[1]The FRB-MIT model is described and discussed in F. deLeeuw and E. Gramlich, "The Federal Reserve-MIT Econometric Model," *Federal Reserve Bulletin*, January 1968, and by the same authors in "The Channels of Monetary Policy: A Further Report on the Federal Reserve-M.I.T. Model," *The Journal of Finance*. The Michigan model is described by S. H. Hymans and H. T. Shapiro, *The DHL-III Quarterly Econometric Model of the U.S. Economy*, Research Seminar in Quantitative Economics, University of Michigan, Ann Arbor, 1970. The Wharton model is an outgrowth of an older model described in M. K. Evans and L. R. Klein, *The Wharton Econometric Forecasting Model*, Economic Research Unit, Wharton School of Finance and Commerce, University of Pennsylvania, 1967. The version used here, which has more price equations and an extended financial sector, exists at the Economic Research Unit at Wharton.

particular policy rule is followed, what is the distribution of possible outcomes?

Our work differs from previous work on the policy implications of large econometric models precisely in that we obtain for each model and each policy a distribution of outcomes. With only partial and rare exceptions, previous studies have been conducted as if the models were deterministic; and, presumably, as if they were exact representations of a deterministic economy, this in spite of the fact that those models were estimated under the assumption that there are random elements in the economy.[2] Consistent with the estimation procedures, we view the economy as random, and accept as descriptive of its randomness the estimates of the residual variances turned out by the models. In addition, we take account of uncertainty about the parameter values and accept as descriptive of that uncertainty the estimates of the variance-covariance matrices of the coefficients. Because we take randomness into account, we come to grips with two related macroeconomic policy issues: the choice between discretionary and nondiscretionary policies, and the choice among instruments for a discretionary policy. The instruments question includes both the monetary-fiscal controversy and the interest-rate–monetary-aggregate controversy. Let's first consider the discretion-nondiscretion issue.

More than 20 years ago, Milton Friedman argued that most discussions of macroeconomic policy were being conducted as if the economy were a deterministic system.[3] He showed that if the policy instrument is connected to the target variable by way of random variables, then the policy action affects the stability (as measured by the variance) of the target variable, and seemingly reasonable policies that depend on recent observations, the usual way of defining discretionary policies, may lead to worse outcomes than

[2]There are few exceptions, but there is the pioneering study by I. Adelman and F. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica,* 1959. Recently three related studies on the FRB-MIT, OBE, and Wharton models were undertaken. They are summarized by Zarnowitz, Boschan, and Moore, "Business Cycle Analysis of Econometric Model Simulations," in *Econometric Models of Cyclical Behavior,* Bert G. Hickman, ed., NBER Conference on Research in Income and Wealth, forthcoming. (See also the references therein.) However, the only random element considered there is the randomness of the additive disturbance. Another approach to the investigation of stochastic properties of econometric models was taken by G. C. Chow and R. E. Levitan, "Nature of Business Cycles Implicit in a Linear Economic Model," *Quarterly Journal of Economics,* Vol. LXXXIII, August 1969, pp. 504-517, and more recently by E. P. Howrey, "Dynamic Properties of a Condensed Version of the Wharton Model," in *Econometric Models of Cyclical Behavior, op. cit.*

[3]See M. Friedman, "The Effects of a Full Employment Policy on Economic Stabilization: A Formal Analysis," in his *Essays in Positive Economics,* University of Chicago Press, 1953.

policies that ignore recent observations, the usual way of defining nondiscretionary policies. In general, the more random the connections between the instruments and the targets, the less the instruments should respond to recent observations. Thus, resolution of the dispute between those who favor discretion and those who do not depends on determining the degree of instability that attaches to the effects of policy actions. Those who favor discretion must argue that the gain outweighs the instability that might result. Those who favor nondiscretion must argue that given the way discretionary policy is currently formulated, the instability dominates. Until now, neither side has presented evidence.

While the discretion versus nondiscretion issue is bound up with the degree to which the connections between instruments and targets are random, the second issue, the choice among instruments for a discretionary policy, depends on how that randomness is distributed. For example, in the monetary versus fiscal instruments controversy, one aspect concerns the degree to which fiscal instruments can be made responsive to macroeconomic policy needs. In this paper we disregard that question and proceed as if the personal and corporate income tax rates were instruments of macroeconomic policy; as if the President had the power to vary those rates each calendar quarter. A second aspect of the controversy involves relative potency, with an imaginary fiscalist arguing that monetary policy has almost no effect and an imaginary monetarist arguing that fiscal policy has almost no effect. The relative potency question has received a great deal of attention, but deserves that attention, if at all, only within a stochastic framework. In a deterministic framework, if a $1 billion open market operation gives an undesirably small effect, then try a $10 billion operation; and, similarly, for tax rate changes. In a deterministic framework, it is only important that instruments work in the desired direction; and, moreover, in a linear system, one instrument is sufficient to attain one target. Once we take randomness into account, however, the variance of the target is affected by the movements of the instruments, and we expect that the use of combinations of instruments will do better than the use of any one instrument, even if there is only one target variable.[4] However, this presumption for monetary and fiscal policy has never been tested.

We present evidence on both the discretion versus nondiscretion issue and on the choice of instruments question primarily in terms of (i) average growth rates of real output and the price level, (ii) dispersion of outcomes around

[4] Brainard shows that diversification among instruments is analogous to diversification among assets. See "Uncertainty and the Effectiveness of Policy," *American Economic Review*, May 1967.

their respective growth rates (within-path variance that serves as a measure of instability), and (iii) degree of uncertainty about the particular growth rates that will occur (among-path variance). We also compute expected utility for a class of utility functions.

We find that our discretionary policies,[5] indeed, affect the average growth rates of real output and the price level in the expected directions. That is, when restrictive policy actions are taken, as they are in the Michigan model, both growth rates decline; while when expansionary policy actions are taken, as they are in the FRB-MIT model, both growth rates increase. As that suggests, we find high correlations between the growth rates for real output and the price level across different policies. Moreover, we find that for given instruments the stronger the action taken, the greater the effects on growth rates. We find that our fiscal policies have a stronger effect than our monetary policies in the FRB-MIT model, but find the reverse in the Michigan model. This may be accounted for by asymmetry in the effects of policy actions; monetary policy, it is often suggested, is more potent when applied to restrict the economy than when applied to stimulate the economy.[6]

Our discretionary policies failed to reduce the within-path (over time) instability of either real income or the price level. Indeed, we persistently find for real income that such instability increases with the strength and, hence, with the average effect of the discretionary policy applied. As among instruments, we have evidence from the FRB-MIT model that within a given range a given effect on the growth rate of real income can be achieved with less accompanying instability of real income by the use of fiscal policy than by the use of either monetary policy or a combination of the two.

Although policies have significant effects on average growth rates, there is great uncertainty about the particular growth rate that will occur under any policy. Moreover, that uncertainty varies across policies. On this score strong policies generally outperform weak policies which, in turn, outperform the nondiscretionary policies. That is, given the uncertainty about the parameters implied by estimation and about the time paths of noninstrument exogenous variables, there is least uncertainty about the particular growth rates that will occur under our strong discretionary policies and most uncertainty under our nondiscretionary policies.

Those findings, however, both for the discretion versus nondiscretion and for the monetary versus fiscal instruments questions are subject to two basic limitations. First, we examine only a few specific discretionary policy rules.

[5] These results are based only on the FRB-MIT and Michigan models.

[6] We gladly acknowledge Bert Hickman for this remark.

Necessarily, there are other discretionary rules that would do better. The problem is finding them. For any model, there exists an optimal policy which is discretionary and which, almost certainly, would involve using both monetary and fiscal instruments. But, deriving that policy for a large, stochastic, dynamic economic model seems beyond present capabilities. It involves solving a horrendously large dynamic programming problem. Thus, we attempted to postulate only plausible rules. Second, our findings with respect to all the rules, both discretionary and nondiscretionary, are only as good as the models from which we infer them. If the models are seriously in error, then our results give no indication of what would happen if any of our rules were actually implemented.[7] One should recognize, however, that, barring experiments on the economy itself, the questions raised above must be studied within the context of estimated models of the economy.

As for the contrast between outcomes from stochastic and nonstochastic simulations, our results show that outcomes from nonstochastic or deterministic policy simulations can be poor estimates of the distributions of outcomes that result from stochastic simulations of the same policies. We find that nonstochastic simulations may not produce reliable estimates of the mean paths of outcomes. For the FRB-MIT model, we generally reject the hypothesis that the nonstochastic outcome for real GNP in any quarter is the mean of the distribution of the corresponding stochastic outcomes. We also find that nonstochastic simulations generally produce estimates of instability that understate the degree of instability found in the stochastic results. And, more serious, they often rank policies on the basis of expected utility differently than do the stochastic outcomes. Thus, even if our results are not accepted as indicative of the outcomes that would result from the application of the rules we study because of skepticism about the models, our experiments are still important because they show how to derive the implications of these models in a way consistent with the underlying stochastic assumptions.

II. THE NATURE OF THE EXPERIMENTS

For each policy rule and each model, we obtain a sample of sixteen-quarter simulation runs for the period 1969-I through 1972-IV. The sample elements are generated by three kinds of randomness: randomness of coefficients, of

[7]While several comparative studies of the forecasting performance of econometric models were published, the forecast errors were not compared to the standard errors of forecasts for the models and, thus, their specifications were not tested against the forecast errors they produced.

additive disturbances, and of exogenous variables. The randomness is such as to generate distributions of outcomes consistent with the estimated models.

The coefficients are taken as random from run to run, but as fixed from quarter to quarter within each sixteen-quarter run, a view consistent with the specifications assumed by the model builders.[8] The random parameters are generated equation by equation.[9] If the ith estimated equation of a model contains a vector of parameters, b_i, then random values of b_i are generated according to the following matrix equation,

$$b_i = \bar{b}_i + R_i \nu,$$

where $\bar{b}_i$ is the vector of point estimates of b_i, R_i is a matrix such that $R_i' R_i$ equals the estimated variance-covariance matrix of $\bar{b}_i$, and ν is a vector of random variables chosen independently of one another, all from a distribution with mean zero and variance one.[10] It follows, then, that b_i has a distribution with mean $\bar{b}_i$ and a variance-covariance matrix equal to the

[8] If the true parameters are, instead, random from period to period, we are understating the degree to which parameters are random and overstating the degree to which residuals are random. Such misassignment of randomness would seem to bias our results in favor of discretionary policies, but such an effect is limited by the nonlinearities of the models we are examining. Disturbances that are additive equation by equation can end up determining solutions multiplicatively, while the converse can be true for parameters. Only in linear systems is there a sharp distinction between the effects of random parameters and the effects of random disturbances.

[9] It was claimed that we should have taken account of possible correlation among disturbances from different equations, which would, of course, imply correlations among parameter estimates from different equations. In the presence of such correlations, both ordinary least squares and two-stage least squares are inefficient procedures. Since the authors of the models did not state explicitly their model assumptions regarding this question, and since they did not choose an estimation procedure to deal with it––e.g., three-stage least squares––we proceeded under the assumption that the authors did not regard possible correlations among disturbances as a problem that warranted action.

[10] The elements of ν and all the underlying random variables used are generated independently from a single distribution, a truncated normal distribution. Let x be a normal zero-one random variable. We draw values of x and accept only those for which $|x|$ is less than 2. The accepted x's have mean zero and variance $(.880)^2$, so that $\nu = (1.137)x$ has mean zero and variance one, the desired distribution.

estimated variance-covariance matrix.[11] This procedure is followed for every estimated equation of every model.

The additive disturbance for each estimated equation is treated as random from run to run and from quarter to quarter within each run. It is chosen independently over time from a distribution with mean zero and variance equal to the estimated residual variance for that equation. The independence assumption is needed if we are to attribute consistency to the estimated variances and covariances that we use.[12]

The third kind of randomness pertains to the noninstrument exogenous variables. The variables in that class differ from model to model, but often include, for example, population, exports, and federal government expenditures. (Since we have chosen to use as fiscal instruments only certain tax rates, all other potential fiscal instruments are treated as uncontrolled.) We assume that all noninstrument exogenous variables are generated by third-order autoregressive schemes of the form:

$$Z_t = a_0 + a_1 Z_{t-1} + a_2 Z_{t-2} + a_3 Z_{t-3} + u_t,$$

where Z_t is the value of a noninstrument exogenous variable in quarter t. We have estimated such equations by ordinary least squares for each of the noninstrument exogenous variables in each of the models.[13] Those equations are treated as are all other estimated equations of the models; the parameters are chosen randomly from run to run, and the disturbance randomly from quarter to quarter. But, here, the residual variance is taken to be one-half the estimated residual variance, the argument being that if one had set out to build a model that explained all noninstrument variables, one could have

[11]We do, however, require that each element of b_i have the same sign as the corresponding element of $\overline{b}_i$. If the sign constraint is violated, we choose a new random vector, ν. The sign constraint expresses our prior views about the distributions of the true parameters. In no instance did it turn out to be binding.

[12]If the true residuals are serially correlated, we are, in a sense, attributing too much variance to them. But, that may be more than offset by the adjustments that should then be made in the estimated variances. Serial dependence has the effect of reducing degrees of freedom, and the implied adjustment would lead to larger estimated variances.

[13]In a few cases, multicollinearity prevented us from inverting the moment matrix of the right-hand side variables. In those cases, we used a lower order scheme.

done better explaining the noninstrument exogenous variables than we do with the autoregressive schemes. Those schemes can imply very large forecast variances when used, as we use them, to forecast many periods ahead; in any particular sixteen-quarter simulation run, the values in each quarter depend on the corresponding values in all previous simulated quarters. Thus, as we proceed quarter by quarter, the effects of the disturbances drawn in the early quarters can be magnified.

We simulate all policy rules for a given model on the same sample of random variables so that differences among the distributions of outcomes for different policies can be attributed entirely to the policies. The basic random sample is determined from simulations of a nondiscretionary policy. Those simulations are conducted as follows. To each set of random parameters a sixteen-quarter set of random disturbances is associated, both for the structural equations and for the autoregressive schemes. If a solution is obtained for all sixteen quarters on such a sample, then all other policies are run on that sample. If a solution is not obtained in any quarter of the sixteen-quarter run, then that sample is discarded and is not used for any other policy.[14] In either case, we proceed to a new random sample of both parameters and disturbances and start a new sixteen-quarter run.

The procedure for a nonconvergence turned out never to be implemented for the Michigan model. For that model, sixteen-quarter solutions were obtained for all random samples tried. Moreover, no nonconvergences resulted when the other policies were applied to that basic sample. The record was very different for the FRB-MIT and Wharton models. There, as shall be described below, for the nondiscretionary policy the model failed to converge to a solution or converged to a nonsensical solution on many random samples and, when other policies were applied to the FRB-MIT model, it did not converge to a sensible solution on some of the samples in the basic sample.

For any policy rule and model, a particular sixteen-quarter element of the sample of outcomes is obtained as follows. The first policy decision is made at the beginning of the first quarter of 1969 and determines the 1969-I values of the instruments on the basis of actual data up through 1968-IV. That first policy action and a sample of values of parameters and disturbances—both for the "structural equations" and for the autoregressive schemes we supply—determine the 1969-I values of all noninstrument variables. Then, those values together with actual data up through 1968-IV determine the 1969-II values of the instruments. The 1969-II values of the instruments and

[14] All three models are nonlinear and are solved by way of an iterative procedure that may not always converge to a solution, let alone a plausible solution.

a new sample of disturbances, both for the "structural" and autoregressive equations, determine the 1969-II values of all noninstrument variables. The process is continued through the fourth quarter of 1972, with the parameters held fixed through all sixteen quarters.

Note that for any discretionary policy rule, the policy actions taken from 1969-II to 1972-IV may differ from sixteen-quarter run to sixteen-quarter run, because the action taken at any time within a particular simulation run depends on what has happened previously within that run. For example, for any policy rule that allows for discretionary fiscal action, tax rates in 1969-II depend on the 1969-I values of endogenous variables which are functions of random variables. The 1969-I policy actions are an exception, because they depend entirely on events prior to 1969-I which are described by actual data. Actual data are used when and only when we need values of variables for dates prior to 1969-I. In that sense, the simulations are entirely endogenous; they could have been performed at the beginning of 1969-I, as soon as data for 1968-IV became available. Moreover, in a sense, there are no exogenous variables in our experiments. The policy rules to be described below are, in effect, the equations for the instrument variables; the autoregressive schemes are the equations for the noninstrument exogenous variables; and the estimated equations and identities that constitute the models are, of course, the equations for the endogenous variables.

Finally, in addition to generating distributions of outcomes for each policy rule, we also obtain one nonstochastic outcome for each policy. The nonstochastic path is obtained by setting all parameters and disturbances equal to their means, both in the structural equations and in the autoregressive schemes.

III. THE MODELS

The three models chosen for our experiments range in size from Michigan with 24 behavioral equations, to Wharton with 51, to FRB-MIT with 75. Each of them was constructed by a group of economists who continuously modify their models and use them to forecast and to evaluate policies. The models differ in the attention they give to different sectors of the economy. For example, in the FRB-MIT model, the largest block of behavioral equations (17) is devoted to the financial sector; in Wharton 6 equations describe the financial sector; while in Michigan there is only one equation, that describing the relationship between long- and short-term interest rates. The models also differ with respect to the degree of interdependence: the FRB-MIT is the most highly interdependent and also has the richest lag structure; Michigan is almost recursive. Despite this, the FRB-MIT model was estimated by ordinary least squares, while Wharton and Michigan were estimated by two-stage least squares.

All the models respond similarly to the usual instruments of economic policy. Monetary policy works via changes in short-term interest rates, with long-term rates affected by way of a distributed lag. Those rates, in turn, affect aggregate demand––investment in residential housing, business and government fixed investment, and, in the FRB-MIT model, consumption. Tax rate changes affect aggregate demand in two ways. They affect disposable incomes of businesses and firms, and, therefore, their expenditures. In the FRB-MIT model, they also affect rates of return.

All three models use quarterly data. A special attempt was made in the FRB-MIT model to take account of serial correlation in the equation residuals. First-order autocorrelation coefficients were estimated, and the implied partial first differences taken. Most of the equations of the Michigan model were estimated in simple first difference form. A special adjustment for autocorrelation was made in only two equations. In Wharton, no account was taken of serial correlation.

The sample periods used for estimating the models also differ. Wharton's sample period starts as early as the first quarter of 1948 and ends in the last quarter of 1968.[15] The FRB-MIT typically uses the post-Korean War period up to late 1966 or early 1967, but a few important equations were fit to data up through 1968-III. In Michigan all stochastic equations were fit to data for the period 1954-I to 1967-IV.[16]

The number of exogenous variables varies with the size of the model, Michigan having less than 20, Wharton 53, and FRB-MIT 70. However, there are only 14 exogenous variables that required autoregressive schemes in Michigan, 34 in Wharton, and 40 in the FRB-MIT model. The others were dummies, time trends, or strictly legally determined variables such as the maximum rate payable on time deposits which, except for the trends, were maintained at their 1968-IV values.

In addition to the model description normally required for simulation experiments, we required estimates of residual variances and of parameter covariance matrices. Wherever possible, the residual variances were taken from the published or mimeographed versions of the models. For Michigan, the coefficient covariance matrices were supplied to us by the authors. For

[15]We have replaced the total labor force equation with one based only on the post-Korean War period. It was given to us by Professor L. Klein.

[16]However, for two equations, the authors supplied us with coefficient estimates different from those that appear in the published version.

the FRB-MIT model, they were estimated for a project being undertaken under the auspices of the Federal Reserve Bank of Minneapolis, and were made available to us.[17] For Wharton, we computed for each structural equation the inverse of the moment matrix of the right-hand side variables and multiplied that by the residual variance reported in the model description. Admittedly, for all equations estimated by two-stage least squares, such moment matrices are not identical to those that would be obtained from the two-stage procedure. However, no disservice was done to Wharton. Any quadratic form of the inverse of the moment matrix computed from the original series cannot exceed the corresponding quadratic form of the inverse computed from the predictors estimated in the first-stage regressions of two-stage least squares. Thus, if anything, we understate the variance of the coefficients.

IV. THE POLICY INSTRUMENTS

Our experiments involve the use of both monetary and fiscal instruments. The fiscal instruments are the personal and corporate income tax rates, which, however, are used as a single instrument; the same percentage change is always imposed on both tax rates. There are two alternative monetary instruments: the rate on 4- to 6-month prime commercial paper and unborrowed reserves. They are alternative instruments because when one of them is the instrument, the other is necessarily endogenous.[18] Both the commercial paper rate and unborrowed reserves are potential instruments because the Federal Reserve can, if it wishes, peg the rate on commercial paper from quarter to quarter, or can, if it wishes, control unborrowed reserves almost perfectly. (It cannot, of course, do both simultaneously.) The rate on commercial paper was chosen as one of the monetary instruments because it is the only potential monetary instrument common to all three models. All discretionary monetary policies are carried out with that rate as the instrument. Nondiscretionary monetary policy is carried out in two ways: with the interest rate exogenous and constant for all sixteen quarters, and with unborrowed reserves exogenous and growing by one per cent per quarter. The unborrowed reserves experiments can be attempted only in the FRB-MIT and Wharton models.

[17]The procedure is described in a forthcoming paper by J. Kareken, T. Muench, T. Supel, and N. Wallace, "Determining the Optimum Monetary Instrument."

[18]If the models are correctly specified, it is valid to use them to inquire about the effects of using alternative instruments. Large structural models of the economy are constructed in order to allow us to study the effects of changes in structure, and a change in instruments is one kind of change in structure.

Unborrowed reserves was chosen as an instrument because it was the model builders' choice as the monetary instrument and because control of it is generally thought to imply approximate control of the money stock. In the models, as in the economy, complete control of unborrowed reserves does not imply complete control of the money stock. One reason for the lack of control is the ability of banks to borrow from the Federal Reserve. In order to reduce changes in bank borrowings from the Federal Reserve, or, to put it differently, to reduce changes in banks' desired holdings of free reserves, the discount rate is always set one-half of a per cent above the rate on commercial paper in the previous quarter.[19] While that rule should make the connection between unborrowed reserves and the money stock closer than it would otherwise be, our stochastic treatment still allows for a number of slippages. We treat required reserves as a stochastic function of the levels of demand and time deposits because given legal reserve requirements—which we hold fixed throughout our experiments—required reserves depend on the distribution of deposits by class of bank, a stochastic element. And we treat the demand for free reserves stochastically, just as we do all other structural relationships.

Tax rates were chosen as the fiscal instrument because we believe they are closer to being actual instruments of macroeconomic policy than are government expenditures, even though legislation would be required in order to allow tax rates to vary quarter by quarter in accord with a macroeconomic criterion.[20] Even assuming such an institutional change, however, serious questions about controllability were posed because the models do not, in general, contain as variables the tax schedules that would be altered by policy. An exception is the FRB-MIT treatment of corporate taxes, and, to a certain extent, its treatment of personal taxes.

In the FRB-MIT model, corporate tax liabilities are determined by an estimated equation in which the maximum corporate tax rate appears as a variable. The treatment of personal taxes is similar. There exists a variable, defined as the average tax rate under Federal personal income tax, which was

[19]The discount rate has an effect on income and the price level only when unborrowed reserves is exogenous. When the commercial paper rate is exogenous, the discount rate is irrelevant; it affects only free reserves and unborrowed reserves.

[20]Again the question arises: Are the models appropriate for studying the effects of such fiscal policies, given that such policies were not in effect during the sample periods? First of all, as will become clear in the next section, even if the fiscal rules are known, one can predict the course of tax rates only by predicting the course of the economy. But, more important, if these models cannot be used to study the effects of such fiscal policies, those effects cannot be studied at all.

constructed independently of current data on tax revenue and which appears in an estimated equation determining revenue. We treat both tax rates as controllable instruments. They appear in structural equations that we treat as we do all other structural equations; namely, we allow randomness in these relationships too.

In the Wharton and Michigan models, tax rates do not appear as variables. There are estimated equations for tax revenues, which for personal and corporate taxes in both models can be represented schematically as,

$$T = a_0 + a_1 B + u,$$

where T is revenue, B is the assumed tax base, and u is a disturbance. The parameters, a_0 and a_1 are estimated from data for short periods over which tax laws are uniform. We adapt such equations as follows. (i) The coefficient a_1 is replaced by the product of a tax rate, τ, and a coefficient, a^*. (ii) The mean of a^* is set at unity. The variance-covariance matrix of (a_1, a^*) and the variance of u are taken to be the relevant estimated variances and covariances in the linear regression of T on B over the whole data period. (iii) The tax rate, τ' is treated as controllable with an initial value equal to the point estimate of a_1 supplied by the model. For example, the Wharton personal tax equation is

$$TP = -12.8 + (.16 + SLTP)(\text{Base}),$$

where TP is personal tax and nontax payments in current dollars, and $SLTP$ is a "slope adjustment." We rewrite the equation as

$$TP = a_0 + a^*(\tau_p)(\text{Base}) + u.$$

The parameters a_0 and a^*, which vary randomly from run to run, are given by the equation,

$$\begin{bmatrix} a_0 \\ a^* \end{bmatrix} = \begin{bmatrix} -12.8 \\ 1.0 \end{bmatrix} + R \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}$$

where R is such that RR' equals the estimated variance-covariance matrix from a linear regression of TP on the base over the whole data period, and where the ν's are random variables, independently chosen from a distribution with mean zero and variance one. The disturbance, u, which varies from quarter to quarter, has mean zero and variance equal to the estimated residual variance from the same regression. The tax rate, τ_p, is assumed controllable, with an initial, 1968-IV value of .16.

A controllable corporate tax rate for the Wharton model and controllable personal and corporate tax rates for the Michigan model are defined in the same way. In each case we use the form of the revenue equation given in the model and use as initial values for the tax rates the point estimates supplied by the model. The initial values of the controllable tax rates for the different models are listed below.

Model	Personal	Corporate
FRB-MIT	.231	.528
Wharton	.16	.46
Michigan	.20	.47

One way to rationalize the differences among models is to say that the models summarize the same tax law in different ways. As a consequence, they apply their resulting "average" rate to different bases. For example, the Wharton personal tax base includes social security contributions of individuals, while the Michigan base does not.

In our experiments, fiscal policy is conducted by making the same percentage changes in the rates for all the models. The implicit assumption is that a given percentage change in all those rates corresponds to a given change in tax laws of the surcharge type. We start, by the way, with the 1968 surcharge fully in effect.

V. THE POLICY RULES

The policy rules we propose to investigate recognize the policy makers' utility tradeoff between growth and inflation and take some account of lags. Our operating criteria are based on the per cent unemployed (un) as a measure of the departure from attainable growth and on the percentage rate of change of the GNP deflator (p) (1958 = 1.0). Thus, for purposes of determining policy, we compute at the beginning of each quarter, t, the following weighted averages:

$$P_t = 100 \sum_{1}^{4} a_i (p_{t-i} - p_{t-i-1})/p_{t-i-1}$$

$$U_t = \sum_{1}^{4} a_i \, un_{t-i}$$

where P_t is the weighted average of the percentage change in the price level, U_t is the weighted average of the unemployment rate, and $a_1 = .4$, $a_2 = .3$, $a_3 = .2$, and $a_4 = .1$.

Action is taken based on the values of P_t and U_t according to the following discrete classes.

U \ P	−.5% – .5%	.5% – 1.5%	1.5% – + ∞
0 – 4%	(1,1) No action	(1,2) Moderate tightening	(1,3) Extreme tightening
4% – 6%	(2,1) Moderate ease	(2,2) No action	(2,3) Moderate tightening
6% – 100%	(3,1) Extreme ease	(3,2) Moderate ease	(3,3) No action

Here, for example, cell (1,1) represents all combinations of P and U such that P, the weighted average of the percentage change in the price level over the past four quarters, is between −.5 and .5 per cent, and U, the weighted average of the unemployment rate over the past four quarters, is less than 4 per cent. If (P_t, U_t) lies in (1,1), no action is taken. Actions are defined as follows.[21]

No action means keep all instruments unchanged.

Moderate action means move the instrument(s) a moderate amount subject to the following proviso. Unless the situation at t is worse than at $t-1$, action may be taken at t only if action in the same direction was not taken either at $t-1$ or $t-2$.

Extreme action means move the instrument(s) an extreme amount subject to the following proviso. Unless the situation at t is worse than that at $t-1$, action may be taken at t only if action in the same direction was not taken at t-1. But this waiting rule is waived if in cell (3,1), $U_t - U_{t-1} > 1$; and if in cell (1,3), $P_t - P_{t-1} > .25$.

[21] We also specified actions for $P < -.5$ per cent, but such values were not encountered.

In the provisos, worse means that (P_t, U_t) lies in a worse cell than (P_{t-1}, U_{t-1}), where for this purpose, iso-utility lines run diagonally southwest in the table above, with cell (1,1) having the highest utility. Thus, for example, if (P_t, U_t) lies in cell (3,1) and (P_{t-1}, U_{t-1}) had been in (2,1), then instruments are moved an extreme amount in the direction of ease even if they had been moved in the direction of ease at t-1, because of the extreme action proviso.

Given that scheme, we now define moderate and extreme actions in terms of our instruments: the interest rate for discretionary monetary policy, tax rates for discretionary fiscal policy. The scheme above applies, of course, only when discretionary policies are in effect. Altogether, we examine ten policy rules defined in terms of percentage changes in the instruments in Table 1. In the table, R stands for the rate on four- to six-month commercial paper (R_{cp}), and τ for both the personal and corporate tax rates. The plus (+) is used whenever tightening action is called for, and the minus (–) whenever easing action is called for.

TABLE 1
Policy Actions at Time t

	$\pm(R_t - R_{t-1})/R_{t-1}$		$\pm(\tau_t - \tau_{t-1})/\tau_{t-1}$	
Policy	Moderate	Extreme	Moderate	Extreme
1. Nondiscretionary, R_{cp} exogenous	0	0	0	0
2. Monetary I, R_{cp} exogenous	.1	.2	0	0
3. Fiscal I, R_{cp} exogenous	0	0	.05	.10
4. Joint I, R_{cp} exogenous	.05	.1	.025	.05
5. Monetary II, R_{cp} exogenous	.2	.4	0	0
6. Fiscal II, R_{cp} exogenous	0	0	.1	.2
7. Joint II, R_{cp} exogenous	.1	.2	.05	.1
8. Nondiscretionary, unborrowed reserves exogenous	Endogenous		0	0
9. Fiscal I, unborrowed reserves exogenous	Endogenous		.05	.10
10. Fiscal II, unborrowed reserves exogeneous	Endogenous		.1	.2

For policies 8, 9, and 10, unborrowed reserves grow one per cent per quarter.[22] Thus all three involve nondiscretionary monetary policy, just as do policies 1, 3, and 6, but of a different kind. Note that extreme action is always twice moderate action, that the Roman two (II) policies, hereafter called strong policies, are always twice the Roman one (I) or weak policies, and that

[22] And the discount rate is set one-half of a percent above the value of R_{cp} in the previous quarter.

the joint policies are a simple average of the corresponding monetary and fiscal policies. Wherever possible, all rules are applied to all three models.

Although the rules are necessarily arbitrary, they seem not unlike those that might be applied.[23] They were chosen prior to any experimentation by us on the three models to which we apply them. It would have been desirable to standardize, on the one hand, all the weak policies and, on the other hand, all the strong policies so that within each group all policies have the same effect on some criterion, say, the two-quarter expected change in the unemployment rate. The problem is that it would take a great deal of experimentation to determine such equivalences, nor is it easy to single out a criterion of equivalence.

VI. SUMMARY STATISTICS

1. Growth Rates and Variances Around Constant Growth Rate Paths

We analyze the results mainly in terms of growth rates of real GNP and of the GNP deflator and in terms of variance around the respective constant growth rate paths. We assume that

$$X_{tj} = X^0 e^{\gamma_j t} e^{\mu_{tj}} \tag{1}$$

where X_{tj} stands either for real GNP in quarter t of the jth sixteen-quarter run, y_{tj}, or for the GNP deflator in quarter t of the jth run, p_{tj} (or for the money stock); X^0 is the 1968-IV value of X, the value in the quarter before our runs begin, and is common to all runs; the disturbance, u_{tj}, is assumed to have expected value zero and variance independent of t, t=1,2, . . . ,16; and, although we shall not always mention it, all tests for significance depend on the assumption that u is normally distributed. Equation 1 says that X_{tj} grows on the average at a constant rate of growth per quarter, γ_j, starting from its 1968-IV value.

We estimate γ_j and σ_u^2 the variance of u, in a least squares regression. Taking the natural logarithm of each side of (1):

[23]These rules seem reasonable to us and seem to conform to Phillip's recommendation: "A strong proportional element is needed as the main basis of the policy, sufficient integral correction should be added to obtain complete correction of an error within a reasonable time and an element of derivative correction is required to overcome the oscillatory tendencies which may be introduced by the other two elements of the policy." (A. P. Phillips, "Stabilization Policy in a Closed Economy," *Economic Journal,* June 1954.) The proportional element in our rule is the relatively heavy weighting given to the most recent observation. The integral element is the positive weighting of lagged observations, which, in any case, seems desirable for a stochastic model. The derivative element is the set of waiting rules.

$$\log X_{tj} - \log X^0 = \gamma_j t + u_{tj}. \qquad (t = 1, 2, \ldots, 16)$$

Thus, we regress $\log X_{tj} - \log X^0$ on t, constraining the intercept to be zero. For example, for the Michigan model, there are 50 sixteen-quarter runs for each of seven policies, so that there are 7(50) such regressions for prices ($X_{tj} = p_{tj}$) and 7(50) such regressions for real output ($X_{tj} = y_{tj}$). We, henceforth, denote the individual growth rate estimates by $\gamma \ldots \gamma_y$ for real income, γ_p for prices, and the individual residual variances by $\sigma^2 \ldots \sigma_y^2$ for the estimated variance of u when $X = y$, σ_p^2 when $X = p$.

For each policy, we compute average growth rates

$$\overline{\gamma}_y = (1/N) \sum_j \gamma_y \qquad (j = 1, 2, \ldots, N)$$

and

$$\overline{\gamma}_p = (1/N) \sum_j \gamma_p \qquad (j = 1, 2, \ldots, N)$$

where N is the number of sixteen-quarter runs for which we have output ($N = 50$) for the Michigan model; $\overline{\gamma}$ is identical to the regression coefficient of time in a pooled regression of $\log X_{tj} - \log X^0$ on t run on all $16N$ observations; and the standard error of estimate of $\overline{\gamma}$, denoted $\sigma(\overline{\gamma})$ and always presented in parentheses beneath the corresponding $\overline{\gamma}$, is, in effect, estimated in that pooled regression.

In addition to the average growth rates, we focus on two kinds of variance, within-path variance and among-path variance. For each sixteen-quarter run, within-path variance is measured by σ^2, a measure of within-path stability. For each policy, we compute

$$\overline{\sigma}_y^2 = (1/N) \sum_j \sigma_y^2$$

and

$$\overline{\sigma}_p^2 = (1/N) \sum_j \sigma_p^2$$

average within-path variance. We report corresponding standard deviations, $\overline{\sigma}_y$ and $\overline{\sigma}_p$. Note that those are average standard deviations based only on the residual variance within each sixteen-quarter regression.

Since the time path of the economy is, according to the particular model under consideration, characterized by a sixteen-quarter path like one in our sample of N, the $\overline{\gamma}$'s are measures of the growth rates that will be experienced, and the $\bar{\sigma}$'s are measures of the instability that will be experienced. According to the models, those statistics are representative of characteristics of the economy. Finally, note that $\bar{\gamma}$'s and $\bar{\sigma}$'s are also computed for the single nonstochastic path produced for each policy.

The second kind of variance we examine, among-path variance, is in part attributable to our lack of knowledge about the economy, in particular, to our uncertainty about the true parameters. For a given policy, the among-path variance is a function of the distribution of the N individual growth rate estimates. As measures of those distributions, we report for each policy

$$\sigma(\gamma) = [\Sigma(\gamma-\bar{\gamma})^2/(N-1)]^{1/2}$$

for both real income, $\sigma(\gamma_y)$, and prices, $\sigma(\gamma_p)$. In addition, we report the correlation among the γ_y's and γ_p's for each policy, denoted by $\rho(\gamma_y,\gamma_p)$.

For testing purposes, we can summarize the above by an analysis of variance table.[24] In the RMS column, the total-residuals entry is the residual standard deviation from the pooled regression. $\sigma(\bar{\gamma})$ is equal to that divided by $(N\Sigma t^2)^{1/2}$. The among-samples entry in that column is the standard deviation of the distribution of the individual γ's multiplied by the square root of the second moment of the independent variable, time, about its 1968-IV value, which is zero. The within-samples entry in the RMS column is simply the average within-path standard deviation.

We wish to test for the following comparisons: (i) $\bar{\gamma}$'s across policies for stochastic and nonstochastic outcomes, (ii) $\bar{\sigma}$'s across policies for stochastic and nonstochastic outcomes, (iii) $\sigma(\gamma)$'s across policies, and, finally, (iv) individual γ's within each policy. The appropriate test statistics are as follows: (i) If $|\bar{\gamma}-\bar{\gamma}'|/[\sigma^2(\bar{\gamma}) + \sigma^2(\bar{\gamma}')]^{1/2} > t_c$, then we accept $\bar{\gamma}\neq\bar{\gamma}'$, where t_c is the critical value from the t distribution with a chosen significance level and the appropriate degrees of freedom. When $\bar{\gamma}$ and $\bar{\gamma}'$ are stochastic means for different policies, there are $2(16N-1)$ degrees of freedom. (ii) If $\bar{\sigma}/\bar{\sigma}' > (F_c)^{1/2}$ (where $\bar{\sigma} > \bar{\sigma}'$), then we accept the hypothesis $\bar{\sigma} > \bar{\sigma}'$, where F_c is the critical value from the F distribution with a chosen significance level and $15N$ and $15N$ degrees of freedom, and, where for nonstochastic outcomes, N=1. (iii) This test is the same as for (ii), except that $\sigma(\gamma)$ replaces $\bar{\sigma}$ and F_c now has $N-1$ and $N-1$ degrees of freedom. (iv) The second entry in the RMS column divided by the third is distributed as the $(F)^{1/2}$ with $N-1$ and $15N$ degrees of freedom, so that if that ratio exceeds the appropriate $(F_c)^{1/2}$, we accept the hypothesis that the individual γ's are not all equal.

[24] The appendix contains a sample table.

Finally, note that all tests for comparisons across policies are only suggestive, because they are based on statistics computed from the same samples of random variables; not from independent samples, the assumption that underlies the tests.

2. Expected Utility

Another related way to summarize outcomes is to compute expected utility for each policy for a class of utility functions. We represent utility by

$$U = (1/16) \sum_{1}^{16} \left[y_t^{1/2} + b\, 10^4 (p_t - p_{t-1} / p_{t-1})^2 \right]$$

where in this subsection y is per capita real GNP in 1958 prices, and p is the GNP deflator. The subscript t ranges over the sixteen simulated quarters.

The parameter b, which takes only nonpositive values, determines the tradeoff between per capita real income and percentage price variance around the value of the price level in the previous quarter. At any time and at a given value of y_t, the tradeoff is given by

$$\left. \frac{dy_t}{10^4 d \left[(p_t - p_{t-1} / p_{t-1})^2 \right]} \right|_{dU = 0} = -2by_t^{1/2}$$

Thus, for y_t =\$3,600, which is approximately the 1968-IV value, and b=–.3, the utility function implies indifference between an addition to per capita real income of one per cent and an addition to the percentage variance of prices of one percentage point. The closer b is to zero, the less concern there is for price variance. We shall compute expected utility for values of b ranging from zero to –.3.

Note that because y appears in the utility function raised to a power less than one, the tradeoff between real income and price variance also depends on y. The larger y is, the more y is willingly given up to reduce price variance. Raising y to the power one-half also imposes risk aversion. At a given value for price variance, fair gambles on y are always rejected. Put differently, raising y to the power one-half makes expected utility inversely dependent on the variance of y. A second-order Taylor expansion of $y^{1/2}$ about the expected value of y implies

$$E(y^{1/2}) = (Ey)^{1/2} \left[1 - (1/8) E \left(\frac{y - Ey}{Ey} \right)^2 \right]$$

Thus, the expected value of $y^{1/2}$ is a decreasing function of the percentage variance of y, but only a mildly decreasing function.[25]

For each policy, we compute expected utility by averaging over the N outcomes of U for that policy. The policies are then ranked. The nonstochastic outcomes are also ranked by U.

VII. RESULTS, THE FRB-MIT MODEL

1. Convergence and the Sample

The basic random sample was generated by applying to random samples, as described in Section II, nondiscretionary policy with tax rates held constant and unborrowed reserves growing one per cent per quarter (policy 8). Out of 83 sixteen-quarter runs attempted, 50 sensible sixteen-quarter solutions were obtained; in 30 cases the model failed to converge during some quarter of the run, and in three of the runs the unemployment rate converged to a negative value which led us to discard those runs. The record of sensible sixteen-quarter solutions that were obtained when the other nine policies were applied to the basic 50 random samples is shown in Table 2.[26]

The result, though, is a common random sample of only 35 on which sensible sixteen-quarter solutions were obtained for all ten policies. All our subsequent analysis and discussion of the FRB-MIT model are based on that sample of 35.

2. Average Stochastic Growth Rates

The average stochastic growth rates, $\overline{\gamma}_y$'s for real GNP and $\overline{\gamma}_p$'s for the GNP deflator, are shown in the first and third columns of Table 3. The $\overline{\gamma}_y$'s range from .05 for policy 1, the policy in which the interest rate and tax rates are held constant, to .84 for policy 6, the strong fiscal policy with the interest rate held constant. That is, they range from a rate slightly above zero to one

[25] For example, the average percentage within-path standard deviation of real output in the FRB-MIT model is as high as 6 per cent. If that represented the percentage standard deviation of per capita income around its expected value, it would imply $E[(y-Ey)/Ey]^2 = (.06)^2$. According to our utility function, there is indifference between, on the one hand, that variance and Ey = \$360.00, and, on the other hand, zero variance of y and Ey = \$359.50.

[26] A solution is not sensible if variables have economically meaningless values. This criterion resulted in discarding a number of runs for which the unemployment rate was negative.

TABLE 2.
FRB–MIT Model, the Convergence Record by Policy

Policy	Number of Sensible Sixteen-Quarter Solutions Out of Fifty Attempts
1. Nondiscretionary, R_{cp} exogenous	47
2. Monetary I, R_{cp} exogenous	47
3. Fiscal I, R_{cp} exogenous	47
4. Joint I, R_{cp} exogenous	46
5. Monetary II, R_{cp} exogenous	44
6. Fiscal II, R_{cp} exogenous	46
7. Joint II, R_{cp} exogenous	46
9. Fiscal I, reserves exogenous	49
10. Fiscal II, reserves exogenous	45

implying an annual growth rate of 3.75 per cent. Note that each $\overline{\gamma}_y$ for the weak version of each policy is substantially lower than that for the corresponding strong version, and that the fiscal policies (3 and 6) produce higher real growth rates than do the corresponding joint policies, which, in turn, produce higher growth rates than do the corresponding monetary policies. (It turned out that after the initial moderate tightening action in 1969-I, which is exogenous and common to all runs, our policy rules implied easing actions, on the average, throughout the remaining fifteen quarters of the stochastic runs of this model.) In contrast, the $\overline{\gamma}_p$'s vary very little. They range from .11 for policy 8 to .27, only slightly greater than 1 per cent per year, for policy 7. The correlation between the $\overline{\gamma}_y$'s and $\overline{\gamma}_p$'s across policies is .89; on average, higher rates of growth of real output are accompanied by higher rates of growth of the price level, as shown in Figure 1. Although the direction of tradeoff between the average growth rates of real output and prices is what one expects to find, the relative unresponsiveness of prices is somewhat surprising. Given our policy rules, the relative constancy of prices helps account for the easing actions taken in this model.

3. Average Stochastic Within-Path Variance

Policies, of course, are not judged solely by growth rates, but also by the degree of stability around the constant growth rate path. For the 35 paths for each policy, instability is measured by the average within-path standard

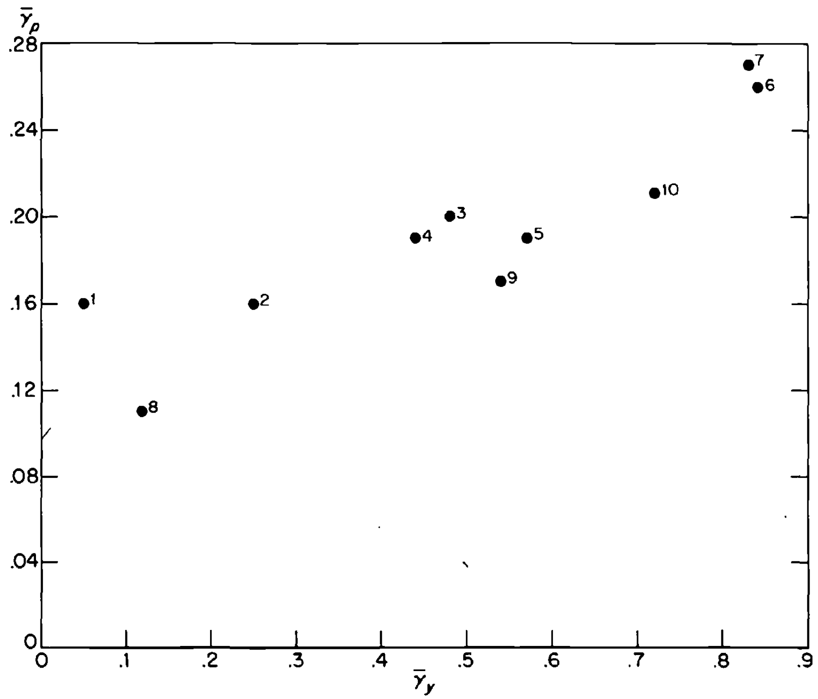
TABLE 3

FRB-MIT Model, Rates of Growth and Standard Deviations of Real Income and Price Level

	Policy	$\bar{\gamma}_y$		$\bar{\gamma}_p$		$\bar{\sigma}_y$		$\bar{\sigma}_p$				Among Divided by Within		
		S	NS	S	NS	S	NS	S	NS	$\sigma(\gamma_y)$	$\sigma(\gamma_p)$	y	p	$\rho(\gamma_y \gamma_p)$
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1.	Nondiscretionary,	.05	.43	.16	.18	2.31	2.84	0.89	0.39	.75	.27	12.5	11.6	.66
	R_{cp} exogenous	(.03)	(.07)	(.01)	(.01)									
2.	Monetary I,	.25	.74	.16	.21	3.83	4.56	0.89	0.42	.70	.23	7.1	10.0	.57
	R_{cp} exogenous	(0.3)	(.12)	(.01)	(.01)									
3.	Fiscal I,	.48	.81	.20	.24	4.32	4.27	0.82	0.45	.54	.20	4.8	9.4	.46
	R_{cp} exogenous	(.03)	(.11)	(.01)	(.01)									
4.	Joint I,	.44	.78	.19	.23	4.64	4.45	0.89	0.44	.57	.21	4.8	9.0	.47
	R_{cp} exogenous	(.03)	(.12)	(.01)	(.01)									
5.	Monetary II,	.57	.96	.19	.25	6.04	5.71	1.34	0.74	.57	.20	3.6	5.6	.34
	R_{cp} exogenous	(.03)	(.15)	(.01)	(.02)									
6.	Fiscal II,	.84	1.04	.26	.29	5.65	5.12	1.35	0.93	.33	.14	2.3	4.2	.09
	R_{cp} exogenous	(.03)	(.13)	(.01)	(.02)									
7.	Joint II,	.83	1.04	.27	.28	6.56	5.56	2.09	0.93	.45	.16	2.7	2.9	.35
	R_{cp} exogenous	(.03)	(.14)	(.01)	(.02)									
8.	Nondiscretionary,	.12	.50	.11	.15	3.56	4.17	0.78	0.40	.62	.20	6.7	10.0	.52
	reserves exogenous	(.03)	(.11)	(.01)	(.01)									
9.	Fiscal I,	.54	.77	.17	.20	5.10	4.81	0.89	0.53	.41	.16	3.1	6.8	.20
	reserves exogenous	(.03)	(.12)	(.01)	(.01)									
10.	Fiscal II,	.72	.92	.21	.25	5.26	4.96	0.95	0.74	.30	.14	2.2	5.9	.12
	reserves exogenous	(.03)	(.13)	(.01)	(.02)									

NOTE: All rates and standard deviations are expressed in per cent per quarter. S is stochastic and NS is nonstochastic. Standard deviations are in parentheses.

Figure 1. FRB-MIT Model, Average Quarterly Growth Rates, Price Level vs Real Income

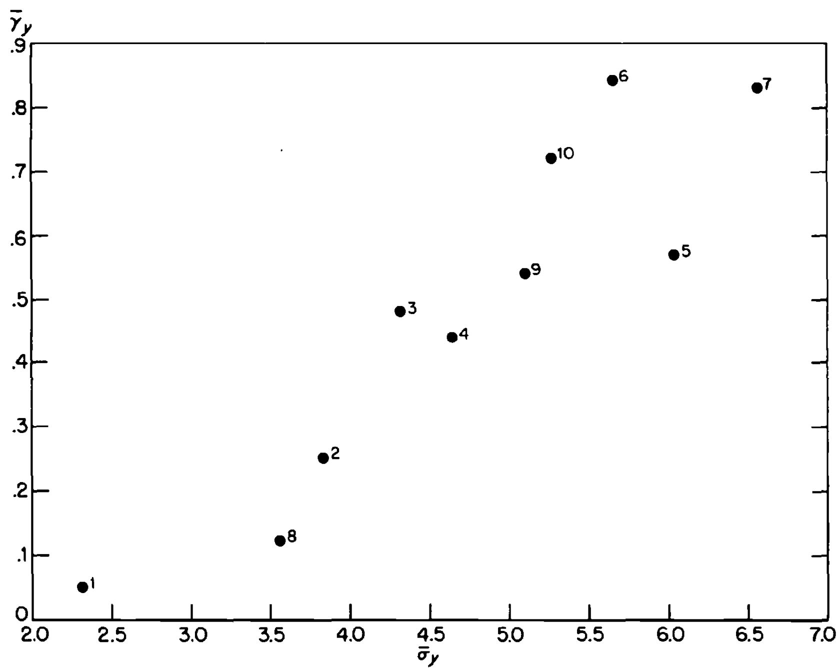


Source: Table 3.

deviations of real output and prices given in columns five and seven of Table 3. For example, for policy 1, the average within-path standard deviation for real GNP is 2.3 per cent, meaning that the square root of the average squared deviation of real income for that policy lies 2.3 per cent above or below its respective constant rate of growth path. If the deviations of log y around each constant growth rate path are normally distributed, almost one-third of the real income outcomes for policy 1 deviate by more than 2.3 per cent from the values determined by those paths.

The $\bar{\sigma}_y$'s are lowest for the nondiscretionary policies, 1 and 8; next lowest for the four weak policies, 2, 3, 4, and 9; and highest for the four strong policies, 5, 6, 7, and 10. (At a 10 per cent significance level, any ratio of those standard deviations in excess of 1.1 is significant.) The pairs $(\bar{\sigma}_y, \bar{\gamma}_y)$ are plotted in Figure 2. There is clearly a positive correlation. Thus, the relatively high growth rates achieved by the strong discretionary policies (6,

Figure 2. FRB-MIT Model, Real Income, Rates of Growth vs Within-Path Standard Deviations



Source: Table 3.

7, and 10) are accompanied by relatively large degrees of instability as we measure it. But care should be taken in interpreting that result. Sixteen quarters may be too short a time interval for measuring instability. Given the lags between changes in the instruments and the effects on real output, each of our runs may constitute only a few "observations," each consisting of an endogenous stimulus to policy, a policy action, and a response. One would like to measure instability over a long sequence of such "observations."

Figure 2 does, however, suggest a pattern among kinds of policies with the interest rate exogenous. The line segment connecting the points for policies 3 and 6 (the fiscal policies) lies above the line segment connecting the points for policies 4 and 7 (the joint policies) which, in turn, lies above a line segment connecting the points for policies 2 and 5 (the monetary policies). If it is assumed that points on those line segments are attainable by different strengths of each respective kind of policy, then those segments suggest that with the interest rate as policy instrument fiscal policies are superior to joint policies, which, in turn, are superior to monetary policies; for any given $\bar{\sigma}_y$ in the range covered by those line segments, the use of fiscal policy gives the highest $\bar{\gamma}_y$, joint policies the next highest, and monetary policies the lowest. Those segments, therefore, are not consistent with the presumption noted in the introduction; namely, that random connections between instruments and targets imply that any expected value can be attained with smaller variance by the use of multiple instruments than by the use of a single instrument. We should note, though, that one way to interpret these results is that fiscal policy works with a shorter lag than monetary policy. Figure 2 also contains a hint that superior combinations of $(\bar{\sigma}_y, \bar{\gamma}_y)$ are attainable with the interest rate constant and various doses of fiscal policy (policies 1, 3, and 6) than with unborrowed reserves growing at 1 per cent per quarter and various doses of fiscal policy (policies 8, 9, and 10).

For prices, the results are less clear-cut, but again the strong policies give rise to the highest $\bar{\sigma}_p$'s. Focusing just on prices, and assuming, of course, that low values for both the growth rate of prices and the variance are preferable to high values, there is a clear-cut preference for each weak policy over its strong counterpart. That being the case, there is no way to rank by kind of policy—monetary, fiscal, and joint—simply on the basis of price performance.

4. Among-Path Variance

For each policy and each variable, among-path variance is a function of the distribution of the 35 individual growth rates. If the true individual growth rates are identical, among-path variance would on average equal within-path variance. If they are not identical, the among variance should exceed the within variance. For each policy, the ratios of the among-path standard deviation to the within-path standard deviation are given in columns 11 and 12 of Table 3. (Ratios that exceed 1.22 are significant at a 10 per cent level of significance.) The hypothesis that the individual growth rates are identical is always rejected for both real output and the price level.

The standard deviation of the distributions of the individual γ's allows us to pose questions like the following: How likely is it that a single path of outcomes for, say, policy 6, is characterized by a real output growth rate smaller than, say, .5 per cent per quarter? The relevant statistic is the difference between the mean growth rate for policy 6, .84, and the posited value, .5, divided by $\sigma(\gamma_y)$ for policy 6, .33. That statistic has the t distribution with 34 degrees of freedom. For the question just posed, its value is just over unity, so the probability for policy 6 of observing a single path with real income growth rate smaller than .5 is about .17. This suggests that even though the mean output growth rates for different policies are estimated quite precisely, for any policy there is substantial uncertainty about the particular path that will occur.

Among-path variance follows a different pattern across policies from that of within-path variance. This may be seen by examining the standard deviations of the distributions of the individual growth rates in columns 9 and 10 of Table 3. Ratios of any $\sigma(\gamma_y)$ to any smaller one, or of any $\sigma(\gamma_p)$ to any smaller one are distributed as the square root of F with 34 and 34 degrees of freedom, the 10 per cent critical value for which is 1.3. There are, therefore, highly significant differences across policies in uncertainty about the growth rates that will prevail.

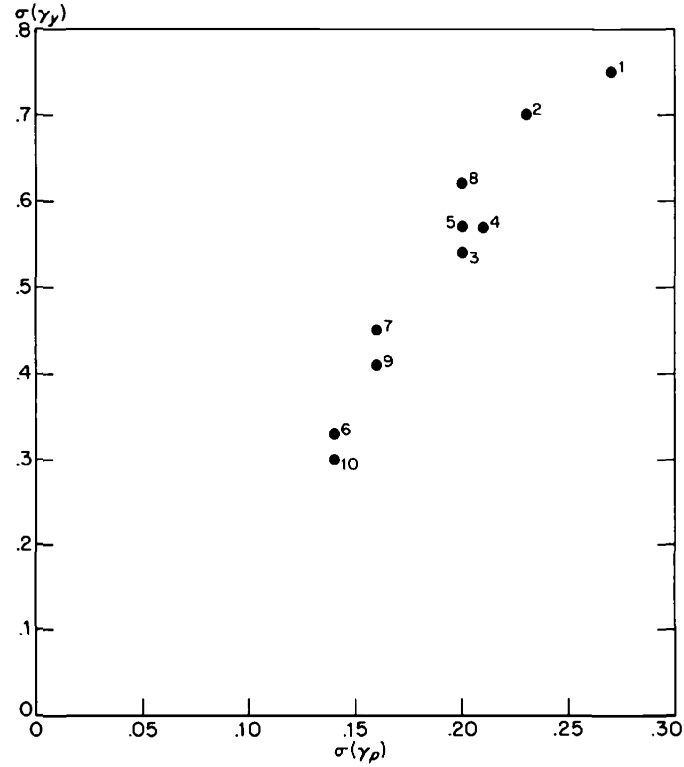
In Figure 3, we plot $[\sigma(\gamma_y), \sigma(\gamma_p)]$ by policy. The strong fiscal policies do best, while the nondiscretionary policies do very poorly. Indeed, in the result for policy 1, we have a qualified confirmation of what many view as a fundamental proposition; namely, that great uncertainty attaches to a policy of holding the interest rate constant over a substantial period of time. The confirmation is qualified because policy 6—strong fiscal policy with the interest rate held constant—is only insignificantly worse than policy 10—strong fiscal policy with unborrowed reserves growing steadily. Those with very strong a priori attachment to what we have just called a fundamental proposition will take our fairly weak confirmation of it as grounds for rejecting the FRB-MIT model. They might find it hard to believe that policy 8—nondiscretionary policy with unborrowed reserves growing steadily—does almost as poorly by these measures of uncertainty as does policy 1. Such skepticism might be reinforced by the surprising stability of prices remarked upon above.

Finally, note that the individual growth rates for real output and prices for a given policy are positively correlated. (See the last column of Table 3.) That means that when a particular random sample implies a higher than average rate of growth of real income, it is likely to imply a higher than average rate of growth of the price level. That is consistent with the positive correlation of average growth rates across policies. The correlation is strongest for the nondiscretionary policies and weakest for the strong policies, a finding consistent with the general unresponsiveness of prices in this model. Policies tend to affect real output without having much effect on the price level.

5. Stochastic Versus Nonstochastic Outcomes

The average stochastic outcomes and the nonstochastic outcomes for each quarter are shown in Tables 4 (for real output) and 5 (for the price level). The

Figure 3. FRB-MIT Model, Among-Path Standard Deviations, Price Level vs Real Income



Source: Table 3.

standard deviation of the distribution of stochastic outcomes for each quarter is presented beneath the corresponding mean. For almost all policies and for both real income and prices, those standard deviations increase quarter by quarter from 1969-I on. That was to be expected because those standard errors are analogous to standard errors of forecast which grow with the forecast span.

Note that the nonstochastic values for real GNP and for prices are equal to or exceed the corresponding mean stochastic outcomes. The discrepancy is small for the price level, fairly large for real output. The consistency of the discrepancies is explained by the dependence among outcomes across policies and over time. Dependence across policies arises because the outcomes for

TABLE 4
FRB-MIT Model, Means and Standard Deviations of Real GNP Obtained Each Quarter by Application of the Various Policy Rules

Year and Quarter	Commercial Paper Rate Exogenous														Unborrowed Reserves Exogenous					
	Non-discretionary		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2		Non-discretionary		Fiscal 1		Fiscal 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1969–I	719	718	719	718	717	716	718	717	718	717	715	715	716	716	718	717	716	716	715	714
		(6.8)		(6.7)		(6.8)		(6.7)		(6.6)		(6.7)		(6.7)		(6.4)		(6.4)		(6.3)
1969–II	724	720	722	719	719	716	720	718	719	717	714	713	716	715	720	718	715	714	713	711
		(13.2)		(12.9)		(13.1)		(13.0)		(12.5)		(12.8)		(12.7)		(11.7)		(11.6)		(11.4)
1969–III	724	717	720	715	718	712	719	713	716	712	710	707	713	710	717	712	711	708	708	704
		(18.4)		(17.7)		(18.0)		(17.9)		(16.9)		(17.3)		(17.2)		(15.3)		(15.0)		(14.5)
1969–IV	722	713	716	709	715	708	716	708	712	706	707	703	709	704	711	706	707	702	704	699
		(26.7)		(25.4)		(25.5)		(25.5)		(24.7)		(24.2)		(24.1)		(21.6)		(20.8)		(20.1)
1970–I	718	709	712	705	713	705	712	705	708	701	707	702	706	701	706	700	703	697	702	697
		(34.1)		(32.2)		(31.9)		(32.2)		(30.5)		(30.1)		(30.0)		(27.0)		(25.7)		(25.0)
1970–II	717	707	712	702	713	705	712	703	709	699	709	705	708	701	704	697	703	697	704	700
		(39.1)		(36.5)		(35.0)		(35.9)		(34.1)		(31.1)		(32.4)		(30.2)		(27.5)		(25.7)
1970–III	707	708	714	703	717	710	715	706	713	701	720	715	716	708	705	697	709	703	714	710
		(43.5)		(40.4)		(37.1)		(38.8)		(37.5)		(31.9)		(33.8)		(33.2)		(28.8)		(26.0)
1970–IV	718	710	718	707	725	717	722	712	721	707	733	728	726	719	708	700	718	712	727	724
		(49.0)		(45.4)		(40.5)		(42.7)		(42.2)		(33.7)		(36.4)		(38.5)		(32.3)		(28.8)

(continued)

Table 4 (continued)

Year and Quarter	Commercial Paper Rate Exogenous														Unborrowed Reserves Exogenous					
	Non-discre-tionary		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2		Non-discre-tionary		Fiscal 1		Fiscal 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1971–I	721	713	727	712	737	726	732	719	734	716	752	744	743	733	714	705	732	724	746	740
		(52.4)		(48.4)		(42.5)		(44.7)		(45.0)		(34.7)		(37.5)		(42.2)		(34.2)		(30.3)
1971–II	727	719	741	722	754	739	748	731	753	730	774	765	765	754	726	713	750	740	768	760
		(57.8)		(53.6)		(47.1)		(49.0)		(50.0)		(37.8)		(42.4)		(48.7)		(39.5)		(35.1)
1971–III	737	725	759	733	773	752	766	744	777	747	801	785	792	776	741	722	771	758	794	780
		(62.3)		(59.2)		(51.5)		(53.2)		(55.3)		(39.9)		(47.7)		(54.6)		(45.5)		(39.2)
1971–IV	748	728	780	743	794	764	788	758	803	766	827	803	820	799	758	729	793	775	818	796
		(66.2)		(65.2)		(54.5)		(57.0)		(59.4)		(40.4)		(52.5)		(58.4)		(50.6)		(41.6)
1972–I	762	730	803	754	815	777	810	773	831	787	850	823	847	824	778	738	816	791	838	811
		(70.8)		(71.8)		(56.5)		(61.0)		(63.7)		(42.6)		(60.8)		(62.9)		(54.2)		(42.9)
1972–II	780	733	828	765	836	791	833	789	862	810	870	841	873	846	800	748	837	805	856	824
		(75.5)		(78.9)		(59.1)		(67.1)		(67.6)		(44.8)		(67.6)		(66.8)		(51.8)		(39.7)
1972–III	798	739	853	778	856	807	856	806	892	838	887	858	896	868	820	762	854	819	868	837
		(79.8)		(83.1)		(60.2)		(70.4)		(71.6)		(46.0)		(67.6)		(71.1)		(46.9)		(35.4)
1972–IV	816	745	878	793	874	824	877	823	924	868	903	874	917	895	835	775	867	830	877	847
		(84.4)		(83.4)		(59.1)		(69.1)		(77.4)		(48.8)		(92.0)		(73.1)		(42.1)		(32.4)

NOTE: NS is nonstochastic and S is stochastic. Standard deviations are in parentheses.

TABLE 5
FRB-MIT Model, Means and Standard Deviations of Prices Obtained Each Quarter by Application of the Various Policy Rules

Year and Quarter	Commercial Paper Rate Exogenous														Unborrowed Reserves Exogenous					
	Non-discretionary		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2		Non-discretionary		Fiscal 1		Fiscal 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1969–I	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124
		(.3)		(.3)		(.3)		(.3)		(.3)		(.3)		(.3)		(.3)		(.3)		(.3)
1969–II	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124	124
		(.5)		(.5)		(.5)		(.5)		(.5)		(.5)		(.5)		(.5)		(.5)		(.5)
1969–III	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	124
		(.7)		(.7)		(.7)		(.7)		(.7)		(.7)		(.7)		(.7)		(.7)		(.7)
1969–IV	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(.9)		(.9)		(.9)		(.9)		(.9)		(.9)		(.9)		(.8)		(.8)		(.8)
1970–I	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(1.2)		(1.2)		(1.1)		(1.2)		(1.1)		(1.1)		(1.1)		(1.1)		(1.1)		(1.0)
1970–II	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(1.5)		(1.4)		(1.4)		(1.4)		(1.3)		(1.3)		(1.3)		(1.3)		(1.2)		(1.2)
1970–III	126	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(1.9)		(1.8)		(1.7)		(1.7)		(1.7)		(1.5)		(1.6)		(1.6)		(1.5)		(1.4)
1970–IV	126	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(2.3)		(2.1)		(2.0)		(2.1)		(2.0)		(1.8)		(1.8)		(1.9)		(1.7)		(1.6)

(continued)

Table 5 (continued)

Year and Quarter	Commercial Paper Rate Exogenous														Unborrowed Reserves Exogenous					
	Non-discretionary		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2		Non-discretionary		Fiscal 1		Fiscal 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1971–I	126	125	125	125	126	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
		(2.8)		(2.5)		(2.3)		(2.4)		(2.3)		(2.0)		(2.0)		(2.2)		(1.9)		(1.8)
1971–II	126	125	125	125	126	125	126	125	125	125	126	125	126	125	125	124	125	125	125	125
		(3.2)		(2.8)		(2.6)		(2.7)		(2.5)		(2.1)		(2.2)		(2.4)		(2.1)		(1.9)
1971–III	126	125	126	125	126	126	126	125	126	125	126	126	126	126	125	124	125	125	126	125
		(3.6)		(3.1)		(2.8)		(2.9)		(2.7)		(2.2)		(2.2)		(2.7)		(2.2)		(2.0)
1971–IV	126	126	126	125	127	126	126	126	126	125	127	127	127	126	125	125	126	125	127	126
		(4.1)		(3.4)		(3.0)		(3.1)		(3.0)		(2.3)		(2.3)		(3.0)		(2.3)		(2.1)
1972–I	126	126	127	126	127	126	127	126	127	126	128	128	128	127	126	125	127	126	128	127
		(4.6)		(3.8)		(3.3)		(3.5)		(3.3)		(2.6)		(2.5)		(3.5)		(2.7)		(2.4)
1972-II	126	126	127	126	128	127	128	127	128	127	129	129	129	129	126	125	127	127	128	128
		(5.2)		(4.4)		(3.8)		(3.9)		(4.0)		(3.0)		(3.3)		(4.0)		(3.0)		(2.8)
1972–III	127	127	128	127	129	128	128	128	129	128	131	130	131	130	126	126	128	128	129	129
		(5.9)		(5.1)		(4.4)		(4.6)		(4.6)		(3.4)		(4.8)		(4.5)		(3.5)		(3.3)
1972–IV	127	127	129	128	130	129	129	129	131	130	132	132	132	134	127	126	129	129	131	130
		(6.8)		(5.8)		(4.9)		(5.2)		(6.4)		(4.9)		(9.0)		(5.0)		(3.9)		(3.8)

NOTE: NS is nonstochastic and S is stochastic. Standard deviations are in parentheses.

different policies are generated from a common random sample, while dependence over time arises because of lags in the model. Nevertheless, two related questions can be posed. First, if the mean and standard deviation of the true distribution of outcomes are given by the statistics for the stochastic runs, how likely is it that the nonstochastic outcomes come from that distribution? Quite likely. All nonstochastic outcomes are within one standard deviation of the mean of the distribution of stochastic outcomes. Second, and more important given the widespread use of nonstochastic forecasts and analyses, it may well be asked, How likely is it that the nonstochastic outcome is the mean of the distribution of outcomes? Quite unlikely. The relevant test statistic is the difference between the nonstochastic outcome and the average stochastic outcome divided by the standard deviation of the stochastic *average,* which in each case is the standard deviation (given in parentheses) divided by the square root of 35. Assuming normality, that statistic is distributed as t with 34 degrees of freedom, the critical value for which at a 5 per cent significance level for a two-tailed test is 2.03. For real income, the test statistic exceeds that by a wide margin for most 1972 observations. Thus, for real income, we must either reject the hypothesis that the nonstochastic outcome represents the true mean, or must reject the hypothesis that the sample of 35 stochastic paths for real income is representative of the distribution of possible solution values of the model. One reason for rejecting the first of these is that there is no a priori support for it. Dependent variables are complicated functions of the underlying random variables—–the parameters and disturbances. The expected or average value of a function of random variables is in general equal to the function of the expected values (the nonstochastic outcome) only if the function is linear in the random variables. We know in our case that the functions determining real income and the price level are not linear.

Summary data for the comparison of stochastic and nonstochastic outcomes are presented in Table 3. The difference between average values for the stochastic and nonstochastic outcomes is revealed in terms of growth rates in columns 1-4. Columns 5-8 of the table allow us to compare stochastic and nonstochastic within-path standard deviations. Surprisingly, there is no general pattern between the nonstochastic within-path standard deviation for real income and the stochastic for each policy. None of the ratios of the stochastic to the nonstochastic is significantly different from unity. (Such ratios have to exceed 1.33 to be significant at the 10 per cent level.) For prices there is a general pattern. The stochastic within-path standard deviation exceeds the nonstochastic and significantly so except perhaps for policy 10.

Obviously, there is no nonstochastic analogue to the $\sigma(\gamma)$'s so that one cannot deduce from nonstochastic results the substantial differences in uncertainty that accompany the different policies.

6. Utility Rankings

The policies are ranked by expected utility in Table 6 according to the utility function described above. Rankings of both the stochastic and nonstochastic outcomes are given. For the stochastic outcomes, policy 6, which produced the highest average growth rate of real output, holds first place when no weight is given to the price variance term in the utility function. Its standing drops steadily the greater the weight given to that term. Policy 10, strong fiscal policy with unborrowed reserves growing steadily, is the best policy when any of the nonzero weights are given to the price variance term.

The nonstochastic and stochastic rankings differ more the greater the weight given the price variance term. That is not surprising, because the less weight given to the price variance term, the more are policies being judged almost solely on the basis of expected values. As noted in Section VI, while our utility function implies risk aversion with respect to y, it implies only very mild concern for the variance of y.

TABLE 6
FRB-MIT Model, Policies Ranked by Expected Utility

	b											
	0.0		−0.6		−.12		−.18		−.24		−.30	
Ranking	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
1. (best)	6	6	10	6	10	6	10	6	10	3	10	3
2.	7	7	6	7	9	7	9	7	9	6	9	4
3.	10	5	9	5	3	5	3	10	3	10	3	10
4.	5	10	3	10	6	10	4	3	4	4	4	2
5.	9	3	4	3	4	3	6	5	2	5	2	6
6.	3	4	5	4	2	4	2	4	8	2	8	9
7.	4	2	7	2	5	2	8	2	1	7	1	5
8.	2	9	2	9	8	9	1	9	6	9	6	7
9.	8	8	8	8	1	1	5	1	5	1	5	1
10. (worst)	1	1	1	1	7	8	7	8	7	8	7	8

NOTE: $\text{Utility} = U = (1/16) \sum_{1}^{16} \left[y_t^{1/2} + b\, 10^4 \, (p_t - p_{t-1}/p_{t-1})^2 \right]$

7. The Money Stock

We begin by describing the behavior of the money stock in the runs with unborrowed reserves exogenous and growing at 1 per cent per quarter, the policy 8, 9, and 10 runs. Table 7 contains summary statistics based on the same computation scheme for growth rates as is used throughout for real output and the price level. The money stock, m, is the sum of demand deposits and currency.

The $\bar{\sigma}_m$'s, which measure the average within-path variance of m, reveal a high degree of instability for the stock of money under policies 8, 9, and 10. If log m is normally distributed, almost one-third of the stochastically determined quarterly values of the money stock deviate by more than 2.5 per cent from their respective constant rate of growth paths, even though unborrowed reserves are growing steadily at 1 per cent per quarter. At a base of about \$200 billion, that amounts to a quarterly "miss" in excess of \$5

TABLE 7

FRB-MIT Model, Growth Rates and Standard Deviations of the Money Stock

Policy	$\bar{\gamma}_m$ S	$\bar{\gamma}_m$ NS	$\bar{\sigma}_m$ S	$\bar{\sigma}_m$ NS	$\sigma(\gamma_m)$	Among Divided by Within	Total Standard Deviation
1. Nondiscretionary, R_{cp} exogenous	.35 (.04)	.61 (.03)	2.37	1.26	.87	14.2	8.60
2. Monetary I, R_{cp} exogenous	1.13 (.03)	1.60 (.15)	4.64	5.64	.48	4.0	6.42
3. Fiscal I, R_{cp} exogenous	.65 (.03)	.90 (.06)	2.88	2.50	.64	8.5	6.67
4. Joint I, R_{cp} exogenous	.91 (.03)	1.21 (.10)	3.93	3.78	.52	5.1	6.25
5. Monetary II, R_{cp} exogenous	2.35 (.06)	2.64 (.25)	9.84	9.79	.90	3.5	12.84
6. Fiscal II, R_{cp} exogenous	.94 (.03)	1.09 (.09)	4.06	3.60	.47	4.5	5.95
7. Joint II, R_{cp} exogenous	1.57 (.04)	1.72 (.15)	6.87	5.94	.51	2.9	8.27
8. Nondiscretionary, reserves exogenous	.55 (.01)	.59 (.06)	2.69	2.18	.21	3.0	3.29
9. Fiscal I, reserves exogenous	.51 (.02)	.61 (.06)	2.55	2.29	.27	4.1	3.56
10. Fiscal II, reserves exogenous	.49 (.02)	.63 (.06)	2.46	2.41	.29	4.5	3.62

NOTE: All rates and standard deviations are expressed in per cents per quarter. S is stochastic and NS is nonstochastic. Standard deviations are in parentheses.

billion about one-third of the time, and that is only the within-path "miss." There is, in addition, significant among-path variance. For example, the probability for policies 8, 9, and 10 of experiencing a growth rate 50 per cent above or below the average for those policies is about .18. The last column of Table 7, which gives the average standard deviation around the pooled growth rate path, combines within- and among-path variance. Given normality, it implies that about one-third of the quarterly observations for policies 9 and 10 lie more than 3.5 per cent from the average constant growth path for those policies.

These results raise questions both about (i) the usefulness of unborrowed reserves as an exogenous monetary instrument in a quarterly model, and about (ii) the validity of the assumption made in these models that unborrowed reserves was an exogenous monetary instrument during the sample period. It is obvious that the Federal Reserve can, if it wishes, control the stock of money much more closely than is suggested by the variances in Table 7. It is no less obvious that the Federal Reserve has never set a value for the average stock of unborrowed reserves for a calendar quarter independent of what was occurring during the quarter. Even if unborrowed reserves is the day-by-day instrument, the Federal Reserve receives information on other financial variables and responds to it almost continuously. That makes the stock of unborrowed reserves over a quarter or even over a month dependent on the values of other variables in that quarter or month. In a quarterly model, the best surrogate for control of the money stock may well be the money stock, in the sense that the use of any other variable understates greatly the degree of control of the quarterly stock of money that can, in fact, be attained. Unfortunately, we did not examine policies with the money stock exogenous.

Despite the erratic behavior of the money stock under policies 8, 9, and 10, there is a sharp contrast between its behavior under those policies and its behavior under policies 1-7. That difference is traceable to the fact that under policies 1-7, the money stock is demand determined, while under policies 8-10 it is largely "supply" determined. For example, the $\bar{\gamma}_m$'s for policies 1, 3, and 6 are directly related to the average growth rates for income under those policies, while under policies 8-10, there is almost no relationship between the average growth rates of the money stock and those of income.

VIII. RESULTS, THE MICHIGAN MODEL

1. The Sample

As noted above, we encountered no convergence difficulties with the Michigan model. Solutions were obtained for nondiscretionary policy 1 for the first 50 sets of random variables we tried, and solutions were obtained from all those sets for the other six policies. (Recall that unborrowed reserves does not appear in the Michigan model, so that we have results only for the seven interest-rate exogenous policies.) Thus, our analysis and discussion of the Michigan model are based on that sample of 50.

2. Average Stochastic Growth Rates

The average stochastic growth rates for real GNP and for the GNP deflator are shown in the first and third columns of Table 8. For policy 1, the average stochastic growth rate for real output is .698 per cent per quarter, while that for the price level is .794 per cent per quarter. Those contrast sharply with what we found for that policy for the FRB-MIT model. There we found that holding the interest rate and tax rates constant for sixteen quarters resulted in an average stochastic growth rate of real output near zero and a growth rate for the price level of .16 per cent per quarter. Thus, for policy 1, the Michigan model predicts an annual real income growth rate 2.6 per cent higher than that for the FRB-MIT model and an annual rate of growth of prices 2.5 per cent higher. Growth rate outcomes for nondiscretionary policy 1 are the only ones comparable across these models, because our policy rules implied very different courses of action in the two models. In FRB-MIT, after the initial tightening common to all models, the course of the economy called for easing actions, on average, throughout the period, while in this model, the opposite occurred. Thus, we find for Michigan that the average growth rates for real output and the price level are lower when discretionary policies are applied.

The pairs $(\bar{\gamma}_y, \bar{\gamma}_p)$ are plotted in Figure 4. Just as in the FRB-MIT model, there is an obvious positive correlation—indeed, a perfect rank correlation—between the growth rates. (The correlation coefficient is .99.) The real output growth rates range from .445 per cent per quarter for policy 5 to .698 per cent for policy 1, while the price level growth rates range from .745 to .794 per cent. Each strong policy is characterized by lower growth rates for real output and prices than its corresponding weak version. Given that tightening actions were taken, that is to be expected. Note, in addition, that the monetary policies produce greater effects than the joint policies, which, in turn, produce greater effects than the fiscal policies. In the FRB-MIT model, the positions of monetary and fiscal policies were exactly the reverse; there fiscal policy had the largest effect and monetary policy the smallest effect. Moreover, while in FRB-MIT each strong policy dominated all weak policies, in Michigan weak monetary policy is more effective in terms of growth rates than both versions of fiscal policy.[27]

[27] This may reveal another inconsistency between the two models, or may, as suggested by Bert Hickman and noted above, arise because of asymmetry in the effects of monetary and fiscal policy.

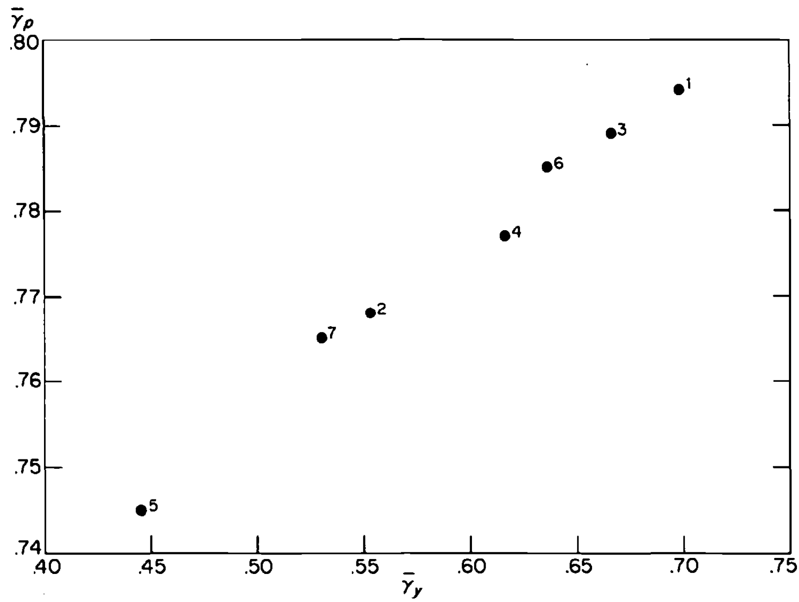
TABLE 8

Michigan Model, Growth Rates and Standard Deviations of Real Income and Price Level

Policy	γ_y S	γ_y NS	γ_p S	γ_p NS	$\bar{\sigma}_y$ S	$\bar{\sigma}_y$ NS	$\bar{\sigma}_p$ S	$\bar{\sigma}_p$ NS	$\sigma(\gamma_y)$	(γ_p)	Among Divided by Within y	Among Divided by Within p	$\rho(\gamma_y \gamma_p)$
1. Nondiscretionary, R_{cp} exogenous	.698 (.016)	.739 (.033)	.794 (.007)	.765 (.023)	1.67	1.27	.97	.87	.445	.164	10.33	6.53	+.323
2. Monetary I, R_{cp} exogenous	.553 (.015)	.562 (.032)	.768 (.005)	.745 (.024)	1.70	1.23	.94	.94	.400	.139	9.05	5.74	+.166
3. Fiscal I, R_{cp} exogenous	.666 (.016)	.696 (.031)	.789 (.006)	.763 (.023)	1.67	1.21	.95	.89	.427	.157	9.90	6.38	+.282
4. Joint I, R_{cp} exogenous	.616 (.015)	.622 (.035)	.777 (.006)	.750 (.024)	1.72	1.37	.94	.92	.420	.146	9.46	6.02	+.223
5. Monetary II, R_{cp} exogenous	.445 (.015)	.459 (.053)	.745 (.005)	.727 (.025)	2.12	2.04	.95	.95	.382	.122	6.95	4.99	+.054
6. Fiscal II, R_{cp} exogenous	.636 (.015)	.652 (.030)	.785 (.006)	.761 (.023)	1.67	1.15	.94	.90	.410	.150	9.45	6.20	+.242
7. Joint II, R_{cp} exogenous	.530 (.015)	.586 (.034)	.765 (.006)	.748 (.024)	1.74	1.33	.94	.93	.385	.136	8.54	5.57	+.126

NOTE: All rates and standard deviations are expressed in per cents per quarter. S is stochastic and NS is nonstochastic. Standard deviations are in parentheses.

Figure 4. Michigan Model, Average Quarterly Growth Rates, Price Level vs Real Income



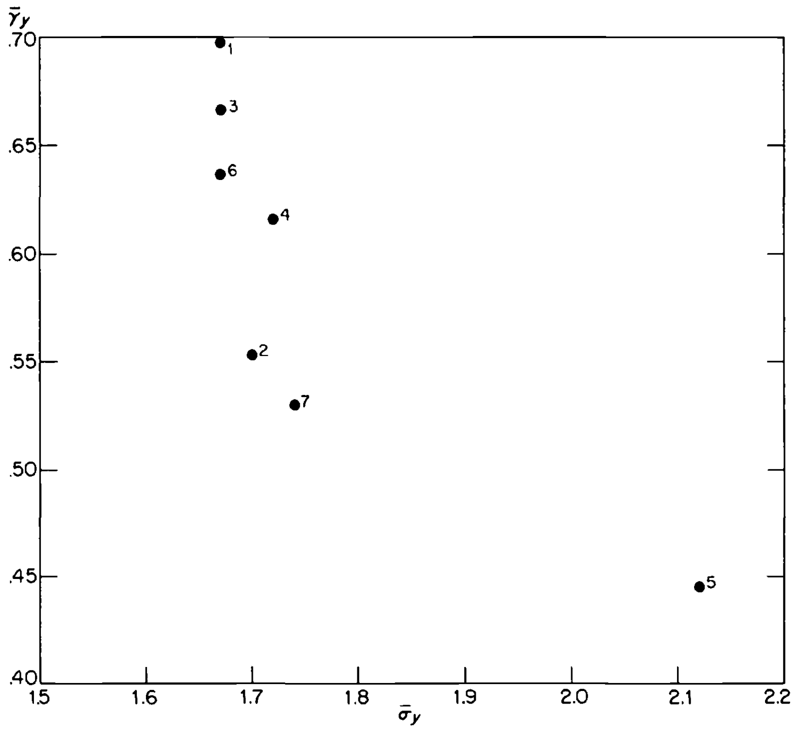
Source: Table 8.

3. Average Stochastic Within-Path Variance

Figure 5 shows that for other than fiscal policy, as discretionary policies are applied, within-path variance of real income increases. That is consistent with the FRB-MIT results, but here the differences are relatively slight. Moreover, policy by policy, within-path variance for real income is lower in the Michigan model than in the FRB-MIT model. In Michigan, the within-path standard deviation of real income ranges from about 1.7 to 2.1 per cent, while in FRB-MIT it ranged from 2.3 to 6.6 per cent. When $\bar{\sigma}_y$'s for strong policies are compared to those for weak policies, significant differences are found only for monetary policy.

The within-path standard deviation of prices varies almost not at all across policies. Its value is, on the average, lower than that found for the FRB-MIT model. Thus, the Michigan model exhibits more within-path stability for both real income and the price level than does the FRB-MIT model.

Figure 5. Michigan Model, Real Income, Rates of Growth vs Within-Path Standard Deviations



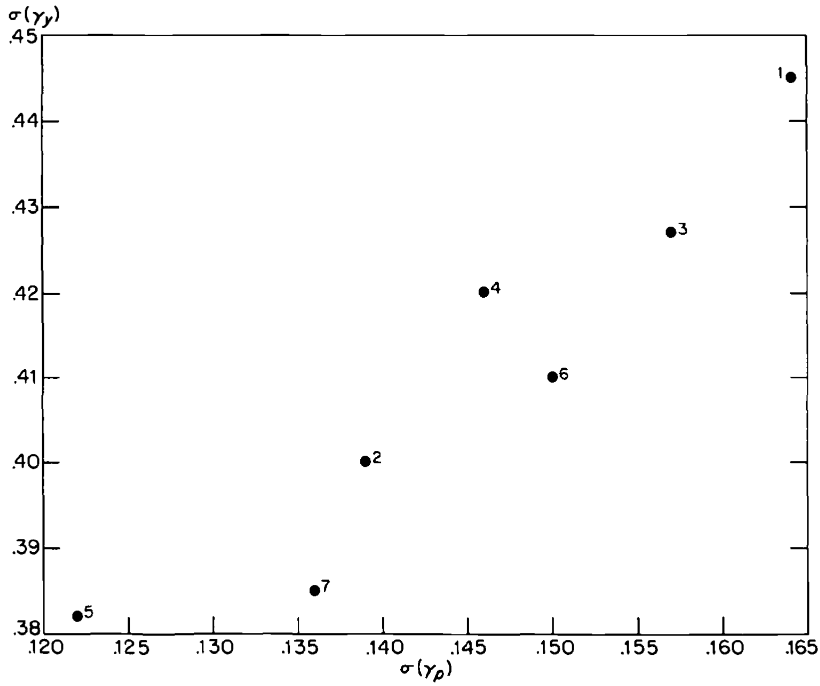
Source: Table 8.

4. Among-Path Variance

On the average, there is also less dispersion among the individual real income growth rates for the Michigan model than for FRB-MIT. Despite that there is still great uncertainty about the particular real income growth that will occur under any policy. For example, the probability under policy 5 of observing a real income growth rate larger than .698—the value of the highest average growth rate obtained, that for policy 1—is about one-third.

In Figure 6, we plot $[\sigma(\gamma_y), \sigma(\gamma_p)]$ by policy. Least uncertainty attaches to policy 5 and most to policy 1. That is consistent with our findings for FRB-MIT in two respects: (i) nondiscretionary policy with the interest rate

Figure 6. Michigan Model, Among-Path Standard Deviations, Price Level vs Real Income



Source: Table 8.

held constant does worst; and (ii) that policy which is most effective in terms of growth rates does best.

As in the FRB-MIT model, individual growth rates for real income and prices for a given policy are positively correlated (see the last column of Table 8).

5. Stochastic Versus Nonstochastic Outcomes

Table 9 for real output and Table 10 for the price level show the average stochastic outcomes (and associated standard deviations) and the non-stochastic outcomes for each quarter. As one expects, the standard deviations

TABLE 9

Michigan Model, Means and Standard Deviations of Real GNP Obtained Each Quarter by Application of the Various Policy Rules

Year and Quarter	Commercial Paper Rate Exogenous													
	Control		Monetary I		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1969–I	717	718	717	718	717	718	717	718	716	717	717	718	717	718
		(6.1)		(6.1)		(6.1)		(6.1)		(6.2)		(6.0)		(6.1)
1969–II	716	717	716	716	716	717	716	716	712	715	716	716	716	716
		(9.7)		(9.7)		(9.7)		(9.6)		(10.2)		(9.7)		(9.6)
1969–III	720	719	719	718	720	719	718	719	714	715	720	719	718	718
		(13.1)		(13.0)		(13.2)		(13.0)		(13.8)		(12.9)		(12.9)
1969–IV	725	724	722	722	725	723	722	722	716	717	724	723	721	721
		(16.7)		(16.4)		(16.6)		(16.4)		(17.6)		(16.4)		(16.3)
1970–I	732	731	727	727	731	730	727	728	718	720	730	729	726	726
		(20.6)		(20.1)		(20.3)		(20.1)		(20.9)		(20.1)		(19.9)
1970–II	739	738	733	732	738	737	733	734	723	724	737	736	732	731
		(26.5)		(26.0)		(26.1)		(25.8)		(26.0)		(25.8)		(25.7)
1970–III	747	745	739	738	746	744	740	740	727	728	744	742	738	736
		(29.5)		(28.6)		(28.9)		(28.5)		(28.0)		(28.3)		(28.0)
1970–IV	756	753	743	744	753	752	747	748	731	733	751	750	745	742
		(32.4)		(30.6)		(31.5)		(30.8)		(29.7)		(30.6)		(29.8)

(continued)

Table 9 (continued)

Year and Quarter	Commercial Paper Rate Exogenous													
	Control		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1971–I	764	762 (36.1)	749	750 (33.7)	761	760 (35.0)	754	755 (34.4)	736	739 (32.9)	759	758 (33.8)	751	748 (32.7)
1971–II	772	770 (38.5)	756	757 (35.0)	769	768 (37.0)	761	763 (36.4)	744	746 (33.9)	766	765 (35.6)	758	755 (33.8)
1971–III	780	780 (42.7)	762	765 (38.2)	776	777 (40.9)	768	772 (40.0)	753	754 (37.0)	773	774 (39.2)	765	763 (36.7)
1971–IV	788	788 (45.8)	770	772 (40.3)	784	785 (43.7)	775	780 (42.6)	762	761 (39.1)	780	782 (41.7)	772	770 (38.5)
1972–I	796	795 (48.5)	777	779 (41.8)	791	792 (46.1)	783	787 (44.8)	773	768 (39.8)	786	788 (43.9)	779	776 (39.9)
1972–II	803	803 (49.7)	784	786 (41.2)	798	799 (46.8)	792	794 (45.1)	784	776 (37.9)	793	796 (44.3)	787	783 (39.2)
1972–III	811	811 (53.1)	792	793 (43.6)	805	806 (49.7)	800	801 (47.9)	793	784 (39.9)	800	802 (46.7)	795	790 (41.3)
1972–IV	819	817 (57.1)	799	800 (45.8)	813	812 (52.9)	808	807 (50.8)	803	791 (41.9)	807	808 (49.4)	803	796 (43.2)

NOTE: S is stochastic and NS is nonstochastic. Standard deviations are in parentheses.

TABLE 10
Michigan Model, Means and Standard Deviations of Price Obtained Each Quarter by Application of the Various Policy Rules

Year and Quarter	Commercial Paper Rate Exogenous													
	Control		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1969–I	126	126	126	126	126	126	126	126	126	126	126	126	126	126
		(.4)		(.4)		(.4)		(.4)		(.4)		(.4)		(.4)
1969–II	127	126	127	126	127	126	127	126	127	126	127	126	127	126
		(.8)		(.8)		(.8)		(.8)		(.8)		(.8)		(.8)
1969–III	128	128	128	128	128	128	128	128	128	128	128	128	128	128
		(1.0)		(1.0)		(1.0)		(1.0)		(1.0)		(1.0)		(1.0)
1969–IV	129	129	129	129	129	129	129	129	129	129	129	129	129	129
		(1.2)		(1.2)		(1.2)		(1.2)		(1.2)		(1.2)		(1.2)
1970–I	130	130	130	130	130	130	130	130	130	130	130	130	130	130
		(1.3)		(1.3)		(1.3)		(1.3)		(1.2)		(1.3)		(1.3)
1970–II	131	131	131	131	131	131	131	131	131	131	131	131	131	131
		(1.4)		(1.4)		(1.4)		(1.4)		(1.3)		(1.4)		(1.4)
1970–III	131	131	131	131	131	131	131	131	131	131	131	131	131	131
		(1.5)		(1.5)		(1.5)		(1.5)		(1.5)		(1.5)		(1.5)
1970–IV	132	132	132	132	132	132	132	132	132	132	132	132	132	132
		(1.7)		(1.7)		(1.7)		(1.7)		(1.6)		(1.7)		(1.7)

(continued)

Table 10 (continued)

Year and Quarter	Commercial Paper Rate Exogenous													
	Control		Monetary 1		Fiscal 1		Joint 1		Monetary 2		Fiscal 2		Joint 2	
	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS	S
1971–I	133	133	133	133	133	133	133	133	132	133	133	133	133	133
		(2.0)		(1.9)		(2.0)		(1.9)		(1.8)		(1.9)		(1.9)
1971–II	134	134	133	134	134	134	133	134	133	133	134	134	133	134
		(2.2)		(2.1)		(2.2)		(2.1)		(1.9)		(2.1)		(2.1)
1971–III	134	135	134	134	134	135	134	135	134	134	134	135	134	134
		(2.4)		(2.2)		(2.4)		(2.3)		(2.0)		(2.3)		(2.2)
1971–IV	135	136	135	135	135	136	135	135	134	135	135	136	135	135
		(2.8)		(2.5)		(2.7)		(2.6)		(2.2)		(2.6)		(2.4)
1972–I	136	137	135	136	136	137	136	136	135	136	136	136	136	136
		(3.3)		(2.7)		(3.1)		(2.9)		(2.3)		(3.0)		(2.7)
1972–II	137	138	136	137	137	137	136	137	136	136	137	137	136	137
		(4.1)		(3.1)		(3.8)		(3.4)		(2.6)		(3.5)		(3.0)
1972–III	138	139	137	138	137	139	137	138	137	137	137	138	137	138
		(4.9)		(3.5)		(4.5)		(3.9)		(2.8)		(4.1)		(3.3)
1972-IV	138	140	138	139	138	140	138	139	137	138	138	139	138	139
		(5.3)		(3.7)		(4.8)		(4.2)		(2.9)		(4.4)		(3.5)

NOTE: S is stochastic and NS is nonstochastic. Standard deviations are in parentheses.

increase continuously from 1969-I on. Although differences among the 1972-IV average stochastic outcomes for real GNP seem slight, the ranking is the same as the ranking by average growth rate in Table 8. Note that for prices the average stochastic values for each quarter are almost the same across policies, which is consistent with the near identity of growth rates for prices across policies. In contrast to what was found in FRB-MIT, for this model the hypothesis that the nonstochastic outcomes represent the means of the distributions of outcomes can not be rejected. Here, each nonstochastic outcome could be the mean of the corresponding distribution of outcomes.

The nonstochastic within-path variance estimates for real GNP are significantly lower than the stochastic except for policy 5. For prices, the differences are not significant (see Table 8). Underestimation of within-path instability is what one expects to find in the nonstochastic runs.

6. Utility Rankings

The policies are ranked by expected utility in Table 11. For the stochastic outcomes, the ranking for values of b near zero is the same as the ranking by real output growth rates. Only when the price variance term is given substantial weight, does the ranking change; the nondiscretionary policy, policy 1, falls in the rankings, ending as the fifth best policy for b–.3. In contrast, the nonstochastic expected utility ranking is the same for all values

TABLE 11
Michigan Model, Policies Ranked by Expected Utility

	b											
	0.0		–0.6		–.12		–.18		–.24		–.30	
Ranking	S	NS	S	NS	S	NS	S	NS	S	NS	S	NS
1. (best)	1	1	1	1	1	1	1	1	3	1	6	1
2.	3	3	3	3	3	3	3	3	6	3	4	3
3.	6	6	6	6	6	6	6	6	1	6	3	6
4.	4	4	4	4	4	4	4	4	4	4	2	4
5.	2	7	2	7	2	7	2	7	2	7	1	7
6.	7	2	7	2	7	2	7	2	7	2	7	2
7. (worst)	5	5	5	5	5	5	5	5	5	5	5	5

$$U = (1/16) \sum_{1}^{16} \left[y_t^{1/2} + b\, 10^4 \, (p_t - p_{t-1} / p_{t-1})^2 \right]$$

of b. It is almost the same as the stochastic for values of b near zero. For b = –.24 and –.30, the discrepancy between the stochastic and nonstochastic ranking can be explained by the fact that the nonstochastic ranking, by definition, takes no account of among-path variance.

IX. RESULTS, THE WHARTON MODEL

We did not carry out our experiments on the Wharton model. The problem centered on the determination of the unemployment rate, *un*. In the Wharton model,

$$un = 1-(x+y)/(z-w),$$

where $z-w$ is the civilian labor force—the total labor force, z, minus the number of military personnel, w, and $x+y$ is total employment—the number of private nonfarm employees, x, plus the sum of civilian government employees, farm workers, and the nonagricultural self-employed, y. We tried a number of extrapolation procedures for the exogenous variables, y and w, but for all of them, *un* took on negative values in a high percentage of the sixteen-quarter runs attempted. We could have constrained the unemployment rate to be positive, but since its value in any quarter helps determine wages, prices, and other variables in that and subsequent quarters, those solutions would depend on the lower bound—necessarily nonzero—chosen by us. Rather than report results dependent on an arbitrarily chosen lower bound for *un*, we chose not to perform the experiments.

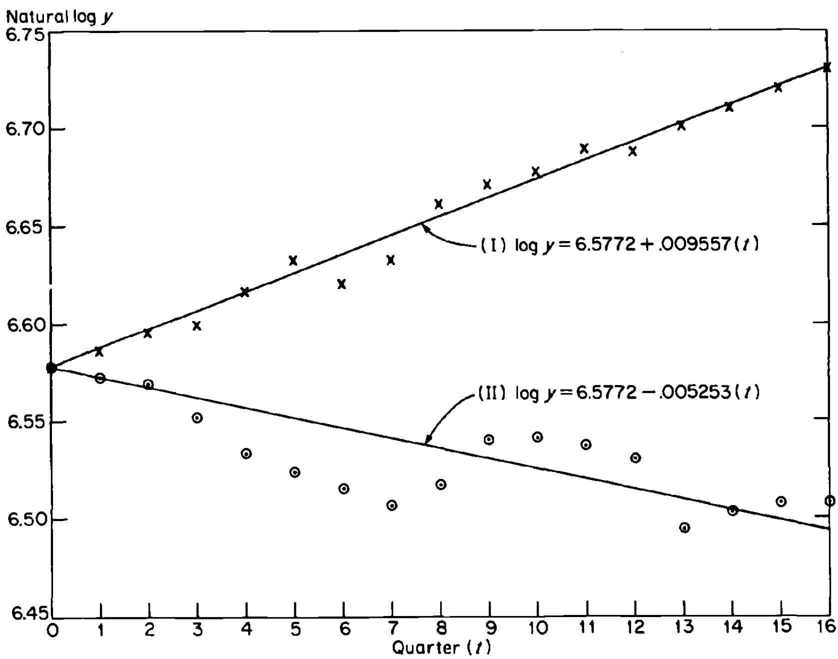
APPENDIX

In this appendix we illustrate the computations required for deriving the summary growth rate and variance statistics. For this purpose we use two sets of results for real income from the FRB-MIT model obtained from the application of nondiscretionary policy with the interest rate exogenous. The natural logarithms of those real income results are plotted in Figure 7.

The constant growth rate paths shown in Figure 7 were computed from least squares regressions as described in part 1 of Section VI. The constant growth rates are .956 and –.525 per cent per quarter for runs I and II, respectively. The residual standard deviations around those constant growth rate paths were used as measures of instability. They are .64 for run I and 1.85 for run II. In the text we report the average (pooled) growth rate, $\overline{\gamma}_y$, and the average standard deviation, $\overline{\sigma}_y$. If a sample consisted of these two runs, then $\overline{\gamma}_y = (.956-.525)/2 = .216$ and $\overline{\sigma}_y = [(.64^2 + 1.85^2)/2]^{1/2} = (1.916)^{1/2}$. We also report the standard deviation of the individual growth rates, which for this sample of two is $\sigma(\gamma_y) = [(.956-.216)^2 + (-.525-.216)^2]^{1/2}/2^{1/2} = (.496)^{1/2}$. The analysis of variance table for this sample of two is as follows:

Source of Variation	Degrees of Freedom	Sum of Squares	Root Mean Square (RMS)
Total residuals from pooled regression	$16N-1 = 31$	$\sigma^2(\gamma_y)\sum_1^{16} t^2 + 15N\bar{\sigma}_y{}^2 = 799.5$	$(799.5)^{1/2} = 28.3$
Among samples	$N-1 = 1$	$\sigma^2(\gamma_y)\sum_1^{16} t^2 = (.496)(1{,}496) = 742.0$	$(742.0)^{1/2} = 27.2$
Within	$15N = 30$	$15N\bar{\sigma}_y{}^2 = 30(1.916) = 57.5$	$(1.916)^{1/2} = 1.38$

Figure 7. Two Stochastic Paths of Real Income and Their Respective Constant Growth Rate Regressions



The standard deviation of the average growth rate is the first entry in the RMS column divided by $(N\ \Sigma\ t^2)^{1/2}$; for this example, .52. In the text in Tables 3 and 9 under the heading "Among Divided by Within," we report ratios of the second entry in the RMS column to the third. For this sample of two that ratio is 19.7. Aside from the correlation coefficient between income and price level growth rates, that completes the description of all growth rate and variance statistics.