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Chapter Eight

The Production of Housing Services from Existing Dwelling Units

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INTRODUCTION

Despite the fact that the bulk of housing services in urban areas is provided by existing dwelling units, our empirical and theoretical understanding of how existing dwelling units are operated over time to produce housing services is meager. Early analyses of this topic focused on the change in quality or in relative quality ranking of existing dwelling units. This change, termed filtering, is typically represented in early studies as a reduced form combination of supply and demand behavior (Lowry 1960; Smith 1964). Recently, models that more fully specify the supply and demand side of the market for existing units have been developed, some of which use simulation techniques to represent the supply and demand decisions of housing producers and households (de Leeuw 1972) while others apply the optimal control theory approach (Sweeney 1974).

An alternative model of the housing market's supply side focuses not on the production of housing services from combinations of existing units and operating inputs, but on the production of dwelling units from combinations of land and capital. Partial empirical tests of that model have been made by examining residential

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density gradients in urban areas (Muth 1969; Mills 1970). That model is used to analyze long-run investments in housing stocks, whereas our concern in this study is with the shorter-run operations of existing units.

The model presented here differs from most other analyses of the supply side of the housing market in its treatment of housing services produced by existing units. The model allows more than one structure type; and existing housing capital as well as housing outputs are assumed to be structure-type-specific. In addition, substitution between housing capital and operating inputs is incorporated in the production of structure quality. This approach can be contrasted with other recent work on housing supply that either treats housing capital as homogeneous or does not allow for substitution between capital and operating inputs (de Leeuw 1972; Sweeney 1974).

The model's major focus in on how housing producers operate existing dwelling units to produce housing services. Existing units are described in terms of three summary characteristics—structure quality, structure type, and neighborhood quality—that differ by mode of production or supply elasticity. The production of structure quality is treated in terms that parallel the choice of optimal plant size in the theory of the firm, and structure quality is produced by heterogeneous capital and operating inputs. Decisions about structure type follow the usual assumptions of investment theory. Neighborhood quality is assumed to be a local public good not produced by housing entrepreneurs. The model also includes some determinants of supply response, such as cash flow constraints, that may have significant impacts upon short-run adjustments in the housing market.

The analytic framework of the model is outlined and estimates of the supply parameters are given. Numerical experiments are performed with the supply model to determine likely values for the magnitude and duration of the increases in short-run average costs of structure quality that might accompany a housing market policy, such as a housing allowance, that suddenly increased the demand for structure quality. The supply model is combined with a demand model in a primitive market-clearing framework, and several comparative static equilibrium sensitivity analyses of model parameters are carried out. Appendix 8A contains a brief description of the estimation of the elasticity of substitution between capital and operating inputs in the production of structure quality.

CHARACTERIZATION OF HOUSING PRODUCTION

In this model, housing is a composite of several characteristics, and dwelling units are available with specific combinations of these characteristics. We define the housing services produced by dwelling units in terms of their characteristics as:

$$\text{Housing services}_s = (Q_s, N, A)$$

where Q_s is the quality of structure services produced by a unit of structure type s; N is the quality of the neighborhood of the unit; and A is the accessibility of the unit. The rents for particular dwelling units are determined by all three characteristics; but for existing units, only Q_s, the structure quality, can be varied. In this analysis we ignore accessibility and incorporate neighborhood quality to the extent that it affects the production of structure quality. We focus on the supply of structure quality, the one characteristic of existing units that is under the control of housing producers.

Structure services are produced from three components—land, capital, and operating inputs—which differ significantly in their durability. At the extremes, land is infinitely durable, and operating inputs are completely consumed during production. The capital that is used to produce structure quality has a range of durabilities. Many basic capital components of a dwelling unit are very durable, while the interior finish of a unit, its mechanical subsystems, and many of its exterior features have relatively short lives. Although an average depreciation rate can be calculated for the capital in a given dwelling unit, that rate is a function of the mix of capital stocks in the unit. It is apparent that a variation of the index number problem exists for the representation of housing capital depreciation.[1]

Besides having a variety of durabilities, the capital in a dwelling unit also differs in terms of its substitutability in the production process that yields structure quality. Structure quality is subscripted by structure type, and we assume some minimum stock and configuration of capital is required in order to produce structure quality of a particular structure type. Minimum stock includes the unit's foundation and shell, and is essentially an entry requirement on the supply side. Once this minimum stock is available, a housing producer can produce structure quality with operating inputs and additional capital. This restriction on production can be summarized as

$$Q_s = \begin{cases} \text{zero}; KT_s < C_s \\ f(K_s, O); KT_s > C_s \end{cases}$$

where C_s is the minimum stock of capital required for the production of structure quality of structure type s, O represents operating inputs, and KT is a unit's total capital.

Categorizing a dwelling unit's capital in terms of its durability and substitutability in the production of structure quality is an interesting topic for research, but one we will not pursue here. Instead, we assume that a dwelling unit's capital can be separated into two categories, as we have done in the foregoing equation. The first category, termed "structure capital," or C_s, is extremely durable and has an (assumed) depreciation rate of zero. The second category, termed "quality capital," or K_s, is the capital required for the production of structure quality of structure type s. Quality capital is not very durable; it depreciates at an annual rate, d; and it can be changed readily by incremental investment, termed "maintenance" (M). Structure capital will not affect the production of quality by existing units and will only be a factor in decisions about new construction or the conversion of a unit from one structure type to another. This categorization of a dwelling unit's capital implies that its depreciation rate, averaged over both capital categories, increases with the level of structure quality the unit produces.

Within this framework we assume that housing producers operate individual dwelling units and try to maximize their profits. During each time period they pick a level of operating inputs for their unit, determine the expenditure on maintenance, and calculate whether it would pay them to alter the structure type of their unit. We assume that in making these decisions, the housing producer is familiar with the operating requirements and debt structure of his building; that he knows the prices of the inputs he uses; and that he has well-defined expectations about the structure of rents in his neighborhood and the way in which rents would vary with the quality of his unit.

In many respects the model we set forth parallels the analysis of optimal plant size in the theory of the firm. In our model the dwelling unit is the "plant," and the housing producer must invest or disinvest in this plant over time so that he produces his "output," dwelling unit quality, in a least-cost way. In addition, housing producers can have different kinds of plants (structure type) and buy and sell in different markets (neighborhoods).

ANALYTICS OF THE SUPPLY MODEL

Time in this model is treated as a series of discrete periods of one year's duration. We assume the housing producer revises his operating strategy at the beginning of each time period on a contracting or market day. At this time he makes three decisions. The first deals with the level of quality he will produce this period and how to do it. The second involves the amount he will invest in the building this period. The third is whether he should transform his unit into some other structure type. We consider each of these decisions in turn.

The Current-Period Operating Decision

During the current period the housing producer's unit has a stock of quality capital, K_t, embodied in it (for clarity, the subscript for structure type is omitted). We assume that any maintenance expenditure this period will not alter the stock of capital until the next period, so K_t is fixed in the current period. In addition, we assume that rental receipts and expenditures on operating inputs are concurrent, with the result that no within-period discounting is required. These two assumptions let us separate the housing producer's choice of operating inputs from his choice of maintenance expenditures in the current period.

The housing producer's opportunities for producing structure quality with various combinations of operating inputs, O_t, and quality capital, K_t, are summarized in a production function,

$$Q_t = f(O_t, K_t) \tag{8-1}$$

where Q_t is the level of structure quality produced in period t. In addition, the housing producer knows how gross rents will vary with structure quality for his unit, and he knows P_o, the price per unit of operating inputs. Since K_t, the unit's stock of quality capital, is fixed for the current period, the housing producer chooses operating inputs to maximize his short-run profits or cash flow, defined as the difference between gross rents and expenditures on operating inputs. The solution involves the usual first-order conditions:

$$\frac{dR(O; K, t)}{dO} = P_o \tag{8-2}$$

where $R(O; K, t)$ is the relation between gross rents and operating inputs obtained by substituting the production function in Equation (8-1) into the relation between gross rents and quality which is known to the housing producer.

The Current-Period Maintenance Decision

The current-period maintenance or investment decision is more complicated than the choice of operating inputs because maintenance expenditures made this period will affect the stock of quality capital of the unit in subsequent periods.[2] Moreover, there are constraints on the rate at which a housing producer can alter the stock of quality capital in his unit. We assume, for example, that maintenance must be non-negative: the housing producer cannot reduce his stock of quality capital at a pace that exceeds the rate of depreciation. In addition, if a housing producer wants to purchase maintenance inputs in amounts that exceed his cash flow, he must pay a higher interest rate than the opportunity cost of his cash flow, a requirement that will tend to reduce his rate of investment. Given these constraints, the housing producer's objective is to pick a maintenance and operating input stream that maximizes the present value of his net revenues. This objective can be stated as

$$\text{maximize} \sum_{t=0}^{\infty} (R_t - P_o O_t - P_m M_t - F_t) \frac{1}{(1+r)^t} \tag{8-3}$$

subject to the production function for quality and the accounting relation for capital, maintenance, and depreciation:

$$K_{t+1} = K_t - dK_t + M_t \tag{8-4}$$

In Equation (8-3), R_t is the gross rent of the unit, P_m is the price per unit of maintenance, F_t are the fixed costs exclusive of financing, and r is the interest rate. If the relation between rents and dwelling unit quality is known over time, a housing producer can calculate his optimal time path for maintenance and operating inputs within the framework of optimal control theory.[3] However, this supply model does not follow the formal optimal control procedure, but rather represents the housing producer's decision about maintenance in a simpler framework.

The major simplification we make is to relax the assumption that a housing producer has perfect knowledge of the relation between rents and quality over time. Instead, we assume he has an expectation about that relation over a planning interval of five periods. After that time, he projects a relation between rents and quality that does not change. This simplification is based on the premise that housing producers have some knowledge or expectation about how prices in their neighborhood will be changing over the next few periods, and what the relation bween rent and quality is likely to be after these changes have occurred. Beyond his planning interval, however, a housing producer's information about prices is poor, and he merely projects his expectations for the fifth year into the future without change.

This assumption about housing producers' price expectations fixes the relation between rents and quality in future periods. By making this relation fixed or stationary beyond the planning interval, we can derive the optimal stock of quality capital that corresponds to the future stationary state without using optimal control techniques, and then choose a maintenance policy that will move the dwelling unit toward the optimal capital stock. This approach may correspond more closely to the way in which housing producers actually behave while still being closely related to the behavior implicit in the optimal control theory representation.

In the stationary state, where the expected relation between rent and quality, the production function, and the input prices do not change, the landlord's choice of O_t, M_t, and K_t will also be unchanging. This stationarity assumption implies, therefore, that maintenance inputs exactly offset depreciation losses. Furthermore, with the stationarity assumption, the optimal stock of quality capital satisfies the usual first-order condition,

$$\frac{d(R - P_o O)}{dK} = P_k \tag{8-5}$$

where P_k is the price of a unit of quality capital for one period, essentially the price of a capital *flow*. Since maintenance inputs augment the stock of quality capital, we know that the price for a unit of quality capital *stock* is P_m. If we keep the unit of capital for one period, we must pay the going rate of interest on the capital, r. In addition, we have assumed that quality capital depreciates at a constant rate, d, during each period. Hence, to keep a unit of quality capital for one period, a landlord must pay $P_m r$ in interest and $P_m d$ in depreciation. The one-period price of quality capital is therefore $P_m(r + d)$, and this is the price that must satisfy the first-order conditions in Equation (8-5). By combining the first-order conditions in Equations (8-5) and (8-2) with the production function in Equation (8-1), we can derive the optimal stock of quality capital, the optimal fow of operating inputs, and the optimal quality level to be produced at the end of the planning interval.

Having determined the optimal capital stock in the future stationary state, we now turn to the maintenance decision. Each dwelling unit has an existing stock of quality capital, K_t, and the unit's maintenance policy should lead to the optimal capital stock, K^*, during the planning interval. Let us consider first a situation in which a unit's current stock of quality capital exceeds its optimal level. Since we have assumed that maintenance inputs cannot be negative, the unit's stock of quality capital cannot be reduced more rapidly than the depreciation rate. That is, over the planning interval of T periods, no maintenance inputs will be purchased this period if

$$\frac{K_t}{(1+d)^T} \geqslant K^* \tag{8-6}$$

for in that case the purchase of maintenace inputs would slow the rate at which the optimal level of capital stock is approached. If Equation (8-6) does not apply, then some maintenance inputs will be provided over the planning interval. In that case, the housing producer determines what the most efficient stock of capital would be for the quality level he has decided to produce during the current period, and he attempts to move to that capital stock by the next time period. If the most efficient stock of capital for quality level L is $K^e(L)$, a housing producer who is producing L this period will have a desired level of maintenance,

$$M = K^e(L) - K_t(1-d) \tag{8-7}$$

Since the most efficient capital stock for any quality level is a function of the interest rate, the desired maintenance will also be a function of the interest rate. In our discussion thus far we have used r as the interest rate, implicitly assuming that this rate is r_g, the opportunity cost of internally generated funds. This assumption will hold in the stationary state if the building is viable. When a housing producer is adjusting his stock of quality capital in the short run, however, maintenance expenditures may exceed cash flow for one or more periods. We assume that there is an external interest rate, r_x, greater than r_g, at which additional funds can be borrowed.[4] In the short run, therefore, a housing producer's interest rate is related to his cash flow, and his supply curve of funds resembles S, shown in Figure 8-1, where CF is available cash flow. This available cash is gross rents less expenditures on operating inputs and other fixed obligations such as taxes. Since fixed costs can vary, otherwise similar units may have different cash flows. Desired maintenance expenditures can be represented as a demand curve on Figure 8-1, and curves M_1, M_2, and M_3 illustrate possible outcomes.

Another constraint on the level of current maintenance expenditures is its consistency with the optimal capital stock at the end of the planning interval. That is, the maintenance for the current period, M_t, must satisfy the condition

$$K_0(1-d)^T + M_t(1-d)^{T-1} \leqslant K^* \tag{8-8}$$

Of course, when the housing producer's decision on maintenance is made within a dynamic framework, the end of the planning interval is never attained. During each time period the planning interval covers the next T periods. This rolling interval is merely a device that lets us represent the more distant future, when expectations are poorly defined, as a stationary state. In each current period the housing producer reformulates his operating and maintenance strategy.

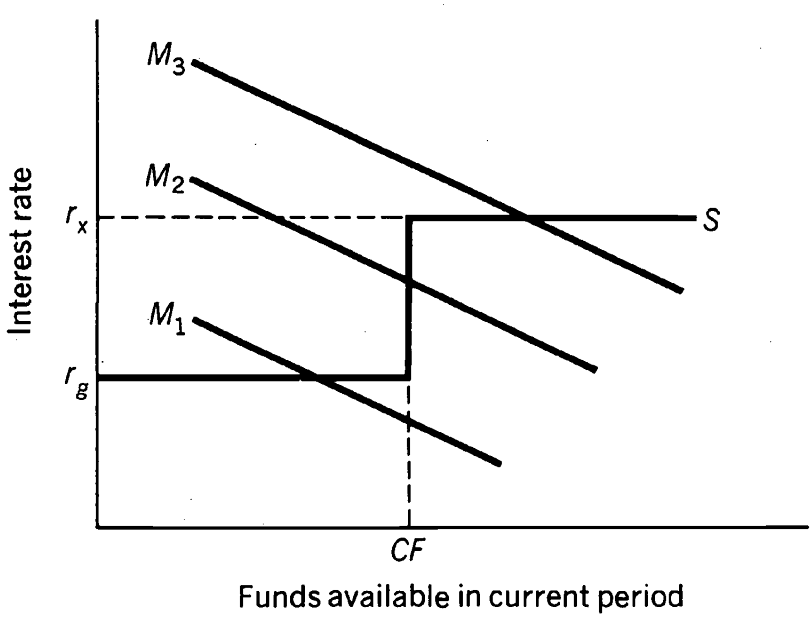


Figure 8-1. Interest Rates, Cash Flow, and Current-Period Maintenance Decisions.

The Structure-Type Decision

The housing producer's decisions about operating inputs and maintenance are conditional on his decision concerning the structure type of his unit. We consider the structure type decision last because its prerequisites are the current period's optimal operating and maintenance decisions. We assume that a housing producer decides to convert a unit from one structure type to another by comparing the value of alternative types with the value of the current type and its conversion cost. In this case, the value of a unit, V, is equal to the discounted sum of net revenues from the unit, or

$$V = \sum_{t=0}^{\infty} (R_t - P_o O_t - P_m M_t - F_t) \frac{1}{(1+r)^t} \qquad (8\text{-}9)$$

where F denotes fixed costs associated with the unit exclusive of financing costs. In order to calculate the maximum value of a unit, the optimal maintenance and operating strategy must be determined. This is required for the structure type the housing producer has now, as well as for alternative types he considers. For his existing unit, the housing producer determines his expected net revenues during the planning interval and during the postplanning-interval stationary state. For alternative structure types, we assume the housing producer immediately attains the optimal level of quality capital during conversion and calculates the value of alternative structure types from their expected net revenues.

Abstracting from changes in lot size, the conversion cost from structure type i to structure type j has two major components: The first is the expenditure for additional structure capital or for removing existing structure capital, or both. The second is the expenditure on quality capital that is required to produce a unit of structure type j with the optimal stock of quality capital. Let us denote this total conversion cost as C_{ij}. For all j structure types, a housing producer will convert from type i to type j if $Z_j > 0$, where

$$Z_j = \max_j \left[V_j - (V_i + C_{ij}) \right] \qquad (8\text{-}10)$$

Although we will not consider it in detail in the model, at any instant in time, a housing producer will be able to construct a new unit of structure type j by purchasing land, structure capital, and quality capital. If we represent the cost of new construction by NC_j, then construction will occur if

$$V_j > NC_j \qquad (8\text{-}11)$$

and NC_j will ultimately limit V_j. Of course, NC_j will vary over time if land prices or the optimal quality level of a unit change. The costs of conversion and new construction can limit possible values of V_i and V_j, but many combinations of the two will produce neither structure-type conversions nor new construction.

Change in Neighborhood Quality

We assume that housing producers do not have direct control over the production of neighborhood quality. They may have expectations about how neighborhood quality will change over time, however, and these expectations might influence their behavior. For the moment we will not specify precisely the determinants of neighborhood quality, although it is undoubtedly some amalgam of the characteristics of dwelling units and households in an area. At this point we will merely assume that neighborhood quality is a neighborhood-specific public good.

SPECIFYING THE SUPPLY MODEL

In this section we specify the dimensions and parameters of a housing supply model that incorporates many of the features described in the previous sections. For the sake of simplicity the model employs only two levels of neighborhood quality and two structure types. Input prices vary by neighborhood, and the production function for quality varies by structure type. Dwelling unit quality can be produced at eight possible levels; so its representation is more continuous than neighborhood quality or structure type.

Although the supply model includes neighborhoods, it does not include location in this simple version. The model can be envisioned as representing the housing market in a metropolitan area that has been stratified into zones according to neighborhood quality level. The model refers to aggregates of zones of each quality level and otherwise ignores the location of particular units.

Since the major goal of the supply model is to represent the operation of existing units, the construction of new units and the conversion from one structure type to another are not endogenous. Accordingly, the major parameters that must be determined in the supply model are those in the production function for structure quality. The data for determining these parameters are not as good as we would like, but the available data do permit us to obtain reasonable estimates of parameter values. We obtain production function parameters for two structure types: a high-rise elevator building and a small multiple unit building.

A CES (constant elasticity of substitution) production function is employed. This is flexible, yet simple, with properties that are attractive for representing the production of structure quality. Using K for quality capital, O for operation inputs, and Q for structure quality, the CES function is

$$Q = A[\alpha K^{-\beta} + (1-\alpha)O^{-\beta}]^{-1/\beta} \tag{8-12}$$

in the case of constant returns to scale. The parameters of this function are A, the scaling or efficiency factor; α, the distribution factor; and β, the substitution factor, which determines $1/(\beta + 1)$, the elasticity of substitution between the factor inputs.

The Elasticity of Substitution

The elasticity of substitution between operating inputs and quality capital can be estimated by regressing the log of the ratio of dwelling unit quality to operating inputs on the log of the ratio of their prices (Arrow et al. 1961). We estimated this elasticity from a time series sample compiled for small multiple units in the Boston area (Key 1973). This sample, which covers the period 1942-1969, includes annual rental and expense information for twenty-nine apartment buildings, the majority of which have fewer than fifty dwelling units. These apartment buildings are operated by large real estate firms that provide good quality middle-income housing and follow conscientious maintenance practices. For this sample structure quality was proxied by the annual average rent per room divided by the Boston-area rent index; operating inputs are measured by the price-adjusted operating expense per room. Both the consumer price index and a fuel and utility index were used as measures of the price of operating inputs.[5] One difficulty with these data is that rent control was in effect from 1942 until March 1956.

Using the data in combination with various post-rent-control dummy variables, we obtained estimates of the elasticity of substitution ranging from 0.32 to 0.65; the results of the regressions are shown in Appendix 8A. Although these estimates left much to be desired, we concluded that the elasticity of substitution for these units was approximately 0.5, and we used this value as the elasticity of substitution for the small, multiple-unit building. We lacked the data for estimating the elasticity of substitution for the high-rise structure, but we assumed that it had a higher elasticity than the small, multiple unit buildings. This stems from our belief that a large "plant" offers more possibilities for substitution than a small "plant." Accordingly, we set the elasticity of substitution for the high rise at 0.6, a value that is still within the range of estimated elasticities for small multiple units. These elasticities of substitution imply values for β of 1.0 for small structures and 0.67 for large ones.

The Distribution Factor

The distribution factor, α, in a CES production function is an important determinant of the share of output that each input factor receives. Applying the usual first-order conditions to Equation (8-12), we obtain the following ratio of factor shares:

$$\frac{P_o \cdot O}{P_k \cdot K} = \frac{P_k}{P_o}^{-\beta/(\beta+1)} \left(\frac{1-\alpha}{\alpha}\right)^{1/\beta+1} \tag{8-13}$$

which shows that β and the input prices also play a role in determining relative factor shares. Having determined β for the two structure types, we went on to specify the prices of the inputs. We set both P_o and P_m equal to unity. Since $P_k = P_m(r + d)$ we also had to specify the interest and depreciation rates. We assumed that the depreciation rate for quality capital was 10 percent per year and that the interest rate was 5 percent per year.[6] Hence, the ratio of P_k to P_o was 0.15.

The ratio of factor shares specified in Equation (8-13) is not directly observable because operating statements do not report $P_k K$, the payment to quality capital, but they do contain enough information to determine the ratio of operating expenses to maintenance expenses. Moreover, we know that the ratio of maintenance expenditures to total expenditures on quality capital is $d/(r + d)$; so we can write the expression for factor shares in terms of observable quantities as

$$\frac{P_o O}{P_m M} = \left(\frac{r+d}{d}\right) \left(\frac{P_k}{P_o}\right)^{-\beta/(\beta+1)} \left(\frac{1-\alpha}{\alpha}\right)^{1/(\beta+1)} \tag{8-14}$$

Operating data were available to us from two sources: the time series on Boston units used to estimate the elasticity of substitution and a recent study of rent control in the Boston region (Sternlieb 1974). These data are shown in Table 8-1, and the ratios of expenditures on operating inputs to expenditures on maintenance are similar for comparable time periods and structure types. The ratio for high-rise units is approximately 1.33 and the ratio for smaller buildings seems to be somewhat lower. During the most recent time periods 1.25 is a representative ratio for our small multiple unit structures. Using these ratios of input shares we obtained values for α of 0.906 for the latter and 0.812 for the high rises.

Other Parameters of the Supply Model

We can calculate the optimal stock of quality capital and the optimal level of operating inputs using the production function in Equation (8-12) and first-order conditions in Equation (8-13). The optimal levels of each input are

$$K^* = \left[\alpha + (1-\alpha)\left(\frac{P_k}{P_o}\,\frac{1-\alpha}{\alpha}\right)^{-\beta/(1+\beta)}\right]^{1/\beta}\frac{Q}{A}$$

$$O^* = \left[(1-\alpha) + \alpha\left(\frac{P_k}{P_o}\,\frac{1-\alpha}{\alpha}\right)^{\beta/(1+\beta)}\right]^{1/\beta}\frac{Q}{A} \qquad (8\text{-}15)$$

Table 8-1. Operating and Maintenance Expenditures as Ratios to Gross Rents and to Each Other,[a] 1942-1969 and 1971-1973

Expenditure Item	*Time Period or Structure Type*					
	Uncontrolled Rents			*Controlled Rents*		
	'65-'69	*'60-'64*	*'55-'59*	*'50-'55*	*'45-'49*	*'42-'44*
			Key Data (1973, p. 45)			
Operating inputs (*O*)	0.26	0.27	0.31	0.32	0.33	0.30
Maintenance and repair (*M*)	0.21	0.18	0.21	0.15	0.14	0.15
Total	0.47	0.45	0.52	0.47	0.47	0.45
Ratio of *O* to *M*	1.24	1.50	1.48	2.13	2.36	2.00

	Period of Uncontrolled Rents		*Period of Controlled Rents*	
	High Rise	*Other*	*High Rise*	*Other*
	Sternlieb Data (1975, pp. 39-43)			
Operating inputs (*O*)	0.16	0.20	0.19	0.21
Maintenance and repair (*M*)	0.12	0.16	0.14	0.16
Total	0.28	0.36	0.33	0.37
Ration of *O* to *M*	1.33	1.25	1.36	1.31

[a]The data from Key are for 29 buildings. The Sternlieb data are for 12,068 units and cover the period 1971-1973.

Moreover, in the CES production function, minimum levels of each input are required to produce a given level of output. In our case this means that the quality isoquant of each dwelling unit is asymptotic to nonzero values of quality capital and operating inputs. For any level of structure quality, Q, the minimum input requirements are

$$K_{\min} = \alpha^{1/\beta}\,(Q/A) \tag{8-16}$$

$$Q_{\min} = (1-\alpha)^{1/\beta}(Q/A)$$

These conditions imply that dwelling units with specific levels of quality capital will not be able to produce quality levels above a certain point in the short run, a very plausible restriction for a model of the housing market.

The final values we specified to complete our supply side model were the prices of factor inputs across neighborhoods and the values of dwelling unit quality to be produced. We assumed that all factor prices across neighborhoods were equal except for the interest rate, r, which we assume is 5 percent in good neighborhoods and 7 percent in bad ones. This interest rate difference reflects differences in risk between the two neighborhoods. Setting the eight levels of dwelling unit quality that will be produced was arbitrary; so we assumed A equalled unity, and we scaled our eight quality levels to increase logarithmically from the lowest to the highest quality level.

The parameters and dimensions of the supply model are summarized in Table 8-2; in Table 8-3, we show the eight quality levels and the long-run annual average cost of supplying each quality level by each structure type in each neighborhood. The factor share data in Table 8-1 show that expenditures on maintenance and operating inputs claim from 28 to 47 percent of gross rents, and they imply that the total cost of supplying quality (using our assumed values of r and d) ranges from 34 to 62 percent of gross rents. Using these ratios as rough guides, the annual rents for the dwelling unit quality levels shown in Table 8-3 will be from 1-2/3 to 3 times the cost of quality for each quality level. That is, a typical gross rent for quality level 8 might range from \$32 to \$72 per month, and a typical gross rent for quality level 1 might range from \$190 to \$430 per month. The factor share data in Table 8-1 also suggest that owners devote a lower share of gross rents to dwelling unit quality in high rises than in small multiple unit structures. Structure capital and land costs are presumably higher for high-rise units than for low-rise ones.

The data that are available for estimating parameters of the

Table 8-2. Supply Model Parameters[a]

Parameter	Structure Type: High Rise	Structure Type: Small Multi
Elasticity of substitution	0.6	0.5
β	0.67	1.0
α	0.812	0.906
A	1.	1.

	Good Neighborhood		Bad Neighborhood	
	High Rise	Small Multi	High Rise	Small Multi
K^*	$1.90Q$	$1.66Q$	$1.83Q$	$1.61Q$
O^*	$0.25Q$	$0.21Q$	$0.26Q$	$0.22Q$
K_{min}	$0.73Q$	$0.91Q$	$0.73Q$	$0.91Q$
O_{min}	$0.08Q$	$0.09Q$	$0.08Q$	$0.09Q$
Long-run marginal cost per unit of Q	0.54	0.46	0.57	0.49
r	0.05	0.05	0.07	0.07
d	0.10	0.10	0.10	0.10
$P_o(O/P_m)M$	1.33	1.25	1.42	1.32

[a]Variables are defined infra.

Table 8-3. Quality Levels and Long-Run Average Cost of Quality per Dwelling Unit per Year

Quality Level No.	Q	Good Neighborhood High Rise	Good Neighborhood Small Multi	Bad Neighborhood High Rise	Bad Neighborhood Small Multi
1	3,000	$1,605	$1,377	$1,713	$1,481
2	2,322	1,242	1,066	1,326	1,146
3	1,798	962	825	1,027	888
4	1,392	745	639	795	687
5	1,078	577	495	616	532
6	834	446	383	476	412
7	646	346	297	369	319
8	500	268	230	286	247

production function for structure quality are not perfect, but they do suggest that different structure types are likely to have different production functions. An alternative interpretation would be that the various parameters and input prices constitute a partial sensitivity test of the production function over a reasonable range of values.

SHORT-RUN ADJUSTMENT PATHS

In this section we perform some experiments with the production functions estimated in the previous section. In these experiments we examine the rate at which housing producers can alter the levels of structure quality produced by their units and the paths of short-run average costs during the adjustment period. We concentrate on cases where changes in patterns of demand will lead housing producers to increase the level of structure quality they produce. This type of demand shift is likely to accompany the introduction of a housing allowance program in a metropolitan housing market. We limit our analysis to the adjustment of structure quality and ignore possible changes in the demand for structure type or neighborhood quality. These latter two characteristics will be dealt with in the next section.

We begin by reviewing market conditions when the supply of structure quality in the housing market is in equilibrium. Since this characteristic is elastic in supply in the long run and is produced by a constant-returns-to-scale technology, the long-run supply curve will be horizontal. All levels of quality produced will be equally profitable; the rent for structure quality will equal the long-run average cost of production; and housing producers at each quality level will have the optimal stock of quality capital. If demands for structure quality suddenly increase, housing producers will immediately be able to increase the quality level they produce by increasing operating inputs. In the short run, however, housing producers will no longer have optimal capital stocks, and their short-run average costs of producing structure quality will rise above long-run average costs. Since the rents for quality will be determined by the costs of marginal producers at each quality level, the rents will rise in those levels with increased demand. Over a number of periods, housing producers will invest in quality capital until their stock of quality capital attains its new optimal level. At this time, rents for structure quality will again equal long-run average costs. In such a scenario at least two questions are of interest: How much will rents for structure quality increase in the short run? How long will it take for these rents to return to their equilibrium levels?

Using our production functions we can estimate rent changes from cost changes under certain pricing rules. It is apparent that the answers to these questions depend on several factors, including the magnitude of the increase in demand and the amount of cash flow available to the housing producer for investment. The cash flow available is in turn related to the pricing rule followed. We will assume that housing producers price their structure quality at short-run average cost. An average cost pricing rule is used because the marginal housing producer in each quality level has an alternative use for his unit: producing a lower quality level.

Short-Run Rent Increases from Demand Shifts

When a unit with the optimal stock of quality capital for quality level L is used to produce quality level $L + i$, operating inputs must be increased because the stock of quality capital is fixed in the short run (the current period). In addition, operating inputs are not perfect substitutes for quality capital; as shown in Equation (8-16), some quality levels may not even be attainable in the short run. The data in the following tabulation were computed using the short-run average cost pricing rule. The figures show the percent increase in the rent for structure quality that would be demanded by a housing producer whose unit was used to produce structure quality with the stock of quality capital that is optimal for a lower quality level:

		Number of Quality Levels Above That for Which Capital Stock Is Optimal			
Type	*0*	*1*	*2*	*3*	*4*
High rise					
Good neighborhood	0	8.1	50.2	312.5	–
Bad neighborhood	0	9.2	56.3	415.0	–
Small multi					
Good neighborhood	0	12.4	162.1	–	–
Bad neighborhood	0	13.5	235.0	–	–

For example, if a high-rise unit in a good neighborhood has an optimal stock of quality capital one level lower, the rent charge for quality will be 8.1 percent above the long-run equilibrium rent for quality. The data also illustrate that high-rise units, with their higher elasticity of substitution, can move up three quality levels in the short run while small multiple units can move up only two levels in the same period. Finally, we see from the table that the cost of having the incorrect capital stock is somewhat greater in bad neighborhoods than in good ones because of the former's higher interest rate.

It is clear that the magnitude of the short-run increase in rents for quality will depend on the size of the demand shift and the distribution over quality levels of the existing stock. For example, if the increase in the demand for quality is moderate, dwelling units might only shift to the next higher quality level, and the data in the table suggest that quality rents would rise only from 8 to 13 percent in the short run. If some units shifted two quality levels, quality rents would rise much more, from 50 to 235 percent, and it would be surprising if the increase in demand from a housing allowance would be large enough to sustain a price rise of that magnitude. We will investigate this question further when we operate the supply model in conjunction with a demand model.

The percent change figures in the foregoing table remind us that even in the very short run the production of dwelling unit quality from the existing stock is not inelastic. Our production functions for dwelling unit quality allow some substitution between inputs and allow a range of adjustments of outputs. At the same time, however, we see that sudden changes in demand are likely to produce rent increases in some quality levels because some housing producers will have nonoptimal capital stocks. These rent increases are not merely income transfers to landlords; they represent payments for production costs that are incurred by housing producers who suddenly find themselves out of equilibrium. These short-run rent increases will continue until housing producers can adjust their capital stocks to new optimal levels. We now try to determine how long this adjustment might take.

Length of the Adjustment Period

To reach a higher level of quality capital, a housing producer must make net investments over time, and the time it takes him to reach the new quality level will be a function of the resources he can devote to investment. We analyze two options for financing net investment that might be available to a housing producer: First we assume that only cash generated by the unit is available for investment and that he cannot borrow; and second, we assume he can borrow funds to finance his investment and that he must pay off his loan with his cash flow.

To simplify the case where investment is paid for directly out of cash flow, we assume that the cash flow available for net investment can be represented as a proportion of the stock of quality capital. If

p is the proportion of quality capital available for net investment, then the rate of change of quality capital, $\dot{K}$, will be

$$\dot{K} = p\,K_t \tag{8-17}$$

which is a simple differential equation. Solving it yields the result,

$$T_{ij} = \frac{1}{p}\,\ln \frac{K_j}{K_i} \tag{8-18}$$

where T_{ij} is the time required to move from capital stock K_i to capital stock K_j, and p is the proportion of the capital stock devoted to net investment.

It will be recalled from Table 8-3 that the quality levels we use in our model are scaled logarithmically; ratios of quality levels are therefore constant multiples. Since the optimal stock of quality capital is a linear function of quality, the ratios of optimal capital stocks for different quality levels will also be constant multiples. As a result, from Equation (8-18) we can see that the time required to move a unit's capital stock up two levels will be twice that required to move it up one level.

These properties of the adjustment period are illustrated in the table below, where we show the number of time periods (in years) required to increase the stock of quality capital of a dwelling unit by one, two, or three quality levels as a function of p, the proportion of quality capital invested each year. (Note that, given the parameters of the model, the optimal stock of quality capital will range from one to two times the annual gross rents.)

Number of Quality Levels Moved		*Proportion of Quality Capital Invested Each Year*						
		0.01	*0.03*	*0.05*	*0.07*	*0.09*	*0.11*	*0.15*
1	1	25.7	8.7	5.2	3.8	3.0	2.5	1.8
2	2	51.4	17.3	10.5	7.6	5.9	4.9	3.7
3	3	77.2	26.0	15.7	11.3	8.9	7.4	5.5

For example, a housing producer with 5 percent of his quality capital available for net investment (roughly 5 percent of gross rents) will take approximately five years to attain the optimal stock of quality capital for the next higher quality level. Note that these time periods of adjustment are independent of structure type and neighborhood quality and depend only on the funds available for investment.

A sense of the path of rents during the adjustment period can be conveyed by examining a particular unit. Figure 8-2 shows the path of quality rents for a high-rise unit in a good neighborhood where the net investment is 7 percent of quality capital. The rent paths are percent increases over long-run levels for a unit that is being shifted one, two, or three quality levels. The short-run rent levels decay exponentially to their long-run equilibrium levels.

In calculating the length of the adjustment period, we have assumed thus far that housing producers try to pay for their increased investments in quality capital on a current basis from their net revenues. This may correspond to the way in which some housing

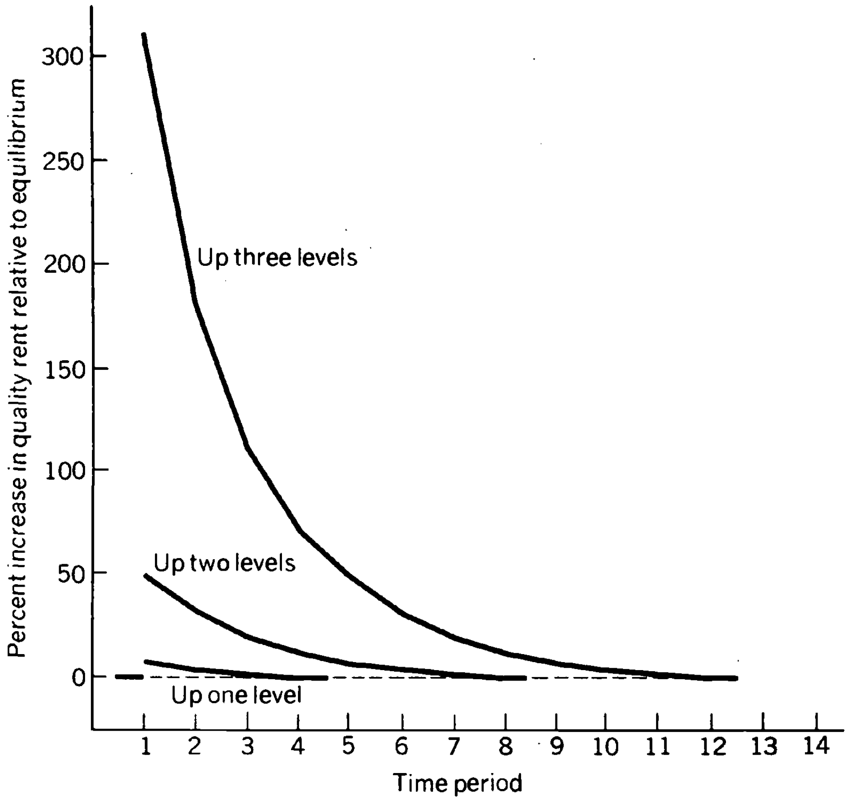


Figure 8-2. Time Path of Rent for Quality during Adjustment with No Borrowing (high rise unit; good neighborhood; p = 0.07).

producers behave, but others may borrow and finance expenditures on additional quality capital.

We now assume that housing producers can borrow money at a rate r_x, that is higher than their long-run opportunity cost of capital, r. Since r_x exceeds r, a housing producer who borrows to increase his stock of quality capital to a level appropriate for quality level j must calculate a new desired capital stock, K_j^x, that reflects r_x. During the current period, a housing producer can invest $p\ K_t$ in additional quality capital. If he wishes to attain his new desired capital stock, he must also borrow

$$B = K_j^x - (K_i' + pK_i') \qquad (8\text{-}19)$$

where K_i' is his existing stock of quality capital.

In the subsequent period the housing producer will produce quality level j with a stock of quality capital equal to K_j^x; hence, there will be some present-worth factor for interest rate r_x that is equal to B/pK_j^x. By searching for this value in a table of present-worth factors for rate r_x, we can determine how many periods it will take to pay back the loan. For the term of the loan to be finite, it is obvious that pK_j^x/r_x must be greater than B. Since we are examining only the supply side of the market we assume that the housing producer wants to minimize the short-run average costs of providing quality level j with his unit. Therefore, he will only take out a loan if borrowing reduces his short-run average costs, and we define B_0 as the largest loan amount that will yield short-run average costs equal to those he would incur if he did not borrow. Combining these limits, we find that the amount of funds a housing producer will borrow is

$$B = \min[K_j^x - (1 + p)K_j'; \frac{pK_1^x}{r_x}; B_0] \qquad (8\text{-}20)$$

If the housing producer does borrow and changes his capital stock to K_j^x, the latter will remain at that level until the loan is repaid. In addition, his short-run average cost will be constant during this period. After repayment, his cost of funds reverts to r; he invests in his unit from current revenues to move it from K_j^x to K_j^* and his short-run average costs approach long-run coverage costs for quality level j.

Let us look at these calculations for the same unit we considered earlier: a high-rise unit in a good neighborhood with cash flow available for investment and amortization equal to 7 percent of quality capital. In addition, let us assume that r_x is 10 percent. In this case, illustrated in Figure 8-3, we find that it does not pay to borrow at all to move up one quality level. However, to move two or three quality levels, borrowing does pay, although the length of the adjustment period is virtually unchanged in both cases.

THE SUPPLY MODEL IN A MARKET-CLEARING CONTEXT

In this section we combine the supply model for dwelling unit quality with household demands for housing. Demand equations in which the independent variables are rents and the neighborhood,

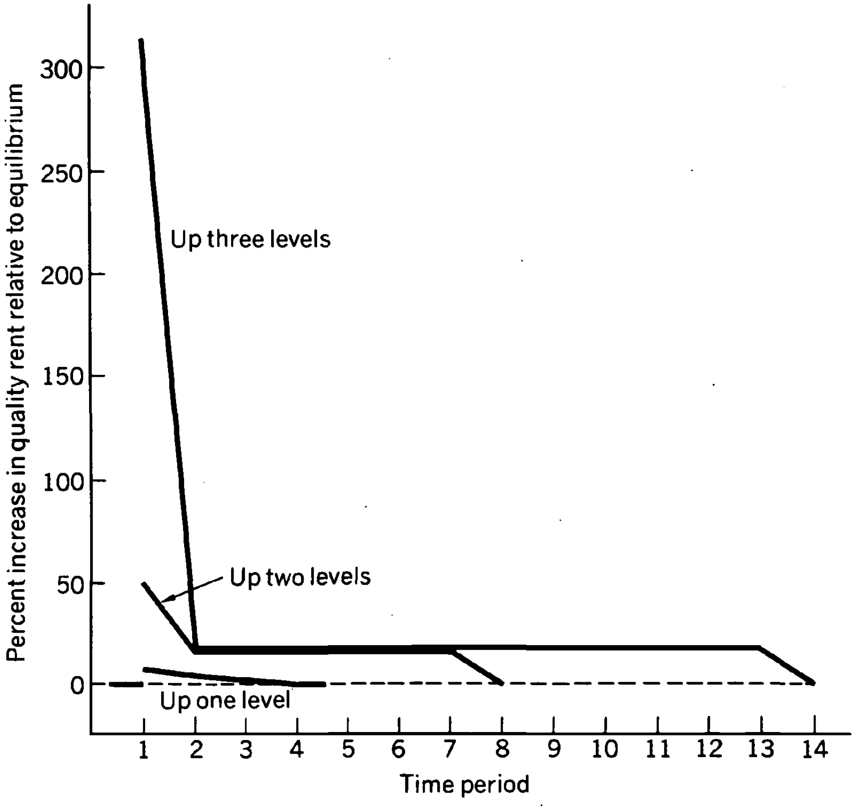


Figure 8-3. Time Path of Rent for Quality during Adjustment Period with Borrowing (high rise unit in good neighborhood; $P = 0.07$; $r_x = 10$ percent).

structure, and quality attributes of dwelling units represent the housing demands of households. Housing producers supply dwelling unit quality with existing units using operating and quality capital inputs as described in earlier sections. We assume that neighborhood quality levels are fixed. New construction and conversions between structure types are not endogenous in the supply sector of the model, but we have carried out some experiments with the model in which new units are added exogenously in response to price changes.

This combination of supply and demand behavior can be envisaged as representing a market for dwelling unit quality over time. At the beginning of each time period 200 households distributed over six income classes face a fixed stock of dwelling units and choose a unit in which to reside for the current period. The market-clearing procedure operates in two steps: First, using current-period expected rents for structure quality, we alter the combined rents for neighborhood and structure type until the market for these two (fixed) attributes is cleared. Once the households are allocated among neighborhoods and structures, a second market-clearing procedure is used to clear the market for dwelling unit quality in each neighborhood-structure combination. Since housing producers can alter the level of dwelling unit quality they produce in the current period, this second market-clearing procedure iterates between demand and supply until the demands of households match the quality levels supplied by housing producers at common rents for dwelling unit quality. On the basis of established rents for dwelling unit quality, housing producers form expectations about what the rents for dwelling quality will be in future time periods, and they use these expectations to determine their maintenance expenditure this period. During the succeeding period, households and housing producers recontract for quality levels, prices are revised, expectations are reformulated, and maintenance decisions are made. When this process has continued for several time periods and no changes have been made in supply or demand parameters, an equilibrium is reached. In addition, model parameters can be altered to generate different equilibriums, which can be analyzed in terms of price and quantity elasticities or other summary statistics.

In this section we present several partial equilibrium analyses or sensitivity tests that involve changing various parameters of the model and determining the effect these changes have on model outcomes. In addition, we test an approximation to a housing allowance program by altering the income distribution of households. Before describing the model experiments, we outline the decisions of households and housing producers as represented in the model.

The Household's Choice of Dwelling Unit

On a given market day 200 households distributed over six income classes participate in the model's market for neighborhoods, structures, and dwelling unit quality. The number of households is equal to the number of available dwelling units in the market. A housing bundle is characterized by the neighborhood, structure type, and quality level it provides. The households have demand equations that are of the form

$$PR(N, S, Q, I) = \frac{\exp\left[\alpha_I N + \beta_I S + \gamma_I Q + \delta_I R(N, S, Q)_t\right]}{\sum_{N,S,Q} \left\{\exp\left[\alpha_I N + \beta_I S + \gamma_I Q + \delta_I R(N, S, Q)_t\right]\right\}} \tag{8-21}$$

where

$PR(N, S, Q, I)$ = proportion of households in income class I choosing a unit of quality level Q, structure type S, and neighborhood N;

$R(N, S, Q)_t$ = rent for a dwelling unit of quality level Q and structure type S in neighborhood N;

$\alpha_I, \beta_I, \gamma_I, \delta_I$ = coefficients of neighborhood quality, structure type, dwelling unit quality, and rent for income class I.

This demand equation is of the logit type, and its exponents can be thought of as utility functions.

The market clearing is done in two steps. In the first step demand and supply classified by neighborhood and structure are equated by changing the rents for these attributes. The rent term, $R(N, S, Q)$, in Equation (8-21) is separated into two components: the expected rent for dwelling unit quality, $R(Q)$, and the rent for neighborhood and structure type, $R(N, S)$.[7] Since the supply of units by structure type and neighborhood is fixed, $R(N, S)$ is adjusted in the first stage of market clearing until household demands are matched up with the dwelling units available with these two attributes. In the second stage of market clearing, $R(N, S)$ is fixed, and demand and supply are iterated within each neighborhood and structure type submarket until the market for dwelling unit quality is cleared.

The Provision of Dwelling Unit Quality

Given a schedule of demands for each of the eight possible quality levels, housing producers must determine the least costly means of meeting the demands at each level. Since each dwelling unit has a different amount of capital associated with it, in terms of operating inputs it is always less costly to produce units at the highest quality level in buildings with the largest amounts of quality capital; to produce the second highest quality level in units with the next largest amounts of quality capital; and so forth. This assignment of dwelling units to quality levels according to the amounts of quality capital embodied in the units maximizes the total cash flow of landlords subject to the constraint that the demand schedule of households must be met.

Given this technique of determining a production schedule that minimizes the total cost of producing the quality levels demanded, each quality level's marginal unit in the supply function can be determined by inspection. Because dwelling units are assigned to quality levels according to the stock of quality capital they embody, the marginal unit in each quality level is the one with the least amount of capital supplying that quality level. The full cost of providing the given quality level with the marginal unit sets the rent for quality, $R(Q)$, in that quality level and is the sum of the cost of the required operating inputs plus the one-period costs of capital embodied in the unit:

$$P_o O' + P_m (r + d)K' \tag{8-22}$$

where O' are the required operating inputs for the quality level and K' is the unit's capital stock. Of course, each quality level will have a different marginal unit and a different cost for its marginal unit.

After the household demands have been matched to the supplies of quality levels and market-clearing rents for quality are formulated, each housing producer revises his rent expectations for the future and determines his maintenance expenditures for the current period. A housing producer expects the long-run equilibrium rents for quality to equal his long-run marginal costs. He must form expectations about rents in the next few periods because these expected rents are the basis for his decision to produce a certain quality level and to purchase additional capital this year. He knows the current and past rents and the expected long-run rent. We assume that the housing producer believes that rents will reach their long-run equilibrium by the end of a planning interval T periods long (in the program we assume that $T = 5$). We believe that the housing producer expects the change in rent next year to be some weighted average of the change in rent this year and the average yearly change needed to achieve the long-run value after T periods. The particular formulation is

$$R(Q)_{t+1} = R(Q)_t + \mu [R(Q)_t - R(Q)_{t-1}] + (1-\mu) \frac{R(Q^*) - R(Q)_t}{T} \qquad (8\text{-}23)$$

where

$R(Q)_t$ = rent for quality level Q for a given neighborhood and structure type;
$R(Q^*)$ = long-run rents of quality level Q; and
μ = parameter of expectations.

Housing producers may have different α's and T's. In the numerical solution we assume they may have different α's only.

After forming his expectations concerning next year's rents, the housing producer calculates his expected profit for each quality level and invests to produce the quality level which he expects will be most profitable to him in the next period. Since it is assumed that a housing producer cannot sell his quality capital, maintenance will be set at zero if his depreciated capital stock will be above the optimal level in the next period.

Parameters for the Dynamic Model

In order to run the dynamic model, parameters must be specified for the demand side of the model. This specification is difficult because we have little empirical evidence about the parameters of demand for quality. Measures of dwelling unit quality are crude, inconsistent, or poorly defined in most data that describe dwelling unit characteristics. The demand parameters we have to choose are the α's, β's, γ's, and δ's in the household demand functions defined in Equation (8-21). One way to insure that the parameters are reasonable is to derive the choice elasticities implicit in Equation (8-21). The elasticity of the probability of choosing a particular quality level can be determined for changes in neighborhood, structure, quality, and rents, as follows:

$$N_N = N\alpha_I (1 - PR) \qquad (8\text{-}24)$$

$$N_S = S\beta_I (1 - PR) \qquad (8\text{-}25)$$

$$N_Q = Q\gamma_I (1 - PR) \qquad (8\text{-}26)$$

$$N_R = R\delta_I (1 - PR) \qquad (8\text{-}27)$$

where *PR* is the probability of choosing a particular combination of dwelling unit attributes; and N_X is the elasticity of the probability with respect to *X*.

The model has six income classes, and the parameters of demand can differ by income class. The parameters we used, shown in Table 8-4, are somewhat arbitrary but they produce reasonable distributions in the model. Table 8-4 also shows representative choice elasticities of the probabilities of each attribute, and the parameters were chosen with some consideration for what would be reasonable elasticities.

Table 8-4. Demand Parameters and Elasticities of Probabilities

Class No.	*Income Class*	α	β	γ	δ
		Parameter Values			
1.	Over $15,000	2.0	2.00	.002	–.00225
2.	$10,000-15,000	2.0	1.90	.002	–.00285
3.	$7,000-10,000	2.0	1.85	.002	–.00345
4.	$5,000-7,000	2.0	1.80	.002	–.00405
5.	$3,000-6,000	2.0	1.75	.002	–.00465
6.	$0-3,000	2.0	1.70	.002	–.00525
		Elasticity Range for *PR* = 0.2			
1.	Over $15,000	1.6-3.2	1.6-3.2	1-6	0.5-5.0
2.	$10,000-15,000	1.6-3.2	1.5-3.0	1-6	0.6-6.3
3.	$7,000-10,000	1.6-3.2	1.5-3.0	1.6	0.7-7.6
4.	$5,000-7,000	1.6-3.2	1.4-2.9	1-6	0.8-8.9
5.	$3,000-6,000	1.6-3.2	1.4-2.8	1-6	0.9-10.2
6.	$0-3,000	1.6-3.2	1.3-2.7	1-6	1.1-11.2

Note: In calculating elasticities, the range of the attributes used was *N*: 1 to 2 (1 = bad neighborhood; 2 = good neighborhood); *S*: 1 to 2 (1 = small multiple; 2 = high rise); *Q*: $500 to $3,000; and *R*: $250 to $2,750.

In addition to demand parameters, the distribution of households by income and of dwelling units by *N*, *S*, and *Q* must be determined. The distributors used in the model were derived from 1970 Census data for the Boston SMSA. In order to generate these distributions we assumed that the City of Boston represented the low-quality neighborhood; and the suburbs, the high-quality neighborhood. In addition, we classified buildings with three to nine units as the small, multifamily structure type, and buildings with ten or more units as high rises. The resultant percent distribution of renter families by income class is shown in the following tabulation; the number of households in each class in the model is shown in parentheses:

Over $15,000	$10,000-$15,000	$7,000-$10,000	$5,000-$7,000	$3,000-$5,000	$3,000 or Less
11.3% (22)	19.4% (39)	20.3% (41)	14.6% (29)	13.4% (27)	21.0% (42)

The percent distribution of dwelling units by neighborhood and structure type and the number of units in each category in the model are as follows:

Boston City		Boston Suburbs	
Small Multi	High Rise	Small Multi	High Rise
26.4% (53)	20.2% (40)	33.3% (67)	20.0% (40)

The Basic Case

When operated together the demand and supply portions of the housing market model produce an equilibrium outcome that we term the "basic case." In this equilibrium the rents expected by housing producers and households equal the short-run market rents, and all dwelling units have their long-run equilibrium capital stocks. In Table 8-5 some results from the equilibrium basic case are summarized.

The first part of Table 8-5 shows that high-income households are more likely to reside in the more desirable structure type ($S = 2$) and the more desirable neighborhood ($N = 2$) than are low-income households. The rents shown for neighborhood and structure type, $R(N, S)$, are essentially demand-determined quasi-rents for the fixed supplies of structure and neighborhood attributes. These quasi-rents obviously would play a key role in determining either the construction of new structures in each neighborhood or the conversions from one structure type to another.

The second part of Table 8-5 shows that the equilibrium distribution of dwelling quality varies across the neighborhood-structure type combinations, a result we would expect given that these combinations attract income classes differentially. The column labeled $r(Q)$ displays the rent per unit of dwelling quality in each neighborhood and structure combination. This per-unit rent is constant across all levels of dwelling quality because we have assumed dwelling quality is produced by a constant returns to scale

Table 8-5. Summary Results from the Equilibrium Solution of the Basic Case

Neighborhood and Structure Type by Income Class

N	S	R(N,S)	Income Class[a] 1	2	3	4	5	6	Total No. of Households
1	1	17.2	2	6	9	8	10	18	53
1	2	415.7	4	8	9	6	5	8	40
2	1	560.1	6	13	15	10	9	14	67
2	2	1038.1	10	12	8	5	3	2	40
	Total		22	39	41	29	27	42	200

Dwelling Unit Quality by Neighborhood and Structure Type

N	S	r(Q)	Quality Level[a] 1	2	3	4	5	6	7	8	Total No. of Households
1	1	.49	7	6	6	7	6	7	7	7	53
1	2	.58	5	4	5	4	5	5	6	6	40
2	1	.45	13	10	9	7	7	7	7	7	67
2	2	.54	8	6	5	5	4	4	4	4	40
	Total		33	26	25	23	22	23	24	24	200

Percent Distribution of Dwelling Unit Quality by Income Class for High Rise Dwelling in Bad Neighborhood (N = 1, S = 2)

Income Class	Quality Level[a] 1	2	3	4	5	6	7	8	Total
1	32%	19%	13%	10%	8%	7%	6%	5%	100%
2	21	16	14	12	10	10	9	8	100
3	14	13	13	12	12	12	12	12	100
4	7	9	11	12	14	15	16	16	100
5	4	6	9	11	14	17	19	20	100
6	2	4	7	10	14	18	21	24	100

[a]The income class equivalents are given in Table 8-4; the quality equivalents, in Table 8-3. $N = 1$ designates a good neighborhood; $N = 2$, a bad one. $S = 1$ designates a small, multiple-unit dwelling; $S = 2$, a high rise.

technology. The per-unit rent of dwelling quality differs by neighborhood and structure type because of the specification of the model: Recall that the parameters of the dwelling quality production functions differ by structure type and that the price of capital differs by neighborhood.

The third part of Table 8-5 shows the percent distribution across quality levels by income class for a particular neighborhood and structure type combination. Note how these distributions change from the highest income class (=1) to the lowest (=6) and that the distribution exhibits some evidence of diagonality.

Although the actual numbers shown in Table 8-5 are primarily illustrative of how the housing market model works when the demand and supply sides are integrated, they do remind us of the implications of the model specifications we have employed. Many other models that represent quality change or the production of housing services over time assume that incremental investment or maintenance applied to existing units is less efficient than new construction, has decreasing returns to scale, or has decreasing returns over time (de Leeuw; Sweeney). Since the supply model presented here assumes constant returns to scale for incremental investment in existing units, a decline in the quality of existing units over time has not (in effect) been specified on the supply side. In this model some changes on the supply side, e.g., an increase in the prices of factor inputs, can change the quality of dwelling units over time, but a more important cause of such changes is likely to be shifts in the demand for the structure and neighborhood attributes of dwelling units. Changes in the composition of demand within neighborhood and structure type submarkets will alter the equilibrium quality levels produced, and the latter can either increase or decrease depending on the demographic or real income changes that occur. If some heterogeneity is introduced into the representation of housing stocks, changes in the quality of particular dwelling units over time can result from demand changes as well as from possible decreasing returns on the supply side. If housing is represented as a homogeneous good, however, quality change over time typically must stem from the specification of diminishing returns to investment in existing units.

Sensitivity Analyses of the Model

In addition to examining the basic case, we made several runs in which parameters were changed and new equilibriums were attained. Elasticities with respect to the parameter changes were calculated to see how sensitive various equilibrium outputs of the model were to such changes. In Table 8-6 we summarize seven sensitivity runs of the model. The columns display the overall average of dwelling quality, the average rent per unit of dwelling quality, the average quasi-rent for neighborhood and structure type, and the average total expenditure on housing. In all sensitivity runs the distribution of households by income class and the distribution of dwelling units by neighbor-

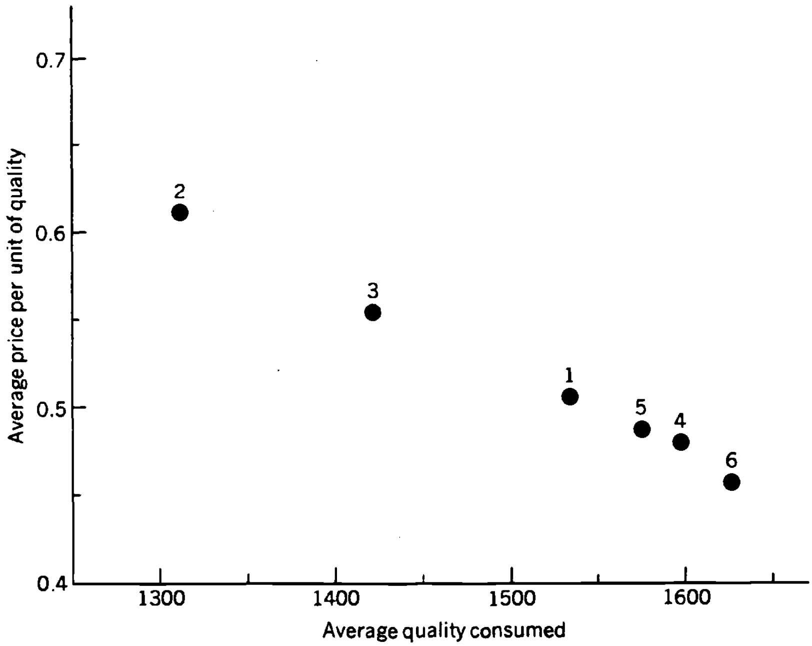
Table 8-6. Sensitivity Tests: Results Aggregated over All Households

Item	$\overline{Q}$	$\overline{r(Q)}$	$\overline{R(N,S)}$	*Average Total Expenditure*
1. Basic Case				
Level	1,533	0.504	463	1,235
2. Increase P_Q by 50% (1.0 to 1.5)				
Level	1,312	0.611	459	1,260
Elasticity	−0.29	0.42	−0.02	0.04
3. Increase Interest Rates 50% (7.0 to 10.5; 5.0 to 7.5)				
Level	1,423	0.554	471	1,259
Elasticity	−0.14	0.19	0.03	0.04
4. Reduce Interest Rate in $N = 2$ by 50% (5.0 to 2.5)				
Level	1,596	0.480	496	1,262
Elasticity	−0.08	0.09	−0.14	−0.04
5. Raise Elasticity of Substitution in $S = 2$ by 10% (0.6 to 0.66)				
Level	1,575	0.486	484	1,250
Elasticity	0.27	−0.36	0.45	0.12
6. Raise α for $S = 2$ by 10% (0.812 to 0.893)				
Level	1,626	0.453	531	1,268
Elasticity	0.60	−1.0	1.5	0.26
7. Demand Coefficient of Q Reduced 25% (0.002 to 0.0015)				
Level	1,244	0.504	470	1,096
Elasticity	0.75	0	−0.06	0.45
8. Demand Coefficient of Rent Reduced 10%				
Level	1,549	0.504	509	1,287
Elasticity	0.07	0	0.99	0.43

Note: Elasticities are calculated with respect to parametric changes.

hood and structure type remain fixed. Runs 2 through 6 alter parameters of the supply side: runs 2 and 3 change the costs of producing quality in all units, and runs 4, 5, and 6 alter supply costs in only one neighborhood or one structure type. Runs 7 and 8 alter demand coefficients for all households.

A comparison of runs 2 through 6 with the basic case shows that average total expenditure is fairly constant, increasing by a maximum of 2 percent. In these runs households respond to changes in $r(Q)$ by varying the amount of quality they purchase. This is shown in Figure 8-4, which displays what is esentially a demand curve for dwelling quality. Of course, this relation is complicated somewhat by changes in $R(N, S)$, the quasi-rents for N and S. Runs 7 and 8, in which demand coefficients are altered, produce somewhat larger changes in total expenditures and $R(N, S)$. In these runs the supply side is



Note: Numbered points refer to runs in Table 8-6.

Figure 8-4. Quality versus Rent for Quality from Sensitivity Tests of Supply Parameters.

unaltered; and $r(Q)$ keeps its basic case value, but the altered demand parameters produce changes in the average level of quality consumed.

Table 8-7 displays average values and elasticities for model outputs by structure type and neighborhood. Supply changes limited to one structure type or neighborhood (runs 4, 5, and 6) tend to produce their largest effects on units with the attributes affected by the altered parameters. Note, for example, that in run 4 the first neighborhod results are very similar to those of the basic case, and in run 5 the structure 1 results are little changed from the basic case. In these two runs supply changes are made in neighborhood 2 and structure 2, respectively.

The sensitivity tests show that model results are, indeed, responsive to parameter values. The importance of the demand parameters, while not surprising, compounds the problems on the demand side of the model, which is especially difficult to estimate.

Table 8-7. Sensitivity Tests: Results by Neighborhood and Structure Type

	Good Neighborhood						Bad Neighborhood					
	Small Multi			High Rise			Small Multi			High Rise		
Item	*Q*	*r(Q)*	*R(N,S)*	*Q*	*r(Q)*	*R(N,S)*	*Q*	*r(Q)*	*R(N,S)*	*Q*	*r(Q)*	*R(N,S)*
1. Basic Case												
Level	1,429	0.487	17.2	1,381	0.576	415.7	1,635	0.454	500.1	1,654	0.538	1,038.1
2. Increase P_O by 50%												
Level	1,201	0.587	17.7	1,173	0.699	408.4	1,416	0.551	498.0	1,424	0.657	1,026.3
Elasticity	−0.32	0.41	0.6	−0.30	0.42	−0.04	−0.26	0.42	−0.01	−0.28	0.18	−0.02
3. Increase Interest Rates 50% (7.0 to 10.5; 5.0 to 7.5)												
Level	1,298	0.542	16.4	1,268	0.638	415.5	1,539	0.495	515.3	1,552	0.585	1,054.5
Elasticity	−0.18	0.22	−0.09	−0.16	0.21	0	−0.12	0.18	0.06	−0.12	0.18	0.03
4. Reduce Interest Rate in $N = 2$ by 50% (5.0 to 2.5)												
Level	1,429	0.487	14.2	1,381	0.576	409.6	1,754	0.412	563.0	1,768		
Elasticity	0	0	0.3	0	0	0.03	−0.15	0.18	−0.25	−0.13	0.18	0.61
5. Raise Elasticity of Substitution in $S = 2$ by 10% (0.6 to 6.6)												
Level	1,429	0.487	15.1	1,477	0.532	469.9	1,635	0.454	496.0	1,764	0.494	1,101.5
Elasticity	0	0	−1.22	0.70	−0.76	1.30	0	0	−0.08	0.67	−0.81	0.61
6. Raise α for $S = 2$ by 10% (0.745 to 0.820)												
Level	1,413	0.487	11.8	1,721	0.454	578.3	1,629	0.454	490.4	1,806	0.409	1,240.2
Elasticity	−0.11	0	−3.1	2.5	0	−8.6	−0.04	0	−0.19	0.92	−2.4	1.95
7. Demand Coefficient of Q Reduced 25% (0.002 to 0.0015)												
Level	1,156	0.487	16.4	1,121	0.576	441.7	1,334	0.454	494.1	1,335	0.538	1,056.5
Elasticity	0.76	0	0.19	0.75	0	−0.25	0.74	0	0.05	0.77	0	−0.08
8. Demand Coefficient of Rent Reduced 10%												
Level	1,372	0.487	13.1	1,402	0.576	458.4	1,622	0.459	548.0	1,781	0.538	1,199.9
Elasticity	−0.40	0	−2.3	0.15	0	1.03	−0.08	0	0.95	0.77	0	1.08

Experimenting with Housing Allowances

The final exercises performed with the model incorporate a representation of a housing allowance program, which is simulated by altering the income distribution of households in the model. The top portion of Table 8-8 displays changes made in the income distribution of the model that roughly correspond to the implementation of a housing allowance program. Two runs were made with the altered income distribution. In the first run no other changes were made, and in particular the distribution of dwelling units by neighborhood and structure type was unchanged. In the second run this distribution was changed by adding two units in the neighborhood 2, structure type 2 category and subtracting two units in the neighborhood 1, structure 1 category; this change is intended to represent new construction that might occur in response to increase in quasi-rents for neighborhood and structural type in the first run.

The lower section of Table 8-8 summarizes results from the two housing allowance runs and presents comparable figures from the basic case run. When the supply is held fixed the change in income distribution produces higher quasi-rents for N and S, an increase in

Table 8-8. Housing Allowance Simulations[a]

Income Distribution Changes for Housing Allowance (number of households)

Run	Income Class 1	2	3	4	5	6
Basic Case	22	39	41	29	27	42
Allowance	22	39	41	37	42	19
Change	0	0	0	+8	+15	−23

Summary of Results from Allowance Simulations

N		S	Basic Case $\bar{Q}$	Basic Case R(N,S)	Basic Case Av. Tot. Expend.	Fixed Supply $\bar{Q}$	Fixed Supply R(N,S)	Fixed Supply Av. Tot. Expend.	Altered Supply $\bar{Q}$	Altered Supply R(N,S)	Altered Supply Av. Tot. Expend.
1		1	1,429	17.2	713	1,458	15.8	726	1,456	16.6	726
1	2	2	1,381	416	1,211	1,413	430	1,244	1,413	420	1,234
2	1	1	1,635	500	1,242	1,672	514	1,273	1,672	505	1,264
2	2	2	1,654	1,038	1,928	1,654	1,060	1,950	1,620	1,029	1,901
	Weighted average		1,533	463	1,235	1,560	474	1,258	1,553	463	1,243

[a]The income class limits are given in Table 8-3. $N = 1$ designates a good neighborhood; $N = 2$, a bad one. $S = 1$ designates a small, multiple-unit dwelling; $S = 2$, a high rise.

average quality levels consumed, and a slight rise in total housing expenditures. It appears that the higher incomes are used to bid up rents for the fixed attributes (*N* and *S*) and to raise quality in three of the four *N* and *S* categories. The slight alteration in supply in the second run returns the quasi-rents nearly to their levels in the basic case, raises the average quality level consumed, and increases total housing expenditures relative to the basic case.

A more detailed description of the two housing allowance runs and a comparison with the basic case are presented in Table 8-9, which shows the distribution over dwelling quality levels for each neighborhood and structure type. In the run with the stock of units

Table 8-9. Housing Allowance Runs and Quality Levels[a] (number of households)

		Quality Level								
N	*S*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Total*
1	1	7	6	6	7	6	7	7	7	53
1	2	5	4	5	4	5	5	6	6	40
2	1	13	10	9	7	7	7	7	7	67
2	2	8	6	5	5	4	4	4	4	40
	Total	33	26	25	23	22	23	24	24	200
					Allowance with Fixed Supply					
1	1	7	7	6	7	6	6	7	7	53
1	2	5	5	4	5	5	5	5	6	40
2	1	14	10	9	7	7	7	7	6	67
2	2	8	6	5	5	4	4	4	4	40
	Total	34	28	24	24	22	22	23	23	200
	Change from basic case	+1	+2	−1	+1	0	−1	−1	−1	
					Allowance with Altered Supply					
1	1	8	7	6	6	5	6	6	7	51
1	2	5	5	4	5	5	5	5	6	40
2	1	14	10	9	7	7	7	7	6	67
2	2	7	6	5	4	5	5	5	5	42
	Total	34	28	24	22	22	23	23	24	200
	Change from basic case	+1	+2	−1	−1	0	0	−1	0	

[a]Quality levels are described in Table 8-3; *N* and *S* in note to Table 8-4.

fixed by N and S, the quality levels consumed increase relative to the basic case, but in a complicated manner as households shift among quality levels. When the stock of units is altered in the second run, the average level of quality consumed still exceeds that in the basic case, but the same number of households consume the lowest quality level, as in the basic case. In addition, the number of units of the lowest quality level has increased in the most desirable neighborhood and structure type.

Although these runs are merely illustrative of the changes that may occur under a housing allowance regime, they do suggest that households may use increased dollars to buy structure or neighborhood attributes they desire rather than dwelling unit quality. In fact, housing allowances might in some cases encourage households to trade off dwelling unit quality for these other attributes. It is obvious, however, that we must have more data to support the demand equation parameters before we can project impacts of housing programs with any confidence.

CONCLUSION

In this study we presented a theoretical representation of housing supply that emphasizes the operation of the existing stock of dwelling units. Three major characteristics of dwelling units were distinguished: structure quality, structure type, and neighborhood quality. The production of dwelling unit quality was treated in terms that parallel the choice of optimal plant size in the theory of the firm; decisions about structure type, although treated less completely, were made in accordance with the usual assumptions of investment theory.

A production function having constant elasticity of substitution was specified for the production of structure quality, and parameters for it were estimated from available data on the operation of dwelling units. The quantitative version of the model distinguished two structure types and two levels of neighborhood quality. Numerical experiments were performed with the model to determine likely ranges of values for short-run increases in rents, as well as their duration, that might accompany any housing market policy that increases demands for structure quality. Although increases in rents could be quite large in the very short run, moderate increases in the demand for structure quality would probably increase gross rents by from 4 to 7 percent in the short run, and these rent increases would likely dissipate in less than six years.

The supply representation was combined in a market-clearing framework with demand equations for housing attributes. Several sensitivity analyses were performed with this market model, and some preliminary housing allowance experiments were reported on. These experiments suggested that housing allowances increase the average quality of housing consumed, but may lead some households to substitute neighborhood quality or structure attributes for dwelling quality.

We stressed the role of operating inputs and maintenance in short-run adjustments of existing units. Expenditures on these factors constitute an important component of the costs of producing housing, and they typically receive little attention in analyses of housing markets.

Finally, we gave much attention here to the short-run dynamics of housing market adjustment processes, not only because of their importance in gauging the response of the housing market to specific policy proposals, now under consideration, but also because more complete representation of these adjustment mechanisms will increase our ability to judge the appropriateness of equilibrium assumptions in the housing market.

APPENDIX 8A: ESTIMATES OF THE ELASTICITY OF SUBSTITUTION BETWEEN OPERATING INPUTS AND CAPITAL

The estimation of the elasticity of substitution between operating inputs and capital in the production of structure quality depends on several assumptions, which we describe here. We begin by deriving the first-order conditions of the CES production function in Equation (8-12) as

$$\frac{dQ}{dO} = \frac{P_o}{P_Q} = -\frac{1}{\beta}[\alpha K^{-\beta} + (1-\alpha)O^{-\beta}]^{-1/(\beta-1)}(1-\alpha)(-\beta)^{-\beta-1} \tag{8A-1}$$

which with substitution leads to

$$\frac{dQ}{dO} = \frac{P_o}{P_Q} = (1-\alpha)(Q/O)^{\beta+1} \tag{8A-2}$$

By rearranging terms, we obtain the equation to be estimated:

$$\frac{Q}{O} = (1-\alpha)^{-1/(\beta+1)}(P_o/P_Q)^{1/(\beta+1)} \tag{8A-3}$$

where P_o and P_Q are the prices (or price indexes) of operating inputs and structure quality, respectively. Estimating Equation (8A-3) is impossible because we do not observe Q directly. However, we do have data on rent payments and expenditures on operating inputs, and we can obtain price indexes for rents and operating inputs, O, over time. We then estimate the equation

$$\frac{R/P_R}{\text{Op.Exp.}/P_o} = \frac{S}{O} = c(P_o/P_R)^b \qquad \text{(8A-4)}$$

and obtain b, which is an estimate of the elasticity of substitution if S is a good proxy for Q. S would be a good proxy if, for example, the ratio Q/S were constant. Such constancy is unlikely throughout the entire period covered by our data, perhaps most notably because rent control was in effect until March 1956. The major control for variations in Q/S used in the estimation is a dummy variable that distinguishes years with rent control from years without, a technique that assumes Q/S was constant within the two periods.

The price index for rents used in the estimation is the Boston rent index drawn from Bureau of Labor Statistics, *Consumer Price Indexes in Selected SMSA's*, and it is available from 1944 to the present. Two price indexes reported in the same publication were used for operating inputs: the Boston consumer price index, available over the whole time span of the sample; and a price index for utilities and fuel, which can be readily constructed from 1953 on. Separate estimates of the elasticity of substitution were made using these two indexes for the price of operating inputs, and a graphical display of the data is shown in Figures 8A-1 and 8A-2. It is clear from Figure 8A-1, in which the CPI is used as the price index of operating inputs, that the sample seems to be broken into a rent control and post-rent-control period, except that both 1956 and 1957 appear to be part of the former period. Both 1956 and 1957 appear to be in the rent control period in Figure 8A-2 as well, where the index of utilities and fuels is used as the price of operating inputs. Because of the difficulty of defining the post-rent-control period with a dummy variable, two definitions were used: $D1$, which covers 1958-1969; and $D2$, for 1956-1969. Regression estimates of the elasticity of substitution using these dummy variables are shown in the following tabulation (figures in parentheses are t statistics):

Constant	Elasticity of Substitution	D1 (1958-1969)	D2 (1956-1969)	R^2
	Based on Consumer Price Index (sample = 1944-1969)			
−1.14	0.65 (7.59)	0.11 (8.04)		.77
−1.13	0.49 (3.83)		0.08 (3.63)	.43
	Based on Utility and Fuel Index (sample = 1953-1969)			
−1.13	0.63 (6.65)	0.08 (3.9)		.80
−1.09	0.32 (3.15)		−0.01 (0.03)	.57

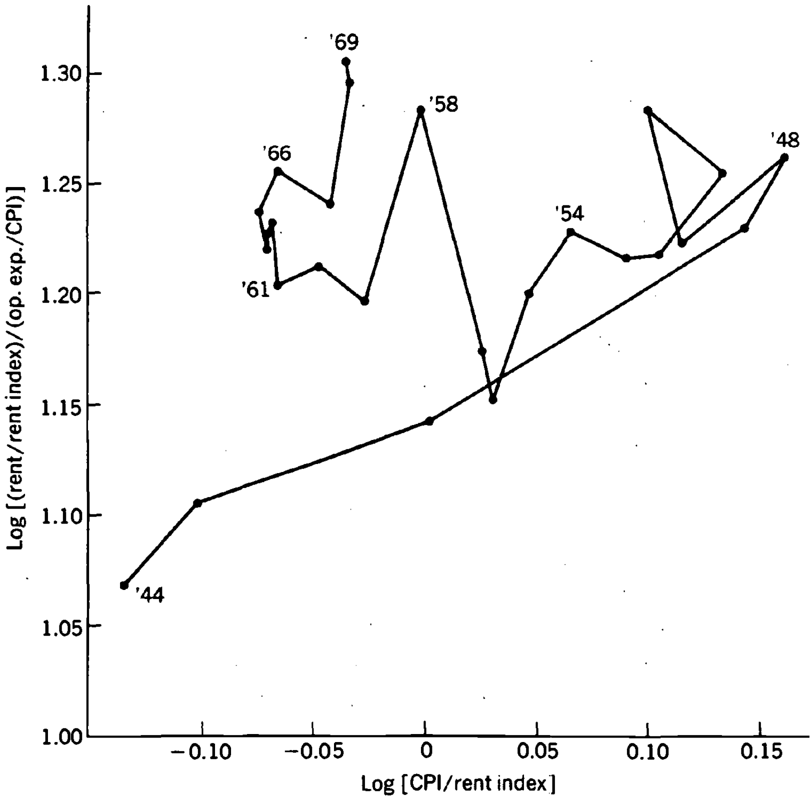


Figure 8A-1. Elasticity of Substitution Using Consumer Price Index (CPI).

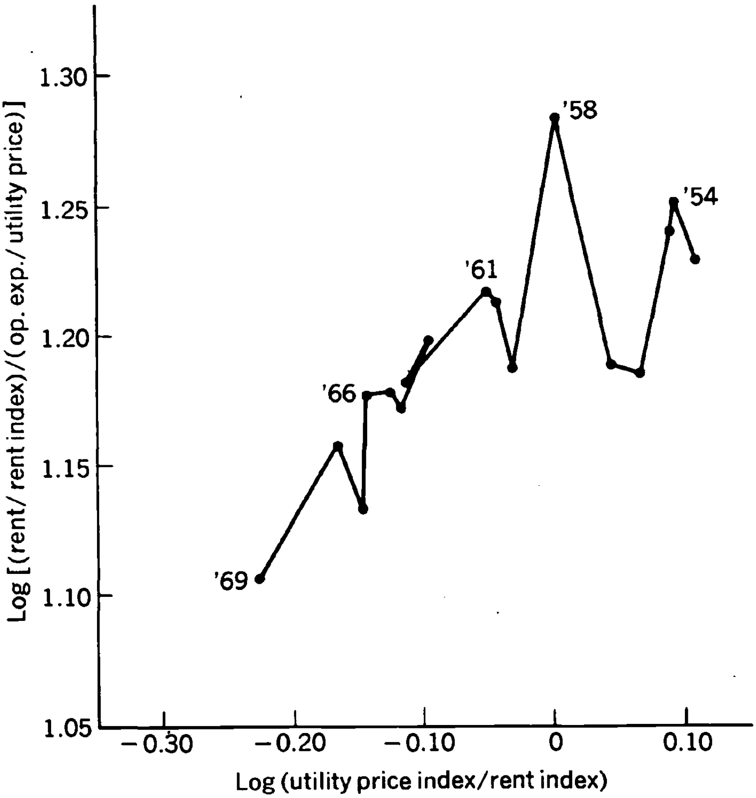


Figure 8A-2. Elasticity of Substitution Using Utility and Fuel Index.

NOTES TO CHAPTER EIGHT

1. Of course, this problem is not limited to housing capital; see Feldstein and Rothschild (1974).

2. Note that maintenance is defined as gross investment in quality capital and excludes operating inputs.

3. An exposition of this approach is given in Dildine and Massey (1974).

4. Alternatively, r_x might increase with the amount borrowed.

5. Operating expenses do not include property taxes. Details of the estimation are presented in Appendix 8A.

6. Since only quality capital depreciates, if the average depreciation rate for housing is 2 percent, we are assuming that quality capital amounts to one-fifth of the value of a unit.

7. Household expected rent for dwelling unit quality this period is $R(Q)$ of the previous period.

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Comments on Chapter Eight

Jerome Rothenberg

This study by Ingram and Oron is ambitious, imaginative, and quite skillfully carried out. Its subject has up to now been only very slightly examined. It is highly suggestive, indicating useful directions for further work.

PRODUCTION INPUTS

The authors concentrate on the supply behavior of owners of existing rental housing. Their chief emphasis is on a distinction among three types of housing structure inputs with notably different longevities: structural capital, quality capital, and operating inputs. The three have different depreciation rates: essentially zero for structural capital, which is assumed to have an indefinite lifetime; one-period durability for operating inputs; and in-between for quality capital. Moreover, structural capital has a discontinuous cost function; and quality capital and operating inputs are deemed to have smooth substitution possibilities with one another.

What is gained from this trichotomy? First, a clear basis for generating the distinction between short- and long-run supply adjustments. Second, an interesting discussion of the transition adjustment process. There is an illuminating simulation application to a demand-supply interaction, with both policy runs and sensitivity analysis that carry a reasonably realistic ring. Thus, the distinctions are instructive; they highlight some important issues in supply—mostly relating to individual production and cost functions, and individual supplier behavior.

But the treatment does at the same time slight other important issues in housing supply: for example, questions about market structure, the aggregate supply behavior of different groups, competing types of supply, and the impact of different forms of competition—across tenure classes, structure types, neighborhoods, different quality levels, and between new construction and conversion. It excludes custom suppliers, i.e., those who are simply direct agents of users. Moreover, the emphasis on rental housing is too insulated. There is no note taken that market opportunities in the rental sector—the relation between rents and quality—depend on what is happening to demand *and supply* for ownership units: households can trade off across tenure classes.

PRODUCTION FUNCTION

Assuming the structural capital of existing units to be fixed, the authors treat the production of structural *quality* as a process involving quality capital and operating inputs in a CES production function. This entails continuous tradeoffs with constant elasticity of substitution and constant returns to scale over an indefinite range of structural quality. While there is some ambiguity as to what is included in the category of quality capital, this treatment reflects what may be a serious misrepresentation of the technical opportunities of changing the quality of existing structures. I shall begin by briefly characterizing what I take to be some salient features of such opportunities, and then relate these to the present treatment.

A given housing structure constrains future changes in quality in a variety of ways: (1) through size, general layout, materials, plumbing; (2) through more specific architectural features, such as number of rooms; (3) through particular installations, decorations, appliances; (4) through a given state of condition (repairs, etc.); (5) through a set of current services associated with occupancy, such as heating, garbage removal, cleanliness, etc. These groups are ordered in terms of the degree of constraint each exercises on future changes in quality. Groups 1 and 2 especially exert important technical, as well as aesthetic, complementarities that are absent, or must be violated, under significant conversion. Thus, in a housing supply model I have been working on for some time, if levels of overall quality are specified, for any given quality level new construction permits achievement of that level at a lower total cost than can be attained by converting an existing unit to the given level from some different starting level. The discrepancy between the two in total costs is greater the greater the required change through conversion from starting to target quality level.

These fundamental features may depreciate in terms of rental earning ability, indeed may even be totally written off in an accounting sense, and yet may continue to exert technical constraints on conversion because they *physically* remain. Not economic relevance but physical relevance is the basis of this influence. Explicit demolition costs are needed fully to undo the constraints, and demolition services should represent a genuine component of the production function.

Groups 3, 4, and 5 exert decreasing constraints on "coversions" in a broad sense of deliberate quality changes in existing units, with 5 alterable almost without constraint. But while these groups of characteristics can be more easily varied, they are also less important components of the housing package from the point of view of the user; and considerable alteration in these components increasingly strains basic complementarities with the more fundamental aspects of the housing package. Thus, increasing the frequency of redecorating does less and less to enhance living in a tiny apartment with abysmal plumbing. Even various combinations of changes in these types of components should have decreasing enhancement effect on user quality as the latter diverges increasingly from conventional balance with the more long-lived features of the housing unit.

Thus, conversion seems to involve a technology of decreasing returns to scale, whatever combination of inputs is involved (and whether or not new construction involves constant returns to scale). The more basic components affect user welfare strongly but are increasingly expensive in real resource costs. Demolition is required to liberate some aspects of the technical opportunities, but demolition requires a heavy use of resources, and this must be included as a genuine cost of the process. The less structurally intrinsic components are easier to vary in different combinations, but they are likely to have progressively less effect on user welfare when attached to unchanged basic features of the housing unit, and thus increasingly large package additions must be made to obtain equal impact on quality—a possibly more pronounced situation of decreasing returns to scale than for more basic conversions.

It is not clear what groups of components are included in "quality capital" as employed in the paper. The selection of a 15 percent depreciation rate in the original version (now reduced to 10 percent after serious questioning) plus oral discussion at the conference suggest that categories 4 and 5 and some shorter-lived items of 3 are intended. This seriously restricts the scope of the analysis, since it omits what many researchers in the field would consider the most important aspects of conversion. Property owners are not likely to attempt more than modest changes in quality by resort to these components alone. Larger quality changes are very likely to involve the more basic components as well. Yet, for the reasons given above, the criticism against a technology of constant returns to scale holds even for this reduced menu of conversions.

The above emphasis on technical constraints and complementarities has a broader significance than merely to suggest that constant returns are inappropriate. It implies that long-run outcomes are *much more* influenced by past and present supply decisions than the authors allow. (In the model the constant low cost of production in the long run wipes out all influence of the past after an extremely modest transition period.)

Given a present stock of housing with specific character, as demand changes to create new market opportunities, the newly attractive quality levels can be obtained at lower total cost through new construction rather than by converting existing units. (Such conversions have rising average cost functions.) But some of these latter might nonetheless outcompete the new units because the decision to convert is based on conversion plus opportunity costs, not total costs, and this sum is often much lower than total costs. (Net revenue at the original quality level, which would be foregone by conversion, comprises the opportunity cost.) Technical constraints on the *particular* units presently existing with help determine where such conversion can successfully compete with new construction and thus help determine the character of the overall response, even though new-construction technology in principle dominates conversion in the long run.

The long run, indeed, in this view, does not totally differ in character from a succession of short-run transitions, unlike in the Ingram-Oron model. Existing units do not disappear in any long run: they are maintained or converted, and so they continue to exist *in every period*, imposing their influence on the overall supply posture. Short-run sequences *are* the long run. Throughout this process, the character of existing stocks in each period is both a reflection of past supply decisions and an important influence on future ones.

Temporal sequences resulting from policy and other changes impinging on the system are an important part of the study. Its sharp cleavage between short-run transitions and long-run equilibriums, one of the presumed fruits of its distinctions among different types of housing input, is in fact based on an overly simple conceptualization. The difficulty stems from the authors' relative neglect of the market aggregation level—with that variety of competitive relationship whose absence we noted early in these remarks.

One final remark should be made about the production function.

A distinction should be made between the technology involved in raising quality and the technology involved in lowering it. This is especially important if quality capital is meant to include elements of the basic structure of the housing unit, with the result that, e.g., changes in the "scale" of the unit are envisaged. Raising quality is typically accomplished by positive investment of resources: a form of construction. Lowering quality can, however, be accomplished by either a comparable form of investment, i.e., by using resources to provide less space in more units, or by allowing the condition or operating services to decline, i.e., by refraining from spending resources. These are likely to involve different cost characteristics and require different gestation periods—an asymmetry between upward and some downward conversions.

ESTIMATION OF THE ELASTICITY OF SUBSTITUTION AND THE DISTRIBUTION FACTOR

The use of annual average rent per room as a measure of structural quality is unsatisfactory. It confounds structural quality with the balance of market forces. Neighborhoods and other aspects of housing units differ and influence average rent. These are not controlled for in the authors' procedure.

The original use of the consumer price index to represent the price of operating inputs was questionable because the latter comprise specific expenditure items and types of labor services that do not move dependently with the index over the business cycle. In the present version of the study, the CPI is supplemented by the fuel and utilities index, and this is an improvement. The problem of proper representation is still not solved, but the present procedure probably gives a more tolerable approximation.

"Guesstimates" for the relative elasticity of substitution for high-rise and low-rise structures may be inappropriate. The authors give 0.5 for low-rise and 0.6 for high-rise structures because of their belief "that a large 'plant' offers more possibilities for substitution than a small 'plant'." In fact, the reverse may be true. A high-rise structure is more complex than a low rise, with many tightly complementary linkages among structural components. Architectural constraints are probably more formidable for high-rise than for low-rise structures, permitting less substitution among components *once the structure is already built* (the authors' belief probably holds for the planning stage). The guesstimates used in the study may err both in absolute and comparative terms. This is important because in the simulations performed much depends on the elasticity of substitution.

In the original version of the study a depreciation rate of 15 percent per year was chosen for quality capital. That seemed much too high if quality capital covered significant aspects of the housing unit. The criterion for depreciation in this use is not marketability (or risk) of the unit but the exercise of constraints in conversions. The authors have now lowered it to 10 percent per year. This is certainly in the right direction. Moreover, if quality capital *is* restricted mostly to appliances and other nonintrinsic installations, as the conference discussion suggested, 10 percent may be appropriate. But, as I suggested above, this would considerably limit the scope and interest of the study. A more inclusive category, to make possible significant conversion possibilities, would call for a still lower depreciation rate.

PRICE EXPECTATIONS AND AGGREGATE GESTATION PERIODS

The analytical model proceeds on the assumption that each supplier (owner) has perfect knowledge of the relation between rent levels and quality levels over at least a planning interval of four or five years.

This appears to neglect some important aspects of rents as market phenomena. For example, housing supply typically has a long gestation period. A producer, whether of new or converted units, will often not know what other similar supply activities are "in the works" at any time. This can affect price—notably through triggering uninformed supply behavior. Secondly, it seems to omit the market effects of types of endogenous supply behavior competitive with the type of conversion focused on, namely, new construction and structural conversion. Changes in these latter supply modes should affect price; hence, when known or anticipated by quality converters, they should influence the behavior of the latter. If supply responses through other modes had short gestation periods, each converter could easily perceive the market price effects, and the assumed correct predictions of these would be apt. But gestation periods for the other modes are also lengthy—some lengthier than for this mode. So the same uninformedness holds here too.

Another issue, in addition to the prediction problem created by long gestation periods, is the aggregate competitiveness across supply modes. The proposed aggregate response of the supply mode in question will influence the aggregate response of the supply modes competitive with it and thus change market opportunities. Unless these two-way intermodal relationships are comprehended in the model in an explicitly dynamic setting, it is difficult to interpret the meaning of the assumption of accurate individual rent-quality predictions over the planning period.

FUNDING AND INTEREST RATES

The model assumes an unlimited amount of funds can be borrowed at a given fixed interest rate. This neglects the possibility of a (smoothly) rising cost of capital as a function of the level of new construction and conversions and thus partially endogenous. Alternatively, it neglects the possibility of credit rationing.

OVERALL EVALUATION

This study represents a useful inquiry into some of the theoretical and empirical issues involved in modeling housing supply. Many important issues have been excluded—as must be the case in all finite research. Unfortunately, however, some of these exclusions bias the results concerning what is *included*. The treatment of some of the empirical issues is defective in a way that shows how inherently difficult the area is to work in.

Comments on Chapter Eight

Marion Steele

This rigorous study by Ingram and Oron is of major importance because of the way it treats heterogeneity in housing. Its treatment is unique among housing models because, on the supply side, the production functions for different characteristics of housing are allowed to differ, and on the demand side, different characteristics of the housing bundle are entered as separate arguments in the utility function. This treatment, and only this treatment, makes it possible to focus on a question of crucial concern for public policy: What bundle of housing characteristics would a household consume after receiving a housing allowance?

To emphasize the importance of this question, let us pose the following: Would voters support a housing allowance if recipients used it merely to purchase better-located housing? If in fact voters would support an allowance only if it were used to increase the quality and size of dwellings, the simulation results suggest great difficulty for any allowance program. For these results show the interaction of demand and supply resulting in a tiny decrease, or no decrease at all, in the amount of the lowest-quality housing. Is this arresting result to be taken seriously? Quite seriously, I believe, but there are some problems in the analysis.

One problem is the incomplete specification of the characteristics of the housing stock. Ingram and Oron distinguish four: structure type, structure quality, neighborhood quality, and accessibility. To be complete, the specification should include another characteristic: dwelling unit size. As the concepts are currently defined, neither structure type nor structure quality encompasses this characteristic.

On the one hand, structure capital refers just to the foundation and shell. There is nothing explicitly disallowing the possibility of a given shell containing, say, three large units instead of four smaller ones; and in the simulation experiments the authors use two structure types which differ in number of dwelling units but not in dwelling unit size. On the other hand, structure quality does not encompass dwelling size. We infer this from the high depreciation rate (10 percent) assumed for quality capital, and from the characterization of increments in quality capital as maintenance. Furthermore, in their empirical estimation, the authors take rent per room to represent quality; this allows quality to reflect the size of rooms, but not their number. In my view, dwelling unit size should be explicitly included as a separate characteristic, rather than merged into either the structure type or structure quality concepts. There is substantial evidence that the demand parameters for these three aspects of housing differ greatly.[1]

A further problem in the authors' treatment of heterogeneity is their lack of complete carryover from the analytical section to the simulations. While structure quality is endogenously determined in the simulations, structure type is not. In addition, in the simulations, as in the analysis, neighborhood quality is exogenous, and of course, dwelling unit size is omitted. Thus, in the simulations we have two polar extremes of supply elasticity: the supply of structure type—and the neighborhood—is perfectly inelastic, while, because of the nature of the assumed production function, the supply of structure quality is perfectly elastic. Ceteris paribus, then, the increase in structure quality resulting from a housing allowance is biased upward, and the change in structure type (and dwelling unit size) is biased downward. In view of this, the actual simulation result—an absence of any change in the number of units of the worst quality level in the less desirable neighborhood (Table 8-9, middle section, rows one and two)—is indeed remarkable.

We suspect that if two kinds of changes were made, the results would be much more favorable to a housing allowance. First, the authors' demand equations assume that (1) the strength of preference for neighborhood quality relative to structure quality is precisely the same for low-income as for high-income households (Table 8-4) and (2) the elasticity of the probability of a household's choosing a particular quality level *declines* as the quality level declines (Equation (8-27)). The Bailey (1966) hypothesis of segregation suggests that assumption (1) is inappropriate and should be changed to make low-income households prefer neighborhood quality relatively less strongly. This, plus a change in assumption (2) to a constant elasticity, would substantially increase the structure quality demand of households in the allowance-receiving groups. And as the authors comment, simulation results for the quantity of quality are certainly sensitive to changes in the demand parameters (Table 8-4 and 8-7).

The second kind of change I suspect would make a substantial difference to the results would be the introduction, as suggested above, of dwelling size as a separate characteristic. Some of the housing allowance used to purchase neighborhood quality would be diverted to increase the number of rooms. Since I suspect that the number of rooms, especially in low-rise structures, is relatively elastic in supply, the simulation result might be a substantial reduction in the number of units with few rooms. Voters would be inclined to accept an allowance program, I believe, if it resulted in more privacy (less crowding) as well as greater structure quality.

These comments point up the importance of the specification of an "additions" function; this would be required where dwelling unit size is taken to refer to square feet of space rather than number of rooms. This problem is, of course, closely associated with the knotty problem of specifying a function for cost of conversion to a different structure type. No such function is given, but perhaps the problem would become more tractable if space and number of rooms were explicitly part of the model.

Finally, I would like to see the supply side of the model modified to accommodate the effects of income and property taxes. These do not properly belong under the heading of "fixed costs" (Equation (8-3)). Often, appraisal practice makes property tax a function of total capital (i.e., here, K_t), so taxes rise with increments in quality capital. This increases the price of quality capital relative to the price of operating inputs, and also increases the price of quality.

NOTE TO COMMENT ON CHAPTER EIGHT

1. See David (1962) and note that to a large extent the owner-tenant split is a proxy for a structure-type split. Also see Straszheim (1975, tables 4.5-4.7) but note that the latter assumes that elasticities do not change with income.

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