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6 Macroeconomic policy design in an interdependent world

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I Introduction

Although most of the literature on macroeconomic policy design has focused on policy questions in the single open economy, there is an important strand that is concerned with the issues raised by interdependence between economies. (See, for example, Hamada and Sakurai (1978), Hamada (1979), Canzoneri and Gray (1983), Cooper (1983), Corden (1983), Miller and Salmon (1983, 1984), Sachs (1983), and Turner (1983, 1984).) This literature emphasises the game-theoretic, strategic aspect of policy-making in the international arena, and the prospects that non-cooperative forms of policy, arising from the elements of externality in the effects of policy internationally, may lead to outcomes markedly inferior to those of cooperative policies.

As Corden (1983) notes, analytical tractability has limited this analysis to static models, or to dynamic models with rather rudimentary dynamics, or to neglect of the longer run dynamics by focusing only on short run outcomes. In view of the complexities of the interactions between countries, whether through prices, real demands, asset prices or the flow of funds, this neglect of dynamics is a clear limitation. In this paper, we seek to overcome it by examining policy interactions and interdependence in a stochastic rational expectations model with developed dynamics within a framework that considers both the short and longer run effects of policy.

In the arena of policy debate, there has been evident the objective of formulating policy in terms of simple rules. Thus we may cite the advocacy of monetary or nominal income targeting as a major plank of policy, or the more recent advocacy of particular forms of decoupled control both for the single open economy (see Vines *et al.* (1983)) and for coping with problems of interdependence (see Meade (1983)). The arguments for simplicity include the perceived advantages in providing an anchor for nominal variables, the gains in understanding derived from stating policy in simple terms, as well as propositions about the relative information advantage in certain variables. (For further discussion, see Currie and Levine (1984).) In the international sphere, such simple rules for the conduct of policy may act to constrain the options for noncooperative behaviour whilst remaining within the agreed rules of the game. Underlying this argument is the notion that it may be easier to secure international agreement on the qualitative aspects of the ground rules for the conduct of policy rather than the quantitative aspects, except within broad ranges. Simple rules, by constraining the range of policy options, may serve to constrain the degree of noncooperative behaviour.

Section II establishes a control theory framework to assist in addressing these questions. Our analysis is conducted throughout in a stochastic context. We also assume rational or consistent expectations, so that the private sector makes no systematic errors in prediction. This provides a useful test-bed for policy regimes, since a policy which performs badly under consistent expectations could perform well only by virtue of systematic forecasting error by the private sector, and this provides an ill-founded and inherently unstable basis for policy. We therefore set out a procedure for deriving optimal linear time-invariant feedback rules in linear stochastic rational expectations models, with the form of the feedback rule constrained as appropriate. This permits, for example, the design of optimal indicator regimes or optimal decoupled systems. We also derive the full optimal rule, noting the rather complex form that this rule takes. To permit analysis of noncooperative game behaviour, we show, for simple rules, how the Nash equilibrium in a two-country game can be derived as the outcome of a Cournot-type adjustment behaviour. However, when extended to the two countries pursuing the full optimal rule this process leads to an ever-expanding state vector, which may provide an additional rationale for simple rules. These procedures are used in Section VI.

In Section III we set up a dynamic stochastic rational expectations model of the open economy with a developed wage/price spiral, a government budget constraint and asset accumulation, and exchange rate dynamics under a floating exchange rate regime. In the context of this model, we review a variety of simple rules for the conduct of monetary and fiscal policy in the single open economy. These include the use of various indicators (the money supply, the exchange rate, nominal income and the price level) for the adjustment of interest rates. They also include a variety of decoupled rules for monetary and fiscal policy, including those advocated by Meade (Meade (1983), Vines *et al.* (1983)). Of the simple rules considered, the price rule dominates the others. Thus it performs well in the face of a variety of disturbances, and in this sense provides 'a horse for all courses', in contrast to the other simple rules. Moreover, its superior performance continues to hold up in the face of wide parameter variation. Relative to full optimal monetary policy, its performance is good, so that the costs of simplicity for this rule are not high. Its performance relative to full optimal monetary and fiscal policy is not so good, but nonetheless for a simple rule it performs rather well. The form of the full optimal feedback rule suggests that some extra refinements, adding, in particular, an element of integral control, may well improve its performance further.

Thus our methods of policy design for the single open economy suggest that the choice lies between two types of policy: full optimal policy, which is rather complicated to specify and implement, but yields the best performance (by definition); and a form of simple price rule, which is much simpler to specify and implement, but at the cost of some (though not excessive) loss in performance. These results are arrived at by means of methods of policy evaluation that, although non-standard in their use of stochastic control under rational expectations, are typical in their focus on the single economy and their consequent neglect of issues of interdependence. They are therefore of a kind that might be derived from a macroeconometric model of the single open economy which properly models expectations. The focus of the policy analysis is in the remaining section of the paper, where we examine cooperative policy design and compare it with design in the single open economy. In Section VI, we examine policy design in an interdependent world of two identical economies. It is shown that the problem of policy design decomposes into two orthogonal problems, the aggregate problem (which applies equally to a world of many identical economies) and the divergence problem. Optimal cooperative policy is bound to involve rather more fiscal activism and less active use of interest rate policy than for the single open economy policy design. Furthermore, the simple monetary rule for interest rates outperforms the nominal income rule, and this in turn outperforms the price rule, which gives no gain from control. This reverses the ranking of these policies when evaluated for the single open economy.

Of equal interest is the global performance of rules designed for the single open economy. Both the monetary and the nominal income rule perform tolerably when evaluated in this different context. By contrast, the price rule formulated for the single open economy results in total global destabilisation if implemented generally; this appears to be because it relies for its effects on the link between the exchange rate and the domestic price level. Since this cannot operate in the aggregate, it triggers an over-reaction of interest rates to inflationary pressures and consequently leads to instability. In this sense, it is a beggar-my-neighbour policy. The full optimal rule designed for the single open economy also relies significantly on the exchange rate influence on prices, and therefore shares in its beggar-my-neighbour consequences.

These results highlight the externalities inherent in macroeconomic policy design in an interdependent world. They suggest that there may be an incentive for single countries to renege on the optimal cooperative policy and adopt free-riding forms of policy. To shed further light on this issue, we report in Section VI the results of Nash games played between two countries with different constraining ground-rules. These confirm that the incentives to renege on the best cooperative design of monetary or nominal income rules, whilst remaining within the overall constraint of such rules, is not strong. By contrast, there is a strong incentive to renege on the cooperative price rule (which amounts to minimal control), particularly since the country which reneges first retains an advantage even in the long run. Moreover, the threat to retaliate in kind is not credible since it leads to total destabilisation of the system. Clearly the price rule, and other rules (such as the full optimal rule) which rely on the exchange rate influence on domestic prices are inimical to international cooperation.

This analysis highlights the importance of analysing policy in an interdependent, rather than a single country, framework. It also leads to the issue of systems of penalties to sustain cooperative solutions to the policy problem. We offer some thoughts on that question in the conclusions to this paper.

II The solution procedure

We consider the following general linear stochastic rational expectations model:

$$\begin{bmatrix} dz \\ dx^e \end{bmatrix} = A \begin{bmatrix} z \\ x \end{bmatrix} dt + Bwdt + dv \qquad \text{(II.1)}$$

where $z(t)$ is an $(n-m)\times 1$ vector of variables predetermined at time t, $x(t)$ is an $m\times 1$ vector of non-predetermined or 'free' variables, $dx^e = x^e(t+dt, t)-x(t)$ where $x^e(t, \tau)$ denotes the expectation of $x(t)$ formed at time τ, $w(t)$ is an $r\times 1$ vector of instruments, A and B are $n\times n$ and $n\times r$ matrices respectively with time-invariant coefficients and dv is an $n\times 1$ vector of white noise disturbances independently distributed with $\text{cov}(dv) = \Sigma dt$ where Σ is a positive definite matrix with time-invariant coefficients. Variables z, x and w are all measured as deviations about the long-run equilibrium.

We see a linear time-invariant feedback rule,

$$w = D\begin{bmatrix} z \\ x \end{bmatrix} \tag{II.2}$$

where D is an $r \times n$ matrix with time-invariant coefficients which minimises the asymptotic quadratic loss function, asy $E(W)$, where

$$W = [z^T x^T] Q \begin{bmatrix} z \\ x \end{bmatrix} + w^T R w \tag{II.3}$$

and Q and R are $n \times n$ and $r \times r$ time-invariant positive definite matrices respectively. By appropriate restrictions on the coefficients of D, (II.2) can represent a simple feedback rule on only some variables of the system. Such simple rules can include indicator and intermediate target regimes and decoupled rules of the kind advocated by Meade (see Vines, Maciejowski and Meade (1983)).

Our solution procedure may be sketched as follows. We first solve the model for a given feedback rule D, which is assumed to be known by economic agents along with the model and the current state vector. The solution yields the asymptotic variances and covariances of all endogenous variables, and hence permits us to evaluate the loss function. We can then implement an iterative search procedure in which the unconstrained elements of D are varied so as to minimise the loss function.

This procedure yields optimal simple linear time-invariant feedback rules. That policy should be expressible in this form is restrictive for two reasons. First, as we indicate in the following, the full optimal policy cannot be implemented in the form of a linear time-invariant feedback rule of the form of (II.2), so that to insist on policy formulated in this form is restrictive. (This is an important difference between control of models with, and those without, free variables. However, as indicated in Section II.2, the optimal rule can be implemented by means of linear feedback on the z vector, together with integral control terms involving z.) Second, restrictions on the coefficients of the D matrix, such as the zero restrictions that arise naturally from indicator regimes or decoupled control, limit policy design further.

It is helpful to be able to assess the costs of these restrictions on policy design. For if the costs are not high, the benefits of design simplicity discussed in the introduction to this paper may make simple policy desirable. This requires some benchmark against which the performance of simple rules can be judged. The obvious benchmark is that of the performance of the full optimal rule.

The full optimal rule is particularly useful as a benchmark since it has the important property of certainty equivalence. This means that the closed loop feedback solution to the deterministic and stochastic control problems are of the same form, so that the solution to the stochastic problem is independent of the disturbance covariance matrix, Σ. Use of the full optimal rule means that policy makers need not assess the combination of shocks likely to perturb the system. In this sense, the full optimal rule provides 'a horse for all courses'.

By contrast, linear time-invariant feedback rules of the form (II.2) do not satisfy certainty equivalence. This is because the optimal choice of the parameters of D depends on the disturbance covariance matrix, Σ. This is a major disadvantage of simplicity in policy design, for policy makers may have no reasonable estimate of Σ. It is not helpful to policy makers to have a 'horse for each course' if the actual course is not known.

However, the lack of certainty equivalence does not rule out the existence of simple rules that are robust in the sense of performing reasonably well whatever the disturbance covariance matrix, Σ. (We here judge performance relative to that of the full optimal rule.) In our design evaluation of Section IV we pay particular attention to whether robust rules of this kind exist. Of course, robustness with respect to changes in the parameters of the disturbance covariance matrix is merely one aspect of the broader question of robustness with respect to other model parameter and specification changes, which we also consider.

Before turning to the detailed derivation of our control methods, we must briefly consider the issue of time-inconsistency. Both the full optimal rule and the linear time-invariant rules considered here are time-inconsistent, because they offer to policy makers a short term incentive to renege on the rule which the private sector have assumed in formulating their plans. However, reneging imposes longer run costs by undermining faith in the ability of governments to keep to commitments. If such faith is undermined altogether, the result is likely to be a closed-loop Nash equilibrium. This may be arrived at by means of a type of Cournot adjustment process, with each side determining its decision sequentially on the assumption of a given feedback rule on the part of the other.[1] Alternatively, it may come about by the private sector assuming that the government will renege if there is any short run incentive to do so, calculating the government's optimal action accordingly, and then determining its optimal plan in the light of this; the government then determines its optimal plan subject to this procedure for private sector decision making. This equilibrium is time consistent, and is necessarily inferior to the time inconsistent full optimal policy considered in this paper. (See Buiter (1983).)

If the closed-loop Nash equilibrium is markedly inferior to the equilibrium under time-inconsistent optimal policy, whether full or simple, this provides a strong incentive for a far-sighted government not to succumb to the temptation of reneging. Our earlier analysis (Levine and Currie (1983)) suggests this to be the case, at least for certain simple models. It also shows that well-designed simple rules may perform significantly better than the closed-loop Nash equilibrium, so that a similar point applies to this class of rules. This does not, of course, mean that a more myopic government will not renege, but it does suggest that the problems of time-inconsistent policies need not be over-stressed.[2]

We now consider in turn the detailed derivation of optimal simple rules and the full optimal rule, together with the two-player games.

II.1 Optimal simple rule

The first step of the solution procedure is to obtain the rational expectations solution to (II.1) for a given feedback rule (II.2) which is assumed to be known by economic agents along with the model and the current endogenous variables.[3] Substituting (II.2) into (II.3), we obtain

$$\begin{bmatrix} dz \\ dx^e \end{bmatrix} = [A+BD]\begin{bmatrix} z \\ x \end{bmatrix} dt + dv \tag{II.4}$$

We are concerned only with solutions to (II.4) which have the saddle-point property that the number of eigenvalues of $A+BD$ with positive part equals m, the remaining $n-m$ eigenvalues having negative real parts. We shall assume that the pair (A, B) is stabilizable in the sense that there exists at least one D such that $A+BD$ has the saddle-point property. Then stochastic stability follows since we only have additive disturbances (Turnovsky (1977)). We now require the immediate response of the non-predetermined variables x to the feedback rule (II.2). This is found by first forming the matrix of left eigenvectors of $A+BD$, M say, with rows ordered so that the first $n-m$ are the eigenvectors associated with the stable eigenvectors. We then partition so that

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{II.5}$$

where M_{11} is an $(n-m)\times(n-m)$ matrix and M_{22} is $m\times m$. Then, provided that $A+BD$ has the saddle-point property, the rational expectations solution places the trajectory on the unique saddle path

$$x = -M_{22}^{-1}M_{21}z = -Nz \tag{II.6}$$

The feedback rule (II.2) now becomes

$$w = (D_1 - D_2 N)z = \tilde{D}z \tag{II.7}$$

where $D = [D_1 \, D_2]$ with D_1 of dimension $r\times(n-m)$ and D_2 of dimension

$r \times m$. We note that (II.7) is a feedback rule only on the predetermined variables. It follows immediately that D is unique only up to choice of $\tilde{D}$, so that without loss of generality we can confine attention to linear time invariant feedback rules on the predetermined variables, z, alone (i.e. $D_2 = 0$).[4] Let A be partitioned as for M, $B = \begin{bmatrix} B^1 \\ B^2 \end{bmatrix}$ where B^1 is $(n-m) \times r$, B^2 is $m \times r$ and $dv = \begin{bmatrix} dv^1 \\ dv^2 \end{bmatrix}$ where dv^1 is $(n-m) \times 1$ and dv^2 is $m \times 1$. Then substituting (II.6) and (II.7) into (II.1), we have from the first $(n-m)$ rows that

$$dz = [A_{11} - A_{12} N + B^1 \tilde{D}]\, z dt + dv^1. \tag{II.8}$$

The solution to (II.8) is

$$z(t) = \int_0^t e^{C(t-s)}\, dv^1(s) + e^{Ct} z(0) \tag{II.9}$$

where $C = [A_{11} - A_{12} N + B^1 \tilde{D}]$. Equations (II.6) and (II.9) constitute the rational expectations solution to (II.1) for a given feedback rule (II.2).

The second step of the control problem is to optimise with respect to $\tilde{D}$. This requires us to allow for the fact that N depends on $\tilde{D}$. Substituting (II.2) into (II.3), we have

$$\text{asy}\, E(W) = \text{tr}(\text{asy}\, E(W)) = \text{tr}\,((Q + D^T RD)\, Y) \tag{II.10}$$

where $Y = \text{asy cov}\begin{pmatrix} z \\ x \end{pmatrix}$ and we have used the result $\text{tr}\,(ABC) = \text{tr}\,(CAB)$. The asymptotic covariance matrix $z = \text{asy}\, E(z^T z)$ satisfies

$$ZC^{\text{I}} + CZ + \Sigma_{11} = 0 \tag{II.11}$$

where $\Sigma_{11}\, \mathrm{d}t = \text{cov}\,(\mathrm{d}v^1)$. Then combining (II.6) and (II.10) we obtain

$$\text{asy}\, E(W) = \text{tr}\,(\tilde{Q}Z) \tag{II.12}$$

where

$$\tilde{Q} = Q_{11} + 2N^T Q_{21} + N^T Q_{22} N + \tilde{D}^T R \tilde{D}. \tag{II.13}$$

The welfare loss asy $E(W)$ can now be minimised with respect to $\tilde{D}$ by a standard numerical gradient method, subject to the constraint that the saddle-point property is preserved (i.e. that the number of eigenvalues of $A + B^1 \tilde{D}$ with positive real part is equal to m).

From the form of (II.11) and (II.12), it is clear that, in general, the optimal choice of $\tilde{D}$ is dependent on Σ_{11}, so that certainty equivalence does not hold.

II.2 The full optimal rule

The following is a solution procedure, employing Pontryagin's maximum principle, first proposed by Calvo (1978) and later developed by Driffill (1982) and Miller and Salmon (1983).[5] We consider first the deterministic finite time-horizon problem with objective function

$$W = \int_0^{\tau} (y^T Q y + w^T R w)\,dt \tag{II.14}$$

and $y = \begin{bmatrix} z \\ x \end{bmatrix}$. Then on introducing the costate row vector $\lambda(t)$, by the maximum principle we minimise

$$J = W + \int_0^{\tau} \lambda(Ay + Bw - \dot{y})\,dt \tag{II.15}$$

with respect to w, y and λ. Define the Hamiltonian

$$H = (y^T Q y + w^T R w) + \lambda(Ay + Bw) \tag{II.16}$$

Then

$$J = \int_0^{\tau} (H - \lambda\dot{y})\,dt \tag{II.17}$$

Hence, considering arbitrary variations in $\lambda, \delta J = 0$ if and only if

$$\frac{\partial H}{\partial \lambda} = \dot{y} \tag{II.18}$$

which, from (II.16), is simply the model (II.1) in the deterministic case.

Now consider variations in J due to independent variations in w and y. Integrating (II.17) by parts, we have

$$J = -\lambda(\tau)\,y(\tau) + \lambda(0)\,y(0) + \int_0^{\tau} (H + y\dot{\lambda})\,dt \tag{II.19}$$

Differentiating (II.19),

$$\delta J = -\lambda(\tau)\,\delta y(\tau) + \lambda(0)\,\delta y(0) + \int_0^{\tau}\left[\left(\frac{\partial H}{\partial y} + \dot{\lambda}\right)\delta y + \frac{\partial H}{\partial w}\delta w\right] dt \tag{II.20}$$

Partition $\lambda(0) = [\lambda_1(0), \lambda_2(0)]$ where λ_1 is $1 \times (n-m)$ and λ_2 is $1 \times m$. Then $\lambda(0)\,\delta y(0) = \lambda_1(0)\,\delta z(0) + \lambda_2(0)\,\delta x(0) = \lambda_2(0)\,\delta x(0)$ since $z(t)$ is predetermined (i.e. $z(0)$ is given). It follows that $\delta J = 0$ for arbitrary changes $\delta y(\tau)$, $\delta x(0)$, δy and δw if and only if

$$\lambda(\tau) = 0 \tag{II.21}$$

$$\lambda_2(0) = 0 \tag{II.22}$$

$$\frac{\partial H}{\partial w} = 0 \tag{II.23}$$

and

$$\dot{\lambda} = -\frac{\partial H}{\partial y} \tag{II.24}$$

The condition (II.22) can also be obtained (Driffill (1982)) by using the standard result that $\frac{\partial W}{\partial y(0)} = \lambda(0)$ at the optimal point. Hence $\frac{\partial W}{\partial x(0)} = \lambda_2(0)$. But the welfare loss must be insensitive to changes in the initial values of the non-predetermined variables $x(0)$. Thus $\frac{\partial W}{\partial x(0)} = 0$ and the result follows.

From (II.23) and (II.24) with H defined by (II.16), we obtain

$$w = -\tfrac{1}{2}R^{-1}B^T\lambda^T \tag{II.25}$$

and

$$\dot{\lambda} = -(2y^T Q + \lambda A) \tag{II.26}$$

Define $p = \frac{1}{2}\lambda^T$. Then the optimal rule for the deterministic control problem is given by

$$w = -R^{-1}B^T p \tag{II.27}$$

where

$$\begin{bmatrix} \dot{y} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} y \\ p \end{bmatrix} = H \begin{bmatrix} y \\ p \end{bmatrix} \tag{II.28}$$

and z, p satisfy the boundary conditions that $z(0)$ is given, $p_2(0) = 0$ and $p(\tau) = 0$.

It is a standard result in control theory (see, for example, Kwakernaak and Sivan, p. 147) that provided H has $2n$ distinct eigenvalues, n of these associated with predetermined variables $[z^T p_2^T]$ will be stable and n associated with non-predetermined variables $[x^T p_1^T]$ will be unstable where $p^T = [p_1^T, p_2^T]$. Then re-arranging (II.28) we have

$$\begin{bmatrix} \dot{z} \\ \dot{p}_2 \\ \dot{p}_1 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A_{11} & -J_{12} & -J_{11} & A_{12} \\ -Q_{21} & -A_{22}^T & -A_{21}^T & -Q_{22} \\ -Q_{11} & -A_{12}^T & -A_{11}^T & -Q_{12} \\ A_{21} & -J_{22} & -J_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z \\ p_2 \\ p_1 \\ x \end{bmatrix} = \tilde{H} \begin{bmatrix} z \\ p_2 \\ p_1 \\ x \end{bmatrix} \tag{II.29}$$

where $J = BR^{-1}B^T$ and matrices A, J and Q are partitioned as before. Equation (II.29) expresses the model in a form analogous to the standard deterministic rational expectations model. The case of the infinite time horizon, $\tau \to \infty$, is analytically tractable. For in this case (using an argument analogous to that leading to (II.6) the rational expectations assumption that $y(t) \to 0$ as $t \to \infty$ (i.e. the model is stable) together with condition (II.21) which implies $p(t) \to 0$ as $t \to \infty$, imposes the relationship

$$\begin{bmatrix} p_1 \\ x \end{bmatrix} = -\tilde{M}_{22}^{-1}\tilde{M}_{21}\begin{bmatrix} z \\ p_2 \end{bmatrix} = -\tilde{N}\begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{II.30}$$

where $\tilde{M}$ is the matrix of left-eigenvectors of H formed and partitioned as for M in (II.6) except that this time we have n stable and n unstable roots.

The feedback rule (II.27) now becomes

$$w = -R^{-1}B^T[-\tilde{N}_{11}z - \tilde{N}_{12}p_2, p_2]^T = D\begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{II.31}$$

where $D = -R^{-1}[-B_1\tilde{N}_{11}, B_2 - B_1\tilde{N}_{12}]$ where $B^T = [B_1, B_2]$ and $\begin{bmatrix} z \\ p_2 \end{bmatrix}$ is given by

$$\begin{bmatrix} \dot{z} \\ \dot{p}_2 \end{bmatrix} = [\tilde{H}_{11} - \tilde{H}_{12}\tilde{N}]\begin{bmatrix} z \\ p_2 \end{bmatrix} = \tilde{C}\begin{bmatrix} z \\ p_2 \end{bmatrix} \tag{II.32}$$

and $\tilde{H}$ is partitioned into four $n \times n$ blocks. The solution to (II.32) is

$$\begin{bmatrix} z \\ p_2 \end{bmatrix} = e^{\tilde{C}t}\begin{bmatrix} z(0) \\ p_2(0) \end{bmatrix} = e^{\tilde{C}t}\begin{bmatrix} z(0) \\ 0 \end{bmatrix} \tag{II.33}$$

which completes the optimal control solution in closed loop and open-loop form.

From the bottom m rows of (II.30) we have that $x = -\tilde{N}_{21}z - \tilde{N}_{22}p_2$ i.e., $p_2 = -\tilde{N}_{22}^{-1}(x + \tilde{N}_{21}z)$. Then substituting into (II.31) we have

$$w = -R^{-1}B^T S\begin{bmatrix} z \\ x \end{bmatrix} \tag{II.34}$$

where

$$S_{11} = -\tilde{N}_{11} + \tilde{N}_{12}\tilde{N}_{22}^{-1}\tilde{N}_{21} \tag{II.35}$$

$$S_{21} = -\tilde{N}_{22}^{-1}\tilde{N}_{21} \tag{II.36}$$

$$S_{12} = \tilde{N}_{12}\tilde{N}_{22}^{-1} \tag{II.37}$$

$$S_{22} = -\tilde{N}_{22}^{-1} \tag{II.38}$$

Matrix S is, of course, the non-negative definite solution to the familiar Riccati equation and (II.35)–(II.38) provide a convenient method of finding such a solution. However the feedback rule (II.34) can only be implemented in conjunction with the nth order system under control, namely (II.28) or (II.29). If the rule were to be announced in the form

(II.34) alone and the private agents' information set consisted of the model (II.1) and current values of z and x, then the argument of Section II.2 shows that it would be equivalent to a feedback rule on z alone, and that the resulting system has order only $(n-m)$. This result contrasts with the standard case where free variables are absent, so that p_2 and x have zero dimension and the optimal rule can be implemented by means of a linear time-invariant feedback rule on the state vector, z. However, the full optimal rule may be implemented by linear feedback on z combined with elements of integral feedback on z; this follows since, from (II.32), p_2 may be expressed as a suitable integral of z.

For the stochastic control problem, we note that it is a standard result of control theory that certainty equivalence applies as between the deterministic and stochastic optimisation problems and this carries over to rational expectations models.[6] This enables us to calculate the loss under the optimal policy for the stochastic case. From (II.30), using $p_2(0) = 0$, we have $p_1(0) = -\tilde{N}_{11} z(0)$. But for the deterministic case, $W = W(z(0))$, with $W(0) = 0$. Also $\dfrac{dW}{dz(0)} = 2p_1(0) = -2\tilde{N}_{11} z(0)$. Hence integrating, we have

$$W = -z^T(0)\,\tilde{N}_{11}\,z(0) \tag{II.39}$$

since $\tilde{N}_{11}$ is symmetric and non-positive definite. The corresponding welfare loss in the stochastic case is the same provided we put $z(0)\,z^T(0) = \Sigma_{11}$ where $\text{cov}\,(\mathrm{d}v^1) = \Sigma_{11}\,\mathrm{d}t$. (See Levine and Currie (1984).) Thus since (II.31) and (II.32) define the optimal rule for the deterministic problem, they also define the optimal rule for the stochastic counterpart quite independently of the covariance matrix of the disturbances perturbing the system.

II.3 A two-country closed-loop Nash game

Equation (II.1) is sufficiently general to represent an n-country model of interdependent economies. In particular for the case of two countries denote as unstarred variables those referring to country 1 (the 'home' country) and starred variables for country 2 (the 'overseas' sector). Then we may write a two-country model in the form

$$\begin{bmatrix} \mathrm{d}z \\ \mathrm{d}z^* \\ \mathrm{d}x \\ \mathrm{d}x^* \end{bmatrix} = A \begin{bmatrix} z \\ z^* \\ x \\ x^* \end{bmatrix} \mathrm{d}t + [B,\ B^*] \begin{bmatrix} w \\ w^* \end{bmatrix} \mathrm{d}t + \begin{bmatrix} \mathrm{d}v_1 \\ \mathrm{d}v_1^* \\ \mathrm{d}v_2 \\ \mathrm{d}v_2^* \end{bmatrix} \tag{II.40}$$

where the dimensions of z, z^*, x and x^* are $(n-m)\times 1$, $(n^*-m^*)\times 1$, $m\times 1$ and $m^*\times 1$ respectively.

Consider first the case of the two countries pursuing simple rules of the form (II.2). Suppose country 2 adopts a policy given by

$$w^* = D^* \begin{bmatrix} z \\ z^* \\ x \\ x^* \end{bmatrix} \tag{II.41}$$

Then substituting (II.41) into (II.40), country 1 faces the dynamic constraint

$$\begin{bmatrix} \mathrm{d}z \\ \mathrm{d}z^* \\ \mathrm{d}x \\ \mathrm{d}x^* \end{bmatrix} = [A + B^* D^*] \begin{bmatrix} z \\ z^* \\ x \\ x^* \end{bmatrix} + Bw dt + dv \tag{II.42}$$

where we denote $dv^T = [dv_1^T \, dv_1^{*T} \, dv_2^T \, dv_2^{*T}]$.

In a Nash game each 'player' chooses its own move taking the current observed 'moves' of other players as given. In a closed-loop Nash game these moves of other players are observed in feedback form on the state vector. For the two-country game presented here this means that country 1 chooses a feedback rule

$$w = D \begin{bmatrix} z \\ z^* \\ x \\ x^* \end{bmatrix} \tag{II.43}$$

taking D^* as *given*. Suppose country 1 has a loss function asy $E(W)$ where

$$W = [z^T \, z^{*T} \, x^T \, x^{*T}] \, Q \begin{bmatrix} z \\ z^* \\ x \\ x^* \end{bmatrix} + w^T R w \tag{II.44}$$

with a similar loss function asy $E(W^*)$ for country 2, Q and R being replaced with Q^* and R^* respectively. Then the optimisation problem of country 1 – to minimise asy $E(W)$ subject to (II.42) – is in the form of the problem solved in II.1 above and leads to an optimal value for D in the form of a *reaction function* $D = f(D^*)$. Similarly country 2 has a reaction function $D^* = g(D)$ and the Nash equilibrium is at the fixed-point of fg for D and gf for D^*.

A plausible adjustment process by which countries move towards a closed-loop Nash equilibrium is the following Cournot-type sequence of moves. Given an initial feedback rule for country 2 with D^* denoted by $D^*(0)$, country 1 chooses $D = D(1) = f(D^*(0))$. Then country 2 revises its choice of D^* to $D^*(1) = g(D(1))$. Country 1 then re-optimises and so on.

This process may or may not converge to an equilibrium depending on the initial value of D^*. Indeed a Nash equilibrium may not exist and if it does it may not be unique. Given the complexity of models with rational expectations, it seems a formidable task to obtain analytical conditions for the existence, uniqueness and convergence of a Nash equilibrium and we do not attempt to do so here.

Consider next the full optimal policy for the two-country model. Suppose country 2 adopts an optimal rule which from section II.2 can be implemented in the form

$$w^* = D^* \begin{bmatrix} z \\ z^* \\ p_2^* \end{bmatrix} \tag{II.45}$$

where

$$dp_2^* = P^* \begin{bmatrix} z \\ z^* \\ p_2^* \end{bmatrix} \tag{II.46}$$

p_2^* has dimensions $(m+m^*)\times 1$ and P^* depends on A, B, B^* and country 2's loss function. (Country 2 could arrive at (II.45) by assuming country 1 was pursuing a simple rule of the type considered above.) In a closed-loop Nash game country 1 takes (II.45) as given and optimises subject to the dynamic constraint

$$\begin{bmatrix} dz \\ dz^* \\ dp_2^* \\ dx \\ dx^* \end{bmatrix} = \tilde{A} \begin{bmatrix} z \\ z^* \\ p_2^* \\ x \\ x^* \end{bmatrix} dt + \begin{bmatrix} B^1 \\ 0 \\ B^2 \end{bmatrix} w dt + \begin{bmatrix} dv_1 \\ dv_1^* \\ 0 \\ dv_2 \\ dv_2^* \end{bmatrix} \tag{II.47}$$

where

$$\tilde{A} = \begin{bmatrix} A_{11} + B^{*1}D_1^* & B^{*1}D_2^* \\ P_1^* & P_2^* \\ A_{21} + B^{*2}D_1^* & B^{*2}D_2^* \end{bmatrix} \begin{bmatrix} A_{12} \\ 0 \\ A_{22} \end{bmatrix} \tag{II.48}$$

and we have partitioned A so that A_{11} is $(n+n^*-m-m^*)\times(n+n^*-mn^*)$, B so that B^1 is $(n+n^*-m-m^*)\times r$, $D^* = [D_1^*, \; D_2^*]$ with $D_1^* r \times (n+n^*-m-m^*)$ and P^* similarly. The optimal feedback rule of country 1 is then of the form

$$w = D \begin{bmatrix} z \\ z^* \\ p_2^* \\ p_2 \end{bmatrix} \tag{II.49}$$

where

$$dp_2^* = P \begin{bmatrix} z \\ z^* \\ p_2^* \\ p_2 \end{bmatrix} \tag{II.50}$$

and p_2 has dimensions $(m+m^*) \times 1$. Comparing (II.45) and (II.49) we see that matrices D and D^* defining the feedback rules are not comparable as they have different dimensions.

The reaction function analysis adopted from simple rules is now not applicable. If we envisaged a Cournot-type adjustment process, then with each iteration the model defining the dynamic constraint for the optimising country increases its dimension by the number of non-predetermined variables $m+m^*$. The game may well converge in the sense that the coefficients of D and D^* relating to the additional p_2 and p_2^* terms tend to zero and the welfare loss tends to a finite quantity. However, the possibility of countries actually engaging in games of such complexity seems remote and it would appear that the full optimal closed-loop Nash game is not a plausible form of non-cooperative behaviour. The problem with the full optimal closed-loop Nash game arises because for a single country the full optimal (time inconsistent) rule cannot be implemented as a linear feedback on the state vector alone (Levine and Currie (1984)). If we confine ourselves to simple (and, in general, time-inconsistent) rules of the form (II.41) and (II.43) the problem does not arise. Nor does it arise if one focuses on time-consistent (but sub-optimal) policies for government or on Nash *open-loop* games between countries (see Miller and Salmon (1984)). All these options form interesting directions for research; but in this paper we shall analyse non-cooperative behaviour only in terms of the first, namely simple policy rules.

III The model

Throughout the rest of this paper we use variants of the following eleven equation continuous time stochastic model:

$$dy = \psi_1[\alpha_1 q - \alpha_2(r - \dot{p}^e) + \alpha_3 v - \alpha_4 s + \alpha_5 y^* - y]\, dt + du_1 \qquad \text{(III.1)}$$

$$dm = \psi_2[\gamma_1 y - \gamma_2 r + p + \gamma_3 v - m]\, dt + du_2 \qquad \text{(III.2)}$$

$$dv = [-\phi_0 s - \phi_1 y + \phi_2 q + \phi_3 y^*]\, dt - dp \qquad \text{(III.3)}$$

$$dw = \psi_3\left[\beta_1 \int_{-\infty}^{t} y(\tau)\, d\tau + p^e - w\right] dt + du_3 \qquad \text{(III.4)}$$

$$\bar{p}^d = c_1 w + (1 - c_1)(w^* + e) \qquad \text{(III.5)}$$

$$\bar{p} = \theta \bar{p}^d + (1 - \theta)(w^* + e) \qquad \text{(III.6)}$$

$$dp = \psi_4(\bar{p} - p)\, dt \qquad \text{(III.7)}$$

$$de^e = (r - r^*)\, dt \qquad \text{(III.8)}$$

$$q = w^* + e - w \tag{III.9}$$

$$dw^* = -\mu_1 w^* dt + du_4 \tag{III.10}$$

$$dr^* = -\mu_2 r^* dt + du_5 \tag{III.11}$$

where the following notation is used:

- e nominalexchangerate(definedasthepriceofforeignexchange)
- y real output
- q competitiveness
- r domestic nominal rate of interest
- s autonomous taxation
- v real net financial wealth of the private sector
- m nominal money supply
- p general price index
- p^d price index of domestic output
- w nominal wages
- w^*nominal wages overseas
- r^* foreign nominal rate of interest
- du_i white noise disturbance

Because we are concerned in this paper only with stabilization of the system around an exogenously given long run equilibrium (which may incorporate trends), all variables are measured in terms of deviations of their logarithm from equilibrium, except for interest rates which are measured as deviations of proportions. All parameters are defined to be positive. An 'e' superscript denotes an expectation formed at time t on the basis of information available up to time t; while a '*' superscript denotes the foreign counterpart to the variable in question. A bar denotes a partial equilibrium value. Equation (III.1) represents the IS curve with output adjusting sluggishly with a mean lag of ψ_1^{-1} to competitiveness, the real interest rate, real financial wealth, autonomous taxes and foreign demand. Equation (III.2) represent the LM curve. The money supply is assumed to be demand determined for any given level of interest rates, and money demand adjusts sluggishly to output, interest rates and real financial wealth with a mean lag of ψ_2^{-1}. Equation (III.3) determines the change in real wealth from the determinants of the sum of the government budget deficit and the current account of the balance of payments. Neglecting interesting payments and approximating this relationship log-linearly, this makes the change in real wealth depend positively on competitiveness and foreign output, and negatively on domestic output, autonomous taxes and inflation. Equation (III.4) determines the level of nominal wages. Taking the derivative of its deterministic part, we have that long-run wage inflation is determined by an expectations-augmented Phillips curve, but with actual wage inflation adjusting sluggishly towards this long-run relationship. The sluggishness of wave adjustment generates fluctuations in real output in the face of demand disturbances, even under rational expectations. (See, for example, Buiter (1980)). Equation (III.5) is a partial equilibrium relationship giving the price index of domestic output in equilibrium as a weighted average of domestic and foreign wages (the influence of the latter variable working partly through a mark-up on costs and partly through competitive pricing effects). The corresponding general price index in partial equilibrium is given from (III.6) as a weighted average of domestic prices and foreign wages. Actual prices adjust quickly but not instantly according to (III.7) where ψ_4 is large.[7] Equation (III.8) models the exchange rate as asset market determined under conditions of perfect capital mobility.[8] The expected rate of depreciation of the exchange rate in an interval dt (denoted by $de^e = e^e(t+dt, t) - e(t)$ where $e^e(\tau, t)$ is the expected exchange rate at time τ, formed at time t) exactly offsets the interest rate differential in favour of the home currency. (Note that $r = 0$ in equilibrium corresponds to the domestic and foreign interest rates being equal.) Unlike other variables, which adjust slowly and are predetermined variables, the exchange rate is non-predetermined and can make discrete jumps in response to changes in exogenous variables or policy rules. Equation (III.9) defines competitiveness in terms of relative costs. Equations (III.10) and (III.11) specify exogenous first order autoregressive processes for foreign wages and foreign interest rates respectively. Some persistence in these disturbances is required if they are to have any impact on domestic variables, and this process involves the minimum additional complication.

Equations (III.1) and (III.11) specify our model of the small open economy. For the two-country analysis of interdependent economies, we assume an identical structure for the overseas economy. Our model of the overseas sector is therefore given by equations (III.1)–(III.9), with unstarred variables being replaced by starred variables and *vice versa*.[9] In the Appendix we set out the single country model and the two country model in the form of equation (II.1).

For our subsequent policy design analysis, we need to define a suitable loss function. Our loss function for the single country is assumed to take the form

$$W = ay^2 + bp^2 + cr^2 + s^2 \qquad \text{(III.12)}$$

while for the two country case we assume an aggregate loss function of the form

$$W^a = a(y^a)^2 + b(p^a)^2 + c(r^a)^2 + (s^a)^2 \qquad \text{(III.13)}$$

where the 'a' superscript denotes the sum of the relevant variables over the two countries (e.g. $y^a = y + y^*$). We may also consider a divergence loss function given by:

$$W^d = a(y^d)^2 + b(p^d)^2 + c(r^d)^2 + (s^d)^2 \qquad \text{(III.14)}$$

where the 'd' superscript denotes the divergence of the relevant variable between the two countries (e.g. $y^d = y - y^*$). As shown in the Appendix, the assumption of two identical countries permits us to decompose the two country model into two orthogonal parts, the aggregate model (given by (A.4)) and the divergence model (given by (A.5)). As explained in Section VI, we may therefore choose aggregate rules to minimise the aggregate loss function, given the aggregate model; and, quite separately, choose divergence rules to minimise the divergence loss function, given the divergence model. Since

$$W^a + W^d = 2(W + W^*)$$

where W^* is the value of (III.12) evaluated over foreign variables, this amounts to minimising a loss function consisting of the sum of the individual countries' loss functions.

Our assumed parameter values are set out in Table 6.1, together with variants of a number of parameter values to test for robustness of policies with respect to parameter change. The parameter α_5 is set equal to the degree of openness, $1 - \theta_1 c_1$. For the parameter of the objective function, we assume $c = 1$, penalising equally variations in r and s. We penalise price fluctuations twice as much ($b = 2$) relative to the instruments, and for our central assumptions penalise output and price fluctuations equally ($a = 2$). We also consider a Keynesian variant in which output fluctuations are penalised much more heavily ($a = 5$). Only results for central parameter values are reported in Tables 6.2–14.

IV The design of rules for monetary and fiscal policy in a small open economy

In this section we review results considered in more detail in Currie and Levine (1985) for the small open economy. We first consider monetary policy alone, thus holding the fiscal instrument, autonomous taxes, constant. Since tax receipts fluctuate with the level of economic activity, this amounts to allowing automatic fiscal stabilisers to operate unimpeded.

Our simple rules for monetary policy alone take the form

$$r = \begin{cases} \beta m & \text{monetary rule} \\ \beta e & \text{exchange rate rule} \\ \beta(y+p) & \text{nominal income rule} \\ \beta p & \text{price level rule} \end{cases} \qquad \text{(IV.1)}$$

where β is chosen optimally by the procedure outlined in Section II.1. Each of the rules provides a long run anchor for expected nominal variables provided that disturbances to the system follow stationary processes.[10] In the case of the exchange rate target, this anchor depends on foreign prices following a stationary process, but trends in foreign prices may be offset by a suitable trend in the exchange rate along the long run equilibrium path.

We next consider the use of fiscal policy (represented in the model by autonomous tax changes) in conjunction with monetary policy. We consider three forms of decoupled control rules given by

$$\begin{aligned} &\text{DCR I} \quad \begin{cases} r = \beta_1(e + w^* - w) \\ s = \beta_2(y + p) \end{cases} \\ &\text{DCR II} \quad \begin{cases} r = \beta_1 e \\ s = \beta_2(y + p) \end{cases} \\ &\text{DCR III} \quad \begin{cases} r = \beta_1 p \\ s = \beta_2 y \end{cases} \end{aligned} \tag{IV.2}$$

The first, DCR I, represents a Meade-type assignment of using fiscal policy to keep nominal income on track and monetary policy to stabilise the real exchange rate. DCR II is similar, except monetary policy now tracks the nominal, rather than the real, exchange rate. DCR III represents an extension of the price rule in (IV.1) whereby monetary policy is assigned to the price level while fiscal policy reacts to real output fluctuations. This rule implies that a rise in output, for example, generates a contraction of fiscal policy, with a subsequent tightening of monetary policy as and when there is an effect of higher output on inflation. All these rules are of a simple proportioned form: as noted below, elements of integral control may also be desirable.

The solution procedure described in Section II.1 can now be used to find values of β for rule (IV.1), and of β_1 and β_2 for (IV.2), that minimise asy (W) with W given by (III.12).[11] As noted in that section certainty equivalence does not apply for simple rules so that the design of each category of rules depends on the covariance matrix of the disturbances. Our approach to this problem is to consider one disturbance at a time and choose the optimal value of the parameter (or parameters) for each rule. We then evaluate the welfare loss for the chosen optimal rule when each of the other shocks in turn hits the system.

For each form of simple rule then there exist up to five optimal rules corresponding to anticipated disturbances u_i, $i = 1, 5$. In Table 6.2 we report only those with superior performance for central parameter values

Table 6.1. *Parameter values*

parameter	low	central	high
ψ_1		0.5	
ψ_2		0.5	
ψ_3		0.5	
ψ_4		10.0	
α_1		0.3	
α_2		0.1	0.5
α_3	0.1	1.0	2.0
α_4		0.4	
γ_1		1.0	
γ_2		1.0	
γ_3	0.1	1.0	
β_1		0.3	2.0
ϕ_0		1.0	
ϕ_1		1.3	
ϕ_2		0.1	
ϕ_3		0.5	
c_1	0.5	0.7	
θ	0.5	0.7	
μ_1		0.5	
μ_2		0.5	
a		2.0	5.0
b		2.0	
c		1.0	

Table 6.2. *Best policy rules and welfare losses for the single open economy*

	Disturbances				
Policy rule	du_1	du_2	du_3	du_4	du_5
Minimal control	2.44	0.00	1.22	0.96	3.83
Optimal monetary	2.28	0.00	0.39	0.09	0.39
Optimal monetary + fiscal	1.50	0.00	0.22	0.09	0.31
$r = 0.56\,(y+p)$	3.08	0.00	0.82	0.16	1.04
$r = 0.77\,m$	3.82	0.48	0.68	0.27	1.09
$r = 4.02\,p$	2.31	0.00	0.47	0.16	0.46
$r = 10\,p$, $s = 0.46\,y$	2.04	0.00	0.38	0.19	0.75
$r = 2.87\,p$, $s = 1.24\,y$	2.49	0.00	0.33	0.15	0.60

and loss function asy $E(2y^2+2p^2+r^2+s^2)$. For monetary policy alone the exchange rate rule, because of the segmentation of the model under this regime, gives no improvement on minimal control for all disturbances except u_5 and even for that disturbance it performs relatively badly.[12] There is little to choose between the best nominal income and monetary rules reported; the former performs better for u_1 and u_2 shocks and the latter for a u_3 shock. but the rule that completely dominates in category (IV.1) is the price rule, i.e. irrespective of the nature of the disturbance the best price rule is superior to the best monetary rule, nominal income or exchange rate rules. In addition, comparison with the performance of the full optimal rule shows that its performance compares quite well with this benchmark, particularly for domestic shocks. Thus the costs of simplicity do not appear to be enormous for this rule. The other simple rules, by contrast, perform rather badly overall.

It is a familiar result from the government budget constraint literature (see, for example, Blinder and Solow (1973), Christ (1979)) that monetary targeting may be unstable if the wealth effects on money demand are large relative to those on expenditure. To check whether this was at the root of the poor performance of the monetary targets, we examined the consequences of increasing β so that the money supply is kept strictly on a fixed track. The results reported in Table 6.3 indicate that there is no tendency for instability of the model under strict monetary targeting, suggesting that monetary targets perform badly because of their failure to dampen volatility rather than because of inherent instability.

For the central parameter set we also find the price rule is rather robust in its performance. A choice of β somewhere in the range between 2 and 10 yields a similar performance that is insensitive to disturbance uncertainty. To examine the consequences of parameter variation, we subjected the model to a variety of parameter changes (see Table 6.1). These include a more Keynesian objective function, an increased degree of openness, a large impact of demand on inflation, an increased effect of wealth on money demand, an increased direct influence of monetary policy on demand, and reduced wealth effects on aggregate demand and money demand. While the details of these results vary, the superiority and robustness of the price level rule remains intact; while the performance of the other simple rules remains poor. The only significant difference is if the influence of demand on inflation is stepped up, when the performance of the price level rule relative to the full optimal rule falls significantly in dealing with domestic shocks.

Turning to rule (IV.2) we find that DCR I and DCR II perform badly and are ruled out as plausible policy rules. DCR III in the form of two variants performs well but in neither case is there a clear-cut improvement

Table 6.3. *The consequences of monetary targeting. Policy rules* $r = \beta m$, *with increasing values of* β

	β	asy var (m)	asy var (r)	asy var (p)	asy var (y)	Welfare loss
du_1	5	0.07	1.72	1.07	1.20	6.25
	10	0.02	2.07	1.17	1.16	6.73
	15	0.01	2.20	1.20	1.15	6.90
	20	0.01	2.27	1.22	1.14	6.99
Optimal = minimal		1.00	0.00	0.03	1.19	2.44
Minimal control		1.00	0.00	0.03	1.19	2.44
du_2	5	0.16	4.11	0.09	0.00	4.29
	10	0.09	9.01	0.06	0.01	9.13
	15	0.06	9.01	0.06	0.01	14.06
	20	0.05	18.95	0.03	0.01	19.03
Optimal = minimal		1.00	0.00	0.00	0.00	0.00
Minimal control		1.00	0.00	0.00	0.00	0.00
du_3	5	0.01	0.22	0.13	0.15	0.77
	10	0.00	0.25	0.13	0.14	0.79
	15	0.00	0.26	0.13	0.14	0.79
	20	0.00	0.27	0.13	0.14	0.68
Optimal ($\beta = 0.77$)		0.12	0.07	0.12	0.19	0.68
Minimal control		0.62	0.00	0.37	0.24	1.22
du_4	5	0.00	0.03	0.07	0.01	0.18
	10	0.00	0.03	0.07	0.01	0.17
	15	0.00	0.03	0.06	0.01	0.17
	20	0.00	0.03	0.06	0.01	0.16
Optimal ($\beta > 40$)		0.00	0.003	0.06	0.01	0.16
Minimal control		0.29	0.00	0.46	0.02	0.96
du_5	5	0.01	0.12	0.28	0.02	0.72
	10	0.00	0.13	0.26	0.02	0.68
	15	0.00	0.13	0.25	0.02	0.66
	20	0.00	0.13	0.24	0.02	0.65
Optimal ($\beta > 40$)		0.00	0.13	0.24	0.02	0.64
Minimal control		1.14	0.00	1.82	0.10	3.83

on the best price rule using monetary policy alone. The first variant of DCR III improves the performance in the face of u_1 and u_3 shocks at the expense of a deterioration in the face of u_4 and u_5. The second variant provides still more improvement in the face of u_3 shocks at the expense of a deterioration for u_1 and u_5 shocks.

The top of Table 6.2 presents comparisons with full optimal policies, both for monetary policy alone and for fiscal and monetary policy together. The additional use of fiscal policy gives a significant gain in handling domestic disturbances. In consequence, optimal policy significantly outperforms our best simple rules with respect to all disturbances. Moreover, additional simulations showed that the performance of the optimal rule is robust with respect to parameter changes. When fiscal policy is considered, therefore, the costs of simplicity are high. However, the price level rule continues to perform well relative to other forms of simple rule. The poorer performance of the price rule relative to optimal policy when fiscal policy is considered may reflect the absence of any elements of integral control in the design of our simple rules. It remains to be investigated whether better rules can be devised that incorporate integral as well as proportional feedback, whilst remaining simple in design.

V The cooperative two country control problem

Hitherto in this paper, we have considered policy design in the small open economy, treating the rest of the world as exogenous. In this section, we examine the consequences of assuming two interdependent economies, each identical to the model described in Section III and assumed throughout our previous analysis.

Our assumed loss function for the aggregate problem is described by (III.13), and penalises deviations in aggregate variables summed over the two countries. An orthogonal divergence problem is that defined by minimising a loss function (III.14) defined over deviations of the differences in variables between the two countries (see Appendix). The joint solution to the aggregate and divergence problem is equivalent to minimising a loss function which is the sum of the individual country loss functions defined by (III.12).

We focus first on the aggregate problem, and consider the performance of simple rules in this context. Since the exchange rate does not enter the aggregate problem, we ignore the exchange rate rule given in IV.1, as well as DCR I and DCR II. We also consider three new variants of rule defined by:

$$\begin{aligned} &\text{NIR} && \begin{cases} r = \beta_1(y+p) \\ s = \beta_2(y+p) \end{cases} \\ &\text{DCR IV} && \begin{cases} r = \beta_1 m \\ s = \beta_2(y+p) \end{cases} \\ &\text{NIFR} && \quad s = \beta_2(y+p) \end{aligned} \qquad \text{(V.1)}$$

NIR represents a nominal income rule where both monetary and fiscal policy are used jointly to track nominal income. DCR IV represents a form of decoupled control, where interest rates are used to track the money supply and fiscal policy tracks nominal income. NIFR represents a nominal income target pursued by fiscal policy alone.

Table 6.4. *The aggregate two country problem: monetary policy alone*

Expected disturbance (unit variance)	Policy rule		Actual disturbance (unit variance) du_1^a	du_3^a
all	Minimal Control	$(r = 0.00\,p)$	19.30	17.05
	Optimal		14.79	14.50
du_1^a	Nominal Income	$(r = 1.01\,(y+p))$	15.45	17.84
	Monetary	$(r = 0.66\,m)$	16.18	15.71
	Price	$(r = 0.006\,p)$	19.26	17.05
du_3^a	Nominal Income	$(r = 0.38\,(y+p))$	16.70	16.40
	Monetary	$(r = 0.50\,m)$	16.27	15.61
	Price	$(r = 0.02\,p)$	19.35	17.05
	Open Economy Best Rules:			
	$r = 0.56\,(y+p)$		16.03	16.53
	$r = 0.77\,m$		16.23	15.86
	$r = 4.02\,p$		unstable	
	Optimal		16.90	23.02

Table 6.4 presents results for the aggregate problem for the rules using monetary policy alone (to be compared with the results of Section IV. Because the scope for monetary policy is much less for the aggregate problem, with the channel of influence via the exchange rate ruled out, the gain from control is not large. This may be seen by comparing the results reported for minimal and optimal control respectively. Of the simple rules, the price rule gives no gain whatever relative to minimal control. Both nominal income and monetary rules give some gain, the nominal income rule coping better with demand (u_1) shocks and the monetary rule performing better for supply shocks (u_3).

The bottom part of Table 6.4 reports the performance of the best simple rules designed for the single economy reported in Section IV when applied in both countries simultaneously. The nominal income and monetary rules are not dissimilar to those devised for the aggregate country. The price rule, by contrast represents a rather active feedback on prices, and for the aggregate economy it is totally destabilising. Thus the best rule designed for the single open economy is disastrous in its performance when applied generally. Similar problems apply to the full optimal rule designed for the single economy, when applied generally. Although it is stable, Table 6.4 shows its performance to be very poor. This highlights the dangers of the usual approach to policy design which focuses on the single open economy.

Table 6.5. *The aggregate two country problem: fiscal and monetary policy*

Expected disturbance (unit variance)	Policy rule		Actual disturbance (unit variance) du_1^a	du_3^1
all	optimal		3.30	8.31
d_1^a	DCR III	$r = 0.0001\, p$	4.48	13.66
		$s = 1.16\, y$		
	NIR	$r = 0.25\, (y+p)$		
		$s = 1.29\, (y+p)$	5.28	38.84
	DCR IV	$r = 0.09\, m$		
		$s = 1.29\, (y+p)$	5.32	37.33
	NI FR	$s = 1.30\, (y+p)$	5.34	37.36
du_3^a	DCR III	$r = 0.0005\, p$	6.54	10.52
		$s = 0.34\, y$		
	NIR	$r = 0.21\, (y+p)$	11.59	14.86
		$s = 0.14\, (y+p)$		
	DCR IV	$r = 0.29\, m$	12.02	14.83
		$s = 0.12\, (y+p)$		
	NI FR	$s = 0.15\, (y+p)$	11.98	15.00
	Open Economy Best Rules			
	$r = 10\, p$		7.36	18.12
	$s = 0.46\, y$			
	$r = 2.87\, p$		5.35	35.70
	$s = 1.24\, y$			
	optimal		unstable	

The results in Table 6.5 for the two country problem with joint fiscal and monetary policy indicate the important role for fiscal policy in the aggregate problem. The optimal policy gives very considerable gain when fiscal policy is used. Of the simple rules examined in Table 6.5, DCR III gives the best performance. However, the parameter in the interest rate part of this rule indicates that monetary policy is playing a minimal part in this rule, and the full burden of control falls on fiscal policy. This rule reduces to simple Keynesian policy of controlling output by fiscal feedback on output. The performance of the other rules is broadly similar, because of the limited influence of monetary policy, and amounts to fiscal feedback on nominal income. This performs poorly relative to optimal policy or DCR III.

The performance of the best rules for monetary and fiscal policy reported in Section IV for the single open economy are reported at the

Table 6.6. *The divergence component of the two-country problem*

Monetary policy alone ($s = 0$)			Actual disturbance	
Expected disturbance	Policy rule		$d\tilde{u}_1$	$d\tilde{u}_3$
all	optimal		1.39	0.10
	minimal control	($r = 0.0001\ m$)	1.40	0.10
du_1	exchange rate		1.40	0.10
	nominal income	($r = 0.0001\ (y+p)$)	1.40	0.10
	monetary	($r = 0.0001\ m$)	1.40	0.10
	price	($r = 0.008\ p$)	1.40	0.10
du_3	exchange rate		1.40	0.10
	nominal income	($r = 0.0001\ (y+p)$)	1.40	0.10
	monetary	($r = 0.0001\ m$)	1.40	0.10
	price	($r = 0.005\ p$)	1.40	0.10

Monetary plus fiscal policy			Actual disturbance	
Expected disturbance	Policy rule		$d\tilde{u}_1$	$d\tilde{u}_3$
all	optimal		1.15	0.07
du_1	DCR III	$r = 0.01\ p$ $s = 0.29\ y$	1.34	0.09
du_3	DCR III	$r = 0.0001\ p$ $s = 0.70\ y$	1.47	0.08

bottom of Table 6.5. In contrast to the use of monetary policy alone, the simple rules do not totally destabilise the system. However, their performance is poor, whilst the full optimal rule designed for the single open economy is totally destabilising when applied in the aggregate.

We have not carried out any thorough testing of the robustness of the results of this section. However, we have examined the effects of giving monetary policy a greater channel of influence in the aggregate problem by choosing the higher variant for α_2 in Table 6.1. Although this gave monetary policy a greater role it did not alter the broad results. In particular, the result that policies devised for the single open economy perform badly in the aggregate continues strikingly to stand out.

We have derived these results for the two-country case, but they have a more general interpretation. This is because the aggregate problem can be derived from the aggregation of the n-country generalisation of the two country analysis presented in the Appendix. The larger the number of countries, the greater will be the difficulty in sustaining the optimal rules for the aggregate stabilisation problem presented in Tables 6.4 and 6.5, and in preventing single countries adopting the optimal rules reported in Section IV. Our results focus attention directly on the incentive to renege on cooperative forms of international behaviour, despite the rather serious consequences for performance if all countries do, indeed, renege.

The results for the cooperative solution to the divergence problem are reported in Table 6.6. For monetary policy alone (reported in the upper part of the table), there is no gain whatever to control, whether simple or full optimal. This contrasts markedly with the solution to the single country problem. The lower part of the Table shows that there is some scope for fiscal action of a mild kind. However, these benefits of control show up only for the optimal rule, and the simple fiscal rules show little or no benefit relative to minimal control.

These results one more highlight the marked differences between the cooperative two country solution to the control problem and that thrown up by single country optimisation. They suggest that there is a serious free-rider problem in the design of international policy. To examine this further, in the next section we consider policy design in an explicit game theoretic framework.

VI Two-country non-cooperative games

In this section we consider non-cooperative behaviour for two identical countries in the form of a Cournot-type adjustment process leading to a closed-loop Nash equilibrium. We assume that each country pursues the same form of simple rule and we consider monetary policy only. To simplify matters still further we consider shocks (u_1, u_1^*), and (u_3, u_3^*) in pairs so that each country is experiencing either an aggregate demand shock or a supply shock in common. We put $\text{var}(du_i) = \text{var}(du_i^*) = 0.5\,dt$ for $i = 1$ and 3 and assume that disturbances in different countries are independent.[13] Then $\text{var}(du_i^a) = \text{var}(du_i^d) = dt$ and the welfare loss may be compared with that for cooperative policies already considered.

Suppose a cooperative policy is agreed in the form of the simple rules examined in Table 6.4. We can investigate the incentive to renege (i.e., the short-term gain) and the long-term consequence by following a Cournot-type sequence of decisions. The results are displayed in Tables 6.7–14 below for central parameter values with loss functions asy $E(2y^2+2p^2+r^2)$ and asy $E(2y^{*2}+2p^{*2}+r^{*2})$, for a monetary rule ($r = \beta m$ and $r^* = \beta m^*$), a nominal income rule ($r = \beta(y+p)$ and $r^* = \beta(y^*+p^*)$) and a price level rule ($r = \beta p$ and $r^* = \beta p^*$). An exchange rate rule ($r = \beta e$ and $r^* = -\beta e$)

Table 6.7. *The Cournot adjustment process for policy rule* $r = \beta m$

	Expected disturbance du_1^a var $(du_1) =$ var $(du_1^*) = 0.5\ dt$ cov $(du_1, du_1^*) = 0$				Expected disturbance du_3^a var $(du_3) =$ var $(du_3^*) = 0.5\ dt$ cov $(du_3, du_3^*) = 0$			
	Country 1		Country 2		Country 1		Country 2	
Iteration number	β	Welfare loss	β	Welfare loss	β	Welfare loss	β	Welfare loss
0	0.66	4.73	0.66	4.73	0.50	3.95	0.50	3.95
1	0.88	4.70	0.66	4.92	3.13	3.16	0.50	6.17
2	0.88	4.92	0.95	4.88	3.13	5.10	3.34	5.03
3	0.98	4.92	0.95	4.97	3.31	5.10	3.34	5.09
4	0.98	4.94	0.98	4.94	3.31	5.10	3.34	5.09
5	0.98	4.94	0.98	4.94	3.32	5.10	3.34	5.10

Parameters: central loss function: asy $E\,(2y^2 + 2p^2 + r^2)$

Table 6.8. *Welfare loss for Nash equilibrium for policy rule: $r = \beta m$*

Expected disturbance (unit variance)	Policy rule	Actual disturbance (unit variance)	
		du_1^a	du_3^a
du_1^a	$r = 0.98\ m$	4.94	4.10
du_3^a	$r = 3.33\ m$	6.44	5.10

is not considered because in this case an optimal β is indeterminate ruling out the existence of unique reaction functions.

Consider first the monetary rule. For a demand disturbance in view of the slight benefits of divergence control the aggregate policy $r^d = 0.71\ m^d$ of Table 6.4 implies that $r = 0.71\ m$ and $r^* = 0.71\ m^*$ for the two countries. From Table 6.7 it can be seen from iteration 1 that the benefits to country 1 from reneging and pursuing an optimal policy given country 2's policy $r^* = 0.71\ m^*$ are very small and the eventual Nash equilibrium results in a rather greater (but still small) welfare loss. By contrast, for a supply disturbance, both the short-term gains and the long-term losses are quite considerable and at the Nash equilibrium the monetary rule is far stronger ($r = 3.32\ m$) than at the cooperative policy ($r = 0.53\ m$). These results suggest that if a joint policy is agreed between two countries in the form of a monetary rule, a supply shock is far more likely to undermine that agreement than a demand shock if the countries indulge in short-sighted behaviour.

For a nominal income rule from Table 6.9 both types of disturbances result in similar incentives to renege with only a slight long-term loss for a demand shock as against a significant long-term loss for a supply shock. This suggests that even a far-sighted country would be tempted to renege when faced with a demand disturbance. An interesting point to note about both monetary and nominal income rules is that if countries are engaged in a Nash closed-loop game it may actually pay if they are wrong about the nature of the shock hitting the two countries. Thus from Table 6.8 a monetary rule designed with u_1^a in mind results in a better outcome than a rule designed form u_3^a even when u_3^a actually occurs. It follows that countries pursuing a monetary rule benefit if they wrongly rule out the possibility of a u_3^a shock. For a nominal income rule Table 6.10 indicates that countries will benefit if they are wrong on all occasions. If a u_1^a shock occurs it is better if countries expect a u_3^a shock and if a u_3^a shock occurs it is better if they expect a u_1^a shock!

Table 6.9. *The Cournot adjustment process for policy rule:* $r = \beta(y+p)$

	Expected disturbance du_1^a, $\text{var}(du_1) = \text{var}(du_1^*) = 0.5\,dt$, $\text{cov}(du_1, du_1^*) = 0$				Expected disturbance du_3^a, $\text{var}(du_3) = \text{var}(du_3^*) = 0.5\,dt$, $\text{cov}(du_3, du_3^*) = 0$			
	Country 1		Country 2		Country 1		Country 2	
Iteration number	β	Welfare loss	β	Welfare loss	β	Welfare loss	β	Welfare loss
0	1.01	4.47	1.01	4.47	0.38	4.13	0.38	4.13
1	0.49	4.20	1.10	4.89	0.98	3.64	0.38	5.10
2	0.49	4.56	0.43	4.51	0.98	4.62	1.10	4.47
3	0.42	4.55	0.43	4.56	1.07	4.61	1.10	4.59
4	0.42	4.56	0.42	4.56	1.07	4.59	1.08	4.59
5	0.42	4.56	0.42	4.56	1.08	4.59	1.08	4.59

Table 6.10. *Welfare loss for Nash equilibrium for policy rule: $r = \beta(y+p)$*

Expected disturbance (unit variance)	Policy rule	Actual disturbance (unit variance)	
		du_1^a	du_3^a
du_1^a	$r = 0.42\ (y+p)$	4.56	4.14
du_3^a	$r = 1.08\ (y+p)$	4.49	4.59

For both monetary and nominal income rules that closed-loop Nash equilibrium is symmetrical. For our final regime, a price level rule, this is no longer the case. Starting at the best cooperative policy which in this case is minimal control, the Cournot process converges to an asymmetrical outcome with the country that moves first benefiting considerably at the expense of the other, in both the short-term and long-term. (Note that we have imposed an upper limit of $\beta = 10$ for all the rules.) For both demand and supply shocks the country that reneges moves immediately to the strongest possible feedback rule $r = 10p$ leaving the second country's best rule as minimal control (see Tables 6.11 and 6.12). Tables 6.13 and 6.13 report results starting at $r = p$ rather than minimal control. For disturbance du_1^a the country that moves *first* loses out and the second country benefits whereas for a u_3^a disturbance, the final outcome is a symmetrical Nash equilibrium at $r = 0.76\,p$. As yet we have not been able to find a starting point which yields a symmetrical equilibrium for a u_1^a shock.

These results suggest that the incentive to renege on the cooperative monetary or nominal income rule, while adhering to the overall constraint of such a simple rule, is not large. By contrast, the price rule offers a very considerable incentive to renege, particularly since the country that reneges first secures long run, not just short run, benefit. It seems that the price rule, and policies like it, are inimical to international cooperation.

VII Conclusions

In this paper, we have been concerned with the design of macroeconomic policy in a stochastic interdependent world. This analysis was conducted in terms of a model that is rather more complex in its interactions and dynamics than is usual in the analytical literature on policy interdependence, allowing for wage/price dynamics, asset accumulation and exchange rate dynamics.

Section II set out techniques for deriving optimal control rules in

Table 6.11. *The Cournot adjustment process for policy rule:* $r = \beta p$

	Expected disturbance du_1^a $\mathrm{var}(du_1) = \mathrm{var}(du_1^*) = 0.5\,dt$ $\mathrm{cov}(du_1, du_1^*) = 0$				Expected disturbance du_3^a $\mathrm{var}(du_3) = \mathrm{var}(du_3^*) = 0.5\,dt$ $\mathrm{cov}(du_3, du_3^*) = 0$			
	Country 1		Country 2		Country 1		Country 2	
Iteration number	β	Welfare loss	β	Welfare loss	β	Welfare loss	β	Welfare loss
0	0.006	5.18	0.006	5.18	0.02	4.23	0.02	4.32
1	10.00	4.74	0.006	8.53	10.00	2.63	0.02	10.68
2	10.00	4.74	0.0002	8.53	10.00	2.55	0.0002	10.64
3	10.00	4.73	0.0002	8.53	10.00	2.55	0.0002	10.64
4	10.00	4.73	0.0002	8.53	10.00	2.55	0.0002	10.64
5	10.00	4.73	0.0002	8.53	10.00	2.55	0.0002	10.64

Table 6.12. *Welfare loss for Nash equilibrium for policy rule: $r = \beta p$*

Expected disturbance (unit variance)	Policy rule	Actual disturbance (unit variance)	
		du_1^a	du_3^a
du_1^a	$r = 10\,p$ $(r = 0.0002\,p)$*	4.73 (8.53)	2.55 (10.64)
du_3^a	$r = 10\,p$ $(r = 0.0002\,p)$*	4.73 (8.53)	2.55 (10.64)

* Country 2 in brackets (where different).

stochastic rational expectations models with complex dynamics, and also showed how restricted or simple optimal rules may be derived. This latter aspect of policy design assumes significance in view of the importance attached to restrictions on policy design (e.g. monetary or nominal income targeting) in current policy debates. But simple or restricted design carries with it the cost that certainty equivalence no longer holds, so that the design of policy is no longer independent of the nature of the shocks perturbing the system. Methods of handling these complications are set out in Sections II and IV. In addition, Section II sets out the method for deriving the Nash solution to a two country policy game as the outcome of a Nash game between the countries pursuing simple rules. The Nash game between the countries pursuing the optimal rule is shown to lead to an ever-expanding state vector.

In our subsequent analysis, we applied these methods to an examination of the effectiveness of the full optimal policy and a variety of simple rules in stabilisation. What our results bring out clearly is the divergence between policy design in the single open economy and in the global economy. This divergence arises from externalities in policy design in an interdependent world. It leads to the possibility of free-riding behaviour; as countries renege on cooperative policy design.

This raises the question as to how best to contain such free-riding behaviour within a system of international policy coordination. Advances in the theory of noncooperative game theory suggest that this may not be as intractable a problem as is usually assumed.[14] This is because forms of tit-for-tat strategy can be shown to be rather robust strategies in dealing with repeated games of the prisoner's dilemma type so frequently encountered in problems of international policy coordination. However, our results also show that not all aspects of the international policy game are of the prisoner's dilemma type. This is illustrated by the Nash game under

Table 6.13. *The Cournot adjustment process for policy rule: $r = \beta p$*

	Expected disturbance du_1^a; $\mathrm{var}(du_1) = \mathrm{var}(du_1^*) = 0.5\,dt$; $\mathrm{cov}(du_1, du_1^*) = 0$				Expected disturbance du_1^a; $\mathrm{var}(du_3) = \mathrm{var}(du_3^*) = 0.5\,dt$; $\mathrm{cov}(du_3, du_3^*) = 0$			
	Country 1		Country 2		Country 1		Country 2	
Iteration number	β	Welfare loss	β	Welfare loss	β	Welfare loss	β	Welfare loss
0	1.00	7.69	1.00	7.69	1.00	7.23	1.00	7.23
1	0.17	6.99	1.00	5.62	0.61	7.03	1.00	5.56
2	0.17	9.30	10.00	5.30	0.61	6.67	0.89	5.55
3	0.0002	8.53	10.00	4.73	0.67	6.66	0.89	5.80
4	0.0002	8.53	10.00	4.73	0.66	6.46	0.84	6.65
5	0.0002	8.53	10.00	4.73	0.70	6.46	0.84	6.65
20	0.0002	8.53	10.00	4.73	0.76	6.16	0.76	6.16

Table 6.14. *Welfare loss for Nash equilibrium for policy rule: $r = \beta p$*

Expected disturbance (unit variance)	Policy rule	Actual disturbance (unit variance)	
		du_1^a	du_3^a
du_1^a	$r = 10\,p$	4.73	2.55
	$(r = 0.0002\,p)$*	(8.53)	(10.64)
du_3^a	$r = 0.76\,p$	6.82	6.16

* Country 2 in brackets (where different).

the price rule, where one country can secure a lasting gain at the expense of the other. This arises because, in this game, tit-for-tat amounts to the threat to destabilise the system totally, and may therefore not be credible. Similar results may well apply more generally to strategies that rely on manipulating the exchange rate under a regime of floating rates to secure domestic objectives at the expense of global aims.

It may be that other forms of macropolicy threats may be credible in containing this form of behaviour, and this is an issue that needs more detailed consideration. Alternatively, it may be that a wider class of threats, involving factors outside the field of international policy, is required to sustain international cooperation. If this is so, our analysis offers a possible additional argument for simple rules, particularly concerning the targeting of the money supply or nominal income: that by formulating policy in these terms, one may rule out the noncooperative forms of behaviour implied by the price level rule and by full optimal behaviour.

Appendix

In this appendix, we set out the models analysed in the main paper in the form of equation (II.1); that is,

$$\begin{bmatrix} dz \\ dx^e \end{bmatrix} = A \begin{bmatrix} z \\ x \end{bmatrix} dt + Bw\,dt + dv \qquad \text{(A.1)}$$

(i) The single country model

We let $n = \int_{-\infty}^{t} y\,ds$, so that $dn = y\,dt$. We also treat y^* as given exogenously, and therefore incorporated into the disturbance terms in

(III.1) and (III.3). Then we may write (III.1)–(III.9) in the form of (A.2), where $x = e$ and:

$$A = \begin{bmatrix} -\psi_1 & 0 & \psi_1\alpha_3 & \psi_1(\psi_4\alpha_2\delta_1-\alpha_1) & -\psi_1\psi_4\alpha_2 & 0 & \psi_1(\alpha_1+\psi_4\alpha_2\delta_2) & 0 & \psi_1(\alpha_1+\psi_4\alpha_2\delta_2) \\ \psi_2\gamma_1 & -\psi_2 & \psi_2\gamma_3 & 0 & \psi_2 & 0 & 0 & 0 & 0 \\ -\phi_1 & 0 & 0 & -(\psi_4\delta_1+\phi_2) & \psi_4 & 0 & \phi_2-\psi_4\delta_2 & 0 & \phi_2-\psi_4\delta_2 \\ 0 & 0 & 0 & -\psi_3 & \psi_3 & \psi_3\beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_4\delta_1 & -\psi_4 & 0 & \psi_4\delta_2 & 0 & \psi_4\delta_2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$z = \begin{bmatrix} y \\ m \\ v \\ w \\ p \\ n \\ w^* \\ r^* \end{bmatrix}, \quad B = \begin{bmatrix} -\psi_1\alpha_2 & -\psi_1\alpha_4 \\ -\psi_2\gamma_2 & 0 \\ 0 & -\phi_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, dv = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} du_1 \\ du_2 \\ du_3 \\ du_4 \\ du_5 \end{bmatrix}$$

$$w = \begin{bmatrix} r \\ \delta \end{bmatrix} \tag{A.2}$$

and where $\delta_1 = \theta c_1$, $\delta_2 = (1-\theta c_1)$.

(ii) The two country model

For this case, from equations (III.1)–(III.7) we may write the system in the form of (A.1) where A is given in Table A.1, where $x = e$, and where:

$$z = \begin{bmatrix} y \\ m \\ v \\ w \\ p \\ n \\ y^* \\ m^* \\ v^* \\ w^* \\ p^* \\ n^* \end{bmatrix}, \quad B = \begin{bmatrix} -\psi_1\alpha_2 & -\psi_1\alpha_4 & 0 & 0 \\ -\psi_2\gamma_2 & 0 & 0 & 0 \\ 0 & -\phi_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\psi_1\alpha_2 & -\psi_1\alpha_4 \\ 0 & 0 & -\psi_2\gamma_2 & 0 \\ 0 & 0 & 0 & -\phi_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}, dv = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} du_1 \\ du_2 \\ du_3 \\ du_1^* \\ du_2^* \\ du_3^* \end{bmatrix}$$

$$w = \begin{bmatrix} r \\ s \\ r^* \\ s^* \end{bmatrix} \tag{A.3}$$

Exploiting the symmetry of this model, we may follow Aoki (1981) by decomposing this model into two orthogonal components, the aggregate model and the divergence model. Thus let $z^a = z + z^*$, $u_i^a = u_i + u_i^*$,

Table 6A.1. *A matrix for the two-country model*

$$
\begin{bmatrix}
-\psi_1 & 0 & \psi_1\alpha_3 & \psi_1(\psi_4\alpha_2\delta_1-\alpha_1) & -\psi_1\psi_4\alpha_2 & 0 & \psi_1\alpha_5 & 0 & 0 & \psi_1(\alpha_1+\psi_4\alpha_2\delta_2) & 0 & 0 & \psi_1(\alpha_1+\psi_4\alpha_2\delta_2) \\
\psi_2\gamma_1 & -\psi_2 & \psi_2\gamma_3 & 0 & \psi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\phi_1 & 0 & 0 & -(\psi_4\delta_1+\phi_2) & \psi_4 & 0 & \phi_3 & 0 & 0 & \phi_2-\psi_4\delta_2 & 0 & 0 & \phi_2-\psi_4\delta_2 \\
0 & 0 & 0 & -\psi_3 & \psi_3 & \psi_3\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \psi_4\delta_1 & -\psi_4 & 0 & 0 & 0 & 0 & \psi_4\delta_2 & 0 & 0 & \psi_4\delta_2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi_1\alpha_5 & 0 & 0 & \psi_1(\alpha_1+\psi_4\alpha_2\delta_2) & 0 & 0 & -\psi_1 & 0 & \psi_1\alpha_3 & \psi_1(\psi_4\alpha_2\delta_1-\alpha_1) & -\psi_1\psi_4\alpha_2 & 0 & -\psi_1(\alpha_1+\psi_4\alpha_2\delta_2) \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_2\gamma_1 & -\psi_2 & \psi_2\gamma_3 & 0 & \psi_2 & 0 & 0 \\
\phi_3 & 0 & 0 & \phi_2-\psi_4\delta_2 & 0 & 0 & -\phi_1 & 0 & 0 & -(\psi_4\delta_1+\phi_2) & \psi_4 & 0 & -(\phi_2-\psi_4\delta_2) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\psi_3 & \psi_3 & \psi_3\beta_1 & 0 \\
0 & 0 & 0 & \psi_4\delta_2 & 0 & 0 & 0 & 0 & 0 & \psi_4\delta_1 & -\psi_4 & 0 & -\psi_4\delta_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$r^a = r + r^*$, $s^a = s + s^*$. Then the aggregate model (denoted by an 'a' superscript) is given by (A.1) with

$$A^a = \begin{bmatrix} -\psi_1(1-\alpha_5) & 0 & \psi_1\alpha_3 & \psi_1\psi_4\alpha_2 & -\psi_1\psi_4\alpha_2 & 0 \\ \psi_2\gamma_1 & -\psi_2 & \psi_2\gamma_2 & 0 & \psi_2 & 0 \\ -(\phi_1-\phi_3) & 0 & 0 & -\psi_4 & \psi_4 & 0 \\ 0 & 0 & 0 & -\psi_3 & \psi_3 & \psi_3\beta_1 \\ 0 & 0 & 0 & \psi_4 & -\psi_4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad z^a = \begin{bmatrix} y^a \\ m^a \\ v^a \\ w^a \\ p^a \\ n^a \end{bmatrix}$$

$$B^a = \begin{bmatrix} -\psi_1\alpha_2 & \psi_1\alpha_4 \\ -\psi_2\alpha_2 & 0 \\ 0 & -\phi_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad w^a = \begin{bmatrix} r^a \\ s^a \end{bmatrix}, dv^a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} du_1^a \\ du_2^a \\ du_3^a \end{bmatrix} \qquad \text{(A.4)}$$

and where the x vector has zero dimension. It will be noted that the aggregate model contains no free variables.

Using a 'd' superscript to denote the divergence model, and letting $z^d = z - z^*$, $u_i^d = u_i - u_i^*$, $r^d = r - r^*$, $s^d = s - s^*$, then the divergence model is given by (A.1) with:

$$A^d = \begin{bmatrix} -\psi_1(1+\alpha_5) & 0 & \psi_1\alpha_3 & \psi_1(\psi_4\alpha_2(\delta_1-\delta_2)-2\alpha_1) & -\psi_1\psi_4\alpha_2 & 0 & 2\psi_1(\alpha_1+\psi_4\alpha_2\delta_2) \\ \psi_2\gamma_1 & -\psi_2 & \psi_2\gamma_3 & 0 & \psi_2 & 0 & 0 \\ -(\phi_1+\phi_3) & 0 & 0 & -(\psi_4(\delta_1-\delta_2)+2\phi_2) & \psi_4 & 0 & 2(\phi_2-\psi_4\delta_2) \\ 0 & 0 & 0 & -\psi_3 & \psi_3 & \psi_3\beta_1 & 0 \\ 0 & 0 & 0 & \psi_4(\delta_1-\delta_2) & -\psi_4 & 0 & 2\psi_4\delta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$z^d = \begin{bmatrix} y^d \\ m^d \\ v^d \\ w^d \\ p^d \\ n^d \end{bmatrix}, \quad B^d \begin{bmatrix} -\psi_1\alpha_2 & -\psi_1\alpha_4 \\ -\psi_2\gamma_2 & 0 \\ 0 & -\phi_0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad dv^d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} du_1^d \\ du_2^d \\ du_3^d \end{bmatrix}, \quad w^d = \begin{bmatrix} r^d \\ s^d \end{bmatrix} \qquad \text{(A.5)}$$

and where $x = e$.

NOTES

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1 The rational expectations equilibria considered in this paper may be thought of as Stackelberg equilibria, with the government as leader and the private

sector as follower. By reneging consistently, the government relinquishes its leadership role.

2 We can formally evaluate the incentive to renege faced by a myopic government by assuming a given discount rate. This requires us to consider the derivation of optimal rules with time-discounting, as in Levine and Currie (1984).

3 This solution is based on Blanchard and Kahn (1980) and Dixit (1980). For further details, see Currie and Levine (1982).

4 This result does not carry over to the discrete time cases. See Levine and Currie (1984).

5 For further discussion, see Levine and Currie (1984).

6 For further analysis, see Levine and Currie (1984).

7 This partial adjustment process is incorporated for technical reasons. If ψ_4 is large, as it is in the subsequent analysis, it is as though (III.5) and (III.6) hold instantaneously.

8 We are concerned with deviations of the system from long run equilibrium. Assuming that the current account is in equilibrium in this long run equilibrium, it is plausible to assume that transitory current account imbalances cause no divergence between *ex ante* domestic and foreign interest rates expressed in the same currency.

9 Since foreign wages and interest rates are now separately determined (with persistence), equations (3.10) and (3.11) are dropped. Note that $e^* = -e$.

10 Non-stationary disturbance require the addition of integral control to tie down nominal variables, though at the cost of making instruments non-stationary. We exclude this case here.

11 FORTRAN programs have been written to implement the solution procedures of section II.

12 Strictly speaking, setting $\beta = 0$ for the other rules leads to indeterminacy of nominal variables. However, this problem can be avoided and the loss made as close to zero as desired by choosing β to be positive but small. We refer to this case as 'minimal control' in what follows.

13 We have also considered the opposite extreme, u_t and u_t^* perfectly correlated. The same qualitative conclusions reached for uncorrelated disturbances still hold.

14 See Axelrod (1983) and Basar and Olsder (1982). For further discussion, see Currie (1985).

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COMMENT DAVID K. H. BEGG

David Currie and Paul Levine have given us an interesting paper. They emphasise the need for *simple* rules, yet, aside from some general remarks about the ease of securing international cooperation, they offer little formal justification for the advantages of simple rules. I begin by arguing that the issues of time consistency and credibility can be used to suggest why we might be interested in formulating rules in a simple way.

Given a deterministic model with full information, there is no distinction between open-loop and closed-loop rules, and I suggest optimal policy design is as follows. If future actions can truly be precommitted, the possibility of time inconsistency simply does not arise. If precommitment is not possible, a time consistent plan can be formulated by adopting the backward recursion of dynamic programming, which recognises that bygones are bygone as real time elapses and as decisions can be reconsidered. In a deterministic perfect foresight model, time inconsistent policies are simply *incredible*.

Now consider a stochastic model. Partition all possible rules into those components based exclusively on today's information set and those innovation-contingent rules which specify how new information is reflected in policy. Notice two things: first, it will generally pay to use new information as it arrives. Innovation-contingent components are valuable to policy makers. Second, the preceding perfect foresight discussion of time consistency and credibility formally carries over to the stochastic model, provided it is applied to the part of the rules which conditions on today's information set.

Suppose, however, that it is not costless for agents to monitor the actions of the policy maker or to diagnose and process new information. There will then arise an ambiguity. Suppose policy this period is different from agents' expectation of policy this period, conditional on information in some previous period. Should agents believe a policy maker who says that this merely represents the implementation of a previously announced innovation-contingent feedback rule? Or does it represent an attempt to renege on that part of the policy rule based purely on previous information? In such circumstances, there may be much to be said for the pursuit of simple rules if they are easier for agents to understand and to monitor. In so doing, they may remove most of the problems which arise in a world in which reneging is easy, whilst simultaneously preferring the policy maker's ability to implement an innovation-contingent feedback rule which blunts the effects of genuine surprises as they hit the system.

Ideally, of course, reneging should be endogenous to the model, and the asymmetric information on which its possibility depends should be made explicit. In such a framework, one could then determine credibility endogenously and thus conduct an examination of the incentives to cheat today at the cost of losing reputation for the future.

Before leaving the overall framework of the paper, I note that the authors adopt an asymptotic approximation to the policy maker's objective function. Thus the paper really complicates the dynamics of the structural constraints for the sake of simplifying the dynamics of the intertemporal objective function. For some purposes this may be an advance; in other cases it may not. For example, many people would argue that a vital component of the Thatcher revolution in policy attitudes in Britain is the changed priority of the future against the present.

I now consider the main sections of the paper. Section II presents a standard linear-quadratic forward looking stochastic model in which the second best time-invariant rule is compared with the 'fully optimal' rule. The latter is time inconsistent, and I have already made clear my objection to this terminology. To be credible, such a rule must rely either on the ability to make binding precommitments of policy or on its voluntary precommitment through an analysis of the cost of losing reputation when this is recognised as endogenous. Section II then specifies a two-country closed-loop Nash game. Although preferable to an open-loop Nash game, this specification still seems to me to have its drawbacks. Individual players are modelled as assessing their own strategies subject to the belief that the other player's reaction function is invariant, yet it is knowable that different strategies by one player will cause the other player to revise that reaction function. The Lucas critique applies in game theory too.

Section III specialises the structural model to a familiar open economy model with sluggish domestic price adjustment and perfect international capital mobility. These seem reasonable. However, the loss function contains a term in the square of the *level* of prices. This is certainly not the same as the notion that inflation is undesirable, nor is it a trivial discrepancy: it is a major conclusion of the paper that simple rules formulated as *price* rules behave rather differently from other types of simple rule. Before attributing too much importance to this result, I should like to know the extent to which it reflects the particular specification of the loss function. Some sensitivity analysis would be valuable here.

Section IV begins with the third best problem of simple monetary rules, for example having nominal interest rates proportional to the price level or to nominal income. At some stage, the authors refer to this as 'targeting'. This seems to me rather misleading. Even if one wishes to target an intermediate variable, it is generally optimal to use all available policy instruments to achieve this end. Next, the robustness of various rules to specific shocks is examined. As I read it, the authors perturb each disturbance separately. Yet in Section II they show that their simple rules do not obey certainty equivalence, and that the parameters of these rules are functions of the variance-covariance matrix of disturbances. That being so, it would be nice if sensitivity analysis utilised shocks with a variance-covariance matrix similar to that implicitly embodied in the parameters of the simple rules whose robustness is being investigated. This leads on to a related point. The authors' search for robust simple rules is very much a hit-and-miss affair. Particular specifications are contemplated, as if out of a hat, and their properties assessed. Why not start from the other end? In Section II we are shown how to calculate the optimal rule. Alternatively, we could calculate the optimal linear time-invariant rule. From one of these more general specifications, could we not examine the coefficient parameters and hence make a more informed judgement about which exclusion restrictions to impose in order to acquire a simple rule?

Having examined at length the question of a monetary rule alone, Currie and Levine then consider the simultaneous choice of simple monetary and fiscal rules. Here simplicity means decoupling: each policy variable has a rule with a single argument. Basically, the authors conclude that fiscal policy matters in their model, but that it should be accompanied by a monetary rule which takes the form of a price rule. Again, my earlier remark applies. Without an examination of alternative loss functions, we cannot be sure at this stage that the preference for a price rule does not reflect the particular way prices enter the loss function.

Section V extends the analysis to two countries, using the Aoki trick of dividing the problem into two orthogonal ones, the first dealing with aggregate variables across countries and the second dealing with divergences between countries. Here I find the conclusions sensible and appealing. There is less scope for monetary policy than fiscal policy to affect aggregate variables. And it is shown that the interaction of unharmonised national policies can easily lead to undesirable outcomes.

Section VI deals briefly with 2-country non-cooperative games, and begins to examine the incentive to renege. Short term gains are compared with long term losses from a reduction in credibility. Given my earlier remarks, I applaud the effort in this section. As yet, as the authors recognise, this work is rather preliminary. For example, time discounting is likely to be important in evaluating how to trade off present and future. Here the simplified intertemporal objective function is concealing more than it reveals. Similarly, conjectures which are based on the Cournot-adjustment of the other player's reaction function are clearly systematically and knowably in error.

COMMENT KOICHI HAMADA

I feel very privileged to have this opportunity to discuss the paper by Professor Currie and Dr Levine, because it is rich in content and neatly written.

In the first part of this paper, the authors sort out various ramifications of optimal control. Being a 'back of the envelope' theorist myself, I learned a lot from their clear exposition of modern control technology combined with the assumption of rational expectations. There are so many layers of categories: constrained variables vs. free variables in initial conditions, full optimisation vs. time-invariant rules, general vs. simple rules, joint utility maximisation vs. Cournot-Nash games, closed loop vs. open loop, time consistent vs. unconstrained rules, and so forth. This paper sorts out these complex layers. The description is compact but gives us a full understanding of the basic structure of the issues.

The second part of this paper develops an expectations augmented Phillips curve model consisting of two countries. Since a time-invariant feedback rule is dependent on the covariance matrix, they examine simple rules corresponding to each type of shock on LM, IS and supply curves, look for the best rule for each particular kind of shock, and examine vulnerability of the system under the simple rule against types of shock other than the system is designed for. The most reliable feedback rule is the price rule for the domestic economy, but this rule collapses when two countries play a game with these simple rules.

When I was reading this paper, I was reminded of a conversation I had with Dr. Hirotsugu Akaike, the founder of AIC of the time series model. He was quite successful in reducing the variance in quality of concrete production by applying some feedbacks to the system according to his methods. He also succeeded in smoothing temperature fluctuations in electricity generation. Incidentally his institute is now studying administrative reform (fiscal consolidation) in Japan. He has to defend the raison d'être of his institute by pointing out that the feedback system saved trillions of Yen. Dr Akaike is interested in the possibility of applying feedback systems to our economy as well. This paper gives us hope that some day, unfortunately seemingly not in the immediate future, we may be able to control the economy like a concrete production process.

One of the most surprising results of this paper is that the simple price rule, which is stabilising in a closed economy, can cause total global destablisation in a two country context. This is somewhat counter-intuitive, because a combination of two stable systems of differential equations will normally result in a stable system unless off-diagonal blocks expressing the interdependence are extremely important. My first question is how, in an analytic as well as an economic sense, the authors interpret this phenomenon.

My next question concerns the statement of the authors on the last page, saying 'in this game, tit-for-tat amounts to the threat of destabilising the system totally, and may therefore not be credible'. This is an interesting remark, but I do not consider that this statement is the result of deduction from the analytical part of the paper. I would like to hear more clarification.

My comments are on the economics of these interesting experiments. My first comment concerns the validity of the asymptotic system. We are all constrained by history, that is by initial conditions at any time. It may take a very long time to achieve the asymptotic state of stochastic equilibrium. I wonder how fast is the convergence speed of the deterministic part of the system to a stationary equilibrium. It would be possible to tell the speed of convergence by examining the magnitude of the stable roots of the system. If it takes a rather short time to reach equilibrium, these experiments are useful although we have to add some learning period for agents to understand the working of the system. If it takes a rather long time, then historical programming taking account of the initial conditions is more relevant than the analysis of stochastic equilibria.

Incidentally, I was talking about this paper with Dr Georges de Ménil on the plane, and his related point, which I should like to quote if he does not mind, was that if we are examining the properties of long run equilibrium, why do we need inertia like the short run Phillips curve? If the asymptotic state is in question, does it not suffice to examine only a new-classical model with a vertical long run Phillips curve?

My second comment concerns the use of simple rules. As a pedagogical device it is fine, but I have an ambivalent feeling about the adoption of very simple rules. In this world of rational expectations, agents know the structure correctly and they know exactly what the government is doing. The simpler the rule, the easier it is for private agents to learn about it.

This is an attractive side of the analysis. On the other hand, I cannot help feeling some uneasiness about the asymmetry in the degree of sophistication assumed between private agents and governments. Private agents know economic structures and government policies completely, and pick the right saddle point paths in spite of various disturbances. Governments, on the other hand, can only choose the value of β. According to Professor Morishima, in the UK the best students go after graduation to the academic world as university or high school teachers, the next best go to public service, and the least good to the private business sector. In Japan, on the other hand, I feel that the best and brightest go to the public service, next to private business, and perhaps the least bright to the academic world. Thus while this model may reflect some aspects of British society, the strong asymmetry in the model does not seem to fit my conception of the real world.

Third, on the simple rules. First of all, though simple minded the government has perfect control of the nominal interest rate. Thus monetary shocks u_2 are completely offset. Imagine a world where a case of missing money appears; can the government still just smooth the shock as assumed here? A more basic question is why one has to choose the *nominal* interest rate. The well known criticism of Keynesian economics developed by the monetarist school was that if one only aims at the nominal interest rate, one cannot distinguish whether a high level of the nominal rate reflects a high real rate of interest that has a depressionary effect, or a high rate of inflationary expectations reflecting a boom. If you stick to the rule of trying to keep nominal interest rates constant in an inflationary situation, you may end up with severe inflation because the attempt may reduce the real rate. Thus it is no wonder that the price rule will do a better job. Here again, even though control of the real rate of interest is hard, the government could be a little smarter.

Finally, there is an interesting structure connecting the choice among simple rules and the possible outcomes under any given rule. Here again temptation arises for the government to renege on the committed rule that private agents are counting on.