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Cost-of-Living Indexes and Price Indexes for U.S. Meat and Produce, 1947–1971 *

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I. INTRODUCTION

CONSUMER price indexes are usually computed as Laspeyres indexes of the components of the consumer budget. For example, the official Consumer Price Index for the U.S. is basically a Laspeyres index, although various ad hoc adjustments are included. A Laspeyres price index provides a comparison of the cost of a fixed bundle of goods relative to the cost in some base period. Such an index is not a "cost-of-living" index because it does not reflect the possibilities of substituting away from goods that become relatively more expensive. A cost-of-living index would provide a comparison of the cost of maintaining a particular level of well-being relative to the cost in the base period. It is well known that a Laspeyres index must be greater than, or equal to, a cost-of-living index, i.e., use of a Laspeyres index as an estimator for a cost-of-living index will result in an upward bias.

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The principal reason that Laspeyres indexes have not been replaced in use by cost-of-living indexes is that construction of the latter requires a knowledge of the utility function representing consumer preferences. In practice, the best that could be done would be to assume a particular utility function, estimate its unknown parameters, and construct a cost-of-living index using the estimated consumer preferences. This has been done by various researchers, for example, Tran Van Hoa (1969), Goldberger and Gamaletsos (1970), and Thangiah (1973).

The utility functions which have been specified for estimating consumer preferences have invariably maintained restrictions of additivity or homotheticity on consumer behavior. If additivity or homotheticity is not consistent with the set of data under consideration, the implied cost-of-living indexes may not be reliable. It would be desirable to specify a utility function which does not maintain additivity or homotheticity. The corresponding estimated cost-of-living indexes could then be compared with cost-of-living indexes from restricted utility functions and with price indexes such as the Laspeyres.

Christensen, Jorgenson, and Lau (1972) have proposed the translog utility function for estimating consumer preferences. The translog function is attractive because it does not maintain additivity or homotheticity. Additivity and homotheticity, however, can be achieved by imposing linear restrictions on the parameters of the translog function, thus permitting statistical tests of their validity. The translog function can be used to represent consumer preferences via a direct utility function or an indirect utility function. The indirect utility function leads more conveniently to a cost-of-living index, thus we use the indirect translog utility function as our most general representation of consumer behavior. For comparison, we also specify that consumer preferences can be represented by the following utility functions: the linear logarithmic utility function; the constant elasticity of substitution (CES) utility function; the Klein-Rubin (1947) utility function; a generalization of the Klein-Rubin utility function; Houthakker's (1960) indirect addilog utility function; the additive indirect translog utility function; and the homothetic indirect translog utility function. We compute the cost-of-living indexes implied by all of these utility functions.

It would be desirable to include many commodities in this study. This is precluded by considerations both of cost and of the availability of computer software. The software we have available would allow for the estimation of a translog utility function with six commodities. The cost of estimation increases much more than in proportion to the number of commodities. Rather than attempting to analyze a single set of six commodities, we decided to analyze two sets of data—each including four commodities. The two data sets are for U.S. consumption of meat and produce, 1947–1971.

We estimate the parameters of eight distinct utility functions for both U.S. meat consumption and U.S. produce consumption. For the functions which do not maintain additivity or homotheticity, we decisively reject the hypotheses that these restrictions hold. The indirect translog function dominates all the other functions in the ability to explain the observed budget shares for meat and produce. Thus, we argue that the translog function provides the most reliable cost-of-living indexes for meat and produce. Using the translog indexes as yardsticks, we compare cost-of-living indexes estimated from restrictive models of consumer behavior. We also compare the estimated cost-of-living indexes with the Laspeyres and other price indexes.

II. NEOCLASSICAL UTILITY FUNCTIONS WITH RESTRICTIONS OF HOMOTHETICITY AND ADDITIVITY

A direct utility function relates the level of consumer utility to the levels of consumption of commodities available. We find it convenient to express the direct utility function in the logarithmic form

$$\ln U = F(\ln X_1, \ldots, \ln X_n) \tag{1}$$

Classical utility theory requires the U be monotonic ($\partial U/\partial X_i > 0$) and strictly quasi-concave (have strictly convex indifference curves). A direct utility function will be strictly quasi-concave if the bordered Hessian matrix for U has all its principal minors of order greater than or equal to three alternating in sign, beginning with a plus.[1] Necessary conditions for the maximization of utility are $\partial U/\partial X_i = \lambda p_i$, where λ is the marginal utility of total expenditure.

A direct utility function, together with the necessary conditions for utility maximization, imply the existence of an indirect utility function, defined on total expenditure and the prices of all commodities.[2] The indirect utility function is homogeneous of degree zero and can be expressed as a function of the ratios of prices of all commodities to total expenditure. For convenience, we express the indirect utility function in the logarithmic form

[1] See Katzner (1970, pp. 210–211) for a clear statement of necessary and sufficient conditions for strict convexity of indifference curves.

[2] The concept of an indirect utility function is due to Hotelling (1932). See Lau (1969ab) and Diewert (1974) for many useful theorems relating the properties of direct and indirect utility functions.

$$\ln V = G(\ln p_1^*, \ldots, \ln p_n^*) \tag{2}$$

where $p_i^* = p_i/M$ are the normalized prices, and M is total expenditure on all commodities $\Sigma p_i X_i$. The indirect utility function has monotonicity and convexity conditions which correspond to those for the direct utility function. The monotonicity conditions are $\partial V/\partial p_i^* < 0$. The indirect utility function will correspond to strictly convex indifference curves if it is strictly quasi-convex. The indirect utility function will be strictly quasi-convex if the bordered Hessian for V has all negative principal minors of order greater than two.

A direct utility function U is said to be homothetic if it can be written as a monotonic transformation of a function which is homogeneous of degree one in the X_i. Similarly, an indirect utility function V is said to be homothetic if it can be written as a monotonic transformation of a function which is homogeneous of degree one in the p_i^*. An indirect utility function is homothetic if, and only if, the corresponding direct utility function is also homothetic.

A direct utility function is said to be additive if it can be written

$$\ln U = \sum_{i=1}^{n} \ln U^i(X_i) \tag{3}$$

where each of the functions U^i depends only on one of the commodities consumed, X_i. Similarly, an indirect utility function is said to be additive if it can be written

$$\ln V = \sum_{i=1}^{n} \ln V^i(p_i^*) \tag{4}$$

In general an additive indirect utility function does not correspond to an additive direct utility function.

The behavioral implications of particular utility functions can be illuminated conveniently by examining the corresponding price and expenditure elasticities of demand. For any utility function there are restrictions among these elasticities as shown by the Slutsky (1915) equations

$$\eta_{ij} = w_j\sigma_{ij} - w_j\eta_{iM} \qquad (i, j = 1, \ldots, n) \tag{5}$$

where $\eta_{ij} = \partial \ln X_i/\partial \ln p_j$ gives the uncompensated response to a price change with other prices and total expenditure held fixed; $w_j\sigma_{ij} = (\partial \ln X_i/\partial \ln p_j)_{\bar{U}}$ gives the compensated response to a price change with other prices held fixed but allowing total expenditure to adjust to maintain the initial utility level; and $\eta_{iM} = \partial \ln X_i/\partial \ln M$ gives the response to a change in total expenditure with prices held fixed. The σ_{ij} are the Allen (1938) Elasticities of Substitution (AES). We summarize here well-known restrictions on the elasticities:[3]

$$\sum_{i=1}^{n} w_i \eta_{ij} = -w_j \qquad (j = 1, \ldots, n) \tag{6}$$

$$\sum_{i=1}^{n} w_i w_j \sigma_{ij} = 0 \qquad (j = 1, \ldots, n) \tag{7}$$

$$\sum_{i=1}^{n} w_i \eta_{iM} = 1 \tag{8}$$

$$\sum_{j=1}^{n} w_j \sigma_{ij} = 0 \qquad (i = 1, \ldots, n) \tag{9}$$

$$\sum_{j=1}^{n} \eta_{ij} = -\eta_{iM} \qquad (i = 1, \ldots, n) \tag{10}$$

Homotheticity and additivity restrictions on utility functions imply additional restrictions on price and expenditure elasticities. Homotheticity of a direct or indirect utility function implies that $\eta_{iM} = 1$ for all commodities, but does not impose any explicit restrictions on the price elasticities.

Direct additivity has several implications for the demand elasticities: Theil (1967) has shown that

$$\eta_{iM} > 0 \qquad (i = 1, \ldots, n) \tag{11}$$

and Goldberger (1967) has shown that

$$w_j \sigma_{ij} > 0 \qquad (i \neq j;\ i, j = 1, \ldots, n) \tag{12}$$

Thus, direct additivity rules out inferior goods and complementary goods. In addition Houthakker (1960) has shown that for direct utility functions

$$\eta_{ik}/\eta_{jk} = \eta_{iM}/\eta_{jM} = \sigma_{ik}/\sigma_{jk} \qquad (i \neq k, j \neq k;\ i, j, k = 1, \ldots, n) \tag{13}$$

Thus, direct additivity implies that the relative percentage response of any two commodities to a price change must be the same as the relative response to a change in total expenditure.

The restriction of additivity on the direct utility function is neither necessary nor sufficient for additivity of the corresponding indirect

[3]This summary is taken from Henderson and Quandt (1971). For derivation and discussion, see, for example, Goldberger (1967).

utility function. Houthakker has shown that indirect additivity implies that all commodities have equal percentage responses to the change in any single commodity price.

$$\eta_{ik} = \eta_{jk} \qquad (i \neq k, j \neq k; i, j, k = 1, \ldots, n) \tag{14}$$

III. ALTERNATIVE UTILITY FUNCTIONS FOR MODELING CONSUMER BEHAVIOR

In this section, we present eight different utility functions which can be used to represent consumer behavior. From each utility function, we derive a set of budget share equations which are functions only of the normalized prices.

Christensen, Jorgenson, and Lau (1972) have proposed the translog function to represent consumer preferences with no a priori restrictions of homotheticity or additivity. The translog function can be used as a direct utility function or an indirect utility function. Since indirect utility functions are convenient for estimating true cost-of-living indexes, we deal only with the indirect translog utility function in this paper.[4] The indirect translog utility function V is conveniently written in the logarithmic form

$$-\ln V = \sum_{i=1}^{n} \alpha_i \ln p_i^* + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln p_i^* \ln p_j^* \tag{15}$$

where $p_i^* = p_i/M$ are the normalized prices, M is total expenditure, $\sum_{i=1}^{n} p_i X_i$, and $\beta_{ij} = \beta_{ji}$

Making use of Roy's (1943) identity

$$\frac{p_i X_i}{M} = -\frac{\partial \ln V}{\partial \ln p_i} \bigg/ \frac{\partial \ln V}{\partial \ln M} \qquad (i = 1, \ldots, n)$$

we obtain a set of budget share equations corresponding to the indirect translog utility function

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \ln p_j^*}{\sum_j \alpha_j + \sum_j \sum_i \beta_{ij} \ln p_j^*} \qquad (i = 1, \ldots, n) \tag{16}$$

where $w_i = p_i X_i / M$.

[4] See Christensen, Jorgenson, and Lau (1972) and Christensen and Manser (1974) for estimates of direct, as well as indirect, translog utility functions.

Homotheticity can be imposed on the indirect translog function by restricting sums of second-order parameters to be equal to zero [5]

$$\sum_i \beta_{ij} = 0 \qquad (j = 1, \ldots, n) \tag{17}$$

Imposition of the homotheticity restrictions reduces (16) to

$$w_i = (\alpha_i + \sum_j \beta_{ij} \ln p_j^*) \Big/ \sum_j \alpha_j \qquad (i = 1, \ldots, n) \tag{18}$$

Additivity can be imposed on the indirect translog function by eliminating all interactions between normalized prices.

$$\beta_{ij} = 0 \qquad (i \neq j;\ i,j = 1, \ldots, n) \tag{19}$$

With these restrictions imposed, (15) becomes

$$-\ln V = \sum_i \alpha_i \ln p_i^* + \sum_i \beta_{ii} (\ln p_i^*)^2 \tag{20}$$

and the budget share equations (16) become

$$w_i = \frac{\alpha_i + \beta_{ii} \ln p_i^*}{\sum_j \alpha_j + \sum_j \beta_{jj} \ln p_j^*} \qquad (i = 1, \ldots, n) \tag{21}$$

The simultaneous imposition of homotheticity and additivity reduces the indirect translog form to the indirect linear logarithmic utility function

$$-\ln V = \sum_i \alpha_i \ln p_i^* \tag{22}$$

and the budget share equations (16) to

$$w_i = \alpha_i \Big/ \sum_j \alpha_j \qquad (i = 1, \ldots, n) \tag{23}$$

The budget share equations (23) implied by the indirect linear logarithmic utility function are identical to those obtained by maximizing the direct linear logarithmic function

$$\ln U = \sum_i \alpha_i \ln X_i \tag{24}$$

subject to a budget constraint.

[5] See Christensen, Jorgenson, and Lau (1972) for detailed discussion of the points in this paragraph.

Several utility functions have been proposed, which are generalizations of the direct linear logarithmic function. The Klein-Rubin (1947) function, which adds n nonhomogeneity parameters, can be written[6]

$$\ln U = \sum_i \alpha_i \ln (X_i - \gamma_i) \tag{25}$$

Budget share equations for the Klein-Rubin function can be written

$$w_i = \gamma_i p_i^* + \frac{\alpha_i}{\sum \alpha_j}\left(1 - \sum_j \gamma_j p_j^*\right) \qquad (i = 1, \ldots, n) \tag{26}$$

The linear logarithmic function (24) entails that all elasticities of substitution are equal to unity. The function can be generalized to the CES function, which has elasticities of substitution which are equal, but not necessarily equal to unity. The CES function, which is homothetic and additive, can be used to represent either the direct or indirect utility function. We write the direct CES function

$$\ln U = \frac{\sigma}{\sigma - 1} \ln \sum_i \delta_i X_i^{\frac{\sigma-1}{\sigma}} \tag{27}$$

Budget share equations for the direct CES form can be written

$$w_i = \frac{\alpha_i (p_i^*)^{1-\sigma}}{\sum_j \alpha_j (p_j^*)^{1-\sigma}} \qquad (i = 1, \ldots, n) \tag{28}$$

where $\alpha_i = \delta_i^\sigma$. Several authors have noted that the generalizations to the linear logarithmic form provided by the Klein-Rubin function and the CES function can be made simultaneously.[7] This results in what we shall call the generalized Klein-Rubin function, which can be written

$$\ln U = \frac{\sigma}{\sigma - 1} \ln \sum_i \delta_i (X_i - \gamma_i)^{\frac{\sigma-1}{\sigma}} \tag{29}$$

Budget share equations for the generalized Klein-Rubin function can be written

$$w_i = \gamma_i p_i^* + \frac{\alpha_i (p_i^*)^{1-\sigma}}{\sum_j \alpha_j (p_j^*)^{1-\sigma}} \left[1 - \sum_j \gamma_j p_j^*\right] \qquad (i = 1, \ldots, n) \tag{30}$$

[6] The expenditure system corresponding to the Klein-Rubin utility function is often referred to as the Stone-Geary linear expenditure system; see Stone (1954) and Geary (1949).

[7] See, for example, Christensen (1968), Johansen (1969), Pollak (1971a), Wales (1971), Gamaletsos (1972), and Brown and Heien (1972).

where $\alpha_i = \delta_i^\sigma$ and σ is the elasticity of substitution. If $\sigma = 1$, (30) reduces to (26); while if $\gamma_i = 0, i = 1, \ldots, n$, (30) reduces to (28). Similarly if $\sigma = 1$, (28) reduces to (23); while if $\gamma_i = 0, i = 1, \ldots, n$, (26) reduces to (23).

The Klein-Rubin, CES, and generalized Klein-Rubin functions are all additive; the CES is homothetic, while the Klein-Rubin and generalized Klein-Rubin functions are marginally homothetic. Houthakker (1960) proposed another additive but nonhomothetic function, which has proved to be popular for empirical work. The indirect addilog utility function can be written

$$-\ln V = \ln \sum_i \delta_i (p_i^*)^{1-\sigma_i} \tag{31}$$

Budget share equations for the indirect addilog function can be written

$$w_i = \frac{\alpha_i (p_i^*)^{1-\sigma_i}}{\sum_j \alpha_j (p_j^*)^{1-\sigma_j}} \tag{32}$$

where $\alpha_i = \delta_i(1 - \sigma_i)$. If all the σ_i are equal, (32) is equivalent to (28), the budget shares for the CES form which is homothetic.

All the budget share systems we have described are homogeneous of degree zero in the α_i's. Thus, only $n - 1$ of the α_i's can be estimated subject to some normalization restriction. It is convenient to use the normalization $\sum_i \alpha_i = 1$ for all budget share systems.[8]

IV. ALTERNATIVE ESTIMATES OF U.S. CONSUMER PREFERENCES FOR MEAT AND PRODUCE, 1947–1971

In the previous section, we derived eight distinct sets of budget share equations, which can be used to represent consumer preferences. In this section we implement the various sets of budget share equations for two sets of time series for United States food consumption.

[8] We impose a second normalization restriction, $\sum\sum_{i \neq j} \beta_{ij} = 0$, on the indirect translog form, which is not required for identification or adding up. The original motivation for this restriction in Christensen, Jorgenson, and Lau (1972) was to assure that a translog approximation to an additive function was itself additive. This restriction is not necessary if the translog function is being used as a utility function in its own right. Christensen and Manser (1974) obtained slightly preferable empirical results by maintaining this second normalization restriction. We maintain the restriction here to avoid presenting results which differ from our earlier paper.

In substantially revising their 1972 paper, Christensen, Jorgenson, and Lau (1975) dropped the second normalization. The publication deadline did not permit us to reflect the new developments in CJL (1975). However, the empirical results of this paper would be only slightly affected by recognition of these developments.

A. Data for U.S. Meat and Produce Consumption 1947–1971

There are no official time series available for United States consumption of major types of food. We have constructed price and quantity indexes for four categories of meat and four categories of produce from data given in U.S. Department of Agriculture (USDA) (1968, 1971). Quantity data are given for numerous commodities in terms of pounds per capita. We convert these data to constant dollar expenditures by multiplying them by the U.S. Bureau of Labor Statistics (BLS) average retail price for 1957–1959. We aggregate subcomponents of meat and produce by summing constant dollar values.

The four meat series are computed as follows:[9] (1) Fish—fresh and frozen plus canned plus cured (Table 9); (2) Beef—beef plus veal (Table 8); (3) Poultry—chicken plus turkey (Table 10); (4) Pork (Table 8). We use the BLS price indexes given in Table 97 of USDA (1968) corresponding to our four types of meat. The price and quantity indexes for meat are presented in Tables 1 and 2. The budget shares constructed from the price and quantity indexes are presented in Table 3. Our data are similar to the meat data analyzed by Brown and Heien (1972).[10]

The four constant dollar series for produce are computed from the 78 subcomponents of fruit and vegetables in Tables 13 through 23 of USDA (1968). The fresh fruit category includes oranges, tangerines and tangelos, grapefruit, lemons and limes, apples, bananas, grapes, peaches, pears, strawberries, cantaloups, watermelons, and other fresh fruit. The processed fruit category includes canned fruit (apples and applesauce, apricots, cherries, citrus segments, cranberries, fruit cocktail, peaches, pineapple, and other), canned fruit juice (orange, grapefruit, blended citrus, pineapple, and other), chilled fruit and juice, frozen fruit and juice (orange juice, other citrus juice, strawberries, peaches, raspberries, and other) and dried fruit (prunes, raisins, and other). The fresh vegetables category includes potatoes, asparagus, snap beans, broccoli, cabbage, carrots, cauliflower, celery, corn, lettuce, onions and shallots, spinach, tomatoes, and other fresh vegetables. The processed vegetables category includes canned vegetables (potatoes, asparagus,

[9] The table numbers given are from *USDA* (1968).

[10] Originally, we used the 1946–1968 meat data constructed by Brown and Heien. In attempting to update their data, we discovered several details which we found unappealing. The most important was that they included edible offals with beef. It seemed to us inappropriate to lump a relatively inexpensive commodity (which is probably not very income elastic) with beef (which is highly income elastic). Thus, we reworked the entire data set, which we present in Tables 1, 2, and 3.

TABLE 1

Price Indexes for Meat, U.S., 1947–1971
(1957–1959 = 1.000)

Year	Fish p_1	Beef p_2	Poultry p_3	Pork p_4
1947	.783	.780	1.260	.932
1948	.903	.944	1.397	.961
1949	.907	.881	1.317	.890
1950	.890	.970	1.261	.878
1951	1.016	1.133	1.321	.931
1952	.990	1.124	1.326	.921
1953	.953	.886	1.293	1.025
1954	.958	.853	1.167	1.057
1955	.939	.844	1.215	.910
1956	.938	.831	1.065	.864
1957	.950	.892	1.038	.995
1958	1.016	1.038	1.026	1.061
1959	1.034	1.069	.935	.944
1960	1.035	1.042	.950	.938
1961	1.058	1.025	.858	.982
1962	1.102	1.062	.907	.991
1963	1.100	1.050	.893	.966
1964	1.074	1.019	.873	.961
1965	1.106	1.068	.900	1.094
1966	1.178	1.124	.949	1.251
1967	1.218	1.131	.889	1.148
1968	1.238	1.177	.917	1.150
1969	1.306	1.295	.969	1.252
1970	1.437	1.352	.964	1.331
1971	1.586	1.413	.969	1.205

green beans, lima beans, corn, peas, pickles, spinach, whole tomatoes, tomato catsup, tomato paste, vegetable juices, and other), frozen vegetables (potatoes, asparagus, green beans, lima beans, broccoli, brussels sprouts, cauliflower, corn, peas, spinach, and other) and other processed potatoes.

The price series corresponding to each of these four categories is constructed from component BLS price indexes. The procedure used is as close as possible to that used by BLS in constructing the con-

TABLE 2

Per Capita Constant Dollar (1957–1959) Expenditures on Meat, U.S., 1947–1971

Year	Fish X_1	Beef X_2	Poultry X_3	Pork X_4
1947	6.734	45.522	10.068	36.879
1948	7.288	41.016	9.893	35.967
1949	7.172	41.134	10.585	35.910
1950	7.766	40.264	11.460	36.708
1951	7.333	35.267	12.114	38.076
1952	7.338	39.051	12.453	38.418
1953	7.460	49.020	12.416	33.687
1954	7.338	50.429	13.087	31.806
1955	6.867	50.752	12.251	35.397
1956	6.817	52.283	13.755	35.625
1957	6.696	51.000	14.622	32.376
1958	6.968	47.160	15.794	31.920
1959	7.162	46.704	16.367	35.796
1960	6.751	48.596	15.900	34.428
1961	7.023	49.270	17.445	32.889
1962	6.963	49.475	17.233	33.687
1963	7.035	51.680	17.443	34.656
1964	6.879	54.612	17.941	34.656
1965	7.156	54.403	19.031	31.122
1966	7.145	56.470	20.407	30.837
1967	6.963	57.210	21.327	33.972
1968	7.206	58.732	21.092	35.055
1969	7.312	59.067	22.025	34.428
1970	7.711	60.307	23.009	35.226
1971	7.323	59.881	23.258	38.646

TABLE 3

Budget Shares and Per Capita Expenditures for Meat, U.S., 1947–1971

	Budget Shares				Total Meat Expenditures, Dollars
Year	Fish	Beef	Poultry	Pork	Per Capita
1947	.060	.404	.144	.391	87.84
1948	.070	.413	.148	.369	93.69
1949	.073	.409	.157	.361	88.65
1950	.075	.422	.156	.348	92.65
1951	.075	.404	.162	.359	98.86
1952	.070	.426	.160	.343	103.05
1953	.070	.429	.159	.341	101.12
1954	.071	.435	.154	.340	98.94
1955	.067	.444	.154	.334	96.38
1956	.067	.456	.154	.323	95.27
1957	.064	.458	.153	.325	99.24
1958	.067	.461	.153	.319	106.10
1959	.070	.469	.144	.318	106.43
1960	.067	.482	.144	.307	105.02
1961	.071	.480	.142	.307	105.20
1962	.070	.481	.143	.306	109.23
1963	.070	.489	.140	.301	111.06
1964	.066	.497	.140	.297	112.00
1965	.068	.496	.146	.291	117.19
1966	.065	.489	.149	.297	129.83
1967	.065	.493	.145	.297	131.14
1968	.065	.502	.140	.293	137.70
1969	.063	.508	.142	.286	150.49
1970	.069	.504	.137	.290	161.68
1971	.070	.512	.136	.282	165.33

sumer price index in order to make the data series for produce comparable with the data series for meat. The price index for each of the four categories in 1947 includes only the few price indexes given for that year; as BLS increased coverage, the new price index was brought into the series using a forward linking procedure. The price and quantity indexes for produce are presented in Tables 4 and 5. The budget shares constructed from the price and quantity indexes are presented in Table 6.

B. Estimation of U.S. Consumer Preferences for Meat and Produce

For convenience of comparison, we draw together the budget share equations derived in Section III and present them in Table 7. We illustrate the relationships among the budget share equations in Figures 1 and 2. The indirect translog form and its special cases are illustrated schematically in Figure 1. The generalized Klein-Rubin form and its special cases are illustrated schematically in Figure 2. The indirect addilog form specializes to the indirect CES form. The budget share equations for the indirect CES have the same form as for the direct CES; thus the direct CES budget share equations can be viewed as a special case of the indirect addilog equations.

We specify classical additive disturbance terms for all the budget share equations in Table 7. To estimate the unknown parameters of the budget share equations, we use the iterative-Zellner (1962, 1963) estimation procedure (IZEF). This procedure has been used by Berndt and Christensen (1973 a, b, c) for the homothetic version of the translog model, which is linear in the parameters. We have followed the same procedures generalized to our nonlinear functional forms. It is well known that IZEF is equivalent to maximum likelihood estimation (assuming a joint normal error structure) for linear models. We have confirmed that this also holds for our nonlinear models. We used a maximum likelihood computer program (by Yonathan Bard, IBM) on our models and obtained results identical to our IZEF results. Our IZEF computations were carried out on the CDC 6600, using program TSP. The nonlinear estimation routine in TSP uses a combination of the Gauss-Newton method and the method of steepest descent.

For convenience in estimation, the price indexes p_1, p_2, p_3, p_4 and per capita total expenditure M were all scaled to equal 1.0 in 1959, for both sets of data. The parameter estimates reported above are not invariant to the scaling of the data, but the fitted budget shares are invariant. It can also be shown that the monotonicity and convexity conditions are invariant to scaling, as are all the implied price and in-

TABLE 4

Price Indexes for Produce, U S., 1947–1971
(1957–1959 = 1.000)

Year	Fresh Fruit p_1	Processed Fruit p_2	Fresh Vegetables p_3	Processed Vegetables p_4
1947	.798	.952	.812	.989
1948	.807	.936	.893	.962
1949	.867	.922	.872	.933
1950	.835	.855	.754	.898
1951	.814	1.006	.875	1.017
1952	.896	.846	1.100	1.005
1953	.929	.869	.890	1.002
1954	.925	.833	.855	.983
1955	.920	.835	.917	.983
1956	.947	.882	.994	1.010
1957	.982	.841	.965	.982
1958	1.025	1.092	1.022	1.008
1959	.994	1.067	1.013	1.010
1960	1.039	.964	1.050	1.018
1961	1.067	1.020	.990	1.072
1962	1.076	.916	1.052	1.062
1963	1.165	1.169	1.070	1.047
1964	1.178	1.203	1.179	1.040
1965	1.129	1.018	1.302	1.065
1966	1.184	.980	1.253	1.096
1967	1.197	.873	1.250	1.142
1968	1.346	.971	1.328	1.177
1969	1.317	1.029	1.401	1.178
1970	1.341	1.008	1.482	1.217
1971	1.422	1.055	1.525	1.261

come elasticities, Allen elasticities of substitution (AES), and test results reported in this paper.[11]

We first estimate by IZEF the translog budget share equations with only the two normalization restrictions imposed. The IZEF estimated covariance matrix is then used to fit the homothetic translog, the additive translog, and linear logarithmic budget share equations. As dis-

[11] See Christensen and Manser (1974) for proof for the translog function. Proofs for the other functions are similar.

TABLE 5

Per Capita Constant Dollar (1957–1959) Expenditures on Produce, U.S., 1947–1971

Year	Fresh Fruit X_1	Processed Fruit X_2	Fresh Vegetables X_3	Processed Vegetables X_4
1947	25.000	9.667	24.375	10.144
1948	23.064	9.994	23.334	9.802
1949	21.934	10.248	22.807	9.989
1950	19.640	11.045	22.296	10.740
1951	21.073	10.802	22.172	11.175
1952	20.081	12.148	21.287	11.718
1953	19.319	12.323	21.309	12.252
1954	18.711	12.564	20.972	12.233
1955	17.791	13.734	20.960	12.994
1956	17.766	13.964	20.716	13.909
1957	17.252	14.326	21.300	14.247
1958	16.946	13.563	20.554	14.976
1959	17.176	13.681	20.475	15.784
1960	16.818	14.242	21.141	16.487
1961	16.317	13.838	21.010	16.783
1962	15.522	14.349	20.400	17.860
1963	14.318	13.164	20.385	18.355
1964	14.928	12.755	19.710	19.446
1965	15.270	13.534	19.266	20.861
1966	15.167	13.593	19.015	21.370
1967	15.216	15.053	18.715	23.714
1968	15.024	14.498	18.990	23.861
1969	15.106	15.411	18.127	25.016
1970	15.471	15.604	18.565	25.778
1971	15.221	15.990	18.222	26.201

TABLE 6

Budget Shares and Per Capita Expenditures for Produce, U.S., 1947–1971

	Budget Shares				Total Produce Expenditures, Dollars Per Capita
Year	Fresh Fruit	Processed Fruit	Fresh Vegetables	Processed Vegetables	
1947	.338	.156	.336	.160	58.96
1948	.320	.161	.358	.162	58.23
1949	.330	.164	.345	.162	57.67
1950	.314	.181	.321	.184	52.30
1951	.292	.185	.330	.193	58.79
1952	.283	.162	.369	.186	63.46
1953	.300	.179	.317	.205	59.91
1954	.300	.181	.310	.208	57.74
1955	.274	.192	.321	.213	59.82
1956	.264	.193	.323	.220	63.78
1957	.267	.190	.324	.220	63.54
1958	.254	.217	.308	.221	68.28
1959	.250	.214	.303	.233	68.34
1960	.249	.195	.316	.239	70.20
1961	.248	.201	.296	.256	70.31
1962	.238	.187	.305	.270	70.28
1963	.228	.211	.298	.263	73.09
1964	.230	.201	.304	.265	76.39
1965	.220	.176	.320	.284	78.32
1966	.229	.170	.303	.298	78.53
1967	.223	.161	.286	.331	81.84
1968	.231	.161	.288	.321	87.60
1969	.220	.175	.280	.325	90.61
1970	.217	.165	.289	.329	95.38
1971	.218	.170	.280	.333	99.35

TABLE 7

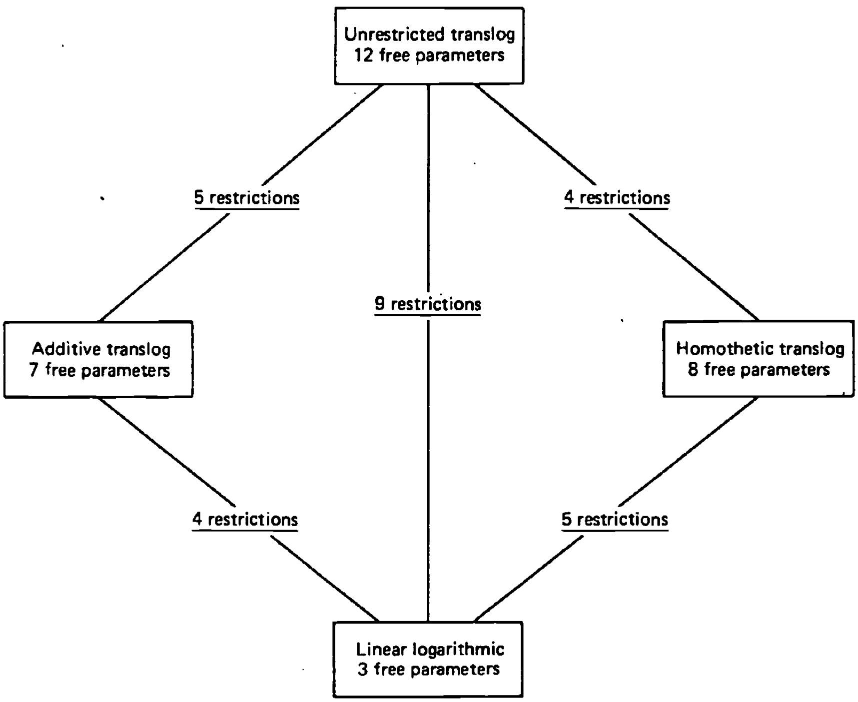
Budget Share Equations for Estimating Consumer Preferences

Utility Function	Budget Share Equations	Number of Parameters: General	Number of Parameters: Four Commodities	Restrictions Used In Estimation	Number of Free Parameters: General	Number of Free Parameters: Four Commodities
Indirect Translog	$w_i = \dfrac{\alpha_i + \sum \beta_{ij} \ln p_j^*}{\sum \alpha_j + \sum \sum \beta_{ij} \ln p_j^*}$ $\beta_{ij} = \beta_{ji}$	$n(n+3)/2$	14	$\sum \alpha_i = 1$ $\sum_{i \neq j} \sum \beta_{ij} = 0$	$[n(n+3) - 4]/2$	12
Homothetic Indirect Translog	$w_i = (\alpha_i + \sum \beta_{ij} \ln p_j^*) \Big/ \sum \alpha_j$ $\beta_{ij} = \beta_{ji}$	$n(n+3)/2$	14	$\sum \alpha_i = 1$ $\sum_{i \neq j} \sum \beta_{ij} = 0$ $\sum_j \beta_{ij} = 0$	$[n(n+3) - 8]/2$	8
Additive Indirect Translog	$w_i = \dfrac{\alpha_i + \beta_{ii} \ln p_i^*}{\sum \alpha_j + \sum \beta_{jj} \ln p_j^*}$	$2n$	8	$\sum \alpha_i = 1$	$2n - 1$	7
Indirect Addilog	$w_i = \dfrac{\alpha_i (p_i^*)^{1-\sigma_i}}{\sum \alpha_j (p_j^*)^{1-\sigma_j}}$	$2n$	8	$\sum \alpha_i = 1$	$2n - 1$	7

Generalized Klein-Rubin	$w_i = \gamma_i p_i^* + \dfrac{\alpha_i (p_i^*)^{1-\sigma}}{\sum_j \alpha_j (p_j^*)^{1-\sigma}} \times [1 - \sum_j \gamma_j p_j^*]$	$2n + 1$	9	$\sum \alpha_i = 1$	$2n$	8
Klein-Rubin	$w_i = \gamma_i p_i^* + \dfrac{\alpha_i}{\sum \alpha_j} (1 - \sum_j \gamma_j p_j^*)$	$2n$	8	$\sum \alpha_i = 1$	$2n - 1$	7
CES	$w_i = \dfrac{\alpha_i (p_i^*)^{1-\sigma}}{\sum_j \alpha_j (p_j^*)^{1-\sigma}}$	$n + 1$	5	$\sum \alpha_i = 1$	n	4
Linear Logarithmic	$w_i = \alpha_i \Big/ \sum \alpha_j$	n	4	$\sum \alpha_i = 1$	$n - 1$	3

FIGURE 1

Schematic Representation of Budget Shares for the Translog Form and Its Special Cases

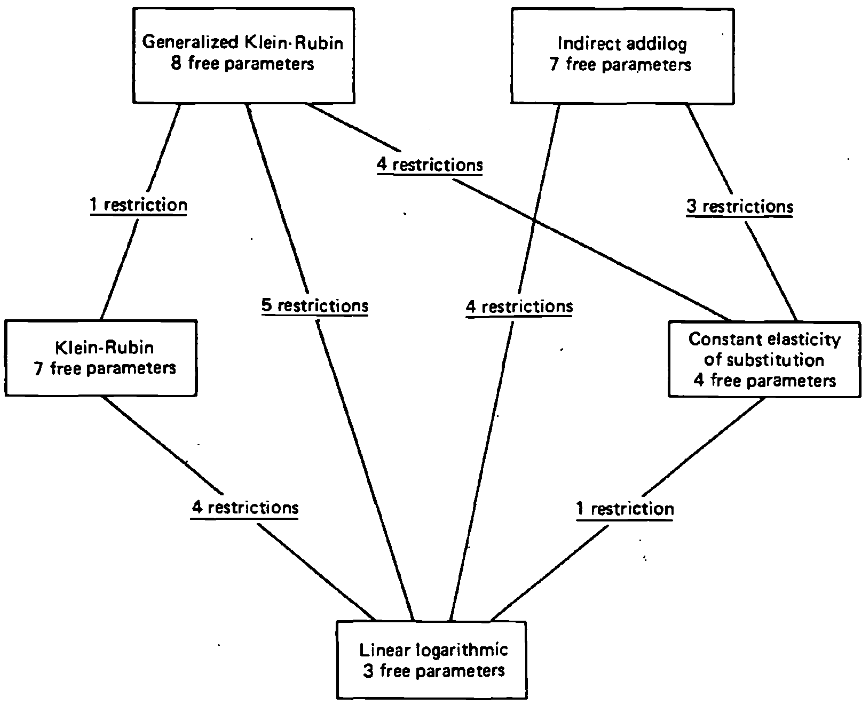


cussed in Christensen and Manser (1974), we test the validity of the homotheticity and additivity restrictions. The test results reported in Table 8 indicate that homotheticity and additivity are decisively rejected. The rejection of additivity does not preclude groupwise additivity. As reported in Christensen and Manser (1974) the hypothesis that (fish) and (beef, poultry, and pork) comprise additive groups cannot be rejected. All other types of groupwise additivity are rejected for both the meat and produce data.

Since the imposition of fish additivity does not result in a significant loss of fit, we adopt this form as our preferred translog specification for the meat data. The preferred specification for the produce data, however, does not have any additivity restrictions imposed. In Table 9 we present parameter estimates for the translog and its special cases for

FIGURE 2

Schematic Representation of Budget Shares for the Generalized Stone-Geary Form, the Indirect Addilog Form, and Their Special Cases



the meat data; in Table 10 we present the corresponding estimates for the produce data. For the meat data, the parameters β_{12}, β_{13}, and β_{14} are equal to zero, since fish additivity is imposed. The monotonicity and convexity conditions for the translog forms are satisfied in all years for the meat data. For the produce data, the montonicity and convexity conditions for the translog forms are also satisfied in all years—with the exception that the convexity conditions are not satisfied for the homothetic translog form for any year.

In Table 11, we present parameter estimates for the indirect addilog function, the generalized Klein-Rubin function and their special cases (except the linear logarithmic function for the meat data). In Table 12 we present the corresponding estimates for the produce data. For the

TABLE 8

Tests of Homotheticity and Additivity Restrictions on the Translog Utility Function

Hypothesis	Number of Restrictions	F Statistic Meat Data	F Statistic Produce Data	Residual Degrees of Freedom	Critical Value (.01 Level)
Homotheticity	4	20.86	11.76	63	3.63
Additivity	5	10.15	13.48	63	3.32
Homotheticity and Additivity	9	31.45	37.61	63	2.71
Homotheticity, Conditional on Additivity	4	34.71	35.34	68	3.61
Additivity, Conditional on Homotheticity	5	18.27	35.48	67	3.61

meat data, we present IZEF estimates for the generalized Klein-Rubin and indirect addilog models. The IZEF estimated covariance matrix from the generalized Klein-Rubin model is used to obtain the Klein-Rubin parameter estimates; similarly, the IZEF estimated covariance matrix from the indirect addilog model is used to obtain the CES parameter estimates. For the produce data, the IZEF estimation procedure did not converge for the generalized Klein-Rubin model. Thus, we report IZEF estimates for the Klein-Rubin and indirect addilog models. The IZEF estimated covariance matrix from the indirect addilog model is used to obtain the CES parameter estimates. The monotonicity and convexity conditions for the generalized Klein-Rubin and indirect addilog utility functions and their special cases are satisfied for all years in both the meat and produce data.

We perform tests of homotheticity on the Klein-Rubin, generalized Klein-Rubin, and indirect addilog functions. The imposition of homotheticity on either the generalized Klein-Rubin or the indirect addilog function results in the CES function; similarly the imposition of homotheticity on the Klein-Rubin function results in the linear logarithmic function. The test results reported in Table 13 indicate that homotheticity is decisively rejected for both the meat and produce data. This implies the rejection of the CES and linear logarithmic functional forms.

TABLE 9

Parameter Estimates for the Indirect Translog Function and Special Cases:
(1) Fish, (2) Beef, (3) Poultry, and (4) Pork
(standard errors in parentheses)

Parameter	Translog	Homothetic Translog	Additive Translog	Linear Logarithmic
α_1	.0710	.0690	.0711	.0683
	(.0006)	(.0012)	(.0008)	(.0016)
α_2	.4621	.4681	.4612	.4626
	(.0033)	(.0043)	(.0042)	(.0055)
α_3	.1449	.1392	.1494	.1481
	(.0016)	(.0019)	(.0016)	(.0022)
α_4	.3220	.3237	.3183	.3210
	(.0037)	(.0039)	(.0038)	(.0043)
β_{11}	.0440	.0207	.0635	0
	(.0130)	(.0147)	(.0158)	
β_{12}	0	−.0018	0	0
		(.0114)		
β_{13}	0	.0042	0	0
		(.0037)		
β_{14}	0	−.0231	0	0
		(.0198)		
β_{22}	−.2606	.0225	−.1923	0
	(.0501)	(.0161)	(.0630)	
β_{23}	−.0582	−.0710	0	0
	(.0128)	(.0151)		
β_{24}	.0103	.0502	0	0
	(.0208)	(.0289)		
β_{33}	.0209	.0253	.0358	0
	(.0053)	(.0063)	(.0048)	
β_{34}	.0479	.0414	0	0
	(.0309)	(.0233)		
β_{44}	.1464	−.0685	.3086	0
	(.0535)	(.0310)	(.0505)	

TABLE 10

Parameter Estimates for the Translog Function and Special Cases: (1) Fresh Fruit, (2) Processed Fruit, (3) Fresh Vegetables, and (4) Processed Vegetables (standard errors in parentheses)

Parameter	Translog	Homothetic Translog	Additive Translog	Linear Logarithmic
α_1	.2693	.2593	.2581	.2614
	(.0055)	(.0058)	(.0053)	(.0114)
α_2	.1879	.2029	.1867	.1818
	(.0048)	(.0049)	(.0048)	(.0170)
α_3	.3166	.3179	.3037	.3132
	(.0030)	(.0034)	(.0030)	(.0166)
α_4	.2262	.2227	.2515	.2436
	(.0056)	(.0063)	(.0054)	(.0086)
β_{11}	−.1228	−.0281	.0943	0
	(.1277)	(.0262)	(.0622)	
β_{12}	.0911	−.0296	0	0
	(.0314)	(.0230)		
β_{13}	−.1250	−.2140	0	0
	(.0924)	(.0229)		
β_{14}	.1087	.2717	0	0
	(.0443)	(.0274)		
β_{22}	.1072	.1539	.0608	0
	(.0275)	(.0213)	(.0257)	
β_{23}	.1167	.0291	0	0
	(.0313)	(.0159)		
β_{24}	−.1486	−.1534	0	0
	(.0344)	(.0304)		
β_{33}	.2692	.0887	.3197	0
	(.0795)	(.0167)	(.0559)	
β_{34}	−.0430	.0962	0	0
	(.0427)	(.0192)		
β_{44}	−.4289	−.2145	−.5086	0
	(.0581)	(.0311)	(.0616)	

TABLE 11

Parameter Estimates for the Indirect Addilog, Generalized Klein-Rubin Function, and Special Cases: (1) Fish, (2) Beef, (3) Poultry, and (4) Pork

Parameter	Generalized Klein-Rubin	Klein-Rubin	CES	Parameter	Indirect Addilog
γ_1	.0508	.0566	0	σ_1	.1176
γ_2	−1.0211	−.1229	0	σ_2	1.1918
γ_3	−.1542	.0334	0	σ_3	.7070
γ_4	.2190	.2698	0	σ_4	.2435
α_1	.0097	.0170	.0684	α_1	.0705
α_2	.7841	.7808	.4726	α_2	.4651
α_3	.1588	.1490	.1432	α_3	.1472
α_4	.0474	.0532	.3157	α_4	.3172
σ	.3267	1	.6634		

TABLE 12

Parameter Estimates for the Indirect Addilog, Klein-Rubin, and CES Functions:[a] (1) Fresh Fruit, (2) Processed Fruit, (3) Fresh Vegetables, and (4) Processed Vegetables

Parameter	Klein-Rubin	CES	Parameter	Indirect Addilog
γ_1	.0269	0	σ_1	.4758
γ_2	.1418	0	σ_2	.7033
γ_3	.2026	0	σ_3	.0156
γ_4	−2.5859	0	σ_4	3.3297
α_1	.0717	.2586	α_1	.2589
α_2	.0164	.1923	α_2	.1868
α_4	.0324	.3093	α_3	.3057
α_4	.8795	.2398	α_4	.2486
σ	1	.5093		

[a] The generalized Klein-Rubin did not provide a statistically significant generalization of the Klein-Rubin function for the produce data.

TABLE 13

Tests of Homotheticity Restrictions on the Klein-Rubin, Generalized Klein-Rubin, and Indirect Addilog Utility Functions

	Number of Restrictions	F Statistic		Residual Degrees of Freedom	Critical Value (0.1 Level)
		Meat Data	Produce Data		
Generalized Klein-Rubin	4	18.76	[a]	67	3.61
Klein-Rubin	4	69.64	72.93	68	3.61
Indirect Addilog	3	12.10	46.89	68	3.61

[a] We did not succeed in obtaining IZEF estimates for the produce data.

To indicate the explanatory power for each of the models, R^2's are presented in Table 14.[12] The R^2's for the indirect translog dominate those for any of the other functional forms for both meat and produce. For the meat data either the Klein-Rubin or generalized Klein-Rubin dominates all forms except the translog; there is no clear dominance among the remaining forms. For the produce data, the homothetic translog dominates the nontranslog forms for fruit, but the pattern is mixed for vegetables.

C. Expenditure, Price, and Substitution Elasticities for Meat and Produce

The implications for consumer behavior of the various estimated budget share equations can be illuminated by computing the expenditure and price elasticities and the AES. The expenditure and price elasticities can be calculated directly from the budget share equations by using the following general formulas

$$\eta_{iM} = 1 + \frac{\partial \ln w_i}{\partial \ln M} \qquad (i = 1, \ldots, n);$$

$$\eta_{ii} = -1 + \frac{\partial \ln w_i}{\partial \ln p_i} \qquad (i = 1, \ldots, n);$$

$$\eta_{ij} = \frac{\partial \ln w_i}{\partial \ln p_j} \qquad (i \neq j;\ i, j = 1, \ldots, n)$$

[12] The R^2's are computed as one minus the ratio of the sum of squared errors to the total sum of squares for the budget shares.

TABLE 14

R^2's for the Estimated Budget Share Equations

A. Meat Data	Number of Free Param-eters	Fish	Beef	Poultry	Pork
Indirect translog [a]	9	.4789	.8828	.5803	.7971
Homothetic indirect translog	8	.2855	.6952	−.1785	.4716
Additive indirect translog	7	.2442	.7023	.3954	.5104
Indirect addilog	7	.2192	.6543	.3194	.3631
Generalized Klein-Rubin	8	.4787	.7619	.5190	.5513
Klein-Rubin	7	.4784	.7924	.3431	.5632
CES	4	−.4267	.2849	.1032	−.1792
Linear logarithmic [b]	3	.0000	.0000	.0000	.0000

B. Produce Data	Number of Free Param-eters	Fresh Fruit	Processed Fruit	Fresh Vegetables	Processed Vegetables
Indirect translog	12	.7884	.3994	.8344	.9073
Homothetic indirect translog	8	.7267	.1965	.6783	.8159
Additive indirect translog	7	.4486	−.4751	.6431	.7811
Indirect addilog	7	.4644	−.4947	.7285	.8158
Klein-Rubin	7	.6046	−.5658	.4797	.8544
CES	4	−.2827	.1922	−.5246	−.2235
Linear logarithmic [b]	3	.0000	.0000	.0000	.0000

[a] Fish additivity is imposed, resulting in 9 rather than 12 free parameters.
[b] R^2's are identically zero, since only an "intercept" is estimated.

In Table 15 we present the expenditure and price elasticity formulas for each of our eight utility functions. The AES can be computed from the Slutsky equation (5). In general, the elasticities are functions of the p_i^*, but for our data the variation over the sample period is not great. Thus, in Tables 16 through 18 we present the elasticities only for the midyear of our sample, 1959. The elasticities for 1947, 1959, and 1971 are presented in Manser (1974).

TABLE 15

Elasticity Formulas for Models of Consumer Preferences [a]

Utility Function	Expenditure Elasticities	Own-Price Elasticities	Cross-Price Elasticities
Indirect translog	$\eta_{iM} = 1 + \dfrac{-\sum_j \beta_{ji} / w_i + \sum_i \sum_j \beta_{ji}}{1 + \sum_i \sum_j \beta_{ij} \ln p_j^*}$	$\eta_{ii} = -1 + \dfrac{\beta_{ii} / w_i - \sum_j \beta_{ji}}{1 + \sum_i \sum_j \beta_{ij} \ln p_j^*}$	$\eta_{ij} = \dfrac{\beta_{ij} / w_i - \sum_i \beta_{ij}}{1 + \sum_i \sum_j \beta_{ij} \ln p_j^*}$
Homothetic indirect translog	$\eta_{iM} = 1$	$\eta_{ii} = -1 + \beta_{ii} / w_i$	$\eta_{ij} = \beta_{ij} / w_i$
Additive indirect translog	$\eta_{iM} = 1 + \dfrac{-\beta_{ii} / w_i + \sum_i \beta_{ii}}{1 + \sum_j \beta_{jj} \ln p_j^*}$	$\eta_{ii} = -1 + \dfrac{\beta_{ii} / w_i - \beta_{ii}}{1 + \sum_j \beta_{jj} \ln p_j^*}$	$\eta_{ij} = \dfrac{-\beta_{jj}}{1 + \sum_j \beta_{jj} \ln p_j^*}$

Indirect addilog	$\eta_{iM} = \sigma_i + \sum_j (1 - \sigma_j) w_j$	$\eta_{ii} = -1 + (1 - \sigma_i)(1 - w_i)$	$\eta_{ij} = -(1 - \sigma_j) w_j$
Generalized Klein-Rubin	$\eta_{iM} = \dfrac{\alpha_i (p_i^*)^{1-\sigma}}{w_i \sum_j \alpha_j (p_j^*)^{1-\sigma}}$	$\eta_{ii} = -w_i \eta_{iM} - \sigma \left(1 - \dfrac{p_i^* \gamma_i}{w_i}\right) \times (1 - w_i \eta_{iM})$	$\eta_{ij} = -p_j^* \gamma_j \eta_{iM} - (1 - \sigma) \eta_{iM} \eta_{jM} w_j$
Klein-Rubin	$\eta_{iM} = \dfrac{\alpha_i}{w_i}$	$\eta_{ii} = -1 + \dfrac{(1 - \alpha_i) p_i^* \gamma_i}{w_i}$	$\eta_{ij} = \dfrac{-\alpha_i p_j^* \gamma_j}{w_i}$
CES	$\eta_{iM} = 1$	$\eta_{ii} = -1 + (1 - \sigma)(1 - w_i)$	$\eta_{ij} = -(1 - \sigma) w_j$
Linear logarithmic	$\eta_{iM} = 1$	$\eta_{ii} = -1$	$\eta_{ij} = 0$

[a] The w_i's refer to fitted budget shares. The normalization $\Sigma \alpha_i = 1$ has been imposed to simplify the formulas in this table.

TABLE 16

Estimated Expenditure Elasticities (η_{iM}) for 1959

A. Budget Share Equations	Fish	Beef	Poultry	Pork
Indirect translog	.331	1.619	.877	.315
Homothetic indirect translog	1.000	1.000	1.000	1.000
Additive indirect translog	.322	1.633	.976	.246
Indirect addilog	.374	1.448	.963	.500
Generalized Klein-Rubin	.140	1.658	1.070	.153
Klein-Rubin	.344	1.651	1.013	.171
CES	1.000	1.000	1.000	1.000
Linear logarithmic	1.000	1.000	1.000	1.000

B. Budget Share Equations	Fresh Fruit	Processed Fruit	Fresh Vege- tables	Processed Vegetables
Indirect translog	1.003	−.062	.136	3.086
Homothetic indirect translog	1.000	1.000	1.000	1.000
Additive indirect translog	.601	.641	−.087	2.988
Indirect addilog	.389	.616	−.071	3.243
Klein-Rubin	.279	.084	.106	3.646
CES	1.000	1.000	1.000	1.000
Linear logarithmic	1.000	1.000	1.000	1.000

In Table 16, we present estimated expenditure elasticities. As expected, all the homothetic forms exhibit unitary expenditure elasticities. For the meat data, the rest of the forms show remarkable agreement: the demand for beef is highly expenditure elastic; the other types of meat are expenditure inelastic, but poultry is more elastic than fish or pork. For the produce data, the agreement is almost as good: processed vegetables are highly expenditure elastic, while fresh vegetables and processed fruits are very inelastic; the translog indicates that fresh fruit is expenditure elastic—an implication not shared by the other models.

In Table 17, we present estimated price elasticities. Beef, fresh fruit, and processed vegetables are found to be own-price elastic; the rest of the commodities are own-price inelastic. The imposition of additivity results in lower own-price elasticities for meat; while those for produce are not strongly affected—with the exception of fresh fruit. Although some of the own-price elasticities are quite large, the cross-price elasticities are, with minor exceptions, all quite moderate. The imposition of homotheticity or additivity severely alters the price elasticities. The cross-price elasticities are arranged in Table 17 so that the restrictions from indirect additivity ($\eta_{ij} = \eta_{jk}$) are clearly displayed.

TABLE 17

Estimated Price Elasticities (η_{ij}) for 1959

IA. Meat: (1) Fish, (2) Beef, (3) Poultry, (4) Pork

Budget Share Equations	η_{11}	η_{22}	η_{33}	η_{44}	η_{21}	η_{31}	η_{41}
(a) Indirect translog	−.425	−1.255	−.866	−.750	−.044	−.044	−.044
(b) Homothetic indirect translog	−.700	−.952	−.818	−1.212	−.004	−.030	−.071
(c) Additive indirect translog	−.170	−1.225	−.796	−.339	−.064	−.064	−.064
(d) Indirect addilog	−.180	−1.103	−.750	−.483	−.062	−.062	−.062
(e) Generalized Klein-Rubin	−.096	−1.007	−.719	−.318	−.105	−.068	−.010
(f) Klein-Rubin	−.200	−1.057	−.807	−.177	−.093	−.057	−.010
(g) CES	−.686	−.823	−.712	−.770	−.023	−.023	−.023
(h) Linear logarithmic	−1.000	−1.000	−1.000	−1.000	.000	.000	.000

IB. Produce: (1) Fresh Fruit, (2) Processed Fruit, (3) Fresh Vegetables, (4) Processed Vegetables

Budget Share Equations	η_{11}	η_{22}	η_{33}	η_{44}	η_{21}	η_{31}	η_{41}
(a) Indirect translog	−1.408	−.596	−.368	−2.383	.533	−.347	.528
(b) Homothetic indirect translog	−1.109	−.242	−.721	−1.975	−.146	−.673	1.236
(c) Additive indirect translog	−.729	−.735	−.267	−2.513	−.094	−.094	−.094
(d) Indirect addilog	−.612	−.759	−.317	−2.751	−.136	−.136	−.136
(e) Klein-Rubin	−.903	−.283	−.361	−2.292	−.002	−.003	−.098
(f) CES	−.636	−.604	−.661	−.627	−.127	−.127	−.127
(g) Linear logarithmic	−1.000	−1.000	−1.000	−1.000	.000	.000	.000

(continued)

TABLE 17 (concluded)

IIA.									
	η_{12}	η_{32}	η_{42}	η_{13}	η_{23}	η_{43}	η_{14}	η_{24}	η_{34}
(a)	.309	−.093	.341	−.011	−.137	.138	−.205	−.182	.126
(b)	−.026	−.510	.155	.061	−.152	.128	−.335	.107	.298
(c)	.192	.192	.192	−.036	−.036	−.036	−.309	−.309	−.309
(d)	.089	.089	.089	−.043	−.043	−.043	−.240	−.240	−.240
(e)	.002	.016	.002	−.007	−.080	−.007	−.040	−.476	−.307
(f)	.030	.124	.021	−.008	−.055	−.006	−.066	−.445	−.273
(g)	−.159	−.159	−.159	−.048	−.048	−.048	−.106	−.106	−.106
(h)	.000	.000	.000	.000	.000	.000	.000	.000	.000

IIB.									
	η_{12}	η_{32}	η_{42}	η_{13}	η_{23}	η_{43}	η_{14}	η_{24}	η_{34}
(a)	.172	.202	−.823	−.682	.404	−.408	.915	−.279	.376
(b)	−.114	.091	−.697	−.825	.143	.437	1.048	−.756	.302
(c)	−.061	−.061	−.061	−.320	−.320	−.320	.509	.509	.509
(d)	−.055	−.055	−.055	−.301	−.301	−.301	.579	.579	.579
(e)	−.039	−.015	−.517	−.056	−.017	−.739	.720	.218	.273
(f)	−.094	−.094	−.094	−.152	−.152	−.152	−.118	−.118	−.118
(g)	.000	.000	.000	.000	.000	.000	.000	.000	.000

In Table 18, we present estimated Allen Elasticities of Substitution. The AES, which are weighted price elasticities computed along an indifference curve, show most clearly the extent of the differences between consumer preferences as estimated by the translog model and the more restrictive models. The translog model indicates that beef, poultry, and pork are all reasonably good substitutes; but of these, only beef is very substitutable with fish. Pork and fish appear to be complements. The variation in the substitution relationships among the types of produce is much stronger than for meat. Fresh fruit and vegetables are complementary, as are processed fruits and vegetables. On the other hand, all four pairs of fresh and processed produce are highly substitutable—the most pronounced is between fresh fruit and processed vegetables.

TABLE 18

Estimated Allen Elasticities of Substitution (σ_{ij})

A. Meat: (1) Fish, (2) Beef, (3) Poultry, (4) Pork

Budget Share Equations	σ_{12}	σ_{13}	σ_{14}	σ_{23}	σ_{24}	σ_{34}
Indirect translog	.999	.258	−.304	.675	1.052	1.269
Homothetic indirect translog	.945	1.437	−.035	−.089	1.332	1.920
Additive indirect translog	.739	.083	−.647	1.393	.663	.007
Indirect addilog	.565	.081	−.383	1.155	.691	.207
Generalized Klein-Rubin	.145	.094	.013	1.104	.158	.102
Klein-Rubin	.308	.189	.032	1.276	.216	.132
CES	.663	.663	.663	.663	.663	.663
Linear logarithmic	1.000	1.000	1.000	1.000	1.000	1.000

B. Produce: (1) Fresh Fruit, (2) Processed Fruit, (3) Fresh Vegetables, (4) Processed Vegetables

Budget Share Equations	σ_{12}	σ_{13}	σ_{14}	σ_{23}	σ_{24}	σ_{34}
Indirect translog	1.919	−1.152	5.048	1.213	−1.295	1.798
Homothetic indirect translog	.438	−1.596	5.765	1.451	−2.438	2.375
Additive indirect translog	.275	−.452	2.623	−.412	2.663	1.935
Indirect addilog	.092	−.596	2.718	−.368	2.946	2.258
Klein-Rubin	.075	.095	3.264	.029	.987	1.238
CES	.509	.509	.509	.509	.509	.509
Linear logarithmic	1.000	1.000	1.000	1.000	1.000	1.000

V. COST-OF-LIVING INDEXES FOR MEAT AND PRODUCE, 1947–1971

The cost-of-living index is the minimum expenditure required to attain a given level of utility in year t relative to the minimum expenditure required to attain the same level of utility in some reference period.[13] We denote the fixed utility level to be attained as V^s; we refer to s as the base period and V^s as the base-period utility level. We denote the

[13] The discussion in this section follows Pollak (1971b).

minimum expenditure to attain V^s in any period t as $M_*^t(p^t, V^s)$. We denote the cost-of-living index

$$I(p^t, p^b, V^s) = \frac{M_*^t(p^t, V^s)}{M_*^b(p^b, V^s)} \tag{33}$$

where b is referred to as the reference period. If the reference period b and base period s coincide, $M_*^s(p^s, V^s) = M^s$ is simply the actual expenditure in period s. The choice of reference period simply results in a particular normalization for the cost-of-living index since the ratio of any two indexes is independent of the reference period

$$\frac{I(p^t, p^b, V^s)}{I(p^{t'}, p^b, V^s)} = \frac{M_*^t(p^t, V^s)}{M_*^{t'}(p^{t'}, V^s)}$$

Calculation of a cost-of-living index requires a functional representation of consumer preferences. The most convenient representation is the indirect utility function $V(p^t/M^t)$. When the base-period utility level is fixed, the indirect utility function can be viewed as an implicit function $V(p^t/M_*^t) - V^s = 0$ with solution M_*^t. The resulting M_*^t's can be plugged into (33) to obtain the cost-of-living index. Utility functions which are homothetic or marginally homothetic imply explicit cost-of-living indexes. We now derive the explicit indexes where possible and describe the numerical computation of the cost-of-living indexes which do not have an explicit form.

Fixing utility at V^s, the indirect translog utility function can be written as an implicit function of M_*^t

$$\frac{1}{2}\sum_i \sum_j \beta_{ij} (\ln M_*^t)^2 - \left(1 + \sum_i \left(\sum_j \beta_{ij}\right) \ln p_i^t\right) (\ln M_*^t)$$
$$+ \left(\ln V^s + \sum \alpha_i \ln p_i^t + \frac{1}{2}\sum \sum \beta_{ij} \ln p_i^t \ln p_j^t\right) = 0 \tag{34}$$

This is a quadratic in $\ln M_*^t$, which can be solved for any given set of prices.[14] We compute cost-of-living indexes for three base years, 1947, 1958, and 1967; it is convenient to use 1947 as the reference year to reveal clearly the rate of increase of the cost-of-living for the three base years. In the first three columns of Table 19, we present the translog cost-of-living indexes for meat; in Table 20, we present the cost-of-living indexes for produce.

[14] Two solutions for $\ln M_*^t$ are obtained. For all results reported below, only one of the solutions was of the proper order of magnitude to be considered. In (34) and in the remainder of the paper, we simplify expressions by using $\sum \alpha_i = 1$.

It is of interest to compute cost-of-living indexes for the models of consumer preferences which are additive or homothetic. We first discuss the models with additive preferences and then those with homothetic preferences. Cost-of-living indexes for the additive translog form can be computed from (34) using the translog estimates with additivity imposed. The results are presented in the fourth, fifth, and sixth columns of Tables 19 and 20.

Fixing utility at V^s, the indirect addilog utility function can be written

$$-\ln V^s = \ln \sum_i \delta_i (p_i/M_*)^{1-\sigma_i} \tag{35}$$

This function cannot be solved analytically for M_*; we use numerical methods to compute M_*. The cost-of-living indexes for the indirect addilog function are presented in the seventh, eighth, and ninth columns of Tables 19 and 20.

The Klein-Rubin (25) and generalized Klein-Rubin (29) functions that we have estimated are direct utility functions. We compute indirect utility functions corresponding to the direct utility functions by substituting the demand functions $X_i = X_i(p_i^t)$ into the direct utility function. The demand functions are obtained by multiplying our budget share equations by M/p_i. The indirect utility function for the generalized Klein-Rubin function can be written

$$\ln V = \ln \left(M - \sum_i \gamma_i p_i\right) - \frac{1}{1-\sigma} \ln \sum \alpha_i p_i^{1-\sigma} \tag{36}$$

Fixing the level of utility at V^s, (31) can be solved explicity for M^*

$$M_* = V^s \left[\sum \alpha_i p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + \sum \gamma_i p_i \tag{37}$$

Similarly the indirect utility function for the Klein-Rubin function can be written

$$\ln V = \sum \alpha_i \ln \alpha_i + \ln \left(M - \sum \gamma_i p_i\right) - \sum \alpha_i \ln p_i \tag{38}$$

which can be solved explicitly for M_*

$$M_* = V^s \prod_i \left(\frac{p_i}{\alpha_i}\right)^{\alpha_i} + \sum \gamma_i p_i \tag{39}$$

For the meat data, generalized Klein-Rubin and Klein-Rubin cost-of-living indexes are presented in columns ten through fifteen of Table 19. Since generalized Klein-Rubin estimates were not obtained for the produce data, only Klein-Rubin cost-of-living indexes are presented in Table 20.

TABLE 19

Cost of Living Indexes for Meat

	Indirect Translog			Additive Indirect Translog			Indirect Addilog		
	(Base Year)			(Base Year)			(Base Year)		
	1947	1958	1967	1947	1958	1967	1947	1958	1967
1947	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1948	1.126	1.127	1.134	1.128	1.129	1.136	1.128	1.129	1.134
1949	1.055	1.056	1.062	1.057	1.058	1.064	1.057	1.058	1.062
1950	1.086	1.088	1.100	1.090	1.092	1.104	1.090	1.092	1.100
1951	1.205	1.209	1.226	1.211	1.214	1.233	1.212	1.214	1.227
1952	1.195	1.198	1.216	1.201	1.204	1.223	1.202	1.205	1.217
1953	1.110	1.111	1.112	1.111	1.111	1.112	1.111	1.111	1.111
1954	1.087	1.086	1.085	1.088	1.088	1.086	1.087	1.087	1.085
1955	1.033	1.034	1.036	1.034	1.034	1.037	1.034	1.034	1.036
1956	.988	.989	.993	.989	.990	.994	.989	.989	.992
1957	1.065	1.066	1.069	1.067	1.068	1.070	1.067	1.067	1.069
1958	1.167	1.169	1.177	1.170	1.171	1.178	1.169	1.170	1.176
1959	1.123	1.125	1.139	1.124	1.127	1.140	1.125	1.127	1.137
1960	1.110	1.113	1.125	1.112	1.114	1.126	1.113	1.115	1.123
1961	1.105	1.107	1.116	1.107	1.108	1.117	1.107	1.108	1.115
1962	1.138	1.141	1.151	1.140	1.142	1.152	1.141	1.142	1.150
1963	1.120	1.123	1.134	1.122	1.124	1.135	1.123	1.124	1.132
1964	1.098	1.100	1.110	1.100	1.102	1.111	1.100	1.101	1.108
1965	1.179	1.180	1.188	1.182	1.183	1.190	1.181	1.182	1.187
1966	1.277	1.278	1.283	1.283	1.283	1.286	1.281	1.282	1.285
1967	1.235	1.237	1.245	1.239	1.240	1.247	1.239	1.240	1.245
1968	1.265	1.268	1.278	1.268	1.270	1.279	1.269	1.270	1.277
1969	1.375	1.378	1.390	1.378	1.380	1.391	1.379	1.380	1.388
1970	1.440	1.442	1.454	1.444	1.446	1.456	1.445	1.446	1.453
1971	1.433	1.437	1.455	1.435	1.438	1.455	1.437	1.440	1.452

TABLE 19 (concluded)

	Generalized Klein-Rubin (Base Year)			Klein-Rubin (Base Year)			Homothetic Indirect Translog	CES	Linear Loga-rith-mic
	1947	1958	1967	1947	1958	1967			
1947	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1948	1.126	1.127	1.134	1.126	1.128	1.135	1.128	1.130	1.131
1949	1.055	1.056	1.063	1.055	1.056	1.063	1.057	1.059	1.060
1950	1.087	1.090	1.102	1.087	1.090	1.103	1.089	1.093	1.095
1951	1.209	1.213	1.230	1.209	1.214	1.233	1.209	1.216	1.218
1952	1.199	1.203	1.220	1.199	1.204	1.223	1.199	1.206	1.208
1953	1.111	1.111	1.111	1.110	1.111	1.112	1.110	1.111	1.113
1954	1.089	1.088	1.084	1.089	1.088	1.086	1.084	1.086	1.088
1955	1.032	1.033	1.036	1.032	1.033	1.037	1.034	1.035	1.036
1956	.987	.988	.991	.987	.988	.993	.989	.990	.992
1957	1.066	1.066	1.066	1.066	1.067	1.070	1.065	1.067	1.070
1958	1.167	1.168	1.173	1.168	1.170	1.177	1.169	1.171	1.175
1959	1.122	1.125	1.136	1.123	1.126	1.140	1.128	1.130	1.133
1960	1.110	1.112	1.122	1.110	1.113	1.126	1.115	1.117	1.120
1961	1.104	1.106	1.112	1.105	1.107	1.117	1.108	1.110	1.113
1962	1.138	1.140	1.147	1.138	1.141	1.152	1.142	1.145	1.147
1963	1.120	1.122	1.130	1.120	1.123	1.135	1.124	1.127	1.129
1964	1.097	1.099	1.106	1.098	1.100	1.111	1.101	1.104	1.106
1965	1.180	1.181	1.184	1.180	1.182	1.189	1.180	1.184	1.186
1966	1.281	1.281	1.280	1.282	1.283	1.286	1.276	1.282	1.283
1967	1.237	1.238	1.241	1.238	1.239	1.247	1.237	1.241	1.243
1968	1.266	1.268	1.273	1.267	1.269	1.279	1.269	1.272	1.274
1969	1.376	1.378	1.385	1.376	1.379	1.391	1.380	1.383	1.384
1970	1.442	1.443	1.449	1.443	1.445	1.455	1.444	1.449	1.449
1971	1.433	1.437	1.450	1.434	1.438	1.456	1.441	1.444	1.443

TABLE 20

Cost of Living Indexes for Produce

	Indirect Translog			Additive Indirect Translog			Indirect Addilog		
	(Base Year)			(Base Year)			(Base Year)		
	1947	1958	1967	1947	1958	1967	1947	1958	1967
1947	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1948	1.029	1.030	1.026	1.024	1.025	1.021	1.024	1.025	1.021
1949	1.033	1.034	1.031	1.028	1.027	1.021	1.026	1.028	1.022
1950	.955	.954	.956	.949	.950	.948	.949	.950	.947
1951	1.048	1.049	1.047	1.047	1.048	1.046	1.047	1.048	1.046
1952	1.124	1.128	1.117	1.117	1.119	1.109	1.118	1.120	1.110
1953	1.067	1.067	1.066	1.057	1.058	1.054	1.057	1.058	1.054
1954	1.041	1.041	1.041	1.030	1.031	1.028	1.030	1.031	1.028
1955	1.062	1.063	1.060	1.051	1.053	1.048	1.052	1.053	1.048
1956	1.115	1.117	1.111	1.105	1.107	1.100	1.106	1.108	1.100
1957	1.100	1.102	1.098	1.089	1.091	1.083	1.089	1.091	1.083
1958	1.194	1.196	1.190	1.183	1.186	1.174	1.184	1.187	1.175
1959	1.176	1.178	1.172	1.166	1.168	1.158	1.167	1.169	1.158
1960	1.184	1.186	1.180	1.173	1.176	1.165	1.174	1.177	1.165
1961	1.198	1.199	1.197	1.187	1.189	1.182	1.188	1.190	1.182
1962	1.199	1.200	1.196	1.186	1.188	1.179	1.187	1.189	1.180
1963	1.280	1.282	1.277	1.267	1.271	1.257	1.268	1.272	1.257
1964	1.325	1.329	1.319	1.312	1.316	1.298	1.312	1.317	1.297
1965	1.318	1.323	1.308	1.310	1.314	1.296	1.310	1.315	1.295
1966	1.320	1.324	1.313	1.310	1.314	1.298	1.311	1.316	1.298
1967	1.314	1.318	1.308	1.302	1.305	1.292	1.303	1.306	1.292
1968	1.415	1.418	1.408	1.403	1.407	1.390	1.404	1.409	1.390
1969	1.442	1.447	1.433	1.433	1.438	1.419	1.434	1.439	1.418
1970	1.482	1.487	1.472	1.474	1.479	1.459	1.475	1.481	1.459
1971	1.544	1.549	1.534	1.535	1.541	1.520	1.536	1.542	1.519

TABLE 20 (concluded)

	Klein-Rubin (Base Year) 1947	Klein-Rubin (Base Year) 1958	Klein-Rubin (Base Year) 1967	Homothetic Indirect Translog	CES	Linear Loga-rith-mic
1947	1.000	1.000	1.000	1.000	1.000	1.000
1948	1.024	1.025	1.021	1.029	1.021	1.023
1949	1.026	1.027	1.021	1.033	1.021	1.025
1950	.949	.950	.947	.954	.946	.947
1951	1.047	1.047	1.046	1.048	1.047	1.047
1952	1.116	1.118	1.111	1.120	1.112	1.114
1953	1.058	1.058	1.055	1.068	1.053	1.057
1954	1.031	1.032	1.029	1.042	1.027	1.030
1955	1.052	1.053	1.049	1.062	1.047	1.051
1956	1.105	1.107	1.101	1.114	1.100	1.105
1957	1.089	1.091	1.084	1.098	1.084	1.088
1958	1.183	1.187	1.174	1.194	1.179	1.182
1959	1.166	1.169	1.158	1.176	1.161	1.165
1960	1.173	1.176	1.165	1.181	1.168	1.173
1961	1.187	1.189	1.182	1.197	1.182	1.186
1962	1.186	1.189	1.180	1.196	1.181	1.185
1963	1.268	1.272	1.256	1.280	1.264	1.267
1964	1.314	1.319	1.299	1.327	1.312	1.315
1965	1.310	1.315	1.297	1.309	1.306	1.309
1966	1.311	1.315	1.299	1.310	1.305	1.309
1967	1.303	1.306	1.295	1.303	1.296	1.298
1968	1.403	1.408	1.392	1.400	1.398	1.401
1969	1.434	1.439	1.420	1.426	1.429	1.431
1970	1.475	1.480	1.462	1.461	1.469	1.470
1971	1.536	1.542	1.522	1.523	1.531	1.532

It is well known that cost-of-living indexes for homothetic preferences are independent of the base-period utility level. This follows because any homothetic indirect utility function can be written

$$V_{HOM} = M \cdot f(p) \tag{40}$$

and thus $I(p^t, p^b, V^s) = M^t_s / M^b_s = f(p^t)/f(p^b) = I(p^t, p^b)$. The homothetic translog form can be written

$$\ln V = \ln M - \sum_i \alpha_i \ln p_i - \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln p_i \ln p_j \tag{41}$$

resulting in cost-of-living indexes

$$I(p^t,p^b) = \exp \left(\sum_i \alpha_i \ln (p_i^b/p_i^t) + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln p_i^b \ln p_j^b - \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln p_i^t \ln p_j^t \right)$$

The indirect CES utility function can be derived by plugging the demand equations into the direct utility function to obtain

$$\ln V = \ln M - \frac{1}{1-\sigma} \ln \sum \alpha_i (p_i)^{1-\sigma} \tag{42}$$

Thus the CES cost-of-living indexes can be written

$$I(p^t,p^b) = \left[\frac{\sum_i \alpha_i (p_i^t)^{1-\sigma}}{\sum_i \alpha_i (p_i^b)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} \tag{43}$$

Finally, the linear logarithmic indirect utility function can be written

$$\ln V = \sum \alpha_i \ln \alpha_i + \ln M - \sum \alpha_i \ln p_i \tag{44}$$

implying the cost-of-living index

$$I(p^t,p^b) = \prod_i (p_i^t/p_i^b)^{\alpha_i} \tag{45}$$

The cost-of-living indexes for these homothetic utility functions are presented in Tables 19 and 20.

Assessing the cost-of-living indexes in Tables 19 and 20, we find that the restriction of additivity has a surprisingly small impact. The indexes for the additive translog, indirect addilog, and Klein-Rubin forms deviate very little from the unrestricted translog indexes. For the meat data, changing the base year has more impact on the indexes than does the imposition of additivity. The restriction of homotheticity requires cost-of-living indexes to coincide for all base years. The homothetic translog, CES, and linear logarithmic indexes are extremely close together. Thus even conditional on homotheticity, the imposition of additivity has little impact. Furthermore, the homothetic indexes do not differ markedly from the nonhomothetic indexes for any of our three base years. We conclude that the loss of information for cost-of-living indexes from use of restrictive neoclassical utility functions is quite small, at least for our two applications.

If we accept that the translog cost-of-living indexes are reliable, we must conclude that quite good estimates of cost-of-living indexes can be obtained from using functional forms with either additivity or homotheticity imposed. The question which now arises is: Do traditional price indexes not motivated by utility functions, such as the Laspeyres, also provide good facsimiles of cost-of-living indexes?[15] We investigate this question by computing various price indexes suggested in the literature and comparing them with our translog cost-of-living indexes.

The best-known price indexes are the Laspeyres and the Paasche. The Laspeyres index can be written

$$L(p^t,p^s) = \frac{\sum x_i^s p_i^t}{\sum x_i^s p_i^s} \sum w_i^s \left(\frac{p_i^t}{p_i^s}\right) \tag{46}$$

where $w_i^s = x_i^s p_i^s \Big/ \sum x_i^s p_i^s$.

The Laspeyres index is of special interest because it provides an upper bound for the cost-of-living index $I(p^t,p^s,V^s)$ defined over any preference field. The Paasche index can be written

$$P(p^t,p^s) = \frac{\sum x_i^t p_i^t}{\sum x_i^t p_i^s} = \frac{1}{\sum w_i^t \left(\frac{p_i^s}{p_i^t}\right)} = \frac{1}{L(p^s,p^t)} \tag{47}$$

A Paasche index is the inverse of a Laspeyres index for a different base period; thus, Paasche indexes do not provide any information on admissible bounds for cost-of-living indexes beyond that provided by Laspeyres indexes. We compute Laspeyres and Paasche price indexes

[15] The widespread use of the Laspeyres index was not motivated by considerations of consumer preferences. However, Pollak (1971b) has shown that the Laspeyres price index is the cost-of-living index corresponding to a fixed coefficients utility function. Unlike the neoclassical utility functions considered in this paper, the fixed coefficients utility function allows no substitution among commodities.

TABLE 21

Laspeyres and Paasche Price Indexes for Meat

	Laspeyres			Paasche		
	(Base Year)			(Base Year)		
	1947	1958	1967	1947	1958	1967
1947	1.000	1.013	1.031	1.000	.992	.986
1948	1.120	1.140	1.163	1.118	1.113	1.106
1949	1.049	1.068	1.090	1.047	1.046	1.041
1950	1.081	1.098	1.122	1.073	1.077	1.072
1951	1.203	1.221	1.247	1.173	1.189	1.188
1952	1.193	1.211	1.238	1.171	1.183	1.181
1953	1.111	1.119	1.136	1.110	1.106	1.102
1954	1.094	1.093	1.105	1.086	1.083	1.081
1955	1.030	1.041	1.059	1.031	1.026	1.023
1956	.986	.992	1.007	.985	.984	.983
1957	1.072	1.068	1.079	1.060	1.064	1.066
1958	1.178	1.169	1.181	1.154	1.169	1.174
1959	1.134	1.128	1.141	1.107	1.124	1.129
1960	1.120	1.115	1.128	1.100	1.113	1.117
1961	1.122	1.108	1.116	1.089	1.107	1.114
1962	1.153	1.142	1.152	1.123	1.141	1.147
1963	1.134	1.124	1.134	1.108	1.124	1.129
1964	1.112	1.101	1.110	1.088	1.101	1.107
1965	1.200	1.182	1.188	1.160	1.179	1.187
1966	1.308	1.281	1.285	1.249	1.271	1.283
1967	1.263	1.240	1.245	1.208	1.233	1.245
1968	1.292	1.271	1.277	1.244	1.266	1.277
1969	1.407	1.382	1.389	1.348	1.376	1.390
1970	1.479	1.449	1.454	1.407	1.439	1.454
1971	1.467	1.446	1.455	1.402	1.436	1.451

for our meat and produce data for the same base years as the cost-of-living indexes. The meat price indexes are presented in Table 21, and the produce price indexes are presented in Table 22. In order to facilitate comparison with the cost-of-living indexes in Tables 19 and 20, we normalize the Laspeyres and Paasche indexes to equal the translog cost-of-living indexes in the base year. Interpreting the Laspeyres and Paasche price indexes as estimates of the translog cost-of-living indexes, we find that they provide poorer estimates than the cost-of-

TABLE 22

Laspeyres and Paasche Price Indexes for Produce

	Laspeyres			Paasche		
	(Base Year)			(Base Year)		
	1947	1958	1967	1947	1958	1967
1947	1.000	1.014	1.041	1.000	.989	.959
1948	1.030	1.036	1.053	1.030	1.020	.993
1949	1.040	1.035	1.047	1.037	1.029	1.004
1950	.961	.958	.975	.953	.951	.934
1951	1.047	1.062	1.087	1.047	1.044	1.023
1952	1.146	1.126	1.131	1.124	1.126	1.112
1953	1.077	1.066	1.081	1.059	1.063	1.052
1954	1.052	1.038	1.054	1.031	1.036	1.027
1955	1.075	1.059	1.071	1.047	1.056	1.053
1956	1.131	1.113	1.123	1.099	1.111	1.109
1957	1.122	1.096	1.102	1.081	1.094	1.093
1958	1.209	1.196	1.200	1.179	1.196	1.192
1959	1.189	1.178	1.184	1.159	1.178	1.174
1960	1.208	1.183	1.185	1.161	1.182	1.180
1961	1.213	1.196	1.207	1.175	1.197	1.194
1962	1.224	1.194	1.199	1.166	1.193	1.194
1963	1.308	1.283	1.281	1.246	1.278	1.273
1964	1.363	1.333	1.323	1.289	1.325	1.317
1965	1.367	1.325	1.313	1.277	1.317	1.313
1966	1.369	1.323	1.314	1.275	1.316	1.313
1967	1.364	1.312	1.308	1.257	1.304	1.308
1968	1.481	1.417	1.404	1.352	1.403	1.404
1969	1.509	1.449	1.433	1.369	1.425	1.432
1970	1.556	1.490	1.474	1.408	1.465	1.471
1971	1.623	1.553	1.534	1.460	1.522	1.531

living indexes from the restrictive utility functions—at least over long time periods. For example, with 1947 as the base year, the Laspeyres index indicates that meat prices increased 46.7 per cent by 1971, and the Paasche index indicates a 40.2 per cent increase; the translog function indicates, however, that the cost of meat increased 43.7 per cent. The discrepancies for the produce data are greater: the Laspeyres index increased 62.3 per cent, the Paasche 46.0 per cent, and the cost-of-living index 54.4 per cent. Over shorter time periods, however, the

Laspeyres indexes appear to provide acceptable approximations to the translog cost-of-living indexes.

The Laspeyres and Paasche price indexes are examples of fixed-weight indexes; the same base period is used for the computation of the index in every year t. The disparity between cost-of-living indexes and Laspeyres and Paasche price indexes typically increases with the distance between the base year s and the comparison year t. It has often been suggested that changing base periods every few years would reduce the disparity with cost-of-living indexes. In fact, the U.S. Bureau of Labor Statistics changes base period approximately every ten years. In order to assess the impact of changing base years, we compute chain-link Laspeyres and Paasche price indexes, where each base period is used to compute the index only for the successive year. The chain-link Laspeyres price index can be written

$$L(t)/L(t-1) = \sum w_i^{t-1}(p_i^t/p_i^{t-1}) \tag{48}$$

and the chain-link Paasche can be written

$$P(t)/P(t-1) = \left[\sum w_i^t(p_i^{t-1}/p_i^t)\right]^{-1} \tag{49}$$

We present chain-link Laspeyres and Paasche indexes for meat and produce in Table 23. The linked indexes for produce are closer to the 1947 base cost-of-living index; the linked indexes for meat, however, differ from the 1947 base cost-of-living index more than the 1947 base nonlinked indexes. Thus the effect of using chain-linked Laspeyres or Paasche indexes does not seem to be predictable.

Among the many index formulas discussed by Fisher (1922) is a chain-linked index, which has subsequently been advocated by Tornqvist (1936) and Theil (1967). The index, which we refer to as the Tornqvist chain-link index, can be written

$$\ln(T(t)/T(t-1)) = \sum [(w_i^t + w_i^{t-1})/2] \ln(p_i^t/p_i^{t-1}) \tag{50}$$

The interesting thing about the Tornqvist chain-link index is that it is very closely related to a cost-of-living index for a particular functional form. Diewert (1973), Sato (1973), and Thangiah (1973) have shown that (50) corresponds to the cost-of-living index from the homothetic indirect translog utility function. The only difference is that (50) uses observed budget shares as weights, while the homothetic translog cost-of-living index uses fitted values from the budget share equations (18) as weights

$$\ln(H(t)/H(t-1)) = \sum_i ((\hat{w}_i^t + \hat{w}_i^{t-1})/2) \ln(p_i^t/p_i^{t-1}) \tag{51}$$

TABLE 23

Chain-Link Price Indexes for Meat and Produce

	Meat			Produce		
	Laspeyres	Paasche	Tornqvist	Laspeyres	Paasche	Tornqvist
1947	1.000	1.000	1.000	1.000	1.000	1.000
1948	1.120	1.118	1.119	1.030	1.030	1.030
1949	1.049	1.047	1.048	1.039	1.037	1.038
1950	1.078	1.074	1.076	.959	.957	.958
1951	1.196	1.186	1.191	1.055	1.050	1.052
1952	1.186	1.176	1.181	1.142	1.127	1.135
1953	1.121	1.085	1.102	1.078	1.064	1.071
1954	1.099	1.061	1.080	1.051	1.037	1.044
1955	1.045	1.005	1.025	1.073	1.058	1.066
1956	1.000	.961	.980	1.129	1.113	1.121
1957	1.081	1.035	1.058	1.112	1.096	1.104
1958	1.188	1.134	1.161	1.216	1.195	1.205
1959	1.146	1.090	1.118	1.197	1.177	1.187
1960	1.133	1.077	1.105	1.202	1.181	1.192
1961	1.127	1.070	1.098	1.217	1.196	1.206
1962	1.162	1.103	1.132	1.214	1.191	1.203
1963	1.144	1.085	1.114	1.303	1.273	1.288
1964	1.120	1.063	1.091	1.351	1.319	1.335
1965	1.202	1.137	1.169	1.348	1.313	1.330
1966	1.299	1.228	1.263	1.349	1.314	1.331
1967	1.261	1.190	1.225	1.343	1.308	1.325
1968	1.293	1.220	1.256	1.442	1.403	1.422
1969	1.407	1.327	1.367	1.472	1.432	1.452
1970	1.472	1.389	1.430	1.512	1.471	1.492
1971	1.474	1.387	1.430	1.574	1.532	1.553

The chain-link Tornqvist indexes are presented in Table 23. As expected, they are very similar to the homothetic translog indexes. Since we argued above that the homothetic translog indexes provided good approximations to the translog cost-of-living indexes, we conclude that Tornqvist chain-link price indexes provide good estimates for cost-of-living indexes.

VI. CONCLUDING REMARKS

We have fitted sets of budget share equations corresponding to eight different utility functions. The translog function is the only one which does not impose homotheticity or additivity on consumer preferences.

The translog budget share equations explain observed budget shares better than the other forms, and have price and expenditure elasticities which differ substantially from the other forms. Furthermore, hypotheses of homotheticity and additivity for the translog form are decisively rejected by statistical tests. In light of these results, one might expect that cost-of-living indexes implied by the various estimated budget share equations would differ substantially. This expectation is not fulfilled; the cost-of-living indexes for homothetic and additive functional forms do not differ substantially from those for the translog form. This finding is analogous to the finding of Berndt and Christensen (1973c), using a translog production function. They found that the imposition of separability restrictions on the translog form led to a substantial loss of fit for the cost share equations. On the other hand, Berndt and Christensen found a very high correlation between output predicted by the separable and nonseparable translog forms. In the present study, the "output" of the indirect utility function must not be substantially altered by the imposition of homotheticity or additivity—or else the implied cost-of-living indexes would differ substantially.

Theil (1971) argued that the necessity of estimating unknown parameters and specifying a base year made the explicit computation of cost-of-living indexes unattractive. He suggested using an approximation to the true cost-of-living index, which is based on observed rather than predicted budget shares—and also allows for substitution possibilities. Based on these and other arguments, Theil advocated the Tornqvist price index (50) as a good estimate for the "true" cost-of-living index. We have found that the Tornqvist index corresponds much better to cost-of-living indexes than do fixed-weight or chain-linked Laspeyres and Paasche indexes. This should not be surprising, since the Tornqvist index corresponds to the cost-of-living index for the homothetic indirect translog utility function.

Pending additional research, extension of our empirical conclusions should be made with caution. We have limited our attention to the meat and produce budgets, thereby excluding large portions of total consumer expenditures. It would be desirable to extend coverage to additional commodities, and to consider additional price indexes. We have not investigated the effects of the degree of commodity aggregation nor have we experimented with the length of interval between linking fixed-weight indexes.

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Comments on "Cost-of-Living Indexes and Price Indexes for U.S. Meat and Produce, 1947–1971"

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IN this paper, Christensen and Manser apply the indirect translog utility function to the demand for meat and to the demand for garden produce with the purpose of using the parameters of the utility function thus estimated to construct "true" cost-of-living indexes for these two components of the consumer's budget. The results with the indirect translog function are compared with ones obtained with several other utility functions, namely, the Klein-Rubin, generalized Klein-Rubin, CES, linear logarithmic, and the indirect addilog of Houthakker. Cost-of-living indexes constructed from these preference orderings for meat and produce for the period 1947–1971 are compared with one another, with traditional Laspeyres and Paasche indexes, and also with the chain-link index proposed by Tornqvist.

Let me begin my discussion with a few remarks about the translog utility function, which has been developed by Laurits Christensen in collaboration with Dale Jorgenson and Lawrence Lau. The exploitation of this function clearly marks an important innovation in applied demand analysis. The function expresses the logarithm of utility as a quadratic function in the logarithms of its arguments – quantities in the case of the direct utility function, and ratios of prices to income in the case of the indirect utility function – and, as such, it can be interpreted as a utility function in its own right or else as an approximation, accurate to the second order, of an arbitrary utility function. Unlike most utility functions currently in use, the translog form can be estimated without imposing homotheticity or additivity restrictions, and it is this feature that makes the translog function especially appealing. In my opinion, the only defect of consequence of the translog function is that it does not appear to lend itself with any ease to dynamization. At least, my own efforts in this direction have so far not led to any success.

The present paper by Christensen and Manser represents an interesting and useful attempt to employ the translog function in the construction of cost-of-living indexes for meat and garden produce. Not unexpectedly, the technical aspects of specification and estimation are very competently executed and I have no comments to make on this front. The cost-of-living indexes that finally emerge seem, on the whole, to be quite sensible, and I think that it is of more than passing interest that their construction has utilized only the indirect utility function. The results here, together with those obtained by Christensen, Jorgenson, and Lau in their 1972 paper, confirm that the indirect utility function has not received its just due in applied work. Yet, despite these positive features, the paper presents a number of problems in my opinion, and, at the risk of appearing unappreciative, I shall address the remainder of my remarks to these.

In terms of exposition, it is not clear to me, the title of the paper notwithstanding, exactly what the central focus of the paper is. Is its purpose primarily further to illustrate the translog utility function by putting it through some demanding new hoops? Or is it the cost-of-living indexes themselves that are of primary interest? In view of the fact that the title of the paper is changed from the one listed in the preliminary program ("Testing for Existence of Distinct Components in the Food Budget"), it would appear that the authors have themselves had some uncertainty as to the central message of their exercise.

Assuming that the principal purpose of the paper is, in fact, the estimation of cost-of-living indexes for subcomponents of the consumer's market basket of goods, the authors unfortunately do not provide any justification in the framework of an overall cost-of-living index for the subindexes that they derive. In a 1971 paper ("Subindices of the Cost of Living Index), Pollak has shown that the overall cost-of-living index can be written as a weighted average of component cost-of-living indexes only in the case where the underlying preference ordering is described by a generalized Cobb-Douglas utility function. Since the translog function is not of this type, partial indexes for meat and produce calculated from it accordingly cannot be interpreted as cost-of-living subindexes.

An alternative procedure, and one which can solve the problem in principle, would be to interpret the partial indexes as conditional cost-of-living indexes in the sense of Pollak. In the case of meat, for example, the conditional cost-of-living index would be interpreted as measuring the ratio of expenditures required to attain a particular indifference curve in a comparison and reference-price situation conditional on given prices and quantities for all other goods in the consumer's market basket. As I have mentioned, viewing the partial indexes as conditional cost-of-living indexes solves the interpretation problem in principle, yet I am very dubious whether, as a practical matter, consumers approach their budgeting for meat and produce in the manner that such an interpretation implies.

One other thing that I am uneasy about, but which I don't have an answer to (and I'm not aware that anyone else has an answer either), is the calculation of cost-of-living indexes from parameters that have been estimated using other price indexes. Christensen and Manser employ Laspeyres subindexes in estimation of their budget share equations and, among other things, I wonder to what extent this may have influenced the final results. More generally, my question here deals with the type of price index that is most appropriate in applied demand analysis. Pollak addressed this question to some degree in his 1971 survey paper, but as far as I am aware, it really still awaits final resolution.

I know from much personal experience that rationalizing empirical results in exercises such as this is always hazardous and problematic. However, some of the authors' results stretch even my understanding. In particular, the results show pork and fish to be complements—well, maybe—but how about fresh fruits and *fresh* vegetables being complements, but fresh fruits and *processed* vegetables being substitutes!