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# Part II
# Market and Nonmarket Aspects of Real Earnings

# Estimates of a Human Capital Production Function Embedded in a Life-Cycle Model of Labor Supply *

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IN this paper, I present estimates of the structural parameters of a human capital production function embedded in a life-cycle model of labor supply. Several recent papers (e.g., Becker and Ghez 1972, Heckman 1971 and Stafford and Stephan 1972) attempt to merge the classical theory of labor supply with models of human capital accumulation. In joining these topics, traditional labor supply theory has been expanded to an inter-temporal framework in which current and future wages and prices determine current labor supply. Human capital theory has been expanded to explicitly incorporate a three-way division of human time among work, investment, and leisure.

Although the marriage of these topics leads to a more complete theory of labor supply behavior and a more "realistic" description of the process of investment in human capital, it has not been empirically fruitful. There are two principal reasons for this: one, mathematical, and the other, statistical in nature. The mathematical problem is that dynamic models are difficult to solve explicitly. The difficulties encountered in the literature on optimal growth are compounded by the particular features that characterize investment in human beings.

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Life is finite, human capital is not resalable, and the presence of both budget constraints and death leads to optimal solutions that are not steady state. Accordingly, the luxury of the long-run path afforded economists dealing with infinite horizon problems is simply not available to economists attempting to characterize the full life-cycle behavior of human beings. Consequently, even the simplest specifications for inter-temporal preferences and human capital production functions lead to mathematically intractable functions and to theories which yield few predictions.

There is also a statistical problem which must be faced even if explicit solutions to the dynamic problem are obtained. The union of labor supply theory and human capital theory introduces an unobservable variable into the analysis: time spent investing. In considering schooling decisions, it seems acceptable to assume that a year in school is a year of complete specialization of nonconsumption time in investment. Even this assumption is open to debate. However, there is no ready way to estimate the proportion of time spent investing in on-the-job training. Since it seems likely that postschool investment occurs on the job (Stafford and Stephan 1972 assume otherwise), measured hours of work are a mixture of pure work time and investment time. Assuming that workers forgo productive work, and the associated earnings, to invest in themselves, *measured* wage rates obtained by dividing weekly income by weekly reported hours on the job systematically *understate* the true wage rate. The understatement is greatest at those ages at which the proportion of work time spent in investing is greatest.

This age-related understatement of wage rates poses a serious problem in testing life-cycle models of labor supply and consumption joined with human capital models. I demonstrate below in one model that the "true" wage rate, even though it is endogenous to the model, retains its role as the price of time in consumption, leisure, and time-investment decisions. However, in the absence of any data on the correct wage rate, tests of the life-cycle model are impossible unless the theory is recast in terms of observable variables.

While it is a relatively straightforward matter to develop testable implications of life-cycle models for consumption and leisure given the wage rate, predictions for investment time are far more difficult to obtain. Since the proportion of work time spent investing is directly related to the amount of time spent investing, the theoretically more appealing route of stating the theory solely in terms of observable phenomena is not available.

An alternative to this procedure, and the approach followed in this paper, is to devise methods for estimating the proportion of time spent investing. Once these estimates are produced, it is possible to measure the true wage rate over the life cycle and, therefore, to test life-cycle models of consumption, saving, and labor supply.

Apart from providing estimates of a corrected wage series, the methodology affords estimates of the time expended in investment over the life cycle, and its dollar value. En route to the final wage series, I estimate the parameters of a human capital production function.

In the first part of the paper, a general model of life-cycle consumption, investment, and labor supply is developed. Specific investment models are considered in an attempt to develop theoretical predictions about the life-cycle variations in the proportion of time spent investing. In the second section, a method for estimating the proportion of time spent investing is proposed and implemented, and the empirical results are discussed. The principal empirical findings are: (1) the neutrality hypothesis of Ben-Porath (1967) is rejected; (2) human capital appears to have negative productivity in its own production; (3) the estimated profile of investment over the life cycle does not follow the simple profile assumed by Mincer (1973) or the profile proposed by Haley (1973).

## I

In this section, I present a simple model of human capital accumulation in a life-cycle model of labor supply. Some of the results in this section have been presented elsewhere in the literature (Becker and Ghez 1972, Heckman 1971, Stafford and Stephan 1972).

### A. The Utility Function

Assume a one-person consumer unit with an instantaneous, strictly concave twice continuously differentiable utility function

$$U[X(t), L(t)]$$

defined for the rate of consumption of goods ($X(t)$) and leisure ($L(t)$). The first partials are assumed to be positive. The assumption of an identical utility function at each point in time is conventional. Assuming an exponential rate of time discount $\rho$, the aggregate of utility over the planning horizon $T$ may be written as

$$\int_0^T e^{-\rho t} U[X(t), L(t)]dt \tag{1}$$

As is well known (Strotz 1956), this utility specification has the convenient property that, in a world of certainty, a consumer's initial decision on his course of action is precisely that yielded by any subsequent maximization of his remaining lifetime utility, subject to the constraints resulting from previous decisions.

*B. The Budget Constraint*

Assuming a perfect credit market with constant and equal marginal costs of borrowing and lending at rate $r$, total financial assets at time $t$ may be written as

$$A(t) = \int_0^t e^{r(t-\tau)}[w(\tau)H(\tau) - P(\tau)X(\tau)]d\tau + A(0),$$

where $H(\tau)$ is hours of work at time $\tau$, at wage rate $w(\tau)$, and where $P(\tau)$ is the price of goods at age $t$. Since in a perfect credit market all loans are repaid, it must be the case that $A(T) \geqslant 0$ (i.e., discounted discrepancies between earnings $(w(t)H(t))$ and consumption $(P(t)X(t))$ must not exceed initial net worth). Note that saving (or dissaving) in financial assets is simply the rate of change of assets at time $t$

$$\dot{A}(t) = w(t)H(t) - P(t)X(t) + rA(t) \tag{2}$$

*C. Human Capital Production*

Very little is known about specific mechanisms for acquiring knowledge. The most widely used model in the human capital literature follows the suggestion of Ben-Porath (1967). Work time $(H(t))$ is distinguished from training time $(I(t))$ although the two may be intermingled in any proportion on a given job. Assuming that a continuum of training opportunities exists in firms and occupations, individuals select optimal quantities of investment time. In this framework, schools are viewed as firms specializing exclusively in training.

Since human capital is assumed to be homogeneous, firms do not pay for any of the costs of training received by their employees. An hour spent investing is an hour not spent working; and for a working individual, its cost is the wage rate $w(t)$. These training costs may be broken into direct costs (e.g., books and tuition) and indirect costs (the value of time not spent at work or consumption). Note that jointness is excluded. If work per se raises future wages, the cost of an hour of leisure is the market wage forgone plus the increment to future earnings of the last unit of work. In the Ben-Porath specification, work time and training time can be varied and are, in principle, distinguishable. Even if training and work come in fixed proportions in one job, these proportions can be varied by selecting alternative jobs (Rosen 1972).

Throughout this paper, I follow the Ben-Porath convention. Within its scope, further hypotheses have become conventional. Without much justification, it has been argued that human capital is self-productive, and that higher stocks of human capital raise the marginal product of time in producing human capital. Letting $\dot{w}$ be the time rate of change in wage rates, and ignoring direct costs, wage growth is assumed to be governed by

$$\dot{w} = F[w(t), I(t)] - \sigma w(t), \; w(0) = w_o \tag{3}$$

where $\sigma$ is an exponential depreciation factor, $w_o$ is the initial value of wage rates, $F$ is a concave twice continuously differentiable production function with positive own partial derivatives, and a *positive* cross partial derivative. An even more extreme view of the role of human capital in self-production has been proposed under the name of the "neutrality hypothesis." This hypothesis restricts $F$ so that the wage function may be written as a strictly concave function of the time cost of investment:

$$\dot{w} = k(Iw) - \sigma w \tag{4}$$

Little justification, other than that of mathematical simplicity, supports the neutrality hypothesis. Empirical work by Ben-Porath (1970) has shown that earnings data are inconsistent with this hypothesis. Nonetheless, its continued popularity makes it an important benchmark case which will be considered in this paper.

### *D. Equilibrium Conditions*

The optimality conditions for a consumer maximizing utility function (1) subject to wage constraint (2) and $A(T) \geqslant 0$ are presented below. Letting $D$ be total time available at $t$, and noting that $H(t) = D - L(t) - I(t)$, the Hamiltonian function becomes

$$e^{-\rho t}U[X(t), L(t)] + \lambda\{w(t)[D - L(t) - I(t)] - P(t)X(t) + rA(t)\} \\ + \mu\{F[w(t), I(t)] - \sigma w(t)\} \tag{5}$$

where $\lambda$ and $\mu$ are dynamic multipliers to be interpreted below. For an interior solution, the optimality conditions are

$$\text{(a)} \quad e^{-\rho t}U_X(t) - \lambda P(t) = 0$$
$$\text{(b)} \quad e^{-\rho t}U_L(t) - \lambda w(t) = 0$$

(c) $\dot{\lambda} + r\lambda = 0$
(d) $\lambda(T)A(T) = 0$

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(e) $\mu F_I - \lambda w(t) = 0$
(f) $\dot{\mu} = (\sigma - F_w)\mu - \lambda[D - L(t) - I(t)]$
(g) $\mu(T)w(T) = 0$ (6)

and equations 2 and 3.

Investment $(I(t))$, leisure $(L(t))$ and goods $(X(t))$ will be nonzero if Inada-type conditions are assumed

$$\lim_{I\to 0} F_I \to \infty \qquad \lim_{X\to 0} U_X \to \infty \qquad \lim_{L\to 0} U_L \to \infty$$

For convenience, these conditions are maintained throughout the paper. These conditions also ensure the existence of an optimal program and concavity ensures its uniqueness. The only possibility of a corner solution is that the sum of leisure plus investment may exhaust the instantaneous time budget $D$.

In this case, during intervals when $L + I = D$, equations 6(b) and 6(e) reduce to

$$e^{-\rho t}U_L = \mu F_I \tag{6(b)$'$}$$

so that the marginal cost of investment time is the (discounted) marginal utility of leisure. Further, equation 6(f) becomes

$$\dot{\mu} = (\sigma - F_w)\mu \tag{6(f)$'$}$$

At the boundary points for these intervals, $\mu(t)$ is a continuous function of time, but $\dot{\mu}$ is not, nor in general are the control variables $X(t)$, $L(t)$, $I(t)$.[1]

Equations 6(a)–6(d) are the familiar consumer equilibrium conditions. Since $U(\cdot)$ is assumed unbounded, $A(T) = 0$, and the marginal utility of income received at $t(\lambda(t))$ is seen to be an exponentially declining function of time

$$\lambda(t) = \lambda(0)e^{-rt}$$

Note that for a given consumer held at his optimal level of utility, $\lambda(0)$ is fixed, and the investment system (equations 6(c)–6(g)) is detachable from the consumer equilibrium system once $L(t)$ is specified.

[1] See, e.g., Hestenes, Theorem 2.1, p. 234.

I now turn to a more detailed examination of the optimality conditions.

*E. Demand for Leisure and Goods*

From the strict concavity of $U$, we solve for the differentiable demand relations

$$\begin{aligned} &\text{(a)} \quad X(t) = X[\lambda(0)e^{(\rho-r)t}P(t),\ \lambda(0)e^{(\rho-r)t}w(t)] \\ &\text{(b)} \quad L(t) = L[\lambda(0)e^{(\rho-r)t}P(t),\ \lambda(0)e^{(\rho-r)t}w(t)] \end{aligned} \tag{7}$$

Since strict concavity of $U$ implies

$$U_{11} < 0,\ U_{22} < 0$$

$$\begin{vmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{vmatrix} > 0$$

a logical consequence of the assumed concavity is $X_1 < 0$, $L_2 < 0$. From the twice continuous differentiability of $U$

$$X_2(t) = L_1(t)$$

In the Ramsey (1928) case of independence in the utility function, $U_{12} = 0$, which implies $X_2(t) = 0 = L_1(t)$.

Several propositions are immediately obvious with respect to the timing of the consumption of goods ($X(t)$) and leisure ($L(t)$). Differentiate 7(a) and (b) to obtain

$$\begin{aligned} &\text{(a)} \quad \dot{X}(t) = \lambda(0)e^{(\rho-r)t}\{(\rho - r)[X_1P(t) + X_2w(t)] + X_1\dot{P}(t) + X_2\dot{w}(t)\} \\ &\text{(b)} \quad \dot{L}(t) = \lambda(0)e^{(\rho-r)t}\{(\rho - r)[L_1P(t) + L_2w(t)] + L_1\dot{P}(t) + L_2\dot{w}(t)\} \end{aligned} \tag{8}$$

If leisure and goods are normal in each time period the terms $X_1P(t) + X_2w(t)$ and $L_1P(t) + L_2w(t)$ are negative.[2]

[2] Normality is used in the following sense. Disregard human capital accumulation and insist that the consumer live within his means for each instant of time, but give him allotment $Y(t)$ to supplement his earnings. Then he maximizes $U[X(t), L(t)] + \lambda(t)\{Y(t) + W(t)[D - L(t)] - P(t)X(t)\}$ and assuming interior solutions, the Hessian for displacement analysis, which yields income and substitution effects, is

$$\begin{Bmatrix} U_{11} & U_{12} & -P(t) \\ U_{21} & U_{22} & -w(t) \\ -P(t) & -w(t) & 0 \end{Bmatrix}$$

For a maximum, the Hessian must have a positive determinant $K$. Then the income effect for goods is

$$\frac{U_{12}w(t) - U_{22}P(t)}{K} = -[X_1P(t) + X_2w(t)]$$

[Concluded on p. 234]

To interpret these equations, suppose, for the moment, that independence in preferences is assumed so that $X_2 = 0$ and $L_1 = 0$, and that the rate of interest ($r$) equals the rate of time preference ($\rho$) so that

$$\dot{X}(t) = \lambda(0)X_1\dot{P}(t)$$
$$\dot{L}(t) = \lambda(0)L_2\dot{w}(t)$$

If wages are assumed to be smooth functions of time, at that age ($\hat{t}$) with peak wages ($\dot{w}(\hat{t}) = 0$), leisure is at a minimum, ($\dot{L}(\hat{t}) = 0$) and as wage rates increase ($\dot{w}(t) > 0$) leisure declines ($\dot{L}(t) < 0$) since $L_2 < 0$. Similar conditions apply to $\dot{X}(t)$ with respect to $\dot{P}(t)$.

Suppose that we retain independence in utility ($X_2 = L_1 = 0$), but allow for $\rho \neq r$. Then

$$\dot{X}(t) = \lambda(0)e^{(\rho-r)t}\{(\rho - r)[X_1P(t)] + X_1\dot{P}(t)\}$$

Then, if $\rho > r$, the peak in consumption, if it occurs, arises after the peak in prices, since $X_1 < 0$. If prices are stable, goods consumption decreases with age $t$. If $\rho < r$, the trough in goods consumption occurs before the peak in the price. Similar conditions apply to the demand for leisure and its price $w(t)$.

In the more general case of nonindependence in utility, the life-cycle profile of consumption of goods and leisure depends on the pattern of wage rates and prices. In contrast to previous work on life-cycle consumption by Modigliani and Brumberg (1954), and Yaari (1964), the trajectories of earnings and consumption expenditure are linked through life-cycle variations in the price of time, $w(t)$. Given data on the life cycle of a representative individual, it is possible to test hypotheses about the signs of the derivatives of $X(t)$ and $L(t)$, and to estimate $\rho - r$.

*F. Investment Relationships*

In contrast to the analysis of life-cycle consumption of goods and leisure, the analysis of investment time leads to few predictions about life-cycle behavior. This ambiguity reduces our ability to test the predictions of the previous section, since the measured price of time is

while the income effect for leisure is

$$\frac{U_{12}w(t) - U_{12}P(t)}{K} = -[L_1P(t) + L_2w(t)]$$

If augmenting income allotment $Y(t)$ raises the consumption of $X(t)$ and $L(t)$, both goods are normal in the sense used in the text, and the proposition in the text follows immediately.

systematically related to the proportion of working time spent investing.

To establish this ambiguity, consider equations 6(e)–6(g). These are more easily understood if $\mu(t)/\lambda(t)$ is replaced by $g(t)$. This substitution is permissible since $\lambda(t)$ is nonzero. The investment equilibrium conditions become

$$w(t) = g(t)F_I(t) \qquad 6(\mathrm{e})'$$

$$\dot{g}(t) = [\sigma + r - F_w(t)]g(t) - [D - L(t) - I(t)] \qquad 6(\mathrm{f})'$$

$$g(t)W(T) = 0 \qquad 6(\mathrm{g})'$$

Since $w(t)$ is nonzero, the expression for $g(t)$ may be written as

$$g(t) = \int_t^T e^{-\int_t^\tau [\sigma + r - F_w(l)]\,dl}\, H(\tau)d\tau$$

so that $g(t)$ is a discounted stock of future working hours. Note that capital productivity $F_w$ tends to offset the depreciation and interest rates in calculating the present value of future hours of work. Equation 6(e)′ is the familiar condition that at an optimum, the marginal cost of investment time ($w(t)$) equals the marginal contribution of investment time to the present value of future earnings.

From these conditions, it is possible to conclude little about optimal investment patterns except the obvious point that at the end of life ($t = T$) no investment will be undertaken. To gain further insight into the nature of optimal investment policies, more structure has to be imposed on the problem. The Ben-Porath neutrality model serves as a convenient benchmark. In the Ben-Porath case, the human capital production function becomes

$$\dot{w} = k(Iw) - \sigma w$$

where $k' > 0$, $k'' < 0$. Letting $\tilde{g}$ be the value of $g(t)$ for the Ben-Porath model, conditions 6(e)′–6(g)′ become

$$1 = \tilde{g}(t)k'[I(t)w(t)] \qquad 6(\mathrm{e})''$$

$$\dot{\tilde{g}} = (\sigma + r)\tilde{g} - [D - L(t)] \qquad 6(\mathrm{f})''$$

$$\tilde{g}(T)w(T) = 0 \qquad 6(\mathrm{g})''$$

In the original Ben-Porath model, leisure is assumed to be fixed at the same value at each point in the consumer's life cycle. Letting $\bar{L}$ be that value, $\tilde{g}(t)$ becomes

$$\tilde{g}(t) = (D - \bar{L}) \int_t^T e^{-(\sigma + r)(\tau - t)} d\tau$$

so that $\dot{\tilde{g}} < 0$. From equation 6(e)″, the dollar cost of time investment, $(I(t)w(t))$, is inversely related to $\tilde{g}(t)$, and hence for ages beyond the period of specialization, gross investment declines with age.[3] If depreciation is zero, the amount of time invested must also decline over the life cycle, so that the understatement of true wage rates by measured wage rates continuously declines with age.

If the path of leisure were to decrease continuously so that total hours spent in the market increase with age, $\tilde{g}$ need not be negative at all post-specialization-period ages of the life cycle, and gross investment, and the proportion of market time spent investing may increase for a while. Of course, the approach of the retirement period eventually causes both gross investment and the proportion of working time alloted to investment to decrease to zero.

To apply the Ben-Porath model to a life-cycle model of labor supply, the assumption of a fixed amount of leisure must be relaxed. Of course, it is possible that leisure is fixed as a result of utility maximizing decisions. An alternative argument, suggested by the work of Michael (1973) shows how leisure time may be neutralized from the analysis of investment decisions in precisely the same way that investment time is neutralized in the Ben-Porath model.

If human capital effectively expands the amount of leisure time available, and does it in such a way that a 10 per cent increase in human capital leads to a 10 per cent increase in the quantity of effective leisure time, the instantaneous utility function may be written as

$$G = G[X(t),\ w(t)L(t)]$$

so that utility is a function of market goods and the dollar cost of time consumption. Condition 6(f)′ becomes

$$\dot{\tilde{g}} = (\sigma + r)\tilde{g} - D$$

so that $\dot{\tilde{g}} < 0$ and all of the implications of the Ben-Porath model concerning the life-cycle profile of investment remain intact. If the rate of depreciation ($\sigma$) is zero, and the rate of time preference is less than the rate of interest ($\rho < r$), work time increases over the life cycle, and the proportion of work time spent investing decreases monotonically.

If Michael neutrality is ignored, it is clear from inspection of the general expression for $\tilde{g}(t)$

[3] During a period of specialization, gross investment must increase, since wages increase and the amount of time spent in investment remains constant.

$$\tilde{g}(t) = \int_t^T e^{-(\sigma + r)(\tau - t)} [D - L(\tau)] d\tau$$

that if hours of time supplied to the market for both work and investment time (i.e., $D - L(t)$) remain constant (Ben-Porath) or decline beyond age $t$, $\dot{\tilde{g}} < 0$, the Ben-Porath implications for investment time remain valid. Only if future hours of market activity increase will gross investment increase with age, and the rate of increase in hours must be "suitably large." Intuitively, $\tilde{g}(t)$ may increase with time if the loss in the total stock of market time due to aging ($\dot{D} - L(t)$) is more than offset by the reduction in the discount factor applied to the remaining stock of future hours.

However, it is not possible, a priori, to rule out the increase in future hours of work, so that even within a very simple model, no prediction about the behavior of the proportion of working time spent investing is possible. Without greater specificity about the structure of preferences and the human capital production function, little can be said about the structure of optimal investment policies. In the next section, a very specific model is analyzed in an attempt to derive refutable propositions about investment behavior.

### *G. A Specific Model*

In the last section, an inconclusive discussion of investment behavior was presented. In this section, much stronger structure is imposed on the problem in an attempt to derive testable implications. Depreciation and time preference are ignored. Human capital is excluded from its own production. These assumptions simplify the analysis and help pinpoint the sources of ambiguity, but as we shall see, they do not yield an unambiguous theory of life-cycle investment.

The instantaneous utility function is specialized to an additive form to ignore complications about substitution between time and goods. Thus, using the same notation for the variables as utilized in the previous section, the instantaneous utility function is

$$U(t) = aX^{\alpha} + fL^{\phi},\ 0 < \alpha < 1,\ 0 < \phi < 1,\ a > 0, f > 0 \quad \text{(G-1)}$$

Saving is as before

$$A = w(D - L - I) - PX + rA,\ A(0) = A_0 \quad \text{(G-2)}$$

Wage growth is governed by

$$\dot{w} = cI^{\gamma},\ 0 < \gamma < 1,\ w(0) = w_o \quad \text{(G-3)}$$

where depreciation is ignored and human capital is excluded from its own production. Using the notation of the previous section, Pontryagin necessary conditions for a maximum for lifetime utility are

$$\alpha a X^{\alpha-1} = \lambda P \tag{G-4a}$$

$$\phi f L^{\phi-1} = \lambda w \tag{G-4b}$$

$$c\gamma\mu I^{\gamma-1} = \lambda w \tag{G-4c}$$

$$\mu(T)w(T) = 0 \tag{G-4d}$$

$$\lambda(T)A(T) = 0 \tag{G-4e}$$

$$\dot{\lambda} = -r\lambda \tag{G-4f}$$

$$\dot{\mu} = -\lambda(D - L - I) \tag{G-4g}$$

Since

$$\lim_{X\to 0} X^{\alpha-1} \to \infty, \qquad \lim_{L\to 0} L^{\phi-1} \to \infty, \qquad \lim_{I\to 0} I^{\gamma-1} \to \infty,$$

the only possibility of a corner solution comes from the constraint $D \geqslant L + I$. If this is binding, equations G-4b and G-4c condense to

$$\beta b L^{\beta-1} = c\gamma\mu I^{\gamma-1} \tag{G-4b)'$$

and equation G-4g′ becomes

$$\dot{\mu} = 0 \tag{G-4g)'$$

Existence of optimal controls is assured by a theorem of Cesari (1965) since the Hamiltonian is concave in the control variables.

From equation $G$-4f, since $A(T) = 0$ (because $U(t)$ is unbounded),

$$\lambda(t) = \lambda(0)e^{-rt}$$

Since $w(T) = 0$

$$\mu(t) = \int_t^T e^{-r\tau}[D - L(\tau) - I(\tau)]d\tau$$

From equations G-4a, G-4b, and G-4c, if $L + I < D$

$$X = \left(\frac{\lambda(0)e^{-rt}P}{a\alpha}\right)^{\frac{1}{\alpha-1}} \tag{G-5a}$$

$$L = \left(\frac{\lambda(0)e^{-rt}w}{\phi f}\right)^{\frac{1}{\phi-1}} \tag{G-5b}$$

$$I = \left(\frac{\lambda(0)e^{-rt}w}{c\gamma\mu}\right)^{\frac{1}{\gamma-1}} \tag{G-5c}$$

During periods of specialization, $L + I = D$. From equation G-4b′, since $\dot{\mu} = 0$, $L$ and $I$ are constant during a period of specialization, and the rate of growth of wages must decrease throughout such a period. During such a period

$$c\gamma\mu(t)I(t)^{\gamma-1} > \lambda(0)e^{-rt}w(t)$$

while at the end of the period, this inequality becomes a strict equality. Since $\mu(t)$ is a continuous function of time, and since $w(t)$ is also continuous, no jump occurs in $I$ at the end of a period of specialization. For the interval to terminate, the *average* rate of growth of wages must exceed the interest rate. Otherwise, a strict equality would never hold, and there would be no end to the period of specialization. But it is clearly never optimal to invest without ever working. Thus, if investment is never sufficiently productive, no period of specialization need arise.[4]

Suppose that time is sufficiently productive so that a period of specialization occurs. Will there be more than one period of specialization? If not, when does it occur? I will show that, at most, one period of specialization occurs, and that if it occurs, it comes at the earliest stage of the life cycle.

Before demonstrating these propositions, assume they are correct, and consider the behavior of the variables at the end of the period of specialization. Since $\mu(t)$ is continuous and $w(t)$ is continuous, $I(t)$ and $L(t)$ are also continuous functions of time. If, coming out of the period of specialization, wages grow at a rate exceeding the interest rate $r$, leisure decreases as does investment time. Thus, time in the market increases, and hours of productive work must increase. Some care must be taken in interpreting this result. If time at school is not counted as work or investment time, the period following the period of specialization will appear to have a sharp jump in working hours. In fact, a large portion of initial market time is, in reality, a continuation of investment time to an alternative institutional arrangement.

To establish these propositions, logarithmically differentiate equations G-5b and G-5c with respect to time to reach

[4] For example, if $cD^{\gamma} < rw(0)$ (i.e., if all available time were devoted to investment and the rate of growth of wages is less than the rate of interest), no specialization would ever occur.

$$\left(\frac{\dot{L}}{L}\right) = \frac{1}{1-\beta}\left(r - \frac{\dot{w}}{w}\right)$$

$$\left(\frac{\dot{I}}{I}\right) = \frac{1}{1-\gamma}\left(r - \frac{\dot{w}}{w} + \frac{\dot{\mu}}{\mu}\right)^{5}$$

Since $1 > \phi > 0$, and $1 > \gamma > 0$, if $\dot{w}/w > r$, leisure declines and, a fortiori, investment declines ($\dot{I} < 0$) since $\dot{\mu} < 0$. Accordingly, the proportion of market time spent investing declines monotonically and earnings rise. Wage growth rates decelerate, since investment is decreasing and the base of the rate is expanding. Sometime before, or at the end of life, $\dot{w}/w < r$, and leisure begins to increase.

As investment decreases, the growth rate in wages falls below the rate of interest, so that the consumption of leisure begins to increase while investment hours and hours of work both decrease. As investment continues to decrease, the wage rate grows at an ever slower rate, so that it is possible (but by no means necessary) that investment time will begin to increase, and total hours of productive work will fall if wage growth is low enough so that

$$\frac{\dot{w}}{w} < r + \frac{\dot{\mu}}{\mu}$$

Intuitively, at this stage in the life cycle, the discounted marginal cost of investment is low. Eventually, investment time must decrease again, since at the end of the horizon, investment terminates $I(T) = 0$, and $I$ is a continuous function of time. (Equivalently, $\dot{\mu}/\mu$ becomes increasingly more negative near the end of the horizon.)

Note that the "position" of wage growth rates defined by

$$\frac{\dot{w}}{w} = r + \frac{\dot{\mu}}{\mu}$$

has a stability property. If the rate of wage growth falls below $r + (\dot{\mu}/\mu)$, investment begins to increase, tending to raise $\dot{w}/w$, and to shut off the growth in investment time. If the wage growth exceeds $r + (\dot{\mu}/\mu)$, investment decreases, and wage growth is slowed.

To gain further insight into this case, it is of some interest to interpret $\dot{\mu}/\mu$. From equation G-4g, it is seen that

[5] Note that although $w(t)$ and $\dot{w}(t)$ are continuous functions of time, $\dot{\mu}$ need not be continuous. Accordingly, right derivatives are used where appropriate.

$$\frac{\dot{\mu}}{\mu} = -\frac{e^{-rt}H(t)}{\int_t^T e^{-r\tau}H(\tau)d\tau}$$

The term in the denominator is that portion of the discounted (from $t = 0$) stock of lifetime hours of work from $t$ to the end of life. Accordingly, $\dot{\mu}/\mu$ is the percentage decline of discounted remaining lifetime hours of work. Near the end of life, this is large in absolute value. Moreover, if hours of work are constant from $t$ on

$$\frac{\dot{\mu}}{\mu} = -\frac{r}{1 - e^{-r(T-t)}} < -r$$

so that if future hours of work are constant or decreasing, $r + (\dot{\mu}/\mu) < 0$. Since $\dot{w} \geqslant 0$, investment can never increase if future hours of work at each instant are less than, or equal to, current hours of work.

Note that the increase in investment time must come at an age after the total amount of time spent in the market has peaked ($\dot{L} = 0$), and at a time when total productive hours of work are decreasing. Moreover, for investment to increase, subsequent hours of work must increase to a level greater than the level of hours worked at the age at which investment time begins to increase. "Greater" is the appropriate term since the paths of all variables are continuous, and in the initial phase of increasing investment, hours of work must decrease. Note, too, that once the rate of growth of wages falls below the rate of interest, it never again exceeds that rate. This follows from the previously stated stability property that $\dot{I} \gtreqless 0$ as $\dot{w}/w \lesseqgtr r + (\dot{\mu}/\mu)$ since $\dot{\mu} < 0$. Accordingly, the lifetime peak in market hours supplied is never reattained, and no further period of specialization arises. Moreover, the period of specialization, if one occurs, must be in the first part of life. For, if investment were to increase until a time when specialization arises, $(\dot{w}/w) < r + (\dot{\mu}/\mu) => (\dot{w}/w) < r$, at the beginning of the specialization interval, and so the specialization interval would never terminate, and hence it is nonoptimal. If investment were to *decrease* or remain constant up to the beginning of the specialization interval level of investment, it must be the case that leisure is increasing, and $(\dot{w}/w) < r$, and hence, the previous argument applies. Thus, investment time is specialized in one period in the first part of life if, indeed, specialization of time ever arises.

Thus far, I have pursued the case of wage growth greater than the rate of interest immediately after the period of specialization. Suppose that wage growth is less than the rate of interest at the end of the specialization period. Leisure increases (i.e., total time spent in the market is at lifetime peak just after the period of specialization is over), investment decreases and hours of work increase. Precisely the same analysis as before holds for this case of $(\dot{w}/w) < r$. Again, investment may fluctuate around the "stability point" for wage growth

$$\frac{\dot{w}}{w} = r + \frac{\dot{\mu}}{\mu}$$

This second case appears to be empirically uninteresting, since it has the very strong, and empirically unacceptable, prediction that the peak in market activity arises just after the completion of schooling.

To summarize this discussion, consider Figure 1, in which the time derivative of investment is plotted against the time derivative of leisure. At any point in time, the graph is broken into three shaded regions. The lower left region ($A$) is the case of wage growth greater than the rate of interest. In this region, both leisure and investment time decrease. If the rate of growth of wages is less than $r$, but greater than $r + (\dot{\mu}/\mu)$, leisure increases, the total amount of time spent in the market contracts, and investment time decreases (region B). Only if the rate of growth of wages falls below $r + (\dot{\mu}/\mu)$ will investment time increase (region C), but the growth in wages will choke off this investment increase, and eventually the path returns to region B. However, the fact that investment is declining (and wage growth falls) keeps the trajectory in region B or region C. It can never reenter region A. One possible path is sketched for a case in which immediately after the period of specialization, the proportional rate of growth in wage rates exceeds the rate of interest.[6]

## II. ESTIMATING A HUMAN CAPITAL PRODUCTION FUNCTION

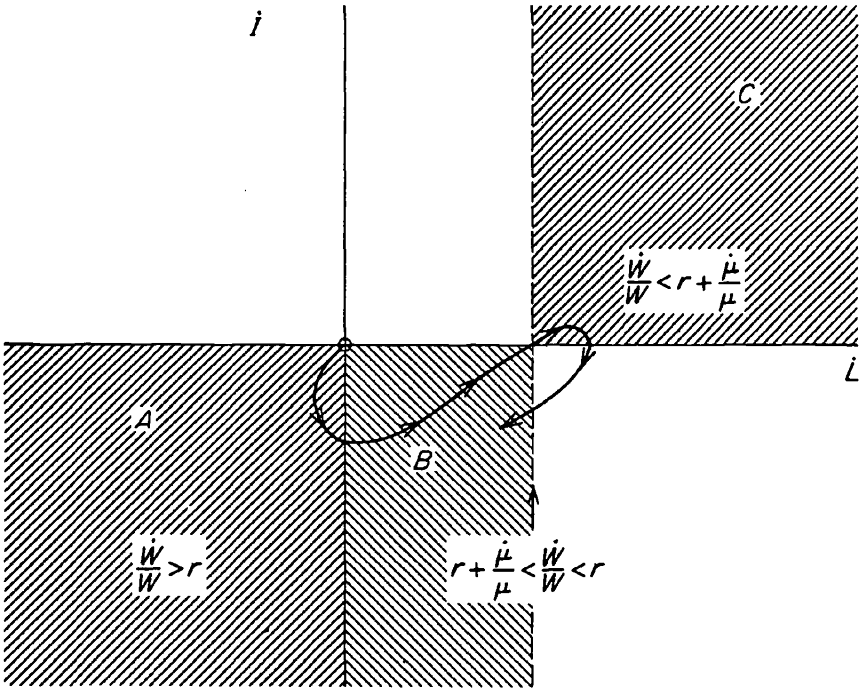
### *A. Choice of Functional Form and Methodology*

No simple profile characterizes human capital accumulation programs. Depending on initial conditions, preferences, and the nature of the production function, almost any accumulation pattern may be generated. If more information is available about the form of the production function, it might be possible to narrow the band of ignorance about capital accumulation profiles, and through simulations with

[6] Remember that the vertical boundary between B and C will shift with time.

FIGURE 1

The Three Phases of the Investment-Leisure Relationship




different initial wage rates and asset levels, to determine typical profiles.

In this section, a Cobb-Douglas production function for human capital is estimated. En route to the production parameter estimates, provisional estimates of the proportion of market time spent investing at each age are generated. The methodology is quite general, and the approach is applicable to human capital models with exogenous or endogenous labor supply.

The human capital production function is specialized to

$$\dot{w} = cI^{\gamma}w^{\beta} - \sigma w,\ 0 < \gamma < 1 \qquad (9)$$

If $\gamma = \beta$, the production function satisfies the neutrality hypothesis. If $\beta = 0$, and $\sigma = 0$, the production function is precisely that analyzed in Section I(G). If $\gamma = \beta = 1/2$, the production function is equivalent to the production functions in the models of Haley (1973) and Rosen (1973).

For postspecialization investment periods, the optimality conditions equivalent to equations 6(e)′ and 6(f)′ become

$$\dot{g}(t) = (\sigma + r)g(t) - \left[D - L(t) - I(t)\left(1 - \frac{\beta}{\gamma}\right)\right] \tag{10}$$

and

$$w(t) = g(t)c\gamma I^{\gamma-1}w^{\beta} \tag{11}$$

Equation 10 dramatically underscores the very special nature of the "neutrality" hypothesis. Only if $\gamma = \beta$ will the shadow price of investment $g(t)$ be independent of the pattern of future investment. Note further that unless leisure is fixed, as in conventional income maximizing models, or as in models in which Michael neutrality is invoked, the shadow price of investment depends on the profile of future time supplied to the market $D - L(t)$. For both reasons, Becker's (1967) conjecture that market bias ($\beta < \gamma$) leads to an accelerated decline in investment compared to the neutrality case ($\gamma = \beta$) is seen to apply only to the cost schedule. When neutrality is relaxed, and labor supply is endogenous, the demand price of human capital need not monotonically decline with age, and so investment may increase with age, at least for some age ranges.[7]

The wage rate plays two distinct roles. Looking backward, the current wage rate is a result of previous investment. Integrating equation 9, the current wage rate is the accounting identity[8]

$$w(t) = e^{-\sigma t}\left[(1 - \beta)\int_0^t cI^{\gamma}(\tau)e^{(1-\beta)\sigma\tau}d\tau + (1 - \beta)w(0)^{1-\beta}\right]^{\frac{1}{1-\beta}} \tag{12}$$

Looking forward, the wage rate is also the marginal cost of current investment time. Integrating equation 10 and substituting into equation 11,

$$w(t) = c\gamma I^{\gamma-1}w(t)^{\beta}\int_t^T e^{-(\sigma+r)(\tau-t)}\left[D - L(\tau) - I(\tau)\left(1 - \frac{\beta}{\gamma}\right)\right]d\tau \tag{13}$$

Suppose that there are data on a typical consumer's profile of time supplied to the market ($M(t) = D - L(t) = H(t) + I(t)$), but that it is

[7] Even if labor supply is set exogenously, and it increases with time, investment may increase for a period in the life cycle.

[8] This equation is analogous to Becker's income identity equation (Becker 1964, eq. 28, and Mincer 1973, eq. 3.1).

not possible to distinguish work time from investment time. Although the true wage rate is unknown, the consumer's earnings ($w(t)H(t)$) are known. Denote the proportion of market time devoted to investment by $S(t)$. Measured wage rates ($w^*(t)$) obtained by dividing earnings by reported hours in the market may be written as

$$w^*(t) = w(t)[1 - S(t)]$$

since only $1 - S(t)$ of the reported working hours have no investment content.

Conditional on a profile of $S(t)$, equations 12 and 13 may be written in terms of observable variables, and upon taking natural logarithms, the following optimality conditions for postschool investment are obtained:

$$\ln w^*(t) = \frac{\ln c\gamma}{1-\beta} + \frac{\gamma - 1}{1-\beta} \ln S(t)M(t) + \ln [1 - S(t)]$$
$$+ \frac{1}{1-\beta} \ln \left\{ \int_t^T e^{-(\sigma+r)(\tau-t)} M(\tau)[1 + (1 - \beta/\gamma)S(\tau)]d\tau \right\} \quad (14)$$

and

$$\ln w^*(t) = \ln [1 - S(t)] - \sigma t$$
$$+ \frac{1}{1-\beta} \ln \left\{ \int_0^t c[S(\tau)M(\tau)]^\gamma e^{(1-\beta)\sigma\tau} d\tau + w(0)^{1-\beta} \right\} \quad (15)$$

Since actual data come in discrete time intervals, it is necessary to make a discrete approximation to the continuous equations to estimate the parameters.[9]

The consumer is assumed to have $T$ years in his postspecialization working life so that there are $T$ observations for $M(t)$ and $w^*(t)$, and $2T$ approximate equations generated by $S(t)$, $\sigma$, $\beta$, $\gamma$, $c$, and $r$. If $S(t)$ is a judiciously parameterized function of time,[10] it is possible to estimate the parameters determining the $S(t)$, as well as $\beta$, $\sigma$, $\gamma$, $c$, and $r$. In this paper, $r$ is assumed to be 10 per cent.[11]

A measure of concordance of the parameters with the data may be defined as the squared deviation of each equation at each age from a

[9] The discrete approximations are discussed more fully in Section II (B), footnote 14.

[10] In particular, if $S(t)$ is a polynomial in time, it is necessary that the degree of the polynomial be less than $T - 4$ so that degrees of freedom are left to determine the remaining parameters $\beta$, $\sigma$, $\gamma$, and $c$, assuming $r$ is fixed. To determine these parameters with any precision, the degree of the polynomial should be much less than $T - 4$.

[11] Given data on consumption expenditure, earnings, and initial assets, an $r$ can be determined which sets $A(t)$ on page 230 to zero. This approach was not pursued in this paper.

perfect fit. Thus, introducing disturbances in the discrete approximation, and letting $V(i)$ be the disturbance for the first equation at age $i$, and letting $U(i)$ be the disturbance for the second equation, parameters may be chosen to minimize

$$\begin{vmatrix} \Sigma V_i^2 & \Sigma U_i V_i \\ \Sigma U_i V_i & \Sigma V_i^2 \end{vmatrix} \tag{16}$$

or its logarithm. Note that this criterion allows for inter-equation correlation in disturbances. If there are errors of measurement in the wage series, it is plausible that there is such inter-equation residual correlation. Minimizing this function conditional on a set of realized values for market time ($M(t)$) is equivalent to maximizing the likelihood function if equation errors are normally distributed, and independent of measurement error in the hours of market-time series.[12]

In such a complicated statistical model, identification of parameters is difficult to determine analytically. Recent work by Smallwood (1970) in estimating a somewhat similar model suggests that even when formal identification is secured, the likelihood function may be virtually indeterminate, and a wide variety of parameter estimates may lead to practically the same value for sample likelihood. Moreover, recent work by Rosen (1973) in estimating models of human capital accumulation suggests that determination of even a limited number of parameters may be a difficult task.

With these considerations in mind, the discrete $S(t)$ series is parameterized in a general logistic form to

$$S(t) = \left(1 + e^{\sum_{i=0}^{R} \chi_i t^i}\right)^{-1} \tag{17}$$

where $R$, and the $\chi_i$, $i = 0, \ldots, R$ are estimable parameters. One advantage of this parameterization is that it constrains the estimated proportions of market time spent investing to lie inside the interval 0 to 1. The procedure followed in this paper is to begin with a simple model, adding successive polynomial time terms until their contribution to sample likelihood is negligible.[13]

### *B. The Data*

In actual practice, no complete life-cycle data on any "representative consumer" exist, and resort to a synthetic cohort is necessary.

[12] Note, however, that the normality assumption is not crucial for establishing desirable asymptotic properties for nonlinear least squares. See Jennrich (1969).

[13] The actual test is a multiple equation heuristic F which asymptotically becomes a likelihood ratio test. For details, see Goldfeld and Quandt, Chapter 2. Each *parameter* was assumed to remove one degree of freedom from the data series.

Following the suggestion of Becker and Ghez (1972) and Rosen (1973), data from a cross section of individuals may be used to approximate the time series for a typical individual. This procedure ignores vintage effects and a variety of historical conditions that might influence the optimal paths of human capital accumulation for individuals of different birth cohorts.[14]

The data used in this study are from the 1970 One in a Hundred Census Tape for white college-educated employed males who are not enrolled in school, and who have not undertaken postgraduate education. These data are plotted in Figures 2A and 2B. The postschool work life is assumed to run from age 23 to age 65. A synthetic representative profile is constructed from computing geometric means of estimated annual hours worked, and geometric means of hourly wage rates. A major defect of these data is that the hourly wage data are generated by dividing reported earnings by estimated annual hours worked. Accordingly, the assumption that measurement error in the wage series is uncorrelated with measurement error in the hours-of-work series may be untenable if there is measurement error in constructing the hours-of-work variable. Since the current study is largely exploratory in nature, a more sophisticated analysis was felt to be inappropriate until some experience with simpler techniques was available.[15]

### C. Empirical Results

Since identification is a touchy issue, a cautious approach is adopted in forming parameter estimates. The first model estimated is one with the production function of Section I(G), in which human capital is excluded from its own production ($\beta = 0$) and the rate of deprecia-

[14] In fairness to Becker and Ghez (1972) and Rosen (1973), it should be stated that both studies propose methods for eliminating "smooth" vintage effects although only the first study actually implements such methods.

[15] In order to facilitate duplication of the reported results, it is necessary to describe the nature of the discrete approximations. Data on a given age group, e.g., 25 year olds, refers to information on people who just turned 25 as well as to information on people almost 26. Since the relevant census data refer to events in the previous calendar year, and since the census is taken one-quarter of a year away from the previous year, the census 25 year old is actually, on average, 24 $3/4$ years old, so far as the relevant data are concerned. In order to generate discrete approximations, this age was used for 25 year olds, as is a similar displaced age for other age groups.

An initial wage rate is needed in equation 12. Age 23 was selected as the initial date. This age is felt to be sufficiently far removed from college graduation and is used only as an initial value. The residuals for the likelihood function were generated from age 24 to the end of life. Note that inclusion of age 65 implies that the retirement age is actually assumed to be 65 $3/4$ on average. The integrals are approximated by finite sums with one year increments in each step, and midpoint values for $S(t)$ assigned.

FIGURE 2A

Hourly Wage Rates by Age, Derived from 1970 U.S. Census Data to Construct a Synthetic Cohort Used in the Empirical Analysis Data are for White College-Educated Employed Males Not Currently Enrolled in School

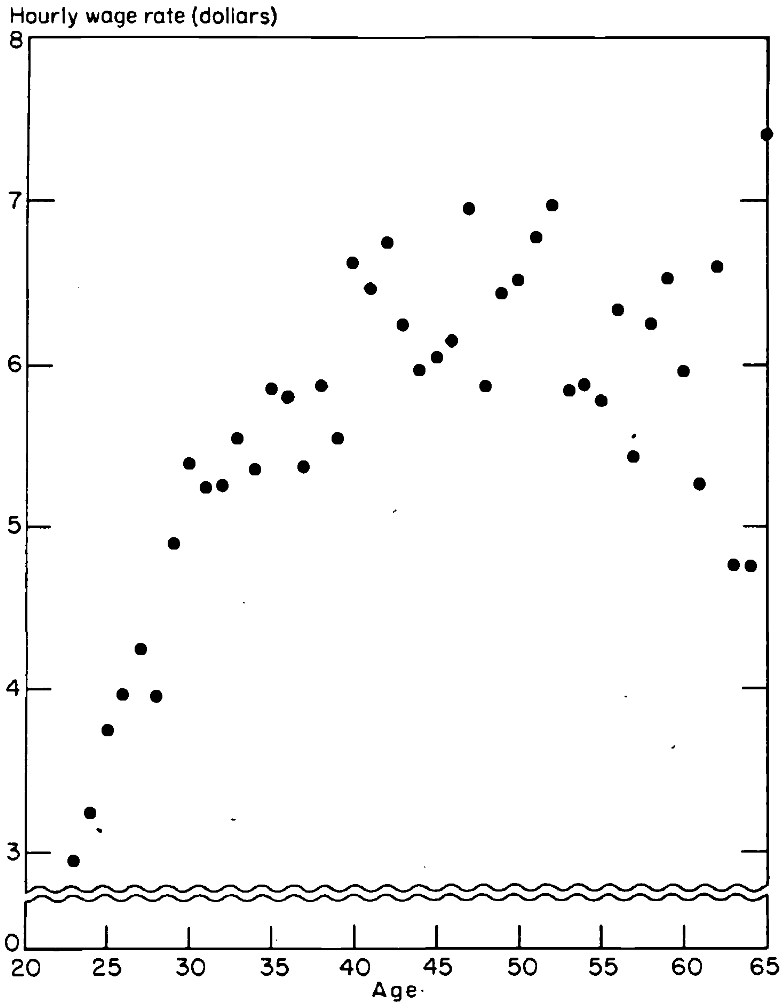


FIGURE 2B

Annual Hours Worked by Age, Derived from 1970 U.S. Census Data to Construct a Synthetic Cohort Used in the Empirical Analysis. Data are for White College-Educated Employed Males Not Currently Enrolled in School

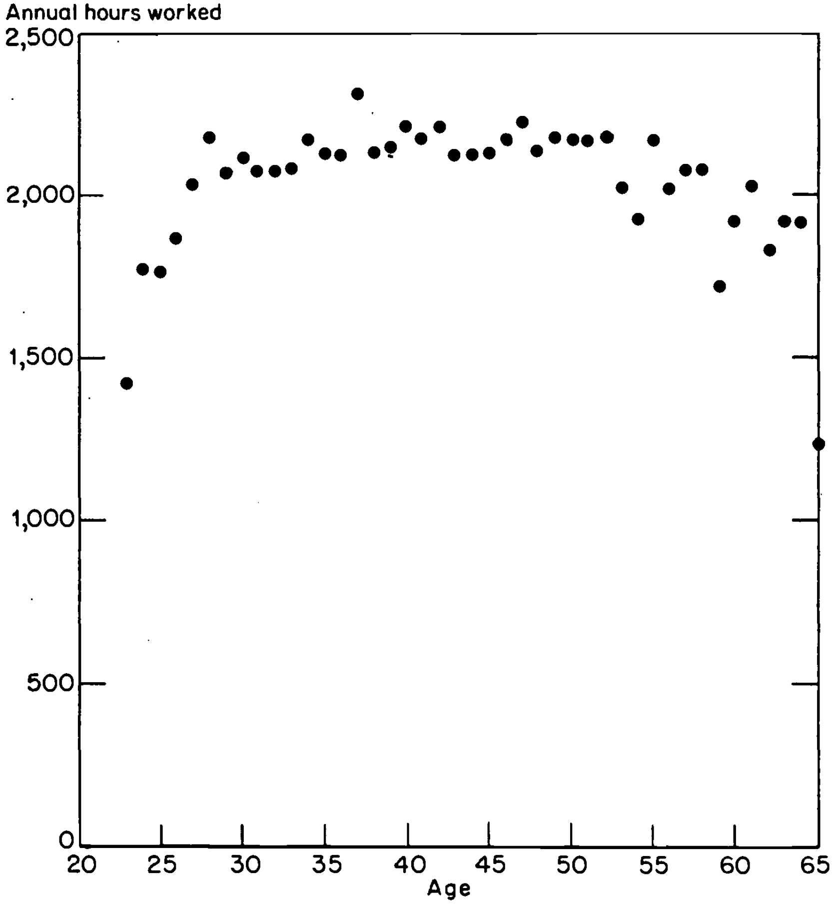


TABLE 1

(asymptotic standard errors in parentheses)

| | Model I (G) | | General Model[a,b] | |
|---|---|---|---|---|
| $c$ | $.14 \times 10^{-2}$ | $(.04 \times 10^{-2})$ | 45.49 | (3.034) |
| $\gamma$ | .67 | (.052) | .99 | (.003) |
| $\beta$ | | - | −6.69 | (.043) |
| $\sigma$ | | - | .0016 | (.00025) |
| $\chi_0$ | .99 | (.16) | .6073 | (.065) |
| $\chi_1$ | .035 | (.010) | .0906 | (.0081) |
| $\chi_2$ | $.39 \times 10^{-2}$ | $(.12 \times 10^{-2})$ | −.00537 | (.0005) |
| $\chi_3$ | - | - | .00240 | (.0007) |
| Value of log likelihood | −12.89 | | 4.69 | |

[a] Test for $\gamma = 1$ rejects that hypothesis. $\left(\frac{\gamma - 1}{.0031}\right) = 3.33.$

[b] Test for neutrality hypothesis $\gamma - \beta = 0$, $\frac{\gamma - \beta}{[\mathrm{Var}(\gamma - \beta)]^{1/2}} = \frac{7.68}{.229} = 33.$

tion ($\sigma$) is set at zero.[16] The interest rate is fixed at 10 per cent. The error sum of squares function (16) was minimized using both the Powell conjugate gradient method and Grad X.[17] Experimentation with polynomial time-trend terms beyond the second power failed to produce any significant improvement in likelihood. The results with this model are reported in the first column of Table 1. The estimated proportion of time spent investing is plotted in Figure 3. The initial proportion of time spent investing is quite high but, by age 47, becomes negligible.

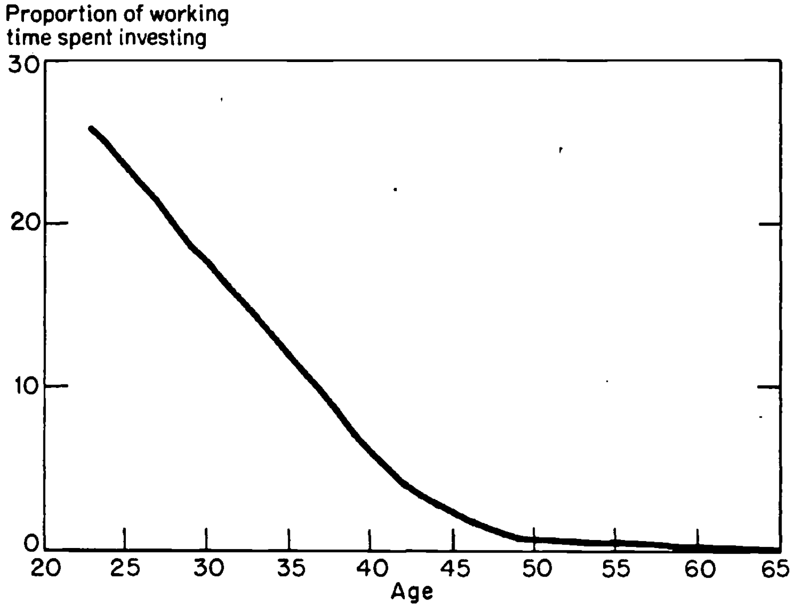
Figure 4 graphs the amount of time invested against age. Note that the *initial* increase in investment time is in apparent contradiction with the pattern predicted in Section I(G). The dollar costs of investment plotted in Figure 5 show a similar initial increase followed by a decline. With the precision afforded by these data, the logically possible

[16] Note that no special assumption is made about preferences. The effect of preferences is embodied in the observed hours-of-work series. However, given an assumption about preferences such as that made in Section I(G), it is possible to make some refutable statements about the time profile of investments.

[17] For a discussion of these techniques, see Goldfeld and Quandt (1972), Chapter 1. Both sets of optima reported in this paper were tested by using "substantial" displacements from the reported optimum parameter estimates. Both optima were stable. Of course, these experiments do not prove that a truly global optimum has been located.

FIGURE 3

Estimate of the Age Profile of the Proportion of Measured Working Hours Devoted to Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive



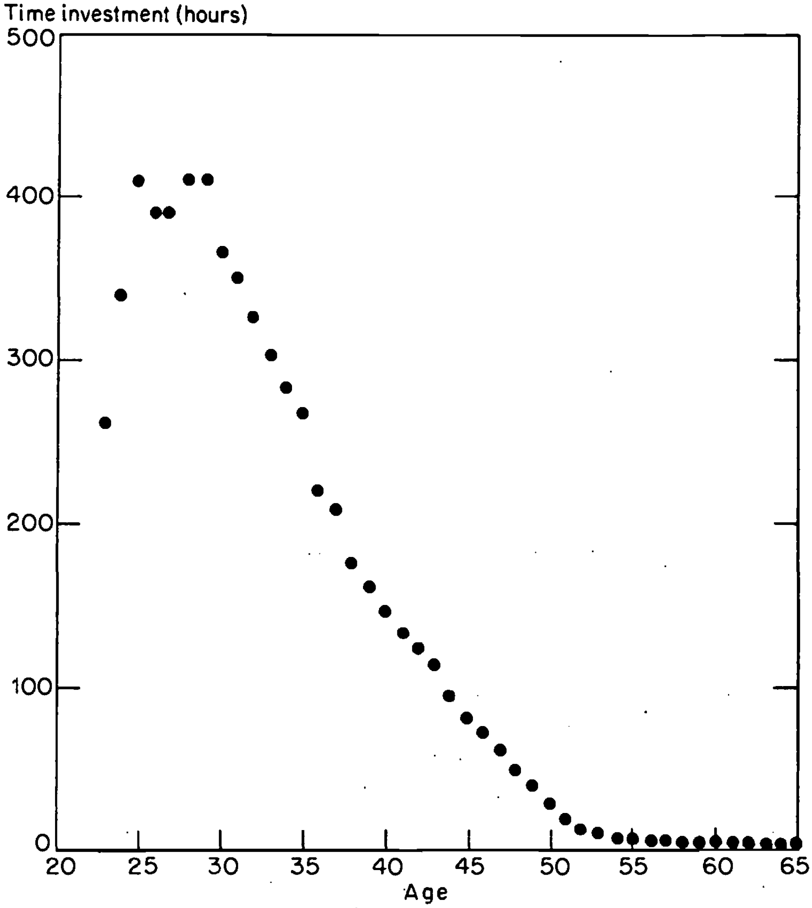

phenomenon of rising investment time following an initial decline in postschool investment appears to be empirically irrelevant.

The increase in postschool investment time immediately after the completion of schooling may be interpreted as a refutation of one of the predictions of model I(G). Either the assumption about preferences or the assumption about technology may be incorrect. With this result in mind, a more general model is also estimated. In this model, both $\beta$ and $\sigma$ are estimated, rather than assumed to be zero. Thus, it becomes possible to make a direct parametric test of the neutrality hypothesis ($\gamma = \beta$).

Empirical results with the revised model are reported in the right-hand column of Table 1. The heuristic F test indicates that a cubic time-trend term should be included. The most dramatic result with the more general model is that the returns-to-scale parameter ($\gamma$) is near

FIGURE 4

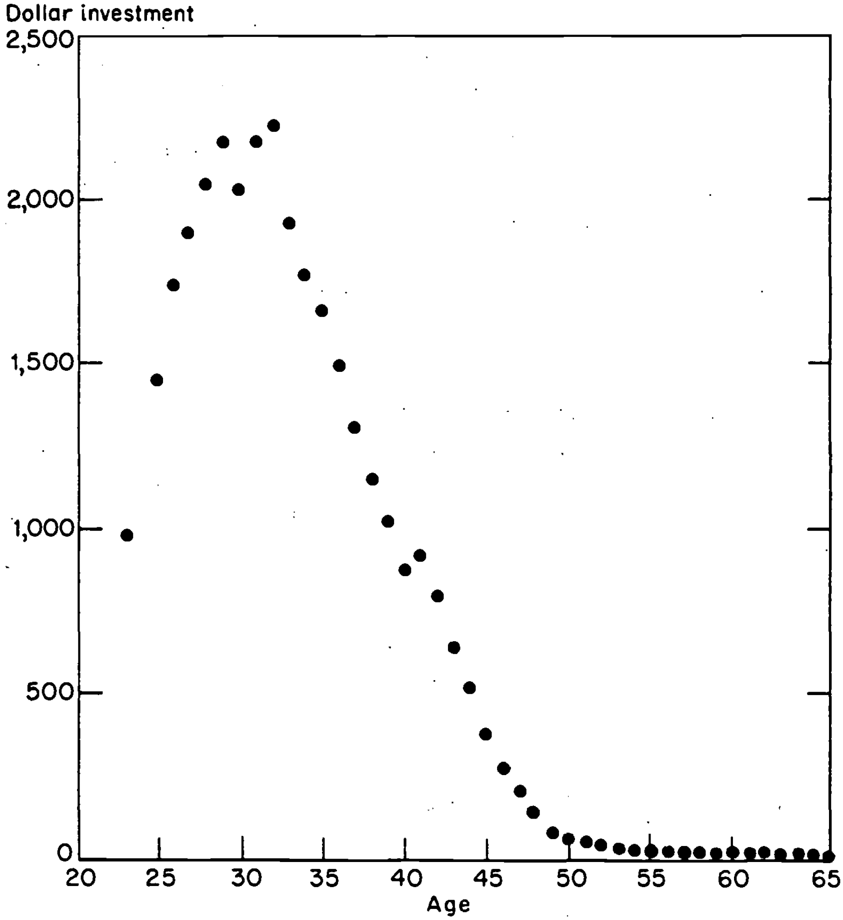
Estimate of the Age Profile of the Number of Hours Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive




unity, although as noted in the first footnote of the table, it is statistically significantly different from unity. If $\gamma$ were unity, the model would collapse into a "bang-bang" control problem, and a continuous post-school investment profile would not exist. Although there are few empirical results with which this finding can be compared, estimates produced in two other papers suggest that $\gamma$ may in fact be near unity.

FIGURE 5

Estimate of the Age Profile of the Dollar Value of the Time Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive




In his commentary on a paper by Ben-Porath, Mincer (1970) estimates the inverse of $\gamma$ as approximately 1.01. No confidence interval is stated. A recent paper by Brown (1973) estimates the inverse of $\gamma$ as 1.15. Again, no confidence interval is available. The comparison between the results in these studies and the results reported here must be qualified since both authors assume neutrality (i.e., $\gamma = \beta$).

The rate of depreciation is estimated to be quite small (one-sixth of 1 per cent or so). Human capital is seen to have *negative* self-productivity. The estimate of $\beta$ is large and negative, and the test in the footnote to the table suggests that the neutrality hypothesis is resoundingly rejected. However, the value of $\beta$ is too negative to be accepted without some skepticism.[18]

The analogues of Figures 3, 4, and 5 are presented in Figures 6, 7, and 8 respectively. Not surprisingly, in view of the large market bias implicit in the estimate of $\beta$, investment time falls off more sharply than in the previous case and, by age 37, becomes negligible.

It is noteworthy that in both models, the profile of the proportion of time spent investing at each age falls off more steeply than the linearly declining profile assumed by Mincer (1972). Moreover, in both models, the curvature of this profile has convexity properties directly opposite to those proposed by Haley (1973).

## CONCLUSION

In this paper, I have discussed life-cycle models of labor supply and human capital accumulation. Methods for estimating the parameters of the human capital production function have been proposed and implemented. By assuming a functional form for the human capital production function, it is possible to utilize both the accounting identity that wage growth is the result of investment, and the optimality condition that the wage rate should equal the marginal benefit of time investment, to estimate the parameters of the underlying production function and to determine the proportion of time spent investing at each age and the empirical relevance of the neutrality hypothesis. The estimates presented in this paper confirm Ben-Porath's (1970) finding that the neutrality hypothesis is not consistent with data on life-cycle profiles, and suggest that the simple time profiles assumed by Mincer (1970) and deduced by Haley (1973) are not empirically relevant for white college-educated males.

[18] One possible reason for the large negative value of $\beta$ may be that market inputs into postschool investment are omitted, tending to overstate the value of $\beta$ (in absolute value) if investment time is negatively correlated with the level of the wage rate. A negative correlation is plausible if $\beta$ is, in fact, negative. For the same reasons, $\gamma$ will be overstated. Another source of bias may be spurious correlation between the disturbances and the hours of work series. However, preliminary results with the Survey of Economic Opportunity data, in which wage and hours data are independently derived, suggest that estimates of $\beta$ remain large and negative.

FIGURE 6

Estimate of the Age Profile of the Proportion of Measured Working Hours Devoted to Learning and Investment Activities from a Model That Allows Human Capital to Enter Its Own Production

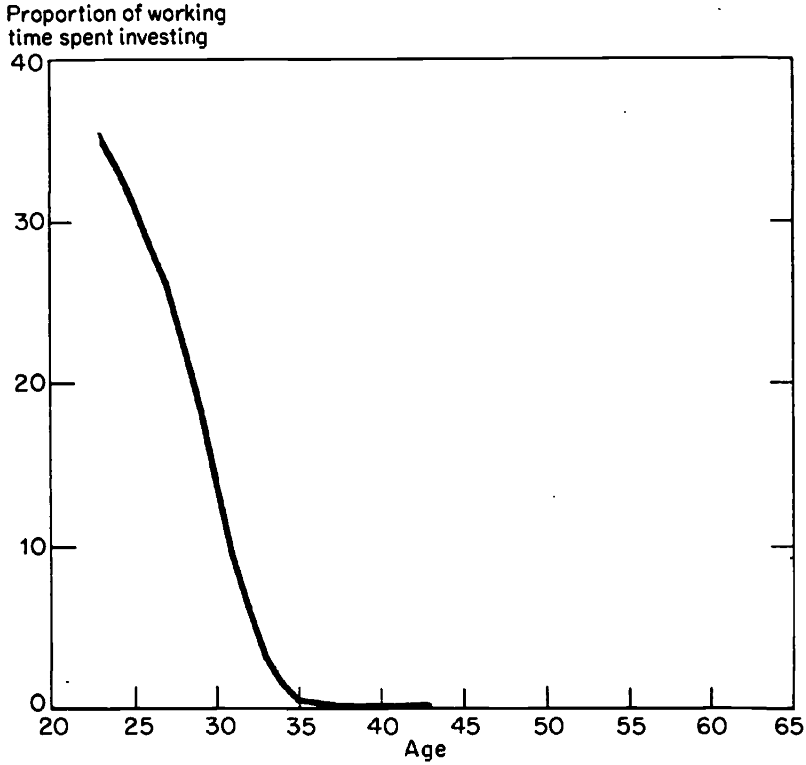


FIGURE 7

Estimate of the Age Profile of the Number of Hours Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Allows Human Capital to Enter Its Own Production

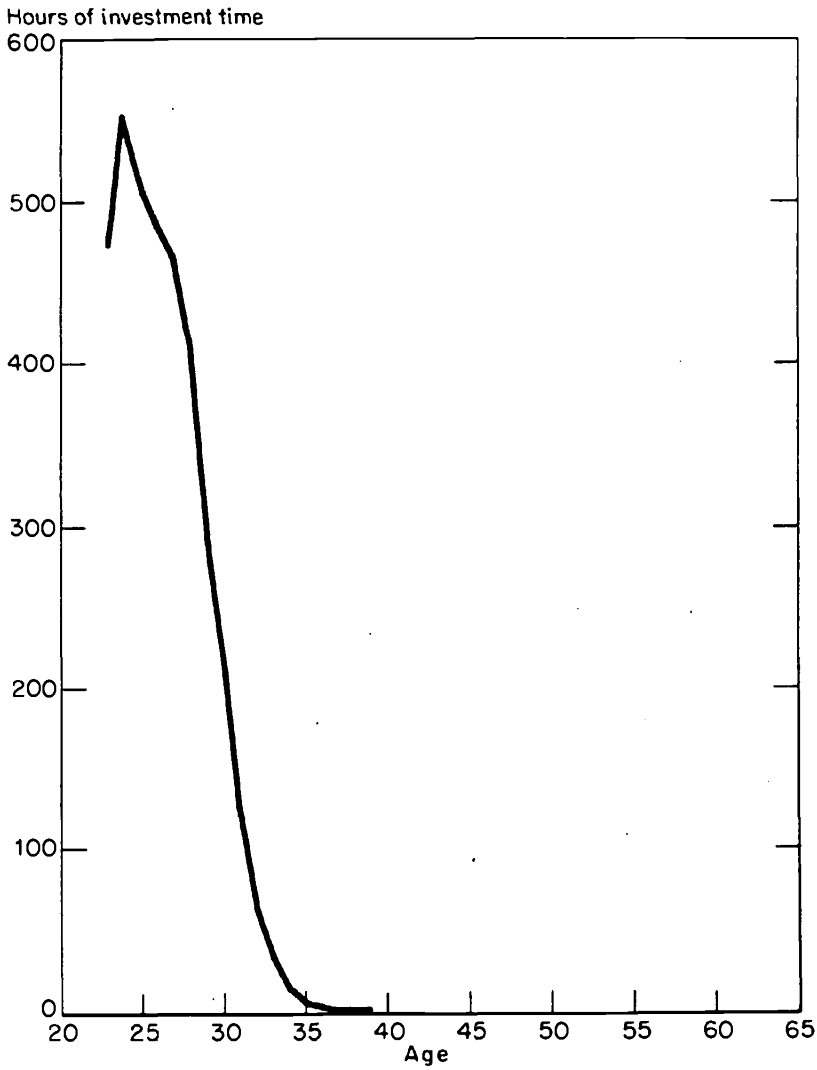
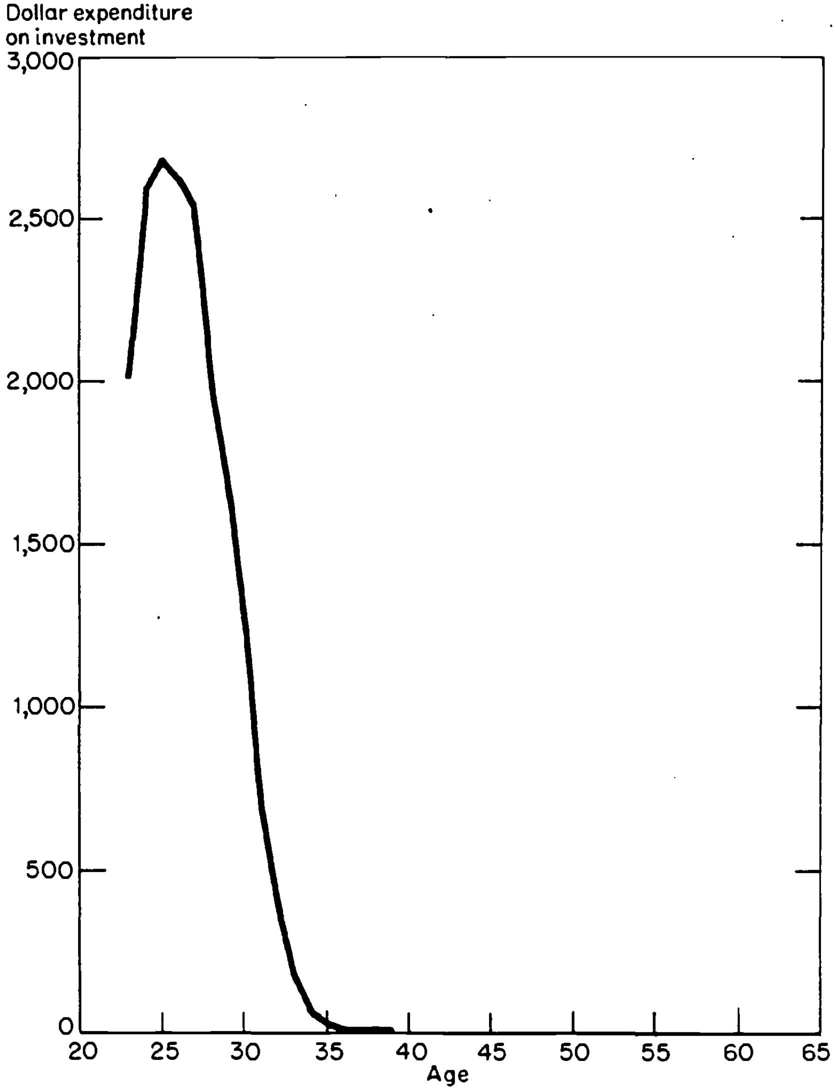


FIGURE 8

Age Profile of the Dollar Value of Time Devoted to On-the-Job Learning, Derived from a Model in Which Human Capital Enters Its Own Production



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# Comments on "Estimates of a Human Capital Production Function Embedded in a Life-Cycle Model of Labor Supply"

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JAMES HECKMAN'S paper presents a dynamic framework for analyzing the life-cycle allocation of time among three ends: activities that augment the individual's market wage (i.e., human capital), market production, and leisure. The process producing human capital is specified such that the optimal path of investment determines the current wage by past investments and equates this wage to the marginal value of future investments of time. The future pattern of investment is relevant because the Ben-Porath neutrality of human capital assumption is relaxed and the labor supply decision is made endogenous. The exposition of the model is clear and thoughtful, and in my view, it represents a unified generalization and extension of the Ben-Porath (1967, 1970) formulation. First, I will restate the problem of underidentification that appears to limit what we can learn about the human capital production function, and describe how Heckman has approached this problem. Then I shall indicate why the econometric techniques and data used by Heckman may not be the most appropriate for estimating the parameters to his formulation of the model.

To understand why market wage rates vary over an individual's life cycle, the human capital framework postulates that investments occur that augment the productive capacity of the human agent, while other processes, such as depreciation and obsolescence, may work to diminish that capacity over time. In the postschooling period of the life cycle, there is, regrettably, no obvious way to measure directly the proportion of time an individual allocates only to increase his future market productivity. Consequently, the general form of the human-capital production function must be restricted if a single time series on market wage rates is to shed much light on the time profile of postschooling investment and the many parameters that can be used to characterize the processes that produce and depreciate the stock of human capital.

Ben-Porath's approach to this problem was to assume that non-working time or leisure was fixed, or alternatively that leisure did not enter the utility function. Heckman relaxes this unappealing assumption, making labor supply decisions endogenous and permitting $\lambda(t)$, the shadow value of time in leisure, to vary independently of $\mu(t)$, the shadow cost of time in human capital investments. The "neutrality" assumption of Ben-Porath, that human capital increases equally the productivity of time in the marketplace and in the production of further human capital, is another restriction that Heckman relaxes, that is, $\gamma$ need not equal $\beta$. Finally, Heckman explores in some detail the effects of depreciation $\sigma$, the market rate of interest $r$, and individual time preference $\rho$ on the optimal regime of life-cycle investment.

From this dynamic allocative model, the demands for leisure and goods can be obtained, with predictions on the signs of the time derivatives of the marginal value of goods and leisure, and on the difference between the individual time preference and the market rate of interest. The parameters to these derived demand equations have already been estimated by Heckman (1971) from individual (synthetic) life-cycle information on the consumption of goods and leisure (or labor supply) and the pattern of wages. With great clarity (and candor), Heckman states why the general model leads to regrettably few predictions about *investment behavior* over the life cycle. The critical question is, therefore, How should the generality of the model be restricted to yield new insights and possible empirical applications for the framework?

In his specific model, Heckman assumes that time preference and depreciation can be eliminated altogether ($\rho$, $\sigma = 0$), and the polar case from that proposed by Ben-Porath is adopted, the assumption being that human capital has *no effect* on the efficiency with which subsequent human capital is produced. Also, an additive utility function is posited that implies strong restrictions. Even with these specific assumptions, testable implications for investment behavior are scarce. Several interesting interrelationships are, nonetheless, explored and diagramed. It is not clear in my mind, however, that this exercise in generalization has not collapsed to a less realistic model than that used by Ben-Porath, and in neither instance are the empirical implications of the restricted model particularly powerful in accounting for anomalous empirical evidence.

In his empirical estimation of the human capital production function, Heckman has set the interest rate to 10 per cent, and initially assumed that depreciation is zero ($\sigma = 0$), and human capital is excluded from its own production ($\beta = 0$). Average information on single age groups of white college-educated males for "hours worked" and "hourly wage rates" from the 1970 Census cross section are used to estimate the model's remaining five parameters: the market productivity of human capital $c$, the returns to scale in human capital investment $\gamma$, an intercept parameter and a quadratic function in age: $\chi_0$, $\chi_1$, $\chi_2$.

The initial restrictions, namely that $\beta$ and $\sigma$ are zero, are then relaxed, and a second set of estimates reported for the general model. These unrestricted estimates might provide one with a better basis for choosing between Ben-Porath's "neutrality" hypothesis and Heckman's strong "market-bias" hypothesis. Getting a better sense of the confidence ellipses obtained around these two parameters, $\gamma$ and $\beta$, might provide a further opportunity to explore the sensitivity (or lack thereof) of the model to these analytically convenient restrictions.

Heckman's approach is conceptually attractive because he incorporates into the human capital investment model additional relevant information about life-cycle labor supply decisions to facilitate identification of his parameterization of the human capital production function. Unfortunately, the expected value of labor supply for United States college-educated white males does not vary much from age 27 to age 52 (see Figure 2). And as Sherwin Rosen pointed out, the variation in labor supply from age 22 to age 26 among this group may be in large part a reflection of frictional lags in labor force entry or part-time and part-year employment by persons still in school. Thus, this critical source of variability in the labor supply series that helps to identify the model may be largely spurious from the point of view of the behavioral process Heckman is trying to study. Reestimating the model without these few younger observations would therefore seem reasonable.

There is also the question of how empirical materials can best be used to discriminate among hypotheses and to estimate more confidently the several interesting parameters to the human capital production function. In my judgment, cross-sectional information from age-specific groups is no longer a satisfactory data base for estimating dynamic models of life-cycle behavior. The synthetic cohort was a tolerable first approximation when the questions asked of earnings data were relatively simple. This is no longer true. Meanwhile, longitudinal data for individuals are becoming publicly available. Interpolating wage rate and labor supply series for all ages from the several jobs reported in the Parnes Longitudinal Labor Force Survey (which Heckman [1972] has already used to great advantage) might permit one to estimate Heckman's general model or more restricted variants thereof. Coleman's life-history survey might also be a useful data base. New problems would also arise, of course, by the nature of these time-series data sources. If such longitudinal data are still unable to identify confidently the parameters to the underlying human capital production function, it may be possible to incorporate information on variation in both cross sections and cohort series, specifying with care the stochastic structure of the resulting composite disturbances. Error-component models can also be formulated to derive information about unobserved variables from common effects across related behavioral equations (Griliches). A variety of such new approaches applied to more appropriate longitudinal individual data may clarify the relevant dimensions of the human capital production function.

Two aspects of the maximum likelihood estimation procedure employed by Heckman deserve note. First, in defining the criterion for estimating the parameters to equations 14 and 15, equal weight is attached to the mean square error of both the past and future investment-wage relationships, plus inter-equation residual correlations. As Zvi Griliches suggested at the Conference, there may be reason to think that people do somewhat better optimizing their current behavior as a function of past outcomes than they do in adjusting their behavior to their (uncertain) knowledge of future outcomes. Perhaps past consistency should be weighted more heavily than future consistency in obtaining the "best" parameter estimates. At least some sensitivity analysis might be warranted, given the arbitrary nature of this criterion.

Second, and much more important, the estimates to equations 14 and 15 are obtained conditional on the realized values of market time ($M(t)$). For this labor supply series to be independent of the disturbances in the investment equations, the investment decision-making process must *not* be jointly and simultaneously determined with life-cycle labor supply. Admittedly, Heckman's assumption of independence from stochastic simultaneity and joint errors of measurement[1]

[1] As is widely recognized in the labor supply literature, the Census wage-rate series is obtained by dividing annual earnings by annual hours worked. Hence there is a negative spurious correlation introduced into the wage and hours series by definition if there are errors in measurement of labor supply. The importance of this bias, of course, is much reduced through aggregation by age, but it must still be present to some degree in Heckman's data.

eases the job of estimation. But, in my view, it also contradicts the essential analytical advance proposed in the paper. I have difficulty imagining how the "Human Capital Production Function" can be "Embedded in a Life-Cycle Model of Labor Supply" and yet permit one to maintain the assumption that these two behavioral processes are not part of a jointly and simultaneously determined system of equations. If the life-cycle labor supply decision has an important bearing on human capital investment behavior, the analytical approach of this paper is interesting. But, in that case, the empirical estimates obtained by taking labor supply as predetermined in estimating the parameters to the human capital production function are subject to simultaneous-equations bias. Can one have it both ways? In my opinion, the analytical approach makes sense but the estimation procedure does not.

What is one to conclude from the empirical estimates? Investment rates decline over the life cycle according to the estimated polynomial time trend, in much the same manner as estimated by Mincer's simpler procedures. Diminishing returns to scale in the production of human capital, i.e., $\gamma < 1$, is implied by the restricted model, whereas constant returns to scale is implied for the generalized model, $\gamma = 1$. The large negative value of $\beta$, in the latter model, suggests that the accumulation of human capital *decreases* substantially the efficiency with which human capital can be subsequently produced. Despite the relatively small asymptotic standard errors, I cannot be confident that the parameter values of the generalized model are precise or even plausible.

One unusual empirical finding is the implied *increase* in the dollar and time costs of investment for the first several years after college, followed by the anticipated monotonic decline. Although dismissed as "empirically irrelevant," this anomaly may deserve further attention, for it is also noted in an empirical investigation of a population of engineering college graduates in Sweden (Klevmarken and Quigley 1973).

One interpretation of these results is that trying to estimate the general parameters of a human capital production function is not now a promising avenue for empirical research. Without additional sources of information that can aid discrimination among the several relevant parameters, the presumption that postschooling investment causes life-cycle variation in wage rates is plausible but also virtually tautological. As a heuristic device for discounting lifetime wage streams, the on-the-job investment hypothesis is a useful accounting mechanism that has not yet been rejected by any empirical tests.[2] What seems clear to me is that we are seeking a great deal of information about several complex processes from very little observable data. Have we any alternative research strategies? How are we more likely to improve our understanding of the related processes of individual time allocation, investment, and savings, which would seem to be responsible for both differences in individual wage rates at one point in time, and differences in wage rates for specific individuals over time?

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[2] Approximately the view expressed in a moment of perverseness by colleague John Hause.