# 9 Energy and Pollution Effects on Productivity: A Putty-Clay Approach

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## 9.1 The Problem

Among the constraints currently affecting production methods in manufacturing, three types deserve special attention: rapid increases in energy prices and interruptions in supplies of specific fuels; fixed time schedules of requirements for water, air, and land pollution abatement; and fixed time schedules of requirements under the Occupational Safety and Health Act. While constraints such as these are not entirely new, their strength, severity, and nearly simultaneous occurrence make the outcome on production patterns, and thus on productivity change, highly uncertain.[1]

One obvious way in which productivity change can be affected is by a diversion of real investment from productivity-enhancing capital additions and replacements to uses that are neutral or even negative in their productivity effects. Examples of the latter are "add-on" investments to reduce heat loss, convert to other fuels, reduce discharge of pollutants, and reduce safety and health hazards to workers. But to the extent that adjustment to the types of constraints mentioned entails the adoption of new production technology, more rapid capital turnover will result and the net effect may well be an acceleration in productivity growth.

In this paper we present the first stage of a project designed to measure the impact of these constraints on individual industries and the

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1. We define productivity as gross output per man-hour. Many of the results we obtain will also apply to broader productivity concepts; we refer to some of these occasionally in the text.

derived effect on productivity change. Our aim here is to construct a mathematical model that captures the most important of these influences (which are treated as coming from outside the industry) and to examine the process of adaptation to them by the industry. The model is dynamic and is designed to represent the succession of changes that will occur over time as an industry reacts to higher energy costs and increased penalties for pollution (hereafter we will use the term "pollution" to refer to all undesirable outputs, including industrial accidents and health hazards). A logically consistent model provides the best guide to what data must be collected in order to test hypotheses and quantify the basic relationships.

The next stage will be to devise tests of the model and to estimate the principal parameters for selected industries. We have observed that, within the manufacturing sector, the impact of each constraint that interests us is highly concentrated among a few industries. For example, eight (four-digit SIC) industries accounted for more than 50% of all energy consumed by the manufacturing sector in 1967 (and these eight include three industries that are highly integrated and can be usefully treated as one industry—pulp, paper, and paperboard mills). The same eight industries accounted for nearly 70% of all water used by manufacturing in 1968, and for a similar proportion of all water pollution by the manufacturing sector. In addition, other forms of pollution and industrial accidents and health hazards appear to be similarly concentrated among the same industries. Successful measurement of the impacts of constraints on energy use and pollution in these industries will go far toward capturing the aggregate effects on the manufacturing sector. A sketch of our measurement procedure is given in section 9.9, which also contains an illustration for the petroleum refining industry.

In the third stage of the study, we will incorporate our empirical results in an existing national model in order to investigate the impact on the entire economy. Specifically, projections of input-output coefficients for individual industries, prepared in stage two, will be utilized to study interindustry reactions and aggregate effects. A working input-output model appears to be the most likely candidate for this application, and we are examining the feasibility of using the INFORUM model of the University of Maryland.

## 9.2 Model and Assumptions

A set of accurately measured elasticities of demand for the outputs of an industry and of the industry's demands for inputs, for relevant periods of adjustment, would provide the basis for estimating the impact we are studying. The size and speed of the changes in prices and regulations suggest that accurate estimation of such elasticities within a conventional neoclassical production function framework is unlikely to be successful.

In the course of studying energy use and water pollution abatement in manufacturing industries (Myers et al. 1974; Gelb and Myers 1976) we have seen the following mechanism at work: research and development, including pilot plant operation, creates a number of discrete bundles of production technology, each with fixed factor proportions. At any given time, the corporate decision-maker is faced with government regulations and prices, both current and prospective, and the choices of producing with present equipment, producing with new equipment, and scrapping old equipment. These choices, aggregated, determine capacity with fixed requirements of labor and raw materials. In concert with demand, productivity is then determined.

The constraints that we are investigating all affect the costs of production. The typical reaction of an industry to changes in production costs is to change its production methods in order to vary input mix or output mix. In the standard comparative statics model, this is described by a simultaneous movement along the production possibilities curve and along the isoquant.

In the real world, production shifts may take place via changes in existing plants or by new plants replacing old. The evidence is that the largest changes take place via replacement.

These changes can be made explicit in a putty-clay production function model. Entrepreneurs have a range of choices regarding input proportions and output mix until the investment is made (putty); thereafter, plant and equipment is fixed in form (clay). The approach most frequently used does not capture the effects we wish to show; in such a putty-putty model, even with embodied technical change, old equipment is just as adaptable as new.

Putty-clay models were developed from the work of Johansen (1959), Solow (1962), Phelps (1963), Pyatt (1965), Boddy and Gort (1971, 1974), Nickell (1974), Adachi (1974), and others. Our model follows those of Salter (1966) and Bliss (1968) to a considerable extent. It represents the behavior of an industry that buys and sells in competitive markets. The production function for each vintage employs investment, $I$, as an input, rather than capital stock. It is thus *ex ante*, describing substitution possibilities before plant and equipment have been put into place.

We make the following assumptions for each industry:

(*a*) Managerial efficiency is equal in all plants.

(*b*) Labor is homogeneous across plants and over time and is paid at the same wage rate in all plants.

(*c*) Once built, a machine embodies technology that determines the input-output coefficients of that machine throughout its life.

(*d*) Machines in production are fully utilized except in marginal plants that are near the age when they will be taken out of service.

(*e*) There are no cyclical or cobweb patterns in price or output.

(*f*) Within each vintage, constant returns to scale obtain.

(*g*) Knowledge of available techniques is general and new machines embody best-practice techniques.

(*h*) All entrepreneurs share the same expectations of future input and output prices.

(*i*) Technical advances are embodied in new plant and equipment but are neutral in their effects, saving each input in the same proportion.

(*j*) Plant and equipment are infinitely durable and are discarded only because of obsolescence.

(*k*) Investment bears fruit without a lag.

Initially, we further assume that input prices are constant.

We begin by presenting a general form of the model, with two inputs and one output. A Cobb-Douglas form follows, also for two inputs and one output; this section incorporates the uncertainty analysis. We then turn to a general form using three inputs and two outputs, which is followed by an examination of the rate of product transformation. We conclude with an examination of a Cobb-Douglas model with three inputs and two outputs.

We are aware that it is possible to present the model in a more sophisticated and compact fashion. (For excellent examples, see Nickell (1974) and Boddy and Gort (1974.) This would make it easier to present cyclic behavior, steadily rising wages, translog production functions, disembodied technical change, and other elements we omitted. At this stage, however, we are primarily concerned with presenting the model in the simplest possible form, so as to enable a step-by-step analysis of the assumptions and implications.

## 9.3 General Form, Two Inputs and One Output

For vintage $v$, the production function is of the general form

$$X(v) = A(v)f[N(v), I(v)], \tag{A}$$

where $X$ is output, $A$ is technical change, $N$ is man-hours, $I$ is investment in constant prices, and $v$ is measured in years. The price measure for investment is for goods of unchanged quality; that is, it measures the change in price of earlier vintage capital goods that are still produced in the given year. Quality changes in capital goods are included in the change, from vintage to vintage, in $A(v)$.

A short-run "utilization function" for vintage $v$, once the investment has become "clay," is of the general form

(B) $$X(v) = A(v)k(v)N(v),$$

where

$$k(v) = \frac{f[N(v),I(v)]}{N(v)}$$

is constant from vintage to vintage as long as input prices and length of life remain unchanged.

Total production from all vintages is given by

(C) $$Y(t) = \int_{v=t-n}^{v=t} A(v)k(v)N(v)dv,$$

where $(t—n)$ is the oldest vintage in production.

Labor productivity of the industry is given by

(D) $$\frac{Y(t)}{\int_{v=t-n}^{v=t} N(v)dv} = \frac{\int_{v=t-n}^{v=t} A(v)k(v)N(v)dv}{\int_{v=t-n}^{v=t} N(v)dv}.$$

Demand for the industry's (homogeneous) product is a function of current price and of the rate of growth of the economy,

(E) $$D(t) = D[p(t),\gamma],$$

where $\gamma$ is the change in demand caused by the aggregate growth rate.

We assume that $A(v)$ is characterized by steady growth at a rate $\alpha$, which represents the rate of technical progress in the industry. The production functions for successive vintages will therefore differ, since fixed amounts of labor and investment will produce greater and greater quantities of output. Given our assumption of constant input prices, technical change will result in lower production costs. And under competition, these will lead to lower output prices. The current price, $p(t)$, will be determined by the cost of production of the most recent vintage in use, $v=t$. And a steady rate of technical change will then lead to a steadily declining supply price at the same rate, $\alpha$. The current price, $p(t)$, is related to the price that held when an older vintage, $v$, was introduced, by the following relation:

(F) $$p(t) = \frac{A(v)}{A(t)}p(v),$$

where

$$\frac{A(v)}{A(t)} = e^{a(v-t)}.$$

The variable cost for any vintage is given by $wN(v)$, where $w$ is the wage rate. A vintage is taken out of production when revenue no longer exceeds variable cost, or

$$\text{(G)} \qquad p(t)X(v) = w\,N(v).$$

Under the stated assumptions, this will occur after a fixed number of years, $n$ (this is shown for a specific form of the production function in the Appendix). This number, $n$, is thus the length of life of a vintage. If we combine (F) and (G), we have the shutdown criterion

$$\text{(H)} \qquad \frac{p(v)X(v)}{e^{an}} = w\,N(v),$$

which indicates that when the initial total revenue of a vintage has been reduced to the wage bill by technical progress, the vintage is taken out of production. This result is shown in figure 9.1.

Here the price of the commodity, $p(t)$, declines from $p(v)$ at time $v$ to $wN(v)/X(v)$, unit variable cost, at time $v+n$, as a result of technical progress. At time $v+n$, variable costs equal revenue and the vintage is taken out of service. The area labeled $b$ represents the total return on the machine over its life, while the rectangle $a$ represents total variable cost over the same period.[2]

In equilibrium, the cost of investment in a vintage will equal the present value of the quasi rents expected over the life of the vintage, $n$. (This analysis is similar to that of Phelps 1963.) The expected quasi rents are discounted at the going, risk-free rate (which we assume to be constant), in order to derive the present value. Now with exogenously

2. A more realistic treatment would show labor cost per unit rising over time for a given vintage, as a result of rising maintenance and repair costs (or of falling technical efficiency as machines deteriorate). This would make the mathematics more complex, but would not change the principal results of the analysis.

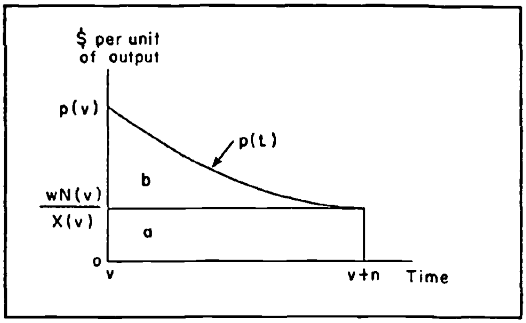


**Fig. 9.1** Revenue and cost of a vintage

given wage rate, production function, and discount rate, the variable that determines the equality between investment cost and present value of quasi rents is the initial price of the (new) vintage, $p(v)$. (It is shown in the Appendix that the length of life, $n$, and $p(v)$ are related by $\alpha$, the rate of technical progress.)

Once $p(v)$ is found, substitution into the demand equation, (E), determines aggregate production from all vintages in service, $Y(t)$. Since we know the capacity taken out of service $[v-(n+1)]$ and the sum of the capacities of the $n$ vintages remaining in service, the volume of investment in vintage $v$ is that which will bring the total capacity of the industry up to the quantity $Y(t)$. This mechanism is illustrated in figure 9.2 (and shown in detail in the Appendix).

In figure 9.2, $p(t)$ is the supply price of the new vintage, $v=t$. This price, together with demand, $D(t)$, determines the equilibrium quantity, $o\,e$. Gross investment in $v=t$ results in new capacity of $\overline{c\,e}$. The height of the supply curve corresponding to the distance $\overline{o\,c}$ shows the variable costs of older vintages, ranging from $v=t-1$ (with the lowest variable cost of the lot, $\overline{o\,h}$) to $v=t-n$ (the oldest in use, with variable cost at $p(t)$).

Figure 9.3 shows the change that takes place in the succeeding year. The demand curve shifts to the right, owing to growth in the economy, from $D(t)$ to $D(t+1)$. The supply price falls from $p(t)$ to $p(t+1)$, owing to technical change. The new supply price, $p(t+1)$, together with the new demand curve, $D(t+1)$, determines the new equilibrium quantity, $\overline{o\,f}$. Vintage $v=t$ is now "clay," so only its variable cost is relevant; the capacity of $v=t$ is $\overline{o\,a}=\overline{c\,e}$, and its variable cost is $\overline{o\,g}$. Gross investment in vintage $v=t+1$ results in production of $\overline{d\,f}$. Because of the decline in price, vintage $v=t-n$ is shut down and capacity $\overline{b\,c}$ is lost. The net addition to capacity is therefore $\overline{d\,f}$ minus $\overline{b\,c}$.

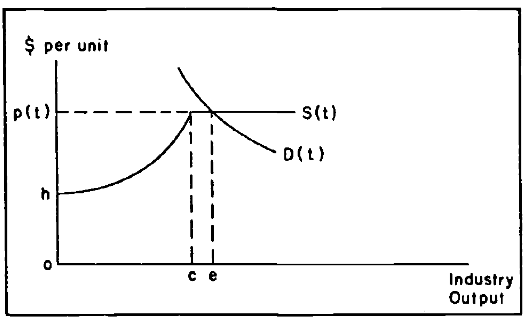


**Fig. 9.2** Static equilibrium

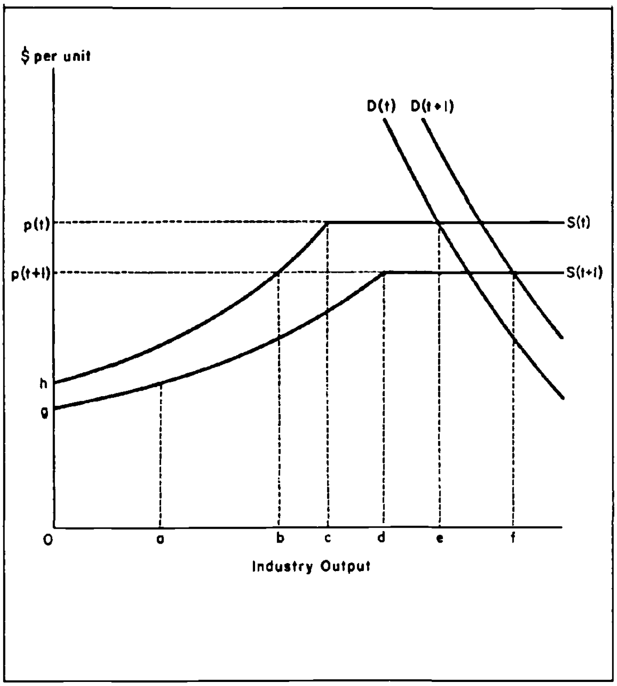


**Fig. 9.3** Comparative statics

Now with steady rates of technical progress and growth in the economy and a constant price elasticity of demand for the product, it follows that the larger (*a*) the rate of technical progress (lowering price), (*b*) the rate of growth of the economy (shifting the demand relation to the right), and (*c*) the price elasticity of demand, the greater will be (i) the amount of investment in the new vintage, and (ii) the net expansion in new capacity.

Suppose now that the price of an input rises unexpectedly and is subsequently expected to remain at the new level.[3] In the simplified form of the model we are now discussing, the price rise is represented by an increase in the wage rate, from $w$ to $w^*$.[4] Examination of the shutdown criterion, ($H$), reveals that the right-hand side (RHS) will now be larger for existing vintages, since the only value affected is the wage rate. On the left-hand side (LHS), the price will increase, but not as much as the wage. The length of life of older vintages will thus be short-

3. The analysis that follows is designed to describe the effects of a rise in the price of energy, such as occurred in 1973.

4. An asterisk indicates the value of a variable after a rise in the price of an input.

ened by a rise in a variable input price; this follows from the putty-clay assumption of fixed input coefficients, which does not permit an adjustment to the new set of relative input prices.

For later vintages, such an adjustment will take place by substitution of investment for labor. A vintage constructed after the input price rise will have a higher labor productivity coefficient, $k^*$, and a higher initial price of output, $p^*$. The life of a new vintage will be longer or shorter, according to whether the numerator of the LHS of the shutdown criterion rises by a smaller or larger proportion than the RHS. In order to obtain a solution, a specific form of the production function must be chosen. This will also permit us to introduce uncertainty.

## 9.4 Cobb-Douglas Form, Two Inputs and One Output

By adopting a simple form of the production function, we can demonstrate that a rise in the wage rate from $w$ to $w^* = \sigma w$ (where $\sigma$ is a number greater than one), will yield the following results for new vintages: the expected length of life is the same as was expected at the time of construction of vintages built before the wage rise; the investment-labor ratio will rise by the factor $\sigma$; and the initial price will rise by a smaller proportion. These results, derived in Part B of the Appendix, are summarized in (I).

(I) $$n^* = n,$$

$$c^* = \sigma c,$$

where $c$ is the investment-labor ratio.

$$p^*(v) = \sigma^b p(v),$$

where $b$ is the elasticity of output with respect to labor.

In order to investigate the extent of the impact of the wage rise on old plants, which results in earlier closings owing to a sudden speeding of obsolescence, a specific form of the demand equation (E) must also be adopted:

(J) $$\mathrm{D}(t) = D(t-1)e^{\gamma}p(t)^{-\epsilon},$$

where $\gamma$ is the change in demand caused by the aggregate growth rate (as in (E)), and $\epsilon$ is the price elasticity of demand.

This demand function will also permit us to examine the equilibrium quantity produced by all vintages, and the portions produced by the new vintage ($v=t$) and the old vintages. The general results of this analysis, which are derived in Part C of the Appendix, are the following:

($a$) Obsolescence will always increase, causing accelerated scrapping, since a new vintage will be profitable at a lower price than that which would permit the oldest equipment to remain in service. This results from the fact that the new vintage benefits from technical progress, which lowers cost, and from a lower investment-labor ratio, which also lowers cost.

(*b*) The more elastic the demand or the greater the wage rise, the smaller is the investment in the new vintage and the smaller the equilibrium quantity produced. The larger the increase in the wage rate, the greater the capacity of old equipment that will be shut down, and the greater the price that will be required to induce new investment. And the greater the price elasticity of demand, the smaller the equilibrium quantity demanded at the new higher price and the smaller the quantity of investment that will be profitable.

(*c*) If the price elasticity of demand or the increase in the wage rate is sufficiently large, no new investment will take place for a time, and the equilibrium price will be below the new vintage price, so all output will be produced from existing plants.

(*d*) Labor productivity will always increase, because accelerated obsolescence will lead to more rapid scrapping of old plants. The productivity increase will be enhanced by the investment, if any, in new plants, which benefit from technical progress and a lower investment-labor ratio.

(*e*) The steady growth of output which characterized the behavior of the model before the wage rise will now be disturbed, and oscillations in growth of labor productivity may ensue.

The impact of a sudden, unexpected rise in the price of a variable input is illustrated in figure 9.4. As a result of the input price rise, the supply curve is shifted to $S^*(t+1)$. The increase in supply price, from $p(t+1)$ to $p^*(t+1)$, is less than the price in variable cost per unit, from $g$ to $g^*$, because the new vintage can partially offset the increase in the wage by varying the investment-labor ratio. Equilibrium output is smaller, at $\overline{o\,f^*}$, than would have held without the input price rise $(\overline{o\,f})$, as the higher price results in a smaller quantity demanded. Accelerated obsolescence is equal to $\overline{d^*d}$, and investment in the new vintage results in new capacity of $\overline{d^*f^*}$. Net expansion in output is $\overline{e\,f^*}$ equal to $\overline{d^*\,f^*}$ less $\overline{d^*\,e}$.

## 9.5 Uncertainty regarding Rise in Price of Variable Input

Instead of the sudden, unexpected rise in a variable input price discussed in the previous two sections, we now consider industry behavior during a period of uncertainty, initially of unknown duration, regarding the price increase. For simplicity, we assume that the probability of the rise in the wage rate to $w^*$ is 0.5 during the uncertainty period, and the probability of no change is also 0.5. This is intended to parallel

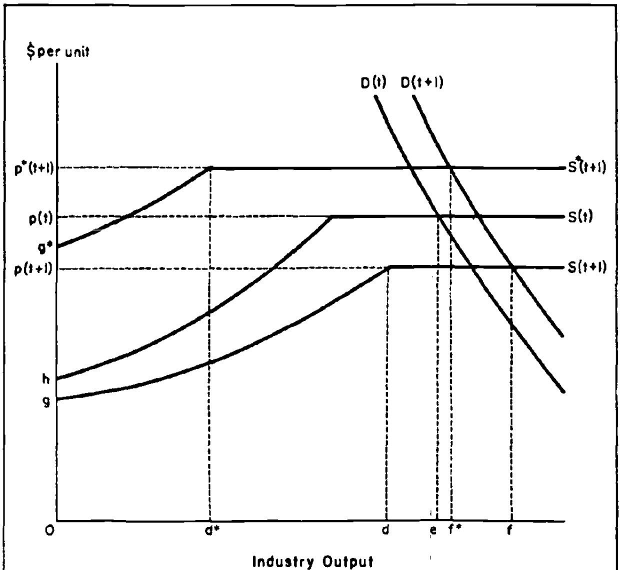


**Fig. 9.4** Impact of rise in price of variable input

states of uncertainty preceding OPEC conferences, or doubts that the cartel can hold together.

Investment made during the period of uncertainty, if guided by the principle of maximizing expected profits, will have an investment-labor ratio intermediate between the two investment-labor ratios that would be appropriate for each possible future wage rate. This ratio will be inappropriate for either of the possible outcomes, yet will be cost-minimizing in view of the uncertainty surrounding either. The initial output price of a vintage constructed during the period of uncertainty must therefore be sufficiently high to recoup the loss that would otherwise ensue once the level of wages is finally determined. Vintages constructed after the period of uncertainty will embody the investment-labor ratio which is appropriate to the finally decided level of the wage, and will therefore have lower variable costs and lower initial output prices than those of the vintage built under uncertainty. So the initial price of output of the "uncertainty vintage" must be sufficiently high to equate the present value of the expected quasi rents under either outcome to the cost of investment, as illustrated in figure 9.5.

Uncertainty prevails over the period from *a* to *b*. The path *c d e f* represents the price pattern with no uncertainty and no wage increase. The path *c d e g h* represents the price pattern with no uncertainty and

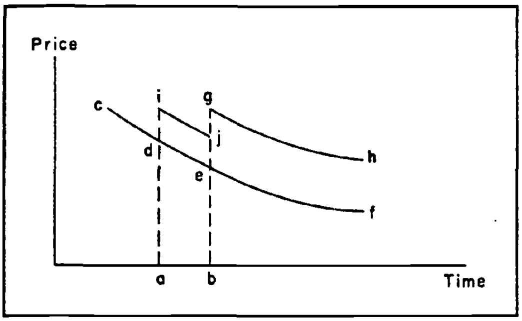


**Fig. 9.5** Price path under uncertainty

a sudden, permanent increase in the wage at time *b*. The path *c d i j e f* represents the uncertainty case followed by no change in the wage. Finally, the path *c d i j g h* is the uncertainty case followed by a permanent wage increase. The rise in price under uncertainty, *d i*, is necessary to recoup the loss that would otherwise be sustained on investment made during the period from *a* to *b*. The minimum price rise that will prevent a net loss is derived for the Cobb-Douglas production function in Part D of the Appendix. This price rise is, in fact, a pure cost of uncertainty.

The uncertainty problem also has an effect on obsolescence. If the wage does in fact rise, vintages built before the period of uncertainty will subsequently obsolesce more rapidly than expected at the time of their construction; this is equivalent to showing that the length of life of these older vintages will be shortened, and is the same result found in the preceding sections. By a similar line of reasoning, a wage rise will result in a shorter length of life for a vintage built under uncertainty than one built thereafter; for the uncertainty vintage, a shorter span of time will ensue before variable cost per unit absorbs product price. But the acceleration of obsolescence for the uncertainty vintage will be less than for the older vintages, because the uncertainty vintage is better adjusted, though not completely so, to the new wage level.

If the wage does not rise, there will be no acceleration in the obsolescence of pre-uncertainty vintages. And the uncertainty vintage will have a longer life than those built either before or after the uncertainty period; this results from the greater investment-labor ratio of the uncertainty vintage, which delays the time when variable cost per unit will absorb the entire product price.

The uncertainty analysis does not result in a distinctive conclusion regarding labor productivity. The results depend mainly on the outcome of the uncertain input price: if the input price does rise, the conclusions of the previous section apply and labor productivity will accelerate; if the input price remains the same, the trend of labor productivity will not change by very much. During the period of uncertainty, it should be noted, the increase in the output price will tend to call back into service the vintage just shut down.

## 9.6 General Form, Three Inputs and Two Outputs

Three inputs are examined in order to cover explicitly the case of variations in energy prices. Study of two outputs permits us to cover the joint production of a "good" and a "bad," the latter referring to pollution, accidents, or health hazards. As before, we assume that substitution is possible among inputs before investment, but not after. Similarly, we assume that product transformation is possible before investment, but not after.[5]

The general form of the production function is given by

$$\text{(K)} \qquad \boldsymbol{X}(v) = [X(1,v),X(2,v)] = A(v)\boldsymbol{f}[N(v),I(v),Z(v)],$$

where boldface type indicates a vector, $X(1,v)$ the good product, $X(2,v)$ the bad product, and $Z$ the other variable input.

A short-run utilization function may be written as

$$\text{(L)} \qquad \boldsymbol{X}(v) = A(v)\boldsymbol{k}[Z(v),I(v)]N(v),$$

or

$$\text{(M)} \qquad \boldsymbol{X}(v) = [A(v)k(1,v)N(v),A(v)k(2,v)N(v)],$$

where

$$\text{(N)} \qquad k(i,v) = \frac{X(i,v)}{A(v)N(v)} \qquad i = 1,2.$$

The value of $k(i,v)$ will vary across vintages, but will be constant over the life of a vintage.

A utilization function can also be written for each output, because of the assumption of joint production after construction of a vintage:

$$\text{(O)} \qquad X(i,v) = A(v)k(i,v)N(v) \qquad i = 1,2.$$

As before, technical progress is assumed to occur at a constant rate, $\alpha$.

The price of the first output is determined, as before, by supply and demand under competitive conditions. The second output, we assume, is subject to a legislative restriction requiring some level of treatment of

5. We chose to express pollution as an output rather than as a cost of production because this approach makes explicit the *ex ante* choice of the ratio of the "good" to the "bad" output; the choice is implemented by the specific product or process adopted. The quantity of the "bad" output that must subsequently be treated before discharge is, of course, dependent on (*a*) the ratio chosen and (*b*) the level of production.

wastes, in the case of pollution, or some level of worker protection, in the case of occupational safety and health. (As mentioned earlier, we are using the term "pollution" to refer to all undesirable outputs.) The cost of this treatment constitutes a negative price per unit of the second output; this price declines, in absolute value, over time as a consequence of improvements in techniques by the industries that supply pollution control equipment.[6]

When a negative price is imposed on the second output, the price of the first output must rise sufficiently to make the present value of a new vintage equal to the cost of investment. This is equivalent to the requirement that total revenue be the same for a unit of investment both before and after imposition of the penalty because input costs are unaffected. We also note that this requirement is independent of the level of $p(2,v)$, since the equal rates of decline of the two output prices will cancel regardless of the initial level of the negative price of the second output. We assume that minimization of the cost of abatement has a neutral effect on the three inputs, $I$, $N$, and $Z$, so that the average products of labor for both outputs can be used to express the constant revenue relationship.

$$\text{(P)} \qquad k(1,v)p(1,v) + k(2,v)p(2,v) = \text{constant, for all } p(2,v).$$

6. We assume, for the sake of simplicity of analysis, that the rate of technical progress is the same in the industry we are studying as in the industry sector that supplies the pollution control equipment. This assumption is obviously inexact, for rates of technical progress vary among industries, so that some will have rates greater than the rate of the pollution abatement equipment sector and some will have rates that are smaller. However, incorporation into our model of a different rate for the using industry than for the equipment sector greatly complicates the analysis without a comparable gain in explanatory power. The main source of this complication is that the length of life of a vintage will vary if different rates of technical progress are assumed in the using sector and the pollution abatement sector. Variations in the length of life will, in general, be very small.

The assumption of equal rates of technical progress in the using industry and in the pollution abatement equipment sector results in both prices, the positive price of the first output and the negative price of the second output, changing at the same rate over time. Both will fall in absolute value, and the ratio of the two prices will remain constant. A vintage designed after imposition of the treatment requirement will utilize the least-cost combination of production process and purchased abatement equipment, so as to maximize profits. As technical progress occurs in our industry, new production processes will permit less production of the second output for a unit produced of the first output. And as technical progress occurs in the abatement equipment sector, the cost of a piece of equipment that will reduce the second output by one unit will also fall. So long as the ratio of prices of two outputs remains constant, the output mix will not change, from vintage to vintage, for new investment.

For vintages constructed before the imposition of $p(2,v)$, change in the production process is no longer possible, so this price will result in decreased revenue, or in plant closing.

The shutdown criterion, after the vintage has been in place for $n$ years, is given by

$$\text{(Q)} \qquad X(1,v)p(1,v+n) + X(2,v)p(2,v+n) - wN(v) - p(Z)Z(v) = 0.$$

This expression, which is parallel to (G), indicates that a vintage will be taken out of service when revenue no longer exceeds variable cost.

In equilibrium, the rate of product transformation (RPT) will be equal to the ratio of the prices of the two products. The RPT can be expressed by means of the output-labor ratios, so we have

$$\text{(R)} \qquad \frac{dk(1,v)}{dk(2,v)} = -\frac{p(2,v)}{p(1,v)}.$$

For new investment, the equilibrium levels of the output-labor ratios are found by equating the marginal product of labor with the ratio of the present value of the wage stream to the present value of the price stream, over the economic life, $n$. In a parallel fashion, the energy-output ratio is found by use of the marginal product of energy (see Part E of the Appendix).

In order to give explicit form to the rate of product transformation, we define two functions, $j(1,v)$ and $j(2,v)$. These relate the two outputs to the three inputs, removing the influences of the levels of inputs and the rate of technical change. They are therefore functionally related to one another:

$$\text{(S)} \qquad j(1,v) = f[j(2,v)].$$

As the production process is changed to reduce the output of pollution, $X(2,v)$, fewer resources are available for output of the desirable product, $X(1,v)$. This is illustrated in figure 9.6, where the terms $j(i,v)$ are used to represent the axes in order to abstract from scale effects.

The curve in figure 9.6 represents the set of possible mixes of the two outputs. We only consider points on the curve to the left of $e$, for this is where the quantity of the first, desirable output is at a maximum. Efforts to reduce the second output in response to penalties will lead to points on the curve that lie to the left of the vertical line above $e$. If the second output, pollution, is reduced to zero, the curve cuts the vertical axis—this is the other boundary of our interest.

Before the imposition of restrictions on pollution, the price of the second output is zero, and the product mix for new vintages is such as to maximize the first output. The price line is thus horizontal and tangent

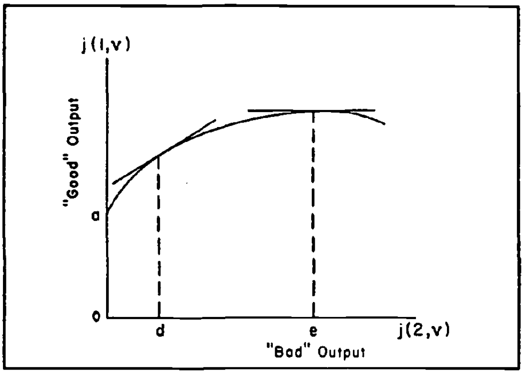


**Fig. 9.6** Determination of output mix

to the RPT above *e*. A negative price of the second output will shift the product mix toward the vertical axis, and the quantities of both goods will fall for a bundle of inputs of equal cost. This is illustrated by the tangency of a price line above *d*.

By adopting particular forms of the production function and the rate of product transformation, we can derive more specific results regarding the interrelationships among an energy price increase; the imposition of restrictions on pollution; input mix; output mix; obsolescence; and product price.

## 9.7 Cobb-Douglas Form, Parabolic Rate of Product Transformation

A parabolic rate of product transformation conforms to our vision of a realistic representation. We define $j(1,v)$ as follows:

$$\text{(T)} \qquad j(1,v) = 1 - \frac{b^2}{4c} + b[j(2,v)] - c[j(2,v)]^2,$$

where $b$ and $c$ are positive and $4c > b^2$.

In terms of figure 9.6, $j(1,v) = 1$, a maximum, when $p(2,v) = 0$; this is the point above *e*. The slope of (T) is nonnegative over the range *o e*, and decreases as $j(2,v)$ increases.

When (T) is combined with a Cobb-Douglas form of the production function, the following results may be demonstrated for new vintages (see Part F of the Appendix):

(1) The energy-labor ratio, $Z/N$, depends only on the relative prices of these two inputs, $p(Z)$ and $w$ (see eq. (F.9) in the Appendix).

(2) Similarly, the investment-labor ratio, $I/N$, depends only on $r$ and $w$ (F.10).

(3) The expected length of life of a new vintage depends only on the rate of technical progress and the discount rate (F.11). It is not affected, therefore, by variations in input or output prices.

If we now consider a sudden, unexpected rise in the price of energy, $p(Z)$, which is expected to continue, we obtain the following results for new vintages (see Part G of the Appendix):

(4) The energy-labor ratio will be reduced (G.1).

(5) The output-labor ratio will be reduced (G.5).

(6) Similarly, the output-investment ratio will be reduced (G.6), but

(7) the output-energy ratio will be increased (G.7).

Our model thus implies that a rise in the price of energy leads to substitution of labor and investment for energy, in new vintages. This result is in accordance with the findings of our case studies of manufacturing industries (Myers et al. 1974).

The increase in the price of energy will also result in a higher price of the desirable output, $p(1,v)$, for new vintages, as would be expected, but the price of pollution, $p(2,v)$, will not be affected (G.10). Therefore, the equilibrium output mix will contain more pollution and more of the desirable output, for inputs of equal cost (G.11). This can be seen by referring to figure 9.6: a rise in the relative price of the desirable output results in a movement to the right along the RPT curve, from the point above $d$ to an intermediate point between that point and the point above $e$. Thus the quantity of pollution (that requires subsequent treatment) per unit of the desirable output increases as the energy price increases.

A parallel implication can be deduced for an increase in restrictions on pollution, equivalent to an increase in the absolute size of $p(2,v)$, holding the price of energy constant. This will result in a change in output mix, reducing the quantities of both outputs, for a bundle of inputs of constant cost. The output-energy ratio will therefore fall. This is in accordance with the claims of industry spokesmen that pollution abatement increases energy requirements.

The impact of an energy price rise on older vintages is to reduce their length of life below that expected at the time of construction (G.8). This is in accordance with the findings in the two-input, one-output case (section 9.3). We have thus generalized our earlier result that an increase in the price of energy will lead to accelerated obsolescence.

A similar result may be shown for the impact on old vintages of the sudden, unexpected imposition of a negative price on pollution. Total revenue of existing vintages will be reduced, thus reducing their life span below that expected at the time of construction (F.23). Obsolescence is thereby increased.

Appropriate adjustments in the three-input, two-output form of the model will yield the results found earlier with respect to ($a$) industry equilibrium following a rise in the price of energy (section 9.4), and (*b*) uncertainty regarding such a rise in price (section 9.5). The mathematics involved, however, are quite tedious. To avoid repetition, we will not present those derivations.

## 9.8 Summary of Model

The principal implications of our model are two: energy price increases and the imposition of requirements for pollution abatement lead to accelerated obsolescence of existing plant and equipment, thus tending to increase labor productivity in an industry; but the same constraints lead to higher requirements of labor and investment per unit of output for new plant and equipment, tending to reduce labor productivity. Which effect dominates over time depends on the elasticity of demand for the product and the nature of the *ex ante* production function. The empirical questions are thus clear-cut.

A reasonable deduction from our analysis is that the obsolescence effect will dominate initially and there will be an acceleration, for a time, in the rate of growth of labor productivity. Subsequent developments will depend on the vintage structure of the industry and on any possible induced acceleration in the rates of technical change in the industry under examination and in the industries supplying pollution abatement equipment.

Our model also implies that reductions in energy use and in pollution are competitive in new vintages.

## 9.9 Sketch of Measurement Procedure

The intent of this project is to derive and project the effects on manufacturing industries of rapid energy price increases and changes in the laws regarding pollution and occupational health and safety hazards. We have in this essay developed a model of industry behavior which leads us to focus on two effects of these changes in the economic environment: acceleration in the rate of obsolescence, and movements along the *ex ante* production function.

Were data available in sufficient detail, we should like to estimate the input and output coefficients of each unit in every plant in the industry. Such estimates, over time, would provide a robust test of the hypotheses and parameters of the model.

For our intention to be realized, however, we do not require such detail. If we can reasonably approximate the efficiency distribution of each industry, then we can derive most of the results we need.

We begin by assuming that investment is retired in order of age, the oldest investments being taken out of production first. (Of course, this assumption is not true, but it will be a good approximation if technical efficiency is a major criterion of obsolescence.)

Next we obtain estimates of the book value of the industry's gross plant and equipment for a recent period, and estimates of current dollar gross investment as far back as possible.

These investment expenditures are then cumulated back in time until their sum equals book value. These are the investments that are assumed to be still in place, and all earlier investments are assumed to have been taken out of service.

After adjusting these figures by appropriate deflators, we obtain real plant and equipment, by vintage, for the industry.

We next wish to obtain the relevant input-output coefficients for the industry, by vintage. These include the output-capital ratio, the labor-capital ratio, the energy-capital ratio, and the various pollution-capital ratios.

If the capital stock grows at a constant rate, prices remain constant, technical change is neutral, and average age is constant, then vintage-to-vintage changes in the ratios will equal year-to-year changes in the average for the entire stock. This will also hold true under the less stringent assumptions that the coefficients change steadily, the capital stock grows at a constant rate, and average age is constant.

However, even the latter assumptions are unlikely to hold over the business cycle. A correction for the cyclical effect may be made by using peak-to-peak values of the various ratios.

In a preliminary test, we applied the above procedure to the petroleum refining industry. Peak-to-peak changes in the capacity output-capital ratios (after correction for changes in the peak-to-peak rate of capacity utilization) were found to be 1.6% from 1951 to 1973.

However, we also found that the average age of capital stock in the industry had been growing for the greater part of the postwar period. The effect of this is to cause the rate of increase in the capacity output-capital ratio, from vintage to vintage, to be underestimated by the year-to-year rate of change in the average.

It also happens that from 1959 to 1966 investment was much lower than in the preceding and following periods. This causes underestimation of the change in the capacity output-capital ratio in the 1959–66 period and overestimation in the 1966–73 period.

The most accurate means for estimating the effects of these disturbances would be a simulation study. For the purposes of this illustration, we merely adjusted the series for the effects of the change in average age, which appear to be by far the most important.

With an estimate of the vintage-to-vintage rate of change in the capacity output-capital ratio, we can construct estimates of the capacity output associated with each vintage still in place.

A similar procedure can then be applied to capacity labor and capacity energy to obtain vintage estimates for these measures. Our previous studies of the petroleum refining industry indicated that a break in the trend of the ratio of energy to output occurred around 1962. At that time, expectations of falling energy prices were replaced by expectations of increasing energy prices. For this industry, therefore, the energy use series is divided into two portions (one before and one after the change in expected prices) which are adjusted separately.

The estimates thus obtained may be cross-checked with data from engineering studies and from plant operations data which are occasionally available from trade and technical journals, as well as from company sources.

Series on air, water, and land pollution for each industry do not exist for the long historical period under study, although there is a historical series on water use in the Census of Manufactures. However, considerable data are available from pollution abatement studies on the industries that interest us. Such studies often provide data on "old," "typical," and "best practice" (or "new") plants which will enable us to form very rough estimates of pollution output by vintage. This material will also be supplemented by data from individual plants, which are available in the technical literature.

We wish to measure the new level of output, the volume of new investment, and the new productivity level. Given the vintage structure outlined above, the input-output coefficients for new equipment, and the price elasticity of demand for the product, we can obtain estimates of these figures.

As a preliminary test of the magnitude of the obsolescence effects, we estimated the vintage structure for petroleum refining, including only the capacity output, energy, and labor coefficients. Since we wish to use petroleum refining as a case study for this project, we used a *ceteris paribus* analysis, in which it was assumed that all prices remained constant with the exception of the price of energy for heat and power.

From engineering studies carried out in 1973–4 (Gordian Associates 1974; Gyftopoulos et al. 1974), we estimated the new input-output coefficients caused by the rise in energy prices (which roughly doubled). This enabled us to obtain an estimate of the change in output price due to the energy price rise.

We then assumed that quasi rents for the oldest vintage in use in 1973 (the capital stock from 1949) were zero in 1973 prices. (Since we wish to estimate only the static obsolescence effect, we used the data to show effects on quasi rents without the passage of time.) Table 9.1 shows quasi rents when energy for heat and power is at 1973 prices and at 1975 prices (roughly double the 1973 level).

**Table 9.1 Quasi Rents of Selected Vintages (Percent of Output Price)**

| | Energy for Heat and Power | |
|---|---|---|
| Vintages | 1973 prices | 1975 prices |
| 1973 | 9.2% | 8.6% |
| 1961 | 6.1% | 4.9% |
| 1951 | 1.4% | 0.2% |
| 1950 | 0.7% | −0.5% |
| 1949 | 0.0% | −1.3% |

As a result of the energy price change, the better part of two vintages go out of production that would not otherwise have done so.

To obtain the new level of output, the volume of investment, and the new productivity level, we also require estimates of the price elasticity of demand for the products of the industry. In the case of petroleum refining, the total effect of the price rises greatly exceeds the capabilities of available price elasticity studies.

At this stage, we have not established a procedure for measurement of uncertainty. There are two routes available. One would be to seek direct and indirect measures of uncertainty by obtaining estimates from decision-makers as to the degrees of uncertainty that prevailed in the past or estimates of errors in actual decisions made. The alternative route is to assume that uncertainty results in underinvestment and higher short-term profits and prices. If our model accurately measures desirable levels of investment in the absence of uncertainty, predicted and actual levels of investment and prices provide an estimate of the effect of uncertainty.

# Appendix A

*Two-Input, One-Output Case*
*General Form*

The *ex ante* production function:

A.1 (A)[7] $X(v) = A(v)f[N(v), I(v)].$

The *ex post* utilization function:

7. Letters in parentheses indicate equations that appear in the text.

A.2 (B) $X(v) = A(v)k(v)N(v)$

$$\text{where } k(v) = f\left[1, \frac{I(v)}{N(v)}\right].$$

Output from all vintages:

A.3 (C) $$Y(t) = \int_{v=t-n}^{v=t} A(v)k(v)N(v)dv.$$

Average labor productivity:

A.4 (D) $$\frac{Y(t)}{\int_{v=t-n}^{v=t} N(v)dv} = \frac{\int_{v=t-n}^{v=t} A(v)k(v)N(v)dv}{\int_{v=t-n}^{v=t} N(v)dv}.$$

Demand equation:

A.5 (E) $D(t) = D[p(t),\gamma]$.

Technical change:

A.6 $A(v) = A(o)e^{av}$,

where the zeroth year is any convenient base year from which to count vintages.

Price:

A.7 (F) $$p(t) = \frac{A(v)}{A(t)}\, p(v).$$

From (A.7) and (A.6):

A.8 $p(t) = p(v)e^{a(v-t)}$.

A machine goes out of production after $t$—$v$ years:

A.9 (G) $p(t)X(v) = wN(v)$.

The shutdown criterion:

A.10 (H) $$\frac{p(v)X(v)}{e^{an}} = wN(v)$$

from (A.8) and (A.9).

Length of life:

A.11 $$n = \frac{1}{\alpha} \ln\left(\frac{p(v)X(v)}{wN(v)}\right)$$

from (A.10).

A.11a $$\frac{p(v)X(v)}{wN(v)} = \frac{p(v)A(v)k(v)}{w}$$

from (A.2).

Since $p(v)A(v)$ is constant, and $k(v)$ is constant as long as input prices are constant, it follows from (A.11) and (A.12) that length of life is constant.

Cost of investment must equal discounted quasi rents:

A.12 $$I(v) = \int_{v}^{v+n} p(t)X(t)e^{-r(t-v)}dt - \int_{v}^{v+n} wN(v)e^{-r(t-v)}dt.$$

A.13 $$I(v) = p(v)A(v)k(v)N(v)\left[\frac{1-e^{-(\alpha+r)n}}{\alpha+r}\right] - wN(v)\left(\frac{1-e^{-rn}}{r}\right).$$

A.14 $$p(v) = \frac{I(v)}{N(v)} + w\left(\frac{1-e^{-rn}}{r}\right) \Big/ A(v)k(v)\left[\frac{1-e^{-(\alpha+r)n}}{\alpha+r}\right].$$

Assuming $I(v)/N(v)$ and $k(v)$ are known from the input prices and the production function, (A.14) and (A.11) together give $p(v)$ and $n$. Thus, to complete the general model, we need (A.15) and (A.16).

*Ex ante* marginal product of labor:

A.15 $$\frac{\partial X(v)}{\partial N(v)} = A(v)\left\{\frac{\partial f[N(v),I(v)]}{\partial N(v)}\right\}.$$

Marginal product of labor is equal to the present value of the wage stream divided by present value of the price stream over the economic life, $n$.

A.16 $$\frac{\partial X(v)}{\partial N(v)} = \frac{\int_{v}^{v+n} we^{(t-v)r}dt}{\int_{v}^{v+n} p(v)e^{(t-v)r}dt} = \frac{w}{p(v)}\left\{\left(\frac{1-e^{-rn}}{r}\right) \Big/ \left[\frac{1-e^{-(\alpha+r)n}}{\alpha+r}\right]\right\}.$$

# Appendix B

*Two-Input, One-Output Case*
*Cobb-Douglas Form*

*Ex ante* production function:

B.1 (A) $X(v) = A(v)N(v)^b I(v)^{1-b}$.

*Ex post* utilization function:

B.2 (B) $X(v) = A(v)\, c^{1-b}\, N(v)$

where $c = I(v)/N(v)$.

Output from all vintages:

B.3 (C) $Y(t) = c^{1-b} \int_{v=t-n}^{v=t} A(v)N(v)dv$.

Length of life:

B.4 $$n = \frac{1}{\alpha} \ln \left[ \frac{A(v)p(v)c^{1-b}}{w} \right].$$

From (A.14):

B.5 $$p(v) = c + w \left( \frac{1-e^{-rn}}{r} \right) \times \left[ A(v)c^{1-b} \frac{1-e^{-(\alpha+r)n}}{\alpha + r} \right]^{-1}.$$

From (A.15):

B.6 $$\frac{\partial X(v)}{\partial N(v)} = bA(v)c^{1-b}.$$

From (B.6) and (A.16):

B.7 $$bA(v)c^{1-b} = w \left( \frac{1-e^{-rn}}{r} \right) \times \left[ p(v) \frac{1-e^{-(\alpha+r)n}}{\alpha + r} \right]^{-1}.$$

Using (B.4) and (B.7),

B.8 $$rbe^{(\alpha+r)n} - (\alpha + r)e^{rn} + (\alpha + r - rb) = 0.$$

Equation (B.8) is the length-of-life formula, showing length of life determined by $\alpha$, the rate of technical progress; $r$, the rate of discount; and $b$, the elasticity of output with respect to labor.

Using (B.7) and (B.5),

B.9 $$c = w\left(\frac{1-b}{b}\right)\left(\frac{1-e^{-rn}}{r}\right).$$

From (B.4),

B.10 $$p(v) = \frac{we^{an}}{A(v)c^{1-b}}.$$

The equilibrium values of the variables of interest for two different wage rates $w$ and $w^* = \sigma w$ bear the following relationships:

B.11 $$n^* = n \quad \text{(from B.8)}.$$

No change in length of life.

B.12 $$c^* = \sigma c \quad \text{(from B.9)}.$$

Investment-labor ratio is proportionate to wage.

B.13 $$p^*(v) = \sigma^b p(v) \quad \text{(from B.10)}.$$

Price rises but less than the wage.

# Appendix C

*Two-Input, One-Output Case*
*Cobb-Douglas with Sudden Wage Rise*

Demand function:

C.1 $$\text{(E)} \quad q(v) = q(o)e^{\gamma v}p(v)^{-\epsilon} = q(o)p(o)^{-\epsilon}e^{v(\gamma+\alpha\epsilon)}.$$

Production from the new vintage:

C.2 $$X(v) = \frac{dq(v)}{dv} + X(v-n).$$

From (C.2) and assuming smooth growth in $X$:

C.3 $$X(v) = q(v)\,\frac{\gamma+\alpha\epsilon}{1-e^{-n(\gamma+\alpha\epsilon)}}.$$

Labor requirement per vintage:

C.4 $$N(v) = \frac{X(v)}{A(v)c^{1-b}}.$$

Total employment:

C.5 $$\int_{v-n}^{v} N(t)dt = \frac{q(v)(\gamma+\alpha\epsilon)(1-e^{-n(\gamma+\alpha(\epsilon-1))})}{A(v)c^{1-b}[\gamma+\alpha(\epsilon-1)][1-e^{-n(\gamma+\alpha\epsilon)}]},$$

unless $\gamma+\alpha(\epsilon-1)=0$, when

C.6 $$\int_{v-n}^{v} N(t)dt = nN(v).$$

Average labor productivity:

C.7 $$\frac{q(v)}{\int_{v-n}^{v} N(t)dt} = A(v)c^{1-b}\left[\frac{\gamma+\alpha(\epsilon-1)}{\gamma+\alpha\epsilon}\right]\left\{\frac{1-e^{-n(\gamma+\alpha\epsilon)}}{1-e^{-n[\gamma+\alpha(\epsilon-1)]}}\right\},$$

unless $\gamma+\alpha(\epsilon-1)=0$, when

C.8 $$\frac{q(v)}{\int_{v-n}^{v} N(t)dt} = A(v)c^{1-b}\left[\frac{1-e^{-n(\gamma+\alpha\epsilon)}}{n(\gamma+\alpha\epsilon)}\right].$$

(C.7) and (C.8) show that labor productivity is proportionate to technical change, $A(v)$, provided input prices are constant and $X(v)$ shows smooth growth.

Suppose now that wages rise from $w$ to $\sigma w$. The price at which new investment will be forthcoming is, from (B.13),

C.9 $$p^*(v) = \sigma^b p(v).$$

Demand corresponding to this price is

C.10 $$q^*_d(v) = q(v)\sigma^{-\epsilon b}.$$

Existing plants shut down can be found from equation (B.4):

C.11 $$n^* = n - \frac{1-b}{\alpha}\ln\sigma.$$

Thus production from existing plants is

C.12 $$q^*_s(v) = q(v) - \int_{v-n}^{v-n^*} X(t)dt.$$

Solving (C.12) with (C.3):

C.13 $$q^*_s(v) = q(v)\left[\frac{1-e^{-n^*(\gamma+\alpha\epsilon)}}{1-e^{-n(\gamma+\alpha\epsilon)}}\right].$$

If $q^*_d(v) \geq q^*_s(v)$, then investment will be forthcoming and the price will in fact be $\sigma^b p(v)$. If not, price will fall until supply and demand balance.

The inequality $q^*_d(v) \geq q^*_s(v)$ holds if C.14 holds:

C.14 $$\sigma^{-\epsilon b} \geq \frac{1 - e^{-n^*(\gamma+\alpha\epsilon)}}{1 - e^{-n(\gamma+\alpha\epsilon)}} \quad \text{(from C.10 and C.13).}$$

In general, the right-hand side of (C.14) will be close to one, since $n - n^*$ will usually not be too large. Thus condition (C.14) holds only for relatively small values of $\epsilon$, that is, for relatively inelastic demand.

We now examine the three cases; (1) $q^*_d(v) = q^*_s(v)$; (2) $q^*_d(v) > q^*_s(v)$; and (3) $q^*_d(v) < q^*_s(v)$.

If $q^*_d(v) = q^*_s(v)$, then price is $p(v)\sigma^{-\epsilon b}$.

Labor employed in the still existing plants is:

C.15 $$\int_{v-n^*}^{v} N(t)dt = \frac{q(v)}{A(v)c^{1-b}} \left[\frac{\gamma + \alpha\epsilon}{\gamma + \alpha(\epsilon - 1)}\right] \times \left\{\frac{1 - e^{-n^*[\gamma+\alpha(\epsilon-1)]}}{1 - e^{-n(\gamma+\alpha\epsilon)}}\right\}.$$

Average labor productivity can be shown to be

C.16 $$\frac{q^*(v)}{\int_{v-n}^{v} N(t)dt} = A(v)c^{1-b}\left[\frac{\gamma + \alpha(\epsilon - 1)}{\gamma + \alpha\epsilon}\right] \times \left\{\frac{1 - e^{-n^*(\gamma+\alpha\epsilon)}}{1 - e^{-n^*[\gamma+\alpha(\epsilon-1)]}}\right\}.$$

Comparing with C.7, since $n < n^*$, labor productivity has increased.

If demand is sufficiently inelastic, and so $q^*_d(v) > q^*_s(v)$, then there will be a one-time spurt in new production, equal to

C.17 $$X^*(v) = q^*_d(v) - q^*_s(v),$$

C.18 $$X^*(v) = q(v)\left[\sigma^{-\epsilon b} - \frac{1 - e^{-n^*(\gamma+\alpha\epsilon)}}{1 - e^{-n(\gamma+\alpha\epsilon)}}\right].$$

Labor productivity of this new production is

C.19 $$\frac{X^*(v)}{N^*(v)} = A(v)(\sigma c)^{1-b}.$$

Since average labor productivity of existing plants is (C.16), the new average labor productivity will exceed the old even further.

If demand is sufficiently elastic that $q^*_d(v) < q^*_s(v)$, then price will fall below $\sigma^b p(v)$ to $\pi^b p(v)$, where $1 < \pi < \sigma$. At this price, plants older than $n^*_\pi$ will fall out of production.

C.20 $$n^*_\pi = n + \frac{b}{\alpha} \ln \pi - \frac{1}{\alpha} \ln \sigma.$$

$\pi$ can be found by solving

C.21 $$\pi^{-\epsilon b} = \frac{1 - e^{-n^*_\pi(\gamma + a\epsilon)}}{1 - e^{-n(\gamma + a_\epsilon)}}.$$

Labor productivity will be

C.22 $$\frac{q^*(v)}{\int_{v^*_\pi}^{v} N(t)dt} = A(v)c^{1-b}\left[\frac{\gamma + \alpha(\epsilon - 1)}{\gamma + \alpha\epsilon}\right]$$

$$\times \left\{\frac{1 - e^{-n^*_\pi(\gamma + a\epsilon)}}{1 - e^{-n^*_\pi[\gamma + a(\epsilon - 1)]}}\right\}.$$

Once price is established at $p(v)\sigma^b$, gross new production will be

C.23 $$X^*(v) = \frac{dq^*(v)}{dv} + X(v - n^*).$$

This can be shown to be

C.24 $$X^*(v) = q(v)(\gamma + \alpha\epsilon)\sigma^{-\epsilon b}$$

$$\times \left[\frac{1 - e^{-n(\gamma + a\epsilon)}\{1 - \sigma^{[\gamma(\frac{1-b}{a}) + \epsilon]}\}}{1 - e^{-n(\gamma + a\epsilon)}}\right].$$

This is faster than the smooth growth we assumed in (C.3), and the industry will exhibit oscillations in labor productivity rather than strict proportionality with technical progress.

# Appendix D

*Cobb-Douglas under Uncertainty*

If the rise in wage to $\sigma w$ is uncertain, then the investment-labor ratio under uncertainty will be

D.1 $$c < c_u < \sigma c.$$

So we may define $\tau$ as

D.2 $$\tau c = c_u,$$

where $1 < \tau < \sigma$.

Suppose, after the fact, it turns out that the wage remains unchanged. In this case, the lifetime of plant built under uncertainty will be

D.3 $$n_u = \frac{1}{\alpha} \ln \left[ \frac{A(v)p(v)(\tau c)^{1-b}}{w} \right],$$

D.4 $$n_u = n + \left( \frac{1-b}{\alpha} \right) \ln \tau.$$

In this case, discounted value of the quasi rents per man-hour will be, using (D.4),

D.5 $$c'_u = A(v)(\tau c)^{1-b}\, p(v) \left\{ \frac{1 - \exp\left[-(\alpha + r)n_u\right]}{\alpha + r} - \exp(-\alpha n_u) \left[ \frac{1 - \exp(-r n_u)}{r} \right] \right\}.$$

If, on the other hand, the wage rises, then the lifetime will be

D.6 $$n^*_u = n - \left( \frac{1-\mathrm{b}}{\alpha} \right) \ln \left( \frac{\sigma}{\tau} \right) = n_u - \left( \frac{1-\mathrm{b}}{\alpha} \right) \ln \sigma.$$

Discounted value of the quasi rents per man-hour is

D.7 $$c^{*\prime}_u = A(v)(\tau c)^{1-b}\, \sigma^b p(v) \left\{ \frac{1 - \exp\left[-(\alpha + r)n^*_u\right]}{\alpha + r} - \exp(-\alpha n^*_u) \left[ \frac{1 - \exp(-r n^*_u)}{r} \right] \right\}.$$

If the probability of the wage rise is assumed to be .5, then the optimum value of $\tau$ will be the one which minimizes

D.8 $$\frac{2\tau c - c'_u - c^{*\prime}_u}{2}.$$

If we differentiate D.8 with respect to $\tau$, it can be shown that

D.9 $$\tau^b = \frac{1}{2} \left\{ \frac{1}{1 - \exp\left[-(\alpha + r)n\right]} \right\} (1 - \exp\left[-(\alpha + r)n_u\right] + \sigma^b \{1 - \exp\left[-(\alpha + r)n^*_u\right]\}).$$

This may be approximated, for $1 < \sigma < 2$:

D.10 $$\tau^b \doteq \frac{1 + \sigma^b}{2},$$

D.11 $$\tau \doteq \left( \frac{1 + \sigma^b}{2} \right) 1/b.$$

Inserting D.9, it can be shown that the expected loss due to uncertainty is

D.12
$$\tau c - \frac{c_u + c^*_u}{2} = \frac{1}{2}\left\{ w \frac{1 - \exp(-rn_u)}{r} + \sigma w \frac{1 - [\exp(-rn^*_u)]}{r} \right\} - \tau w \left[ \frac{1 - \exp(rn)}{r} \right].$$

If the uncertainty period is one year, the price rise at the start of the period necessary to call forth investment is

D.13
$$\Delta p(v) = \left\{ \frac{1}{2}\left[ w \frac{1 - \exp(-rn_u)}{r} + \sigma w \frac{1 - \exp(-rn^*_u)}{r} \right] - \tau w \frac{1 - \exp(rn)}{r} \right\} / A(v)(\tau c)^{1-b}.$$

# Appendix E

*Three-Input, Two-Output Case*
*General Form*

*Ex ante* production function:

E.1 (A) $\boldsymbol{X}(v) = A(v)\boldsymbol{f}[N(v), I(v), Z(v)].$

The bold type indicates vectors:

E.2 $\boldsymbol{X}(v) = [x(1,v), x(2,v)].$

The *ex post* utilization function:

E.3 (B) $\boldsymbol{X}(v) = A(v)\, \boldsymbol{k}\,(v) N(v).$

*Ex post* utilization function for each output:

E.4 $X(i,v) = A(v)\ k(i,v)\ N(v) \qquad i = 1, 2,$

where

E.5 $$k(i,v) = \frac{X(i,v)}{A(v)N(v)} \qquad i = 1, 2.$$

Technical progress:

E.6 $A(v) = A(o)e^{av}.$

Price:

E.7 (F) $p(1,v) = p(1,0)e^{-av}.$

E.8 $$\text{(F)} \quad p(2,v) = p(2,0)e^{-av}.$$

It is assumed that:

E.9 $$A(v)[k(1,v)p(1,v) + k(2,v)p(2,v)] = \text{constant},$$

for all $p(2,0)$ and all $v$.

Define:

E.10 $$h(Z) = \frac{Z(v)}{N(v)},$$

E.11 $$h(I) = \frac{I(v)}{N(v)}.$$

Length of life:

E.12 $$n = \frac{1}{\alpha} \ln \left\{ \frac{A(v)[k(1,v)p(1,v) + k(2,v)p(2,v)]}{w - p(Z)h(Z)} \right\}.$$

Parallel to (A.14):

E.13 $$k(1,v)p(1,v) + k(2,v)p(2,v)$$
$$= h(I) + [w + p(Z)h(Z)] \left(\frac{1 - e^{-rn}}{r}\right) \Big/$$
$$\left\{ A(v) \left[ \frac{1 - e^{-(\alpha+r)n}}{\alpha + r} \right] \right\}.$$

Rate of product transformation:

E.14 $$\frac{dk(1,v)}{dk(2,v)} = -\frac{p(2,v)}{p(1,v)}.$$

Marginal productivity:

E.15 $$\frac{\partial X(1,v)}{\partial N(v)} = w \left(\frac{1 - e^{-rn}}{r}\right) \Big/$$
$$\left\{ \left[ p(1,v) + \frac{k(2,v)}{k(1,v)} p(2,v) \right] \left[ \frac{1 - e^{-(\alpha+r)n}}{\alpha + r} \right] \right\},$$

E.16 $$\frac{\partial X(1,v)}{\partial Z(v)} = p(Z) \left(\frac{1 - e^{-rn}}{r}\right) \Big/$$
$$\left\{ \left[ p(1,v) + \frac{k(2,v)}{k(1,v)} p(2,v) \right] \left[ \frac{1 - e^{-(\alpha+r)n}}{\alpha + r} \right] \right\}.$$

Define rates of product transformation:

E.17 $$j(1,v) = \frac{k(1,v)}{f[h(I),h(Z)]},$$

E.18 $$j(2,v) = \frac{k(2,v)}{f[h(I),h(Z)]}.$$

We wish to make $j(1,v)$ a function of $j(2,v)$ such that conditions (E.19) to (E.21) are fulfilled:

E.19 $$\frac{dj(1,v)}{dj(2,v)} \geq 0,$$

E.20 $$\frac{d^2j(1,v)}{dj(2,v)^2} < 0,$$

E.21 $$j(1,v) = 1$$

when $p(2,v) = 0$.

These are fulfilled by:

E.22 $$j(1,v) = 1 - \frac{b^2}{4c} + b[j(2,v)] - c[j(2,v)]^2,$$

where $b$, $c$ are positive and $4c > b^2$.

From (E.22):

E.23 $$\frac{dj(1,v)}{dj(2,v)} = b - 2cj(2,v).$$

From (E.14):

E.24 $$b - 2cj(2,v) = -\frac{p(2,v)}{p(1,v)}.$$

# Appendix F

*Three-Input, Two-Output Case*
*Cobb-Douglas Form*

*Ex ante* production function:

F.1 $$\text{(A)} \quad X(v) = A(v)f[p(1,v),p(2,v)]N(v)^{1-s-s'} \times I(v)^s Z(v)^{s'}.$$

*Ex post* utilization functions:

F.2 $$\text{(B)} \quad X(1,v) = A(v)[1 - (b^2/4c) + bj(2,v) - cj^2(2,v)]h(I)^s h(Z)^{s'} N(v),$$

F.3 $$\text{(B)} \quad X(2,v) = A(v)j(2,v)h(I)^s h(Z)^{s'} N(v).$$

If $p(2,v) = 0$, then

F.4 $$j(1,v) = 1, \qquad j(2,v) = \frac{b}{2c}.$$

So (F.2) and (F.3) become

F.5 $$X(1,v) = A(v)h(I)^{s}h(Z)^{s'}N(v),$$

F.6 $$X(2,v) = A(v)(b/2c)h(I)^{s}h(Z)^{s'}N(v).$$

Marginal product of labor, from (F.1):

F.7 $$\frac{\partial X(1,v)}{\partial N(v)} = (1 - s - s')A(v)h(I)^{s}h(Z)^{s'}.$$

Marginal product of $Z$:

F.8 $$\frac{\partial X(1,v)}{\partial Z(v)} = s'A(v)h(I)^{s}h(Z)^{s'-1}.$$

Using (F.7), (F.8), (E.15), and (E.16) we obtain

F.9 $$h(Z) = \frac{ws'}{p(Z)(1 - s - s')}.$$

Using (E.13), (F.9), (F.8), and (E.16) we obtain

F.10 $$h(I) = \frac{ws}{1 - s - s'}\left(\frac{1 - e^{-rn}}{r}\right).$$

Using (E.12), (F.7), (E.15), and (F.9) we obtain:

F.11 $$(r - sr)e^{(\alpha + r)n} - (\alpha + r)e^{rn} + sr + \alpha = 0.$$

And also:

F.12 $$p(1,v) = \frac{e^{an}(1 - s)w}{h(I)^{s}h(Z)^{s'}A(v)(1 - s - s')}.$$

We are now ready to introduce $p(2,v) < 0$. Using (E.24) and (E.22),

F.13 $$j(2,v) = \frac{p(2,v)}{2cp(1,v)} + \frac{b}{2c},$$

F.14 $$j(1,v) = 1 - \frac{1}{4c}\,\frac{p(2,v)}{p(1,v)}^{2}.$$

Defining:

F.15 $$p_o(1,v) = p(1,v) \qquad \text{when } p(2,v) = 0.$$

Using (E.9):

F.16 $$j(1,v)p(1,v) + j(2,v)p(2,v) = p_o(1,v).$$

Using (F.13), (F.14), and (F.16),

F.17 $$p(1,v) + \frac{p^2(2,v)}{4cp(1,v)} + \frac{b}{2c}\, p(2,v) = p_o(1,v).$$

Using the quadratic formula to solve (F.17),

F.18 $$p(1,v) = \frac{1}{2}\left\{p_o(1,v) - \frac{b}{2c}\, p(2,v) + \right.$$
$$\left.[p^2{}_o(1,v) - \frac{b}{c}\, p_o(1,v)p(2,v) + \frac{b^2 - 4c}{4c^2}\, p^2(2,v)]^{1/2}\right\}.$$

This formula, of course, holds only for positive values of $j(2,v)$.

Let us now consider the effect on existing plant of the imposition $p(2,v) < 0$. The price of $X(1,v)$ will rise from $p_o(1,v)$ to $p(1,v)$.

For existing equipment, revenue with $p(2,v) = 0$:

F.19 $$p_o(1,v)[A(v)h(I)^s h(Z)^{s'} N(v)].$$

The new revenue, with $p(2,v) < 0$:

F.20 $$p(1,v)X(1,v) + p(2,v)X(2,v) = [p(1,v)$$
$$+ (b/2c)p(2,v)]\,[A(v)h(I)^s h(Z)^{s'} N(v)].$$

(F.19) is greater than (F.20) since, using (F.17),

F.21 $$p(1,v) + \frac{b}{2c}\, p(2,v) = p_o(1,v) - \frac{p^2(2,v)}{4cp(1,v)}\,.$$

Length of life of existing equipment will therefore decline to

F.22 $$n = \frac{1}{\alpha}\ln\left\{A(v)h(I)^s h(Z)^{s'}\left[p_o(1,v) - \frac{p^2(2,v)}{4cp(1,v)}\right]\right/$$
$$\left.[w - p(Z)h(Z)]\right\}.$$

# Appendix G

*Three-Input, Two-Output Case*
*Cobb-Douglas, Energy Price Increase*

We turn to the case of an energy price rise, represented as an increase in $p(Z)$ to $\sigma p(Z)$. We consider the comparative dynamics, and begin with the case $p(2,v) = 0$.

The energy-labor ratio, from (F.9):

G.1 $$h^*(Z) = (1/\sigma)h(Z).$$

The price $p_o(1,v)$ for $p(2,v) = 0$, using (F.12):

G.2 $$p^*{}_o(1,v) = \sigma^{s'} p_o(1,v).$$

From (F.10),

G.3 $$h^*(I) = h(I).$$

Define

G.4 $$h(X) = \frac{X_o(1,v)}{N(v)}.$$

From (G.1), (G.3), and (F.3),

G.5 $$h^*(X) = \sigma^{-s'} h(X).$$

Using (G.3) and (G.5),

G.6 $$\frac{I^*(v)}{X^*{}_o(1,v)} = \sigma^{s'} \frac{I(v)}{X_o(1,v)}.$$

Using (G.5) and (G.1),

G.7 $$\frac{Z^*(v)}{X^*{}_o(1,v)} = \sigma^{-(1-s')} \frac{Z(v)}{X_o(1,v)}.$$

Using (F.9) in (E.12), we obtain the length of life for existing capital faced with the energy price increase:

G.8 $$n^* = n + \frac{1}{\alpha} \ln \left[ \frac{\sigma^{s'}(1-s)}{(1-s) + (\sigma - 1)s'} \right].$$

Since the term in brackets in (G.8) is less than 1, the energy price increase lowers the length of life of existing capital.

Relaxing the assumption that $p(2,v) = 0$, we can evaluate (F.17) to show that

G.9 $$1 < \frac{p^*(1,v)}{p(1,v)} < \sigma^{s'} = \frac{p^*{}_o(1,v)}{p_o(1,v)}.$$

Since $p(2,v)$ does not change while $p(1,v)$ increases, we have

G.10 $$-\frac{p(2,v)}{p^*(1,v)} < -\frac{p(2,v)}{p(1,v)}.$$

By (E.14) and (G.10),

G.11 $$\frac{dj^*(1,v)}{dj^*(2,v)} < \frac{dj(1,v)}{dj(2,v)}.$$

Since $d^2j(1,v)/[dj(2,v)^2]$ is negative, (G.11) results in higher values of both $j(1,v)$ and $j(2,v)$. Whether this results in increased pollution output depends on the elasticity of demand.

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## Comment John E. Cremeans

### Summary of the Paper

#### *Purpose of the Paper*

The purpose of the paper is to examine the effects on investment and productivity of (*a*) rapid and unexpected increases in the price of

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energy, and ($b$) fixed time schedules for meeting requirements for air and water pollution abatement and occupational safety and health.

The paper reports on the first "stage" of a project of three stages: The first stage is to construct a mathematical model that captures the most important influences and may be used as a guide to data to be collected to test hypotheses; the second stage is to devise tests of the model and estimate principal parameters for selected industries; and the third stage is to incorporate empirical results into an existing national model to determine its effect on the economy.

*Description of the Model*

The first section of the paper is a review of existing putty-clay literature, mostly from Salter and Bliss (see Myers's and Nakamura's references). The paper does not include an analysis of data or a discussion of relevant data, but the authors' previous research indicated to them that changes in proportion of inputs take place largely through new plants replacing old, with little chance to adjust proportions of inputs in existing plants. Once built, a plant embodies technology that determines input and output coefficients. *Ex ante*, there may be considerable choice, but *ex post*, there is little choice. Therefore, they reject the "putty-putty" approach and use the "putty-clay" approach. This is, of course, a critical choice.

Assumptions (abbreviated) are as follows: ($a$) competition is pure, returns to scale are constant, input prices are constant; ($b$) labor and managerial efficiency is equal everywhere; ($c$) technology advances at a steady, predictable pace so that a series of vintages is produced over time (each vintage is more efficient than the preceding—each vintage permits production at lower real cost than the preceding—therefore product prices steadily fall as new vintages appear); ($d$) advancing technology is neutral; and ($e$) demand increases at a constant predictable pace.

*The ex ante investment decision.* The *ex ante* investment decision (see fig. 9.1) is made with the expectation that the assumptions just reviewed will hold over the life of the vintage. A vintage is expected to produce at its designated level of output and will continue producing at that level until the market price [$p(t)$] is less than the variable costs per unit of production. The price line $p(t)$ is a predictable result of steadily advancing technology and pure competition. The investor estimates the life of the plant and makes his decision based on expectations for the costs and returns over the life of the investment.

Capital (fixed) costs are "sunk costs" and therefore do not affect the decision to produce or to close the plant. Once investment is made there is no turning back. Therefore, ($a$) variable costs only are entered into the decision to produce or not. The (*ex ante*) investment decision is based on return over total cost—but the decision to stop production is based on variable costs only. (*b*) The *ex ante* investor's decision involves the proportions of the inputs and designed capacity of the plant —the *ex post* operator's decision is a simple on-off. Shall I close the plant or not? The vintage is operated so long as the current price is greater than or equal to the "quit price."

*The ex post market decisions.* The short-term supply curve $h$ is made up of two segments (see fig. 9.2). The segment to the left is the composite output schedule of all vintages now producing at their minimum supply prices—i.e., at their quit prices. $S(t)$, the effective supply price, is set by the unit cost (fixed and variable) of the latest vintage as shown in the segment to the right. $S(t)$, in conjunction with the demand schedule, determines the market price and quantity.

*Replacement of capital over time.* Moving to the next period $(t + 1)$, (see fig. 9.3) the vintage $(V = t)$ is constructed and goes into place (it now has the lowest quit price). The oldest vintage $(V = t - N)$ in the previous time period goes out of production and a new vintage $(V = t + 1)$ is being considered. Demand is expected to shift from $D(t)$ to $D(t + 1)$ and investors now consider the construction of plants with new technology $(V = t + 1)$ with price $p(t + 1)$, $[p(t + 1) < p(t)]$. Vintage(s) with quit price $\geq p(t + 1)$ drop out and the supply curve shifts to "$g$."

The new vintage is built to produce quantity $df$ and the process continues. The size of vintage $V = t + 1$ is determined by the quantity demanded at $p(t + 1)$ less the quantity produced by all plants with quit price $\geq p(t + 1)$.

*Myers-Nakamura Modifications*

The Salter-Bliss model with two inputs and one output is modified to three inputs and two outputs. The conventional inputs are labor and capital with one output good. The Myers-Nakamura modification permits the treatment of labor, capital, and energy as inputs and one good and one bad as outputs. The model thus explicitly treats energy as an important and separate input and recognizes that pollutants and safety hazards are joint products with the production of goods. Economic models have rarely, if ever, recognized this phenomenon in the past, even though it has always been with us. This modification is important and is a part of a number of similar changes that should be made in our models and our thinking.

The modified putty-clay model is actually a putty-double-clay because, *ex post*, not only are input proportions fixed, but so are the proportions of the outputs. That is, energy requirement per unit of output are fixed as are requirements for labor; the output of bads (i.e., pollution and safety hazards) is also fixed. The input and the output coefficients are determined for the vintage once and for all and change can only be effected *ex ante* as new vintages are planned and put in place. Indeed, the only change that can be made to an existing plant is to close it. This is really putty-brick.

Regulation of pollution and OSHA hazards is treated in the model as a penalty price—that is, the effective price is a weighted average of the price of the good and the negative price of the "bad." Thus the quit price formula includes a negative price for the "bad" outputs. Production will continue as long as

$$(P_g G + P_b B) \geqslant V,$$

where $P_g =$ price of goods; $P_b =$ price of "bads" $(P_b < 0)$; $G =$ quantity of goods; $B =$ quantity of bads; $V =$ variable costs. Myers-Nakamura thus depart from the strict consequences of their model when discussing pollution abatement. Few jurisdictions handle pollution by taxing pollutants, even though this procedure is beloved of economists. Normally, pollution regulations forbid the emission of more than a stated amount. The authors handle this with an additional assumption, namely that the polluting plant can contract for treatment by others or construct a separate treatment plant at a fixed fee which equates to the negative price discussed above.

While the authors don't discuss it, it can be expected that the price $(P_g)$ will go up immediately after the imposition of the new energy prices and regulations. The market price is established by the total costs of the last vintage put in place and, presumably, the new price $(P^*_g)$ would be set as

$$P^*_g = P_g - (P_b B)/G,$$

where the value of $(P_b B)/G$ is determined by the costs incurred by the most recent vintage plants. That is, the higher cost of energy and of pollution abatement and safety will be passed on to the consumer.

The condition for an existing plant to remain in production can be restated as

$$P^*_g G \geqslant V^*,$$

where $P^*_g =$ the new higher price and $V^* =$ the new higher variable costs including the payments to contractors for the removal of pollutants.[1]

1. More accurately, the right-hand side will be $(V^* + F^*)$, where $F^*$ is the value of fixed charges for any new energy, pollution-abatement, or safety capital expenditures that might be required.

*Conclusion*

On the one hand, a sudden and unexpected change in the relative price of energy will accelerate the obsolescence of existing plants, and their replacement with new plants will increase productivity. The imposition of pollution abatement and OSHA requirements will also accelerate obsolescence for an analogous reason—namely that the new weighted price of outputs will be taken into account. The "bads" now get their proper negative price. On the other hand, the new vintage will have lower rates of labor productivity than it otherwise would because labor will be substituted for energy, and (in effect) more labor will be expended to produce the same goods because the new negative price on pollutants will result in a shift to a position on the product transformation curve in which less than the maximum amount of product (labor fixed) will be produced. In the authors' view, the question is whether the increased productivity brought about by the new plants, or the decreased productivity brought about by energy prices and pollution/safety restrictions, will dominate.

## Critique

The purpose of the project is certainly an important and timely one. We do need a careful examination of the impact of the new higher energy prices and the stricter pollution, safety, and health regulations. I agree wholeheartedly with the general approach of the project, namely that of modifying existing models to consider explicitly energy inputs and undesirable joint products. As much as I approve of the purpose and the general approach, I do not think the paper advances the objective very much. In my view, the authors fail to examine in detail the decision to build a new vintage, given these new considerations, and so have not designed a useful model or research plan. The model and the research plan are based on the assumption that new plants will enjoy a *significant* cost advantage with respect to energy, pollution, and safety, and this cost advantage will greatly accelerate the closing of existing plants. In my view, this is an unexamined assumption that must be tested systematically against empirical data.

The key question to be answered is, How much will the price of goods produced be reduced by the new vintage and, of that reduction, how much will be due to the energy, pollution, and safety considerations? Recall the condition for a plant remaining in business:

$$P^*_g G \geqslant V^* \text{ or } P^*_g \geqslant V^*/G.$$

According to the model, a condition for putting a new vintage into production is

$$(P^*_g)_t \geqslant (V/G)_{t+1} + (F/G)_{t+1}.$$

That is, the sum of the average variable and fixed costs for the new vintage must be less than or equal to the market price established by the previous vintage.

The model also tells us that with the startup of the new vintage the market price falls to

$$(P_g)_{t+1} = (V/G)_{t+1} + (F/G)_{t+1}.$$

If the new price is less than the "quit price" of the older vintages, then those older vintages will be closed. Thus, the obsolescence of existing plants is determined by

$$\Delta P_g = (P_g)_{t+1} - (P^*{}_g)_t.$$

It is the magnitude of $\Delta P_g$ that concerns us, since that determines whether obsolescence accelerates, decelerates, or remains unchanged. That is, we must be concerned with the differential

$$dP_g = \left[\frac{\partial V/G}{\partial T} + \frac{\partial F/G}{\partial T}\right] dT + \left[\frac{\partial V/G}{\partial S} + \frac{\partial F/G}{\partial S}\right] dS,$$

where $dT$ is the change in technology and $dS$ is the change due to energy, pollution, and safety considerations. In words, the change in price from one vintage to the next will equal the change in variable and fixed costs due to technological improvements plus the change in variable and fixed costs due to the higher energy prices, pollution regulations, and safety considerations.

It is clear that the improvement implied by the first term will (under assumptions of a constant rate of technological improvement) have occurred anyway and will not cause an acceleration of obsolescence. It is the second term that must be carefully evaluated. If it has a significant negative value, acceleration of obsolescence will occur. If it is positive and greater than the absolute value of the first term, obsolescence will decelerate. If it is insignificant (i.e., near zero), it will have a small effect on obsolescence.

### *The Effect of Energy, Pollution, and Safety Considerations on Variable Costs*

The authors assume that

$$\frac{\partial V/G}{\partial S} dS < 0.$$

That is, variable costs will be reduced (significantly) by a shift along the production mix possibility curve. While it appears, a priori, that costs will be lower in a new vintage, in practice they are often higher.

For example, it is not uncommon for EPA and local regulatory agencies to make exceptions for existing plants—particularly if their closing would add to local unemployment—while simultaneously insisting on the most stringent standards for new plants. Also, while there are dramatic examples in which reduction in pollution or increases in safety follow process changes, there are also many cases in which the regulations are met in new plants by adding substantially the same equipment, and with the same costs, that are required to bring existing plants into compliance. Thus, it cannot be assumed that new vintages will show significant reductions in variable costs due to the energy, pollution, and safety considerations. Each industry must be carefully studied on the basis of the empirical evidence.

Although many authorities have predicted that process change will play a large role in pollution abatement, available statistics indicate that process change is neither a large nor a growing proportion of the total effort. At BEA we have conducted three surveys of pollution abatement capital spending and are now beginning a fourth (see *Survey of Current Business*, July 1976). We ask respondents to estimate their expenditures for "change in production process," i.e., changes that alter the process to reduce pollutants, in contrast to "end-of-line" equipment that treats pollutants after they are generated. Respondents reported that 24% of their pollution abatement capital spending was for change in production process in 1973, 21% in 1974, and 18% in 1975. While this is certainly not conclusive (e.g., one could argue that those who used process change got more abatement for less capital expended), it does not support the idea that process change has great competitive advantage.

*The Effect of Energy, Pollution, and Safety Considerations on Fixed Charges*

The authors do not explicitly discuss the effect of energy, pollution, and safety considerations on capital spending for new plants or on the cost and availability of capital. It appears that they assume

$$\frac{\partial F/G}{\partial S}\,ds \leqslant 0.$$

Many businessmen argue, however, that this term is significantly positive—that is, that more capital is required and that this increases the cost (and reduces the availability) of capital for conventional purposes.

It is clear that higher energy prices, pollution regulation, and safety requirements will tend to increase prices and that higher equipment costs will result in higher fixed charges for all investment. If interest charges also rise, then this effect will be even greater and new plants will be relatively more expensive.

Thus, it is possible that the change in fixed charges will be positive, and it is conceivable that

$$\left[\frac{\partial V/G}{\partial S} + \frac{\partial F/G}{\partial S}\right] dS \geqslant 0.$$

If there are industries in which this is the case, obsolescence will not accelerate. It may even decelerate.

*The Effect of Uncertainty on Investment in New Vintages*

In addition, it is my opinion that uncertainty will often decelerate obsolescence. The authors consider uncertainty in terms of two possible prices for an input and find that the solution is to design the plant for the expected price, where the expected price is the sum of the products of prices and their probability of becoming effective.

Turning again to the realm of pollution abatement, uncertainty is frequently experienced, not as an unknown future input price, but as an unknown future abatement standard. A familiar example will illustrate this point. There is a series of standards for automobile pollution emissions identified by production years, e.g., the 1977 standards specify certain maximum emissions of hydrocarbons, carbon monoxide, and oxides of nitrogen. Gasoline engines with "add-on" devices can meet the 1977 (and earlier) standards with some loss of fuel economy and performance. Conventional diesel engines meet the 1977 (and earlier) standards with excellent fuel economy. Authorities generally agree that the diesel engine is a superior way to meet the need for pollution abatement (as expressed in the 1977 standards) and the need for energy conservation. Why don't U.S. manufacturers invest massively in plants to produce diesel engines for passenger cars?

Significantly more stringent standards have been planned (and postponed) for many years and are currently scheduled for 1978.[2] These standards call for significant reduction in the emission of oxides of nitrogen. Diesel engines obtain their greater fuel economy and their reduced emissions of hydrocarbons and carbon monoxides in part because of their higher combustion temperatures which in turn generate oxides of nitrogen. Thus, the diesel engine will meet the 1977 and

2. Since this Comment was first prepared, the "1978 standard" of 0.1 gram of oxides of nitrogen per mile has been postponed again and is now included in "the 1981 standard." Only one U.S. manufacturer is offering diesel powered automobiles and four manufacturers (General Motors Corporation, Volkswagen, Inc., Daimer-Benz, and Automobiles Peugot, Incorporated) have requested a waiver from the Environmental Protection Agency, claiming that enforcement of the standard would force them to stop selling diesel-powered cars (*The Environmental Reporter*, 8 June 1979, p. 200).

earlier standards without loss in fuel efficiency, but will not meet the 1978 standards for oxides of nitrogen even with special devices. EPA and the Congress have postponed the imposition of the more stringent oxides of nitrogen standard in recent years, but they have not yet agreed to eliminate it entirely. Postponement rather than elimination is justified on the grounds that the auto manufacturers will be encouraged to keep trying.

The auto manufacturers claim that there is no known way to reduce emissions of oxides of nitrogen to the levels required, and so have temporized with "add-on" devices for conventional gasoline engines. Had it been certain that the "1978 standards" would not be imposed, the manufacturers might have invested in diesel engine plants and so accelerated the obsolescence of existing gasoline engine plants. While the manufacturers deny it, it is also possible that, had it been certain that the "1978 standards" would be imposed, the manufacturers would have invested in plants to build some other type of engine such as steam or electric.

Uncertainty with respect to pollution abatement and safety regulations makes it difficult, if not impossible, to estimate the useful life of proposed new investment. In effect, the short expected life of the plant makes the fixed charges so high that the condition for putting a new vintage into production is not met. In the face of this uncertainty, investment is reduced and obsolescence of existing plants is retarded.

*Proposed Measurement Procedure*

A proposed measurement procedure is sketched in a final section of the paper that was prepared after the conference. In my view, the procedure leans all too heavily on assumptions of uniform growth and smoothly changing ratios of capital to output, labor, energy, and pollution. It depends on the very assumptions that must be tested empirically. In any event, a useful measurement procedure should include a test of the results. I find neither a plan for testing the measures obtained nor acknowledgment that one is needed. In fact, the last sentence of the paper suggests that the difference between predicted and actual levels of investment is an estimate of uncertainty.

*Summary*

It is my opinion that the model and the research plan should be revised. The assumption that new vintages have significant energy, pollution, and safety cost advantages over existing vintages should be examined empirically, industry by industry. The model should be revised to consider the cost and availability of capital explicitly. The effect of uncertainty should be reconsidered. The problems of measurement must be considered as an integral part of model design.