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A SIMULATION MODEL OF THE ECONOMY OF BRAZIL*

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INTRODUCTION

In the summer of 1970, the Fundaçao Getulio Vargas embarked on a project to develop a series of socioeconomic models of Brazil. This paper describes a small-scale computer simulation model of the economy of Brazil, which was constructed by economists from the Fundaçao Getulio Vargas (FGV) in collaboration with economists from the Catholic University of Rio de Janeiro (PUC), the Brazilian Census Bureau (IBGE), and the Ministry of Planning (IPEA). An earlier version of this model estimated with ordinary least squares appeared in [7]. The version described in this paper has been estimated using two-stage least squares.

The objectives of this research were: (1) to formulate a model of the Brazilian economy which could be used to perform policy simulation experiments to test the effects of alternative economic policies on the behavior of the economy of Brazil, (2) to develop a tool which could be used to check the consistency of existing sources of time-series data, (3) to suggest new data series which might possibly be collected by IBGE or FGV in the future, and (4) to provide students of economics with a meaningful way of obtaining a better understanding of the operations of the Brazilian economy.

The model consists of 16 equations, of which 5 are behavioral equations and the remaining 11 are identities. There are 4 exogenous variables and 9 policy variables. All data which are in monetary units have been deflated and expressed in 1953 cruzeiros. The variables and equations of the model are given below.

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VARIABLES

Endogenous Variables

- B Deficit in balance of trade in billion 1953 Cr$
- C Private and public consumption in billion 1953 Cr$
- Cp Private consumption in billion 1953 Cr$
- D Government deficit in billion 1953 Cr$
- G Government expenditure in billion 1953 Cr$
- GDP Gross domestic product in billion 1953 Cr$
- ΔGDP Change in gross domestic product in billion 1953 Cr$
- I Total gross fixed-capital formation in billion 1953 Cr$
- I_p Private gross investment in billion 1953 Cr$
- K Value of capital stock in billion 1953 Cr$
- M Total imports of goods and services plus net factor payments abroad in billion 1953 Cr$
- $\Delta P/P$ Percentage change in implicit GDP deflator (P = 100 in 1953)
- S Gross domestic savings in billion 1953 Cr$
- T Total direct and indirect taxes in billion 1953 Cr$
- Y Net domestic product at factor cost in billion 1953 Cr$
- Y_d Disposable domestic income in billion 1953 Cr$

Policy Variables

- C_g Public consumption expenditure in billion 1953 Cr$
- I_g Gross public investment in billion 1953 Cr$
- O Other government receipts in billion 1953 Cr$
- Q Money supply in billion 1953 Cr$
- R Implicit exchange rate Cr. per U.S.$ (1953)
- T_d Direct tax in billion 1953 Cr$
- T_i Indirect tax in billion 1953 Cr$
- U Government subsidies in billion 1953 Cr$
- V Government transfer payments in billion 1953 Cr$

Exogenous Variables

- L Size of labor force in millions
- t 47, 48, 49, . . . corresponding to 1947, 1948, 1949, . . .
- W Depreciation in billion 1953 Cr$
- X Import capacity of goods and services in billion 1953 Cr$

THE MODEL

Production function

(1)
$$\log Y = 0.803 + 0.022t + \underset{(0.015)}{0.728} \log L + 0.272 \log K$$

Consumption function

(2)
$$C_p = \underset{(7.706)}{1.9894} + \underset{(0.0192)}{0.8949}\, Y_d - \underset{(38.0788)}{131.1489}\, \Delta P/P$$
$\bar{R}^2 = 0.996$ $S.E. = 10.9652$ $DW = 2.6527$

Investment function

(3)
$$I_p = \underset{(8.0057)}{25.9421} + \underset{(25.1985)}{118.3066}\, \Delta P/P + \underset{(0.1283)}{0.6367}\, \Delta GDP$$
$\bar{R}^2 = 0.7028$ $S.E. = 11.6130$ $DW = 0.9692$

Import function

(4)
$$\log M = \underset{(0.7750)}{-0.7459} - \underset{(0.0353)}{0.0894} \log R + \underset{(0.2046)}{1.0872} \log I$$
$\bar{R}^2 = 0.8484$ $S.E. = 0.0831$ $DW = 1.8067$

Price determination

(5)
$$\Delta P/P = \underset{(0.0357)}{0.0035} + \underset{(0.1521)}{0.4705}\, \Delta Q/Q - \underset{(0.0006)}{0.0005}\, \Delta GDP + \underset{(0.1373)}{0.5372}\, (\Delta P/P)_{-1}$$
$\bar{R}^2 = 0.8030$ $S.E. = 0.0503$ $DW = 2.3184$

THE MODEL (*continued*)

Identities

(6)	$C \equiv C_p + C_g$
(7)	$I \equiv I_p + I_g$
(8)	$D \equiv C_g + I_g + V + U - T - O$
(9)	$G \equiv C_g + I_g$
(10)	$B \equiv M - X$
(11)	$S \equiv GDP - C$
(12)	$GDP \equiv Y + T_i - U + W$
(13)	$\Delta GDP \equiv GDP - GDP_{-1}$
(14)	$K \equiv K_{-1} + I$
(15)	$T \equiv T_i + T_d$
(16)	$Y_d \equiv GDP - T - O + V + U$

SPECIFICATION OF THE MODEL

There are five behavioral equations in the model: production function, consumption function, investment function, import function, and price determination equation. For each equation, we examined the specifications of all of the previous econometric models of Brazil and attempted to incorporate into our model those features of previous models which: (1) seemed plausible from a theoretical standpoint, and (2) gave empirical results which were statistically significant.

The production function (1) is based on a modified version of a production function estimated by Maneschi and Nunes [3]. The general form is given by:

$$Y_t = me^{\lambda t}L^{\alpha}K^{\beta}, \qquad \alpha + \beta = 1 \tag{17}$$

where α and β were estimated indirectly by a method attributed to Wolfson [9], and m and λ were estimated by a method developed by Tinbergen.

The specification and estimation of the equation explaining private consumption were somewhat more straightforward. We simply employed a modified version of the consumption function contained in the World Bank Model [1]. Private consumption is expressed as a function of disposable domestic income and the percentage change in the price index over the last period. Private consumption varies directly with disposable domestic income, a basic relationship in most consumption theories, and inversely with the percentage change in the price index. This inverse relationship seems to reflect the expectations that price level changes will not continue to be of the same magnitude, in percentage terms, or that savings are increased to compensate for the drop in future income from savings as the price level continues to increase. Also, since price increases are associated with a shift in income distribution against wage earners[1] and fixed-income recipients, the tendency is for a drop in the consumption of both groups. Such a drop in consumption is greater than the increase in the consumption of the profit recipients due to taxes, retained earnings, and, possibly, their lower propensity to consume. In addition, the drop in consumption can also be traced to the decrease in credit availability as the price level rises, with a consequent increase in the transaction demand; it is assumed that the level of real income does not drop.

[1] The shift in income distribution against wage earners rests on the assumption that price-level increases would not be associated with sufficient increases in employment to offset the drop in the real income of the currently employed.

TABLE 1
VALUES OF EIGHT ENDOGENOUS VARIABLES, 1949–1968

	ΔP/P			GDP		
Year	Simulated	Actual	% Error	Simulated	Actual	% Error
1949	.120	.100	−20.371	358.809	373.800	4.010
1950	.178	.100	−78.104	379.823	397.800	4.519
1951	.152	.110	−38.183	408.738	421.300	2.982
1952	.128	.120	−7.042	439.773	458.200	4.022
1953	.138	.130	−6.149	460.292	469.500	1.961
1954	.139	.180	22.783	506.009	516.800	2.088
1955	.136	.140	2.505	532.388	552.500	3.640
1956	.149	.190	21.796	558.239	570.100	2.081
1957	.181	.120	−50.559	602.554	616.100	2.199
1958	.158	.100	−57.606	654.071	663.400	1.406
1959	.202	.230	12.243	704.316	700.700	−0.516
1960	.220	.210	−4.591	748.607	768.400	2.576
1961	.261	.250	−4.411	785.711	847.500	7.291
1962	.305	.350	12.865	828.321	892.200	7.160
1963	.330	.440	24.890	869.003	905.900	4.073
1964	.368	.470	21.696	929.760	932.400	0.283
1965	.374	.360	−3.784	992.382	957.900	−3.600
1966	.223	.280	20.198	1,072.473	1,006.800	−6.523
1967	.240	.210	−14.326	1,120.735	1,054.900	−6.241
1968	.220	.220	0.019	1,229.006	1,143.500	−7.478
Mean absolute % error			21.206			3.732

The standard errors of the coefficients are shown within the parentheses below the coefficient estimates. $\bar{R}^2$ denotes the coefficient of multiple determination; *DW* denotes the Durbin-Watson statistic; and *S.E.* denotes the standard error of the estimate.

The private investment equation (3) was adapted from the World Bank Model [1]. Private investment is found to vary directly with the changes in gross domestic product and with the percentage change in the price index. This relationship makes use of the accelerator principle and suggests an outward shift in the marginal efficiency of capital schedule as the price level rises. This shift in the schedule implies expectations of a continuation of price-level increases, in absolute terms.

The import equation (4) is based on the specifications proposed in the Three-Year-Plan Model and the Ten-Year-Plan Model [4]. Imports are directly related to investment, since a rise in investment is associated with an increase in capital goods importation and the secondary effect of a rise in income. Imports are inversely related to the exchange rate since that rate is an ability-to-pay constraint. The data for the implicit exchange rate *R* are not considered to be very reliable and, therefore, the entire equation should be viewed with some skepticism.

The price equation (5) represents the only linkage with the monetary sector in our model. The percentage change in the price index is a distributed lag on the changes in output and the quantity of money. The supply of money is treated as a policy variable. This is a rather serious limitation of the model because, in fact,

TABLE 1 (*continued*)

Y			C		
Simulated	Actual	% Error	Simulated	Actual	% Error
303.409	318.400	4.708	298.880	317.000	5.716
322.223	340.200	5.284	314.235	330.100	4.806
341.138	353.700	3.552	335.696	353.600	5.063
370.573	389.000	4.737	369.753	385.300	4.035
389.593	398.800	2.309	397.448	396.000	−0.366
420.310	431.100	2.503	426.190	435.700	2.183
449.088	469.200	4.286	457.109	465.900	1.887
468.739	480.600	2.468	484.361	487.500	0.644
505.454	518.700	2.554	515.789	515.500	−0.056
536.671	546.000	1.709	548.350	562.600	2.533
572.717	569.100	−0.636	586.391	578.600	−1.346
606.507	626.300	3.160	621.467	656.100	5.279
645.111	706.900	8.741	672.510	717.100	6.218
683.922	747.800	8.542	717.091	757.300	5.309
723.803	760.700	4.850	746.518	779.100	4.182
769.160	771.800	0.342	799.697	787.300	−1.575
818.882	784.400	−4.396	845.294	792.600	−6.648
869.774	804.100	−8.167	918.269	857.200	−7.124
921.335	855.500	−7.695	981.594	902.700	−8.740
980.606	895.100	−9.553	1,054.057	965.700	−9.149
		4.510			4.143

the money supply depends on a number of policy variables administered by the Central Bank.

The remaining set of identities are fairly conventional and require no additional explanation.

VALIDATION OF THE MODEL

Although our primary objective in building this model was not forecasting, if one is going to use such a model for policy-simulation experiments, or as a pedagogical tool for students of economics, one would like to know to what extent it is capable of emulating the actual behavior of the Brazilian economy. For this reason, we did subject our model to the following validity test.

Starting in 1949 (since the price equation has a two-period lag), we solved our sixteen-equation model each year for the sixteen endogenous variables of the system in terms of the given values of the exogenous variables and the policy variables, as well as of the lagged values of the endogenous variables generated in previous time periods. Since the model is nonlinear in variables, we used the Gauss-Seidel Method [5] to solve the simultaneous nonlinear equations. In this manner, we simulated the behavior of the Brazilian economy over the data base period 1949 through 1968.

The simulation results were indeed encouraging, for although our simulation ran for a total of twenty years, it gave better predictive results than any previous

TABLE 1 (*continued*)

Year	S Simulated	S Actual	S % Error	I Simulated	I Actual	I % Error
1949	59.929	53.900	−11.185	61.591	59.600	−3.341
1950	65.588	65.100	−0.749	78.693	64.400	−22.194
1951	73.042	65.600	−11.345	78.335	78.900	0.716
1952	70.021	72.100	2.884	77.099	84.800	9.082
1953	62.845	69.900	10.093	70.432	71.600	1.631
1954	79.819	77.300	−3.259	86.894	77.900	−11.545
1955	75.279	82.500	8.753	74.285	73.400	−1.206
1956	73.878	78.600	6.008	74.980	79.100	5.208
1957	86.765	97.200	10.736	100.132	89.700	−11.630
1958	105.721	96.300	−9.783	104.889	95.000	−10.409
1959	117.926	116.900	−0.877	106.712	107.300	0.548
1960	127.140	106.200	−19.717	111.427	111.700	0.245
1961	113.201	125.500	9.800	109.548	117.400	6.689
1962	111.230	126.500	12.071	119.652	121.000	1.114
1963	122.486	121.400	−0.894	118.343	117.600	−0.632
1964	130.062	139.000	6.430	138.666	120.500	−15.075
1965	147.088	155.100	5.166	145.115	117.400	−23.608
1966	154.205	140.100	−10.068	140.172	141.000	0.587
1967	139.140	140.600	1.038	129.474	143.700	9.900
1968	174.949	167.400	−4.510	164.301	174.700	5.953
Mean absolute % error			7.268			7.066

simulations with econometric models of Brazil, all of which were of a shorter duration than twenty years. Table 1 contains the simulated values, actual values, and percentage error for the eight most important endogenous variables of our model over the period 1949 through 1968. These variables correspond to the eight variables of the World Bank Model [1] for which the simulation results were reported in [6].

Using the mean absolute value of the percentage error as a criterion of validity, we find that our model gives better predictive results than the World Bank Model for five of the eight variables in Table 2. It should be noted that since we have made use of gross domestic product (*GDP*) in our model rather than gross national product (*GNP*), which was used in the World Bank Model, two of the variables are not strictly comparable in Table 2. *GNP* and *S* in the World Bank Model were defined respectively as:

$$GNP = Y + T_i - U + W + F; \tag{18}$$

$$S = GNP - C; \tag{19}$$

where *F* denotes net factor payments abroad. Our definitions of *GDP* and *S* are given in equations (12) and (11) respectively.

Examining Tables 1 and 2, we observe that our model outperforms the World Bank Model in predicting $\Delta P/P$, *S*, *I*, *G*, and *M*. Although our ability to predict

TABLE 1 (*concluded*)

G			M		
Simulated	Actual	% Error	Simulated	Actual	% Error
60.000	60.000	.000	31.192	28.600	−9.063
65.100	65.100	.000	41.547	33.800	−22.921
65.700	65.700	.000	40.957	50.600	19.057
68.200	68.200	.000	39.639	44.400	10.722
80.100	80.100	.000	34.966	32.400	−7.921
77.400	77.400	.000	43.793	42.400	−3.286
81.700	81.700	.000	35.758	35.500	−0.726
89.600	89.600	.000	35.634	37.900	5.979
101.800	101.800	.000	50.037	43.100	−16.095
109.700	109.700	.000	52.318	40.900	−27.918
112.600	112.600	.000	51.631	49.000	−5.369
133.600	133.600	.000	52.827	56.000	5.665
141.600	141.600	.000	49.910	51.900	3.833
143.400	143.400	.000	53.001	50.700	−4.538
148.300	148.300	.000	49.260	47.300	−4.144
148.600	148.600	.000	55.575	43.100	−28.943
145.100	145.100	.000	56.240	43.700	−28.695
154.000	154.000	.000	52.282	52.400	0.226
169.600	169.600	.000	46.396	58.500	20.691
175.000	175.000	.000	60.534	68.000	10.980
		.000			11.839

Note: For definition of variables see p. 152.

price changes is not particularly good, it is substantially better than that of the World Bank Model. We also achieved considerable improvement over the World Bank Model in forecasting investment and imports. It is not surprising that there was no predictive error with regard to government expenditures in our model, for *G* is determined exogenously in our model, whereas it was endogenously determined in the World Bank Model. Although we did not do as well as the World Bank Model in predicting *GDP*, *Y*, and *C*, our results do not differ substantially from the World Bank Model results with respect to these variables.

It should be mentioned, however, that even though our predictions of *Y*, as well as of *GDP* and *C* (both of which are closely linked to *Y*), are not quite as good as those of the World Bank Model, our production function (1) certainly rests on a more solid theoretical foundation than does the production function of the World Bank Model. The World Bank Model contains an extremely naive production function which explains production in the agricultural, industrial, and tertiary sectors, respectively, as three separate functions of acreage, consumption, and government spending, respectively. Both labor and capital are excluded from the production function of the World Bank Model. By selecting the production function which we have used, we may have traded predictive power for explanatory power.

To be sure, we do not feel that the results displayed in Tables 1 and 2 constitute proof of the validity of our model, but they are sufficiently encouraging to cause

TABLE 2
COMPARISON OF PREDICTIVE PERFORMANCE OF THE WORLD BANK MODEL AND OUR MODEL USING MEAN ABSOLUTE PERCENTAGE ERROR AS THE CRITERION

World Bank Model		Our Model	
$\Delta P/P$	45.3%	$\Delta P/P$	21.2%
GNP	3.6	GDP	3.7
Y	3.4	Y	4.5
C	4.0	C	4.1
S	11.2	S	7.3
I	15.7	I	7.1
G	4.9	G	0.0
M	18.1	M	11.8

Note: For definition of variables see p. 152.

us to continue the development of the model. In the meantime, we shall feel reasonably confident in using the model as a training tool and as a vehicle for running simple policy simulation experiments to test the effects of various monetary and fiscal policies on the behavior of the Brazilian economy.

FUTURE DEVELOPMENTS

Obviously there is room for considerable improvement in the model which we have described here. We shall outline some of the steps which we plan to take to improve it.

1. Andrea Maneschi of IPE [Institute of Economic Research], in Sao Paulo, has proposed and, indeed, estimated a number of alternative formulations of consumption, production, and investment functions. It is quite likely that one or more of these alternative specifications may lead to significant improvements in the predictive performance of our model. We plan to run simulation experiments to test the effects of some of these alternative formulations of consumption, production, and investment functions.

2. Our import equation rests on rather shaky ground, and we shall, therefore, try several other specifications developed by IPEA.

3. The absence of a monetary sector is a serious omission which must be corrected. Drawing heavily on the work of Pastore at IPE, in Sao Paulo, we hope to develop a complete model of the monetary sector of Brazil. This model will then be appended to the present model.

4. In a country whose economy is characterized by a high rate of population growth and a high rate of inflation, it makes little sense to construct a model which excludes such variables as wages, employment, population, migration, fertility, mortality, and so on. We expect to devote considerable attention to the inclusion of variables of this type in our model.

5. As important as the agricultural sector is (particularly coffee) to the economy of Brazil, we hope to be able to develop an agricultural model which can be integrated into the total model. We already have a model of the coffee industry developed by Mary Lee Epps, at Duke University, which may be of some use in this regard.

6. Following the work of Fukuchi [2] we may eventually attempt to disaggregate our model by regions.

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