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Synthesis and Further Applications of the Empirical Analysis

In Chapter 2, Ghez applied the theory developed in Chapter 1 to cross-sectional data on family consumption of market goods at different ages. In Chapter 3, Becker applied it to cross-sectional data on hours worked by men of different ages. In this chapter, we first use the results for consumption and male time to estimate various parameters of the theory, such as the elasticities of substitution in production and consumption. We then use these estimates to predict the effects on the allocation of goods and time of seasonal, cyclical, and especially secular changes in wage rates. This chapter is more speculative than was the case of the previous two, and we attempt only to suggest how our approach could be applied to time series and other types of data. Therefore, no attention is paid to significance levels, confidence intervals, or other measures of uncertainty of the different estimates and predictions we present here.

4.1 PARAMETER ESTIMATES

The implication of the theory in Chapter 1 is that the elasticity of response of a family's consumption of goods to a change in the

NOTE: We are equally responsible for this chapter.

husband's wage rate is given by

$$b_1 = s_1(\sigma_f - \sigma_c) \gtreqless 0 \text{ as } \sigma_f \gtreqless \sigma_c, \tag{4.1}$$

where

σ_f = elasticity of substitution in production between any two inputs at a particular time;

σ_c = elasticity of substitution in consumption between commodities in different time periods;

s_1 = share of husband's time in cost of producing commodities.

Similarly, the elasticity of response of the husband's time in the nonmarket sector to a change in his wage rate is

$$a_1 = -[(1 - s_1)\sigma_f + s_1\sigma_c] \leqslant 0, \tag{4.2}$$

with

$$\sigma_f, \sigma_c \geqslant 0. \qquad 0 \leqslant s_1 \leqslant 1 \tag{4.3}$$

Equations (4.1) and (4.2) provide only two equations to determine the three parameters σ_f, σ_c, s_1; hence, even if a_1 and b_1 were reliably estimated, not enough information would be available to determine all three. However, a_1 and b_1 are sufficient to determine σ_f, for clearly

$$\sigma_f = b_1 - a_1. \tag{4.4}$$

The relation between σ_c and s_1 is found by substituting equation (4.4) into either equation (4.1) or (4.2):

$$\sigma_c = (b_1 - a_1) - b_1 \frac{1}{s_1} = b_1 \left(1 - \frac{1}{s_1}\right) - a_1. \tag{4.5}$$

Equations (4.3)–(4.5) place several important restrictions on the values of s_1 and σ_c.[1]

1. From equation (4.5),

$$\sigma_c \gtreqless -a_1 \text{ as } b_1 \lesseqgtr 0$$

since $s_1 \leqslant 1$. By subtracting equation (4.5) from equation (4.4), we obtain $|\sigma_f - \sigma_c| \geqslant b_1$. Finally,

$$\frac{b_1}{b_1 - a_1} \leqslant s_1 \leqslant 1, \tag{i}$$

since $\sigma_c \geqslant 0$.

The theory in Chapter 1 predicts that an increase in age has the same effect on the consumption of goods and of time (one qualification is developed below). In cross-sectional data of the kind considered in chapters 2 and 3, a unit increase in age changes goods and time by the following amount, if nonmarket efficiency does not change with age:

$$a_4 = b_4 = r'\sigma_c - [s_3 + (1 - s_3)\sigma_c]g_w, \qquad (4.6)$$

where

r' = difference between rate of interest and time preference for present;

s_3 = share of goods in cost of producing commodities;

g_w = expected rate of growth over time in real wage rate at given age.

Equation (4.6) helps place useful limits on $r'\sigma_c$, the effect of age on consumption within a given cohort.[2]

As discussed in Chapter 3, a_1 and a_4 are estimated from data in the 1/1,000 sample of the 1960 census. This sample provides information on hours worked, earnings, age, family size, and other income of men. Their consumption time in 1959 is assumed to equal the total time in a year net of estimated time spent on sleep and other personal care minus time spent at work.

The values of a_1, the own wage coefficient, and a_4, the age coefficient, that result from the regression of consumption time of white men on their wage rate, age, and several other variables are reproduced in columns 1 and 2 of Table 4.1; the complete results are given in Table 3.1. The coefficient a_1 is -0.28 for all classes combined, -0.17 for grade and for high school persons, and $+0.05$ for

2. If the share of total costs due to wife's time approximately equals the share due to goods, $s_3 \cong (1/2)(1 - s_1)$. Then by substitution of equation (4.5) into equation (4.6), we get

$$r'\sigma_c = a_4 + \frac{1}{2}\left[1 - a_1 + s_1(b_1 - a_1 - 1)\frac{-b_1}{s_1}\right] g_w.$$

By substitution of equation (i), from note 1, above, we obtain,

$$a_4 + \frac{1}{2}\frac{-a_1}{b_1 - a_1} g_w \leq r'\sigma_c \leq a_4 + (-a_1)g_w, \qquad \text{(i)}$$

if $\sigma_c (= b_1 - a_1) \geq 1/2$. The inequality signs are reversed if $\sigma_c < 1/2$, and these limits are identical if $\sigma_c = 1/2$.

TABLE 4.1
REGRESSION COEFFICIENTS FROM TIME AND GOODS REGRESSIONS: 1/1,000 AND BLS SAMPLES

Group	Effect of Male Wage Rate on Male Time (a_1) (1)	Effect of Age on Male Time (a_4) (2)	Effect of Earnings on Goods (b_1) (3)	Effect of Age on Goods (b_4) (4)
All persons	−.28	.002	.55	.002
Grade school	−.17	−.001	.51	.003
High school	−.17	−.001	.48	.005
College	.05	−.004	.61	.003

SOURCE: Tables 3.1 and 2.4.

college persons; a_4 is +0.002 for all classes combined and negative for each of the three education classes (see the discussion in Chapter 3 for the effects of measurement and systematic error on these estimates).

A completely independent source, the Bureau of Labor Statistics Survey of Consumer Expenditures for 1960–61, provides information on family consumption classified by age of the head, family size, family income, and family earnings; the latter are not broken down to show wage rates and hours worked of different family members separately. The results of a regression of family consumption on the head's age, family size, property income, and family earnings used as a proxy for the head's wage rate are presented in Chapter 2. The coefficients of earnings, b_1, and of age, b_4, are reproduced in columns 3 and 4 of Table 4.1: b_1 ranges from +0.48 for high school persons to +0.61 for college persons, and b_4 ranges from +0.002 to +0.005.[3]

3. Since the BLS Survey does not report wage rates but annual earnings, which are affected by the substitution toward market time induced by a rise in wage rates, Ghez also developed a regression of family consumption on wage rates, age, and other variables from the 1/1,000 Census sample. The estimates of b_1 range only from +0.46 to +0.52 for the three education classes, or slightly below those using earnings and other BLS data (the estimate of b_1 for all classes combined is, however, +1.04 with the Census and only +0.55 with the BLS data). Similarly, the estimates of b_4 range from +0.001 to +0.009, somewhat larger than the range using BLS data (again, however,

Using equation (4.4) and the estimates of a_1 and b_1 in columns 1 and 3 of Table 4.1, we derived the estimates of σ_f, the elasticity of substitution in production, shown in column 1 of Table 4.2. The elasticity equals +0.83 for all education classes combined, and ranges from +0.56 to +0.68 for the individual classes. The upper bounds on σ_c, the elasticity of substitution in consumption, shown in column 2 of Table 4.2, are derived by using equation (4.6), the estimates of a_1, and the fact that $b_1 > 0$. These upper bounds are always small, never exceeding 0.28. Lower bounds on the difference between σ_f and σ_c are shown in column 3: substitution is apparently much easier in production than in consumption since the difference is never less than about one-half.

A comparison of columns 2 and 4 of Table 4.1 shows that a_4 and b_4 are the same for all education classes combined, whereas b_4 is different in sign and several tenths of a percentage point larger for the separate classes. Much of the difference is due to the secular decline in family size, a variable not included in equation (4.6). An increase in family size significantly increases the consumption of goods (see, for example, Table 2.2), and slightly reduces the time allocated to the nonmarket sector by men (see Table 3.1 and the discussion in that chapter). Therefore, a secular decline in family size would reduce the consumption of goods and increase the consumption time of men in younger cohorts relative to older ones. That is, in cross-cohort data typified by the Census or BLS surveys, a secular decline in family size would increase the effect of age on consumption (b_4) and reduce its effect on men's time (a_4). Since the elasticity of goods with respect to family size averages about +0.25, and that of male time about −0.02, the 0.72 per cent annual decline in birth rates since 1909 would result in an increase of 0.0018 in b_4 and a decrease of 0.00014 in a_4. The algebraic difference between these effects, +0.002, is more than a third of the observed difference between b_4 and a_4.

This explanation suggests that a_4 is a better estimate of the variables included in equation (4.6) than b_4 because a_4 is hardly affected by changes in family size. Then, by using equation (i) from note 1

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the estimates for all classes combined are quite different: −0.003 and +0.002). Since, apparently, the biases in estimating b_1 and b_4 by using earnings rather than wage rates are not very great, we rely in this chapter on the estimates obtained by using the BLS data.

TABLE 4.2
ESTIMATES OF PARAMETERS OF MODEL

Group	Elasticity of Substitution in Production ($\sigma_f = b_1 - a_1$) (1)	Upper Bound on Elasticity of Substitution in Consumption ($\sigma_c \leq -a_1$) (2)	Lower Bound on Difference Between σ_f and σ_c ($\sigma_f - \sigma_c \geq b_1$) (3)	Bounds on Effect of Age on Consumption Within Cohort[a] (4)	Lower Bound on Share of Husband's Time in Cost of Producing Commodities $\left(s_1 \geq \frac{b_1}{b_1 - a_1}\right)$ (5)
All	.83	.28	.55	.007 ≤ .010	0.66
Grade school	.68	.17	.51	.003 ≤ .004	0.75
High school	.65	.17	.48	.003 ≤ .004	0.74
College	.56	−.05	.61	−.007 ≤ .007	1.09

SOURCE: Table 4.1. Symbols are defined in Table 4.1 and in accompanying text.

a. Bounds are determined by equation (i) in note 2 of text.

above, and the estimates of a_4, we can derive the bounds on $r'\sigma_c$, the effect of age on consumption within a given cohort, shown in column 4 of Table 4.2.[4] The range is quite small within each education class but differs considerably from class to class: the midpoint declines from +0.0085 for all persons to +0.0035 for both elementary and high school persons to negative values for college persons. The size of all the midpoints except the last one suggests a significant positive effect of interest and time preference combined on consumption. If r', the difference between interest rates and time preference, is taken to be +0.10, the size of the midpoint for all persons implies that $\sigma_c = 0.085$; if $r' = +0.05$, $\sigma_c = 0.17$.

How do these estimates compare with those obtained in other ways? The elasticity of substitution in production can be estimated directly by combining the BLS and Census samples to get ratios of factor quantities and factor prices. The ratio of factor quantities equals family consumption of goods and services (from the BLS) divided by the consumption time of men (from the Census); the ratio of factor prices equals the wage rate of men (Census) divided by unity, the numeraire price of goods and services. In a regression of the log of the ratio of quantities on the log of the ratio of prices, the resulting regression coefficient is a direct estimate of the elasticity of substitution in production.

Table 4.3 contains the results when family size, age, other male income, and earnings of other family members are also included as independent variables in the preceding regression. These direct estimates of the elasticity of substitution are very similar for each of the education classes; they average about +0.47, and are about 0.15 less than the indirect estimates reported in column 1 of Table 4.2. The direct estimate for all classes combined, on the other hand, is about 0.25 larger than the indirect estimate.[5] These regressions, like those run separately for goods and time, indicate that an increase in family size or age, especially the former, increases the consumption of goods relative to male time.

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4. The value of g_w is set equal to 0.0274, the actual rate of growth in real wages in the United States between 1909 and 1967. See U.S. Bureau of the Census, *Long Term Economic Growth, 1860–1965* (1966) and various BLS reports for the figures from 1964–67.

5. Both the direct and the indirect estimates are biased by systematic and random errors in hours worked. It is not clear which set of estimates contains the larger bias.

TABLE 4.3

DIRECT ESTIMATES OF ELASTICITY OF SUBSTITUTION IN PRODUCTION

		Independent Variables (t values in parentheses)							
Education Class[a]	Inter-cept	Log Wage Rate	Log Male Nonwage Income	Log Other Family Income	Log Family Size	Age	R^2	Adj. R^2	Durbin-Watson
All	−0.308	1.09	−0.06	0.28	0.60	−0.002	.90	.78	1.37
		(3.16)	(−1.01)	(3.31)	(2.34)	(−0.297)			
Grade school	−2.45	0.48	−0.05	0.18	0.74	0.007	.74	.48	1.63
		(1.83)	(−1.04)	(1.59)	(3.19)	(1.29)			
High school	−2.18	0.50	−0.06	0.15	0.75	0.010	.86	.71	1.34
		(1.46)	(−1.31)	(1.90)	(4.28)	(1.65)			
College	−2.50	0.42	0.09	0.11	0.96	0.006	.93	.85	1.27
		(1.72)	(1.88)	(1.45)	(4.75)	(1.072)			

SOURCE: Gilbert R. Ghez, "A Theory of Life Cycle Consumption" (Ph.D. diss., Columbia University, 1970).

a. Age range of head of household is 22 to 65.

The low estimates of the elasticity of substitution in consumption between different time periods are consistent with the dominant opinion of professional economists, which is based on a belief that consumption is not very responsive to changes in interest rates. If the elasticity of substitution in consumption were small, the rate of growth over time in consumption would not be very responsive to limited changes in the rate of interest since

$$d\frac{d \log C}{dt} = d\frac{d \log X}{dt} = d\frac{d \log L}{dt} = \sigma_c dr, \tag{4.7}$$

where C, X, and L are the consumption of commodities, goods, and time, respectively, and dr is the change in the rate of interest. If σ_c were about +0.10, a change in the interest rate by two percentage points would change the rate of growth in consumption by only two-tenths of one percentage point. If, however, the difference between the interest rate and time preference were reduced to zero, the rate of growth in consumption also would be significantly reduced, by more than four-fifths of a percentage point according to the midpoint estimate of $r'\sigma_c$ for all education classes combined.

Estimated lower bounds on s_1, the share of husband's time in the cost of producing commodities implied by the values of a_1 and b_1 in Table 4.1, are presented in column 5 of Table 4.2. These bounds are all extremely high, the lowest being 0.66. An upper bound on s_1 can be obtained if property income and the value of wife's time are assumed to be negligible, and if s_1 is equated to the ratio of the time spent by men in the nonmarket sector exclusive of personal care to the total time of men exclusive of personal care. This ratio is about 0.60 for all men in the Census sample (see notes 24 and 25 in Chapter 3), less even than the lowest bound in Table 4.2. The estimates of s_1 derived from the estimates of a_1 and b_1 are, therefore, implausibly high, which again indicates that $-a_1$ is underestimated (relative to b_1). The upward bias in s_1 implies a downward bias in s_3, and thus, by equation (i) in note 1, above, a downward bias in the estimated effects of age on consumption within a given cohort.

4.2 SEASONAL, CYCLICAL, AND SECULAR CHANGES

Although both the theoretical and empirical analysis in Chapters 1, 2, and 3 focuses exclusively on life cycle variations, it has important implications for the effects of seasonal, cyclical, or secular changes in wage rates and other parameters. We first consider seasonal changes, which offer the most promising test of our approach. It is a reasonable assumption that knowledge of the presence of a persistent seasonal in wage rates is usually widespread. Our theory predicts that if $\sigma_f > \sigma_c$, then the consumption of goods and hours worked would both increase during the seasons when wage rates are relatively high. If our empirical estimates of these elasticities also hold for seasonal changes, they imply, in particular, that a 25 per cent seasonal increase in wage rates would increase the consumption of goods by about 12 per cent and hours worked by perhaps 10 per cent.

Casual evidence strongly indicates that employment responds positively to a seasonal in wage rates, but little is known about the seasonal response of consumption. A significant seasonal response of total consumption would be difficult to explain by errors in predicting the future, changes in permanent income, capital market rationing, or other variables used in consumption functions, and would provide a powerful confirmation of our theory.

The well-known procyclical responses of aggregate consumption and employment can also be explained by our theory, but they can be plausibly explained too by errors in forecasting cyclical fluctuations in incomes, capital market rationing, and other considerations. More persuasive evidence of a substitution of goods for time when wage rates are cyclically high, and time for goods when they are cyclically low, can be found in a study by Grossman of the effects of unemployment on the relative consumption of different goods.[6] He argues that male unemployment should cause a relatively large reduction in expenditures that are close substitutes in household production for male time, whereas female unemployment should cause a relatively large reduction in expenditures that are close substitutes for female time. In Table 4.4, we compare the reductions in expenditures on different goods when heads of families (mainly men) are unemployed with the reductions when other family members (mainly women) are unemployed. Unemployment of the latter reduces expenditures on household operations and personal care by a large amount relative to the effects on these categories of unemployment

6. See Michael Grossman, "Unemployment and Consumption: A Note," *American Economic Review* (March 1973).

of the head, presumably because a woman's own time is the best substitute for these expenditures. The same is true for clothing, but to a smaller extent. Substitution of time for goods in household production would result in a procyclical fluctuation in goods and market time even if "permanent" income and interest rates did not fluctuate cyclically.

Still another important application is to secular trends in consumption and "leisure" (i.e., nonmarket uses of time). Although both trends have been studied extensively, they have not been related to each other. Columns 1 and 2 of Table 4.5 contain annual rates of growth in real consumer expenditures and in nonmarket time between 1909 and 1967, and for various subperiods. Both time and goods have grown significantly over the whole period and in the different subperiods, but goods always have grown much faster. The difference between these rates of growth, shown in column 3, indicates that the annual rate of growth of goods has exceeded that of time by about 1.6 percentage points for the period as a whole.

TABLE 4.4

PERCENTAGE REDUCTIONS IN EXPENDITURES ON VARIOUS CONSUMPTION ITEMS AS A RESULT OF UNEMPLOYMENT RELATIVE TO CHANGE IN ALL EXPENDITURES

Item	Head Unemployed	Member Other than Head Unemployed
Food	0.7	0.7
Housing and utilities	0.8	0.4
Household operations	1.0	4.2
House furnishings	1.0	0.6
Clothing	1.6	2.5
Medical care	1.4	0.5
Transportation	1.1	0.9
Personal care	0	1.4
Tobacco and alcohol	1.0	0.9
Reading and recreation	0.9	1.3
Other	1.4	0
Saving	30.4	33.1

SOURCE: Michael Grossman, "Unemployment and Consumption: A Note," *American Economic Review* (March 1973).

TABLE 4.5
ANNUAL RATES OF GROWTH IN REAL CONSUMER EXPENDITURES, NONMARKET TIME NET OF PERSONAL CARE, AND REAL WAGE RATE, 1909–67

Period	Real Consumer Expenditures per Capita ("Goods") (1)	Nonmarket Time (2)	Difference Between Goods and Time (3)	Real Wage Rate (4)	Predicted Difference Between Goods and Time (real wage rate × σ_f) $\sigma_f = .8$ (5)	$\sigma_f = .65$ (6)
1909–67	1.99	.40	1.59	2.74	2.19	1.78
1909–29	2.14	.33	1.81	2.47	1.98	1.61
1930–48	1.62	.65	0.97	3.44	2.85	2.34
1949–67	2.18	.23	1.95	2.60	2.08	1.69
1930–67	1.89	.45	1.45	3.03	2.48	1.89

σ_f = elasticity of substitution between goods and time.

SOURCE: Col. 1 from U.S. Bureau of the Census, *Long Term Economic Growth, 1860–1965* (1966). Col. 2: 1909–58 from Edward Denison, *The Sources of Economic Growth in the United States and the Alternatives Before Us* (New York: Committee for Economic Development, 1962); 1958–67 from U.S. Dept. of Labor, *Manpower Report of the President* (various dates, 1958–67). Col. 4: 1909–63 from Census, *Long Term Economic Growth;* 1964–67 from various BLS reports.

According to a simple version of our theory, a secular rise in real full wealth *alone* would cause both goods and time to rise at the same rate; therefore, if real wage rates also grew secularly, goods would rise faster than time because goods would be substituted for time in the production of commodities. The predicted difference between the rates of growth in goods and time would equal the rate of growth in real wage rates (column 4 of Table 4.5) multiplied by the elasticity of substitution in production (σ_f).

We take 0.8 as the estimated elasticity (σ_f) for all white men and 0.65 for men with elementary or high school education (see Table 4.2, column 1), and multiply each by the rate of growth in wage rates. We can then derive the predicted differences between the rates of growth

TABLE 4.6
PREDICTED RATES OF GROWTH IN GOODS AND MALE TIME, 1909–67

	Predicted Growth in Goods		Predicted Growth in Male Time	
Period	$\sigma_f = .8$ (1)	$\sigma_f = .65$ (2)	$\sigma_f = .8$ (3)	$\sigma_f = .65$ (4)
1909–67	2.41	2.16	.22	.38
1909–29	2.17	1.95	.20	.35
1930–48	3.03	2.72	.28	.48
1949–67	2.34	2.05	.21	.36
1930–67	2.76	2.48	.25	.44

σ_f = elasticity of substitution between goods and time.

in goods and male time shown in columns 5 and 6 of Table 4.5.[7] The predicted differences always exceed the actual ones (in column 3) when the elasticity is assumed equal to 0.8: by small amounts during the periods 1909–29 and 1949–67, and by large amounts during 1909–67 and 1930–48. When the elasticity is assumed equal to 0.65, the predicted differences are close to the actual ones except during 1930–48.[8]

If time series observations on goods and nonmarket time can be interpreted as the successful working out of "lifetime" plans, and if all wealth elasticities equal unity, the rate of growth in goods will equal the sum of the rate of growth in real wealth and the substitution in production toward goods induced by the growth in real wage rates.[9] Similarly, the rate of growth in nonmarket male time will equal the difference between the rate of growth in real wealth and the net substitution away from male time induced by the growth in real wage rates.

7. For these predictions, we assume that male and female wage rates grow at the same rate, and that the elasticity of substitution between aggregate goods and male time equals that between aggregate goods and female time.

8. The predicted differences are overstated because the secular decline in family size, which lowers the rate of growth of goods relative to male time, is ignored.

9. We ignore any secular change in the productivity of household production.

Rates of growth in goods predicted from the rates of growth in real wealth and wage rates are shown in Table 4.6, columns 1 and 2. Real wealth is assumed to grow at a rate equal to the difference between the rates of growth in money wage rates and a weighted average of goods prices and these money wage rates, the weights being the share of each in household production.[10] Since we have not been successful in estimating these shares, the share of time is arbitrarily set equal to 0.60 and that of goods to 0.40. The substitution effect toward goods equals the product of the share of time, the rate of growth in "real" wage rates, and the elasticity of substitution in production between goods and time.[11] For column 1, we assume an elasticity of substitution of 0.8, whereas for column 2, we assume 0.65 (these are the estimates reported in Table 4.2). Actual and predicted rates of growth are rather close for the periods 1909–29, 1949–67, and 1909–67, especially when 0.65 is used, but the predictions are far above the actual values for the depression-war period 1930–48.

Predicted rates of growth in nonmarket male time equal the difference between rates of growth in real wealth and the substitution in production away from male time induced by the growth in male wage rates; the latter equals the product of the share of goods, the rate of growth in wage rates, and the elasticity of substitution in production between goods and male time. Again, the predictions based on an elasticity of substitution equal to 0.65, given in column 4 of Table 4.6, are usually closer to the actual values: very close for 1909–67, 1909–29, and 1930–67 as a whole, but much too low in 1930–48 and too high in 1949–67.

The secular trends in goods and male nonmarket time are, therefore, reasonably well explained even by a simple version of our theory. These trends have not been integrated by the traditional analysis because the substitution between time and goods in the production of commodities induced by the secular rise in wage rates has been ignored.

10. That is, $\tilde{W} = \tilde{w} - (s_g\tilde{p}_g + s_t\tilde{w})$. But since

$$\begin{aligned}\tilde{w} - (s_g\tilde{p}_g + s_t\tilde{w}) &= \tilde{w} - \tilde{p}_g - [(s_g - 1)\tilde{p}_g + s_t\tilde{w}] \\ &= \tilde{w} - \tilde{p}_g - (s_t\tilde{w} - s_t\tilde{p}_g) \\ &= \tilde{w} - \tilde{p}_g - [s_t(\tilde{w} - \tilde{p}_g)] = s_g(\tilde{w} - \tilde{p}_g),\end{aligned}$$

therefore,

$$\tilde{W} = s_g(\tilde{w} - \tilde{p}_g).$$

11. The assumptions made in note 8 also apply to these estimates.