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Macroeconometric Models

E 3.27
Rs in Period $t + 1$ for GNP58 Components

Product	Cross Product	Total
<i>Constant Adjustment, 1st Q '53-4th Q '64</i>		
67	0.93	6.60
80	-0.60	4.18
<i>Constant Adjustment, 1st Q '65-4th Q '68</i>		
81	-57.74	6.88
43	-53.87	14.56
<i>Constant Adjustment,^a 1st Q '53-4th Q '64</i>		
10	1.39	6.79
71	0.04	4.75
22	3.82	10.03
<i>Constant Adjustment,^a 1st Q '65-4th Q '68</i>		
37	-5.52	9.84
10	-6.88	5.53
09	-0.69	13.40

ed for 2 to conform with the forecasting

Part 2

FORECASTS AND ERROR DECOMPOSITION

The Decomposition of Forecasting Errors: A Methodology

4.1 INTRODUCTION

The forecasting errors of the Wharton model are decomposed into their sources via individual structural equations. Here we explain our procedure, which decomposes forecasting errors into the following categories: (a) the part attributable to the structural equations; (b) the part attributable to the rest of the model; (c) the part due to the error in (a) after its reversal; (d) the error attributable to the forecasting errors of the structural equations fully by adjusting for the errors of the forecasting equations; (e) the forecaster's incorrect guesses of the values of the exogenous variable in the model; (f) the error due to the endogenous variables in multiperiod forecasting.

The breakdown of forecasting errors into these categories will answer a number of questions. What is the most important factor primarily responsible for the errors in forecasting? Which of the errors systematically tend to be the largest? How much does the simultaneous man-

The Decomposition of Forecasting Error: Methodology

4.1 INTRODUCTION

The forecasting errors of the Wharton and OBE models are traced to their sources via individual structural equations in the next two chapters. Here we explain our procedure, which permits a decomposition of forecasting errors into the following components of error: (a) the part attributable to the structural equation explaining the variable in question; (b) the part attributable to the rest of the system, including the portion due to the error in (a) after its reverberation throughout the system; (c) the error attributable to the forecaster's failure to correct the stochastic equations fully by adjusting for these problems; (d) the error caused by the forecaster's incorrect guesses as to the future values of the exogenous variable in the model; and (e) the error caused by lagged endogenous variables in multiperiod forecasts.

The breakdown of forecasting errors along these lines enables us to answer a number of questions. Which sector or which specification is primarily responsible for the errors in the model? To what extent do some of the errors systematically tend to cancel or intensify each other? How much does the simultaneous manipulation of both the constant adjust-

ments and the exogenous values improve or hurt the forecasts? To what degree can error be attributed to lags in multiperiod forecasts?

In the presentation that follows we feature a decomposition of GNP, since the GNP series may be considered as an overall single measure of the forecasting performance of a model. The forecasting error in GNP is decomposed into the errors originating in the structural equations describing the endogenous determinants of GNP (these include its demand components), the components of disposable income (evaluated from the supply side), and the effect of the price level forecast. This procedure allows us to break down the observed forecast error into the five components listed above. We illustrate our arguments with a simple linear model. Nonlinearities in the solution of the model will be dealt with only in a heuristic way. It will be shown later that the effect of non-linearity within the range of our interest appears to be inconsequential.

Our procedure uses the values of coefficients as estimated by the model builders. Thus, we trace the effects of observed error in individual equations on forecast error, using the specification of the model and the estimates of the structural parameters used for the forecast in question. Since the values of the true parameters for the equations are unknowable, they cannot be used. Furthermore, the adjustments of the individual equations by the econometric forecasters were based on the estimated parameters of the model. It should be noted that, while our procedure is appropriate for the systems forecasts we analyze, other procedures should be used to estimate what portion of forecast error is attributable to the inherent need for estimating structural parameters derived from a short sample period rather than using the true values of these parameters.¹

4.2 ILLUSTRATION WITH A SIMPLE LINEAR MODEL²

The first step is a condensed version of the model presented in Chapter 3, with the following structural equations:

¹ See, for example, T. M. Brown, "Standard Errors of Forecast of a Complete Econometric Model," *Econometrica*, Vol. 22, April 1954, pp. 178-92; J. W. Hooper and A. Zellner, "The Error of Forecast for Multivariate Regression Models," *Econometrica*, Vol. 29, October 1961, pp. 544-55; George R. Schink, "Small Sample Estimates of the Variance Covariance Matrix of Forecast Error For Large Econometric Models: The Stochastic Simulation Technique," Ph.D. dissertation, University of Pennsylvania, 1971.

² For material similar to some of the ideas in this section, see also P. Paulopoulos, *A*

The Decomposition of Forecast

Aggregate consumption:

$$C_t = \alpha + \beta D_t$$

Aggregate investment:

$$I_t = \gamma Y_t +$$

Net of transfers and retained earnings:

$$D_t = \xi Y_t +$$

National income identity:

$$Y_t = C_t +$$

Disposable income identity:

$$D_t = Y_t + D_t$$

Government expenditure:

$$G_t = \text{Exo}$$

Tax revenues:

$$T_t = \text{Exo}$$

From these one can derive the reduced

$$C_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{ \alpha(1 - \gamma) + \beta(1 - \xi) \}$$

$$I_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{ \alpha\gamma - \beta \}$$

$$Y_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{ \alpha - \beta(1 - \xi) \}$$

$$D_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{ \alpha\xi - \beta \}$$

Statistical Model for the Greek Economy 19 Company, 1966.

prove or hurt the forecasts? To what
 effects in multiperiod forecasts?

we feature a decomposition of GNP,
 considered as an overall single measure of
 model. The forecasting error in GNP is
 originating in the structural equations
 components of GNP (these include its
 components of disposable income (evaluated
 effect of the price level forecast. This
 the observed forecast error into the
 illustrate our arguments with a simple
 solution of the model will be dealt
 be shown later that the effect of non-
 interest appears to be inconsequen-

of coefficients as estimated by the
 effects of observed error in individual
 specification of the model and the
 errors used for the forecast in question.
 errors for the equations are unknow-
 more, the adjustments of the indi-
 cator forecasters were based on the
 . It should be noted that, while our
 systems forecasts we analyze, other
 rate what portion of forecast error
 used for estimating structural param-
 multiperiod rather than using the true

COMPLETE LINEAR MODEL²

version of the model presented in
 structural equations:

² Errors of Forecast of a Complete Econometric
 78-92; J. W. Hooper and A. Zellner, "The Error
 in," *Econometrica*, Vol. 29, October 1961, pp.
 estimates of the Variance Covariance Matrix of
 The Stochastic Simulation Technique," Ph.D.

as in this section, see also P. Paulopoulos, A

Aggregate consumption:

$$C_t = \alpha + \beta D_t + U_t \quad (4.1)$$

Aggregate investment:

$$I_t = \gamma Y_t + W_t \quad (4.2)$$

Net of transfers and retained earnings:

$$D_t = \xi Y_t + V_t \quad (4.3)$$

National income identity:

$$Y_t = C_t + I_t + G_t \quad (4.4)$$

Disposable income identity:

$$D_t = Y_t + D_t - T_t \quad (4.5)$$

Government expenditure:

$$G_t = \text{Exogenous} \quad (4.6)$$

Tax revenues:

$$T_t = \text{Exogenous} \quad (4.7)$$

From these one can derive the reduced form:

$$C_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha(1 - \gamma) - \beta(1 - \gamma)(T_t + V_t) + \beta(1 - \xi)(G_t + W_t) + (1 - \gamma)U_t] \quad (4.8)$$

$$I_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{ \alpha\gamma - \beta\gamma(T_t + V_t) + \gamma G_t + [1 - \beta(1 - \xi)]W_t + \gamma U_t \} \quad (4.9)$$

$$Y_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha - \beta(T_t + V_t) + G_t + W_t + U_t] \quad (4.10)$$

$$D_t = \frac{1}{1 - \beta(1 - \xi) - \gamma} [\alpha\xi - \beta\xi T_t + (1 - \gamma - \beta)V_t + \xi G_t + \xi W_t + \xi U_t] \quad (4.11)$$

Statistical Model for the Greek Economy 1949-1959. Amsterdam, North-Holland Publishing Company, 1966.

Let us compare four types of forecasts, which will differ from each other in two ways: (a) with respect to assumptions regarding knowledge of the values of the exogenous variables essential for the forecasts; and (b) with respect to the ad hoc adjustments made on the structural equations in specific forecasts.

In the first case we shall distinguish between ex post forecasts, where it is assumed that the exogenous values are known, and ex ante forecasts, where the forecaster provides his best guesses about future exogenous values. In the second case, we will show different adjustments made on the structural equations. These usually take the form of an additive disturbance inserted into the single equations in order to account for either the development of exogenous factors *not* included in the model in accordance with the forecaster's judgments, or the patterns in the residuals of the particular equation and any shifts the forecaster may observe in the latest observable periods. This adjustment is usually termed "constant adjustment" because it is accomplished by changing the constant term (intercept) in the structural equations. Thus, if we consider two types of constant adjustments, we have four combinations of both types of forecasts:

1. Ex post plus constant adjustment type I;
2. Ex post plus constant adjustment type II;
3. Ex ante plus constant adjustment type I; and
4. Ex ante plus constant adjustment type II.

Let the ex post exogenous values be denoted by the superscript *p*, the ex ante guesses of the exogenous values by the superscript *a*, and their difference by δ . Thus,

$$T^a - T^p = \delta T, \quad T^p - T^a = -\delta T,$$

$$G^a - G^p = \delta G, \quad G^p - G^a = -\delta G,$$

and, similarly,

$$\delta W = W^I - W^{II}, \quad \delta U = U^I - U^{II}, \quad \delta V = V^I - V^{II}$$

for the constant adjustments types I and II.

Thus, the difference between these forecasts is obtained, with the appropriate change of the sign, by

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \gamma)(\delta T + \delta V) + \beta(1 - \xi)(\delta G + \delta W) + (1 - \gamma)\delta U] \quad (4.12)$$

The Decomposition of Forecasts

$$\delta I = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\gamma(\delta T + \dots) + [1 \dots]$$

$$\delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(\delta T + \dots)]$$

and

$$\delta RE = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\xi\delta T \dots]$$

The system of the last four equations tracing the forecasting errors of the linear systems. In particular, it can be used to compare the ex ante and ex post forecasts (without lags) for different constant adjustments to calculate the difference between ex post type I and ex ante with constant adjustment $-\delta T$ and $-\delta G$ (the ex post-ex ante discrepancies in constant adjustments of the estimated parameters β, γ, ξ). We also investigate the pure effect of exogenous variables or differences in the constant adjustments. For the former $\delta T = \delta G = 0$ the linear systems is that the total effect of exogenous discrepancies and the constant adjustments is the sum of the pure effects. For the latter, the effect is attributable solely to the ex ante and ex post exogenous variables in the system.

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta \dots]$$

where δT and δG are the errors in the constant adjustments respectively, the tax revenues T and G . Similarly, if we want to investigate the effect of exogenous variables in the system.

forecasts, which will differ from each other due to assumptions regarding knowledge of variables essential for the forecasts; and adjustments made on the structural

distinguish between ex post forecasts, where exogenous values are known, and ex ante forecasts where the forecaster provides his best guesses about future values. In this case, we will show different adjustments for ex post forecasts. These usually take the form of adjustments to the single equations in order to account for exogenous factors *not* included in the forecaster's judgments, or the patterns of constant adjustment and any shifts the forecaster makes in the variable periods. This adjustment is usually made because it is accomplished by changing the constant terms in the structural equations. Thus, if we make adjustments, we have four combinations

- 1. constant adjustment type I;
- 2. constant adjustment type II;
- 3. constant adjustment type I; and
- 4. constant adjustment type II.

These values be denoted by the superscript *p*, for post, and ex ante values by the superscript *a*, and

$$T^p - T^a = -\delta T,$$

$$G^p - G^a = -\delta G,$$

$$U^I - U^{II}, \quad \delta V = V^I - V^{II}$$

For cases I and II, these forecasts are obtained, with the

$$-\gamma)(\delta T + \delta V) \tag{4.12}$$

$$\xi)(\delta G + \delta W) + (1 - \gamma)\delta U]$$

$$\delta I = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\gamma(\delta T + \delta V) + \gamma\delta G + [1 - \beta(1 - \xi)]\delta W + \gamma\delta U\} \tag{4.13}$$

$$\delta Y = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(\delta T + \delta V) + \delta G + \delta W + \delta U] \tag{4.14}$$

and

$$\delta RE = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\xi\delta T + (1 - \gamma - \beta)\delta V + \xi\delta G + \xi\delta W + \xi\delta U]. \tag{4.15}$$

The system of the last four equations provides the framework for tracing the forecasting errors of the different forecasting methods in linear systems. In particular, it can be used to explain the differences between the ex ante and ex post single-period forecasts (in systems without lags) for different constant adjustments. For instance, if we want to calculate the difference between ex post with constant adjustments type I and ex ante with constant adjustments type II, we need to insert $-\delta T$ and $-\delta G$ (the ex post-ex ante discrepancies) and δU , δW , and δV (the discrepancies in constant adjustments) and then, knowing the values of the estimated parameters β , γ , ξ , compute δC , δI , δY and δRE . We can also investigate the *pure* effect of either ex post versus ex ante forecast, or differences in the constant adjustments, by setting $\delta U = \delta W = \delta V = 0$ for the former and $\delta T = \delta G = 0$ for the latter. The special feature of linear systems is that the total effect of the ex post versus ex ante discrepancies and the constant adjustment differences is made up of the sum of the pure effects. For instance, the forecasting error in *C* attributable solely to the ex ante-ex post discrepancies in the two exogenous variables in the system, *G* and *T*, is given by

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \xi)\delta T + \beta(1 - \xi)\delta G], \tag{4.12a}$$

where δT and δG are the errors the forecaster made in guessing, respectively, the tax revenues *T* and government expenditures *G*. Similarly, if we want to investigate the additional forecasting error

attributable to the "no constant adjustments" beyond that of *AR* constant adjustment³ in ex post forecasts, we merely interpret type I and II "constant adjustments" above as *AR* and "no constant adjustments," respectively, and set $\delta T = \delta G = 0$, obtaining

$$\delta C = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \gamma)\delta V + \beta(1 - \xi)\delta W + (1 - \gamma)\delta U]. \quad (4.12b)$$

The combined effect of the forecasting error attributable to both ex ante-ex post discrepancies and to the two types of constant adjustments is obtained by adding equations (4.12a) and (4.12b). If, on the other hand, the model were nonlinear, the two additive effects would account only approximately for the combined effect, depending upon the degree of nonlinearity.

A special kind of constant adjustment, an artificial one designed to facilitate the forecasting error analysis, is obtained by computing for a particular *t*:

$$U_t^* = C_t - \alpha - \beta(Y_t - D_t - T_t) \quad (4.16)$$

$$W_t^* = I - \gamma T_t \quad (4.17)$$

and

$$V_t^* = D_t - \xi Y_t. \quad (4.18)$$

The residuals on the left hand side of equations (4.16)–(4.18) are called "structural equation residuals." They are obtained by substituting the realized values for the endogenous variables appearing on the right hand side of structural equations (4.1)–(4.3), rather than the model solution values used in both ex post and ex ante forecasts. The concept of "structural equation residuals" (henceforth denoted by *SER*) has the advantage of isolating forecasting errors due to the specification of the equation under investigation from the error due to the simultaneity of the model. Furthermore, the *SERs* can be incorporated easily into the general framework developed above. For this, an alternative interpretation for the *SERs* should be adopted—namely, that if the *SERs* were simultaneously employed as constant adjustments for all stochastic equations in the model, the ex post forecasting errors would vanish. Thus, δC , δI , δD , and δY now become the forecasting errors of the forecasts with the constant

³ See Chapter 1, p. 9.

The Decomposition of forecast

adjustments under investigation. That viewed here as the sum of the discrepancy *SERs* (U_t^* , W_t^* , V_t^*) and the constant forecast, plus the discrepancies between the exogenous values, all weighted

4.3 A NONLINEAR EXAMPLE

The additive feature just described these systems there are additional exogenous variable discrepancies and as well as interactions within these two

One simple example of a nonlinear problems involved. Despite its simplicity that are usually present in macroeconomic in the endogenous variables are introduced in the consumption function

$$C_t = \alpha^* + \beta^* P_t D_t$$

where $P_t = \mu' \{Y_t / (Y_t - Y_{MAX_t})\}$ is maximum attainable gross national assumed to be exogenous in our simple inversely related to the percentage of attainable level at a particular time *t*

If we solve now for real GNP Y_t

$$Y_t^2 [1 - \beta^*(1 - \xi) - \gamma] - Y_t [\alpha^* + \mu^* + U_t + V_t]$$

where $\mu^* = \mu/\mu'$, and, thus,⁴

$$Y_t = \frac{1}{1 - \beta^*(1 - \xi) - \gamma} \frac{1}{2} \left(\alpha^* + \mu^* + U_t + W_t + G_t + \sqrt{[\alpha^* - \beta^*(V_t + T_t)]^2 - 4[1 - \beta^*(1 - \xi) - \gamma](\alpha^* + \mu^* + U_t + W_t + G_t)} \right)$$

⁴ Another solution exists, too, with a negative sign. Only the one presented here will be economically meaningful.

adjustments" beyond that of *AR* forecasts, we merely interpret type I and *AR* and "no constant adjustments," obtaining

$$\gamma\delta V \tag{4.12b}$$

$$(1 - \xi)\delta W + (1 - \gamma)\delta U.$$

Forecasting error attributable to both exogenous variable discrepancies and constant adjustments (4.12a) and (4.12b). If, on the other hand, two additive effects would account for the total effect, depending upon the degree

of adjustment, an artificial one designed to simulate the effect of constant adjustments is obtained by computing for a

$$\beta(Y_t - D_t - T_t) \tag{4.16}$$

$$\tag{4.17}$$

$$\tag{4.18}$$

side of equations (4.16)–(4.18) are obtained by substituting the variables appearing on the right (4.1)–(4.3), rather than the model and ex ante forecasts. The concept henceforth denoted by *SER*) has the errors due to the specification of the model and the error due to the simultaneity of the variables incorporated easily into the general model, an alternative interpretation for the model is that if the *SERs* were simultaneously defined for all stochastic equations in the model, the errors would vanish. Thus, δC , δI , δD , and δT of the forecasts with the constant

adjustments under investigation. That is, the forecasting errors are viewed here as the sum of the discrepancies between the "observed" *SERs* (U_t^* , W_t^* , V_t^*) and the constant adjustments actually used in the forecast, plus the discrepancies between the actual and guessed values of the exogenous values, all weighted by their respective multipliers.

4.3 A NONLINEAR EXAMPLE

The additive feature just described is lost in nonlinear systems. In these systems there are additional terms of interaction between exogenous variable discrepancies and constant adjustment differences, as well as interactions within these two groups.

One simple example of a nonlinear model will serve to illustrate the problems involved. Despite its simplicity it is typical of the nonlinearities that are usually present in macroeconomic models. The nonlinearities in the endogenous variables are introduced via a "money illusion" effect in the consumption function

$$C_t = \alpha^* + \beta^* D_t + \mu(1/P_t) + U_t, \tag{4.19}$$

where $P_t = \mu'(Y_t/(Y_t - YMAX_t))$ is the price level and $YMAX$ is the maximum attainable gross national product in real terms, which is assumed to be exogenous in our simple model. That is, the price level is inversely related to the percentage that real GNP falls short of its highest attainable level at a particular time period.

If we solve now for real GNP we obtain:

$$Y_t^2 [1 - \beta^*(1 - \xi) - \gamma] - Y_t [\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t] + \mu^* YMAX_t = 0, \tag{4.20}$$

where $\mu^* = \mu/\mu'$, and, thus,⁴

$$Y_t = \frac{1}{1 - \beta^*(1 - \xi) - \gamma} \frac{1}{2} \left(\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t + \sqrt{[\alpha^* - \beta^*(V_t + T_t) + \mu^* + U_t + W_t + G_t]^2 - 4[1 - \beta^*(1 - \xi) - \gamma]\mu^* YMAX_t} \right) \tag{4.21}$$

⁴ Another solution exists, too, with a negative sign preceding the square root, but usually only the one presented here will be economically feasible.

The differences operators can be applied to the nonlinear model. Ignoring second order differencing and using the above relationship (4.21), we arrive at an expression of the discrepancy between two predicted values of real GNP with two different constant adjustments and different guesses about the values of the exogenous variables ($YMAX$ was treated here as a definition in which no error can occur):

$$\delta Y \approx \frac{-\beta^*(\delta V + \delta T) + \delta U + \delta W + \delta G}{2[1 - \beta^*(1 - \xi) - \gamma]\{1 - 1/(1 + \sqrt{1 - D})\}} \quad (4.22)$$

where $D = \frac{4[1 - \beta^*(1 - \xi) - \gamma]\mu^*YMAX_t}{[\alpha^* + \mu^* - \beta^*(V_t + T_t) + W_t + G_t + U_t]^2}$.

Notice that by setting $\mu^* = 0$ (or $YMAX_t = 0$) we eliminate the nonlinearity in the model. Indeed, equation (4.22) can be reduced to the expression for the corresponding linear system, i.e., to equation (4.14). More important, it shows that, although the multiplier, being a function of the random variables V, T, W, G, U , and $UMAX$, is itself a random variable, it will hardly change with small variations in those variables. The next two chapters will demonstrate that the ex post-ex ante discrepancies varied only slightly when different constant adjustments were made. This last point is important because it allows us to decompose the total forecasting error into its additive components, and to ignore the nonlinear effect of the slight difference in "initial conditions" for alternative forecasts of the same equation.

4.4 STRUCTURAL EQUATION RESIDUALS VERSUS FORECASTING ERROR

In the second set of tables of Chapters 5 and 6, column I lists the *SER* minus the constant adjustments⁵ for the stochastic equations of all endogenous components of GNP and disposable income, respectively. In our notation these values can be properly expressed by δU and δW (the discrepancies between two adjustment procedures for the residuals of GNP's endogenous components) and by δV (the discrepancies in the

⁵ No constant adjustment is considered here a special case of constant adjustment, where the constant adjustment is equal to zero.

The Decomposition of Forecasting Error

endogenous component of disposable income, constant type I, and the constant adjustment type II in the decomposition tables the values in the first column are the forecasting errors, which are listed in the second column we get

$$\delta C - \delta U = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\delta V + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta T + \delta G\}$$

and

$$\delta I - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma}$$

and the sum of both:

$$\begin{aligned} \delta C + \delta I - \delta U - \delta W &= \frac{1}{1 - \beta(1 - \xi) - \gamma} \\ &\quad \{-\beta\delta V + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta T + \delta G\} \\ &= \frac{-\beta\delta V + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta T + \delta G}{1 - \beta(1 - \xi) - \gamma} \end{aligned}$$

Equations (4.23) to (4.25) show the decomposition of the forecasting error into (a) the error due to the particular equations and (b) that due to the constant adjustments. The former is the error attributable to the constant adjustments because it is calculated under the assumption that the variables on the right hand side of the equations are constant. The indirect effect resulting from the constant adjustments, through the reverberation throughout the system, is calculated by the appropriate multipliers.

For instance, the indirect effect of the constant adjustment on the induced (indirect) effect of U on C is given by multiplying δU by $\beta(1 - \xi)$.

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be applied to the nonlinear model, and using the above relationship of the discrepancy between two different constant adjustments values of the exogenous variables (in which no error can occur):

$$\frac{\delta U + \delta W + \delta G}{\gamma \{1 - 1/(1 + \sqrt{1 - D})\}} \quad (4.22)$$

$$- \gamma \mu^* YMAX_t$$

$$+ W_t + G_t + U_t]^2$$

(or $YMAX_t = 0$) we eliminate the equation (4.22) can be reduced to the linear system, i.e., to equation (4.14). Though the multiplier, being a function of U , and $UMAX$, is itself a random variable, variations in those variables. The fact that the ex post-ex ante discrepancy constant adjustments were made, it allows us to decompose the total forecasting error into components, and to ignore the difference in "initial conditions" for forecasting.

RESIDUALS VERSUS FORECASTING ERROR

Chapters 5 and 6, column I lists the forecasting errors for the stochastic equations of all variables, respectively. In particular, the error in the consumption function is expressed by δU and δW (the latter is the discrepancy in the forecasting procedure for the residuals of the consumption function) and by δV (the discrepancies in the

⁵ a special case of constant adjustment, where

endogenous component of disposable income). Now we assign the *SER* to constant type I, and the constant adjustment under consideration to constant adjustment type II in the definitions of the δ operator. In our tables the values in the first column are subtracted from the corresponding forecasting errors, which are listed in the third column. Thus, in the second column we get

$$\delta C - \delta U = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta(1 - \gamma)\delta V + \beta(1 - \xi)\delta W + \beta(1 - \xi)\delta U] \quad (4.23)$$

and

$$\delta I - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} [-\beta\delta V + \delta W + \delta U], \quad (4.24)$$

and the sum of both:

$$\delta C + \delta I - \delta U - \delta W = \frac{1}{1 - \beta(1 - \xi) - \gamma} \{-\beta\delta V + [\gamma + \beta(1 - \xi)](\delta W + \delta U)\} \quad (4.25)$$

$$= \frac{-\beta\delta V + \delta W + \delta U}{1 - \beta(1 - \xi) - \gamma} - \delta W - \delta U. \quad (4.25a)$$

Equations (4.23) to (4.25) show us how to decompose the ex post forecasting error into (a) the error due to the specifications of the particular equations and (b) that due to the simultaneity of the model. The former is the error attributable directly to the equation in question because it is calculated under the pretense that the "true" values of the variables on the right hand side of the equation are known. The latter is the indirect effect resulting from the reverberation of the *SER*, adjusted by the constant adjustments, throughout the system. The effect of the reverberation throughout the system is given in equations (4.23)–(4.24) by the appropriate multipliers.

For instance, the indirect effect of δU on the consumption error is given by multiplying δU by $\beta(1 - \xi)/[1 - \beta(1 - \xi) - \gamma]$; this is the induced (indirect) effect of U on consumption. When we add up all the

errors in the endogenous components of GNP (see equation 4.25a) we get the total (indirect plus direct) effect of the errors in these components less the direct effect, leaving only the indirect effect after their reverberation through the system. Moreover, the induced effect of the errors included in disposable income, $-\delta V$ in our simple example, should be added. Therefore, the direct effects of the errors in the disposable income components are listed in the same set of tables. They are multiplied by the appropriate multipliers and then summed up.

4.5 THE PRICE EFFECT

Next, it is interesting to isolate the price effect from the real value effect on nominal GNP. To this end we use the formula

$$GNP_t = P_t \cdot Y_t, \tag{4.26}$$

and thus

$$\delta GNP = (Y_t + \delta Y)\delta P + P\delta Y. \tag{4.27}$$

We call the first term on the right hand side of (4.27) "error due to price." This error can be further decomposed into the errors in the components of real GNP and their corresponding prices. For this, let us denote the real GNP components by Y_i (i.e., $Y = \sum Y_i$) and their corresponding prices by P_i . We have

$$P = GNP/Y = \sum Y_i P_i / \sum Y_i \tag{4.28}$$

and, applying the δ operator, we get

$$(P + \delta P) [\sum (Y_i + \delta Y_i)] = \sum (Y_i + \delta Y_i)(P_i + \delta P_i). \tag{4.29}$$

Subtracting (4.28) from (4.29), we get

$$\delta P \sum (Y_i + \delta Y_i) = \sum \delta Y_i (P_i - P) + \sum (Y_i + \delta Y_i) \delta P_i. \tag{4.30}$$

Unless we have peculiar situations in which large discrepancies in δY_i are systematically associated with positive or negative discrepancies between the corresponding prices, P_i and P , the first term on the right hand side of (4.30) can be ignored, and we finally get

$$\delta P \sum (Y_i + \delta Y_i) \approx \sum (Y_i + \delta Y_i) \delta P_i. \tag{4.31}$$

The left hand side is nothing more than the "error due to price." The

expression on the right of (4.31) consists of sets of endogenous and exogenous variables mentioned above we have further decomposed into the exogenous and endogenous components. The ex post-ex ante error decomposition

4.6 MULTIPERIOD FORECASTS

We have deliberately avoided introducing them. To illustrate errors we resort to our simple linear consumption and investment equations to include

$$C_t = \alpha + \sum_{i=0}^p \beta_i D_i$$

$$I_t = \sum_{j=0}^q \gamma_j Y_{t-j}$$

where (say) $q > p$ and $\beta_{p+1} = \dots$ for Y becomes

$$Y_t = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \alpha + \sum_{i=0}^q \beta_i (V_{t-i} - \xi V_{t-i-1}) \right\}$$

and, applying the differencing operator

$$\delta Y = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^q \beta_i (\delta V_{t-i} - \xi \delta V_{t-i-1}) \right\}$$

That is, the multiperiod forecasting errors made in Y in the earlier periods are influenced by the marginal propensities to consume and marginal propensities to invest. Errors in the retained earnings forecasts by the forecasters in guessing the values

ponents of GNP (see equation 4.25a) we
 effect of the errors in these components
 g only the indirect effect after their
 em. Moreover, the induced effect of the
 income, $-\delta V$ in our simple example,
 e direct effects of the errors in the dis-
 re listed in the same set of tables. They
 te multipliers and then summed up.

plate the price effect from the real value
 end we use the formula

$$P_t = P_t \cdot Y_t \quad (4.26)$$

$$Y_t + \delta Y_t \delta P_t + P_t \delta Y_t \quad (4.27)$$

t hand side of (4.27) "error due to price."
 posed into the errors in the components
 ling prices. For this, let us denote the real
 $\sum Y_t$ and their corresponding prices by

$$Y = \sum Y_t P_t / \sum Y_t \quad (4.28)$$

get

$$Y_t = \sum (Y_t + \delta Y_t)(P_t + \delta P_t) \quad (4.29)$$

ve get

$$Y_t(P_t - P) + \sum (Y_t + \delta Y_t)\delta P_t \quad (4.30)$$

uations in which large discrepancies in
 with positive or negative discrepancies
 s, P_t and P , the first term on the right
 ad, and we finally get

$$Y_t \approx \sum (Y_t + \delta Y_t)\delta P_t \quad (4.31)$$

re than the "error due to price." The

expression on the right of (4.31) can be further decomposed into two sets of endogenous and exogenous prices. In the two sets of tables mentioned above we have further decomposed the "error due to price" into the exogenous and endogenous price effects, which serve as part of the ex post-ex ante error decomposition.

4.6 MULTIPERIOD FORECASTS

We have deliberately avoided lags in our simple model, but it is time now to introduce them. To illustrate the effect of lags on forecasting errors we resort to our simple linear models, but modify the consumption and investment equations to include lags:

$$C_t = \alpha + \sum_{i=0}^p \beta_i D I_{t-i} + U_t \quad (4.32)$$

$$I_t = \sum_{j=0}^q \gamma_j Y_{t-j} + W_t \quad (4.33)$$

where (say) $q > p$ and $\beta_{p+1} = \dots = \beta_q = 0$. The reduced form equation for Y becomes

$$Y_t = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \alpha + \sum_{i=1}^q Y_{t-i} [\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^q \beta_i (V_{t-i} + T_{t-i}) + U_t + W_t + G_t \right\} \quad (4.34)$$

and, applying the differencing operator δ , we finally get

$$\delta Y_t = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^q \delta Y_{t-i} [\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^q \beta_i (\delta V_{t-i} + \delta T_{t-i}) + \delta U_t + \delta W_t + \delta G_t \right\} \quad (4.35)$$

That is, the multiperiod forecasting error in period t is the sum of (a) the errors made in Y in the earlier periods, weighted by the respective marginal propensities to consume times the leakage in retained earnings and marginal propensities to invest appropriate to each period, (b) the errors in the retained earnings equation and the errors made by the forecasters in guessing the values of the exogenous variable T in their ex

ante forecasts, weighted by the marginal propensity to consume, and (c) the contemporaneous errors in consumption, investment, and the exogenous variable government expenditure (in ex ante forecasts)—all multiplied by the multiplier.

Alternatively, equation (4.35) can be rewritten as

$$\delta Y = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^q \gamma_i (\delta Y)_{t-i} - \beta_0 (\delta V + \delta T) + \sum_{i=1}^q \beta_i (\delta DI)_{t-i} + \delta U + \delta W + \delta G \right\} \quad (4.36)$$

This, again, is a function of the earlier errors in real income and disposable income and of the contemporaneous errors in the real income and disposable income components, all weighted by the appropriate marginal propensities and the multiplier.

Our analytical scheme can be used to detect the effect of lags on forecasting errors. This is done by comparing a long-span forecast aimed at a particular period with shorter-span forecasts made later, and aimed at the same period. This comparison is useful for the error decomposition because the *SERs* pertaining to the same period are the same, irrespective of the forecasting span they represent, and thus the remaining error is due to the different lags. In order to put this formally we write

$$\delta Y_{t(s)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^r \delta Y_{t(s-i)} [\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^r \beta_i (\delta V_{t(s-i)} + \delta T_{t(s-i)}) + \delta U_{t(s)} + \delta W_{t(s)} + \delta G_{t(s)} \right\} \quad (4.37)$$

where $r = \min(s, q)$ and the subscript in parentheses denotes the forecasting span, while the subscript preceding it denotes the jump-off period (the latest period for which data were available). Thus, the period for which the forecast was made is given by adding up both subscripts. Notice that by definition $\delta Y_{t(0)} = 0$.

Now we may decrease the forecasting span by one period and move the jump-off period ahead, since we wish to compare forecasts made for the same period. We obtain

The Decomposition of Foreca

$$\delta Y_{t+1(s-1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} - \sum_{i=0}^{r-1} \beta_i (\delta V_{t+1(s-1-i)} + \delta T_{t+1(s-1-i)})$$

and now subtract (4.38) from (4.37)

$$\delta Y_{t(s)} - \delta Y_{t+1(s-1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^{r-1} (\delta Y_{t(i)} + \delta Y_{t(s-r)} [\beta_r(1 - \xi) + \gamma_r] - \sum_{i=0}^{r-1} \beta_i (\delta V_{t(s-i)} + \delta T_{t(s-i)} - \delta T_{t+1(s-1-i)}) - \beta_r (\delta U_{t(s)} - \delta U_{t+1(s-r)} + \delta W_{t(s)} + \delta G_{t(s)} - \delta G_{t+1(s-1)}) \right\}$$

For instance, if our consumption fu 2) and our investment function (4. compare a two-quarters-ahead forecast, equation (4.39) reduces t

$$\delta Y_{t(2)} - \delta Y_{t+1(1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \delta Y_{t(1)} - \beta_0 (\delta V_{t(2)} - \delta V_{t+1(1)}) + \delta T_{t(2)} - \delta T_{t+1(1)} - \beta_1 (\delta U_{t(2)} - \delta U_{t+1(1)} + \delta W_{t(2)} + \delta G_{t(2)} - \delta G_{t+1(1)}) \right\}$$

The formula can be easily four-quarter forecast with a first later and pertaining to the same

original propensity to consume, and (c) consumption, investment, and the expenditure (in ex ante forecasts)—all

can be rewritten as

$$(\delta Y)_{t-t} - \beta_0(\delta V + \delta T) \quad (4.36)$$

$$\{ (\delta DI)_{t-t} + \delta U + \delta W + \delta G \}.$$

earlier errors in real income and sporaneous errors in the real income are, all weighted by the appropriate multiplier.

used to detect the effect of lags on comparing a long-span forecast aimed at long-span forecasts made later, and comparison is useful for the error pertaining to the same period are the long span they represent, and thus different lags. In order to put this for-

$$\delta Y_{t(s-i)}[\beta_i(1 - \xi) + \gamma_i] - \beta_0(\delta V_{t(s)} + \delta W_{t(s)} + \delta G_{t(s)}) \quad (4.37)$$

script in parentheses denotes the period preceding it denotes the jump-off (if data were available). Thus, the period given by adding up both subscripts.

forecasting span by one period and since we wish to compare forecasts

$$\delta Y_{t+1(s-1)} = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^{r-1} \delta Y_{t+1(s-1-i)}[\beta_i(1 - \xi) + \gamma_i] - \sum_{i=0}^{r-1} \beta_i(\delta V_{t+1(s-1-i)} + \delta T_{t+1(s-1-i)}) + \delta U_{t+1(s-1)} + \delta W_{t+1(s-1)} + \delta G_{t+1(s-1)} \right\} \quad (4.38)$$

and now subtract (4.38) from (4.37):

$$\begin{aligned} & \delta Y_{t(s)} - \delta Y_{t+1(s-1)} \\ &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \sum_{i=1}^{r-1} (\delta Y_{t(s-i)} - \delta Y_{t+1(s-1-i)})[\beta_i(1 - \xi) + \gamma_i] + \delta Y_{t(s-r)}[\beta_r(1 - \xi) + \gamma_r] - \sum_{i=0}^{r-1} \beta_i(\delta V_{t(s-i)} - \delta V_{t+1(s-1-i)}) + \delta T_{t(s-i)} - \delta T_{t+1(s-1-i)} - \beta_r(\delta V_{t(s-r)} + \delta T_{t(s-r)}) + \delta U_{t(s)} - \delta U_{t+1(s-r)} + \delta W_{t(s)} - \delta W_{t+1(s-1)} + \delta G_{t(s)} - \delta G_{t+1(s-1)} \right\}. \end{aligned} \quad (4.39)$$

For instance, if our consumption function (4.32) contains two lags ($p = 2$) and our investment function (4.33), 3 lags ($q = 3$), and we wish to compare a two-quarters-ahead forecast with a one-quarter-ahead forecast, equation (4.39) reduces to

$$\begin{aligned} & \delta Y_{t(2)} - \delta Y_{t+1(1)} \\ &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \delta Y_{t(1)}[\beta_1(1 - \xi) + \gamma_1] - \beta_0(\delta V_{t(2)} - \delta V_{t+1(1)}) + \delta T_{t(2)} - \delta T_{t+1(1)} - \beta_1(\delta V_{t(1)} + \delta T_{t(1)}) + \delta U_{t(2)} - \delta U_{t+1(1)} + \delta W_{t(2)} - \delta W_{t(1)} + \delta G_{t(2)} - \delta G_{t+1(1)} \right\}. \end{aligned} \quad (4.40)$$

The formula can be easily extended to compare, say, also a four-quarter forecast with a first-quarter forecast made three quarters later and pertaining to the same quarter that the four-quarter forecast

was aiming at, i.e.:

$$\begin{aligned} \delta Y_{t(4)} - \delta Y_{t+3(1)} &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ [\beta_1(1 - \xi) + \gamma_1] \delta Y_{t(3)} \right. \\ &\quad + [\beta_2(1 - \xi) + \gamma_2] \delta Y_{t(2)} \\ &\quad + \gamma_3 \delta Y_{t(1)} - \beta_0 [\delta(V + T)_{t(4)} - \delta(V + T)_{t+3(1)}] \\ &\quad - \beta_1 \delta(V + T)_{t(3)} - \beta_2 \delta(V + T)_{t(2)} \\ &\quad \left. + \delta(U + W + C)_{t(4)} + \delta(U + W + G)_{t+3(1)} \right\}. \end{aligned} \tag{4.41}$$

Thus, as expected, the difference between any two forecasts made at two different points in time, but referring to the same time period, includes the errors in the endogenous and exogenous components of GNP which enter into the lags and, of course, the contemporaneous errors.

However, expressions (4.40) and (4.41) can be simplified in ex post no constant adjustments, since—as was pointed out before—there are no errors in the exogenous variables in ex post forecasts and the *SERs* are the same, irrespective of the forecasting span. Thus, (4.40) and (4.41), respectively, become

$$\begin{aligned} \delta Y_{t(2)} - \delta Y_{t+1(1)} &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \delta Y_{t(1)} [\beta_1(1 - \xi) + \gamma_1] - \beta_1 \delta V_{t(1)} \right\} \\ &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \gamma_1 \delta Y_{t(1)} + \beta_1 \gamma \delta D I_{t(1)} \right\} \end{aligned} \tag{4.42}$$

and

$$\begin{aligned} \delta Y_{t(4)} - \delta Y_{t+3(1)} &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ [\beta_1(1 - \xi) + \gamma_1] \delta Y_{t(3)} \right. \\ &\quad + [\beta_2(1 - \xi) + \gamma_2] \delta Y_{t(2)} \\ &\quad \left. + \gamma_3 \delta Y_{t(1)} - \beta_1 \delta V_{t(3)} - \beta_2 \delta V_{t(2)} \right\} \end{aligned} \tag{4.43}$$

$$\begin{aligned} &= \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \beta_1 \delta D I_{t(3)} \right. \\ &\quad \left. + \gamma_2 \delta Y_{t(2)} + \gamma_3 \delta Y_{t(1)} \right\}. \end{aligned}$$

Thus, to isolate the pure effect of lags effects of different constant adjustments the forecaster's wrong guesses as to need only subtract the ex post "no co in a later period from one pertaining to This will be demonstrated in the last se

$$\begin{aligned}
 & - \xi) + \gamma_1 \delta Y_{t(3)} \\
 & \delta(V + T)_{t+3(1)} \\
 & T)_{t(2)} \\
 & - W + G)_{t+3(1)}.
 \end{aligned}
 \tag{4.41}$$

nce between any two forecasts made
 ut referring to the same time period,
 ous and exogenous components of
 nd, of course, the contemporaneous

nd (4.41) can be simplified in ex post
 s was pointed out before—there are
 as in ex post forecasts and the *SERs*
 forecasting span. Thus, (4.40) and

$$\begin{aligned}
 & (1 - \xi) + \gamma_1) - \beta_1 \delta V_{t(1)} \} \\
 & + \beta_1 \gamma \delta D I_{t(1)} \}
 \end{aligned}
 \tag{4.42}$$

$$\begin{aligned}
 & - \xi) + \gamma_1 \delta Y_{t(3)} \\
 & \}
 \end{aligned}
 \tag{4.43}$$

$$\begin{aligned}
 & = \frac{1}{1 - \beta_0(1 - \xi) - \gamma_0} \left\{ \beta_1 \delta D I_{t(3)} + \beta_2 \delta D I_{t(2)} + \gamma_1 \delta Y_{t(3)} \right. \\
 & \left. + \gamma_2 \delta Y_{t(2)} + \gamma_3 \delta Y_{t(1)} \right\}.
 \end{aligned}$$

Thus, to isolate the pure effect of lags in multiperiod forecasts, with the effects of different constant adjustments in the various forecasts and of the forecaster's wrong guesses as to exogenous values filtered off, one need only subtract the ex post "no constant adjustment" forecast made in a later period from one pertaining to the same period but made earlier. This will be demonstrated in the last set of tables in Chapters 5 and 6.